Extension of Non-Local Means (NLM) Algorithm with Gaussian Filtering for Highly Noisy Images

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***Abstract*—The denoising performance of the Non-Local Means (NLM) method decreases as the variance of additive white Gaussian noise becomes higher. In this paper, we explain this phenomenon and propose a modified version of the Non-Local Means (NLM) method, called the Enhanced-Weights NLM (EWNLM) algorithm, to denoise highly noisy images. The EWNLM algorithm evaluates weights from a pre-filtered image using the Gaussian kernel, which in turn result in more robust weight contributions from similar pixels in the search window. Experimental results are given to demonstrate the superior performance of the EWNLM scheme when the standard deviation of the additive white Gaussian noise (AWGN) is greater than 20.**

***Index Terms*—Image denoising, Non-Local Means (NLM), enhanced weights, uniform weights.**

I. Introduction

Image denoising is a classical problem that has attracted attention from the research community since 60s. Some of the pioneering work in the field was discussed and a new algorithm called the Non-Local Means (NLM) filtering was proposed in [1]. Since the inception of the NLM algorithm, it has gained great popularity not only in the field of denoising but also in other image processing applications such as super-resolution, demosaicing and image segmentation.

The NLM algorithm produces denoised results with a higher peak signal-to-noise ratio (PSNR) value as well as good perceptual quality. It estimates each pixel based on the weighted average of all pixels inside a search window. The weight of a contributing pixel is evaluated on the basis of "similarity measure" of a Gaussian neighborhood between the contributing and the target pixels. Higher weights are given to pixels that have a neighborhood which is more similar to that of the target one. Since the essence of the NLM lies in finding "good" similar pixels with high weights, its denoising performance outperforms prior denoising algorithms by a significant margin in edged and structured regions.

The denoising performance of the Non-Local Means (NLM) method decreases as the variance of the additive white Gaussian noise (AWGN) is higher. In this paper, we first provide a theoretical explanation to this phenomenon. Then, we propose a modified version of the NLM method, called the Enhanced-Weights NLM (EWNLM) algorithm, that targets at the denoising of highly noisy images. The EWNLM algorithm evaluates weights from the Gaussian filtered version of the noisy image to obtain more uniform contributions from similar pixels in the search window. Several methods have been suggested to refine the weights in NLM filters. For example, [2] extends the idea of bilateral filtering to NLM, [3] suggests probabilistic refinement of weights iteratively, [4] uses Zernike moments to improve the weight estimation. A similar concept can be found in [5], where a pre-processed image is used in the weight estimation with Weiner filtering on 3-D transformed groups of similar blocks. As compared with previous works, EWNLM is simpler and computationally more efficient.

The Gaussian filtered image comes at the cost of the blurring artifact at image singularities and textural regions, which tend to contaminate the weight estimation process when the noise variance is lower. In this work, we show experimentally that the standard NLM works better if the standard deviation of AGWN is less than 15 while EWNLM outperforms the standard NLM if the standard deviation of AGWN is greater than 20. When the standard deviation of AGWN is between 15 and 20, the performance of NLM and EWNLM is comparable.

This research is supported by a research grant from Xerox Corporation

II. Impact of Noise Variance on Weight Estimation

Consider a noisy image, $\mathbf{X}$, that is composed of original image $\mathbf{I}$ and AWGN $\mathbf{N}$ in form of

$$\mathbf{X} = \mathbf{I} + \mathbf{N}. \tag{1}$$

An NLM estimate $\hat{\mathbf{I}}_{nlm}(i)$ of pixel i is given by [1]

$$\hat{\mathbf{I}}_{nlm}(i) = \sum_{j \in \Omega_X} w(i,j)\mathbf{X}(j), \tag{2}$$

where Ω_X in an image space. Although Ω_X in Eq. (2) could be the entire input image, the associated computation would be too high to be practical. Hence, it is often restricted to a square window of size $s \times s$ centered around target pixel i. The weight, $w(i,j)$, between target pixel $\mathbf{I}(i)$ and contributing pixel $\mathbf{X}(j)$ is given by

$$w(i,j) = \frac{1}{C_i} \exp\left\{ - \frac{\|\mathbf{X}(\mathcal{N}_i) - \mathbf{X}(\mathcal{N}_j)\|_{2,a}^2}{h} \right\}, \tag{3}$$

where C_i is a normalizing factor such that $\sum w(i,j) = 1$, $\mathbf{X}(\mathcal{N}_i)$ denotes a neighborhood, N_i, centered around pixel i in noisy image $\mathbf{X}$, $\|\cdot\|_{2,a}^2$ is the Euclidean norm weighted by a Gaussian kernel of standard deviation a, and h is a parameter that adjusts the decay of the weight.

Let σ be the standard deviation of $\mathbf{N}(i)$. We derive the mean and the variance of the sum of weighted squared differences (SWSD) term in Eq. (3) below. Its mean can be written as

$$\mathbb{E}\|\mathbf{X}(\mathcal{N}_i) - \mathbf{X}(\mathcal{N}_j)\|_{2,a}^2 = \|\mathbf{I}(\mathcal{N}_i) - \mathbf{I}(\mathcal{N}_j)\|_{2,a}^2 + 2\sigma^2. \tag{4}$$

If $\|\mathbf{I}(\mathcal{N}_i) - \mathbf{I}(\mathcal{N}_j)\|_{2,a}^2$ is significantly larger than $2\sigma^2$, higher weights will be assigned to pixels that have a similar neighborhood. However, there exists a multiplicative factor, *i.e.*, $e^{-\frac{2\sigma^2}{h}}$, in the weight estimate due to the existence of noise. This factor will deviate from one as σ^2 becomes larger.

Next, the variance of the SWSD term can be simplified as

$$
\begin{aligned}
&\mathrm{VAR}\,\|\mathbf{X}(\mathcal{N}_i)-\mathbf{X}(\mathcal{N}_j)\|_{2,a}^2 = \mathrm{VAR}\,\|\mathbf{N}(\mathcal{N}_i)-\mathbf{N}(\mathcal{N}_j)\|_{2,a}^2\\
&= \mathbb{E}\{\|\mathbf{N}(\mathcal{N}_i)-\mathbf{N}(\mathcal{N}_j)\|_{2,a}^2\}^2 - \{\mathbb{E}\|\mathbf{N}(\mathcal{N}_i)-\mathbf{N}(\mathcal{N}_j)\|_{2,a}^2\}^2\\
&= \mathbb{E}\|\mathbf{N}(\mathcal{N}_i)\|_{2,a}^4 + \mathbb{E}\|\mathbf{N}(\mathcal{N}_j)\|_{2,a}^4\\
&\quad + 6\ \mathbb{E}\|\mathbf{N}(\mathcal{N}_i)\|_{2,a}^2 \mathbb{E}\|\mathbf{N}(\mathcal{N}_j)\|_{2,a}^2 - 4\sigma^4\\
&= 3\sigma^4 + 3\sigma^4 + 6(\sigma^2)^2 - 4\sigma^4 = 8\sigma^4
\end{aligned}
\tag{5}
$$

In above, we use the fact that $\mathbb{E}Z^4 = 3\sigma^4$ for a zero-mean Gaussian random variable Z with variance σ^2, and $\mathbf{N}(i)$ & $\mathbf{N}(j)$ are independent samples if $i \neq j$. This calculation is done for similar neighborhood weight evaluation and, hence, the true signal variance can be ignored. However, if the neighborhood is not similar, it can be shown easily that the variance of SWSD term in Eq. (5) contains an additional term $\mathrm{VAR}\,\|\mathbf{I}(\mathcal{N}_i)-\mathbf{I}(\mathcal{N}_j)\|_{2,a}^2$. As shown in Eq. (5), the variance of the SWSD term can be very high for noise with a large variance value. The high variance of the SWSD term affects the robustness of estimated weights as shown in Eq. (3) and, in turn, the denoised image $\hat{\mathbf{I}}_{nlm}(i)$ in Eq. (2).

III. Enhanced-Weights Non-Local Means (EWNLM) Algorithm

High noise variance adversely affects the weight estimation due to the high variance of its contribution to the SWSD term as derived in Eq. (5). In this section, we propose a method to improve the weight estimation, thereby making the estimation process more robust.

The basic idea is to evaluate the weight based on a pre-filtered image, $\mathbf{X}_g$, rather than the original noisy image, $\mathbf{X}$, itself. The image pre-filtered with a Gaussian kernel can be written as

$$\mathbf{X}_g = \mathbf{G}_b * \mathbf{X} \tag{6}$$

where $\mathbf{G}_b$ is a Gaussian kernel with parameter b and $*$ is the convolution operation. Now, the weight is given by

$$\mathbf{w}_g(i,j) = \frac{1}{C_{ig}} \exp\left\{ -\frac{\|\mathbf{X}_g(\mathcal{N}_i)-\mathbf{X}_g(\mathcal{N}_j)\|_{2,a}^2}{h} \right\}. \tag{7}$$

The denoising filter uses the enhanced weight as given in Eq. (7) along with the noisy image to determine the denoised pixel value:

$$\hat{\mathbf{I}}_g(i) = \sum_{j\in\Omega_X} \mathbf{w}_g(i,j)\mathbf{X}(j) \tag{8}$$

Once the image is pre-filtered by the Gaussian Kernel, $\mathbf{G}_b$, the weight is improved by a factor depending on the kernel size and parameter b. It was shown in [6] that the Gaussian filtering reduces the noise variance by a factor

$$\delta(\mathrm{VAR}) = \frac{\epsilon^2}{8\pi b^2}, \tag{9}$$

where $b = k\epsilon$, and k is the number of samples of noise in an interval of length b. Typically, $k \gg 1$. Because of the reduced variance in noise, the variance of the weight is also reduced from $2\sigma^2$ to $\frac{2\epsilon^2\sigma^2}{8\pi b^2}$.

It is best to use an example to illustrate the above idea. Consider two locations in the Lena image as shown in Fig. 1. One lies in the flat surface of the face region and the other lies in the edge of the hat region. Their enlarged views are shown in Fig. 2(a) and (d), respectively. In this example, we adopt a search window of size 23×23 and a neighborhood window of size 7×7.

Suppose that the image is corrupted by AWGN with high variance (say, $\sigma = 30$). The corresponding weight distributions using the standard NLM are shown in Figs. 2 (b) and (e) while the corresponding weight distributions using the proposed EWNLM scheme are shown in Figs. 2 (c) and (f), respectively. In Figs. 2(b) and (c), the intensities are normalized on a common scale for the display purpose. Similarly, intensities in Figs. 2 (e) and (f) are normalized on another common scale. In the flat region, the proposed EWNLM scheme provides a weight distribution that is more uniform than the original NLM scheme. This means that EWNLM provides more "similar" pixels than NLM as the weights for similar pixels have a uniform distribution, thereby assigning lower weights to the remaining pixels in the search window. The denoised output will not be influenced much by noisy neighboring pixels so as to yield a better result. In the edged region, since the proposed scheme allows contributions from more pixels that have a similar neighborhood in the search window, a better denoising effect can be achieved.

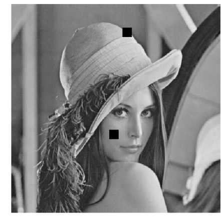

Fig. 1. Two locations in the Lena image: (a) a pixel lying in the edge of the hat region and its search window, and (b) a pixel lying in the flat surface of the face region and its search window.

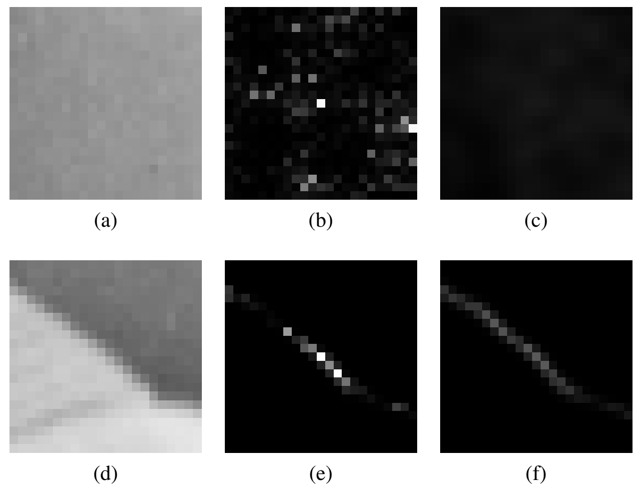

Fig. 2. The original patches in the smooth and edged regions are shown in (a) and (d), the weight distributions for the central pixel denoising using the standard NLM are shown in (b) and (e), and the weight distributions for the central pixel denoising using the proposed EWNLM are shown in (c) and (f), respectively.

Furthermore, we plot the standard deviation of weights at each pixel location for NLM and EWNLM in Figs. 3(a) and (b), respectively. Again, the standard deviation values have been normalized to a common scale for the display purpose. A high intensity location in these images indicates that the standard deviation of the estimated weights at this location is high. We see that EWNLM has a significantly lower standard deviation of estimated weights in the smooth region. We expect a higher standard deviation of the weight distribution in the edged region. This is consistent with the results shown in Figs. 3(a) and (b). It is however worthwhile to point out that the standard deviation of pixels in the edged region is also lower for EWNLM.

It is worthwhile to examine the relation between the variance of

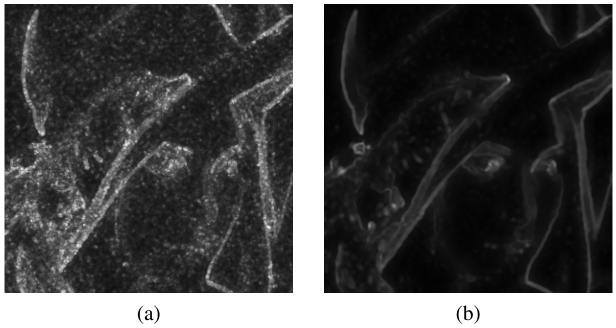

Fig. 3. The standard deviation of weights at each pixel location for (a) NLM and (b) EWNLM, where the intensities are normalized on a common scale for the display purpose.

the SWSD term in Eq. (5) and Figs. 2 (c) and (f). Eq. (5) shows that the variance of the SWSD term is directly proportional to the noise variance. However, this should not be interpreted as a constant variance of the SWSD term for the entire image. As mentioned earlier, the variance of the SWSD term also includes the variance of the true signal. In a flat region, the signal variance is low, resulting in a low variance of weights which depends on the factor given in Eq. (5). In an edged region, the signal variance interferes with the weight estimation, resulting in a high variance in edged regions. This is confirmed by Figs. 2 (c) and (f).

IV. Experimental Results

We used five test images of size 512×512 in the experiment to compare the performance of NLM and EWNLM. They are Airplane, Girlface, Lena, Peppers and Zelda. For the efficient implementation of NLM and EWNLM, a fast comparison algorithm based on the fast Fourier transform (FFT) was used [7]. In this experiment, we used a search window of size 15×15, a neighborhood window of size 7×7 and set the filter parameter to $h = 10\sigma$ as recommended in [1]. The parameter, a, for the Gaussian smoothing kernel in the weight decay function was set to 1.5 and filter parameter b in the Gaussian pre-filtering kernel, $\mathbf{G}_b$, in the proposed EWNLM was set to 1.

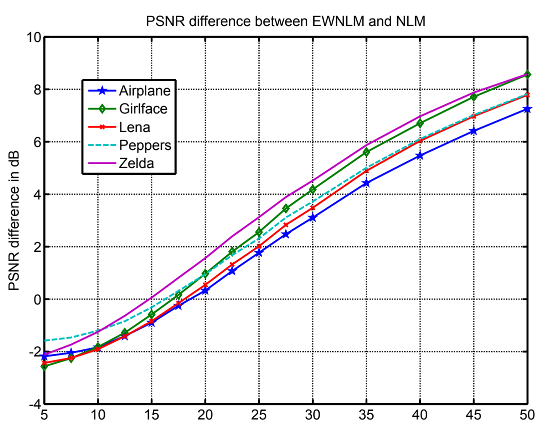

Fig. 4. The performance gain (in dB) of EWNLM over NLM for the Lena Image as a function of standard deviation σ of AWGN.

We show the PSNR values of denoised images using NLM and EWNLM with four standard deviation values (σ=20, 22.5, 25 and 27.5) in Table I, where the difference of the PSNR values between EWNLM and NLM Column is listed in column Δ. We see that EWNLM outperforms NLM in the denoising performance for all test images. The average PSNR performance improvement for the case of $\sigma = 20$ is 0.87dB. The performance gain of EWNLM over NLM becomes larger as the σ value is higher. This is consistent with our earlier discussion. That is, the increase in non-uniformity of the weight distribution caused by noise samples with a higher σ value results in performance degradation of NLM. In contrast, the performance of EWNLM is more robust in the presence of high noisy images. The increase in σ does not drastically affect the performance of the denoising filter due to the pre-filtering operation.

Next, we compare the denoising performance of NLM and EWNLM with respect to a broader range of noise variance (σ=5 to 50) for all the test images. We show the performance gain of EWNLM over NLM in Fig. 4, where a positive value means that EWNLM outperforms NLM. We see clearly from the figure that NLM outperforms EWNLM when $\sigma \leq 15$ while EWNLM outperforms NLM when $\sigma \geq 20$. When $15 < \sigma < 20$, the performance of NLM and EWNLM is comparable.

Finally, we show denoised images using NLM and EWNLM in Figs. 5 and 6 for side-by-side visual comparison. Clearly, EWNLM outperforms NLM in smooth regions. EWNLM also performs well in edge regions. For example, letters "F-16" in the airplane image and edges of peppers in the Peppers image are better preserved by EWNLM. Overall, the denoised images of EWNLM are perceptually more appealing than those of NLM when the noise level is high.

V. Conclusion

In this work, we first pointed out that the denoising performance of the NLM algorithm is not effective when the AWGN of the input image has a higher variance and explained this phenomenon mathematically. Then, we proposed the EWNLM algorithm to improve the denoising performance. A Gaussian pre-filtered image is adopted for the weight estimation in EWNLM. It was shown analytically that the Gaussian pre-filtering operation can reduce the variance of denoising weighting coefficients, thereby making the EWNLM algorithm more robust than the standard NLM in the presence of high noise. Experimental results were given to support our analysis. It was demonstrated that EWNLM gives better results in terms of both perceptual quality and the PSNR measure if the standard deviation of the AWGN is higher than 20.

References

[1] A. Buades, B. Coll, and J. Morel, "A non-local algorithm for image denoising," in *IEEE Computer Society Conference on Computer Vision and Pattern Recognition, 2005. CVPR 2005*, vol. 2, 2005.

[2] C. Kervrann and J. Boulanger, "Optimal spatial adaptation for patch-based image denoising," *IEEE Transactions on Image Processing*, vol. 15, no. 10, pp. 2866–2878, 2006.

[3] C. Deledalle, L. Denis, and F. Tupin, "Iterative weighted maximum likelihood denoising with probabilistic patch-based weights," *IEEE Transactions on Image Processing*, vol. 18, no. 12, pp. 2661–2672, 2009.

[4] Z. Ji, Q. Chen, Q. Sun, and D. Xia, "A moment-based nonlocal-means algorithm for image denoising," *Information Processing Letters*, vol. 109, no. 23-24, pp. 1238–1244, 2009.

[5] K. Dabov, A. Foi, V. Katkovnik, and K. Egiazarian, "Image denoising by sparse 3-D transform-domain collaborative filtering," *IEEE Transactions on Image Processing*, vol. 16, no. 8, pp. 2080–2095, 2007.

[6] A. Buades, "Image and film denoising by non-local means," Ph.D. dissertation, Universitat de les Illes Balears, 2006. [Online]. Available: http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.109.4948&rep=rep1&type=pdf

[7] R. Vignesh, B. Oh, and C.-C. J. Kuo, "Fast Non-Local Means (NLM) Computation With Probabilistic Early Termination," *IEEE Signal Processing Letters*, vol. 17, no. 3, p. 277, 2010.

TABLE I
PSNR COMPARISON BETWEEN EWNLM AND NLM

Image	PSNR in dB											
	$\sigma = 20$			$\sigma = 22.5$			$\sigma = 25$			$\sigma = 27.5$		
	NLM	EWNLM	Δ	NLM	EWNLM	Δ	NLM	EWNLM	Δ	NLM	EWNLM	Δ
Airplane	30.40	30.73	0.33	29.38	30.45	1.07	28.40	30.17	1.77	27.33	29.80	2.47
Girlface	31.13	32.10	0.97	30.01	31.82	1.81	28.93	31.48	2.55	27.83	31.29	3.46
Lena	30.54	31.09	0.55	29.51	30.83	1.32	28.52	30.54	2.02	27.47	30.31	2.84
Peppers	30.47	31.42	0.95	29.43	31.11	1.68	28.46	30.77	2.31	27.42	30.53	3.11
Zelda	31.02	32.58	1.56	29.88	32.27	2.39	28.82	31.95	3.13	27.70	31.59	3.89
Average	30.71	31.58	**0.87**	29.64	31.30	**1.65**	28.63	30.98	**2.36**	27.55	30.70	**3.15**

(a) (b) (c)

Fig. 5. (a) The original airplane image, and two denoised results when applied to a noisy airplane image with $\sigma = 20$: (b)NLM (PSNR=30.40dB) and (c)EWNLM (PSNR=30.73dB).

(a) (b) (c)

Fig. 6. (a) The original pepper image, and two denoised results when applied to a noisy pepper image with $\sigma = 20$: (b)NLM (PSNR=30.47dB) and (c)EWNLM (PSNR=31.42dB).