

## Background

In the United States of America coastal communities suffer huge yearly losses from flooding. One of the causes of such flooding is storm surges, this is the sea level rise caused by tropical hurricanes and cyclones. Studies have shown that between 1963 and 2012 49% of all deaths from hurricanes in the US were caused by the storm surge (Fig. 1) (Rappaport, 2014). The economic impact has also been huge with estimates showing that with no further improvements to coastal flood defenses by 2050 yearly losses from storm surge flooding could exceed US\$1trillion (Hallegatte et al., 2013). Following this it makes sense to model and simulate these storm surge events to facilitate better planning for flood defence spending. Most storm surge models require finely tuned parameters which either requires direct measurement which can be expensive and difficult to carry out or to be inferred from previous data. Inference of these parameters requires huge numbers of time consuming model runs. To address this we investigate the use of various surrogate models that approximate the model outputs in much less time so this inference can be carried out. We focus on the coefficients of friction at varying water depths as our parameters.

## Aims and Objectives

- Implement a storm surge model using GeoClaw (Clawpack Development Team, 2020; Mandli et al., 2016; Berger et al., 2011)
- Implement and use the kriging surrogate modelling method to infer friction parameters
- Implement Polynomial Chaos Expansion (PCE) as a surrogate method to infer the friction parameters
- Implement the hybrid Polynomial Chaos Kriging (PC-Kriging) surrogate model as a hybrid to infer the friction parameters.
- Analyse the inferred parameters from each surrogate model and compare the accuracy when used in the full model, as well as the uncertainty in the approximations.

## Causes of Hurricane Deaths

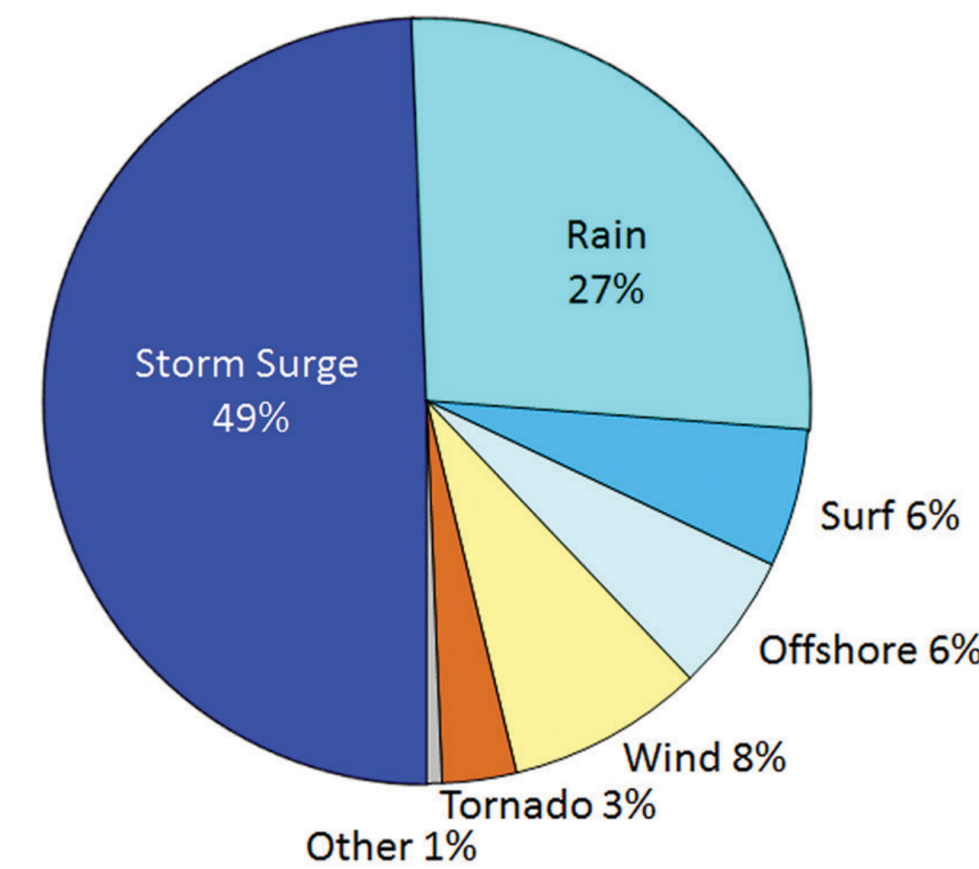


Fig. 1: Deaths in the US caused by hurricanes between 1963-2012 (Rappaport, 2014)

## Method

- Formulate the parameter inference problem as an instance of Bayesian inference:

$$P(n_1, n_2, n_3, \sigma_i^2 | \eta_i) \propto L(\eta | n) q(\sigma^2) q(n_1) q(n_2) q(n_3)$$

$$L(\eta | n) = \frac{1}{\sqrt{2\pi\sigma^2}} \prod_i \exp\left(-\frac{(\eta_i - M_i(n))^2}{2\sigma^2}\right)$$

$n_i$  represents friction parameters,  $\eta_i$  represents actual observed sea level rise,  $M_i$  is the model output for sea level rise, and  $q(k)$  are prior distributions of model parameters.

- Using the formulation by repeatedly sampling from the posterior it is possible to infer the distribution of friction parameters. This sampling will be carried out using the Markov Chain Monte Carlo (MCMC) algorithm due to the multidimensionality of the problem.
- Updating and sampling from the posterior will be done cheaply by using the surrogate models rather than the GeoClaw model.
- After inferring the parameters with each surrogate model, they will be used with the GeoClaw model and their accuracies compared when simulating the resulting storm surge from Hurricane Ike (2008).

## Surrogate Models

- The kriging surrogate is defined as (Warder, 2020):

$$\mathbf{f}_* | \mathbf{f} \sim \mathcal{N}(\boldsymbol{\mu}_* + \boldsymbol{\Sigma}_*^T \boldsymbol{\Sigma}^{-1}(\mathbf{f} - \boldsymbol{\mu}), \boldsymbol{\Sigma}_{**} - \boldsymbol{\Sigma}_*^T \boldsymbol{\Sigma}^{-1} \boldsymbol{\Sigma}_*)$$

$$\boldsymbol{\mu} = g(\mathbf{X})^T \cdot \boldsymbol{\beta}$$

$$\boldsymbol{\mu}_* = g(\mathbf{X}_*)^T \cdot \boldsymbol{\beta}$$

- The PCE surrogate is defined as (Sraj et al., 2014):

$$U(\mathbf{x}, t, \zeta) = \sum_{k=0}^P U_k(\mathbf{x}, t) \Psi_k(\zeta)$$

- The PC-Kriging model is based on that of Schobi et al. (2015), it is identical to the kriging model except that:

$$\boldsymbol{\mu} = \sum_{k=0}^P U_k(\mathbf{x}, t) \Psi_k(\zeta)$$

## Validity

The proposed methodology is valid as the surrogates are closely based of off models used in the literature to solve similar hydrodynamic problems. The data shown in Fig. 1 demonstrates that this is an important problem to solve and the necessity of using surrogate models to do so is presented by Sraj et al. (2014).

## References

- Berger, M. J., George, D. L., LeVeque, R. J. and Mandli, K. T. (2011), 'The GeoClaw software for depth-averaged flows with adaptive refinement', *Adv. Water Res.* **34**, 1195–1206.  
 URL: [www.clawpack.org/links/papers/awr11](http://www.clawpack.org/links/papers/awr11)
- Clawpack Development Team (2020), 'Clawpack software'. Version 5.7.1.  
 URL: <http://www.clawpack.org>
- Hallegatte, S., Green, C., Nicholls, R. J. and Corfee-Morlot, J. (2013), 'Future flood losses in major coastal cities', *Nature Climate Change* **3**(9), 802–806.
- Mandli, K. T., Ahmadi, A. J., Berger, M., Calhoun, D., George, D. L., Hadjimichael, Y., Ketcheson, D. I., Lemoine, G. I. and LeVeque, R. J. (2016), 'Clawpack: building an open source ecosystem for solving hyperbolic pdes', *PeerJ Computer Science* **2**, e68.
- Rappaport, E. N. (2014), 'Fatalities in the united states from atlantic tropical cyclones: New data and interpretation', *Bulletin of the American Meteorological Society* **95**(3), 341–346.
- Schobi, R., Sudret, B. and Wiart, J. (2015), 'Polynomial-chaos-based kriging', *International Journal for Uncertainty Quantification* **5**(2), 171–193.
- Sraj, I., Mandli, K. T., Knio, O. M., Dawson, C. N. and Hoteit, I. (2014), 'Uncertainty quantification and inference of manning's friction coefficients using dart buoy data during the tohoku tsunami', *Ocean Modelling* **83**, 82–97.
- Warder, S. C. (2020), Sensitivity analysis, uncertainty quantification and parameter estimation for a numerical tide and storm surge model, PhD thesis, Imperial College London.