

2) $\text{Ann } M \subseteq \text{Ann } N \subseteq \text{Ann } (M \oplus N)$ $X \times \{ \pm 1 \}$ is D - 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830,

①

$$\forall x \in X \quad h_0(x) \begin{cases} 1 & P(y=1|x) \geq \frac{1}{2} \\ -1 & \text{o.w.} \end{cases}$$

$$h_D = \alpha / y \max_{y \in \{+1\}} P(x|y) P(y).$$

$$P(y|x)P(x) = P(x|y)P(y)$$

$$\arg \max_{y \in \{ \pm 1 \}} P(x|y)P(y) = \arg \max_{y \in \{ \pm 1 \}} \underset{P(x)}{P(y|x)}P(x) = \arg \max_{y \in \{ \pm 1 \}} P(y|x)P(x)$$

כאשר y הוא ערך האחדות נקבע מכך ש $P(x) \leq 0$ והוא קבוע גדול
בחירת y של P גם $(y=0|x)$ ו- $(y=1|x)$ נמצאים
משלבים $\frac{P(y=0|x)}{P(y=1|x)} \geq \frac{1}{\epsilon} \Leftrightarrow P(y=0|x) \leq \frac{\epsilon}{2}$
נקודה

$$\arg \max_{y \in \{+1, -1\}} P(x|y) P(y) = \begin{cases} +1 & P(y=+1|x) \geq \frac{1}{2} \\ -1 & P(y=-1|x) < \frac{1}{2} \end{cases} = \begin{cases} 1 & P(y=+1|x) \geq \frac{1}{2} \\ -1 & \text{o.w.} \end{cases}$$

$$= h_0(x)$$

f.l.w

\mathbb{R}^d / μ_y אחרת וכל $x|y \sim \mathcal{N}(y, \Sigma)$. $X = \mathbb{R}^d$ \Rightarrow נ"ל

$$f(x|y) = \frac{1}{\sqrt{(2\pi)^d \det(\Sigma)}} e^{-\frac{1}{2}(x-\mu_y)^T \Sigma^{-1}(x-\mu_y)}$$

for $\mu_{n+1}, \mu_{n-1} \in \Sigma$ for each $n \in \mathbb{N}$

$$h_0(x) = -\arg \max_{y \in \{ \pm 1 \}} f_y(x)$$

also pick $\mathbb{R}^d \rightarrow \mathbb{R}$ s.t. $\mu = \delta_{-1} + \delta_{+1}$ $y \in \mathbb{R}^d$

$$g_y(x) = x^T \Sigma^{-1} \mu y - \frac{1}{2} \mu y^T \Sigma^{-1} \mu y + \ln p(y) \quad y \in \{-1, 1\}$$

$$h_0(x) = \arg \max_{y \in \{1,2\}} \delta_y(x) = \arg \max_{y \in \{1,2\}} P(x|y)P(y)$$

ln → "lll"lll "lll"lll "lll"lll

$$\ln(h_0) = \arg \max_{y \in \{1,2\}} \ln P(x|y)P(y) =$$

$$= \arg \max_{y \in \{1,2\}} (\ln(P(x|y)) + \ln(P(y))) =$$

$$= \arg \max_{y \in \{1,2\}} \left(\ln \left(\frac{1}{(\sqrt{2\pi})^d \det(\Sigma)} \right) e^{-\frac{1}{2}(x-\mu_y)^T \Sigma^{-1} (x-\mu_y)} + \ln(P(y)) \right) =$$

$$= \arg \max_{y \in \{1,2\}} \left(-\frac{1}{2} (x-\mu_y)^T \Sigma^{-1} (x-\mu_y) + \ln(P(y)) \right)$$

$$= -\frac{1}{2} \left(-\frac{1}{2} x^T \Sigma^{-1} x + \frac{1}{2} \mu_y^T \Sigma^{-1} x + \frac{1}{2} x^T \Sigma^{-1} \mu_y - \frac{1}{2} \mu_y^T \Sigma^{-1} \mu_y + \ln(P(y)) \right)$$

$$= -\frac{1}{2} \left(x^T \Sigma^{-1} \mu_y - \frac{1}{2} \mu_y^T \Sigma^{-1} \mu_y + \ln(P(y)) \right) =$$

$$= \arg \max_{y \in \{1,2\}} \delta_y(x)$$

l.l.l

$$S = (x_1, y_1), \dots, (x_m, y_m)$$

x
③

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$$\hat{p}(y) = \frac{1}{m} \sum_{i=1}^m \delta(y_i = y)$$

$$\hat{\Sigma} = \frac{1}{m} \sum_{\substack{i=1 \\ y_i \in \{1,2\}}}^m (x_i - \mu_y)(x_i - \mu_y)^T$$

$$\hat{\mu}_y = \frac{1}{m} \sum_{i=1}^m (x_i - \mu_y)(x_i - \mu_y)^T$$

4) $\text{argmin}_{v \in \mathbb{R}^n} \|v\|_2$ subject to $v^T v = 1$

1) $\text{argmin}_{v \in \mathbb{R}^n} \|v\|_2$ subject to $v^T v = 1$

2) $\text{argmin}_{v \in \mathbb{R}^n} \|v\|_2$ subject to $v^T v = 1$

3) $\text{argmin}_{v \in \mathbb{R}^n} \|v\|_2$ subject to $v^T v = 1$

4) $\text{argmin}_{v \in \mathbb{R}^n} \|v\|_2$ subject to $v^T v = 1$

5) $\text{argmin}_{v \in \mathbb{R}^n} \|v\|_2$ subject to $v^T v = 1$

5

$$QP: \argmin_{v \in \mathbb{R}^n} \left(\frac{1}{2} v^T Q v + a^T v \right) \text{ s.t. } A v \leq d,$$

$$Q \in \mathbb{R}^{n \times n}, A \in \mathbb{R}^{m \times n}, d \in \mathbb{R}^m - \text{fixed}$$

Hard-SVM $\text{argmin}_{w, b} \|w\|_2$ subject to $y_i w \cdot x_i + b \geq 1$

$$\argmin_{(w, b)} \|w\|_2^2 \text{ s.t. } y_i w \cdot x_i + b \geq 1$$

$$\argmin_{(w, b)} \begin{pmatrix} w \\ b \end{pmatrix}^T \begin{pmatrix} w \\ b \end{pmatrix} \text{ s.t. } \begin{pmatrix} y_1 x_1 & 1 \\ \vdots & \vdots \\ y_m x_m & 1 \end{pmatrix} \begin{pmatrix} w \\ b \end{pmatrix} \geq \begin{pmatrix} 1 \\ \vdots \\ 1 \end{pmatrix}$$

$$= \argmin_{(w, b)} \begin{pmatrix} w \\ b \end{pmatrix}^T \begin{pmatrix} 2I \\ 0 \end{pmatrix} \begin{pmatrix} w \\ b \end{pmatrix} \text{ s.t. } \begin{pmatrix} y_1 x_1 & 1 \\ \vdots & \vdots \\ y_m x_m & 1 \end{pmatrix} \begin{pmatrix} w \\ b \end{pmatrix} \leq \begin{pmatrix} 1 \\ \vdots \\ 1 \end{pmatrix}$$

6) $\text{argmin}_{w, b} \|w\|_2$ subject to $y_i w \cdot x_i + b \geq 1$

$$A = \begin{pmatrix} y_1 x_1 \\ \vdots \\ y_m x_m \end{pmatrix} \quad v = \begin{pmatrix} w \\ b \end{pmatrix} \quad a = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad Q = 2I$$

Soft-SVM Problem:

(6)

$$\arg \min_{w, \xi} \sum_{i=1}^m \xi_i \quad \text{s.t.} \quad y_i \langle w, x_i \rangle \geq 1 - \xi_i \quad \text{and} \quad \xi_i \geq 0$$

$$\arg \min_w \sum_{i=1}^m \frac{1}{2} \|w\|^2 + \frac{1}{m} \sum_{i=1}^m \ell^{\text{hinge}}(y_i \langle w, x_i \rangle)$$

(11)

$$\ell^{\text{hinge}}(a) = \max\{0, 1 - a\}$$

$$\xi_i \geq 1 - y_i \langle w, x_i \rangle \quad \text{for } i = 1, \dots, m$$

$$\xi_i \geq \ell^{\text{hinge}}(y_i \langle w, x_i \rangle)$$

$$\xi_i \geq \ell^{\text{hinge}}(y_i \langle w, x_i \rangle) \Leftrightarrow \xi_i \geq \max\{0, 1 - y_i \langle w, x_i \rangle\}$$

$$\ell^{\text{hinge}}(\xi_i) \geq \ell^{\text{hinge}}(1 - y_i \langle w, x_i \rangle)$$

$$y_i \langle w, x_i \rangle \geq 1 - \xi_i$$

$$\Downarrow$$

$$\xi_i = \ell^{\text{hinge}}(y_i \langle w, x_i \rangle)$$

Lemma 1: The soft margin maximization problem is equivalent to the following problem: