EE 1473 - Digital Communication Systems
Homework Set 3

Due: Thursday, January 29, 2015

The purpose of the first problem on this assignment is to get everyone started on the Design Project. In it, you will write various pieces of MATLAB code that could form part of the baseband signaling block of your simulated system. For your project, you may ultimately end up with a different design than the one considered here, but this assignment should give you a good idea of how some of the blocks could be written.

Although you are permitted to work in pairs on the project, this is a homework assignment, and it is meant to be completed individually. Of course you may discuss your design strategies with other members of the class, but **do not share your MATLAB code with other students. Making use of code obtained from someone other than yourself, including students who are currently taking the class or those who completed it in the past, is plagiarism and will not be tolerated.**

Many of the steps below require that you write MATLAB functions; recall that a function takes certain variables as input and produces other variables as output. This is different from a script, which simply contains a list of commands to be executed. For example, the following function returns the mean-square value of N elements of a vector x:

$$\sigma = \frac{1}{N}\sum_{n=1}^{N} x_n^2.$$

```
function sigma = ms_value(x,N)

if N > length(x)
    disp('Error: the input vector contains fewer than N elements')
    sigma = -1;
    return
end

temp = x(1:N);

sigma = mean(temp.^2);
```

Please remember that you must submit hard copies of all MATLAB figures and other elements as identified below, and you must also submit your MATLAB code electronically to courseweb. An assignment will be created on the Homework page for this purpose.

1. Simulation of Baseband Digital Signaling

   (a) Write a MATLAB function that will produce random binary data values. The inputs to this function should be the number of bits to generate, and the data values a_n to assign to the bits. Call the `rand` function to generate a pseudo-random sequence of numbers in the range $[0, 1]$ and then map them to the correct data values. Your function should be equally likely to produce a binary 0 or a binary 1.

Call your function to produce 100 bits, and assign the value $a_n = -1$ for a binary 0 and $a_n = +1$ for a binary 1. Use the MATLAB `stem` function to plot the data sequence a_n versus n for $1 \leq n \leq 100$, and determine the relative frequency of each data value.

(b) Write a MATLAB function that will produce a rectangular pulse,

$$h(t) = \begin{cases} 1, & |t| < \frac{T_b}{2} \\ 0, & \frac{T_b}{2} < |t| < 5T_b \end{cases}$$

where T_b is the bit period. Note that the above specification has you compute the pulse over 10 bit periods. (The reason for this will become clear in future homework assignments.)

The inputs to your function should be T_b and the number of samples per bit, i.e. the number of instants during each bit period at which to compute the value of the pulse. The outputs of your function should be the pulse samples and a vector of times at which the pulse was computed.

Call your function with $T_b = 1$ second and with at least 16 samples per bit. Plot the resulting pulse versus time.

(c) Write a MATLAB script that will call your random data function from part (a) with at least 20 bits and data values $a_n = \pm 1$. Then call the rectangular pulse function from part (a) with $T_b = 1$ second and at least 16 samples per bit. Form a baseband signal from the data and pulse according to the equation

$$s(t) = \sum_n a_n h(t - nT_b).$$

Plot the resulting signal versus time. If you have done this correctly, your signal should be equal to zero for 4.5 seconds before and after the part of signal corresponding to the data values.

(d) Repeat part (c) using sinc pulses,

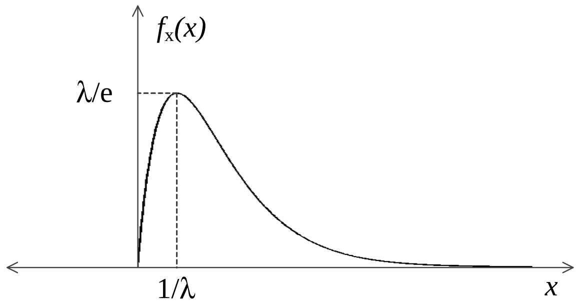
$$h(t) = \frac{\sin \pi R t}{\pi R t},$$

where R is the bit rate.

2. An Erlang-2 random variable has pdf

$$f_{\mathrm{x}}(x) = \lambda^2 x e^{-\lambda x} u(x),$$

where $u(\cdot)$ is the unit step function. A plot of this function is shown below. The *rate parameter* λ controls the peak height of the pdf, as well as the rate at which it falls off as $x \to \infty$.



(a) Show that $f_x(x)$ is a valid pdf, i.e. that it is nonnegative and integrates to 1.

(b) Determine the mean.

(c) Determine the second moment.

(d) Determine the variance.

Hint: The integrals you need can be found in Couch, Section A-5.

3. An analog signal with a bandwidth of 32 MHz is to be transmitted over a PCM system. The peak signal-to-noise ratio due to quantization must be at least 40 dB, and the probability of a bit error during transmission is $P_e = 0$.

   (a) Determine the minimum sampling rate required for this system.

   (b) Determine the number of bits required for each PCM word in order to meet the SNR constraint, and the corresponding number of quantization levels.

   (c) Determine the bit rate of the system if 2 times oversampling is applied to the analog signal.

   (d) Determine the null bandwidth of the PCM signal Polar NRZ signaling is used, and rectangular pulses are used to represent the bits.

   (e) How will the answers to parts (a) through (d) change if the *average* quantization SNR must be at least 40 dB?

4. Consider again the system from Problem 4, in which the peak quantization SNR must be at least 40 dB, but now suppose that the probability of a bit error is not zero.

   (a) Suppose that $P_e = 10^{-6}$. Determine the number of bits per sample n and the number of quantization levels M necessary to meet the constraint. Comment on your results.

   (b) Repeat part (a) for the case when $P_e = 2 \times 10^{-5}$.

   (c) Determine the minimum value for P_e such that $n = 9$ quantization bits and $M = 512$ quantization levels are required.

   (d) Repeat part (a) for the case when $P_e = 10^{-4}$.

   (e) Examine Couch, Figure 7-17. Interpret your answers to parts (a) through (d), making specific reference to this figure.