EE 1473 - Digital Communication Systems
Homework Set 6

Due: Thursday, February 19, 2015

1. The purpose of this problem is to simulate a baseband digital communications system. The simulation that you prepare could be a significant part of your submission for the project that is due at the end of the term. Please submit your MATLAB code to the courseweb site.

   (a) Write a MATLAB function that will produce the Root Raised Cosine Rolloff pulse for $|t| \leq k_T T_b$, where T_b is the bit period and $2k_T$ represents the number of bit periods over which you will compute the pulse. The equation for the Root RCRO pulse is

$$h(t) = \begin{cases} 1 - r + \dfrac{4r}{\pi} & t = 0 \\ \dfrac{r}{\sqrt{2}} \left[\left(1 + \dfrac{2}{\pi}\right) \sin\left(\dfrac{\pi}{4r}\right) + \left(1 - \dfrac{2}{\pi}\right) \cos\left(\dfrac{\pi}{4r}\right) \right] & t = \pm \dfrac{T_b}{4r} \\ \dfrac{\sin\left[\pi R t (1-r)\right] + 4Rrt \cos\left[\pi R t (1+r)\right]}{\pi R t \left[1 - (4Rrt)^2\right]} & \text{otherwise,} \end{cases}$$

   where $R = 1/T_b$ is the bit rate. The inputs to your function should be k_T, T_b, the number of samples per bit, and the rolloff factor r.

   Call your function with $k_T = 5$, $T_b = 1$ second, at least 16 samples per bit, and rolloff factors $r = 0, 0.1, 0.2, \ldots 1$. Plot the resulting pulses versus time on a single pair of axes, and compare with Figure 3-26. Does this pulse satisfy the zero-ISI condition?

   (b) Write a MATLAB script that will generate random data with at least 20 bits and data values $a_n = \pm 1$. Then call the Root-RCRO pulse function from part (a) with $T_b = 1$ second and at least 16 samples per bit. You may choose any rolloff factor other than 0, i.e., $0 < r \leq 1$. State clearly the value you have chosen for r. Form a baseband signal from the data and pulse, and plot the resulting signal versus time.

   (c) Take the baseband signal from part (b) process it with a filter whose impulse response is the same Root-RCRO pulse you used to form the signal. Confirm that the filtered signal has zero ISI.

(d) Add Gaussian noise to your signal from part (b) (use the `randn` function in MATLAB), and filter the noisy signal as in part (c). Can you recover the data from the filtered noisy signal? Experiment with different noise variances to see the effects of noise. Show at least one case where the noise is strong enough to be noticeable, but does not cause any bit errors. Show another case where at least one bit error occurs.

(e) Repeat the simulation from Problem 1 many times (how many?) and compile the bit error rate in each case. Your simulation plots the signals at various stages of the process, but you may want to turn that off for this part of the problem. Compare your results with Couch Figure 7-5 and equation (7-26b). How will you compute E_b/N_0 for your simulation?

2. Consider a binary signaling system in which, when a binary 1 is to be sent, a positive pulse $s_1(t)$ with amplitude +1V is transmitted. When a binary 0 is to be sent, a negative version of the same pulse, with amplitude -1V, $s_0(t) = -s_1(t)$ is transmitted. These pulses are received in the presence of AWGN with PSD

$$P_n(f) = \frac{N_0}{2} = 0.1\,\mathrm{V}^2.$$

(a) Sketch a block diagram of the MAP receiver in the form of a matched filter whose output is sampled. Specify the signal to which the filter is matched.

(b) Determine the optimal detection threshold, γ_0, for the MAP receiver in terms of the prior probabilities and the energy of the transmitted pulse.

(c) Evaluate γ_0 for the case when $P_1 = 0.5$ and the pulses are rectangular.

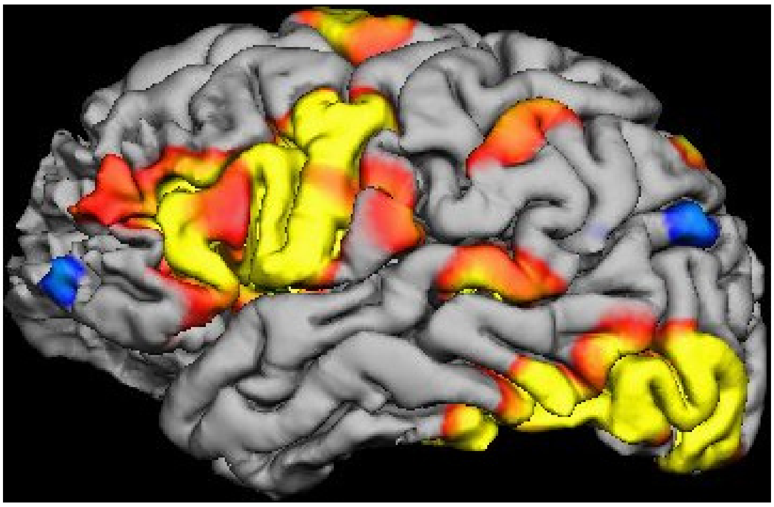
(d) Repeat part (c) for $P_1 = 0.7$.

(e) Repeat part (c) for $P_1 = 0.2$.

(f) Explain the effect of the prior probabilities on γ_0.

(g) Determine the bit error rates for the cases considered in parts (c), (d) and (e). In each case, compare to the bit error rate that would be achieved by a maximum-likelihood (ML) receiver.

3. In *functional magnetic resonance imaging* (fMRI), NMR spectroscopy is used to determine localization of function within the human brain. When a given region of the brain experiences an increase in neural activity, this region also experiences a temporary increase in blood flow, and specifically an increase in the relative concentration of oxygenated blood. Because the ferromagnetic properties of oxygenated and deoxygenated hemoglobin are quite different, an MRI scanner can be made to be sensitive to these changes, allowing for the creation of three-dimensional images of brain activity. For example, the figure below shows a view of the side of the brain, where the color of the image corresponds to the fMRI signal level. The illuminated regions are involved in reading and language processing.



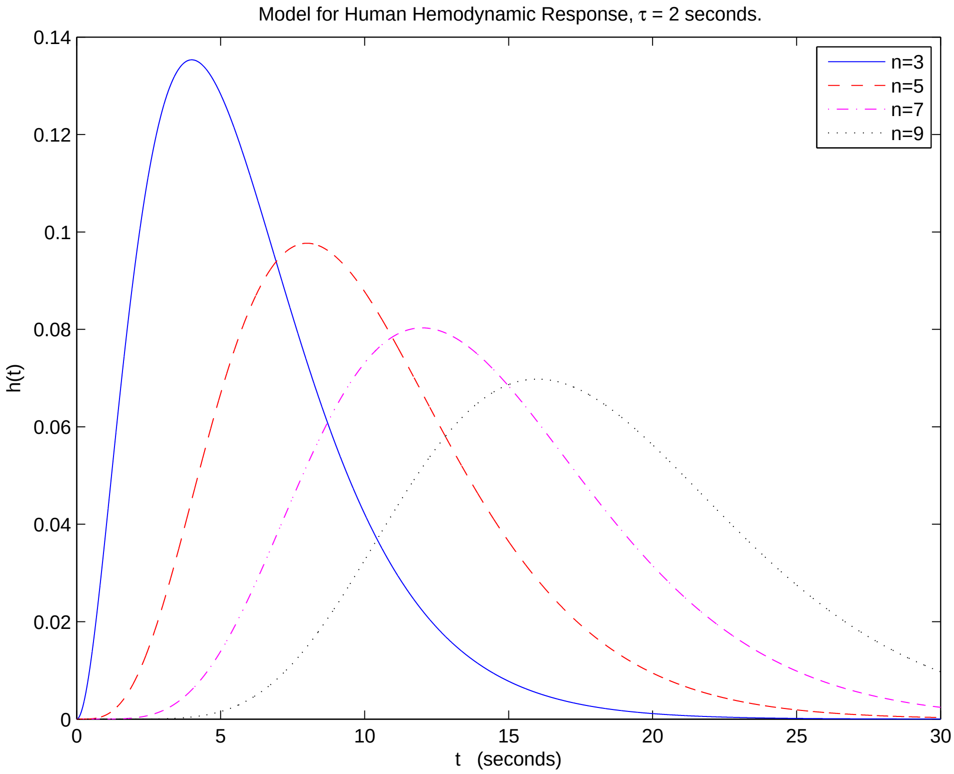
The resolution of these images is on the cubic-millimeter scale. The physics underlying MRI and the signal processing concepts involved in extracting information from different spatial regions are fascinating areas of study, but are not the focus of this problem.

There is a measurable time delay between the onset of neural activity and the corresponding peak in local blood flow. Studying the time course of this relationship is important, because fMRI allows one to study neural activation only through signals that arise from the changes in blood flow. In one area of research, a deterministic signal model has been proposed, known as the *hemodynamic response*, for the fMRI signal at time t, in response to a brief (ideally, impulsive) stimulus presented at time 0. Boynton, et al.[1] model the hemodynamic response using the following equation

$$h(t) = \begin{cases} \dfrac{(t/\tau)^{n-1}\, e^{-t/\tau}}{\tau(n-1)!} & t \geq 0 \\ 0 & t < 0, \end{cases}$$

where τ is the time constant for the exponential, which dominates for large t, and n is an integer-valued parameter that helps determine the response for small t. Typical values are $1 \leq \tau \leq 3$ seconds and $3 \leq n \leq 11$. One can quickly verify that $h(0) = 0$, $h(t) \to 0$ for large t, and the peak of the response occurs at $t = \tau(n-1)$. The following figure shows plots of $h(t)$ for $\tau = 2$ seconds and $n = 3, 5, 7$ and 9.

[1] Boyton, G. M., Engel, S. A., Glover, G. H. & Heeger, D. J. (1996) J. Neurosci. 16, pp. 4207-4221.



In a typical fMRI experiment, a human subject is positioned with their head in an MRI scanner and is given a series of simple cognitive tasks, such as reading a list of words. Statistical analysis is performed on the fMRI signals acquired to determine, for each voxel[2] in the brain space, whether that portion of the brain has become active as a result of the stimulus, and if so to determine the level of activation. In this problem, we map this problem to a signal detection problem. To simplify the analysis, we consider only a single voxel. Under H_0, the voxel is not activated by the stimulus, and the observed fMRI signal is

$$H_0: \ r(t) = n(t),$$

where $n(t)$ is a Gaussian white noise process with intensity $\frac{N_0}{2}$. Under H_1, let A represent the level of activation in response to the stimulus, so that the observed fMRI signal is

$$H_1: \ r(t) = A\sqrt{E}s(t) + n(t),$$

and the energy in the signal portion is A^2E. Assume that A, τ and n are all known, that $r(t)$ is observed over the interval $0 \leq t \leq T$, and that T is chosen such that $h(t) \approx 0$ for $t > T$.

(a) Let $h(t) = \sqrt{E}s(t)$, where $s(t)$ has unit energy on $[0, \infty)$ and $h(t)$ has energy E.

[2] A *voxel* is the three-dimensional analogue to a pixel in a two-dimensional image.

Express E as a function of τ and n, and plot the result for $\tau = 1.5$ seconds and $3 \leq n \leq 11$.

Hint:

$$\int_0^\infty x^a e^{-bx}\, dx = \frac{a!}{b^{a+1}},$$

(b) Derive the optimal receiver for the binary detection problem. Use a minimum probability of error criterion, and assume that the priors are equal. Give your solution both in the form of a correlator-integrator system and a matched-filter-sampler system. Also determine the threshold to which the output of the system should be compared to decide H_0 or H_1.

(c) Now consider the extension of this problem to the M-ary case. Under H_0, there is no activation, and the signal model is unchanged. Under H_i for $1 \leq i \leq M-1$, the level of activation is A_i, and the signal model is

$$H_i: \; r(t) = A_i\sqrt{E}s(t) + n(t) \quad \text{for } i \neq 0,$$

where the A_i are known and ordered, $0 < A_1 < \ldots < A_{M-1}$. Derive the minimum probability of error receiver for this case.