EE 1473 - Digital Communication Systems
Homework Set 7

Due: Thursday, March 5, 2015

1. In this problem, you will approximate the PSD for random polar NRZ data through simulation. (Read Couch, Section 6-2, with particular emphasis on Subsection "Measurement of PSD, Numerical Computation of the PSD.")

   (a) Use the `rand` function in MATLAB to generate a sequence of 32 random bits. You will have to round the output of `rand` to 0 or 1.

   (b) Use the random data to generate a polar NRZ signal in MATLAB, using rectangular pulses. Create 16 samples of the signal for each bit, using a value of +5 to represent a 1, and -5 to represent a 0. The total number of samples in your signal vector should be $32 \times 16 = 512$. If your signal includes "tails" of zero values prior to the first bit, and after the last bit, then you should delete those samples from your signal vector before proceeding.

   (c) Use MATLAB to compute the spectrum of your signal using the FFT, with the following parameters: $N = 512$, $R = 100$ bits per second, $f_s = 16R$, $\Delta t = 1/f_s$, $T = N\Delta t$.

   (d) Let $s(t)$ be the signal you generated in part (b), and let $S(f)$ be the spectrum you computed in part (c). Compute the approximate the PSD of $s(t)$ using

   $$P_s(f) = \frac{|S(f)|^2}{T},$$

   where T is the duration of $s(t)$ in seconds. Plot the PSD versus frequency in Hertz. Also plot on the same pair of axes the theoretical PSD given by equation (3-41).

   (e) The method you have used to approximate the PSD is really only valid if the results are averaged over many random signals. Modify your MATLAB code so that it computes an average PSD for 10 signals. You should create a loop, such that each time through the loop your code generates new data, creates a new signal $s(t)$, computes a new spectrum $S(f)$, and updates the average PSD. Plot the average PSD for 10 signals versus f, and plot equation (3-41) on the same axes.

   (f) Repeat part (e) with 100 signals.

   (g) Does the agreement between theory and simulation increase as more signals are generated? Is the agreement equivalent for all frequencies? Why or why not?

(h) For the PSD using 100 signals, prepare a plot showing both the theoretical and simulation results for frequencies from 200 to 500 Hertz. Comment on the source of any disagreement here.

2. Couch 3-29. In both parts, the data are equally likely to be $a_n = \pm A$, and all bits are independent. In part (a), the pulse shape is

$$f(t) = \begin{cases} 1 & -\frac{\tau}{2} < t < \frac{\tau}{2} \\ 0 & \text{otherwise,} \end{cases}$$

and in part (b), the pulse shape is

$$f(t) = \begin{cases} 1 & -\tau < t < 0 \\ -1 & 0 < t < \tau \\ 0 & \text{otherwise.} \end{cases}$$

3. Let $x(t)$ be a Gaussian random process that is wide-sense stationary, with mean $\mu_x(t) = 0$ and autocorrelation function

$$R_x(\tau) = 4e^{-2|\tau|}.$$

(a) Determine the normalized average power in $x(t)$.

(b) Determine the power spectral density for $x(t)$.

(c) Determine the probability $P(x(t) \leq 3)$. (*Hint*: For t fixed, $x(t)$ is a Gaussian random variable.)

(d) Determine $E\left\{[x(t+1) - x(t-1)]^2\right\}$ (*Hint*: How can this be expressed in terms of the autocorrelation function?)

4. We can define a discrete-time random process $x[n]$ as a random function of $n \in \mathbb{Z}$, where each $x[n]$ is a random variable. If we say that these random variables are *independent and identically distributed* (abbreviated i.i.d.), then we are assuming that:

- $x[n]$ and $x[m]$ are independent random variables for $n \neq m$, and
- all of the $x[n]$ have the same pdf.

For this problem, assume that the $x[n]$ are i.i.d. and Gaussian, with mean 0 and variance σ^2.

(a) Determine the mean and the autocorrelation function of the random process $x[n]$,

$$\begin{aligned} \mu_{\mathrm{x}}[n] &= E\{x[n]\} \\ R_{\mathrm{x}}[n, n+k] &= E\{x[n]x[n+k]\}. \end{aligned}$$

Is $x[n]$ WSS?

(b) Determine the Power spectral density for $x[n]$, which is defined as

$$P_x(f) = \sum_{k=-\infty}^{\infty} R_x[k] e^{-j2\pi f k}.$$

What name would you give to this random process?