Introduction

Chapter

ONE

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1.1 INTRODUCTION

It is generally considered that mathematics which is mostly applied in daily routine need not require the deep knowledge of mathematics. It is true in some respect since it is now possible to make excellent and effective use of computer without having a background of mathematics. However, for the better understanding of the subject the knowledge of mathematics is essential. The reason being that mathematics provides a clear, exact solution to the problem in a precise manner. It has been observed that most of fields of knowledge other than mathematics can be now subjected to use of computer. To prepare the problem for the use of computer requires some skill of mathematics called as Discrete Mathematics. This chapter has been designed to throw some light on this subject as well as the areas it cover to solve their problems.

1.2 DISCRETE MATHEMATICS

Under the new development in the scientific field mathematics is now classified in two major branches

(a) Discrete Mathematics (b) Continuous Mathematics

(a) Discrete Mathematics:

This branch of mathematics mostly deals with those real number that are integers and rational numbers, that is those number where there is room for gapping between any two number just as in the numbers ..., -1, -2, 0, 1, 2 ... there are infinite numbers between any two such numbers. Consequently a discrete variables are such variables which can not become infinitely small as we have in Continuous Mathematics. In fact the concept of continuity is not available in Discrete Mathematics. Hence Discrete Mathematics is such a branch of mathematics devoted to the study of discrete objects that uses arithmetic and algebra but do not require the use of calculus and analysis whose main concern are with continuous functions. However, it is very difficult do draw a line of separation between any two types of mathematics, because there are some branches such as numerical analysis and linear algebra which have both continuous and discrete components.

We will consider some examples to explain the underlying concept of discrete mathematics as given below:

(a) A drawar contain ten black and ten white socks. What is the least number of socks one must pull out to be sure to get matching pair?

(b) How can a circuit that add two numbers and circuit that compare two number can be designed?

(c) In any programming language the variable name must begin with some letter from Alphabet or letter followed by a decimal digit. How many different variable names are there in such programming language?

(d) Every computer system consists of five subsystems the failure of any subsystem independently has the probability of .2. The failure of any subsystem will fail the whole system. Given that the computer system fails. What is probability that subsystem I and only subsystem I may fail?

(e) What is the shortest route starting and ending at a point A and making stops at points B,C, D & E by using the distances assigned to the lines on the diagram below and justify the solution?

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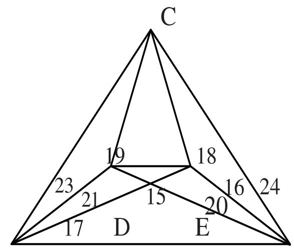

1.3 IMPORTANCE OF DISCRETE MATHEMATICS

This branch of mathematics came into prominence with the invention of computer whose designing involves the elements of discrete mathematics. As the use of computer is growing rapidly in every sphere of life so the importance of discrete mathematics is increasing with many practical and relevant application. The significant of discrete mathematics lies in the fact that modern mathematics deals with sets not number or more precisely finite sets with additional structure because computer can only provide finite solution to the problem. The growth of non-numerical mathematics as well as from the point of view practical application discrete mathematics has taken an important place in today's mathematics. The advances in computer technology have given discrete mathematics a big boost. The key reason of this branch is due to the fact that the information is stored and manipulated in computer machine in a discrete fashion. Even floating point numbers become discrete under computer system having a finite range. Thus the ideas of discrete mathematics are fundamental to science and technology being a part of computer system. It is now the key prerequisite for many advanced work in many branch of mathematics and computer sciences which includes data structure, database theory, automata theory, compiler design, operating system etc. In nutshell the popularity of discrete mathematics is growing with the increased use of computer in different aspects of human activity.

1.4 CONTENTS OF DISCRETE MATHEMATICS

It is very difficult to suggest the clear cuts topics that should be the contents of the discrete mathematics. However, there is general agreement that some of the topics of mathematics are essential that must be included in this branch of mathematics. A brief description of these topics as the part of this text book are highlighted below:

1.4.1 Mathematical logic:

This topic is the most essential part of discrete mathematics. The main reason is the key role played by it in the designing of computer. Basically we know that logic is the science of dealing with the method of reasoning. However, this abstract concept has been converted into symbolic language by G. Boole and De-Margan to express its principles in precise and unambiguous terms known as mathematical logic. It is one of the basic tool of computer technology, therefore, the understanding the basic principle of this topic is utmost necessary to learn the working principle of computer system ranging from hardware concept to artificial intelligence. This requires that one must learn to use logically valid form of argument to avoid common logical errors. One has to know how to use both direct and indirect arguments to derive new results from those already know to be true. In this book two basic topics (i) proposition calculus and (ii) predicate calculus of mathematical logic have been covered in second and third chapters. Since these topic are widely used in the development of computer technology, programming language as well as in may computer applications, so they have been discussed in details in these chapters.

1.4.2 Boolean Algebra:

Another most significant tool which plays a key role and is the backbone of logical circuit in the computer is Boolean Algebra. This topic was developed by G Boole in 1853 as a pure mathematics without having any application in mind. He was very much criticized for this sort of algebra without having any application. But what a surprise! it found its application after nearly one century in 1948 in the designing of computer based on electronic technology. It is based on two discrete values (1 or 0/ true or false). Since electronic circuits used inverters and gates as digital devices and digital circuits operate in binary number system so they make it possible to use Boolean algebra as a mathematical tool for the analysis and design of digital circuit of computer system. Now Boolean algebra is considered as simple and systematic way of representing and performing of logical circuit in any modern digital system. The problem of optimizing the design of these circuits is essentially a problem of Boolean algebra. This topic has been included in this text book.

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1.4.3 Set Theory:

The old definition of Mathematics as given by Greeks was that it is mother of all sciences. The latest definition of mathematics is that it is the study of sets and function. With the introduction of set in 1850 due to German mathematician G-Cantor, it has revolutionized the approach to deal with mathematical problems. All mathematical objects can be defined in term of sets and the language of set theory is used in every mathematical subject. The focal point of today mathematics is not numbers but sets. Due to this concept mathematics is no longer confined to deal with problem of science and technology but of social sciences as well like Economics, Psychology, Sociology etc. It also helps to explain the concept of mathematical logic and Boolean Algebra.

Since it is the basic tool to explain every topic of mathematics so we will highlight some elementary concepts of sets so required for mathematical logic and Boolen Algebra. Precise definition of Set is that it is the collection of well defined object. Just as Sofa set, Tea set, Class, Bunch of keys are simple examples of Set. To have its mathematical form the capital letter A to Z of English alphabet are used to denote the set and small letter a to z are used to denote the members of set. These members are enclosed in a middle brackets (curly brackets) separated by commas and assigned to some capital letter through the sign of equality '='. For example if *A* be set to denote first five letter of English alphabets, we have $A = \{a, b, c, d, e\}$. This representation is called Roster/ Tabular form of the set. But when the members of set are large enough then we select one member as representation of the whole set usually denoted by x and if P(x) be some statement highlighting the members of set and if A be the set to denote this statement then it is expressed as 'A is the set of all x such that x is the member of P(x)' denoted by

$$A = \{x: x \text{ is the member of } P(x)\} \qquad (1)$$

where ':' or '/' are symbol used to denote 'such that'. Again the expression 'is the member of is symbolized by the fifth letter of Greek alphabet 'ε' (epsilon) called as 'belongs to', hence, x is member of $P(x)$ is denoted by $x \in P(x)$. Thus (1) becomes

$$A = \{x \,;\, x \in P(x)\} \qquad (2)$$

This representation is a called set-builders form of the set. Geometrically set is represented by any plane figure usually taken as circle A called as Venn Diagram after the name of Italian mathematician "Venn".

Moreover, a set can be finite or infinite. According to Cantor any set A is said to be finite if on counting the members of the set the counting process comes to an end otherwise it is an infinite se. For example the sands on sea beach, star in the sky are the examples of finite sets because if one could count the grain of sands the counting process would terminate and there is a definite formula in astronomy which gives the number of star in the sky. Now the question arises which can be considered as infinite set? Cantor says that infinite set can be found in the field of mathematics. For example the set of counting number {1, 2, 3...} denoted

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by N is one example of infinite set because for every $n \in N$ there is $(n + 1) \in N$. So there is no end of this process. Similarly other standard infinite sets in mathematics are: set of integers denoted by I and expressed as

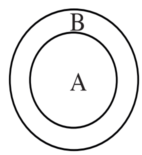
$$I = \{... -3, -2, -1, 0, 1, 2, 3, ...\}$$

Again, we have infinite set as set of rational number of the form $r = a/b$ where $a, b \in I\ b \neq 0$ denoted by Q and is expressed as

$$Q = \{r : r = a/b\ a, b \in I\ b \neq 0\}$$

In addition to this there is a set of real number R as infinite set which consists of rational number as well as irrational number where the irrational numbers can be $\sqrt{3}, \sqrt[3]{2}, \pi,$ log x, e^x, etc.

Any part of a set is called subset. If A and B be sets such that A is the subset of B then it is expressed as A is contained in B denoted by $A \subseteq B$ and B is called super set of A. Some authors denote the subset through line diagram as $\begin{array}{c}B\\ |\\ A\end{array}$ by taking subset A below the set B. By Venn diagram it can be shown as



Now the two symbols '$\in$' and '$\subseteq$' in set theory are very significant. The first one is always used for any element being the member of set in the sense of belonging while the second symbol is used for a part of a set as subset. For example if a be any element of set X then it is denote by $a \in X$ but if we take {a} then it become subset of X. denoted by $\{a\} \subseteq X$. Similarly if any element is not the member of a set or any set is not the subset then they are denoted by the symbols '$\notin$' and '$\not\subset$' respectively.

Again, any set having no element is called null set denoted by 'ϕ' (Phi) expressed as $\phi = \{\ \}$. where set {a} is called singleton set having one element. But note that $\{a\} \neq a$. Moreover the equality of two set are defined as two sets A, B are said to we equal if every member of A is the member of B and every member of B is the member of A, that is A is the subset of B and B is the subset of A expressed as $A \subseteq B$ and $B \subseteq A \equiv A = B$. From this definition of equality of set if we consider the following examples:

A = {1, 2, 3, 4}
B = {1, 2, 2, 3, 3, 4}
C = {4, 3, 2, 1}

Now all these three sets are equal because every member of A is the member of B as well as C and every member of B and C are the member of A. Again every member of B is the member of C and every member of C is the member of A. So we have

A = B = C

The consequence of these conditions given in the set A, B, and C is that (i) repetition of elements does not change the equality of set (ii) the order of element does not change the equality of set, that is, the repetition is allowed in the set without affecting the equality of set and the order of element does not affect the equality of set, that is, set has no ordered condition.

1.4.4 Relations & Functions:

Another significant element of set theory is the function. But the concept of function is better understood by the notion of relation because function is a particular type of relation these two concepts have been extensively explained in this book. Both are basically essential from the point of discrete mathematics because both have application in computer science as well as in real life. For example there is concept of relation database

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developed in computer science as an application in the booking of Airlines and in the diagnoses of hospital patients. Theory of function applies to mathematical modeling where mathematical objects represent certain feature of another or some nonmathematical objects. For the information of readers there is very important result in computer science based on the concept of recurrence function which says that the general recursive functions coincide with the function defined by Touring-machine a concept defined by Alan-Turing "can machine think. The study of computable function is the domain of recursive function theory-an active branch of mathematics.

1.4.5 Abstract and Linear Algebra:

Algebra generally deals with discrete objects and is therefore, a natural part of discrete mathematics. With the introduction of sets there is revolutionary change in the study of mathematics. Most of the topic are becoming abstract rather than concrete as was the odd practice of solving numerical problems. Mathematical system has been now classified into three major three structures (i) Algebraic structure (ii) Topological structure (iii) Ordered structure. Algebraic structure can be considered as the generalization of old algebra. The most important algebraic structures are classified as follows: (a) Groups (b) Rings (c) Field (d) Linear (Vector) space (e) Algebra. Form the above classification one can easily see that Algebra is the part of algebra structure. Similarly topological structure is the generalization of Geometric in abstraction. Since in every structure set is the main entity and we know that the elements of set have no order conditions so whenever some order condition is required it is provided by ordered structure. Since we are concerned with discrete quantities so we well confine mainly to algebraic structure. The abstract algebra has many applications in computer science for example semi-groups helps to explain the concept of formal language and automata theory. Groups are applied in the construction of codes for computer systems as well as coding theory has developed techniques that help in detecting and correcting errors in the transmitted data through the computer system.

Linear space (algebra) is one of the most important algebraic structure covering almost 90% of mathematical system. In fact it is the generalization of vectors and one of its consequence is matrices which is the discrete form and has its application not only in Mathematics, Science and Engineering branches but also has been applied in the many problems of Medical Sciences as well as Social Sciences.

Conclusion:

It is brief Introduction to discrete mathematics. The importance of this subject will continue to grow as the application of computers permeates more and more aspect of Science, Technology and everyday life problems.

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