$$\int x \cdot \operatorname{arctg}(x+1)\,dx = \left| \begin{matrix} \operatorname{arctg}(x+1) & x \\ \frac{1}{(x+1)^2+1} & \frac{x^2}{2} \end{matrix} \right|$$

$$\overset{PP}{=} \operatorname{arctg}(x+1)\cdot\frac{x^2}{2} - \int \left((x+1)^2+1\right)^{-1}\cdot\frac{x^2}{2}\,dx =$$

$$= \frac{x^2}{2}\cdot\operatorname{arctg}(x+1) - \frac{1}{2}\cdot\int \frac{x^2}{x^2+2x+2}\,dx = -\|- - \frac{1}{2}\cdot\int \frac{x^2+2x+2}{x^2+2x+2} - \frac{2x+2}{x^2+2x+2}\,dx$$

$$= -\|- - \frac{1}{2}\cdot\left(x - \int \frac{2+2x}{2+2x+x^2}\,dx\right) = \frac{x^2}{2}\cdot\operatorname{arctg}(x+1)\cdot\frac{1}{2}\cdot\left(x - \ln|x^2+2x+1|\right) + C$$

$$\underline{-\ln|x^2+2x+2| + C}$$

$$1+(x+1)^2 > 0$$

Př. 16152

$$\int \frac{4x-5}{(x-2)(x-3)}\,dx = \int \frac{-3}{(x-2)} + \frac{7}{(x-3)}\,dx = \underline{-3\cdot\ln|x-2| + 7\cdot\ln|x-3| + C}$$

$$x \in \mathbb{R} \setminus \{2;3\}$$

$$\frac{4x-5}{(x-2)(x-3)} = \frac{A}{(x-2)} + \frac{B}{(x-3)}$$

$$\frac{Ax-3A+2Bx-2B}{(x-2)(x-3)} = \frac{(A+2B)x+(-3A-2B)}{(x-2)(x-3)}$$

$$\begin{array}{cc} A & B \end{array}$$

$$\left(\begin{array}{cc|c} 1 & 1 & 4 \\ +3 & +2 & +5 \end{array}\right) \sim \left(\begin{array}{cc|c} 1 & 1 & 4 \\ 0 & -1 & -7 \end{array}\right) \quad \begin{array}{l} B = 7 \\ A = -3 \end{array}$$

Př. 16144

$$\int \frac{1}{e^x+e^{-x}}\,dx = \left| \begin{matrix} u = e^x \\ du = e^x\,du \end{matrix} \right| = \int \frac{1}{e^x+e^{-x}} \cdot \frac{e^x}{e^x}\,dx =$$

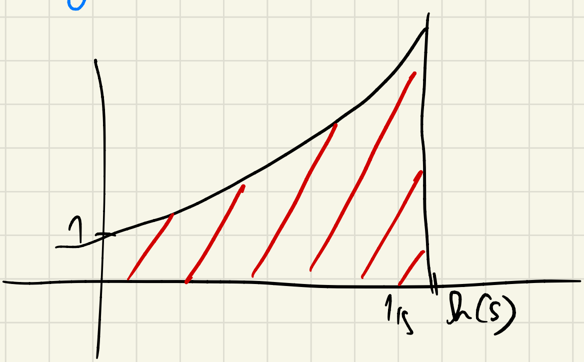
$$\int \frac{1}{u+\frac{1}{u}} \cdot \frac{1}{u}\,du = \int \frac{1}{u^2+1}\,du = \operatorname{arctg}(u)+C =$$

$\operatorname{arctg}(e^x)+C$

Určitý integrál

Př. 16167

$$\int_0^{\ln(5)} e^x\,dx = \left[e^x +1\right]_0^{\ln 5} = e^{\ln 5} +1 - e^0 -1 = 5-1 = 4$$

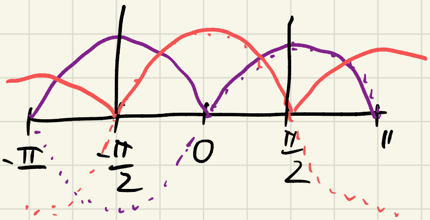


Př. 16162

$$\int_{-\pi}^{\pi} |\sin x| + |\cos(x)|\, dx = 8\cdot\int_0^{\frac{\pi}{2}} \sin(x)\, dx = 8\cdot\left[-\cos(x)\right]_0^{\frac{\pi}{2}}$$

$$= 8\cdot\left(-\cos\left(\tfrac{\pi}{2}\right) - \cos(0)\right)$$

$$= 8\cdot\left(0-(-1)\right) = \underline{\underline{8}}$$



Př. 16180

$$\begin{vmatrix} x^2 & e^x \\ 2x & e^x \end{vmatrix} \qquad \begin{vmatrix} 2x & e^x \\ 2 & e^x \end{vmatrix}$$

$$\int x^2\cdot e^x\, dx \overset{PP}{=} x^2\cdot e^x - \int 2x\cdot e^x\, dx \overset{PP}{=} \dots\dots\ e^x\cdot\left(x^2-2x+2\right)+C$$

$$① = \left[e^x\cdot(x^2-2x+2)+C\right]_0^1 = \underline{\underline{e-2}}$$

$$② \int_0^1 x^2\cdot e^x\, dx \overset{PP}{=} \underbrace{\left[x^2\cdot e^x\right]_0^1}_{e} - \int_0^1 2x\, e^x\, dx = e - \underbrace{\left[2x\cdot e^x\right]_0^1}_{2e} + \int_0^1 2\cdot e^x\, dx$$

$$= e - 2e + \underbrace{\left[2e^x\right]_0^1}_{2e-2} = \underline{\underline{e-2}}$$

① – napřed spočítat $F(x)$ a na konci udělat ten určitý

② – mezi dosazovat postupně

Pr: 16174

$$-\frac{1}{2}\cdot\int \frac{x^2}{x^2+1}\,dx = -\frac{1}{2}\cdot\int \frac{x^2+1}{x^2+1} - \frac{1}{x^2+1}\,dx = -\frac{1}{2}\int 1 - \frac{1}{x^2+1} = -\frac{1}{2}(x - \operatorname{arctg} x)$$

$$\int_0^1 x \cdot \operatorname{arctg} x \, dx \overset{PP}{=} \left| \begin{matrix} \operatorname{arctg}(x) & x \\ \frac{1}{x^2+1} & \frac{x^2}{2} \end{matrix} \right| = \underbrace{\left[\operatorname{arctg}(x)\cdot\frac{x^2}{2}\right]_0^1}_{\operatorname{arctg}(1)\cdot\frac{1}{2} = \frac{\pi}{8}} - \int \frac{1}{x^2+1}\cdot\frac{x^2}{2}\,dx$$

$$= \frac{\pi}{8} - \frac{1}{2}\cdot\underbrace{\left[x - \operatorname{arctg}(x)\right]_0^1}_{\left(1-\frac{\pi}{4}\right)-(0-0)} = \frac{\pi}{8} - \frac{1}{2}\cdot\left(1-\frac{\pi}{4}\right) = \underline{\underline{\frac{\pi}{4} - \frac{1}{2}}}$$