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Pr. 16113

$F_1(x) = \sin(2x) = 2\cdot\sin(x)\cdot\cos(x)$

$F_2(x) = (\cos x + \sin x)^2 = \cos^2 x + 2\cdot\sin x\cdot\cos x + \sin^2 x = 2\sin\cdot\cos x + 1$

$(F_1(x))' = 2\cdot\sin x\cdot\cos x$

$\frac{1}{3}+1$

Pr. 16116

$\int\left(4+\frac{1}{x}+\sqrt[3]{x}\right)dx = 4x + \ln|x| + x^{\frac{4}{3}}\cdot\frac{3}{4}+C$

Pr. 16120

$\int(2+x^3)^2dx = \int(4+4x^3+x^6)dx = 4x + 4\cdot\frac{x^4}{4} + \frac{x^7}{7}+C = 4x+x^4+\frac{x^7}{7}+C$

Per partes $\left(\int u'v\,dx = -\int(u\,v')dx + u\cdot v\right)$

Pr. $\int(x^5\cdot\ln(x))\,dx = \left| \begin{matrix} \ln(x) & x^5 \\ \frac{1}{x} & \frac{x^6}{6} \end{matrix} \right| = \ln(x)\cdot\frac{x^6}{6} - \int\frac{1}{x}\cdot\frac{x^6}{6} =$

$= \ln(x)\cdot\frac{x^6}{6} - \int\frac{x^5}{6}dx = \ln(x)\cdot\frac{x^6}{6} - \frac{1}{6}\cdot\frac{x^6}{6}+C = \ln(x)\cdot\frac{x^6}{6} - \frac{x^6}{36}$

Př. 16123

$$\begin{vmatrix} \cos x & e^t \\ -\sin x & e^t \end{vmatrix}$$

$$t = \int e^t \cdot \sin x = \begin{vmatrix} \sin x & e^t \\ \cos x & e^t \end{vmatrix} = \sin x \cdot e^t - \int (\cos(x) \cdot e^t)\, dx =$$

$$= e^t \cdot \sin x - \left(\cos x \cdot e^t + \int \sin x \cdot e^t\right) = e^t \cdot (\sin x - \cos x) - t$$

$$t = e^t \cdot (\sin x - \cos x) - t$$

$$2t = e^t (\sin x - \cos x)$$

$$t = \frac{1}{2} \cdot e^t \cdot (\sin x - \cos x)$$

$$\int e^{4x+3}\, dx = \begin{vmatrix} u = 4x+3 \\ du = 4dx \end{vmatrix}$$

$$\frac{1}{4} e^{4x+3}$$

Př. 16121

člen s vyšším stupněm

$$\int \left((e^{4x+3}) \cdot (x + x^2) \right) dx = \begin{vmatrix} x^2 + x & e^{4x+3} \\ 2x & \frac{1}{4} e^{4x+3} \end{vmatrix}$$

$$= (x^2+x) \cdot \frac{1}{4} \cdot e^{4x+3} - \int 2x \cdot \frac{1}{4} e^{4x+3} = (x^2+x) \cdot \frac{1}{4} e^{4x+3} - \frac{1}{2} \int x \cdot e^{4x+3}$$

$$= \begin{vmatrix} x & e^{4x+3} \\ 1 & \frac{1}{4} e^{4x+3} \end{vmatrix} (x^2+x) \cdot \frac{1}{4} \cdot e^{4x+3} - \frac{1}{2} \cdot \left[x \cdot \frac{1}{4} e^{4x+3} - \frac{1}{4} \int e^{4x+3} \right] =$$

$$= (x^2+x) \cdot \frac{1}{4} \cdot e^{4x+3} - \frac{1}{2} \cdot \left(x \cdot \frac{1}{4} e^{4x+3} + \frac{1}{4} \cdot \frac{1}{4} e^{4x+3} \right) = (x^2+x) \cdot \frac{1}{4} e^{4x+3} - \frac{1}{2} \cdot \frac{1}{4} \cdot x \cdot e^{4x+3} -$$

$$\frac{1}{2} \cdot \frac{1}{4} \cdot \frac{1}{4} \cdot e^{4x+3} = \frac{1}{4} e^{4x+3} \cdot \left((x^2+x) - \frac{1}{2} x - \frac{1}{8} \right) + C$$

$e^{4x+3} \cdot (ax^2+bx+c)$ (ukázkové 😮

$F'(x) = f(x) = e^{4x+3} \cdot (x^2(4a) + x(4b+2a) + 1(4c+b))$

$\downarrow a = \frac{1}{4}$ $\downarrow b = \frac{1}{8}$ $\downarrow 0$ $c = -\frac{1}{32}$

Př. Custom

$$\int (e^{4x+3} \cdot x^{10})\, dx = NN$$

Substituce

V: f má prim. f. F na (a,b)
φ je diferenc. na (α,β)

$(f(g(x)))' = f'(g(x)) \cdot g'(x)$ $\quad \varphi((\alpha,\beta)) \subset (a,b)$

Př. 16132

oboru hodnot
$\varphi(\mathbb{R}) = \langle 1, +\infty) \subset (0, \infty)$

$$\int (x \cdot \sqrt{x^2+1})\, dx = \left| \begin{aligned} u &= x^2+1 \\ u' &= 2x\,dx \end{aligned} \right|$$

$x \in \mathbb{R}$

$$= \frac{1}{2} \int \sqrt{u+1}\, du = \frac{2}{6} \cdot (u+1)^{\frac{3}{2}} + C = \frac{1}{3} \cdot (x^2+1)^{\frac{3}{2}} + C$$

$u > 0$

$$\int e^{4x+3}\,dx = \left|\begin{array}{l} u = 4x+3 \\ du = 4dx \end{array}\right.$$

$$\frac{1}{4}\int e^{u}\,du = \frac{1}{4}e^{4x+3}$$

Pr. 16145

$$\int \left(\frac{x}{x^4+1}\right)dx = \left|\begin{array}{l} u = x^2 \\ du = 2x\,dx \end{array}\right.$$

$$= \int \frac{\cancel{x}}{u^2+1}\cdot\frac{1}{2\cancel{x}}\,du = \frac{1}{2}\cdot\int\frac{1}{u^2+1}\,du = \frac{1}{2}\operatorname{arctg}(u)+C =$$

$$= \frac{1}{2}\operatorname{arctg}(x^2)+C$$

substituce

$$\int (\cos x \cdot e^{\sin x})\, dx = \left| \begin{matrix} y = \sin x \\ dy = \cos x\, dx \end{matrix} \right|$$

$$= \int e^y\, dy = e^y + C = e^{\sin x} + C$$

• $$\int x \cdot \sqrt{x^2+1}\, dx = \left| \begin{matrix} y = x^2+1 \\ dy = 2x\, dx \end{matrix} \right|$$

$$\frac{1}{2} \cdot \int 2x \cdot \sqrt{x^2+1}\, dx = \frac{1}{2} \int \sqrt{y} = \frac{1}{2} \cdot \frac{y^{\frac{3}{2}}+C}{\frac{3}{2}} = \frac{\sqrt{x^2+1}}{3} + C$$

• $$\int \left(\frac{x}{x^4+1}\right) dx = \left| \begin{matrix} y = x^2 \\ dy = 2x \end{matrix} \right| \frac{1}{2} \int \cdot 2x \frac{1}{x^4+1} = \frac{1}{2} \cdot \int \frac{1}{y^2+1} = \frac{1}{2} \operatorname{arctg} y + C =$$

$$= \frac{1}{2} \operatorname{arctg} x^2 + C$$

• $$\int \frac{\sin \ln x}{x}\, dx = \left| \begin{matrix} y = \ln x \\ dy = \frac{1}{x}\, dx \end{matrix} \right| \int \sin \ln x \cdot \frac{1}{x}\, dx$$

$$\int \sin(y)\, dy = -\cos(y) + C = -\cos(\ln(x)) + C$$

Př. 16147

modulace

• $$\int \sqrt{1-x^2}\, dx \overset{SUB.}{=} \left| \begin{matrix} x = \sin t \\ dx = \cos t\, dt \end{matrix} \right| \int \sqrt{1-\sin^2 t} \cdot \cos dt =$$

$$\int (\sqrt{\cos^2 t} \cdot \cos t)\, dt = \int \cos t \cdot \cos t\, dt = \int (\cos^2 t)\, dt =$$

$$\left| \begin{matrix} \cos t & \cos t \\ -\sin t & \sin t \end{matrix} \right| \overset{PP.}{=} \cos t \cdot \sin t + \int 1 - \cos^2 t\, dt$$

$$= \cos t \cdot \sin t + t - \int \cos^2 t\, dt \rightarrow \frac{1}{2} \cdot (\cos t \cdot \sin t + t) \ldots\ldots$$?

$\frac{1}{2} \cdot (\cos t \cdot \sin t + t)$

$x = \sin(t)$