Seq

Def.

Limita posloupnosti: a je limita posloupnosti $(a_n)_{n=1}^{\infty} \Leftrightarrow$

$$(\forall U_a)(\exists N \in \mathbb{N})(\forall n \in \mathbb{N})(n \geq N \Rightarrow a_n \in U_a)$$

f(x)

Def.

Limita funkce: b je limita funkce $f: A \to \mathbb{R}$, a je HM (hromadný bod) $\Leftrightarrow$

$$(\forall U_b)(\exists U_a)(\forall x \in (A \cap U_a \setminus \{a\})(f(x) \in U_b)$$

Věta — Heine

Limita $\lim_{x \to a} f(x) = b \Leftrightarrow a \in HM_{D_f}$ a pro každou $(x_n)_{n=1}^{\infty}$ s limitou a: $x_n \in D_f \setminus \{a\}$ i $\lim_{n \to +\infty} f(x_n) = b$

Věta — Složená funkce

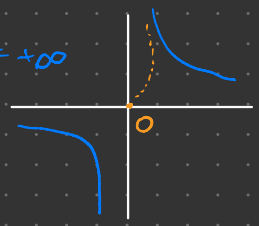
f, g jsou fce: $\lim_{x \to a} g(x) = b \Rightarrow \lim_{x \to b} f(x) = c$ i

a je HB: $(\exists U_a)(\forall x \in U_a \setminus \{a\})(g(x) \neq b) \vee (b \in D_f \wedge f(b) = c)$

$\Downarrow \lim_{x \to a} (f \circ g) = c$

② $\lim_{x\to 1} \frac{\sin(x)}{x} = \sin(1)$ → Buchen! $\lim_{x\to 0} \frac{\sin(x)}{x} = 1$

③ $\lim_{x\to 0+} \frac{1}{x} = +\infty$



④ $\lim_{x\to 0} x \cdot \ln(x)$ →

$\lim_{x\to 0+} x \ln(x) = \lim_{x\to 0+} \frac{\ln(x)}{\frac{1}{x}} \overset{l'H}{=} \lim_{x\to 0+} \frac{\frac{1}{x}}{-\frac{1}{x^2}} \overset{-x}{=} 0$

$\lim_{x\to 0-} x \ln(x)$ → nicht definiert

⑤ $\lim_{x\to+\infty} \frac{\sin(x)}{x} = 0$ ⇒

$-1 \le \sin(x) \le 1$

$-\frac{1}{x} \le \frac{\sin(x)}{x} \le \frac{1}{x} \quad x \to +\infty$

$0 \le \frac{\sin(x)}{x} \le 0$

⑥ $\infty - \infty$ = bah

$\lim_{x\to+\infty} 2x^4 - x^2 - 8x = \lim_{x\to+\infty} x^4 \left(2 - \frac{1}{x^2} - \frac{8}{x^3}\right) = \infty \cdot (2 - 0 - 0) = +\infty$

⑦ $\lim_{x\to+\infty} \operatorname{arctan}(e^x) = \frac{\pi}{2}$

↓

$e^x \overset{x\to+\infty}{\to} +\infty$

$\lim_{x\to+\infty} \operatorname{arctan}(x) = \frac{\pi}{2}$

(8)

$$\lim_{x\to+\infty} \ln\left(\frac{1}{x}\right) = \lim_{x\to+\infty} -\ln(x) = -\infty$$

(9)

$$\lim_{x\to+\infty} 2^{-e^{-x}} = -\lim_{x\to+\infty} e^{\ln(2)\cdot(-e^{-x})} = e^{0} = 1$$

$$\lim_{x\to+\infty} \ln(2)\cdot(-e^{-x}) = \ln(2)\cdot 0 = 0$$

$$\lim_{x\to+\infty} -\frac{1}{e^{x}} = 0$$

(10)

$$\lim_{x\to+\infty} \sqrt{9x^2+x} - 3x = \lim_{x\to+\infty} x\sqrt{9+\frac{1}{x}} - 3x = \lim_{x\to+\infty} x\left(\sqrt{9+\frac{1}{x}} - 3\right) = +\infty\cdot(3-3)$$

$$\lim_{x\to+\infty} \sqrt{9x^2+x} - 3x \cdot \frac{\sqrt{9x^2+x}+3x}{\sqrt{9x^2+x}+3x} = \lim_{x\to+\infty} \frac{9x^2+x-9x^2}{\sqrt{9x^2+x}+3x} = \lim_{x\to+\infty} \frac{1}{\sqrt{9+\frac{1}{x}}+3} = \frac{1}{3+3} = \frac{1}{6}$$

(11)

$+\infty\cdot 0 = nn$

$$\lim_{x\to+\infty} \sqrt{x}\,\sin\left(\frac{1}{\sqrt{x}}\right) = \lim_{x\to+\infty} \frac{\sin\left(\frac{1}{\sqrt{x}}\right)}{\frac{1}{\sqrt{x}}}$$

$$\lim_{x\to+\infty} \frac{1}{\sqrt{x}} = 0 \Rightarrow \lim_{x\to 0} \frac{\sin x}{x} = 1$$