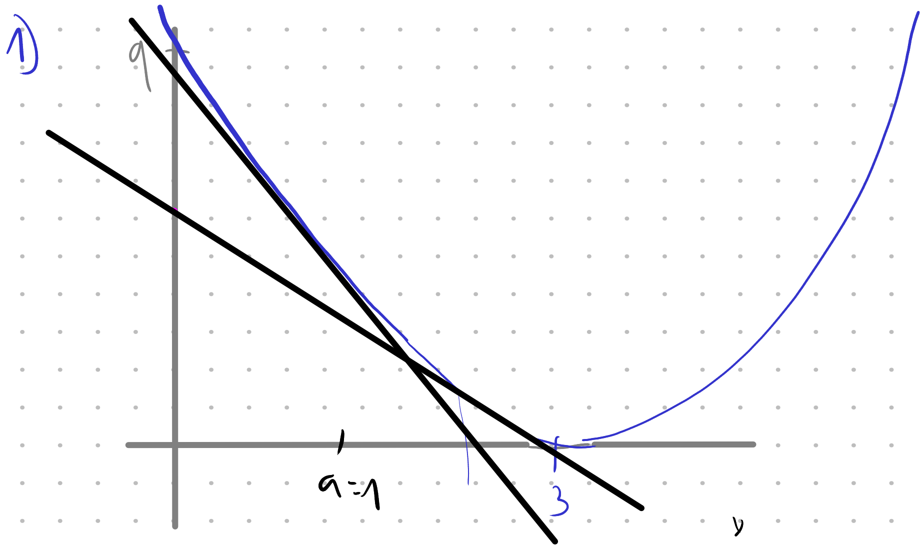
1)



$f(x) = (x-3)^2$

$\downarrow$

$f(a) = (a-3)^2$

$$S = \frac{x \cdot y}{2}$$

Tečna z $f(x) = y = f(a) + f'(a)\cdot(x-a)$; pro $a \in (0,3)$

$f'(a) = 2\cdot a - 6$

$y = (a-3)^2 + (2a-6)\cdot(x-a)$

$\downarrow$

$y=0$	$x=0$
$0 = (a-3)^2 + (2a-6)\cdot(x-a)$	$y = (a-3)^2 + (2a-6)\cdot(-a)$
$0 = \cancel{a^2} - \cancel{6a} + 9 + 2ax - 2a^2 - 6x + \cancel{6a}$	$y = a^2 - 6a + 9 - 2a^2 + 6a$
$0 = -a^2 + 2ax + 9 - 6x$	$y = -a^2 + \cancel{6a} - \cancel{6a} + 9$
$a^2 - 9 = x\cdot(2a-6)$	$y = -a^2 + 9$
$x = \frac{a^2-9}{2a-6} =$	$y = -(a^2 - 3^2) = -(a-3)\cdot(a+3)$
$x = \frac{\cancel{(a-3)}\cdot(a+3)}{2\cdot\cancel{(a-3)}}$	$y = -(a-3)(a+3)$
$x = (a+3)\cdot\frac{1}{2}$	

$$S(a) = \frac{(a+3)\cdot(-(a-3)\cdot(a+3))}{2\cdot 2} = \frac{(a+3)\cdot(a^2-9)}{-4}$$

$a \in$	$(0,1)$	$(1,3)$
	$+$	$-$

$\Downarrow$

$\boxed{\text{ostré lokální maximum pro } a = 1}$

maximum $S(a)$

$f'(a) = \left(-\frac{1}{4}\cdot(a+3)(a^2-9)\right)' = -\frac{1}{4}\left((a^2-9) + (a+3)\cdot 2a\right) =$

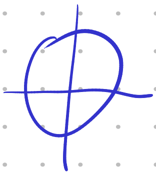
$= -\frac{1}{4}\cdot(a^2 - 9 + 2a^2 + 6a) = -\frac{1}{4}(3a^2 + 6a - 9) = -\frac{3}{4}\cdot(a^2 + 2a - 3) = -\frac{3}{4}\overset{+}{(a+3)}\overset{-}{(a-1)}$

2) $\lim_{x\to\frac{\pi}{2}} tg(x)\cdot(2x-\pi) = \lim_{x\to\frac{\pi}{2}} \frac{(2x-\pi)}{\frac{1}{tg(x)}} = \lim_{x\to\frac{\pi}{2}} \frac{2x-\frac{\pi}{2}}{\frac{\cos x}{\sin x}} = \lim_{x\to\frac{\pi}{2}} \frac{(2x-\frac{\pi}{2})\cdot\sin(x)}{\cos x} = l'H$ ✓

má smysl pro okolí $(\frac{\pi}{2}-1;\frac{\pi}{2}+1)$

$\frac{0}{0}$

existuje limita



$= \lim_{x\to\frac{\pi}{2}} \frac{2\sin x+(2x+\frac{\pi}{2})\cdot\cos x}{-\sin x} = \lim_{x\to\frac{\pi}{2}} \frac{2\sin x+2x\cdot\cos x+\frac{\pi}{2}\cdot\cos x}{-\sin x} = \lim_{x\to\frac{\pi}{2}} -2+\frac{2x\cos x+\frac{\pi}{2}\cos x}{-\sin x}$

$\neq 0$ ✓

$-2+\frac{0+0}{-1} = \underline{-2}$ ✓

3) $f(x)=3x^4\cdot 8x^3$

$f(-2)=3\cdot(-2)^4+8\cdot(-2)^3 = 3\cdot 16+8\cdot(-8)$
$= 48-64=-16$

$D_f=\mathbb{R}$

$f'(x)=12x^3+24x^2=12x^2\cdot(x+2)$ $D_{f'}=\mathbb{R}$

$f'(x)=0 \Rightarrow 12x^2\cdot(x+2)=0 \Rightarrow x=0 \lor x=-2$

$x\in$	$(-\infty,-2)$	$(-2,0)$	$(2,+\infty)$
$sgn(f'(x))$	$-$	$+$	$+$

$f(x)$ je ostře rostoucí na $\langle -2,0\rangle \cup \langle -2,\infty)$

$f(x)$ je ostře klesající na $(-\infty,-2\rangle$

ostré lokální minimum v bodě -2, $\underline{f(-2)=-16}$

4) $f(x)=(1-x)\cdot e^{-x}$ $\quad (1-2)\cdot e^{-2} = -e^{-2}$

$y=0$ $(1-x)\cdot e^{-x} \Rightarrow x=1$

$x=0 \Rightarrow e^{-x}$

$D_f=\mathbb{R} \Rightarrow$ není periodická, sudá, lichá

$f'(x)=(-1)\cdot e^{-x}+(1-x)\cdot e^{-x}\cdot(-1) = -e^{-x}-e^{-x}+x\cdot e^{-x} = -2e^{-x}+x\cdot e^{-x}$
$= \underline{e^{-x}\cdot(-2+x)}$

$f'(x)=0 \Rightarrow (-2+x)=0$
$x=2$ $\quad D_{f'}=\mathbb{R}$

$f''(x) = -e^{-x}\cdot(-2+x)+e^{-x}(1) = 3e^{-x}-e^{-x}\cdot x \rightarrow e^{-x}\cdot(3-x)$ $\quad D_{f''}=\mathbb{R}$

$f''(x)=0 \Rightarrow 2x-x^2=0$
$x\cdot(2-x)=0 \Rightarrow x=0 \lor x=2$

$-2-1$
$6-9$

$x\in$	$(-\infty,2)$	$(2,+\infty)$
$sgn(f'(x))$	$-$	$+$

$x\in$	$(-\infty,0)$	$(0,3)$	$(3,+\infty)$
$sgn(f''(x))$	$-$	$+$	$-$

$f(x)$ je ostře klesající na $(-\infty,2\rangle$

$f(x)$ je ostře rostoucí na $\langle 2,+\infty)$

$f(x)$ je konvexní na $\langle 0,3\rangle$

$f(x)$ je konkávní na $(-\infty,0\rangle \cup \langle 3,+\infty)$

$f(x)$ má ostré lokální minimum v bodě 2, $f(2)=-e^{-2}$

asymptoty:

nebude mít přímou asymptotu v 3 =>

$$\lim_{x\to 2^\pm} (1-x)e^{-x} = -1\cdot e^{-2} \neq \mp\infty \Rightarrow \text{není asymptota}$$

$$(1-x)\cdot e^{-x} = 0$$
$$e^{-x} - x e^{-x} = 0 \Rightarrow e^{-x}\cdot(1-x) = 0$$

v $\mp\infty$

$$k_\mp = \lim_{x\to\mp\infty} \frac{f(x)}{x} = \lim_{x\to\mp\infty} \frac{(1-x)\cdot e^{-x}}{x} = \frac{(1-x)}{x\cdot e^{x}} \overset{l'H}{=} \lim_{x\to\mp\infty} \frac{-1}{e^{x}}$$

$0^{\mp}$ $\frac{\mp\infty}{\mp\infty}$

$x\to+\infty$: 0

$x\to-\infty$: $-e^{\infty} = -\infty$ → není

$$q_- = \lim_{x\to\infty} f(x) - kx = 0$$

↓

asymptota v $+\infty$ => $y=0$