# Program Reasoning

## 3. Concepts in Program Verification

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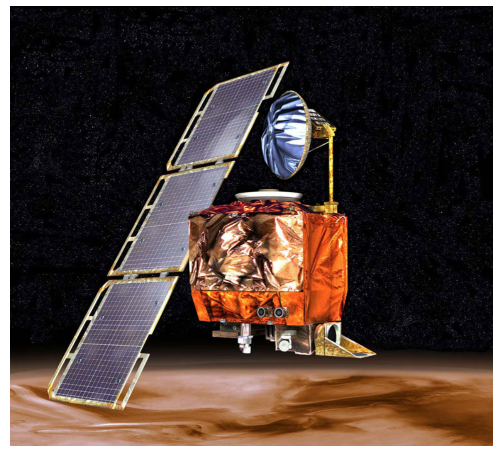
# Impact of Poor Software Quality

The Patriot Missile (1991)
Floating-point roundoff
28 soldiers died

The Ariane-5 Rocket (1996)
Integer Overflow
$100M



NASA's Mars Climate Orbiter (1999)
Meters-Inches Miscalculation
$125M

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By Anthony Spadafora November 23, 2019

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# Towards Error-free SW

*"Program testing can be used to show the **presence** of bugs, but **never** to show their **absence**!"*

**- Edsger W. Dijkstra, 1970**

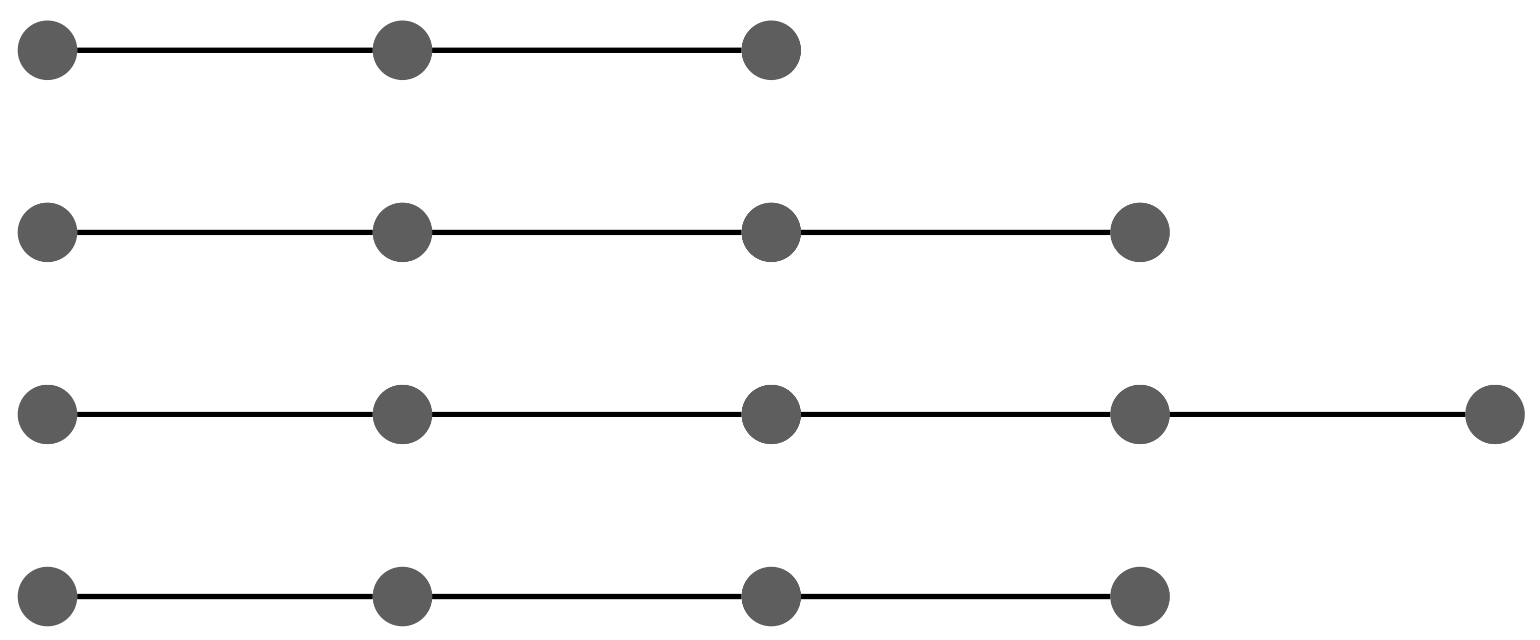
# Properties

- Points of interest in programs
  - for verification, bug detection, optimization, understanding, etc
  - E.g.,“p == NULL?”, “idx < size?”, “fp can be only f, g, or h?”, “value of x”, etc
- Two categories:
  - Trace properties = properties of individual execution traces
    - safety properties + liveness properties
  - Information-flow properties = properties of multiple execution traces

# Trace

- Trace = a list of states
- Recall small-step operational semantics
- A program can have a set of (infinite) set of traces
- $[\![P]\!]$ : a set of all possible execution traces

$$
\begin{aligned}
&(2 \times 2 \times 2) \times (2+1) \\
&\rightarrow (4 \times 2) \times (2+1) \\
&\rightarrow 8 \times (2+1) \\
&\rightarrow 8 \times 3 \\
&\rightarrow 24
\end{aligned}
$$



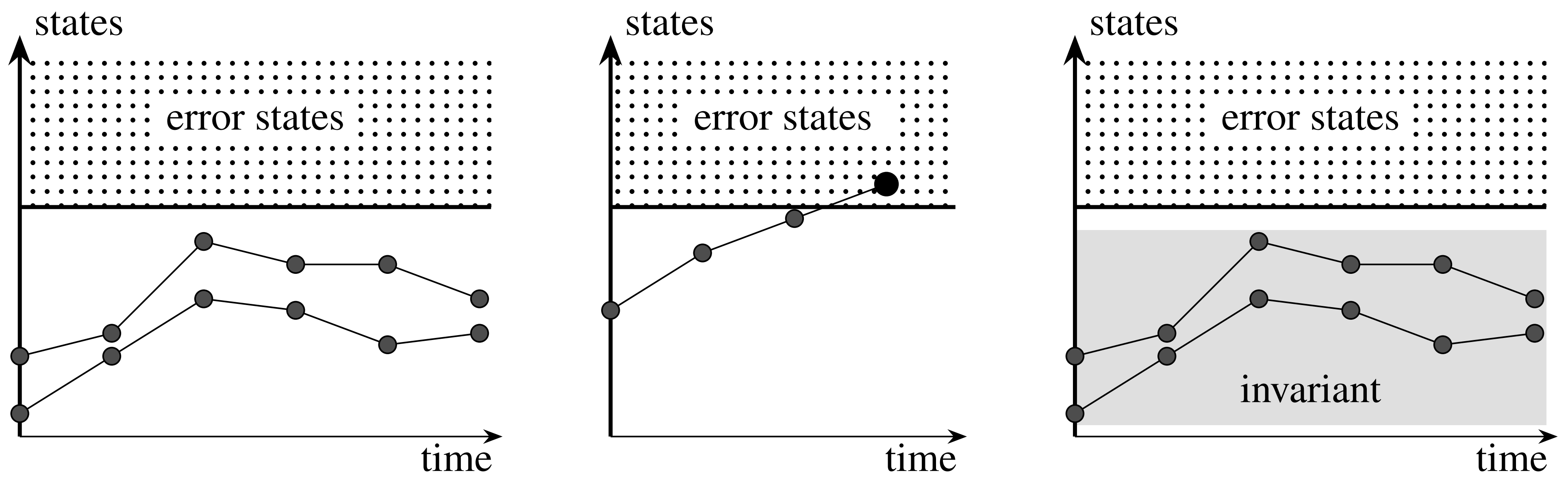
...

# Trace Properties

- A semantic property $\mathscr{P}$ that can be defined by a **set of execution traces** that satisfies $\mathscr{P}$
  - Ex1: “all traces that satisfies x != 0 at line 10”
  - Ex2: “all traces where the value of y at line 97 is the same as the one in the entry point”
- Program $P$ satisfies property $\mathscr{P}$ iff $[\![P]\!] \subseteq T_{\mathscr{P}}$
- State properties: defined by a set of states (so, obviously trace properties)
  - E.g., division-by-zero, integer overflow
- Any trace property: the conjunction of a safety and a liveness property

# Safety Property

- A program **never** exhibit a behavior observable within **finite time**
    - "Bad things will never occur"
    - Bad things: integer overflow, buffer overrun, deadlock, etc
- If false, then there exists a **finite counterexample**
- To prove: all executions never reach error states




(a) Correct executions (b) An incorrect execution (c) Proof by invariance

# Invariant

- Assertions supposed to be **always true**
  - Starting from a state in the invariant, any computation step also leads to another state in the invariant (i.e., fixed point!)
  - E.g., “x has an int value during the execution”, “y is larger than 1 at line 5”
- Loop invariant: assertion to be true at the beginning of every loop iteration

```
x = 0;
while (x < 10) {
  x = x + 1;
}
assert(x > 0);
assert(x == 10);
```

Loop invariant 1: “x is an integer”

Loop invariant 2: “x >= 0”

Loop invariant 3: “0 <= x <= 10”

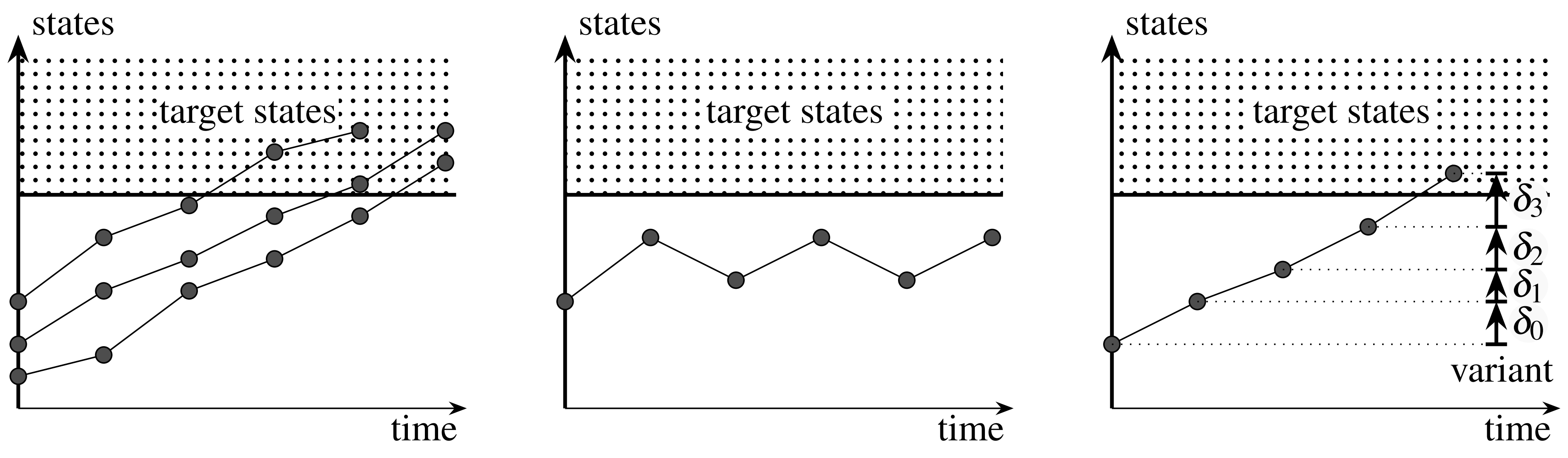
# Example: Division-by-Zero

```c
int main(){
  int x = input();
  x = 2 * x - 1;
  while (x > 0) {
    x = x - 2;
  }
  assert(x != 0);
  return 10 / x;
}
```

```c
int main(){
  int x = input();
  x = 2 * x;
  while (x > 0) {
    x = x - 2;
  }
  assert(x != 0);
  return 10 / x;
}
```

# Liveness Property

- A program will **never** exhibit a behavior observable only after **infinite time**
  (A program will **eventually** exhibit a behavior observable within **finite time**)
  - “Good things will eventually occur”
  - Good things: termination, fairness, etc
- If false then there exists an **infinite counterexample**
- To prove: all executions eventually reach target states




(a) Correct executions (b) An incorrect execution (c) Proof by variance

# Variant

- A quantity that **evolves towards** the set of target states
  (so guarantee any execution eventually reach the set)
- Usually, a value that is strictly decreasing for some well-founded order relation
  - Well-founded order: there exists a minimal element
  - E.g., an integer value is always positive and strictly decreasing

```
x = pos_int();
while (x > 0) {
  x = x - 1;
}
```

**x is always a positive integer** $\wedge$ **x is strictly decreasing** $\Rightarrow$ **The program terminates**

# Example: Termination

- Introduce variable c that stores the value of "step counter"
  - Initially, c is equal to zero
  - Each program execution step increments c by one

```
// A factorial program
i = n;
r = 1;
while (i > 0) {
  r = r * i;
  i = i - 1;
}
```

```
// An instrumented program
i = n;
r = 1;
c = 2;
while (i > 0) {
  r = r * i;
  i = i - 1;
  c = c + 3;
}
// what is the value of c in the loop?
```

c <= 3n + 2

**0 <= 3n + 2 - c ∧ 3n + 2 - c is strictly decreasing ⇒ termination**

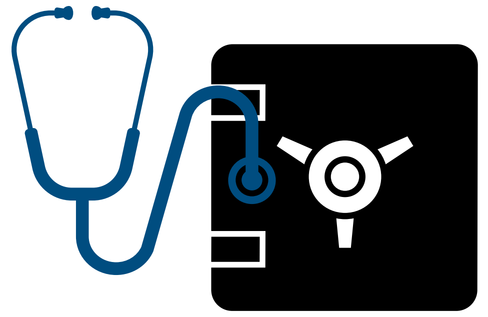
# Example

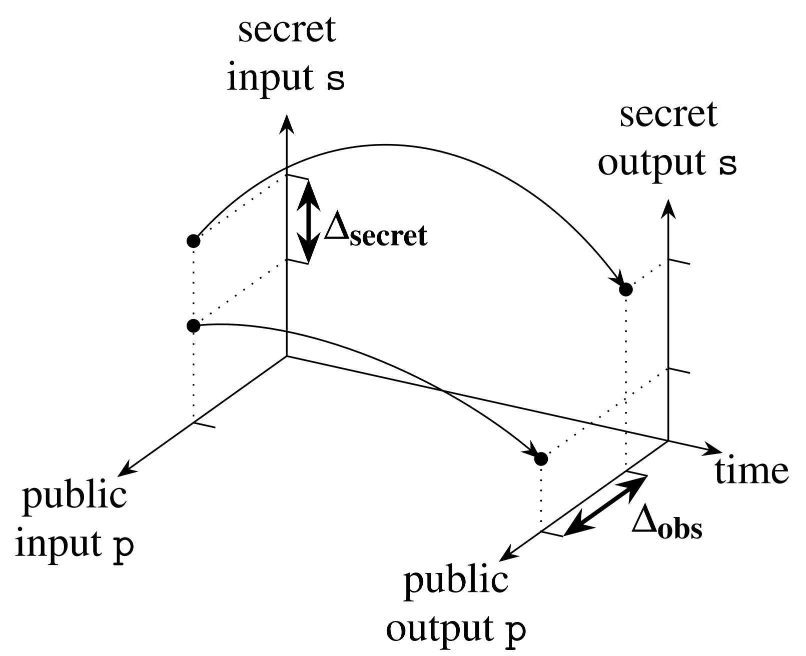
- Correctness of a sorting algorithm as trace property

| Property | Safety or Liveness? | State? |
|---|---|---|
| Should not fail with a run-time error | | |
| Should terminate | | |
| Should return a sorted array (if terminated) | | |
| Should return an array with the same elements and multiplicity (if terminated) | | |

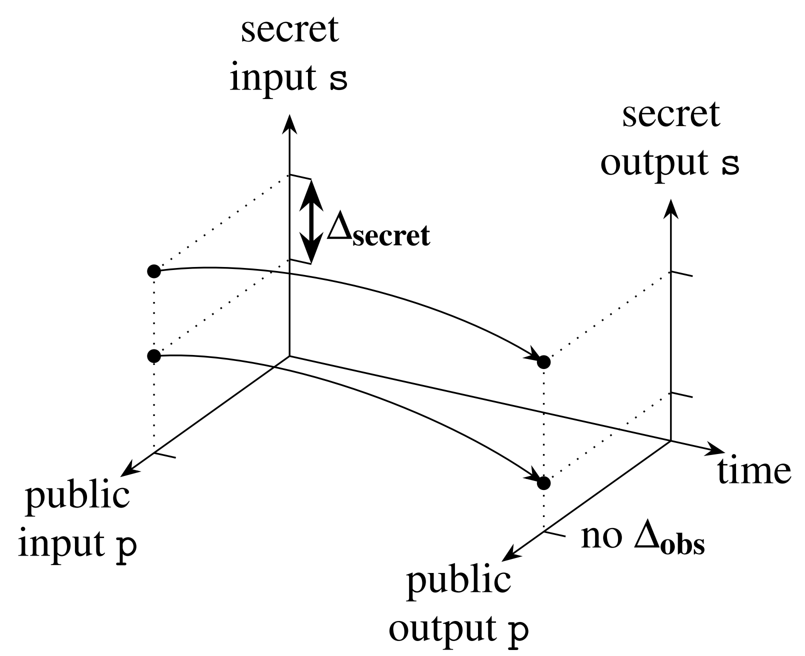
# Information Flow Properties

- Properties stating the absence of dependence between **pairs of executions**
  - Beyond trace properties: so called **hyper-properties**
- Mostly for security: multiple executions with public data should not derive private data
- E.g., a door lock beeps louder if a right digit is pressed at the right position






A pair of executions with insecure information flow




A pair of executions without insecure information flow

# Example

- Assume that variables s (secret) and p (public) take only 0 and 1

```
// Program 0
p_out := p_in * [0, 1]
```

```
// Program 1
p_out := p_in * s * [0, 1]
```

```
// Program 2
p_out := p_in + [0, 1] - s
```

| Input | | Output |
|---|---|---|
| p | s | p |
| 0 | 0 | {0, 1} |
| 0 | 1 | {0, 1} |
| 1 | 0 | {0, 1} |
| 1 | 1 | {0, 1} |

| Input | | Output |
|---|---|---|
| p | s | p |
| 0 | 0 | {0} |
| 0 | 1 | {0} |
| 1 | 0 | {0} |
| 1 | 1 | {0, 1} |

| Input | | Output |
|---|---|---|
| p | s | p |
| 0 | 0 | {0, 1} |
| 0 | 1 | {0, 1} |
| 1 | 0 | {0, 1} |
| 1 | 1 | {0, 1} |

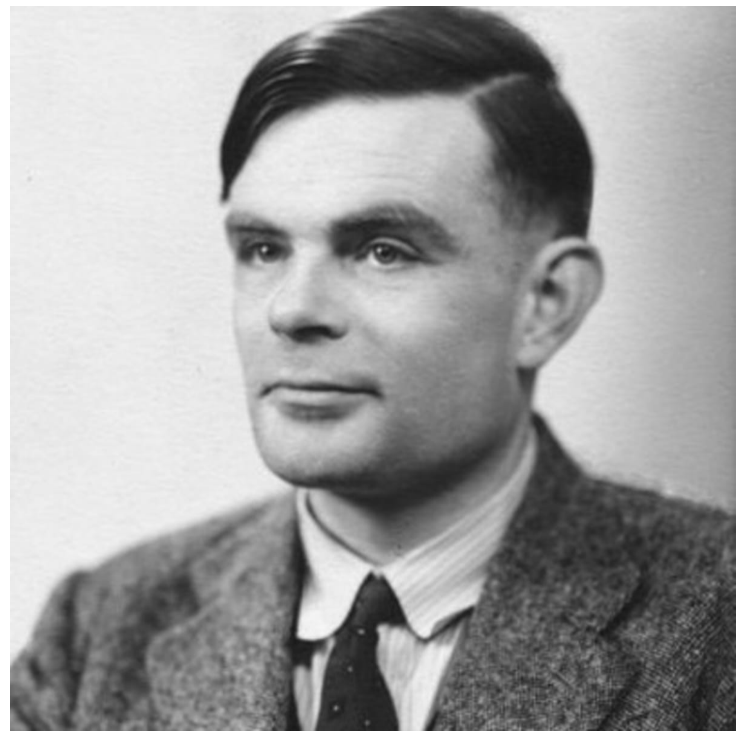
# A Hard Limit: Undecidability

**Theorem (Rice's theorem).** Any non-trivial semantic properties are undecidable.

- Non-trivial property: worth the effort of designing a program analyzer for
  - trivial: true or false for all programs
- Undecidable? If decidable, it can solves the Halting problem!

HP: Given a Turing machine $T$ and an input $i$, does $T$ eventually halt on $i$?

Undecidable: There is no Turing machine that can solve HP!



# Informal Proof of Undecidability of HP

HP: Given a Turing machine $T$ and an input $i$, does $T$ eventually halt on $i$?

- Assume $H(T, i)$ returns true or false
- Let $F(x) = \text{if } H(x, x) \text{ then } \texttt{loop}() \text{ else } \texttt{halt}()$
- Does $F(F)$ terminate?

# Informal Proof of Rice's Theorem

- Assumption: HP is undecidable
- An analyzer **A** for a property: "*This program always prints 1 and finishes*"
- Given a program **P**, generate **P' =** "**P**; print 1;"
- Analyze **P'** using **A: A(P')**
  - **A(P')** says "Yes": **P** halts,
  - **A(P')** says "No": **P** does not halt
- HP is decidable if we use **A** : contradiction!

# Toward Computability

**Undecidable**

**⇒ Automatic, terminating, and exact reasoning is impossible**

**⇒ If we give up one of them, it is computable!**

- **Manual** rather than **automatic**: assisted proving
    - require expertise and manual effort
- **Possibly nonterminating** rather than **terminating**: model checking, testing
    - require stopping mechanisms such as timeout
- **Approximate** rather than **exact**: static analysis
    - report spurious results

# Soundness and Completeness

- Given a semantic property $\mathscr{P}$, and an analysis tool $\mathbf{A}$

- If $\mathbf{A}$ were perfectly accurate,

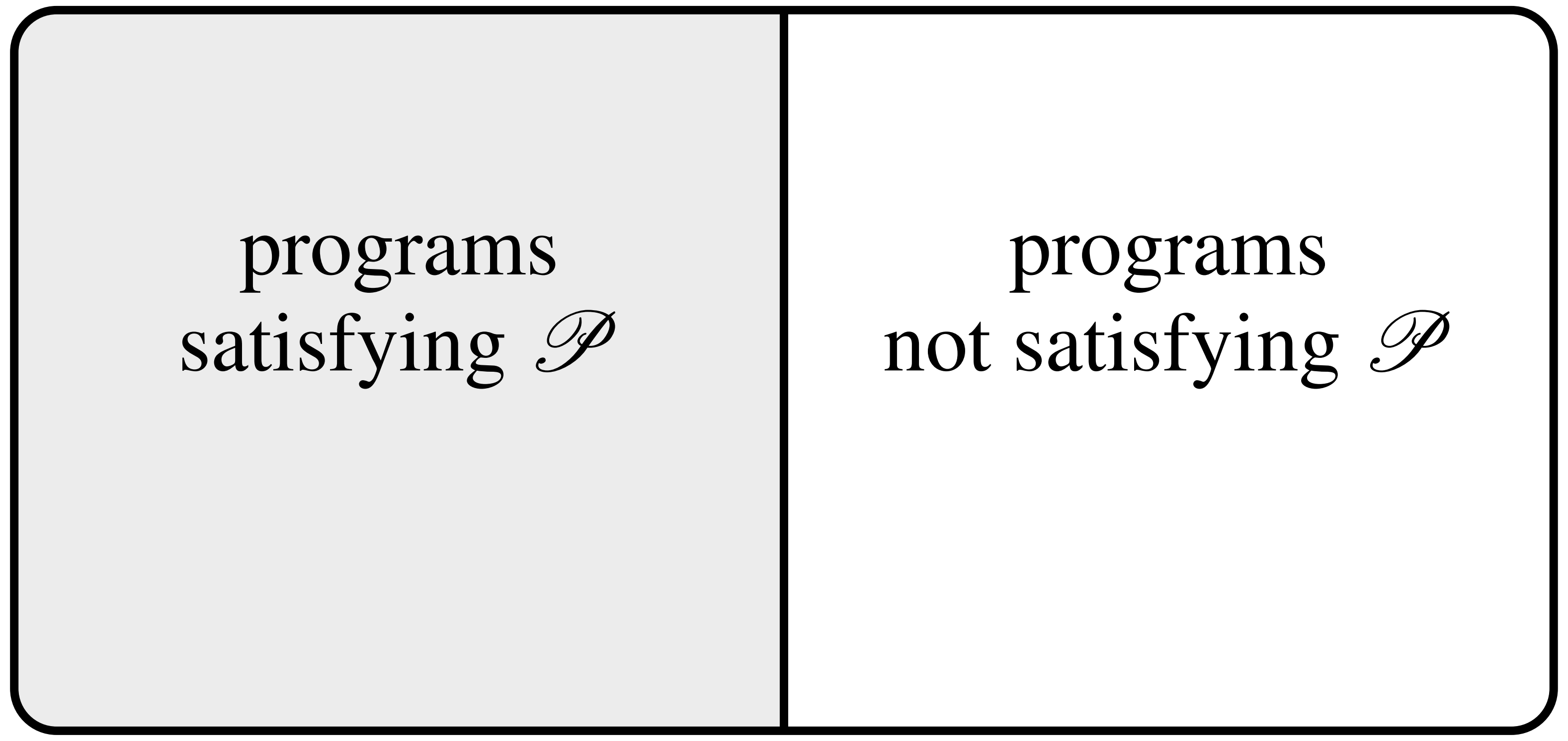
$$\text{For all program p, } \mathbf{A}(\text{p}) = \mathbf{true} \iff \text{p satisfies } \mathscr{P}$$

which consists of

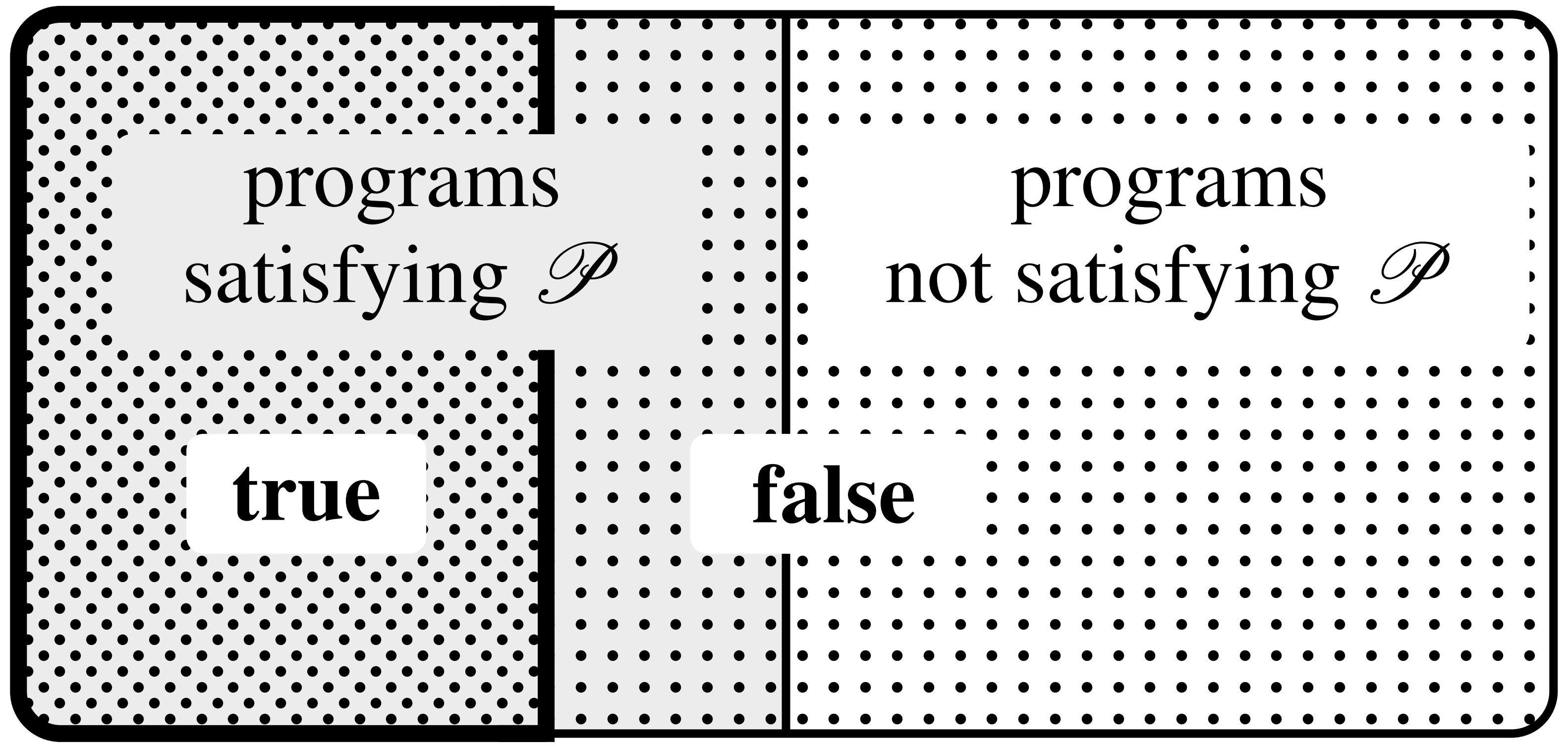
$$\text{For all program p, } \mathbf{A}(\text{p}) = \mathbf{true} \Rightarrow \text{p satisfies } \mathscr{P} \qquad \textbf{(soundness)}$$

$$\text{For all program p, } \mathbf{A}(\text{p}) = \mathbf{true} \Leftarrow \text{p satisfies } \mathscr{P} \qquad \textbf{(completeness)}$$

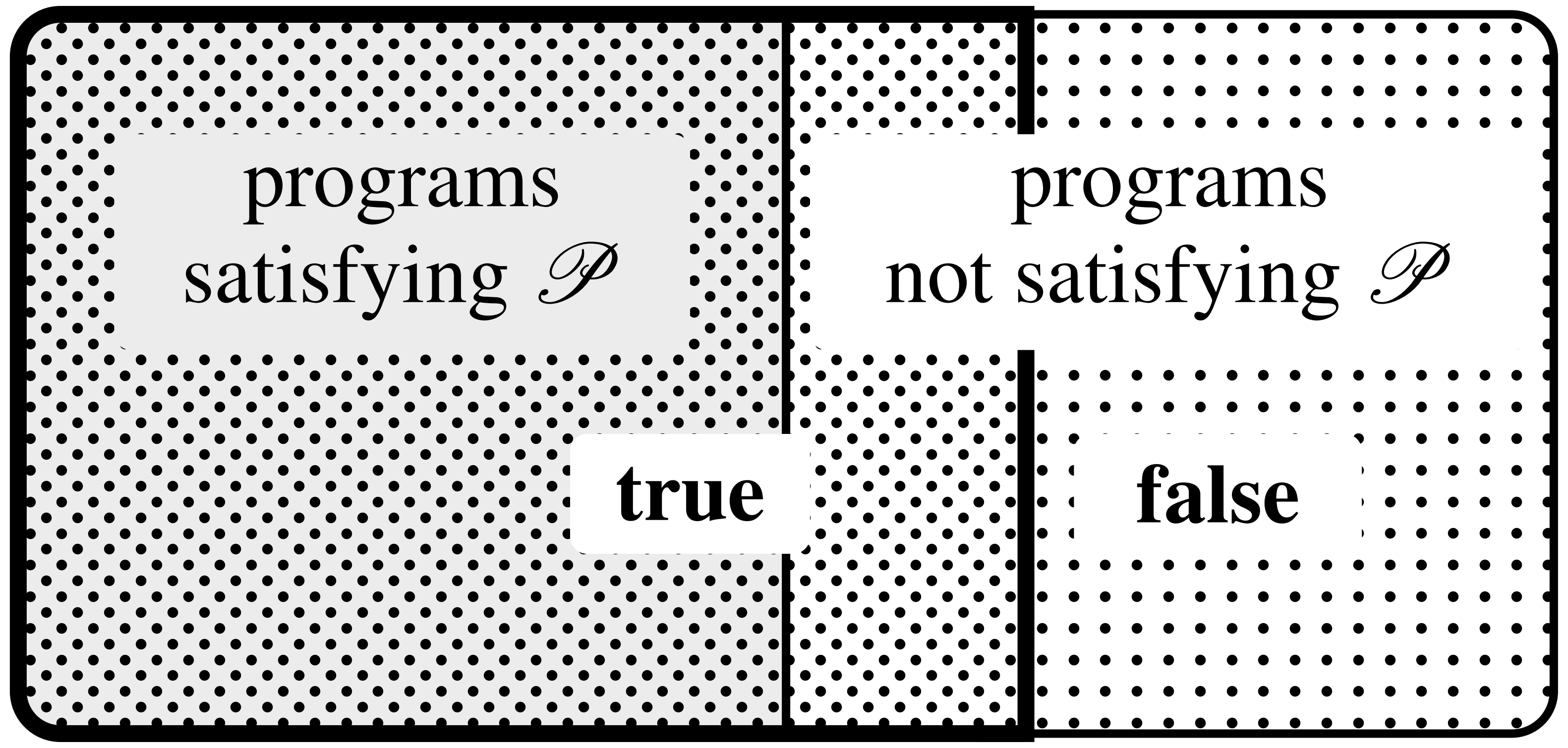
# Soundness and Completeness



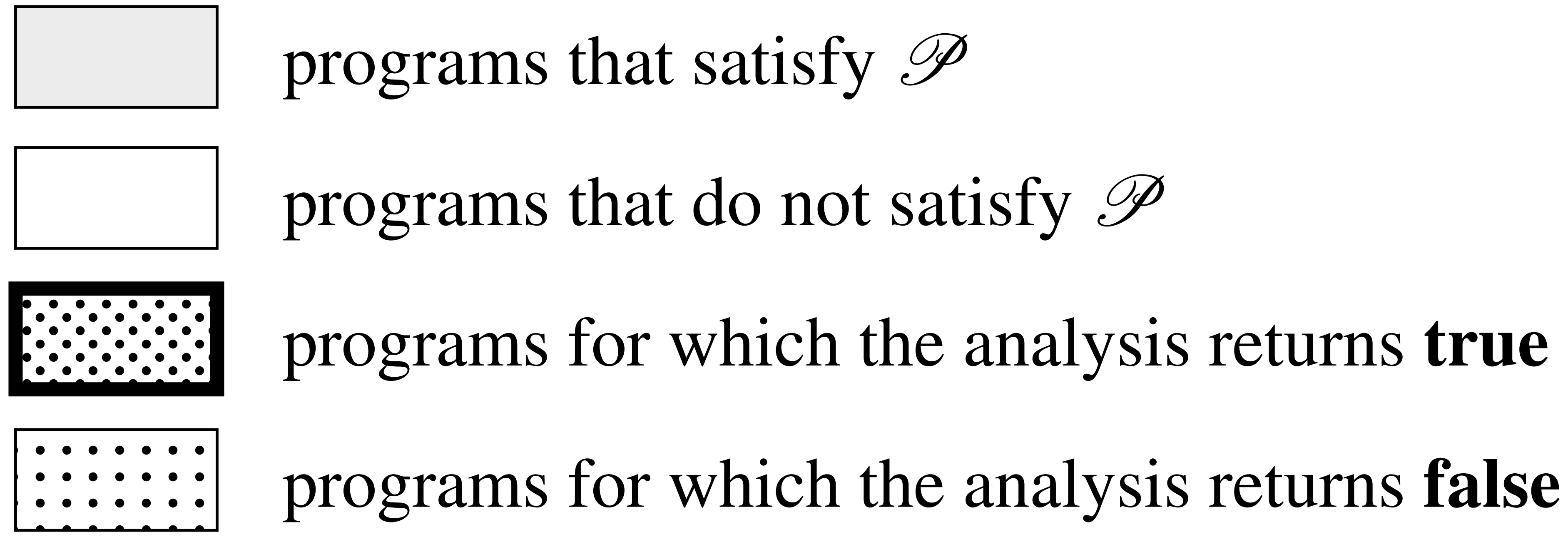

(a) Programs




(b) Sound, incomplete analysis




(c) Unsound, complete analysis




(d) Legend

# Program Verification

- Prove a given program satisfies the target properties
    - Loop invariants provided by the user or another program analyzer
- **Sound and complete** if a “good” invariant is provided
- **Sound and incomplete** if an imprecise invariant is provided
- **Sound, complete, yet non-terminating** if the invariant generation does not terminate
- How to describe the target property (specification)?
- How to prove the target property (specification)?

**A: Program Logic**

# Summary

- Property: point of interest in a program (safety, liveness, information flow, etc)
- Program verification: check whether a property is satisfied or not
- Hard limit of program analysis: generally undecidable problem
- Practical solutions:
    - **Manual** rather than **automatic**
    - **Possibly nonterminating** rather than **terminating**
    - **Approximate** rather than **exact**