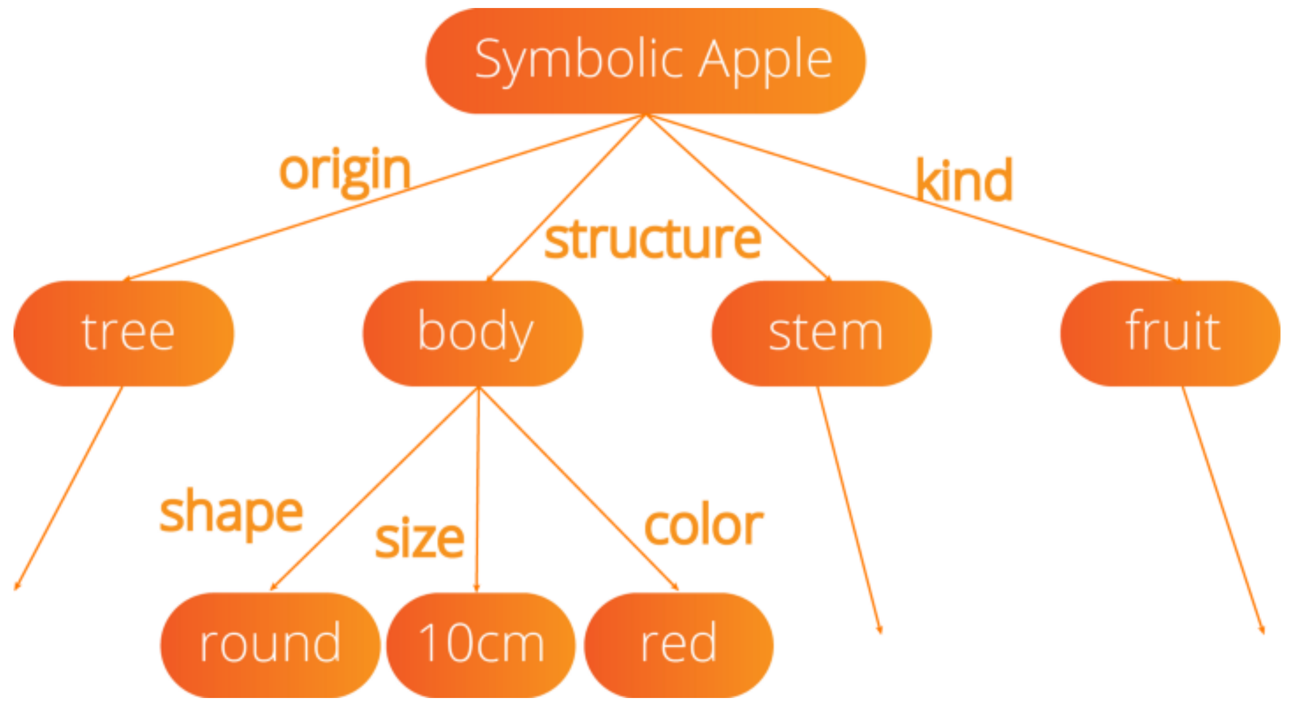
Program Reasoning

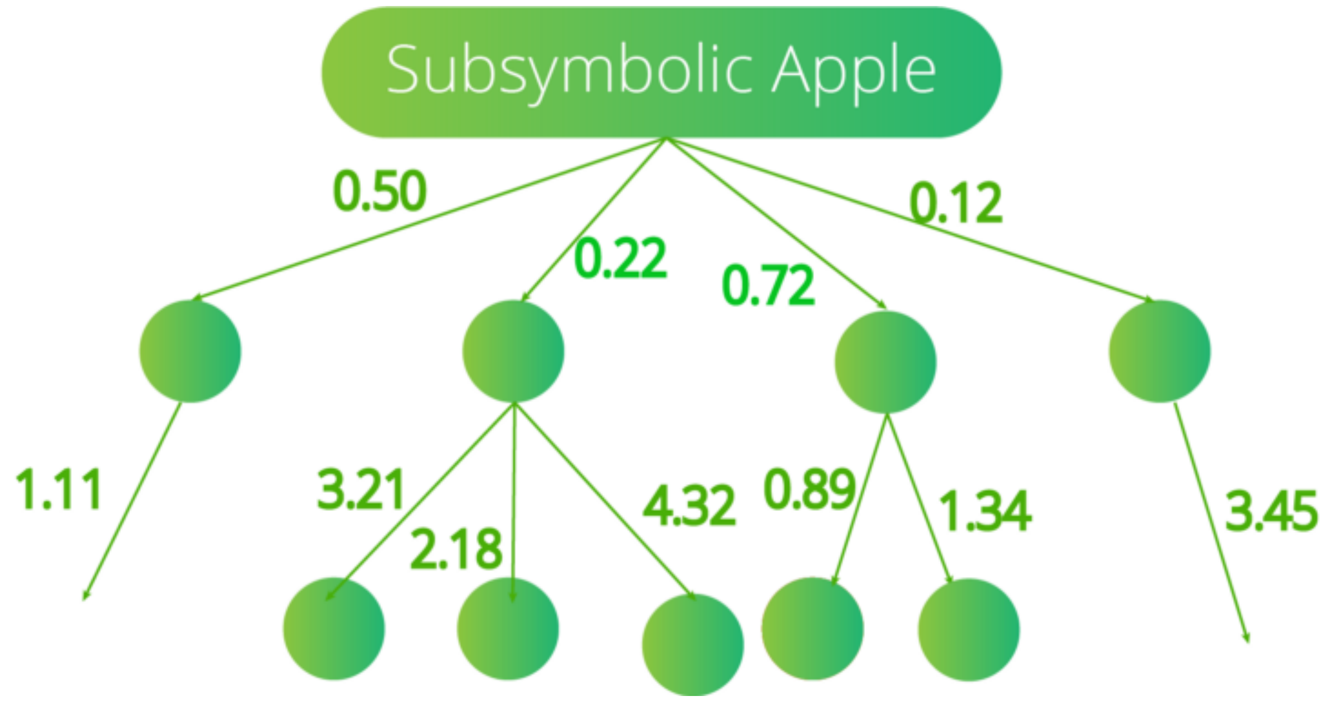
16. Neuro-Symbolic AI

Kihong Heo

KAIST

Two Major Camps of AI





- Symbolist
- 1950s - 1980s
- High-level symbolic representation
- Human-readable & interpretable
- Logic- & search- based

- Connectionist
- 1980s - present
- Low-level neural representation
- Efficient learning
- Differentiation-based

Artificial Ignorance (Then)

by Copilot

by Copilot

by Dall-E

by Dall-E

Artificial Ignorance (Now)

The hidden dangers of AI coding

How AI-generated code is changing cybersecurity — and what developers and “vibe coders” should expect.

Stan Kaminsky

October 10, 2025

AI • CODING

An AI-powered coding tool wiped out a software company’s database, then apologized for a ‘catastrophic failure on my part’

By Beatrice Nolan

Tech Reporter

July 23, 2025, 7:22 AM ET

Add us on G

Daniel Stenberg

curl CEO. Code Emitting Organism

6개월 · 수정됨

That's it. I've had it. I'm putting my foot down on this craziness.

1. Every reporter submitting security reports on #Hackerone for #curl now needs to answer this question:

"Did you use an AI to find the problem or generate this submission?"

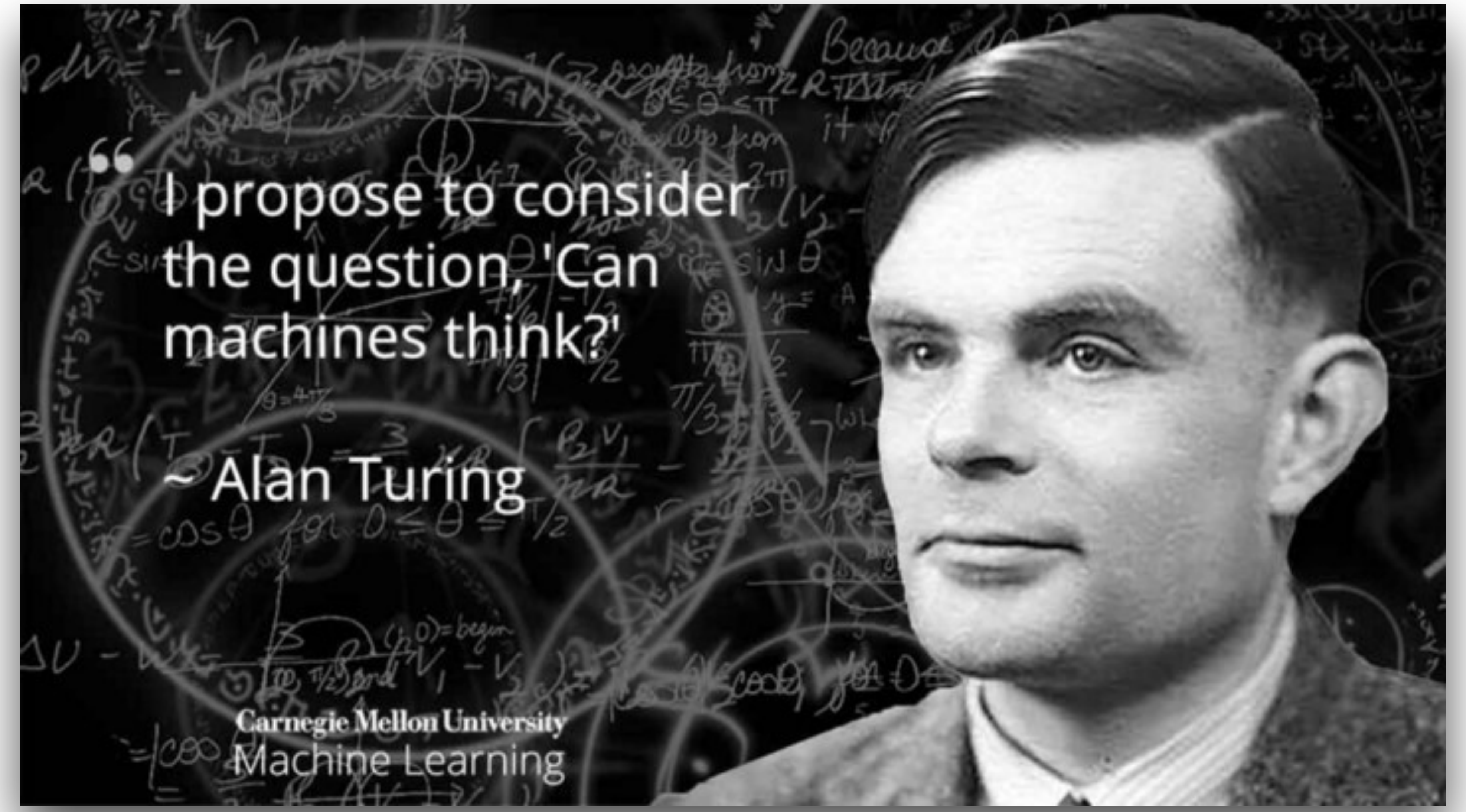
(and if they do select it, they can expect a stream of proof of actual intelligence follow-up questions)

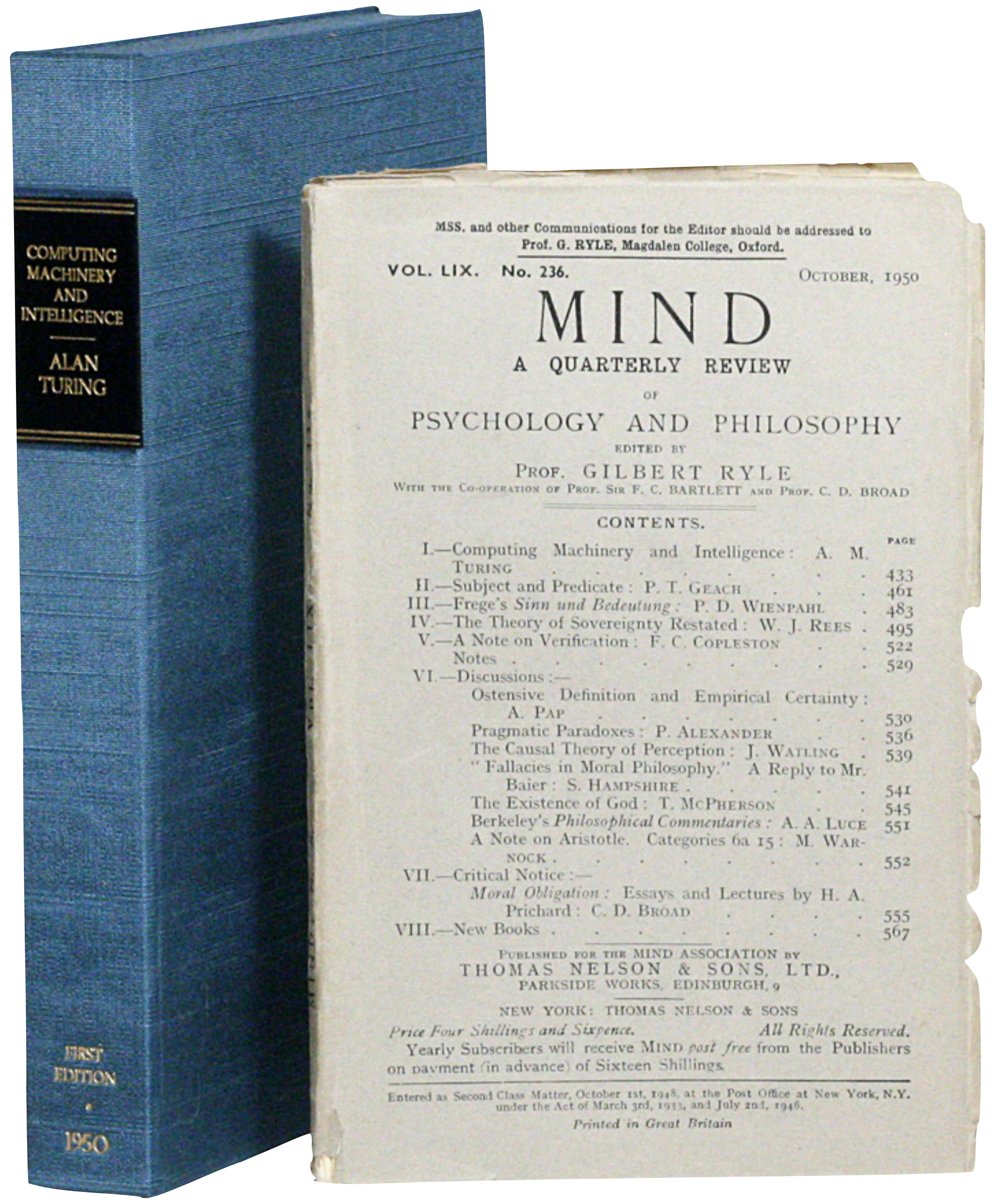
2. We now ban every reporter INSTANTLY who submits reports we deem AI slop. A threshold has been reached. We are effectively being DDoSed. If we could, we would charge them for this waste of our time.

We still have not seen a single valid security report done with AI help.

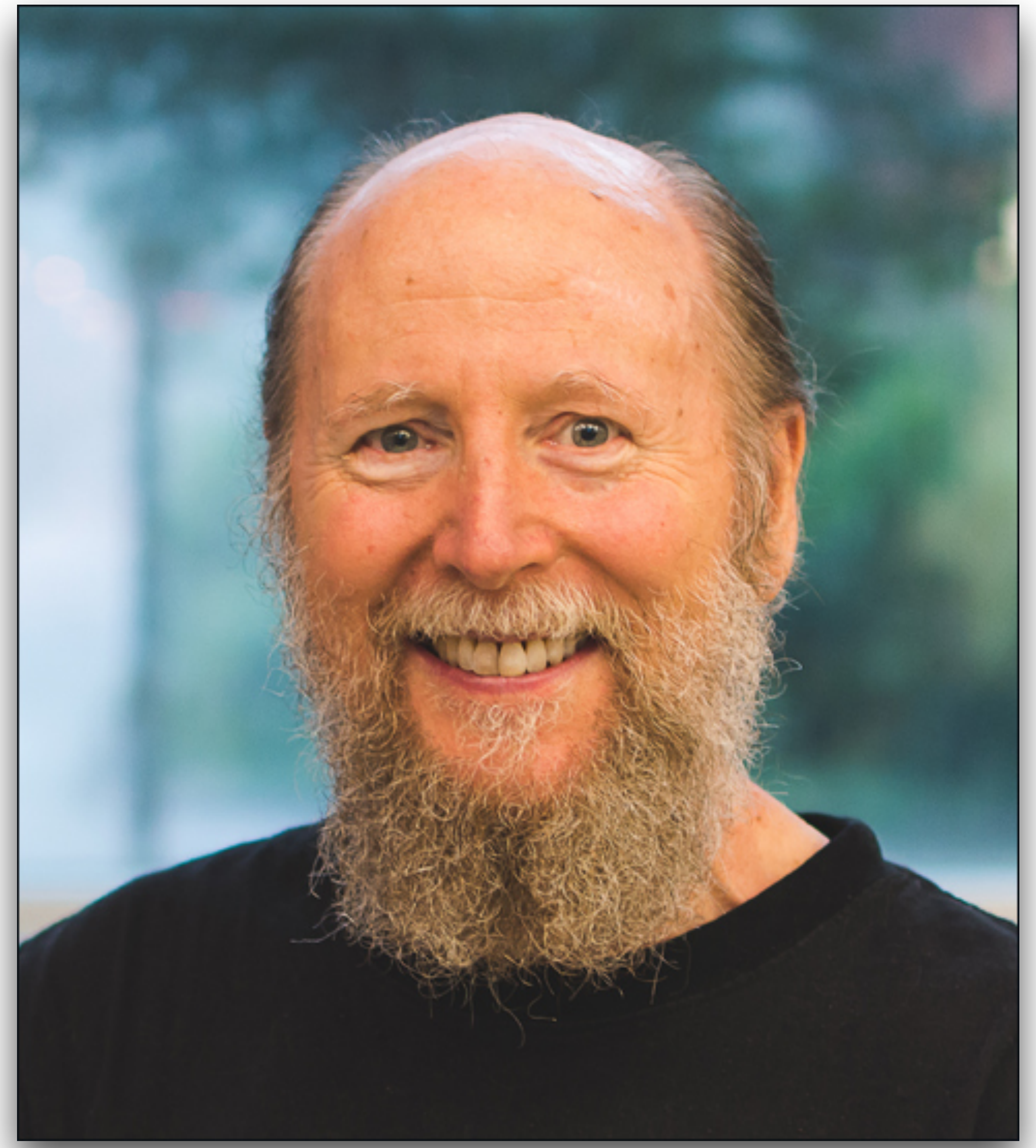
Program Synthesis & AI

- Turing's dream
- Program: explainable & executable & verifiable
- Future of AI/SW/CS?





Program Verification & AI



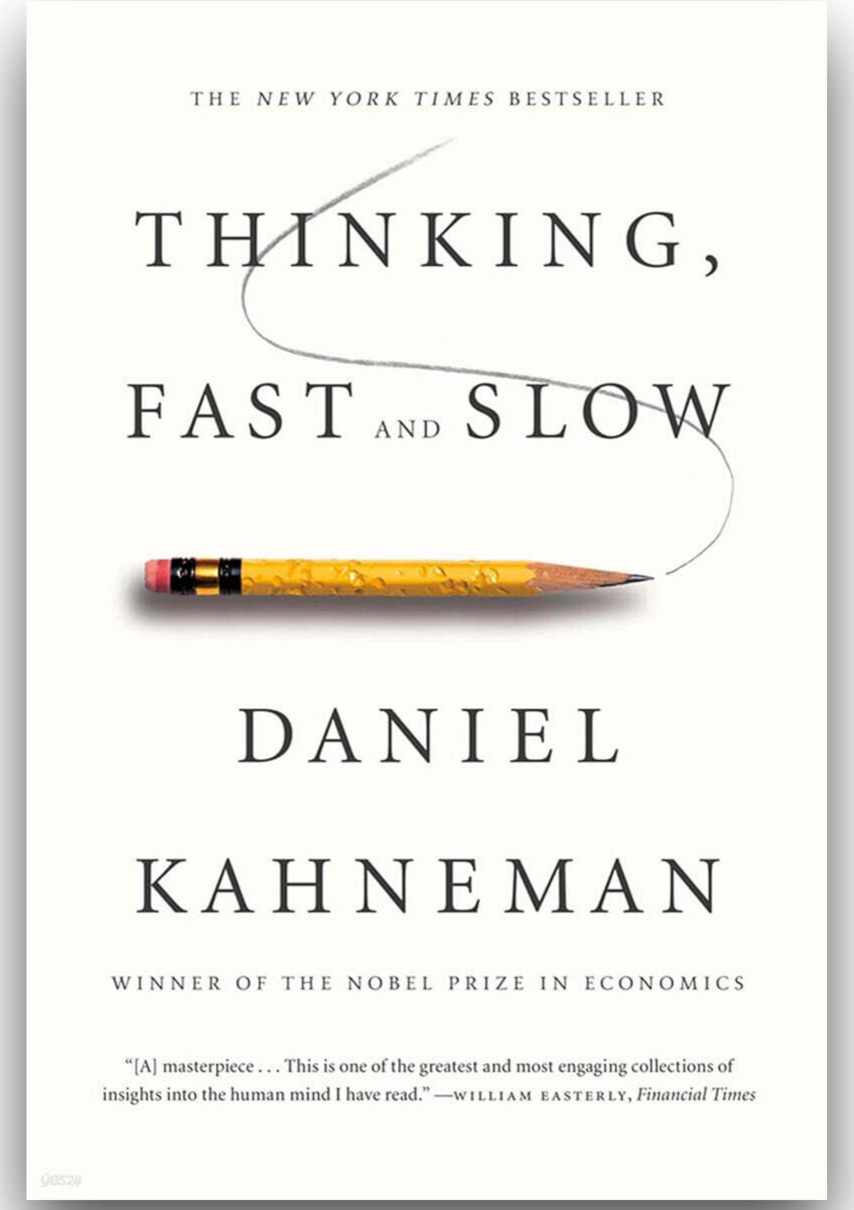
Richard Sutton
2024 Turing Award Winner

"An AI system can create and maintain knowledge only to the extent that it can verify that knowledge itself."

- "Verification, The Key to AI", 2001

Then, what is our mission?

Neuro-Symbolic AI



- What is the future of AI (or programming)?
  - Combining the strengths of the two camps
- Two directions
  - Symbolic engine + neural guide: Program synthesis using numerical optimization
  - Neural engine + symbolic guide: Constrained decoding

Inductive Logic Programming (ILP)

- A subfield of symbolic artificial intelligence
- Goal: given a dataset (inductive), infer a set of rules (logic programming)
- Synthesizing programs in logic programming languages
  - E.g., Prolog, Datalog

Datalog Programs

- A set of Horn clause rules ($X_1 \wedge X_2 \wedge \cdots \wedge X_n \rightarrow H$)
- Input & output: a set of tuples
- Applications: big data analysis, network protocol, program analysis, etc

```
edge(1,2)
edge(2,3)
edge(3,4)
```

```
path(X,Y) :- edge(X,Y)
path(X,Y) :- edge(X,Y) & path(X,Y)
```

```
path(1,2)  path(1,3)
path(1,4)  path(2,3)
path(2,4)  path(3,4)
```

Example

- Parent

Dataset

```
parent(a,b)    parent(a,c)    parent(d,b)
father(a,b)    father(a,c)    mother(d,b)
male(a)        female(c)      female(d)
```

Rules

```
father(X,Y) :- parent(X,Y) & male(X)
mother(X,Y) :- parent(X,Y) & female(X)
```

- Transitive closure

Dataset

```
edge(1,2)    edge(2,3)    edge(3,4)
path(1,2)    path(1,3)    path(1,4)
path(2,3)    path(2,4)    path(3,4)
```

Rules

```
path(X,Y) :- edge(X,Y)
path(X,Y) :- edge(X,Z) & path(Z,Y)
```

Datalog Program Synthesis

- Given a set of candidate rules, find a subset that is consistent with a given set of examples
  - A typical combinatorial optimization problem (i.e., NP-hard)
- In this lecture, we assume a set of candidate rules is predefined

```
edge(1,2)
edge(2,3)
edge(3,4)
```

I

```
✓ path(x,y) :- edge(x,y).
✗ path(x,x) :- edge(x,y).
✓ path(x,z) :- edge(x,y), path(y,z).
✗ path(x,y) :- path(y,x).
                  …
```

O^+

```
path(1,2)  path(1,3)
path(1,4)  path(2,3)
path(2,4)  path(3,4)
```

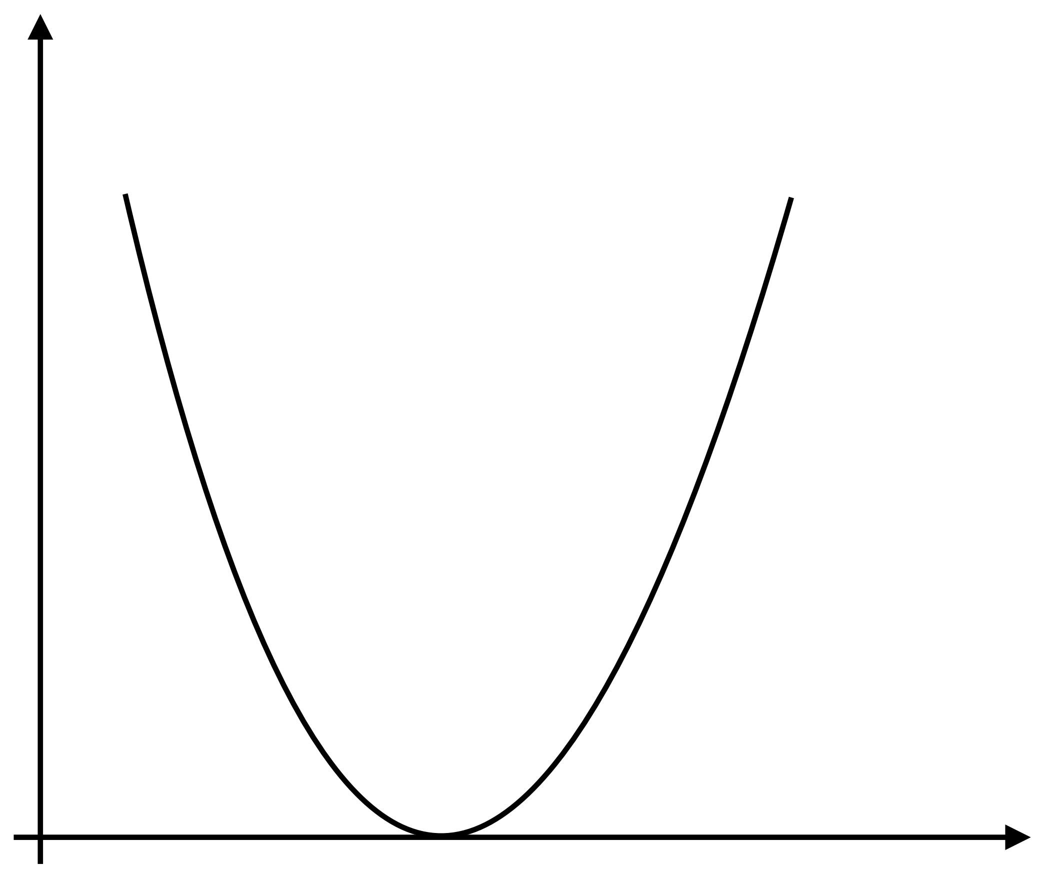
O^-

```
path(2,1)  path(3,1)
path(4,1)  path(4,2)
```

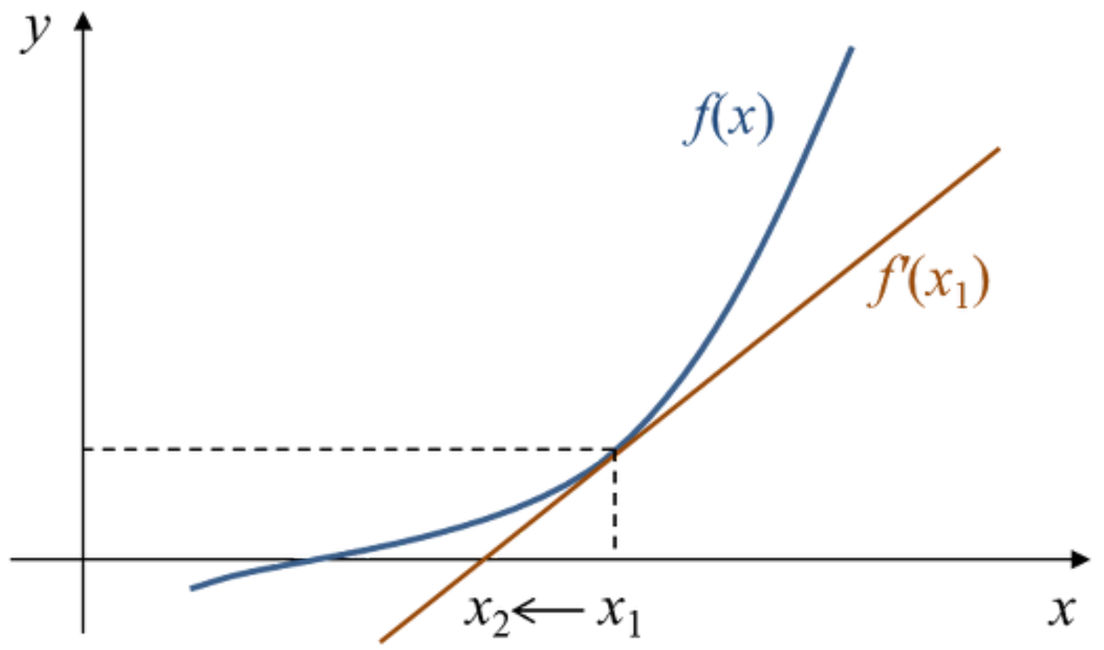
Challenges

- Huge search space
  - E.g., # of possible combinations of 50 candidate rules?
- If a wrong rule is chosen? The program produces a wrong tuple
- If a correct rule is missed? The program does not produce a correct tuple
- That is, how to find a point where the "loss" is zero?
  - Loss: the amount of derived wrong tuples + underived correct tuples
- Can the synthesizer learn from failures?

Analogies

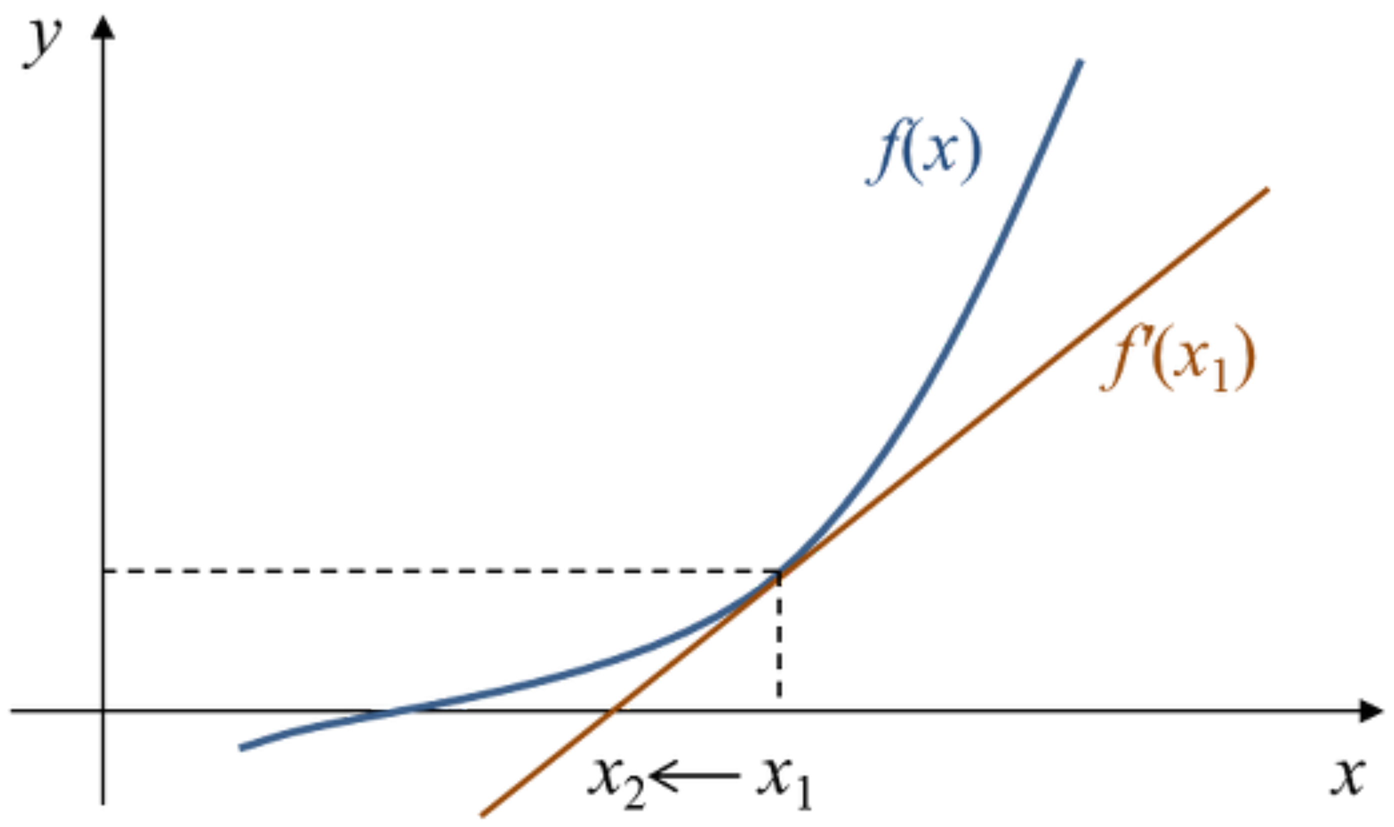


- In other fields of mathematics, how to find a zero point?
- Enumerative approach: general but not inefficient
  - E.g., 0, 1, 2, 3, 4…
- Algebraic approach: efficient but not general
  - E.g., $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ for quadratic equation, but ≥ 5-degree?
- Approximative approach: moderately efficient and general
  - E.g., Newton's method



A Solution: Difflog*

- A Datalog synthesis algorithm using numerical optimization
- Key idea: solving combinatorial optimization via numerical optimization
- Why numerical optimization? Many powerful algorithms exist!
  - E.g., Newton's method for differentiable loss functions
- Problem: Datalog programs are not differentiable



*Si et al., Synthesizing Datalog Programs using Numerical Relaxation, IJCAI 2019

Idea 1: Provenance

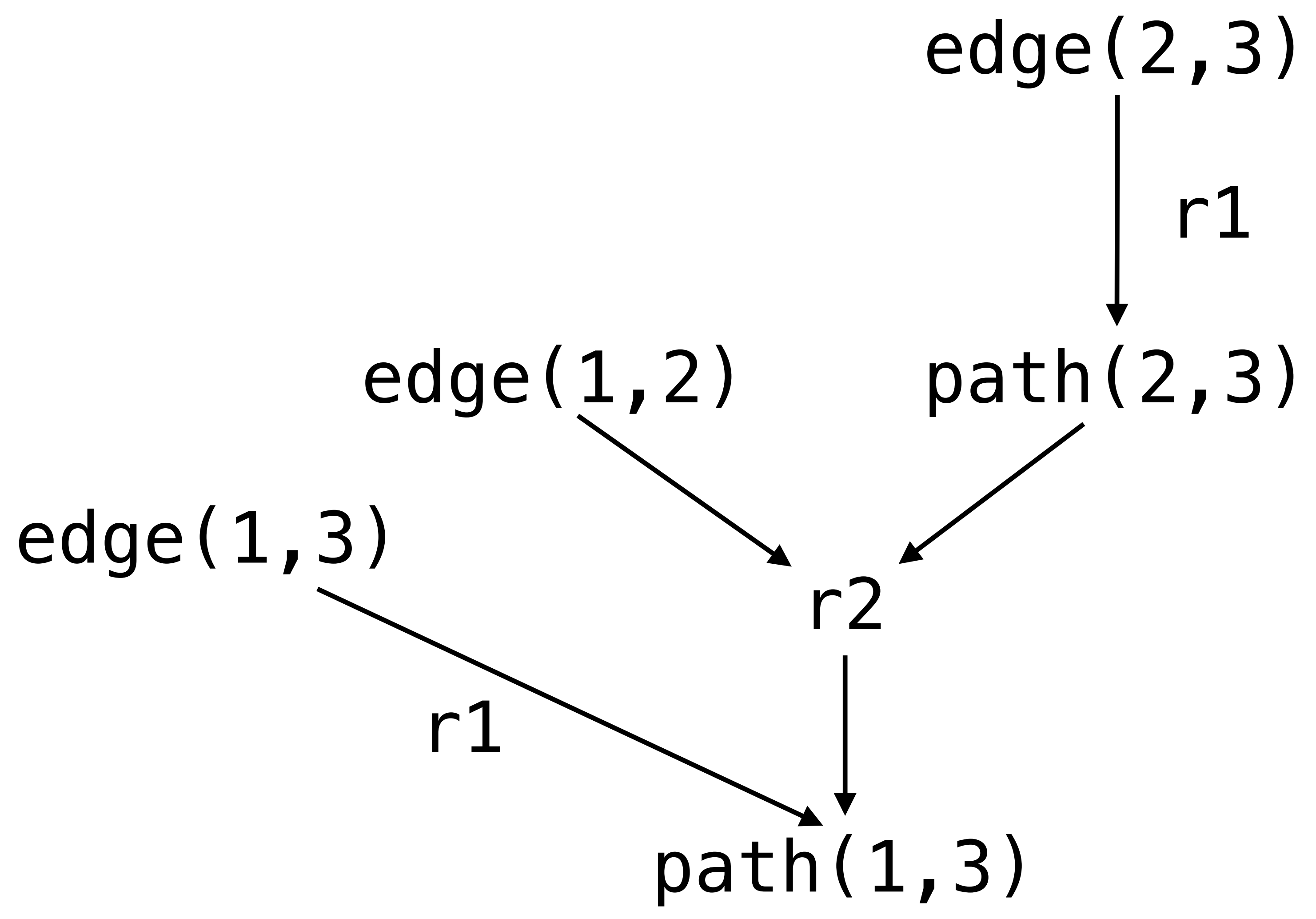
- Datalog programs produce provenance as well as output tuples
- Provenance: a proof that explains why a tuple is derived
  - If an undesired tuple is derived, we can see the reason

Rules

```
r1:  path(x,y) :- edge(x,y).
r2:  path(x,z) :- edge(x,y), path(y,z).
```

Input tuples

```
{edge(1,2), edge(2,3), edge(1,3)}
```



Idea 2: Continuous Semantics (1)

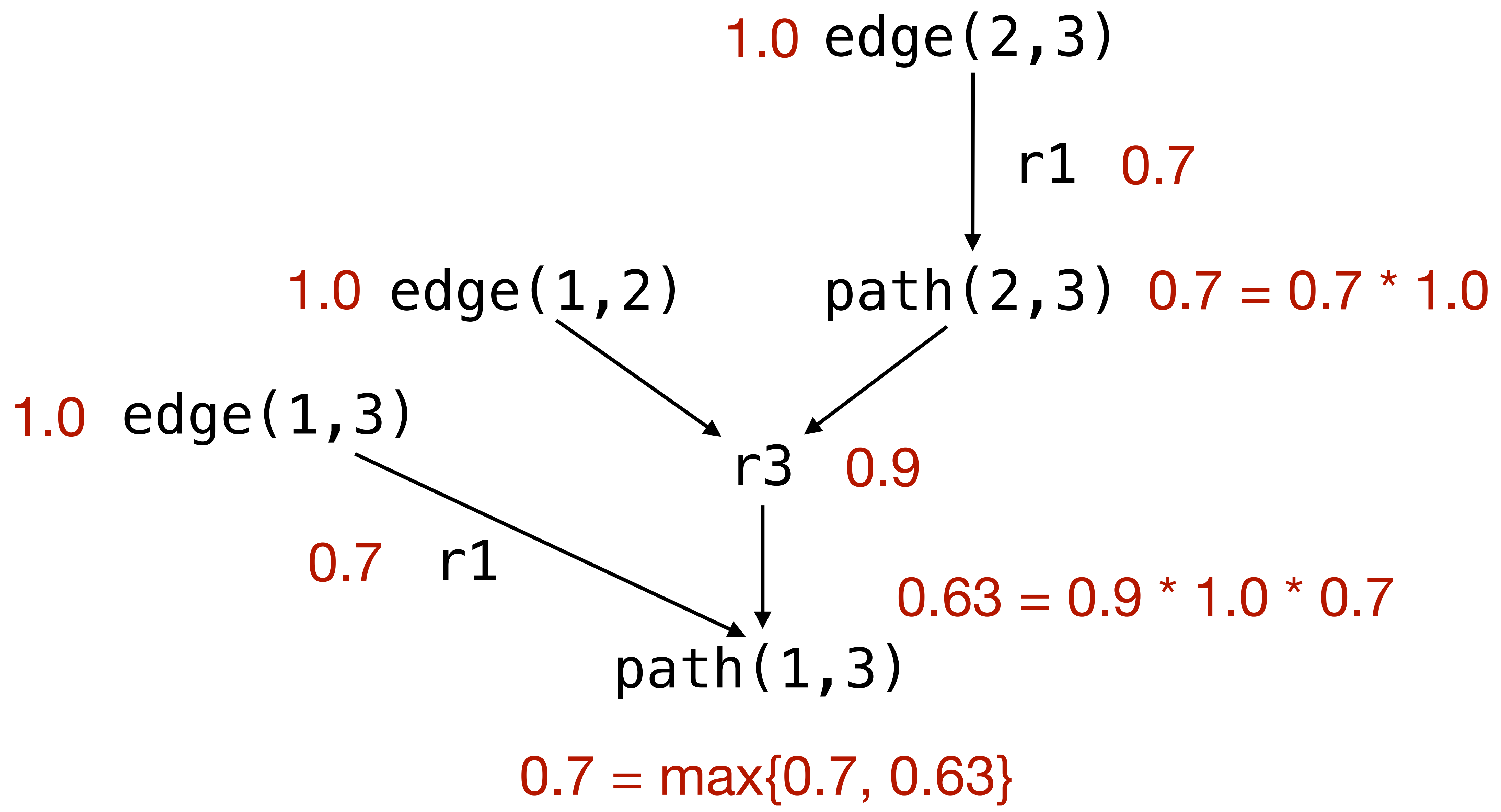
- Interpret Datalog programs (non-continuous function) as continuous functions
  - Existence of tuple {0, 1} to weight of tuple [0, 1]
  - Each rule is associated with a weight
  - The weight of a tuple is computed using the weights of rules on the provenance
- Then, combinatorial optimization problem → numerical optimization problem
  - Many existing algorithms applicable

Example (1)

Parameters: $\vec{W}$

```
0.7  path(x,y) :- edge(x,y).
0.1  path(x,x) :- edge(x,y).
0.9  path(x,z) :- edge(x,y), path(y,z).
0.3  path(x,y) :- path(y,x).
```

Input	edge(1,2)	edge(2,3)
Weight	1.0	1.0



Discrete semantics

$$v_t = \bigvee_g (v_{a_1} \wedge v_{a_2} \wedge \cdots \wedge v_{a_k})$$

Continuous semantics

$$v_t = \max_g (w_g \times v_{a_1} \times v_{a_2} \times \cdots \times v_{a_k})$$

Example (2)

Parameters: $\vec{W}$

```
0.7 path(x,y) :- edge(x,y).
0.1 path(x,x) :- edge(x,y).
0.9 path(x,z) :- edge(x,y), path(y,z).
0.3 path(x,y) :- path(y,x).
```

Input	edge(1,2)	edge(2,3)
Weight	1.0	1.0

Output	path(1,2)	path(2,3)	path(1,3)	path(1,1)	path(2,1)
Weight (v_t)	0.7	0.7	0.63	0.1	0.21
Expectation	1	1	1	0	0

Discrete semantics

$$v_t = \bigvee_g (v_{a_1} \wedge v_{a_2} \wedge \cdots \wedge v_{a_k})$$

Continuous semantics

$$v_t = \max_g (w_g \times v_{a_1} \times v_{a_2} \times \cdots \times v_{a_k})$$

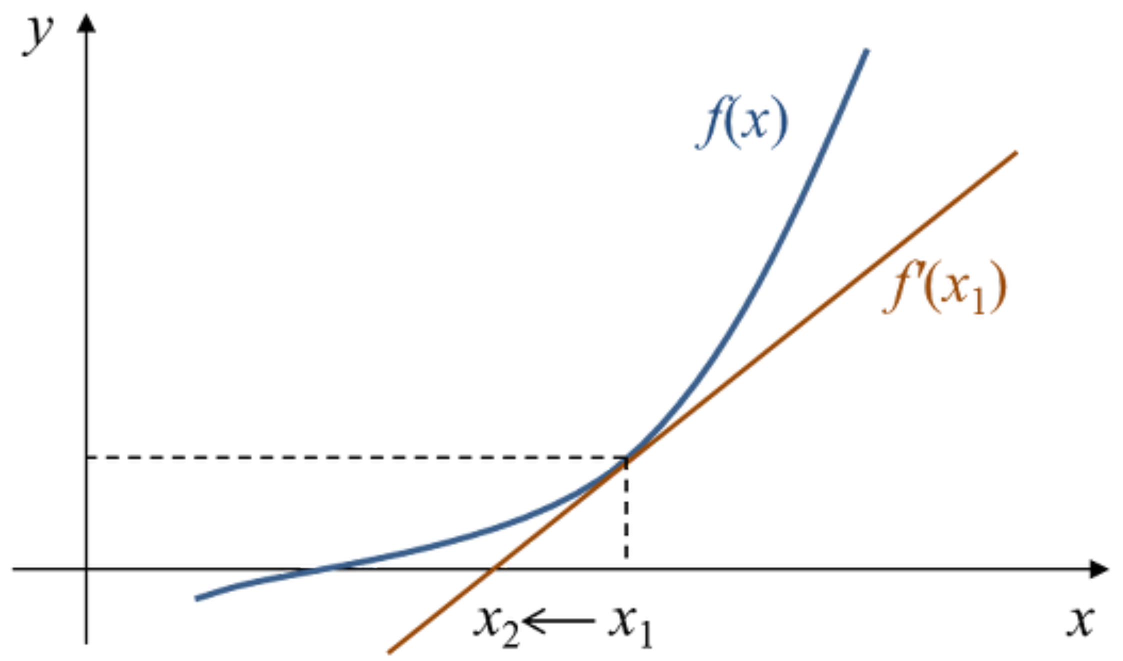
Optimization problem:
find $\vec{W}$ that minimizes the following loss

$$loss = \Sigma_{t \in pos}(1 - v_t)^2 + \Sigma_{t \in neg}(0 - v_t)^2$$

Numerical Optimization

$$loss = \Sigma_{t \in pos}(1 - v_t)^2 + \Sigma_{t \in neg}(0 - v_t)^2$$

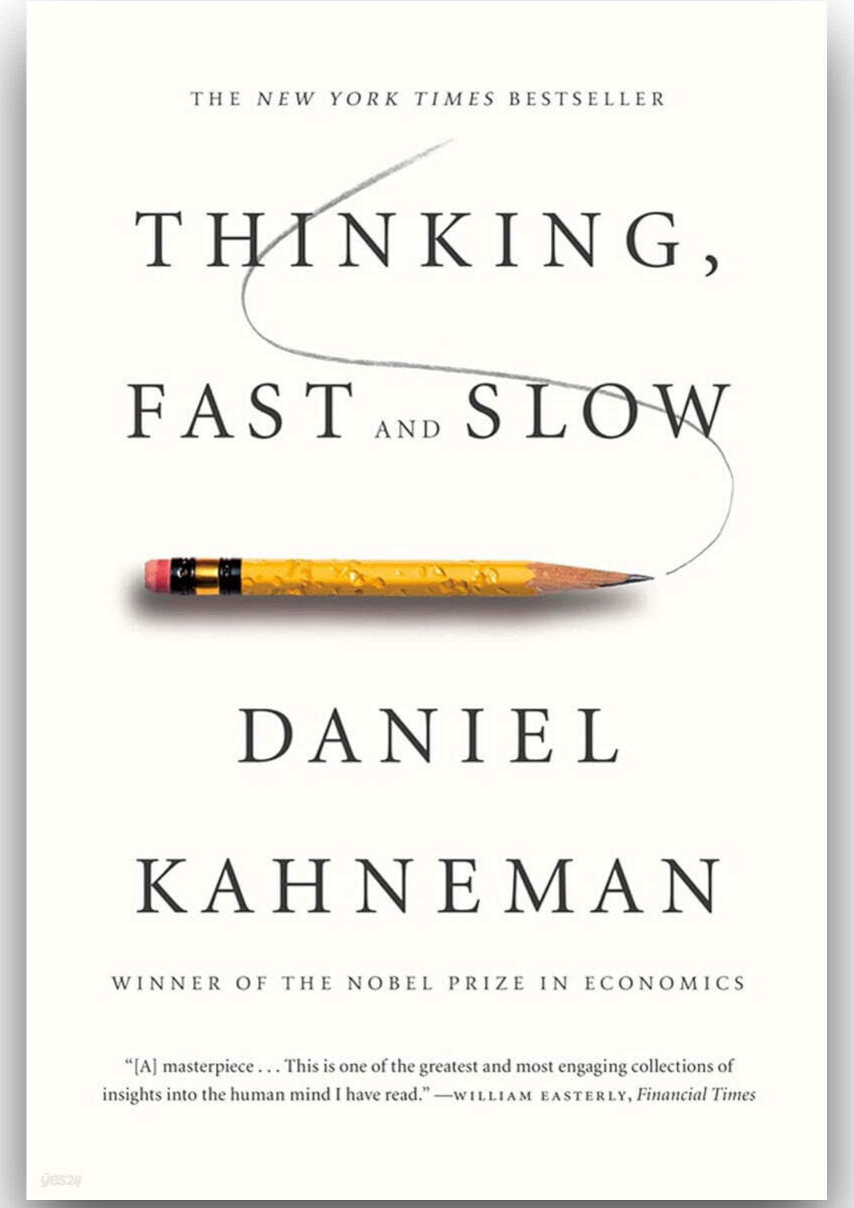
- In continuous semantics, v_t is a polynomial with w_r (differentiable)
- Then, the loss function is a polynomial with w_r (differentiable)
- Solve using a well-known algorithm (e.g., Newton's method)
    - loss is 0 = consistent with all the examples = a desired program



Effectiveness

Benchmark	Rel	Rule		Tuple		DIFFLOG			ALPS
		Exp	Cnd	In	Out	Iter	Smpl	Time	Time
inflamation	7	2	134	640	49	1	0	**1**	2
abduce	4	3	80	12	20	1	0	$<$ **1**	2
animals	13	4	336	50	64	1	0	**1**	40
ancestor	4	4	80	8	27	1	0	$<$ **1**	14
buildWall	5	4	472	30	4	5	1	**7**	67
samegen	3	3	188	7	22	1	0	**2**	12
scc	3	3	384	9	68	6	1	**28**	56
polysite	6	3	552	97	27	17	1	**27**	84
downcast	9	4	1,267	89	175	5	1	**30**	1,646
rv-check	5	5	335	74	2	1,205	41	**22**	195
andersen	5	4	175	7	7	1	0	**4**	27
1-call-site	9	4	173	28	16	4	1	**4**	106
2-call-site	9	4	122	30	15	25	1	**53**	676
1-object	11	4	46	40	13	3	1	**3**	345
1-type	12	4	70	48	22	3	1	**4**	13
escape	10	6	140	13	19	2	1	**1**	5
modref	13	10	129	18	34	1	0	**1**	2,836
sql-10	3	2	734	10	2	7	1	**11**	41
sql-14	4	3	23	11	6	1	0	$<$ **1**	54
sql-15	4	2	186	50	7	902	31	875	**11**

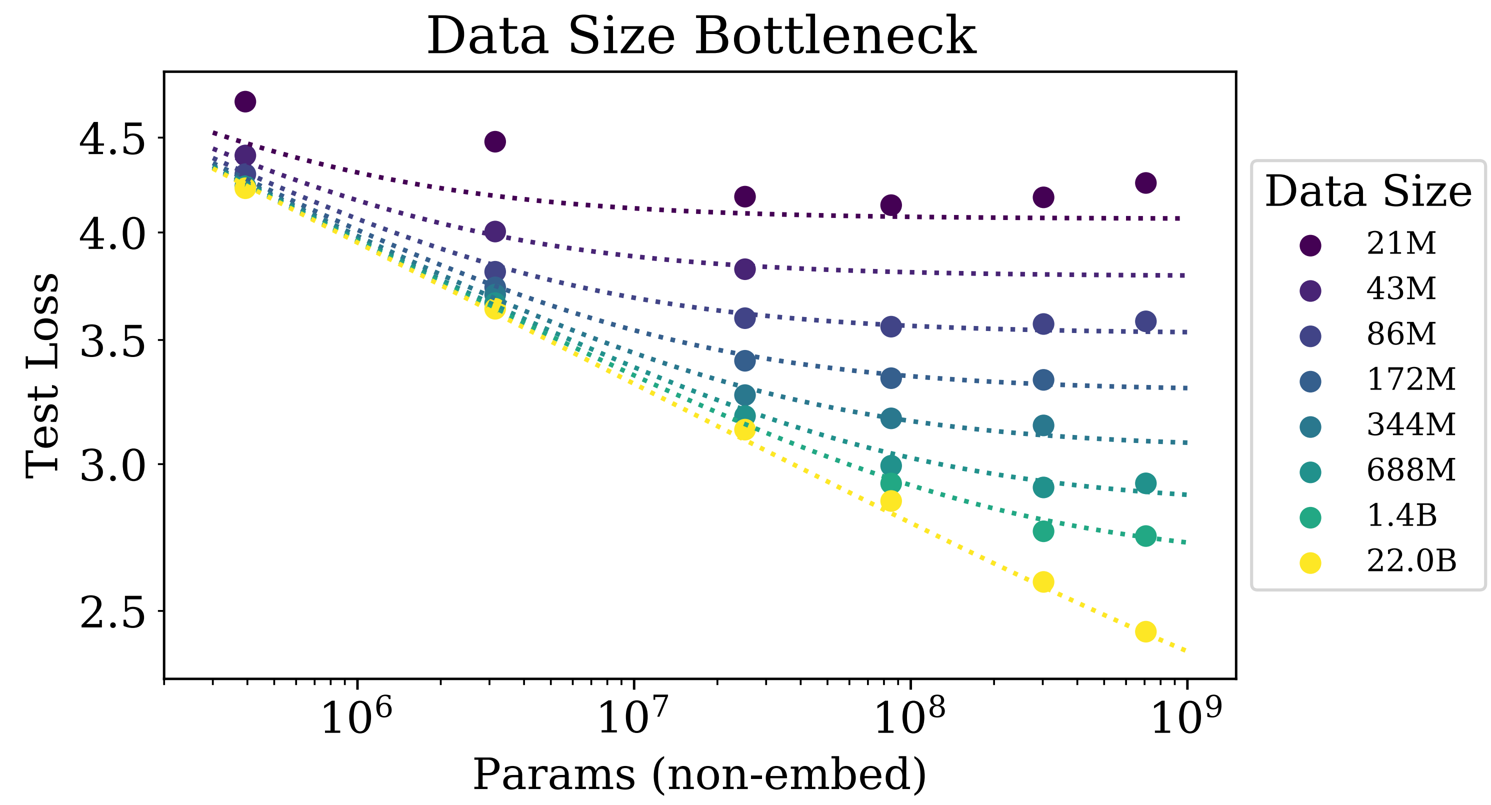
Neuro-Symbolic AI

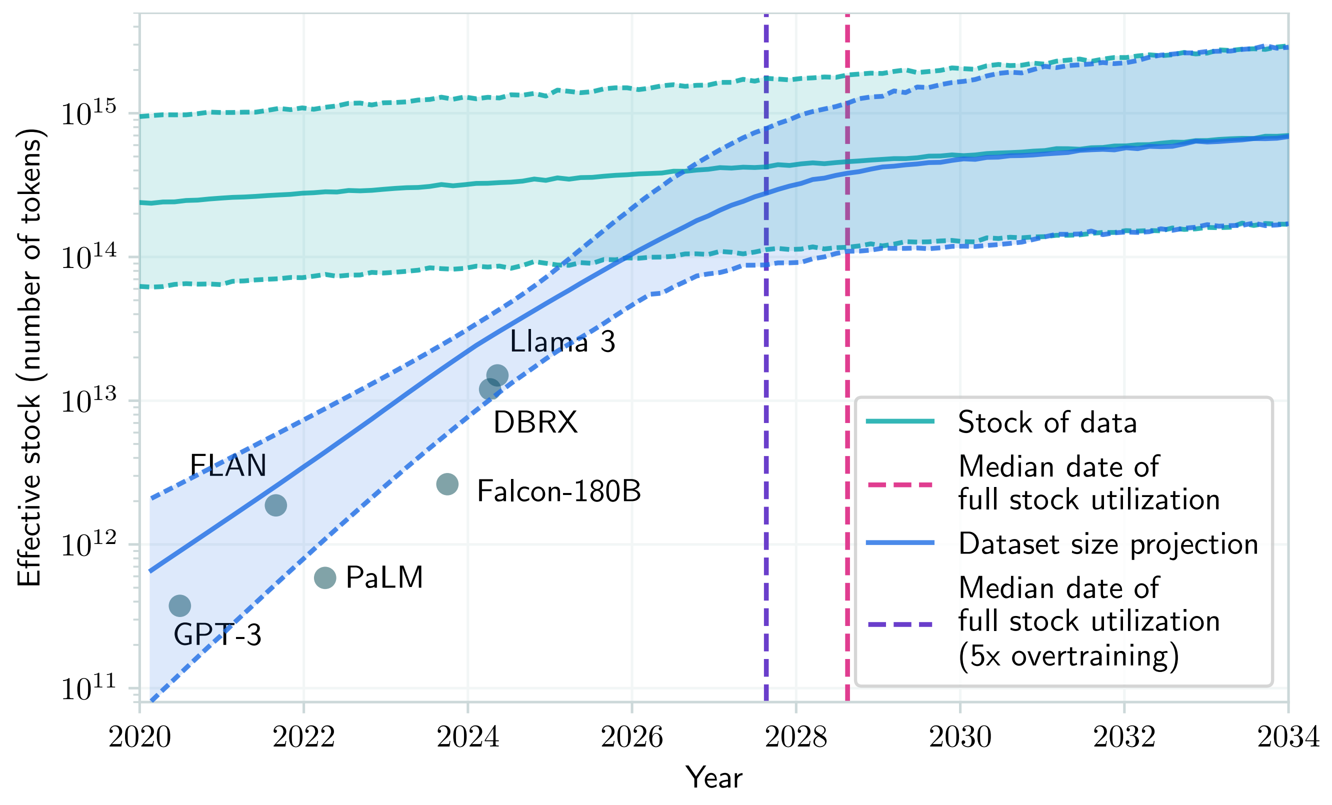


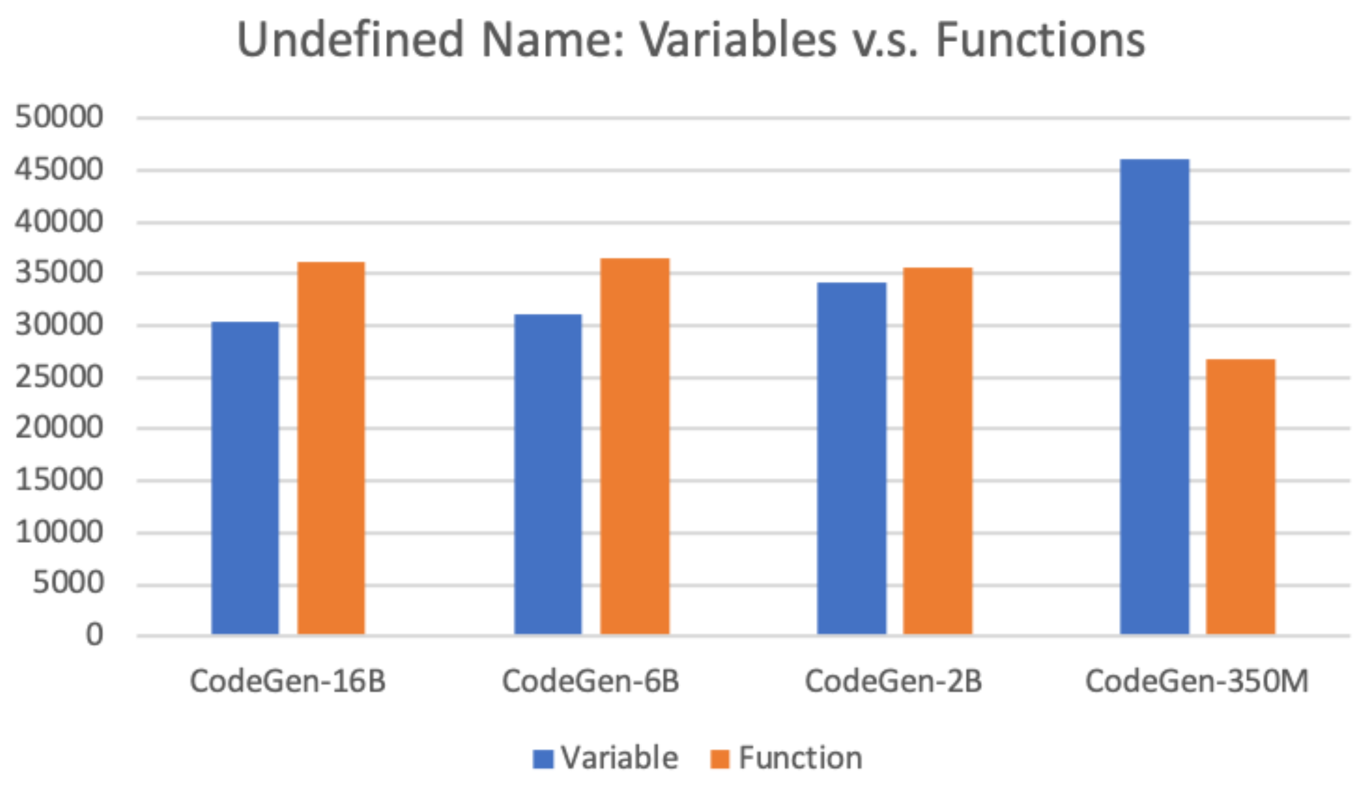
- What is the future of AI (or programming)?
    - Combining the strengths of the two camps
- Two directions
    - Symbolic engine + neural guide: Program synthesis using numerical optimization
    - Neural engine + symbolic guide: Constrained decoding

Problems of Language Models for Code

- No guarantee on important properties (e.g., syntax, semantics, safety, security, etc)
- More data needed for training larger models
- All human-generated data will be exhausted by 2026-2032
- Larger model does not always mean better performance







Kaplan et al., Scaling Laws for Neural Language Models, 2020
Villalobos et al., Position: Will We Run out of Data? Limits of LLM Scaling Based on Human-Generated Data, ICML’24
Ding et al., A Static Evaluation of Code Completion by Large Language Models, ACL’23

Constrained Decoding조건부 생성

- How to make LMs to generate *correct* outputs?
- Guide LLMs in generating content that satisfies a given constraint
  - Syntax: Json, XML, Python, OCaml, etc
  - Semantics: type, security vulnerability, etc
- Idea: attach a filter at the backend of LMs
  - Combination of LM and symbolic checker

Language Models in a Nutshell

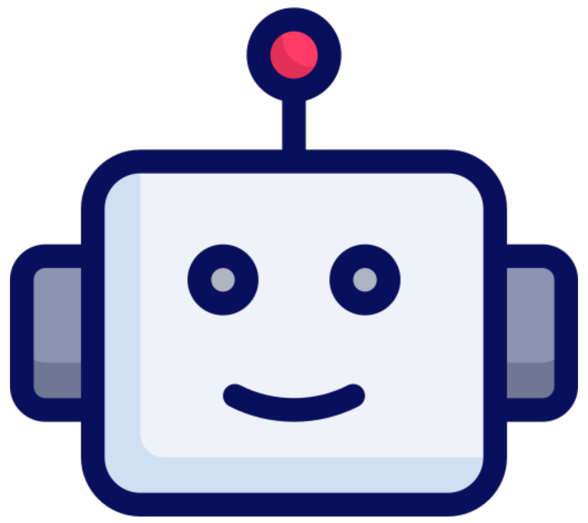
- T is a finite set of tokens (vocabulary)
- LM $\in String \rightarrow \Pr(T)$: given a prefix, return a probability distribution on T

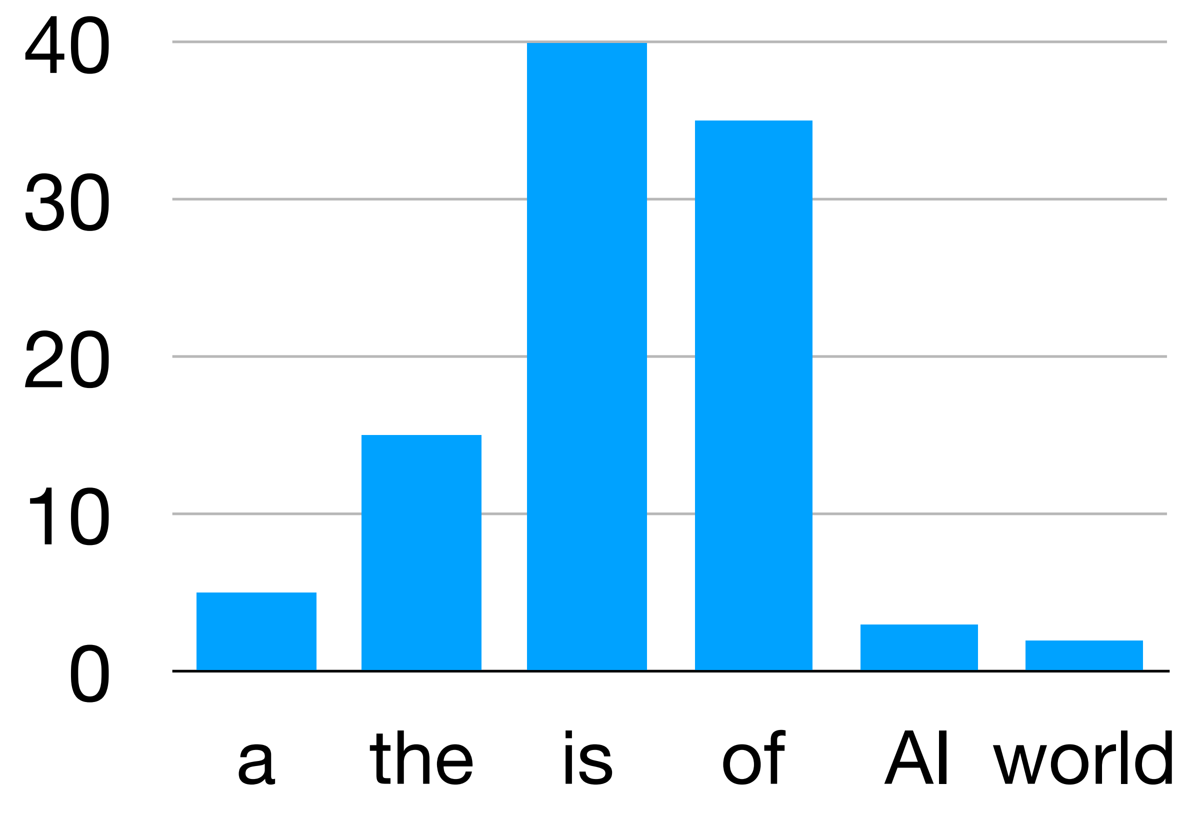
```
LMSynthesizer(prompt):
  s := initialize(prompt)
  while true do
    v := LM(prompt + s)
    t := Sample(v)
    if t = EOS then break
    s := s + t
  return s
```

```
Prompt: “Which university is the best?”
S0: “”
S1: “The”
S2: “The best”
S2: “The best university”
S3: “The best university is”
```

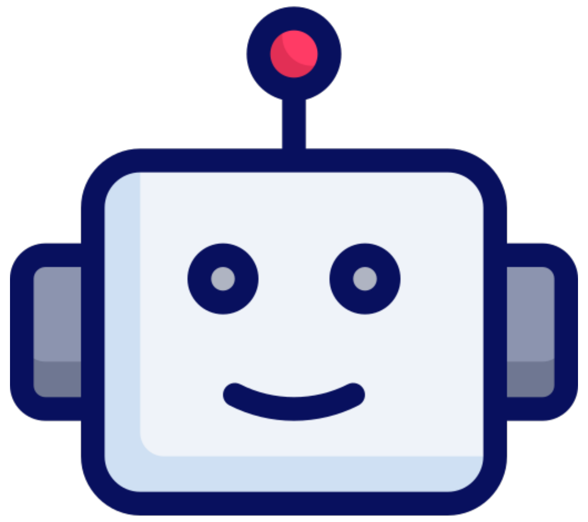
Inference & Sampling

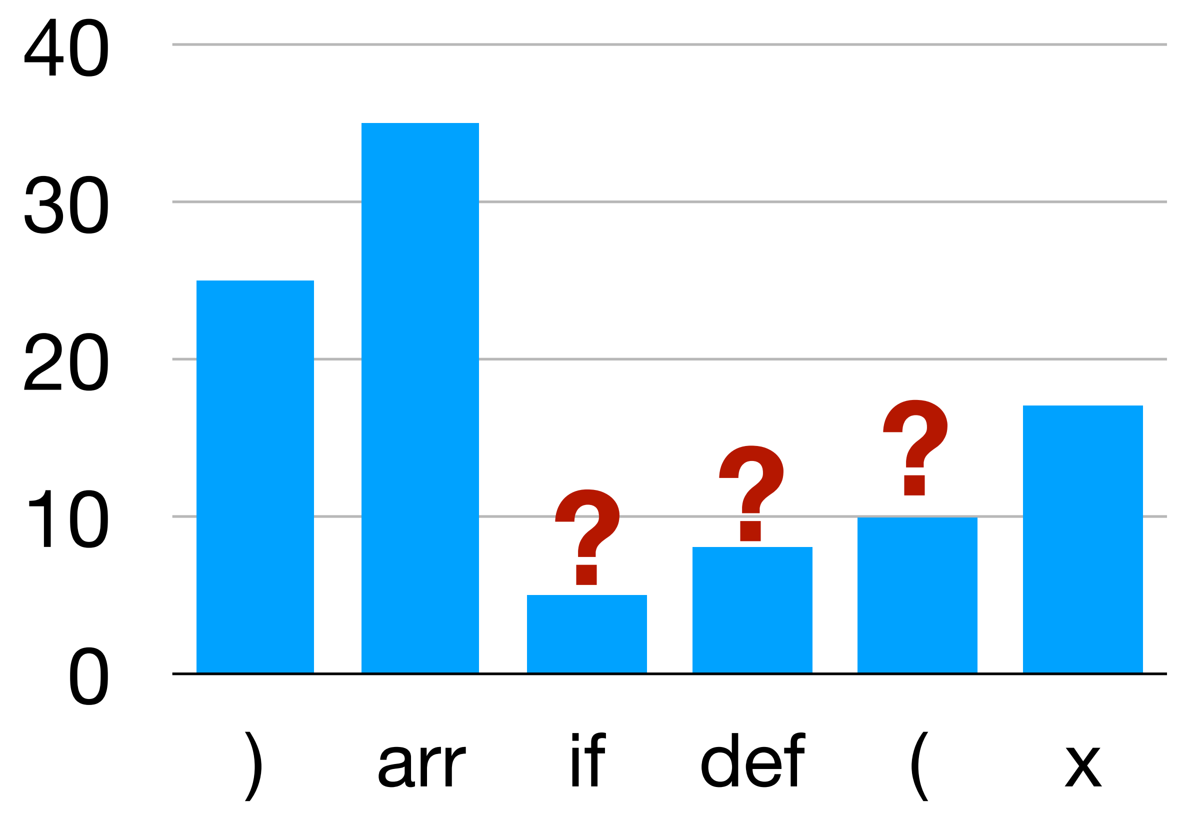
“The best university”





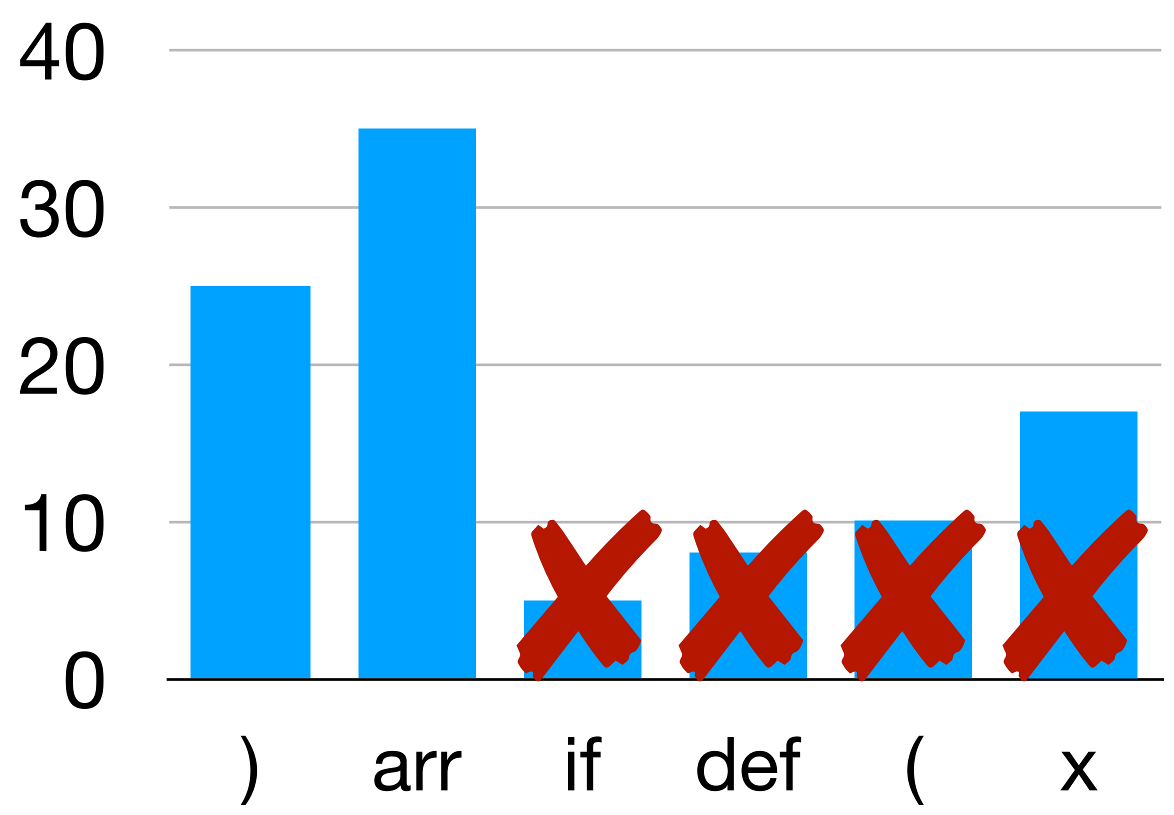
“def sort(”





Grammar-Constrained Decoding

- Guide LM to generate grammatically correct text
- Idea: combine parser (i.e., syntactic analysis) with LM
  - Mask incorrect tokens from the distribution
  - Normalize the masked distribution
  - Sample



Preliminary: Bottom-up Parsing

- Two basic actions
  - Shift: shift the next input symbol onto the top of the stack
  - Reduce: replace symbols at the top of the stack to a nonterminal
- Example:

Grammar

$$
\begin{array}{rcl}
e & \rightarrow & x \\
 & \mid & n \\
 & \mid & e+e \\
 & \mid & e-e \\
 & \mid & e*e \\
 & \mid & e \;/\; e \\
 & \mid & (e)
\end{array}
$$

Parsing

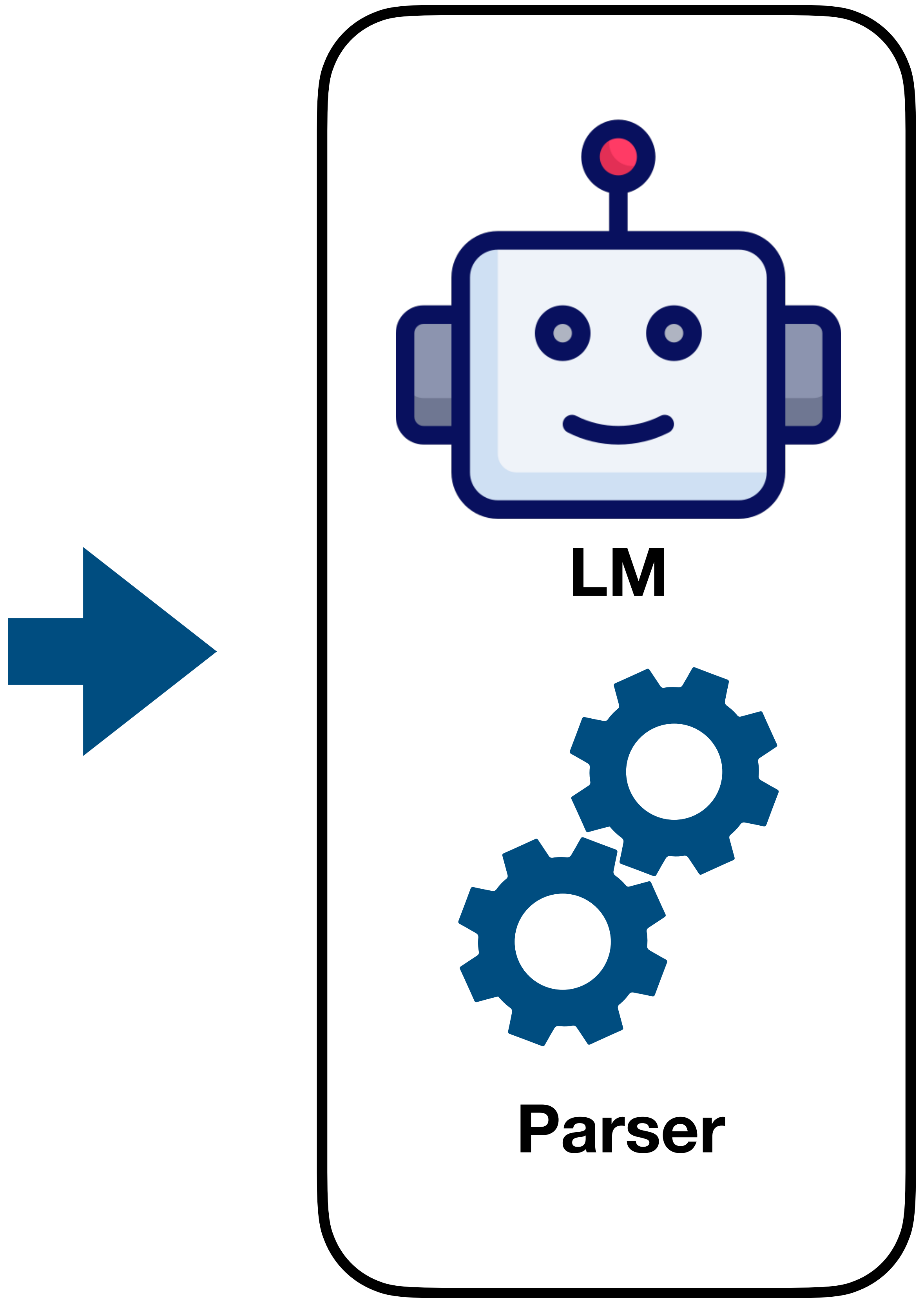
Stack	Input	Action
	(x + 1) / 2	shift
(	x + 1) / 2	shift
(x	+ 1) / 2	reduce
(e	+ 1) / 2	shift
(e +	1) / 2	shift
(e + 1	) / 2	reduce
(e + e	) / 2	reduce
(e	) / 2	shift
(e)	/ 2	reduce
e	/ 2	shift
…	…	…
e		accept

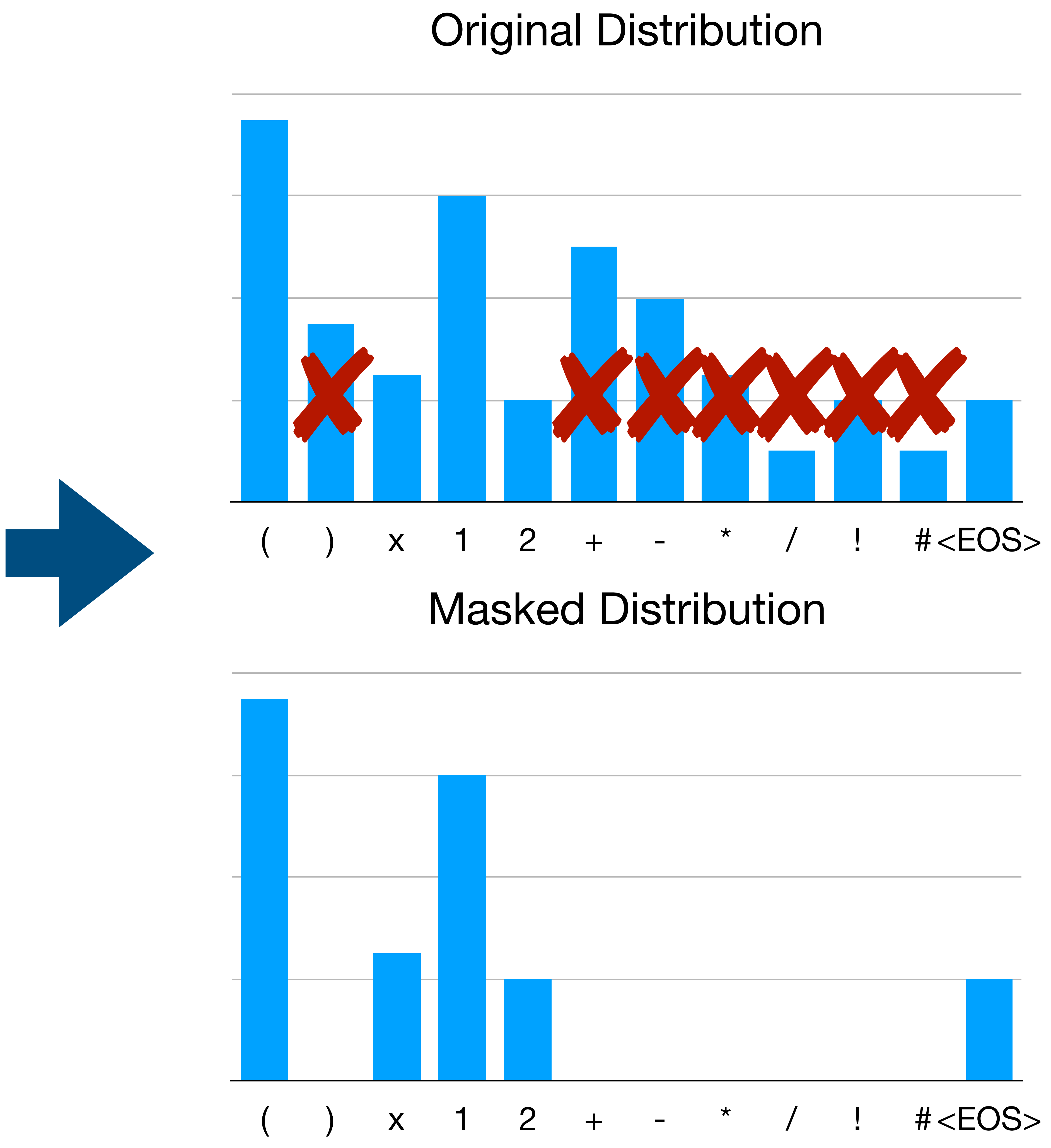
Grammar-Constrained Decoding

Grammar

$$
\begin{array}{lcl}
e & \rightarrow & x \\
 & \mid & n \\
 & \mid & e + e \\
 & \mid & e - e \\
 & \mid & e * e \\
 & \mid & e \;/\; e \\
 & \mid & (e)
\end{array}
$$

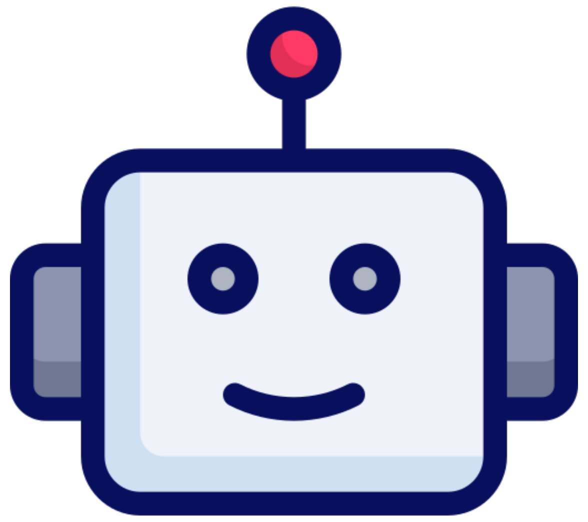
```
Prompt: “What is the sum of
integers from 1 to x?”

S0: “”
```

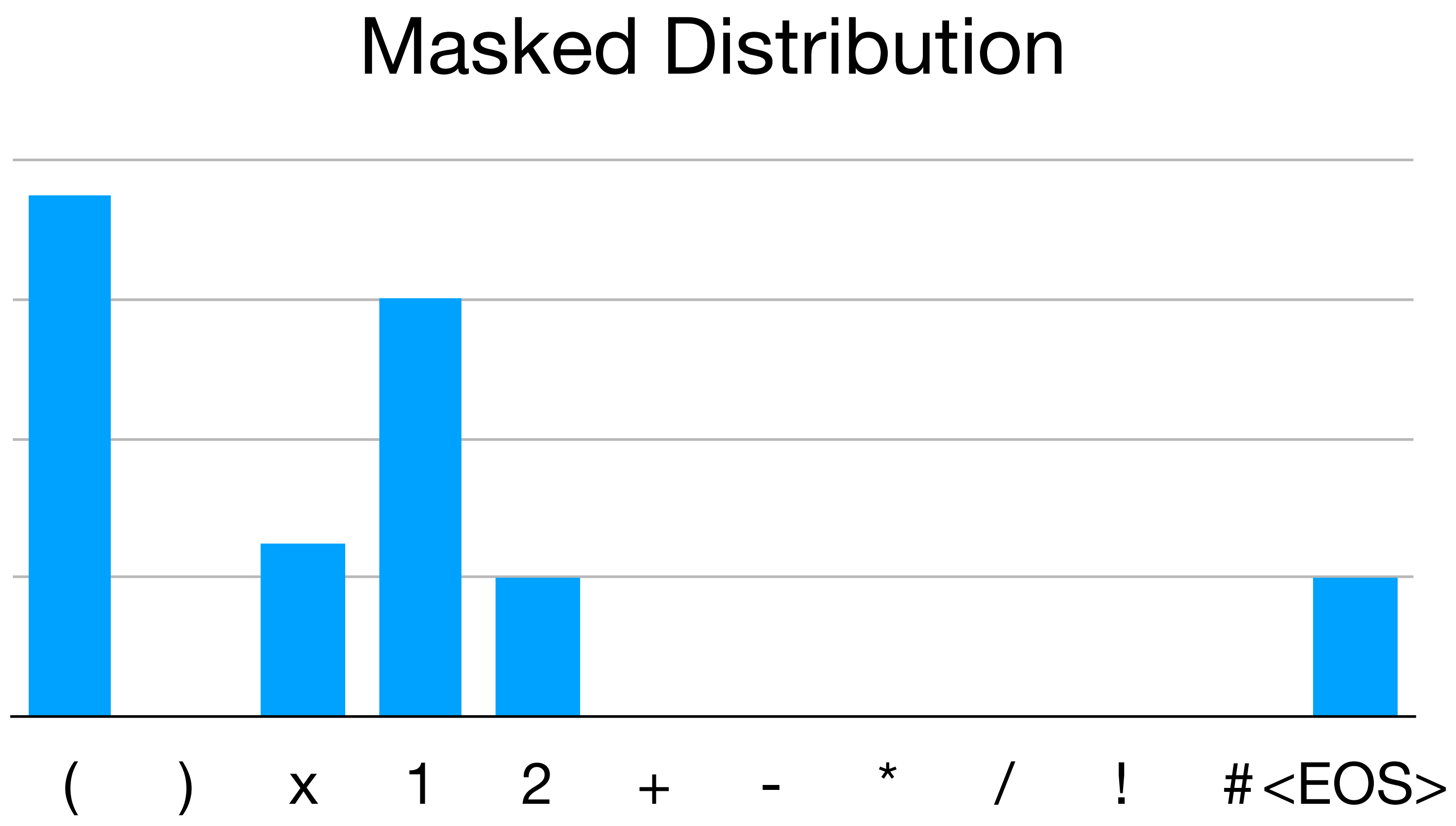




Example: Arithmetic Expression

Prompt: “What is the sum of integers from 1 to x?”
S0: “”





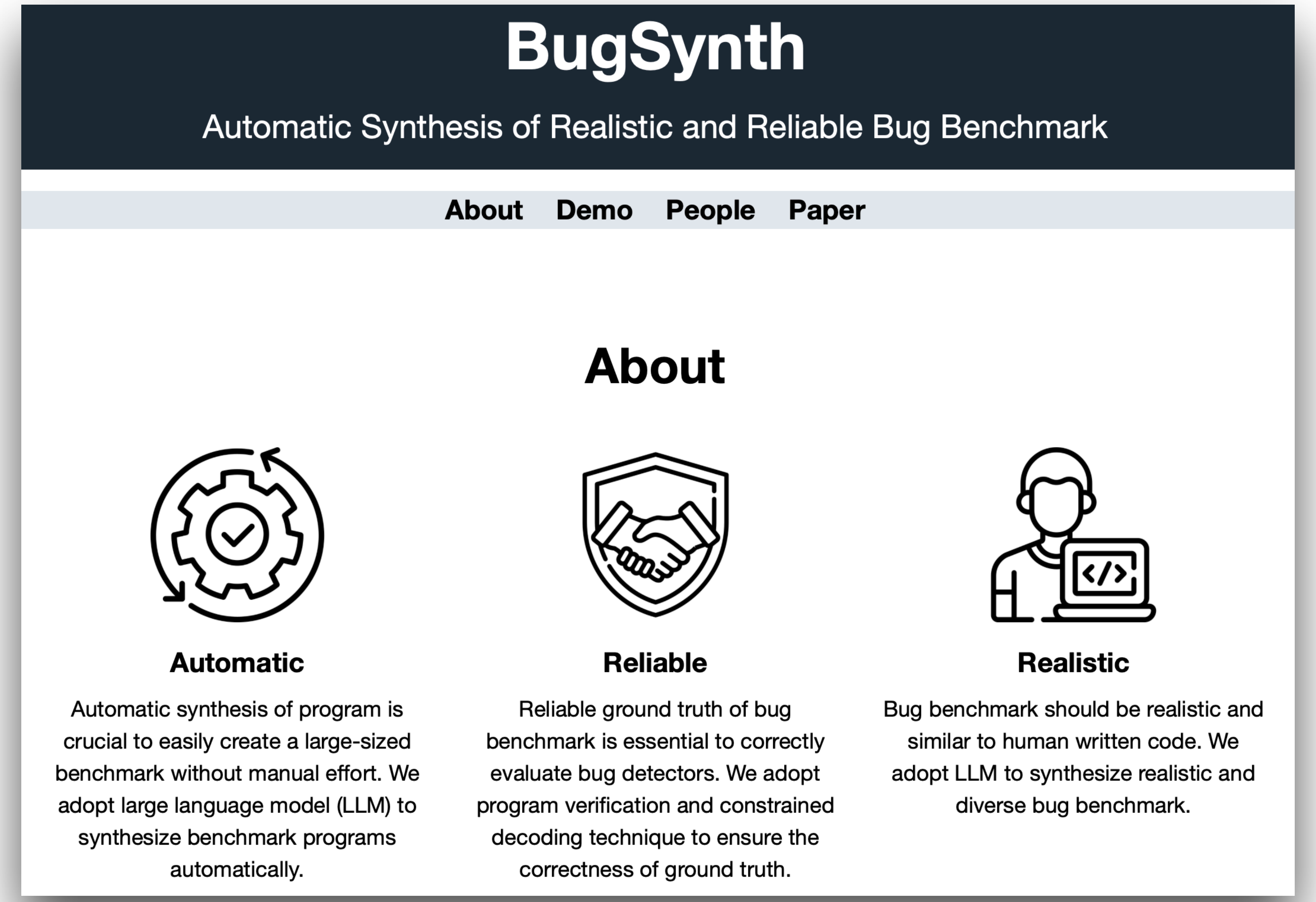
Stack	Possible Input	Input	Action
	(, x, 1, 2, <EOS>	(	shift
(	(, x, 1, 2, <EOS>	x	shift
(x			reduce
(e	+, -, *, /	+	shift
(e +	x, 1, 2	1	shift
(e + 1		)	reduce
(e + e	)	)	shift
(e + e)			reduce
e		/	shift
…		…	…
e	+, -, *, /, <EOS>	<EOS>	accept

Practical Aspect

- Different processing units between parser and LM
  - Lexeme (parser): defined by lexer using regular expression, e.g., while, length, if, {
  - Token (LM): learned during training, e.g., wh, ile, len, th, if{
- Important: correctly and efficiently handle the mismatch

Application: Buggy Program Synthesizer

- Automatically generate buggy programs for program analysis assignments
- Constraints
  - Grammartically correct C programs
  - Limited features of C
  - Presence of bugs at the desired program point
  - Absence of bugs for all the other program points
- Used in CS348, CS424, CS524 at KAIST



https://prosys.kaist.ac.kr/bugsynth/

When & Why Constrained Decoding?

- Low-resource programming languages
  - E.g., 69.8% parse errors of GPT-4 Dafny code (2025)
- New domain-specific languages and rules
  - E.g., coding guidelines
- Other domains that have constraints
  - E.g., robots, automobiles, biochemistry, etc

Summary

- A long-standing argument in AI: connectionism vs symbolism
  - Connectionism (neural network): good at image recognition, machine translation, etc
  - Symbolism (logic): good at equation solving, logical reasoning, etc
- What is the future of AI (or programming)?
  - Neuro-symbolic AI: how to effectively combine both paradigms?