Program Reasoning

3. Concepts in Program Verification

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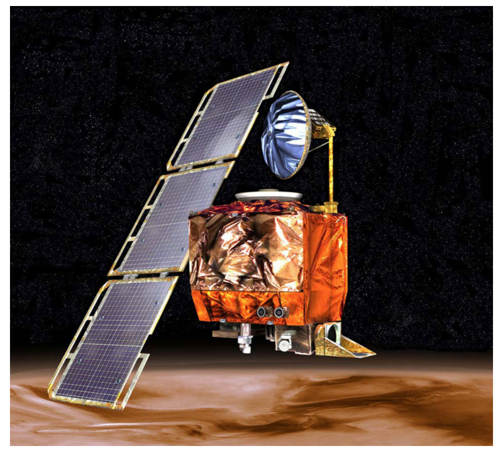
Impact of Poor Software Quality

The Patriot Missile (1991)
Floating-point roundoff
28 soldiers died

The Ariane-5 Rocket (1996)
Integer Overflow
$100M



NASA’s Mars Climate Orbiter (1999)
Meters-Inches Miscalculation
$125M

Towards Error-free SW

"Program testing can be used to show the presence of bugs, but never to show their absence!"

- Edsger W. Dijkstra, 1970

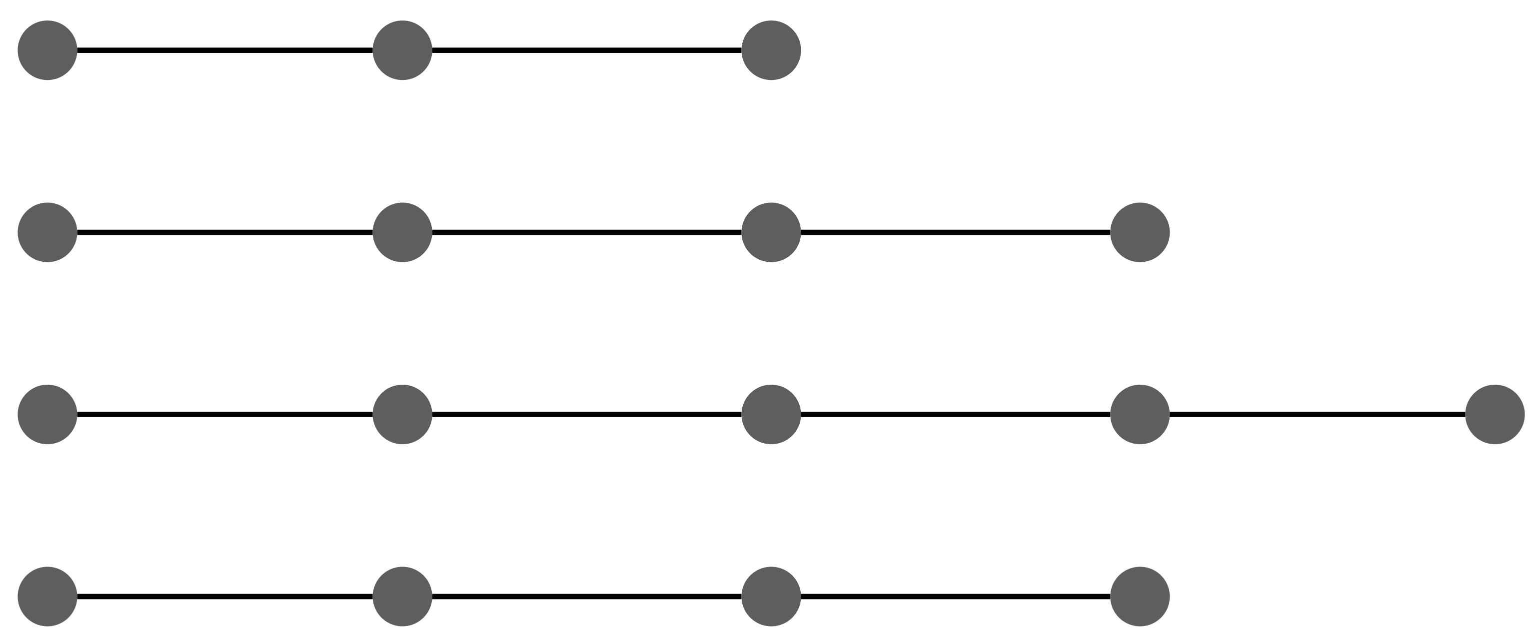
Properties성질

- Points of interest in programs
  - for verification, bug detection, optimization, understanding, etc
  - E.g.,“p == NULL?”, “idx < size?”, “fp can be only f, g, or h?”, “value of x”, etc
- Two categories:
  - Trace properties = properties of individual execution traces
    - safety properties + liveness properties
  - Information-flow properties = properties of multiple execution traces

Trace실행경로

- Trace = a list of states
- Recall small-step operational semantics
- A program can have an (infinite) set of traces
- $[\![P]\!]$: a set of all possible execution traces

$$
\begin{aligned}
&(2 \times 2 \times 2) \times (2 + 1) \\
&\rightarrow (4 \times 2) \times (2 + 1) \\
&\rightarrow 8 \times (2 + 1) \\
&\rightarrow 8 \times 3 \\
&\rightarrow 24
\end{aligned}
$$



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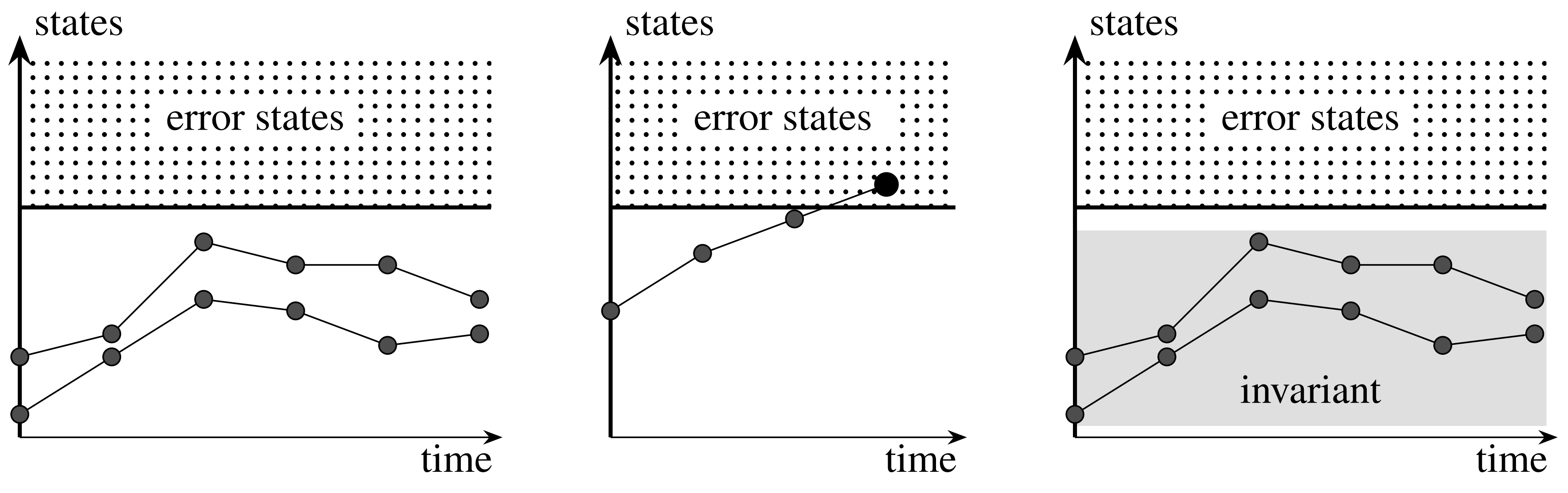
Trace Properties개별경로 성질

- A semantic property $\mathscr{P}$ that can be defined by a **set of execution traces** that satisfies $\mathscr{P}$
  - Ex1: “all traces that satisfies x != 0 at line 10”
  - Ex2: “all traces where the value of y at line 97 is the same as the one in the entry point”
- Program P satisfies property $\mathscr{P}$ iff $[\![P]\!] \subseteq T_{\mathscr{P}}$
- State properties: defined by a set of states (so, obviously trace properties)
  - E.g., division-by-zero, integer overflow
- Any trace property: the conjunction of a safety and a liveness property

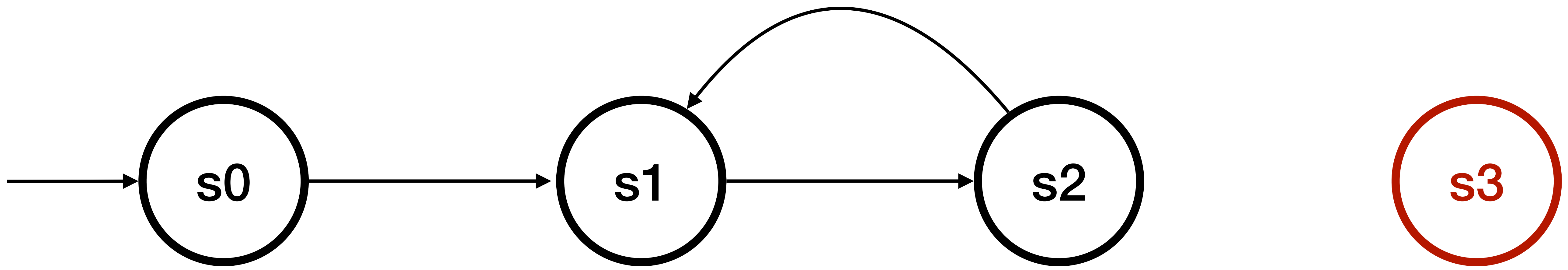
Safety Property항상성질

- A program **never** exhibit a behavior observable within **finite time**
  - “Bad things will never occur”
  - Bad things: integer overflow, buffer overrun, deadlock, etc
- If false, then there exists a **finite counterexample**
- To prove: all executions never reach error states



(a) Correct executions (b) An incorrect execution (c) Proof by invariance

Example



Reachable states <= 0 step : {s0}
Reachable states <= 1 step : {s0, s1}
Reachable states <= 2 steps : {s0, s1, s2}
Reachable states <= 3 steps : {s0, s1, s2}
Reachable states <= 4 steps : {s0, s1, s2}
…
Reachable states <= 100 steps : {s0, s1, s2}
…
Reachable states <= ∞ steps : {s0, s1, s2}

Invariant불변식

- Assertions supposed to be **always true** and **remain unchanged** after any operations
  - Starting from a state in the invariant, any computation step also leads to another state in the invariant (i.e., fixed point!)
  - E.g., “x has an int value during the execution”, “y is larger than 1 at line 5”
- Loop invariant: assertion to be true at the beginning of every loop iteration

```
x = 0;
while (x < 10) {
  x = x + 1;
}
assert(x > 0);
assert(x == 10);
```

Loop invariant 1: “x is an integer”

Loop invariant 2: “x >= 0”

Loop invariant 3: “0 <= x <= 10”

Invariant in Art

by hyerinshelly, 2022

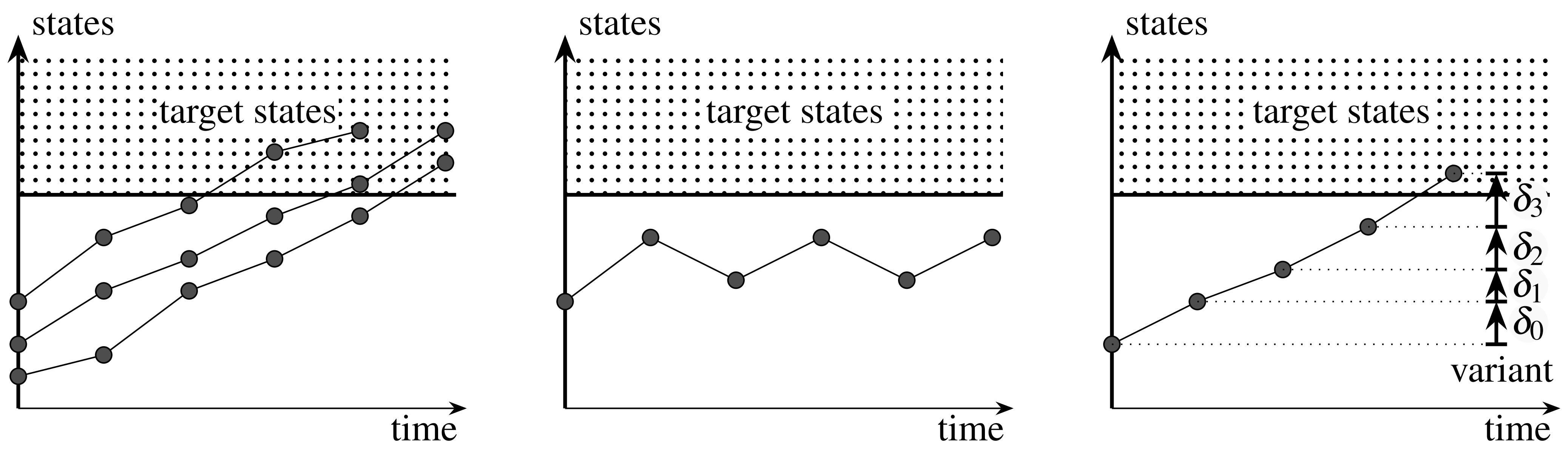
Example: Division-by-Zero

```c
int main(){
  int x = input();
  x = 2 * x - 1;
  while (x > 0) {
    x = x - 2;
  }
  assert(x != 0);
  return 10 / x;
}
```

```c
int main(){
  int x = input();
  x = 2 * x;
  while (x > 0) {
    x = x - 2;
  }
  assert(x != 0);
  return 10 / x;
}
```

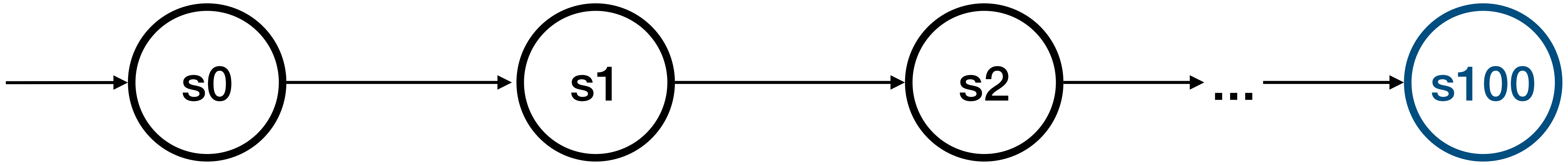
Liveness Property 결국성질

- A program will **never** exhibit a behavior observable only after **infinite time**
  (A program will **eventually** exhibit a behavior observable within **finite time**)
  - "Good things will eventually occur"
  - Good things: termination, fairness, etc
- If false then there exists an **infinite counterexample**
- To prove: all executions eventually reach target states



(a) Correct executions (b) An incorrect execution (c) Proof by variance

Example



Shortest distance after 0 step : 100
Shortest distance after 1 step : 99
Shortest distance after 2 steps : 98
Shortest distance after 3 steps : 97

…

(if we are sure that the distance will keep decreasing)

…

Reachable states after 100 steps : 0

Variant불변식

- A quantity that **evolves towards** the set of target states
  (so guarantee any execution eventually reaches the set)
- Usually, a value that is strictly decreasing for some well-founded order relation
  - Well-founded order: there is no infinite decreasing chain
  - E.g., an integer value that is always positive and strictly decreasing

```
x = pos_int();
while (x > 0) {
  x = x - 1;
}
```

x is always a positive integer $\land$ **x is strictly decreasing** $\Rightarrow$ **The program terminates**

Example: Termination

- Introduce variable c that stores the value of “step counter”
  - Initially, c is equal to zero
  - Each program execution step increments c by one

```
// A factorial program
i = n;
r = 1;
while (i > 0) {
  r = r * i;
  i = i - 1;
}
```

```
// An instrumented program
i = n;
r = 1;
c = 2;
while (i > 0) {
  r = r * i;
  i = i - 1;
  c = c + 3;
}
// what is the value of c in the loop?
```

c <= 3n + 2

0 <= 3n + 2 - c ⋀ 3n + 2 - c is strictly decreasing ⇒ termination

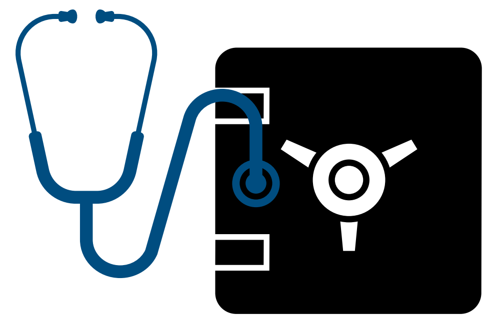
Example

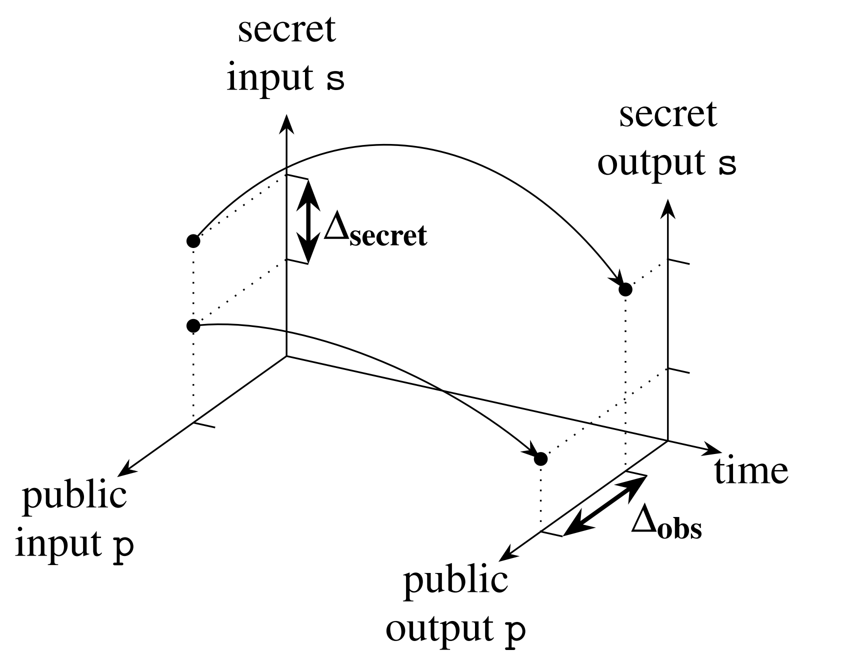
- Correctness of a sorting algorithm as trace property

Property	Safety or Liveness?	State?
Should not fail with a run-time error		
Should terminate		
Should return a sorted array (if terminated)		
Should return an array with the same elements and multiplicity (if terminated)		

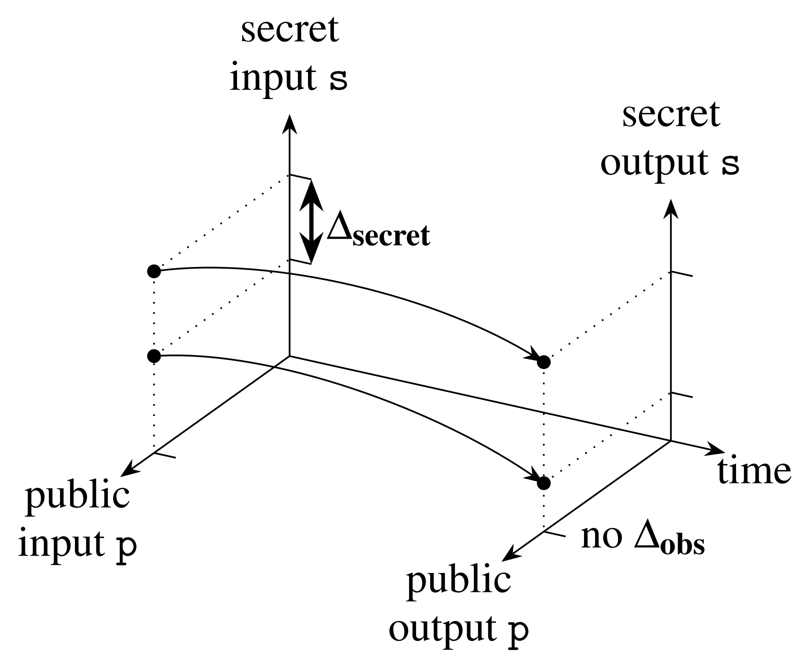
Information Flow Properties실행 간섭 성질

- Properties stating the absence of dependence between **pairs of executions**
    - Beyond trace properties: so called **hyper-properties**
- Mostly for security: multiple executions with public data should not derive private data
- E.g., a door lock beeps louder if a right digit is pressed at the right position





A pair of executions with insecure information flow



A pair of executions without insecure information flow

Example

- Assume that variables s (secret) and p (public) take only 0 and 1

```
// Program 0
p_out := p_in * [0, 1]
```

```
// Program 1
p_out := p_in * s * [0, 1]
```

```
// Program 2
p_out := p_in + [0, 1] - s
```

Input		Output
p	**s**	**p**
0	0	{0, 1}
0	1	{0, 1}
1	0	{0, 1}
1	1	{0, 1}

Input		Output
p	**s**	**p**
0	0	{0}
0	1	{0}
1	0	{0}
1	1	{0, 1}

Input		Output
p	**s**	**p**
0	0	{0, 1}
0	1	{0, 1}
1	0	{0, 1}
1	1	{0, 1}

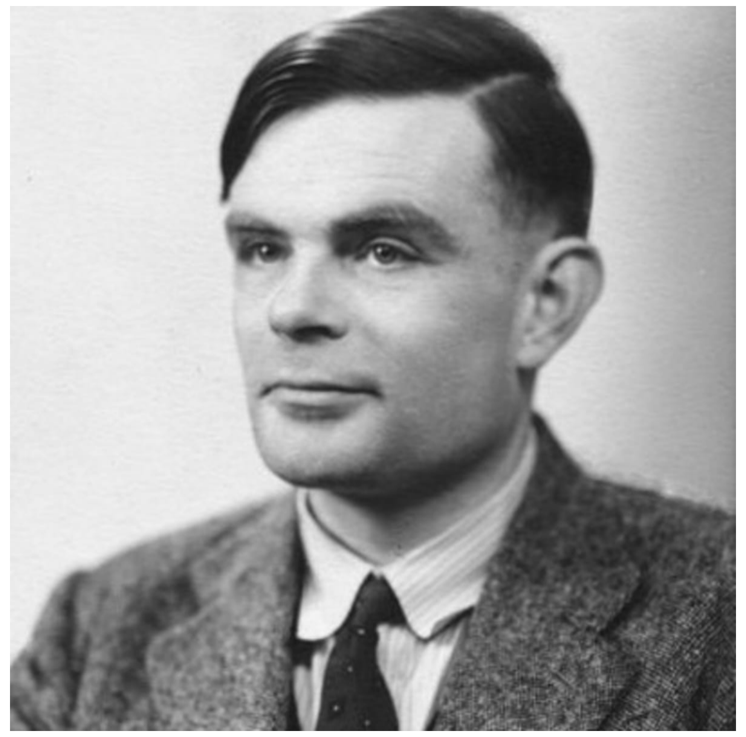
A Hard Limit: Undecidability결정불가능성

Theorem (Rice's theorem). Any non-trivial semantic properties are undecidable.

- Non-trivial property: worth the effort of designing a program analyzer for
  - trivial: true or false for all programs
- Undecidable? If decidable, it can solves the Halting problem!

HP: Given a Turing machine T and an input i, does T eventually halt on i?

Undecidable: There is no Turing machine that can solve HP!



Informal Proof of Undecidability of HP

HP: Given a Turing machine T and an input i, does T eventually halt on i?

- Assume $H(T, i)$ returns true or false
- Let $F(x) = \text{if } H(x, x) \text{ then } \texttt{loop}() \text{ else } \texttt{halt}()$
- Does $F(F)$ terminate?

If Decidable?

- Many mathematical problems become trivial!

```
// Fermat's last theorem

for (a, b, c, n) in ℕ^4 do
  if n > 2 && a^n + b^n = c^n then
    exit()
```

```
// Goldbach's conjecture

for k in Even do
  for (p, q) in Prime^2
    if k != p + q then
      exit()
```

…

…

…

Informal Proof of Rice's Theorem

- Assumption: HP is undecidable
- An analyzer **A** for a property: "*This program always prints 1 and finishes*"
- Given a program **P**, generate **P' =** "**P**; print 1;"
- Analyze **P'** using **A: A(P')**
  - **A(P')** says "Yes": **P** halts,
  - **A(P')** says "No": **P** does not halt
- HP is decidable if we use **A** : contradiction!

Toward Computability

Undecidable

⇒ Automatic, terminating, and exact reasoning is impossible

⇒ If we give up one of them, it is computable!

- **Manual** rather than **automatic**: assisted proving
  - require expertise and manual effort
- **Possibly nonterminating** rather than **terminating**: model checking, testing
  - require stopping mechanisms such as timeout
- **Approximate** rather than **exact**: static analysis
  - report spurious results

Soundness안전성 and Completeness완전성

- Given a semantic property $\mathscr{P}$, and an analysis tool $\mathbf{A}$

- If $\mathbf{A}$ were perfectly accurate,

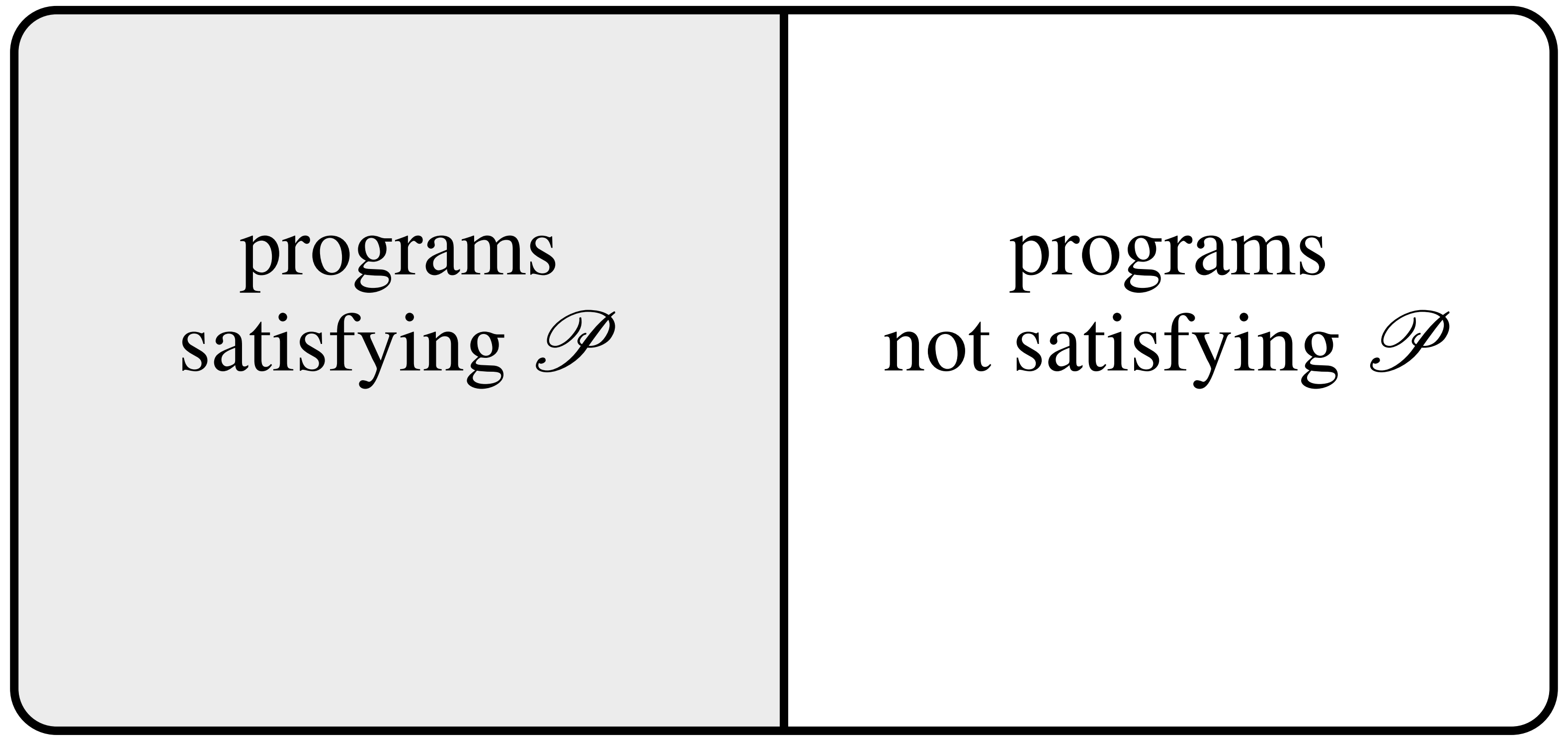
$$\text{For all program p, } \mathbf{A}(\text{p}) = \mathbf{true} \iff \text{p satisfies } \mathscr{P}$$

which consists of

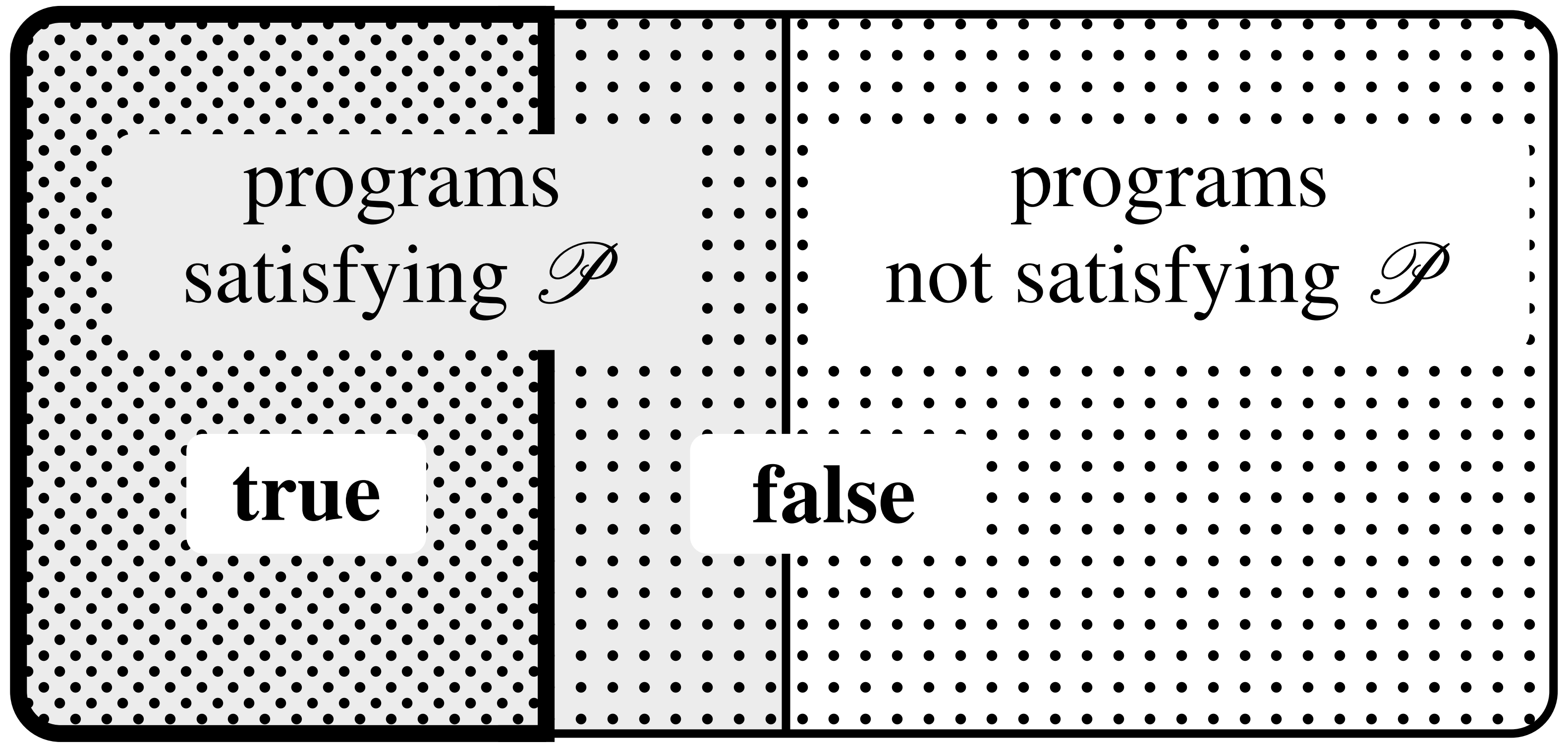
$$\text{For all program p, } \mathbf{A}(\text{p}) = \mathbf{true} \Rightarrow \text{p satisfies } \mathscr{P} \qquad \textbf{(soundness)}$$

$$\text{For all program p, } \mathbf{A}(\text{p}) = \mathbf{true} \Leftarrow \text{p satisfies } \mathscr{P} \qquad \textbf{(completeness)}$$

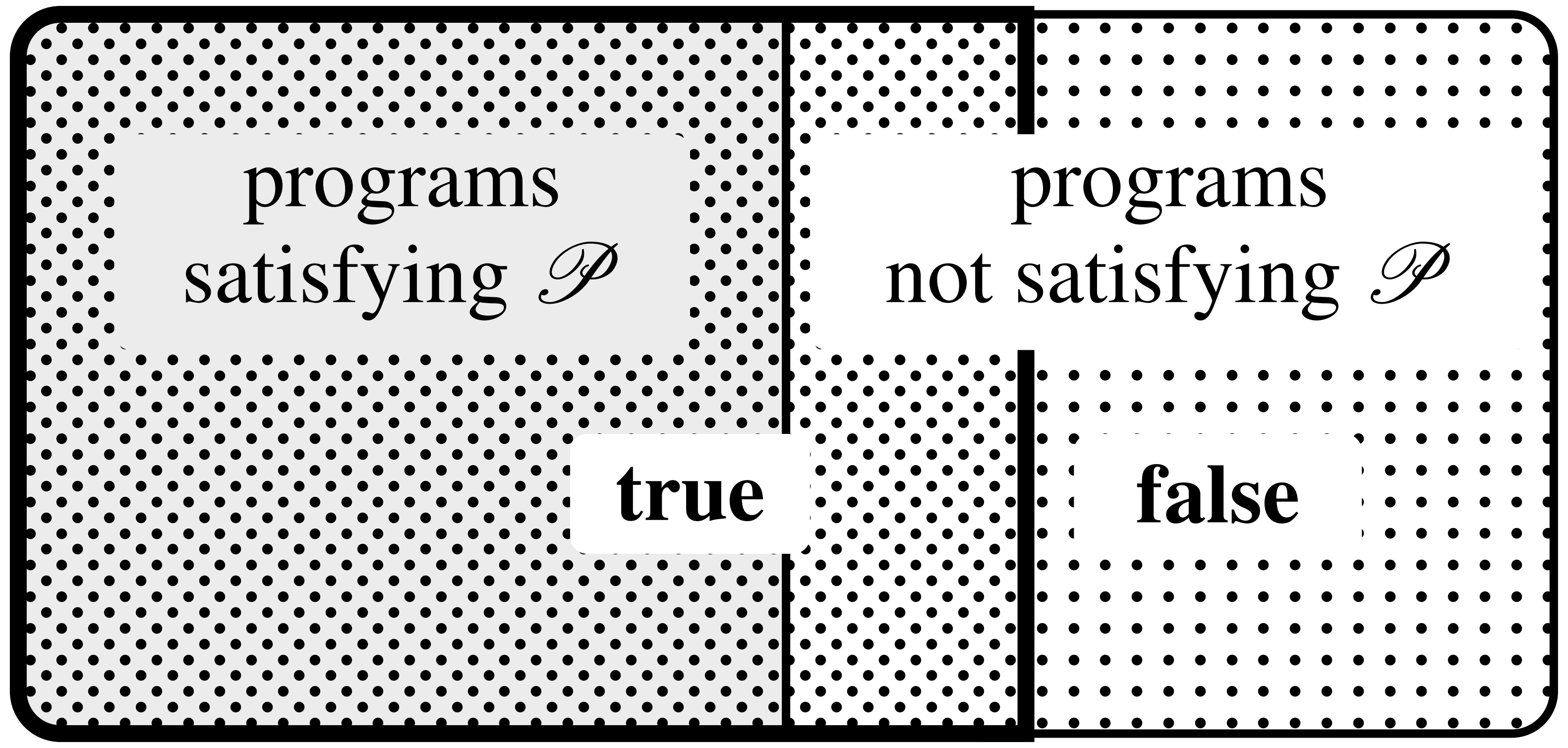
Soundness and Completeness



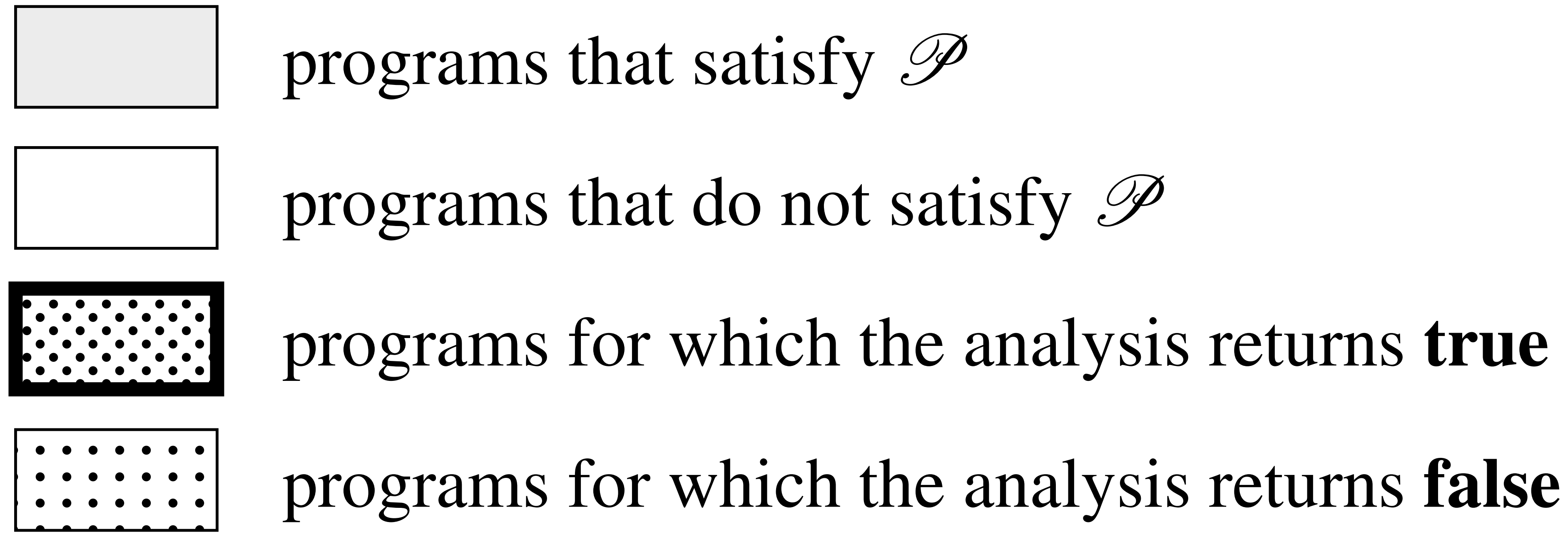
(a) Programs



(b) Sound, incomplete analysis



(c) Unsound, complete analysis



(d) Legend

Program Verification

- Prove a given program satisfies the target properties
  - Loop invariants provided by the user or another program analyzer
- **Sound and complete** if a “good” invariant is provided
- **Sound and incomplete** if an imprecise invariant is provided
- **Sound, complete, yet non-terminating** if the invariant generation does not terminate
- How to describe the target property (specification)?
- How to prove the target property (specification)?

A: Program Logic

Summary

- Property: point of interest in a program (safety, liveness, information flow, etc)
- Program verification: check whether a property is satisfied or not
- Hard limit of program analysis: generally undecidable problem
- Practical solutions:
  - **Manual** rather than **automatic**
  - **Possibly nonterminating** rather than **terminating**
  - **Approximate** rather than **exact**