2024. 05. 14. Paper Presentation Talk

Optimizing Homomorphic Evaluation Circuits by Program Synthesis and Term Rewriting

Presented by Geon Park

KAIST

Programming Systems Laboratory

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Original paper by DongKwon Lee, Woosuk Lee, Hakjoo Oh, and Kwangkeun Yi

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Motivation

- Homomorphic Encryption, by example

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Age	Figure
100	5

Motivation

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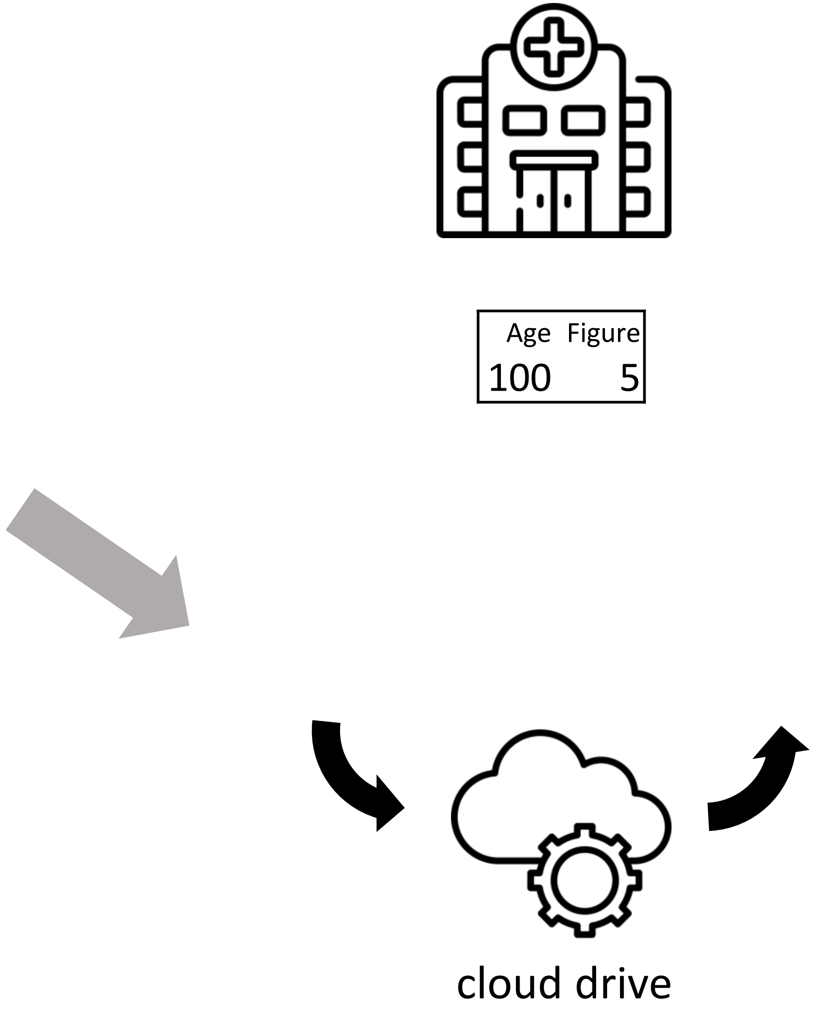

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cloud drive

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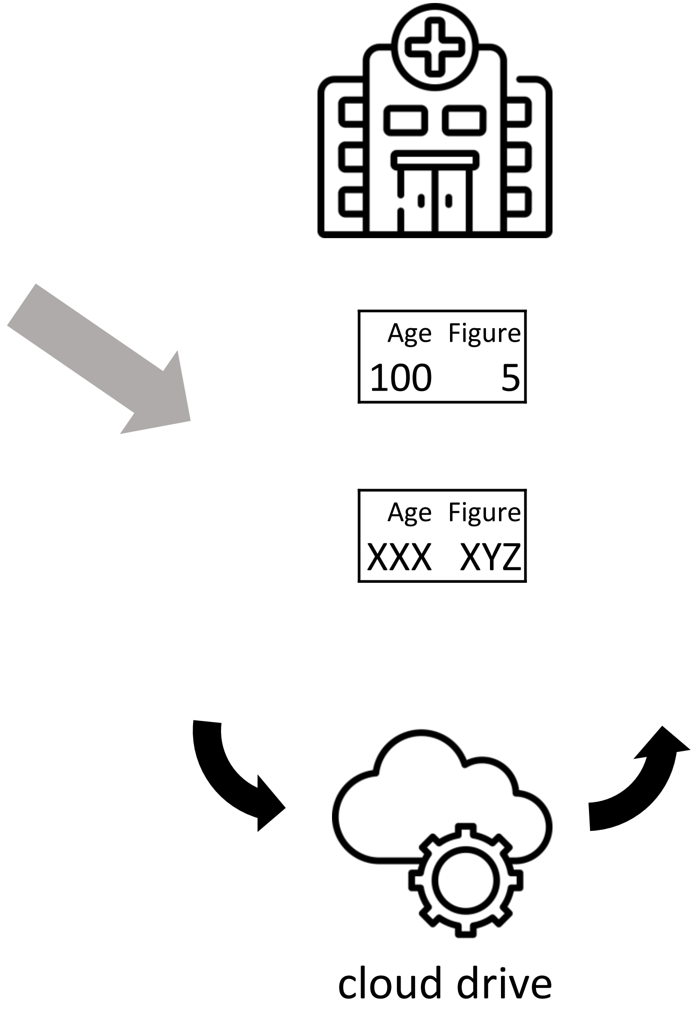

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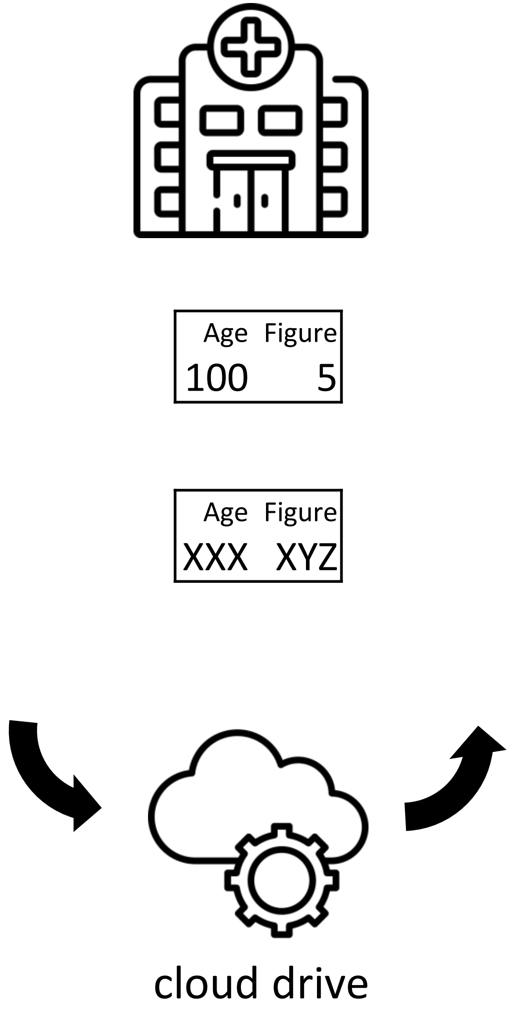

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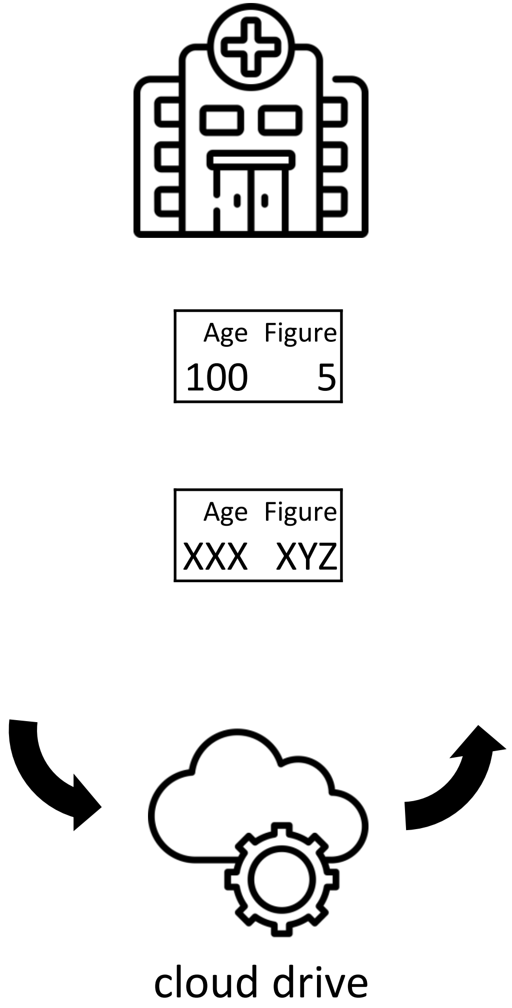
  - Q: Can we do operation on encrypted data?



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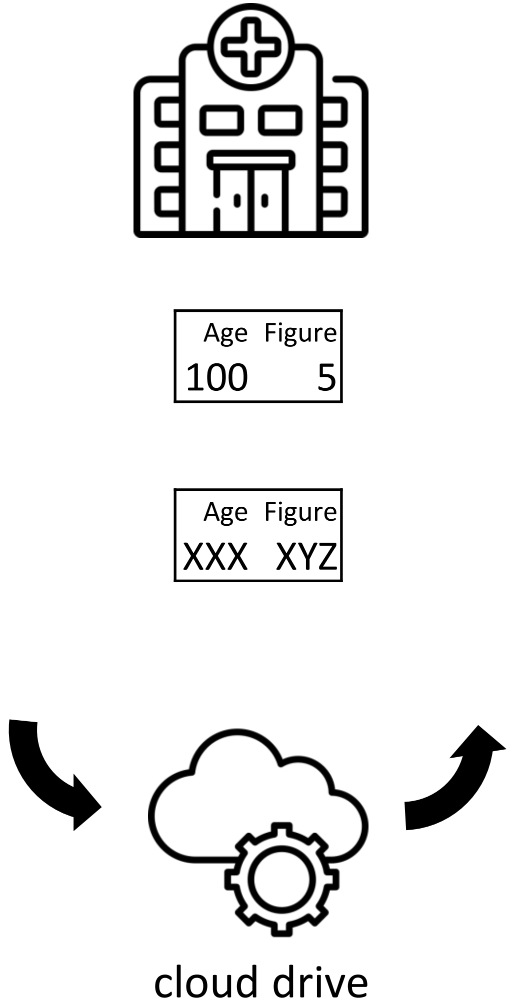
  - A: Yes, for RSA encryption scheme



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        - Decrypting operation (×) result gives same result as done for plaintext

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*Kihong Heo, “CS348 Information Security – Lecture 7”, 2022

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    - Cloud can't decrypt personal data(100) from encrypted data(100^7 % 707)

Age	Figure
100	5

Age	Figure
XXX	XYZ

Age
100
Figure
5

(707, 7)

↑

lock selected by RSA scheme

Age(Enc.)
100^7 % 707
Figure(Enc.)
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cloud drive

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(707, 343)

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Age	Figure
XXX	XYZ

Age
100
Figure
5

(707, 7)

↑

lock selected by RSA scheme

Age(Enc.)
$100^7 \% 707$
Figure(Enc.)
$5^7 \% 707$

(707, 343)

↑

open key selected by RSA scheme

$(\text{Age}_{\text{(Enc.)}} \times \text{Figure}_{\text{(Enc.)}})^{343} \% 707$

$= 100 \times 5$

$= \text{Age} \times \text{Figure}$

(Works because $100 \times 5 < 707$)

cloud drive

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Fully Homomorphic Encryption

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AI and Privacy: The privacy concerns surrounding AI, its potential impact on personal data

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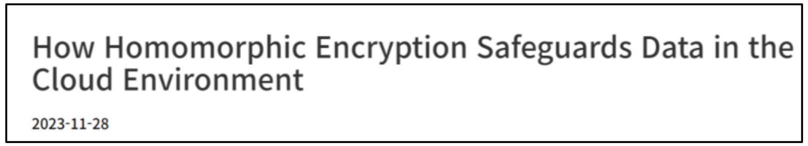
How Homomorphic Encryption Safeguards Data in the Cloud Environment

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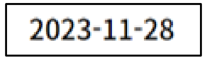
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Cyber Security

Can fully homomorphic encryption solve AI's privacy problem?

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Need of general optimizing technique

- The computational model, called **circuits**, are of great interest

Age	Figure
100	5

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XXX	XYZ

cloud drive

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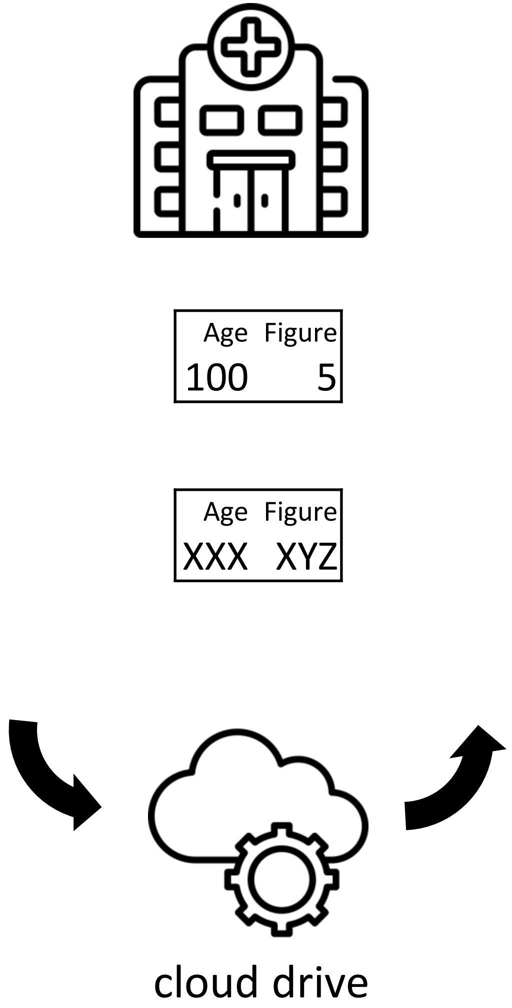

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  - Will be shown in killer example

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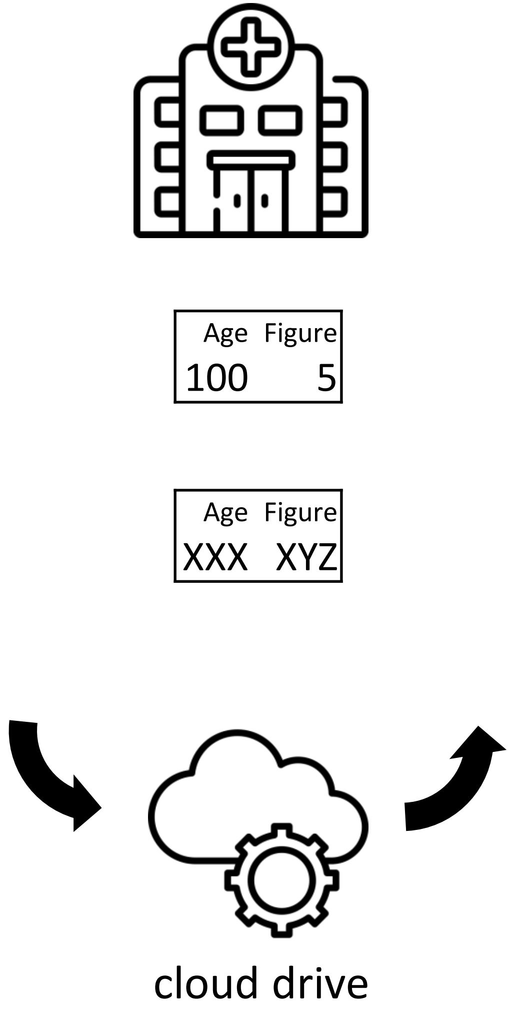

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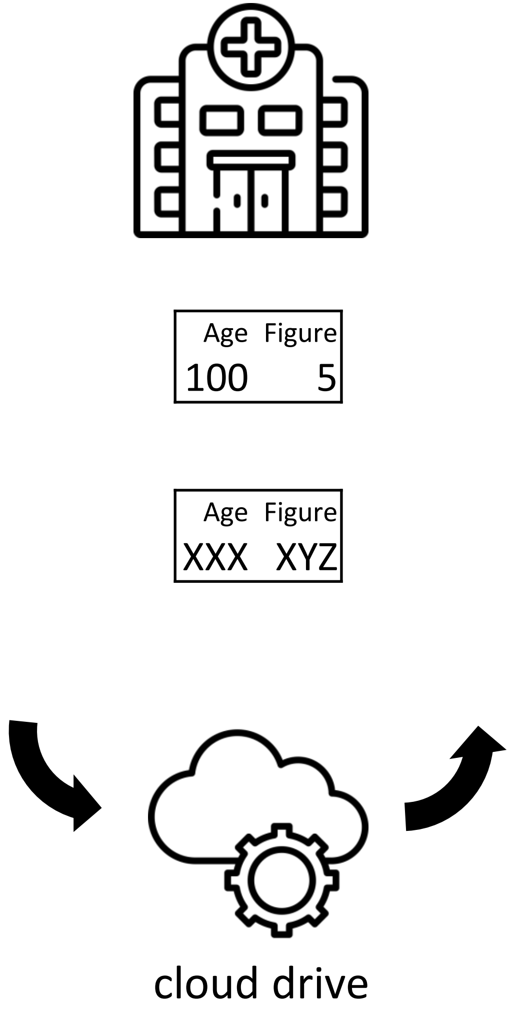

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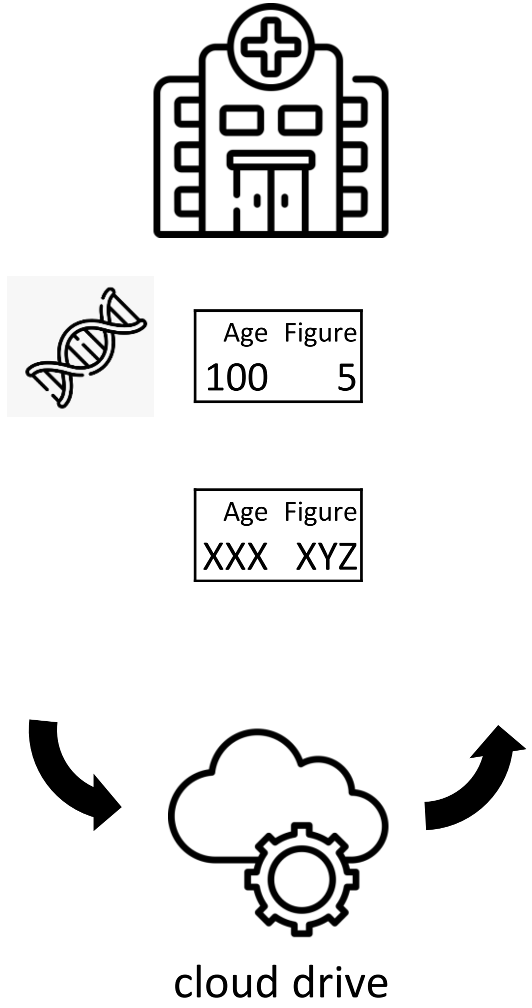

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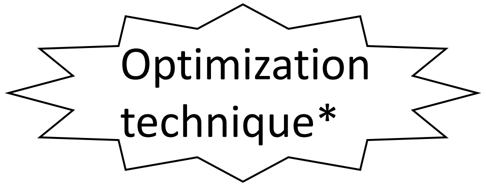

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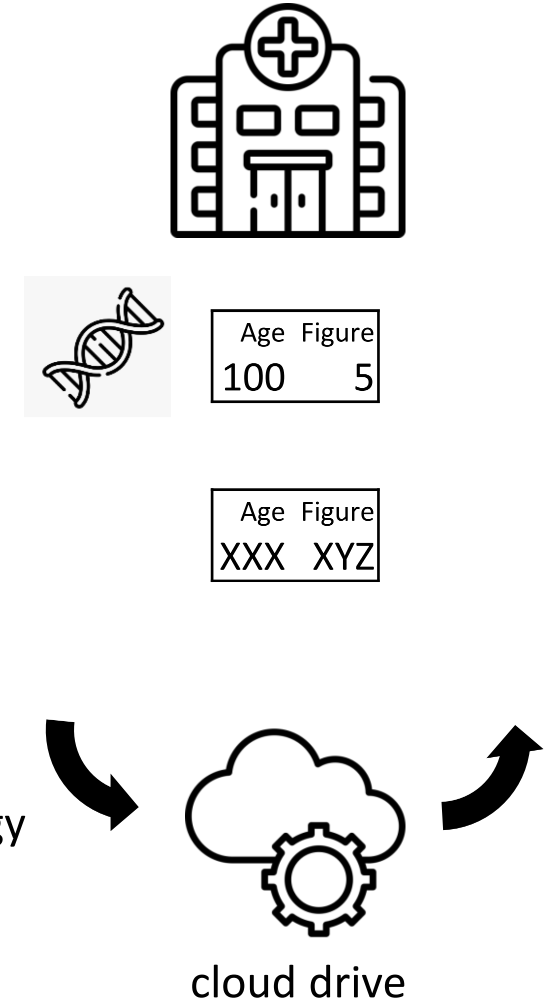

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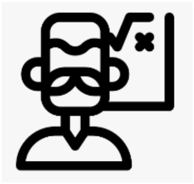

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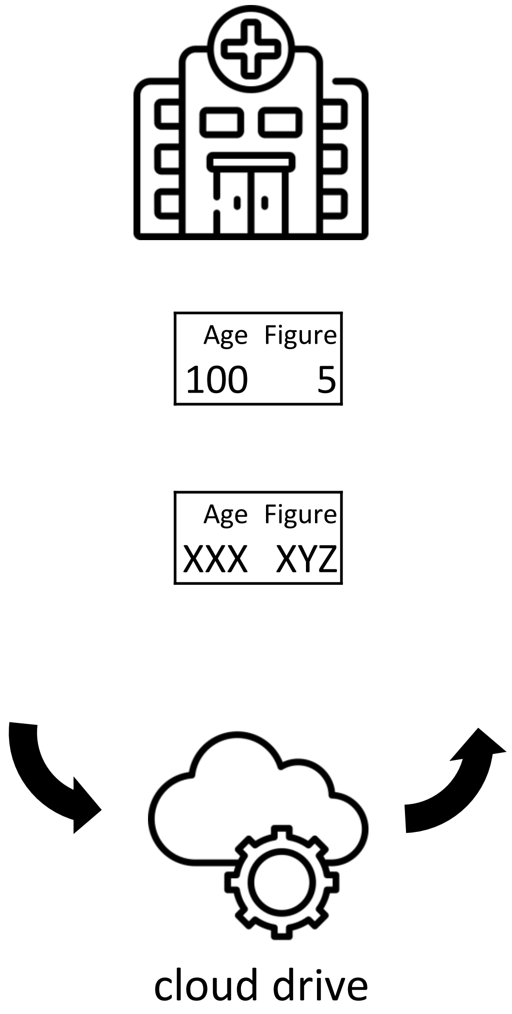

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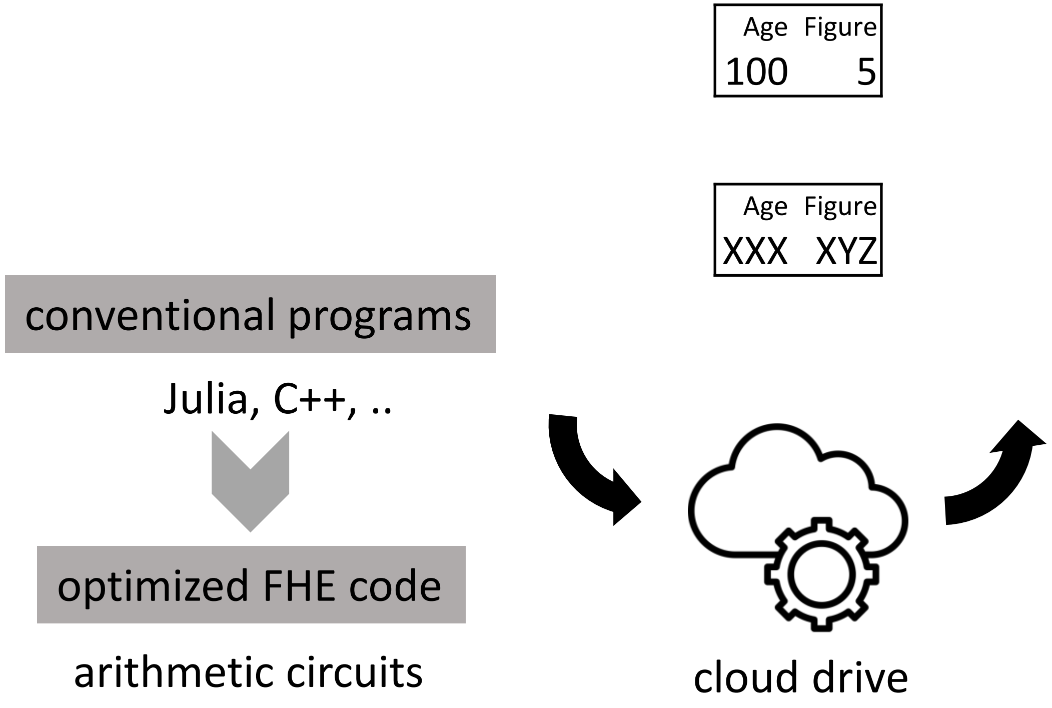

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- One Solution : Domain-specific optimizations
  - But, require an expertise in cryptography
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  - Compiler will equip circuit-to-circuit optimization
  - But, current FHE compilers still favor FHE-friendly programs
  - Also, only hand-written optimization rules yet

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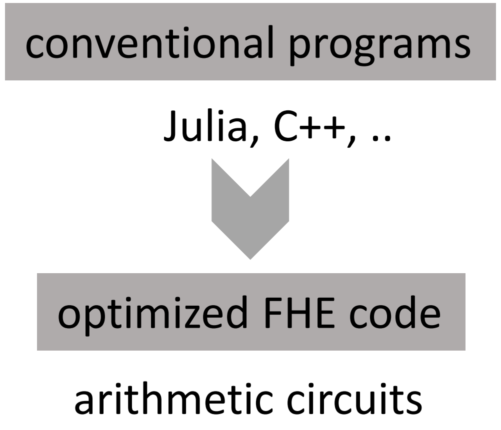

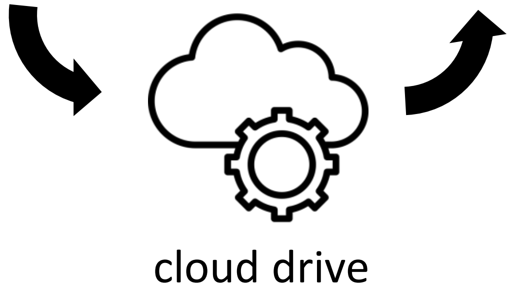

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- **Reduce multiplicative(×) depth** which is the main overhead of program

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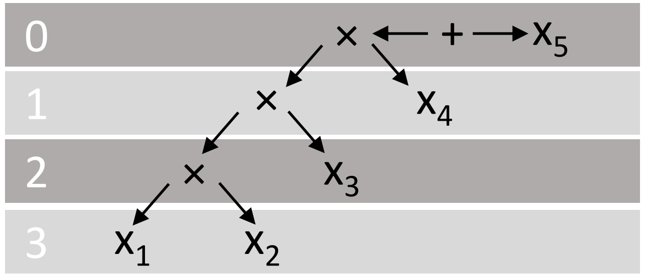
circuit of × depth 3

$c(x_1,x_2,x_3,x_4,x_5) = ((x_1 \times x_2) \times x_3) \times x_4 + x_5$



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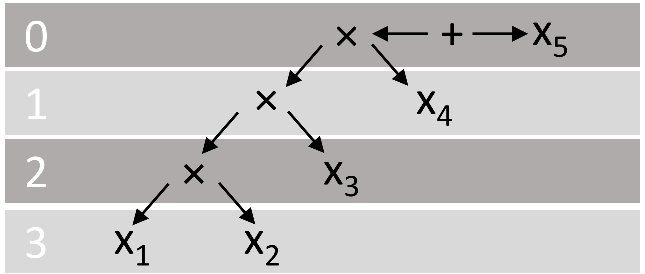
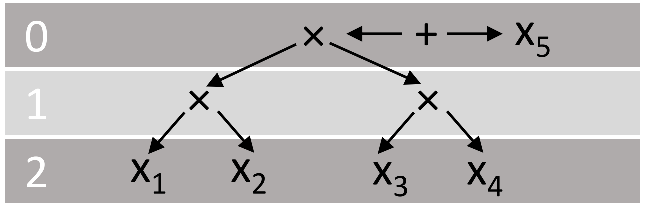


circuit of × depth 2

$c'(x_1,x_2,x_3,x_4,x_5) = (x_1 \times x_2) \times (x_3 \times x_4) + x_5$



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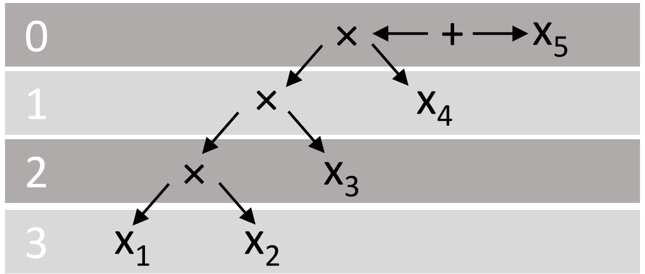

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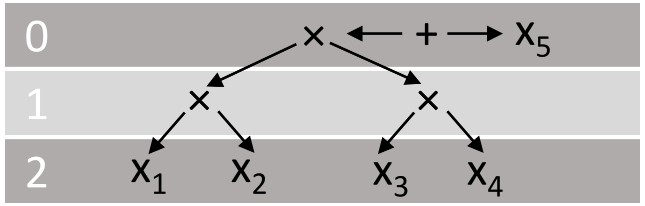

- **Syntax-guided synthesis** to find depth-decreasing rules

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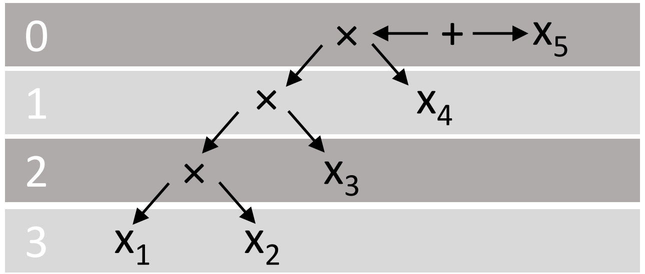

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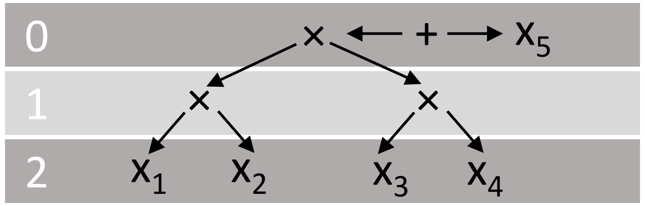

- **Syntax-guided synthesis** to find depth-decreasing rules
- Then, can apply rule generally

Appealing result: A new best attempt

1. Speedup

For 4 sorting benchmarks, average 2x speedup, outperforming state-of-the-art FHE compiler (1.1x)

*Dongkwon Lee et al., Optimizing homomorphic evaluation circuits by program synthesis and term rewriting, PLDI, 2020.

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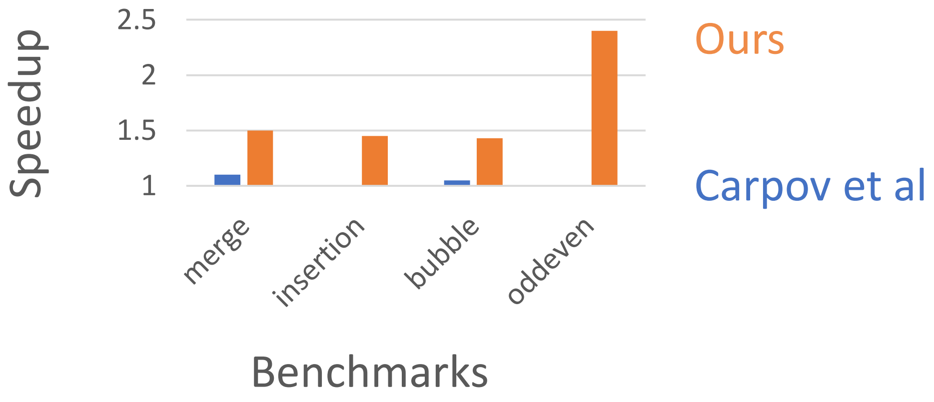

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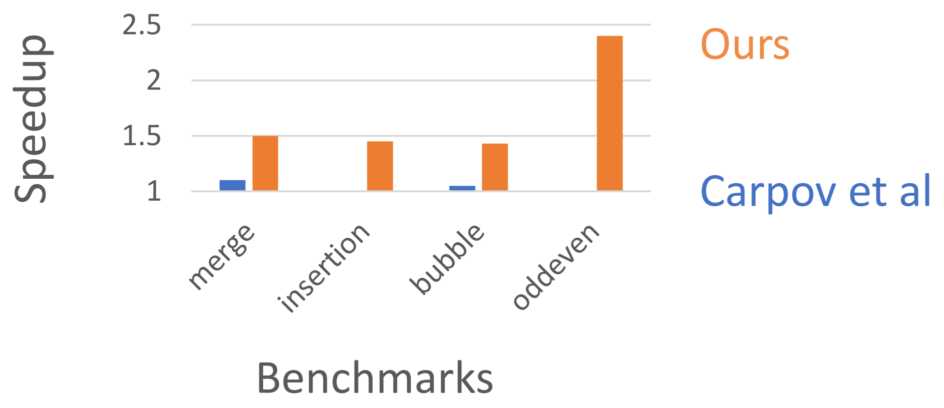

2. Decreasing multiplicative (×) depth

For 4 sorting benchmarks, average 20% decrease, outperforming state-of-the-art FHE compiler (4.4%)

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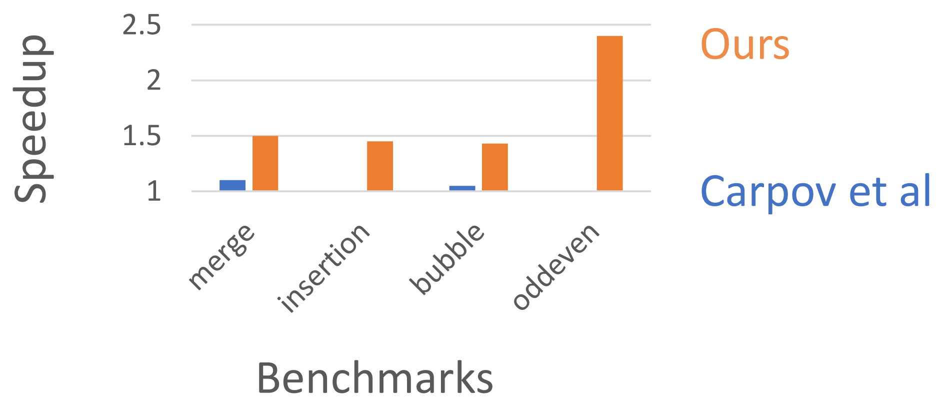

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	Original	**Carpov et al**	**Ours**
merge	45	41	36
insertion	45	45	36
bubble	45	41	36
oddeven	25	25	20

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Killer example – why ×?

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- Observe multiplicative depth is the main overhead
  - The noise which stacks for each operation, grows much higher in × than +

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FHE scheme to encode 1 bit (m)

Killer example – why ×?

- Observe multiplicative depth is the main overhead
  - The noise which stacks for each operation, grows much higher in × than +

FHE scheme to encode 1 bit (m)

- m, + and × are on $Z_2=\{0, 1\}$
- secret key : *p*, random number : *q*, *r* ($r \ll p$)

plaintext

$$m \xrightarrow{\text{Enc}} pq + 2r + m$$

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$m \xrightarrow{\text{Enc}} pq + 2r + m$

$((c \bmod p) \bmod 2) \xleftarrow{\text{Dec}} c$

Killer example – why ×?

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FHE scheme to encode 1 bit (m)

- m, + and × are on $Z_2=\{0, 1\}$
- secret key : *p*, random number : *q*, *r* (r ≪ p)

plaintext

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plaintext

$$m \xrightarrow{\text{Enc}} pq + 2r + m$$

$$((c \bmod p) \bmod 2) \xleftarrow{\text{Dec}} c$$

$$m \xleftarrow{\text{Dec}} pq + 2r + m \qquad (2r + m < 2r + 2 \ll p)$$

$$\begin{aligned} \therefore \mathrm{Dec}(pq + 2r + m) &= (((2r + m) \bmod p) \bmod 2) \\ &= ((2r + m) \bmod 2) \\ &= m \end{aligned}$$

Killer example – why ×?

- Observe multiplicative depth is the main overhead
  - The noise which stacks for each operation, grows much higher in × than +

FHE scheme to encode 1 bit (m)

- m, + and × are on $Z_2=\{0, 1\}$
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plaintext

$m \xrightarrow{\text{Enc}} pq + 2r + m$

$((c \bmod p) \bmod 2) \xleftarrow{\text{Dec}} c$

$m \xleftarrow{\text{Dec}} pq + 2r + m$ ($2r + m < 2r + 2 \ll p$)

$\therefore \text{Dec}(pq + 2r + m) = (((2r + m) \bmod p) \bmod 2)$
$= ((2r + m) \bmod 2)$
$= m$

$c_1 + c_2 = p(q_1+q_2) + 2(r_1+r_2) + m_1 + m_2$

$c_1 \times c_2 =$

Killer example – why ×?

- Observe multiplicative depth is the main overhead
  - The noise which stacks for each operation, grows much higher in × than +

FHE scheme to encode 1 bit (m)

- m, + and × are on $Z_2=\{0, 1\}$
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$((c \bmod p) \bmod 2) \xleftarrow{\text{Dec}} c$

$m \xleftarrow{\text{Dec}} pq + 2r + m$ $(2r + m < 2r + 2 \ll p)$

$$\therefore \mathrm{Dec}(pq + 2r + m) = (((2r + m) \bmod p) \bmod 2)$$
$$= ((2r + m) \bmod 2)$$
$$= m$$

noise after (mod p)

$c_1 + c_2 = p(q_1+q_2) + 2(r_1+r_2) + m_1 + m_2$

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$m \xrightarrow{\text{Enc}} pq + 2r + m$

$((c \bmod p) \bmod 2) \xleftarrow{\text{Dec}} c$

$m \xleftarrow{\text{Dec}} pq + 2r + m$ $\quad (2r + m < 2r + 2 \ll p)$

$$\therefore \mathrm{Dec}(pq + 2r + m) = (((2r + m) \bmod p) \bmod 2) = ((2r + m) \bmod 2) = m$$

noise after (mod p)

$c_1 + c_2 = p(q_1+q_2) + 2(r_1+r_2) + m_1 + m_2$

$c_1 \times c_2 = p(pq_1q_2 + \ldots) + 2(2r_1r_2 + r_1m_2 + r_2m_1) + m_1m_2$

Killer example – why ×?

- Observe multiplicative depth is the main overhead
  - The noise which stacks for each operation, grows much higher in × than +

FHE scheme to encode 1 bit (m)

- m, + and × are on Z_2={0, 1}
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plaintext

$$m \xrightarrow{\text{Enc}} pq + 2r + m$$

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$$\therefore \text{Dec}(pq + 2r + m) = (((2r + m) \bmod p) \bmod 2)$$
$$= ((2r + m) \bmod 2)$$
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noise after (mod p)

$$c_1 + c_2 = p(q_1+q_2) + 2(r_1+r_2) + m_1 + m_2$$

$$c_1 \times c_2 = p(pq_1q_2 + \ldots) + 2(2r_1r_2 + r_1m_2 + r_2m_1) + m_1m_2$$

noise after (mod p)

Structure

- Step 1 - Offline Learning (with training circuits)

Training set

Rule

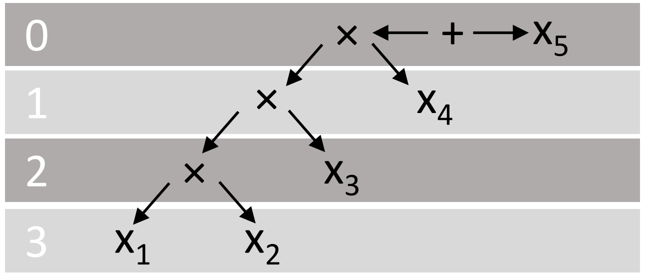
Structure

- Step 1 - Offline Learning (with training circuits)

Training set

- $((x_1 \times x_2) \times x_3) \times x_4 + x_5$
- …

Rule $\emptyset$



Structure

- Step 1 - Offline Learning (with training circuits)

Training set

- $((x_1 \times x_2) \times x_3) \times x_4 + x_5$
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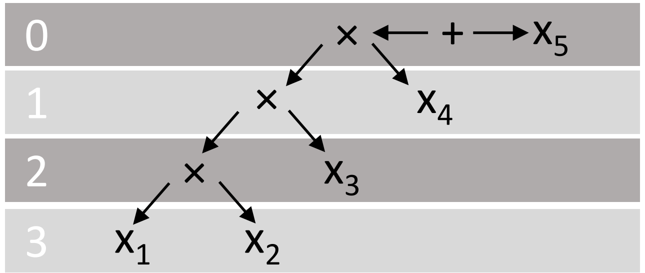
Rule $\emptyset$



nodes of path that leads to deepest part

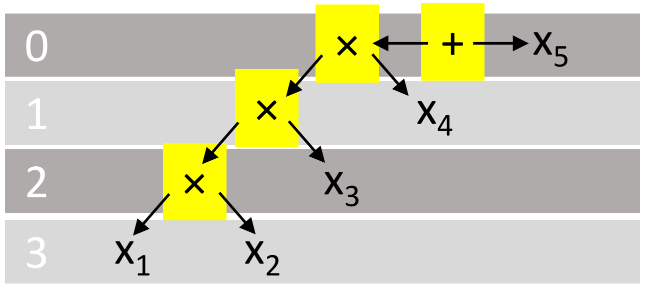
Structure

- Step 1 - Offline Learning (with training circuits)

Training set

- $((x_1 \times x_2) \times x_3) \times x_4 + x_5$
- …

Rule ∅



nodes of path that leads to deepest part

Structure

- Step 1 - Offline Learning (with training circuits)

Training set

- $((x_1 \times x_2) \times x_3) \times x_4 + x_5$
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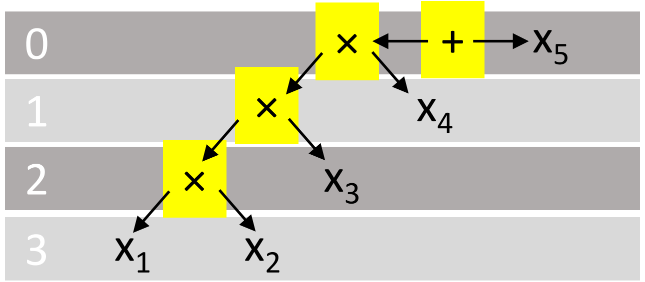
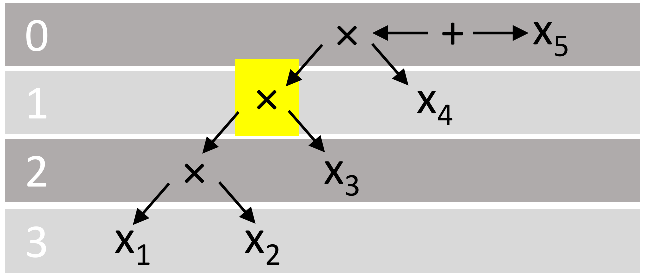
Rule $\emptyset$



nodes of path that leads to deepest part



select any node

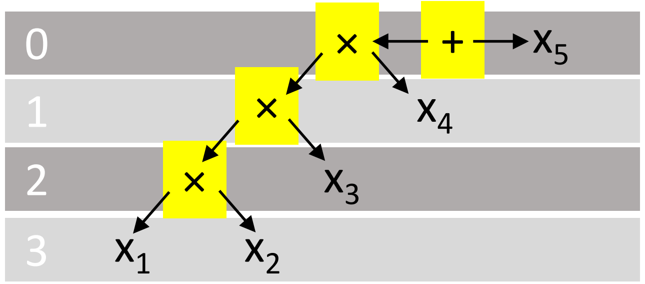
Structure

- Step 1 - Offline Learning (with training circuits)

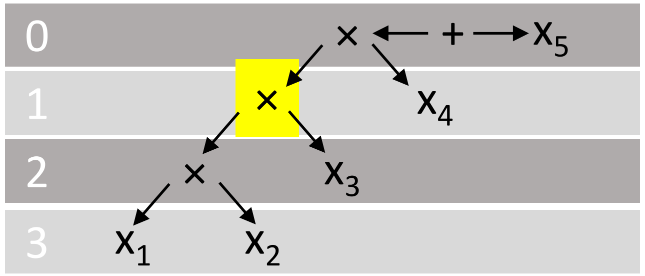
Training set

- $((x_1 \times x_2) \times x_3) \times x_4 + x_5$
- …

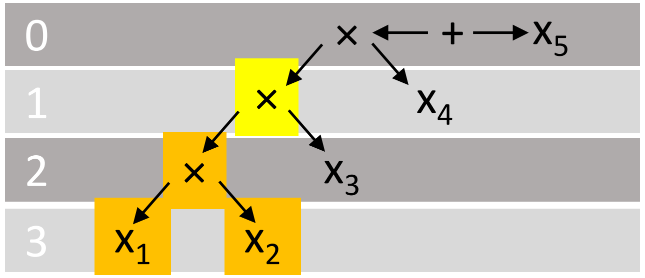
Rule $\emptyset$



nodes of path that leads to deepest part



select any node



select left or right (one with larger depth)

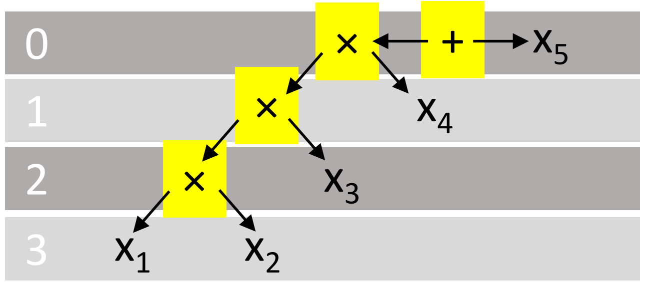
Structure

- Step 1 - Offline Learning (with training circuits)

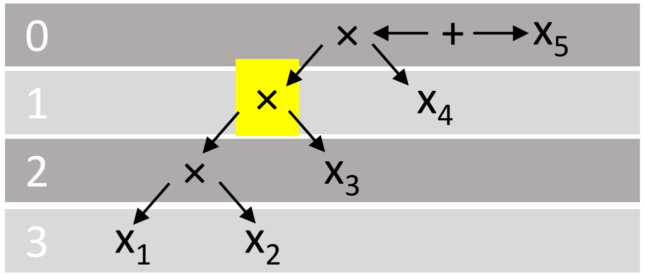
Training set

- $((x_1 \times x_2) \times x_3) \times x_4 + x_5$
- …

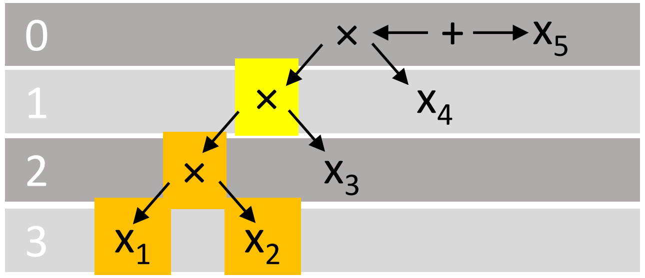
Rule $\emptyset$



nodes of path that leads to deepest part



select any node



select left or right (one with larger depth)

1. $\forall x_1,x_2.x_1 \times x_2 \Leftrightarrow r'(x_1,x_2)$
2. $\text{depth}(r') < \text{depth}(x_1 \times x_2)$

synthesize a rewrite rule if possible

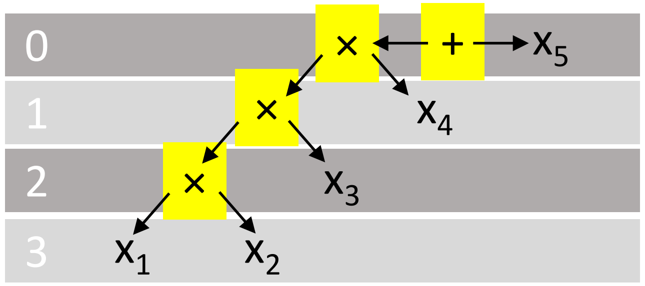
Structure

- Step 1 - Offline Learning (with training circuits)

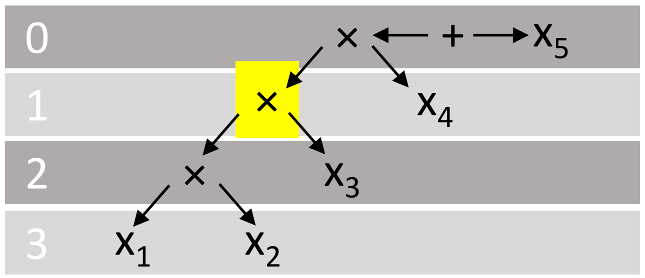
Training set

- $((x_1 \times x_2) \times x_3) \times x_4 + x_5$
- …

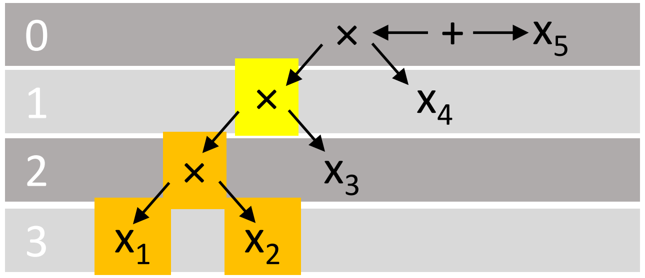
Rule $\emptyset$



nodes of path that leads to deepest part



select any node



select left or right (one with larger depth)

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2. $\mathrm{depth}(r') < \mathrm{depth}(x_1 \times x_2)$

synthesize a rewrite rule if possible

$\perp$

(no rewrite rule)

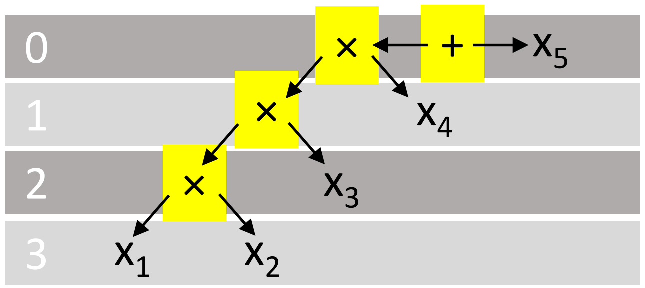
Structure

- Step 1 - Offline Learning (with training circuits)

Training set

- $((x_1 \times x_2) \times x_3) \times x_4 + x_5$
- ...

Rule $\emptyset$



nodes of path that leads to deepest part

select any node

select left or right (one with larger depth)

synthesize a rewrite rule if possible

(found rewrite rule)

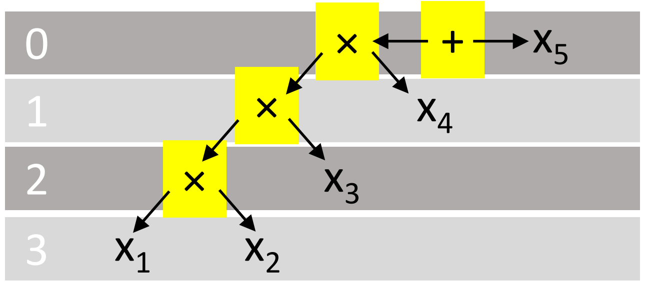
Structure

- Step 1 - Offline Learning (with training circuits)

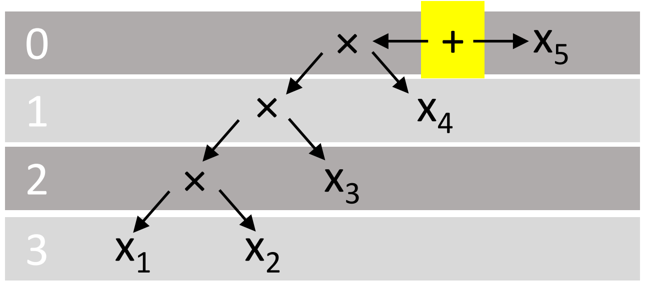
Training set

- $((x_1 \times x_2) \times x_3) \times x_4 + x_5$
- ...

Rule Ø



nodes of path that leads to deepest part



select any node

select left or right (one with larger depth)

synthesize a rewrite rule if possible

(found rewrite rule)

Structure

- Step 1 - Offline Learning (with training circuits)

Training set

- $((x_1 \times x_2) \times x_3) \times x_4 + x_5$
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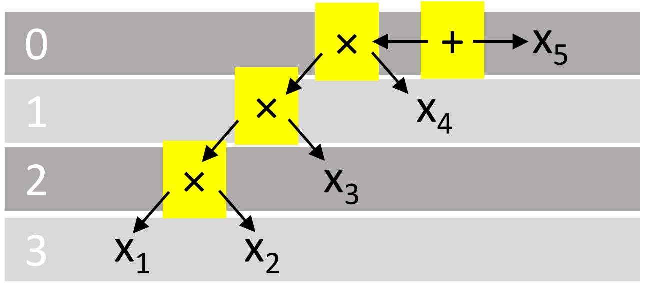
Rule $\emptyset$



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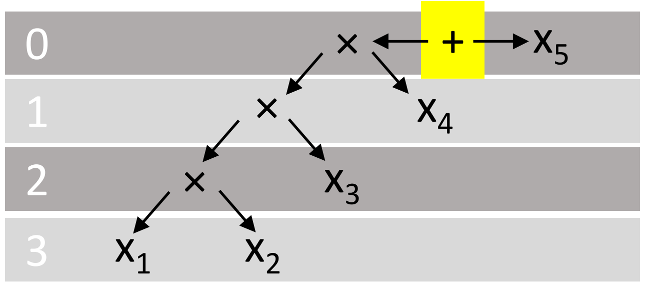
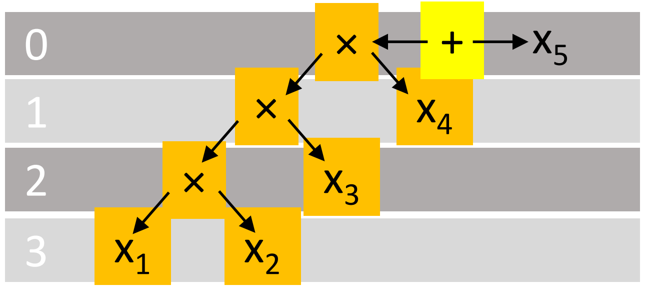


select any node



select left or right (one with larger depth)

synthesize a rewrite rule if possible

(found rewrite rule)

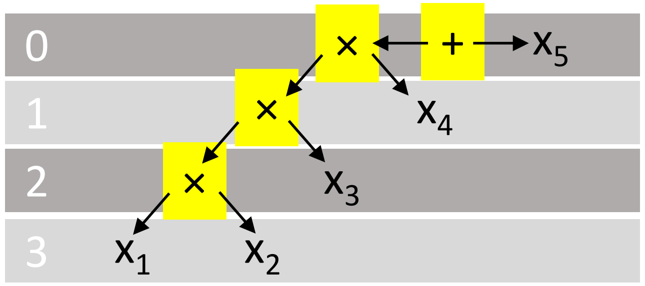
Structure

- Step 1 - Offline Learning (with training circuits)

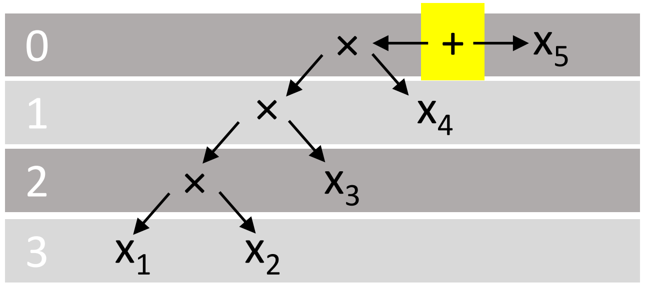
Training set

- $((x_1 \times x_2) \times x_3) \times x_4 + x_5$
- …

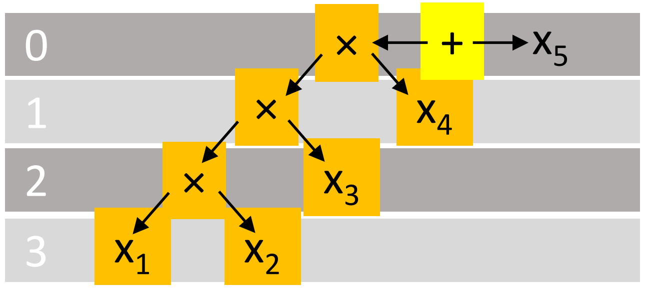
Rule $\emptyset$



nodes of path that leads to deepest part



select any node



select left or right (one with larger depth)

1. $\forall x_i.(((x_1 \times x_2)) \times x_3) \times x_4) \Leftrightarrow r'(x_1,x_2,x_3,x_4)$
2. $\mathrm{depth}(r') < \mathrm{depth}(((x_1 \times x_2)) \times x_3) \times x_4)$

synthesize a rewrite rule if possible

(found rewrite rule)

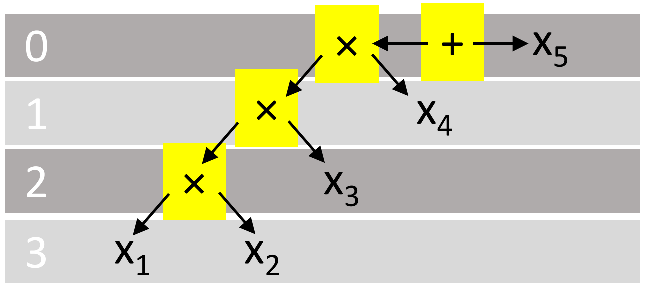
Structure

- Step 1 - Offline Learning (with training circuits)

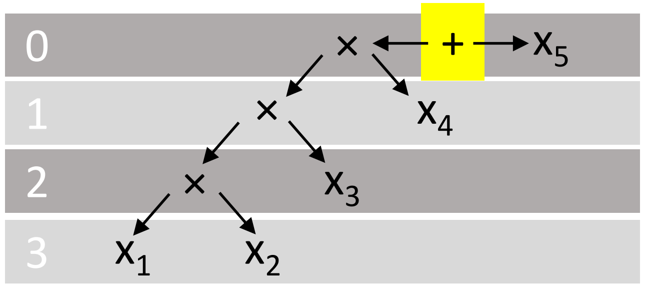
Training set

- $((x_1 \times x_2) \times x_3) \times x_4 + x_5$
- ...

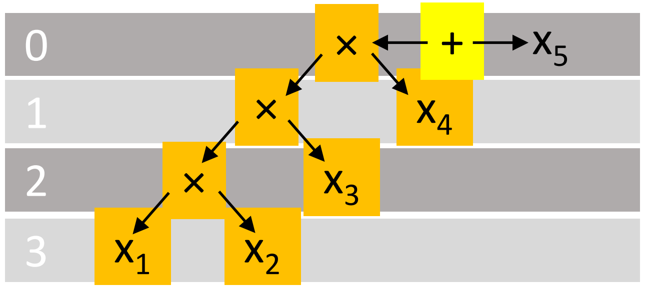
Rule $\emptyset$



nodes of path that leads to deepest part



select any node



select left or right (one with larger depth)

1. $\forall x_i.(((x_1 \times x_2)) \times x_3) \times x_4) \Leftrightarrow r'(x_1,x_2,x_3,x_4)$
2. $\mathrm{depth}(r') < \mathrm{depth}(((x_1 \times x_2)) \times x_3) \times x_4)$

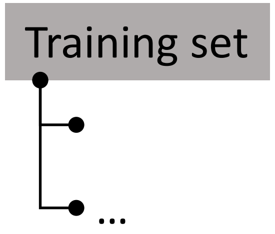
synthesize a rewrite rule if possible

$r' = (x_1 \times x_2) \times (x_3 \times x_4)$

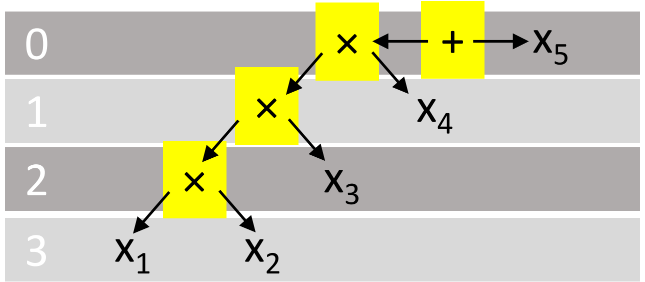
(found rewrite rule)

Structure

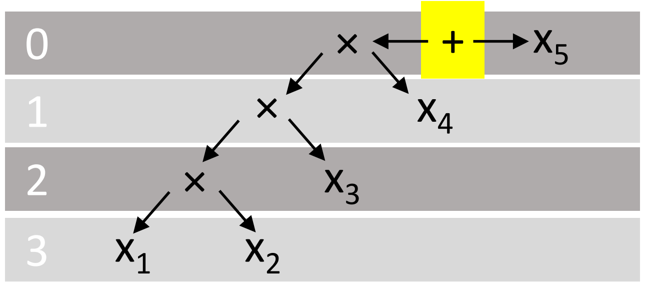
- Step 1 - Offline Learning (with training circuits)



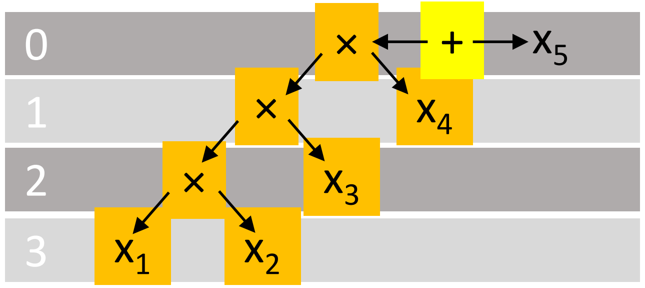
Rule Ø



nodes of path that leads to deepest part



select any node



select left or right (one with larger depth)

1. $\forall x_i.(((x_1 \times x_2)) \times x_3) \times x_4) \Leftrightarrow r'(x_1,x_2,x_3,x_4)$
2. $\text{depth}(r') < \text{depth}(((x_1 \times x_2)) \times x_3) \times x_4)$

synthesize a rewrite rule if possible

$r' = (x_1 \times x_2) \times (x_3 \times x_4)$

(found rewrite rule)

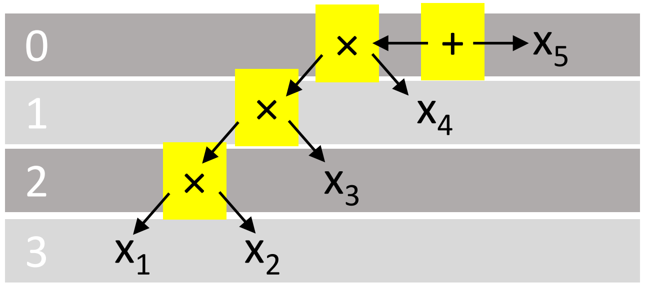
Structure

- Step 1 - Offline Learning (with training circuits)

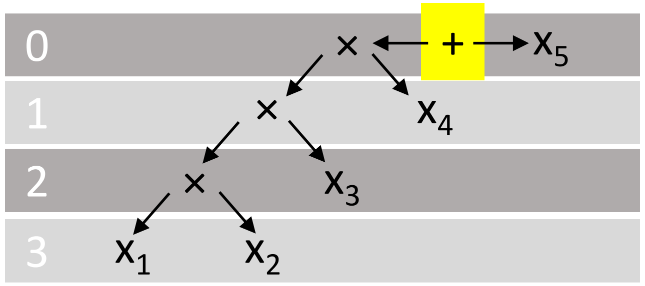
Training set

- $(x_1 \times x_2) \times (x_3 \times x_4) + x_5$
- …

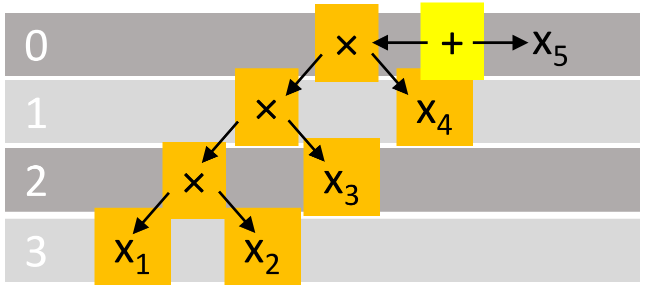
Rule $\emptyset$



nodes of path that leads to deepest part



select any node



select left or right (one with larger depth)

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2. $\mathrm{depth}(r') < \mathrm{depth}(((x_1 \times x_2)) \times x_3) \times x_4)$

synthesize a rewrite rule if possible

$r' = (x_1 \times x_2) \times (x_3 \times x_4)$

(found rewrite rule)

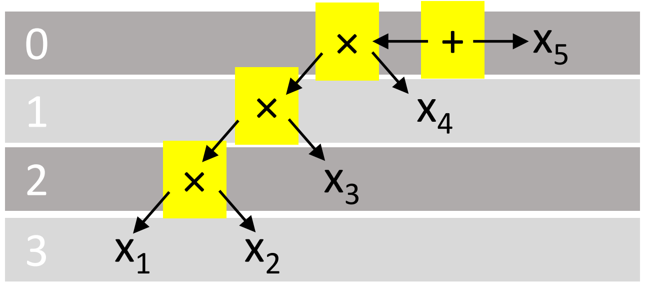
Structure

- Step 1 - Offline Learning (with training circuits)

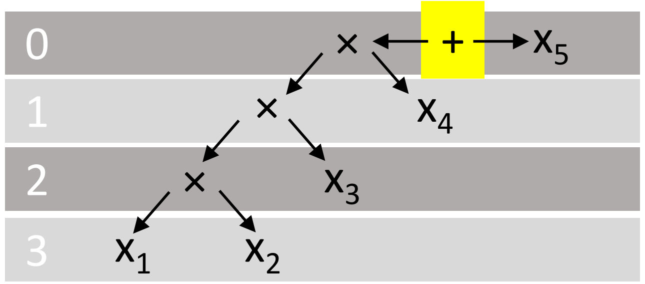
Training set

- $(x_1 \times x_2) \times (x_3 \times x_4) + x_5$
- …

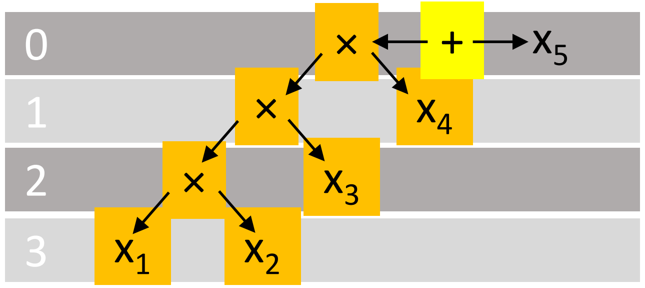
Rule



nodes of path that leads to deepest part



select any node



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2. $\mathrm{depth}(r') < \mathrm{depth}(((x_1 \times x_2)) \times x_3) \times x_4)$

synthesize a rewrite rule if possible

$r' = (x_1 \times x_2) \times (x_3 \times x_4)$

(found rewrite rule)

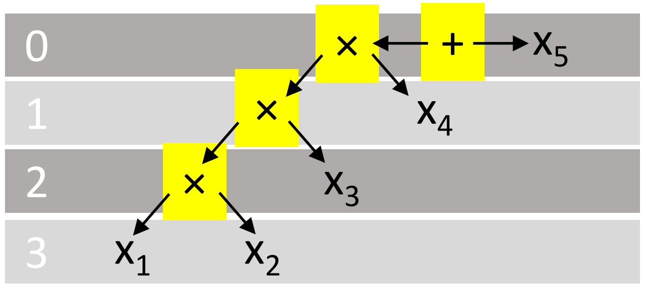
Structure

- Step 1 - Offline Learning (with training circuits)
  - Simplify each rule by changing sub-formula to new variable, then running SAT solver

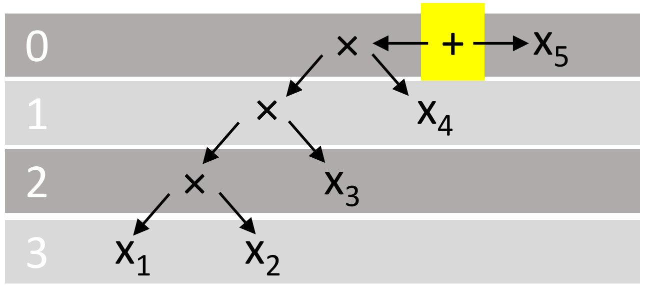
Training set

- $(x_1 \times x_2) \times (x_3 \times x_4) + x_5$
- …

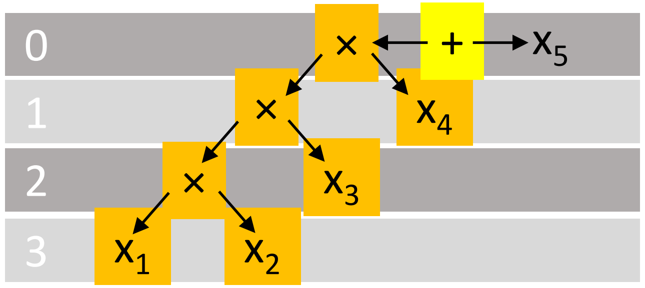
Rule 1 rule (simplified)



nodes of path that leads to deepest part



select any node



select left or right (one with larger depth)

1. $\forall x_i.(((x_1 \times x_2)) \times x_3) \times x_4) \Leftrightarrow r'(x_1,x_2,x_3,x_4)$
2. $\mathrm{depth}(r') < \mathrm{depth}(((x_1 \times x_2)) \times x_3) \times x_4)$

synthesize a rewrite rule if possible

$r' = (x_1 \times x_2) \times (x_3 \times x_4)$

(found rewrite rule)

Structure

- Step 1 - Offline Learning (with training circuits)
  - Simplify each rule by changing sub-formula to new variable, then running SAT solver

Training set

- $(x_1 \times x_2) \times (x_3 \times x_4) + x_5$
- …

Rule 1 rule (simplified)

nodes of path that leads to deepest part

select any node

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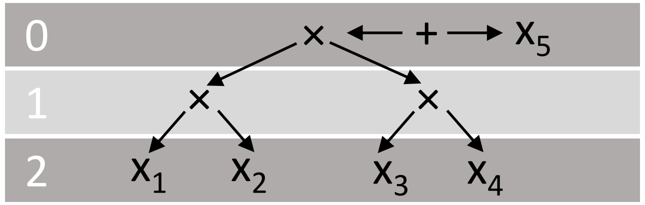

nodes of path that leads to deepest part

start again…

select any node

select left or right (one with larger depth)

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Structure

- Step 2 - Online Optimization
  - Find possible substitution that matches circuit to rule's (LHS)
  - Apply only when depth decreases

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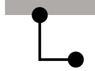

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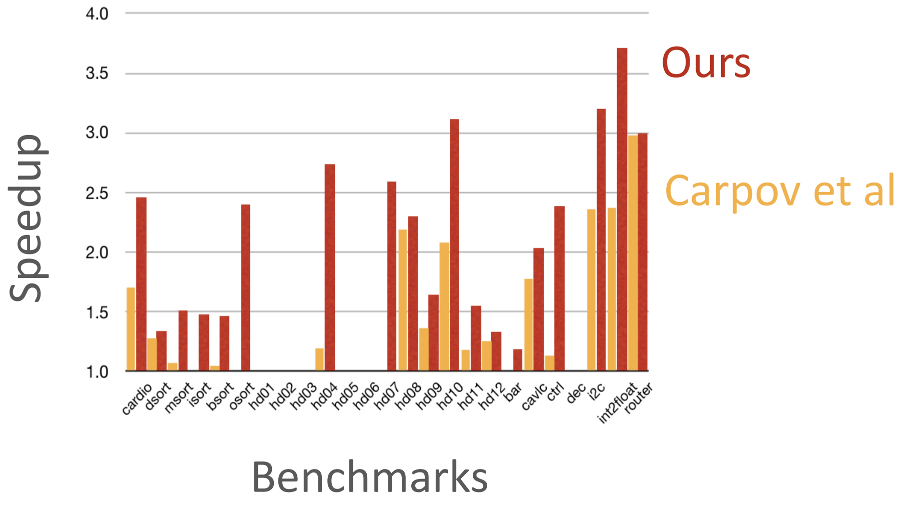

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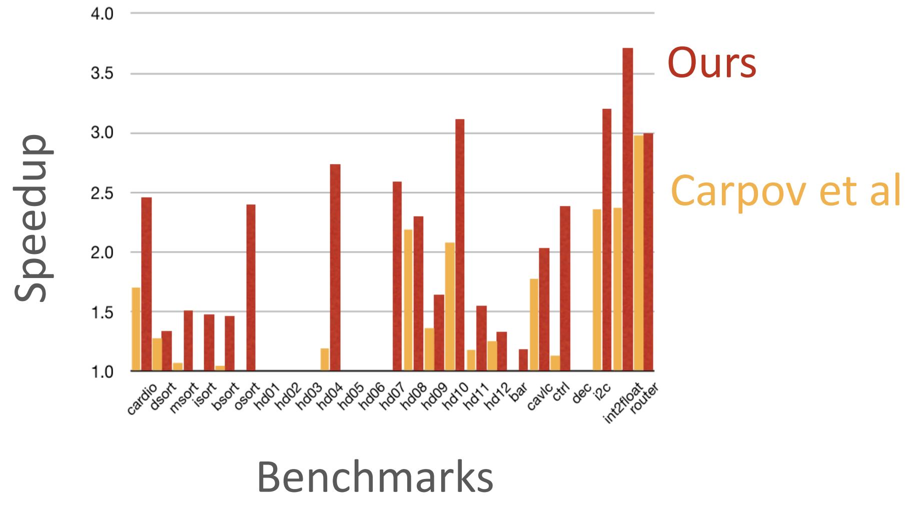

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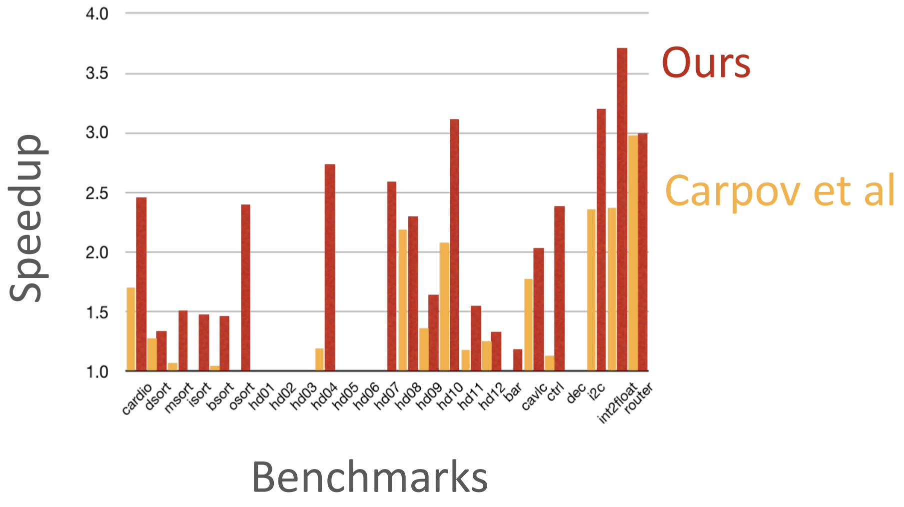

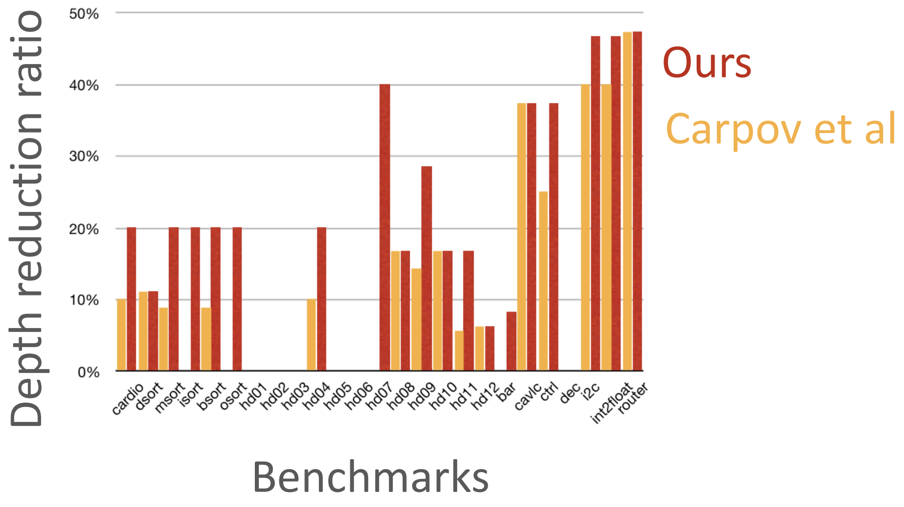

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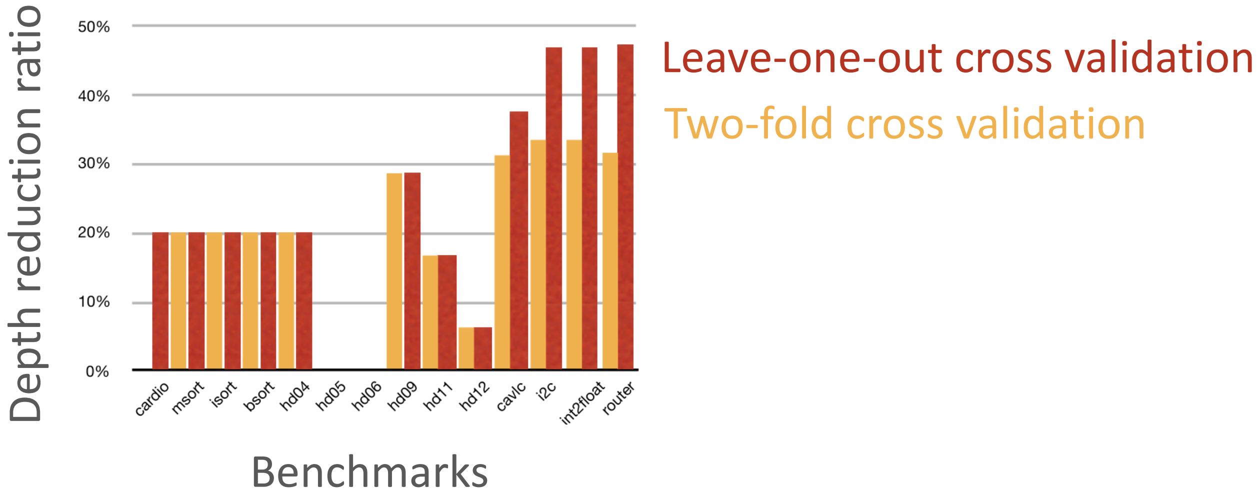

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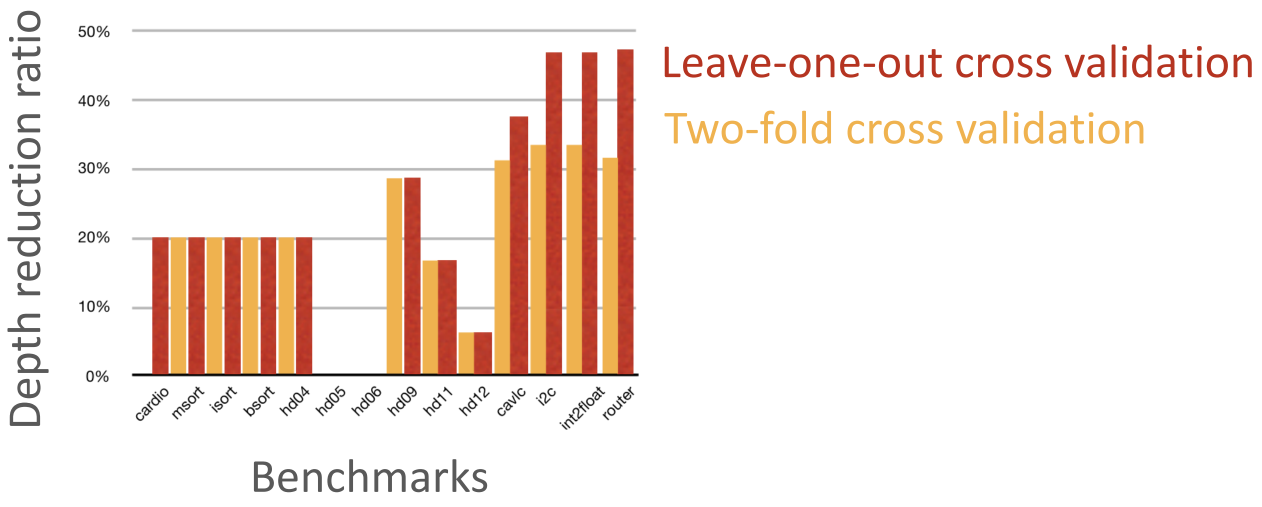

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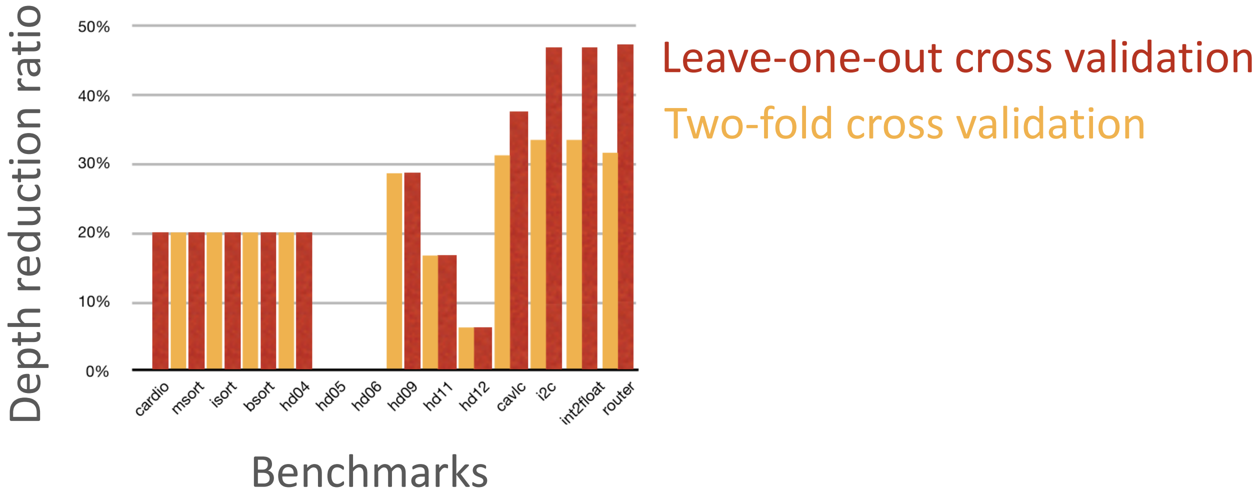
Learning rewrite rules is meaningful



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