Solutions to Exercises

Lab 1 Exercise Solutions

1.1 A letter is chosen at random from the word STATISTICS.

a) P(vowel) = $\frac{3}{10}$
b) The complement of the vowels is clearly the consonants.

P(consonant) = $\frac{7}{10} = 1 - \frac{3}{10}$, using the law of complements.

1.2 The possible outcomes are,

You	Friend1	Friend2	You pay	All Pay
H	H	H		√
H	H	T		
H	T	H		
H	T	T	√	
T	H	H	√	
T	H	T		
T	T	H		
T	T	T		√

a) P(You pay) = $\frac{2}{8} = \frac{1}{4}$

b) P(Share) = $\frac{2}{8} = \frac{1}{4}$

Suppose the bill is £b. If you do this sort of thing often, a quarter of the time you'll pay the whole £b, a quarter of the time you'll pay one third of the bill, i.e. £$\frac{b}{3}$ and the rest of the time you'll pay nothing. Therefore, on average you'll pay

£$\frac{1}{4} \times b + \frac{1}{4} \times \frac{b}{3} + \frac{1}{2} \times 0 = £\frac{b}{3}$. Thus, in theory, this is a fair way to split the bill.

1.3 a) Some trial and error, or use of LCMs, shows that the following values fit the bill,

Event	e_1	e_2	e_3	e_4
Probability	$\frac{1}{8}$	$\frac{2}{8}$	$\frac{4}{8}$	$\frac{1}{8}$

b) If E={e_1,e_3}, then P(E) = $\frac{1}{8}+\frac{4}{8}=\frac{5}{8}$.

1.4 a) A little thought shows that you should have a tree with three branches on the left, and, from each branch, three branches spring. Each branch has probability $\frac{1}{3}$ so each of the possible outcomes has probability $\frac{1}{9}$. The sample space is, S = {$PP,PB,PL,BP,BB,BL,LP,LB,LL$}.

b) Find the probability that,

(i) P(PP) = $\frac{1}{9}$
(ii) P(LL) = $\frac{1}{9}$
(iii) P(at least one profit) = $\frac{5}{9}$.

c) There would be 3^3 = 27 different possible outcomes.

1.5 A set of cards consists of the standard suits ♣, ♦, ♥, ♠ with 13 cards in each suit.

a) Suppose one card is drawn at random. Find the probability that it is

(i) $\mathrm{P}(\heartsuit)=\frac{13}{52}=\frac{1}{4}$
(ii) P(K♠) $=\frac{1}{52}$
(iii) P(Any picture card.)=$\frac{12}{52}=\frac{3}{13}$

b) There are a total of 52×52 = 2704 different ways of drawing two cards with replacement (for each possible 52 cards drawn first, there are 52 possible second draws)

(i) P(Both K$\heartsuit$) $=\frac{1}{2704}$.
(ii) There are lots of ways of drawing two aces, in fact sixteen altogether,ranging from, {(A♠,A♠),(A♠,A♥),...,(A♣,A♦),(A♣,A♣)}. Therefore, P(Draw two aces) = $\frac{16}{2704}=0.0059$

1.6 a) The table should read :

	Male	Female	Total
Agree	30	40	70
Disagree	20	10	30
Total	50	50	100

b) If a student is selected at random, find the probability that they,

(i) P(Agree)= $\frac{70}{100}=0.7.$
(ii) P(Female)= $\frac{50}{100}=0.5.$
(iii) P(Male)= $\frac{50}{100}=0.5.$

(iv) P(Male and Agree)= $\frac{30}{100} = 0.3.$ (v) P(Female and Agree)= $\frac{40}{100} = 0.4.$

1.7 a) The completed table is:

	Stressed	Not stressed	Total
Manager	14	6	20
Shopfloor	62	38	100
Total	76	44	120

b) Assuming an individual is drawn at random, find the probability they are

(i) P(Stressed)= $\frac{76}{120} = 0.633$

(ii) P(Shopfloor worker)= $\frac{100}{120} = 0.833$

(iii) P(Stressed manager)= $\frac{14}{120} = 0.17$

(iv) P(Unstressed Shopfloor worker)= $\frac{38}{120} = 0.32$

1.8 a) ${}^5P_3 = 60$ b) ${}^7P_4 = 840$ c) ${}^6P_4 = 360$

1.9 Require,

$$\frac{(n+1)!}{(n+1-3)!} = \frac{n!}{(n-4)!}$$

$$\Rightarrow \frac{n+1}{(n-2)!} = \frac{1}{(n-4)!}$$

$$\Rightarrow n + 1 = (n-2)(n-3) \Rightarrow n = 5$$

1.10 The books within subjects can be arranged in 4!, 5! and 3! ways, respectively. However, the subjects can be arranged in 3! ways. The total number of permutations is therefore 4! × 5! × 3! × 3! = 103680.

1.11 a) ${}^7C_6 = 7$ b) ${}^5C_3 = 10$ c) ${}^9C_5 = {}^9C_4 = 126$

1.12 a) The man can be selected in one of 8 ways and the women in ${}^6C_4$ ways. The total number is then $8 \times {}^6C_4 = 120$

b) Clearly, ${}^8C_4 \times {}^6C_4 = 1050$

c) ${}^8C_5 \times {}^6C_3 = 1120$

1.13 a) ${}^{10}C_4$

b) (i) There are two cases where we can have a majority of X:

- All four from X: ${}^6C_4$
- Three from X, one from Y: ${}^6C_3 \times {}^4C_1$

Since the two cases cannot happen at the same time, by the rule of sum: ${}_6C_4 + {}_6C_3 \times {}_4C_1$.

(ii) ${}_4C_4 + {}_4C_3 \times {}_6C_1$

(iii) ${}^6C_2 \times {}^4C_2$

1.14 $\frac{{}^3C_2 \times {}^7C_2}{{}^{10}C4} = 0.3$

1.15 Total number of possible orderings = $(2n)!$. Number of ways books can be ordered within subject = $n!$, but subjects can be ordered in two ways on the shelf, hence

$$P(\text{Separated}) = \frac{2 \times n!^2}{(2n)!}$$

1.16 a) Need to solve,

$$\begin{aligned} 0.5 &= 1 - \left(\frac{364}{365}\right)^n \\ \Rightarrow n &= \frac{\ln(0.5)}{\ln(0.9973)} \approx 253 \end{aligned}$$

b) Need to solve

$$\begin{aligned} 1 - \left(\frac{364}{365}\right)^n &= \frac{1}{n} \\ \Rightarrow n &\approx 19 \end{aligned}$$

1.17 a) ${}^{20}C_4 = 4845$

b) ${}^{20}P_4 = 116280$

c) ${}^{26}P_4 \times {}^{10}P_3 = 258336000$

d) ${}^{13}C_4 \times {}^{13}C_3 = 204490$

e) ${}^{15}P_2 \times {}^{13}C_4 = 150150$

f) $3 \times {}^{7}C_3 = 105$

1.18 The total number of possible hands is ${}^{52}C_5$.

a) Only 4 hands give a royal flush, so that P(Royal Flush) = $4/{}^{52}C_5$ = 0.000001539

b) There are only 13 ways of getting four of a kind. The fifth card in thehand can be any one of the remaining 48. Thus, P(Four of a kind) = $13 \times 48/{}^{52}C_5 = 0.00024009$

c) The pairs can be chosen in ${}^{13}C_2$ different ways. Each pair selects any 2 from 4 cards in ${}^{4}C_2$ ways. The remaining card can be any one of 11 different values and there are 4 of each value. Therefore, P(Two pairs) = ${}^{13}C_2 \times ({}^{4}C_2)^2 \times 11 \times 4/{}^{52}C_5 = 0.047539$

d) The values can be chosen in ${}^{13}C_2$ different ways. The triple selects 3 from 4 in ${}^{4}C_3$ ways and the pair selects 2 in ${}^{4}C_2$ ways. However, the triple and pair can also swap, so that P(Full house) = ${}^{13}C_2 \times {}^{4}C_3 \times {}^{4}C_2 \times 2/{}^{52}C_5 = 0.001441$

e) Quite difficult! The pair can be any one of 13 values and it selects any 2 from 4 cards. Hence number of ways of selecting the paired cards is ${}^{13}C_1 \times {}^{4}C_2$. The remaining 3 cards can be chosen from any one of the 12 distinct values not in the pair, but each card can be any one of 4 suits, i.e. ${}^{12}C_3 \times ({}^{4}C_1)^3$ ways. Hence, P(One pair) = ${}^{13}C_1 \times {}^{4}C_2 \times {}^{12}C_3 \times ({}^{4}C_1)^3/{}^{52}C_5 = 0.42257$