Lecture 8 Exercises

8.1 A recursion formula for the Poisson distribution. Show that,

$$P(X = x) = \frac{\mu}{x} \times P(X = x - 1)$$

8.2 A biologist on a field study has found that the number of plant species in a $1m^2$ square follows a Poisson distribution with mean 6.

    a) Find the probability that the number of plant species in a given $1m^2$ quadrant is,

        (i) at least 8

        (ii) less than or equal to 8

        (iii) exactly 8

        (iv) between 6 and 12 inclusive

    b) Find the probability that in a quadrant of area 0.5 m^2, the number of plant species is

        (i) at least 3

        (ii) fewer than 5

        (iii) exactly 4

        (iv) between 3 and 6 inclusive

8.3 When a car leaves a production line it is carefully examined for any signs of imperfection in the paintwork. Previous experience has shown the number of blemishes per car follows a Poisson distribution with mean 0.4.

    a) Find the probability that a car has,

        (i) at least one blemish

        (ii) more than one blemish

        (iii) exactly 1 blemish

        (iv) no blemishes

    b) In 1 hour, an inspector can examine 20 cars. Assuming that blemishes occur independently, find the probability that in 1 hour an inspector finds,

        (i) fewer than 5 blemishes

        (ii) exactly 5 blemishes

        (iii) at least 1 blemish

8.4 A traffic survey found that buses pass a checkpoint at an average rate of 4.5 per hour. Lorries pass the same checkpoint at the rate of 5 per hour and coaches at the rate of 1.5 per hour.

    a) Find the probability that in 1 hour,

      (i) 5 or more buses pass the checkpoint

     (ii) between 10 and 15 lorries inclusive pass the checkpoint

    (iii) fewer than 3 buses pass the checkpoint

  b) Find the probability that,

      (i) at least 8 buses or coaches will pass the checkpoint in one hour

     (ii) exactly 15 buses or coaches will pass the checkpoint in 2 hours

    (iii) ten or fewer buses, lorries or coaches will pass the checkpoint in half an hour.

8.5 In each of the previous questions, use the Poisson formula to evaluate probabilities involving equalities.