# Lecture 4 Exercises

4.1 Suppose that two events $A$ and $B$ are independent. Show that $A$ and $B^c$ are also independent.

4.2 Suppose that two events $A$ and $B$ are independent, show that $A^C$ and $B^C$ are also independent.

4.3 Assume the appropriate sets form an exhaustive partition for $S$.

a) Given the following probabilities, calculate $P(E)$.

$$P(A) = 0.4 \quad P(B) = 0.6 \quad P(E \mid A) = 0.1 \quad P(E \mid B) = 0.05$$

b) Given the following probabilities, calculate $P(E)$.

$$P(A) = 0.6 \quad P(B) = 0.1 \quad P(C) = 0.3$$
$$P(E \mid A) = 0.1 \quad P(E \mid B) = 0.05 \quad P(E \mid C) = 0.05$$

4.4 Forty percent of of new employees hired by a large company have degrees. Seventy percent of employees with degrees are promoted within two years. Of those without degrees, only 30 percent are promoted within two years.

a) What is the probability that a new employee will be promoted?

b) If an employee has been promoted, what is the probabilty that they have a degree?

4.5 A manufacturing company is considering adopting six-sigma techniques for all of its four production lines. At present, the production lines use different quality control techniques and have the following characteristics:

| Line | Quality method | Proportion of Output | Percentage of Defective items |
|---|---|---|---|
| 1 | Six-sigma | 40 | 1 |
| 2 | Control charts | 30 | 3 |
| 3 | Visual inspection | 20 | 5 |
| 4 | None | 10 | 15 |

a) Find the percentage of defective items produced.

b) Given that an item is defective, find the probability that it was produced by each of the four production lines.

4.6 Three chefs prepare food in a busy restaurant. The percentages of meals they cook and praise they receive from diners are summarised in the following table.

| Chef | Percentage of meals | Percentage praised |
|---|---|---|
| Jamie Bolivar | 35 | 64 |
| Gordon Ransom | 25 | 78 |
| Camelia Smith | 40 | 15 |

a) What proportion of meals that are cooked receive praise from diners?

b) Given that a meal received praise, find the probability that it came from each of the three chefs.

4.7 Mr Big is suspected of committing a crime. Your analysis of the evidence so far points to a probability of guilt of 0.9. However, Mr Big demands to take a lie-detector test, that you know has a seventy percent accuracy rate. Thus the characteristics of the test are P(Positive | Guilty) = 0.7, and P(Negative | Innocent)=0.7. Mr Big's test comes back negative.

a) How should you update your probability of guilt for the suspect?

b) Suppose the test had an accuracy of 0.999, how would this have affected your conclusions?

4.8 An urn contains 3 red balls and 5 blue balls.

a) A ball is picked at random and then two balls of the same colour are replaced in the urn, so that the urn now contains 9 balls.

(i) If a second ball is chosen, what is the probability it is red?

(ii) Given that a red ball was chosen second, what is the probability that the first ball chosen was blue?

b) A ball is picked at random and then two balls of the other colour are replaced in the urn.

(i) If a second ball is chosen, what is the probability it is blue?

(ii) Given that a red ball was chosen second, what is the probability that the first ball chosen was blue?

4.9 A bag contains 3 coins; two are normal unbiased coins while the third is two-headed. A coin is chosen at random from the bag and tossed.

a) If heads appears, what is the probability that the two-headed coin has been chosen?

b) If the coin tossed 4 times and heads turned up each time, what is the probability that it is the two-heade coin?

4.10 Approximately 25% of males over 50 have some form of heart problem. A clinic has observed that males with a heart problem are three times as likely to be smokers as males with no heart problem. What is the probability that a male over 50 has a heart problem given that he is a smoker?

4.11 (tricky) Cage A contains five diseased mice and six healthy mice. Cage B contains two diseased mice and five healthy mice. Two mice are chosen at random from cage A and transferred to cage B. A mouse is now chosen at random from cage B and found to be diseased. Find the probability that the two mice that were transferred were,

a) both diseased;

b) both healthy.