Lecture 3 Exercises

3.1 The following data shows the distribution of male and female students on various degree courses at a University.

	Accountancy	Economics	Finance
Male	330	360	90
Female	120	390	60

Suppose a student is selected at random. Find the probability that,

a) they are female,

b) are studying economics

c) are male or studying economics

d) are male given they are studying economics

e) are female given they are studying economics

f) are studying economics given they are female

g) are the events 'student is male' and 'studying economics' independent?

3.2 The following table shows the lung cancer data for females in the same 1950 study given in Example 3.2

	Smoker		
	No	Yes	Total
Lung Cancer	19	41	60
No Cancer	32	28	60
	51	69	120

a) Calculate the RR of cancer for female smokers compared to non-smokers.

b) Can you suggest any reason for the discrepancy in the figures between males and females? (Hint: It's not because women are immune to the effects of smoking. Think about when the study took place).

3.3 Two electrical components, X and Y, have probabilities of working 3/4 and 7/8, respectively. They also function independently of each other. Two devices D_1 and D_2, are constructed. In D_1, X and Y are wired in series and in D_2, they are wired in parallel.

a) (i) Find the probability that D_1 works.
(ii) Find the probability that D_2 works.

b) Suppose D_1 works, find the probability that
(i) X is working,

(ii) only X is working,
(iii) that both X and Y are working

c) Suppose D_2 works, find the probability that
(i) X is working,
(ii) only X is working,
(iii) that both X and Y are working

3.4 An urn contains two green balls and three red balls. Suppose two balls will be drawn at random one after another and without replacement, i.e. the first ball is not returned to the urn after being drawn. Find the probabilities that,

a) A green ball appears on the first draw.

b) A green ball appears in the second draw. (Hint: draw a tree diagram)

3.5 The following table shows the *fear factor* for children attending the dentist.

	School		
	Infant	Junior	Senior
Afraid	0.12	0.08	0.05
Not Afraid	0.28	0.25	0.22

For a child selected at random define the events A={Afraid}, N={Not Afraid}, and I, J, S being the school level in an obvious fashion. Calculate the following probabilities,

a) P(A), P(N), P(A∪I)

b) P(A|I), P(I|A)

c) P(A|S), P(N|S). What do you notice about these two probabilities? Can you offer an explanation for the result?

d) Are A and I independent?

3.6 A survey by an electrical retailer determines that 40% of customers who seek advice from sales staff buy an appliance and only 20% who do not seek advice buy an appliance. If 30% of customers seek advice, what is the probability that a customer entering the warehouse buys an appliance? Hint: draw a tree diagram.

3.7 Four cards are drawn at random without replacement from a deck of 52 cards. What is the probability that the sequence is,

a) ♡♡♣♠

b) ♡♡♣♣

[Hint: use the formula for higher order conditional probability]

3.8 A student comes back from a night at the pub with a bunch of keys, only one of which works. They try one key at random in the lock and discard it if it doesn't fit.

a) Suppose the bunch contains 2 keys. Find the probability they open the door on

(i) the first attempt

(ii) the second attempt

b) Repeat for a bunch of three keys being successful at the first, second and third attempts.

c) Suppose now that the bunch contains n keys. Find the probability that the door is opened on the rth attempt ($1 \leq r \leq n$).

3.9 To ascertain the proportion of people who have had a sexually transmitted disease, the following survey procedure was used on 1000 individuals.

They were asked to think but not divulge which day of the week their most recent birthday fell on. If their last birthday was on a Monday, Tuesday or Wednesday they were to answer the question 'have you ever had a sexually transmitted disease?'

If their last birthday was on a Thursday, Friday Saturday or Sunday they were to answer the question 'Is your age an even number?'

In the survey 290 people answered 'yes'. Assuming that ages and birthdays are randomly distributed, can you estimate the proportion of people who have had a sexually transmitted disease? (Hint: rather like Example 3.7 but with some figures changed).