

Lecture 1 Exercises

1.1 A letter is chosen at random from the word STATISTICS.

- a) What is the probability that it is a vowel?
- b) What is the complement of the event in a)? What is the probability of the complementary event occurring?

1.2 Suppose you are eating in a restaurant with two friends. You agree to pay the bill as follows. Each person tosses a coin. The person who gets a result different from the other two will pay all the bill. If all three tosses are the same, the bill will be shared equally. Find the probability that:

- a) Only you will pay the bill
- b) All three will share the bill

Do you think this is a *fair* way to split the bill?

1.3 One of four students, A, B, C or D, will be awarded a prize for hard work next week, so that the sample space consists of the four elementary events $e_1 = \{A\}$, $e_2 = \{B\}$, $e_3 = \{C\}$ and $e_4 = \{D\}$. You are told that e_1 and e_4 are equally likely, e_2 is twice as likely as e_1 and e_3 is four times as likely as e_1 .

- a) Find the probabilities of each of the elementary events.
- b) If $E = \{e_1, e_3\}$, find $P(E)$.

1.4 An investment can either make a profit (P), break even (B), or make a loss (L), each with the same probability. If two separate investments are made,

- a) List the sample space by drawing a tree diagram.
- b) Find the probability that,
 - (i) both investments make a profit
 - (ii) both investments make a loss
 - (iii) at least one investment makes a profit
- c) How big would the sample space be if three separate investments were made? (don't try to list them all!)

1.5 A set of cards consists of the standard suits ♣, ♦, ♥, ♠ with 13 cards in each suit.

- a) Suppose one card is drawn at random. Find the probability that it is
 - (i) A ♥
 - (ii) The king of ♠
 - (iii) Any picture card.

- b) (getting harder) Suppose two cards are drawn at random, but with the first being replaced and the deck shuffled before the second is drawn (this is called sampling with replacement). Find the probability that

- (i) Both cards are the King of \heartsuit .
(ii) Both cards are Aces.

To answer this question, think about how many elementary events there are and what they consist of. There's no need to try and list them all!!

- 1.6 Fifty male and fifty female students were asked whether they agreed with the statement "Statistics are often misleading". Seventy students, thirty of whom were male, agreed.

- a) Complete the following table:

	Male	Female	Total
Agree			
Disagree			
Total			

- b) If a student is selected at random, find the probability that they,
- (i) Agree
(ii) Are female
(iii) Are male
(iv) Are male and agree
(v) Agree and are female

- 1.7 Interviews with 120 working people revealed that 76 were stressed, 20 were managers and 14 were both managers and stressed.

- a) Complete the following table

	Stressed	Not stressed	Total
Manager			
Shopfloor			
Total			

- b) Assuming an individual is drawn at random, find the probability they are
- (i) Stressed
(ii) A shopfloor worker
(iii) A manager who is stressed
(iv) A shopfloor worker who is not stressed

- 1.8 Evaluate a) 5P_3 b) 7P_4 c) 6P_4 .

- 1.9 For what value of n is ${}^{n+1}P_3 = {}^nP_4$?

- 1.10 Four different mathematics books, 5 different statistics books and 3 different computing books are to be arranged on a shelf. In how many ways can the books be arranged if

- a) The books in each subject must stand together
 - b) Only the statistics books must stand together.
- 1.11 Evaluate a) 7C_6 b) 5C_3 c) 9C_5 d) 9C_4
- 1.12 How many different committees can be formed from 8 men and 6 women if the committee consists of
- a) 1 man and 4 women
 - b) 4 men and 4 women
 - c) 5 men and 3 women
- 1.13 A council consists of 10 members, 6 from Party X and 4 from Party Y.
- a) In how many ways can a committee of 4 be formed?
 - b) In how many ways can a committee of 4 be formed so that;
 - (i) Party X has the majority
 - (ii) Party Y has the majority
 - (iii) Neither party has the majority.
- 1.14 Ten equally qualified assistant managers are lined up for promotion; seven are men and three are women. If the company promotes four of the ten at random, what is the probability that exactly two of the four chosen are women?
- 1.15 Suppose that a library bookshelf contains an equal number, n each say, of mathematics and physics books. If the bookshelf is emptied and the books placed back at random, what is the probability that the books for each subject are separated?
- 1.16 Suppose a room contains n people.
- a) Approximately how large must n be for the probability to be 0.5 that someone shares your birthday
 - b) What value of n makes the probability that someone shares your birthday closest to $\frac{1}{n}$ (Hint: draw the function for some values of n).
- 1.17 A miscellany of combinations and permutations:
- a) From a group of 20 employees, 4 are chosen for promotion. In how many ways can they be chosen?
 - b) From a group of 20 employees, 4 are chosen for promotion, but each to a different role. In how many ways can they be chosen?
 - c) A product code consists of 4 letters followed by 3 digits. How many codes are possible if repetitions are not allowed?
 - d) A 7-card hand is dealt from a normal pack of 52. How many hands will contain 4 clubs and 3 hearts?

- e) How many ways can merit awards be allocated to a group of 15 students if there is one first prize, one second prize and 4 identical third prizes.
 - f) Four students (each with one letter A,B,C,... assigned to them) are to be chosen from a group of 10. If exactly one of A, B or C must be chosen, how many ways are there of choosing the four students?
- 1.18 In the game of poker, five cards from a standard deck of 52 cards are dealt in a hand. Find the probability that a hand contains,
- a) A royal flush (ace, king, queen, jack and 10 of the same suit)
 - b) Four of a kind (e.g. all four 5's)
 - c) Two pairs
 - d) A full house (i.e. three of one kind and two of another)
 - e) One pair