Lecture 2 Exercises

2.1 A sample space consists of seven elementary outcomes e_1, e_2, ..., e_7. Two events are defined as A=$\{e_2, e_4, e_5, e_7\}$ and B=$\{e_1, e_2, e_5\}$.

   a) Draw a Venn diagram to illustrate the events A and B.

   b) Determine the elements of the events

   (i) A∩B
   (ii) B^c
   (iii) A∩B^c
   (iv) A∪B

2.2 A sample space consists of 9 elementary outcomes e_1, e_2, ..., e_9 whose probabilities are:

$$P(e_1) = P(e_2) = 0.08 \quad P(e_3) = P(e_4) = P(e_5) = 0.1$$
$$P(e_6) = P(e_7) = 0.2 \quad P(e_8) = P(e_9) = 0.07$$

Suppose A=$\{e_1, e_5, e_8\}$ and B=$\{e_2, e_5, e_8, e_9)$

   a) Calculate P(A), P(B) and P(A∩B).

   b) Using the addition law of probability, calculate P(A∪B).

   c) List the elements of the event A∪B and calculate P(A∪B) by adding the probabilities of the elementary outcomes.

   d) Calculate P(B^c) from P(B) and also by listing the elementary outcomes of B^c.

2.3 The following table shows the probabilities concerning two events, A and B.

	B	B^c
A	0.25	0.12
A^c		
	0.40	

   a) Determine the missing entries.

   b) What is the probability that A occurs and B does not occur?

   c) Find the probability that either A or B occurs.

   d) Find the probability that one of these events occurs and the other does not.

2.4 If P(A) = 0.6 and P(B) = 0.5, can A and B be mutually exclusive?

2.5 From the probabilities shown in this Venn diagram, determine the probabilities of the following events,

   a) A does not occur

   b) A occurs and B does not occur

   c) Exactly one of the events A and B occurs
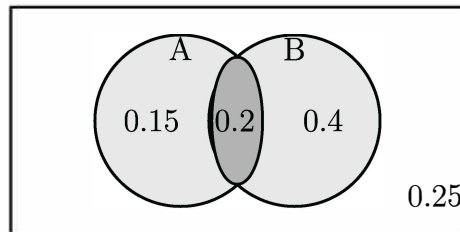
2.6 Fifty workers work on a production line. Of these, 5 produce work late, 6 assemble defective products and 2 produce both late work and assemble defective products. One worker is picked at random, what is the probability that,

   a) They produce work late.

   b) They produce work late and they assemble defective products.

   c) They produce work late or assemble defective products.

   d) Their work is satisfactory in both respects.

2.7 I toss a fair coin and then toss a die.

   a) Are these events independent?

   b) What is the probability I obtain a head and a 6.

2.8 A torch uses two batteries in series. Each battery works with probability 0.95, independently of the other. What is the probability that,

   a) The torch will work

   b) Both batteries fail

   c) Only one of the batteries will work

2.9 The medical records of 100 male diabetic patients reporting to a clinic during one year provided the following classification.

Age of Patient	Light Case Diabetes in Parents		Serious Case Diabetes in Parents	
	Yes	No	Yes	No
Below 40	15	10	8	2
Above 40	15	20	20	10

Suppose a patient is chosen at random from this group and the events A, B and C are defined as,

A: He has a serious case
B: He is below 40.
C: His parents are diabetic.

a) Find the probabilities P(A), P(B), P(A∩B), P(B∩C), P(A∩B∩C).

b) Describe the following events verbally and find their probabilities; $A^c \cap B^c$, $A^c \cup C^c$, $A^c \cap B \cap C^c$

Hint: imagine each patient is represented by a ticket to be drawn at random from a hat, this time with three pieces of information on each one.

2.10 Whether a student gets up on time depends on whether or not he has remembered to set his alarm the night before. 90% of the time he remembers, the other 10% he forgets. When the clock is set, he will get up on time 95% of occasions. If it is not set, the chance that he will oversleep is 35%.

Use a tree diagram to find the probability that he will oversleep.