MA5710 MMI Assignment 01

"CASE STUDY I: Traffic Dynamics - A Micro Mode"

Preamble (as discussed in the class)

$$\text{I} \Rightarrow \quad \boxed{y_i(t) = Vt - (i-1)(d+L)}$$

$\Uparrow$

Displacement associated with the "Equilibrium Speed" V of the i^{th} car at time t.

$$\boxed{\dot{y}_i(t) = V}$$

$$\text{II} \Rightarrow \quad V_1(t) = \begin{cases} V & \text{for } t \leq 0 \\ V(1-b(t)) & \text{for } t > 0 \end{cases}$$

where $$b(t) = \begin{cases} 0 & \text{for } t \leq 0 \\ k t e^{(t_s - t)/t_s} & \text{for } t > 0 \end{cases}$$

$\Uparrow$

Speed of the leading car when perturbed smoothly for shorter time.

t_s is the time at which decelerate stops.

With $x_1(0) = 0$ $\Uparrow$

Point at which decelerate

$$x_1(t) = \begin{cases} vt & \text{for } t \leq 0 \\ v(t - B(t)) & \text{for } t > 0 \end{cases}$$

Where $B(t) = \int_0^t b(s)\,ds$

$\Uparrow$

Corresponding displacement of the Lead Car

where
$$B(t) = \begin{cases} 0 & \text{for } t \leq 0 \\ kt_s\left(t_s - (t+t_s)e^{-t/t_s}\right)e & \text{for } t > 0. \end{cases}$$

$$z_1(t) = x_1(t) - y_1(t)$$
$$= \begin{cases} 0 & \text{for } t \leq 0 \\ -vB(t) & \text{for } t > 0 \end{cases}$$

$\Uparrow$

Perturbation displacement of the Lead Car.

III $\Rightarrow$ To avoid a collision (Remember this model fails after the point where cars collide and after that time onwards), the remaining cars are forced to brake as well.

For $i \geq 2$, the perturbation displacement of the i^{th} car is $\Downarrow$

$$\boxed{\begin{aligned} Z_i(t) &= x_i(t) - y_i(t) \\ &= \begin{cases} 0 & \text{for } t \leq 0 \\ x_i(t) - vt + (i-1)\dfrac{e}{\rho_{max}} & \text{for } t > 0 \end{cases} \end{aligned}}$$

where $\dfrac{e}{\rho_{max}} = d + L$.

Remember $x_{i-1} - x_i > L \Rightarrow$

$$L < x_{i-1}(t) - x_i(t) = Z_{i-1}(t) - Z_i(t) + d + L$$

$$\Rightarrow \boxed{Z_{i-1}(t) + d > Z_i(t)}$$

$\Uparrow$

Violation of this Condition $\Rightarrow$ Collision and Model is NOT VALID

From

$$V_i(t+\tau) = V \ln\left(\ell_{max}\left(x_{i-1}(t) - x_i(t)\right)\right)$$

Taking

$$x_i(t) = y_i(t) + z_i(t)$$

$$V_i(t+\tau) = \frac{d x_i(t+\tau)}{dt} = \frac{d}{dt}\left[y_i(t+\tau) + z_i(t+\tau)\right]$$

$$= V \ln\left\{\ell_{max}\left[y_{i-1}(t) + z_{i-1}(t) - y_i(t) - z_i(t)\right]\right\}$$

$$\Downarrow$$

$$V + \frac{d}{dt} z_i(t+\tau) = V \ln\left\{\ell_{max}\left[\frac{e}{\ell_{max}} + z_{i-1}(t) - z_i(t)\right]\right\}$$

for $2 \le i \le N$ with initial conditions

$z_i(t) = 0$ for $t < 0$, $1 \le i \le N$

and $z_1(t)$ as given in Page 2

$$\Uparrow$$

Speed of Perturbation Final Model

"Delay-Differential Equation"

Question 1 Construct Explicit Solution by substituting $t \to t-\tau$ in the above model of Delay-DE on the intervals $[-\tau, 0)$, $[0, \tau)$ ... $[k\tau, (k+1)\tau)$ ...

What is the Limitation of this Solution Procedure?

Question 2 Using Euler's method* and Runge-Kutta 4th order* method to solve the model (Delay-DE) with the following Data: Cars are travelling at an equilibrium Speed of 100 Km/hr with a distance of about 3 car lengths between successive vechicles. Take $L = 6\,m$, with the choice of $\rho_{max} = 40$ Cars/Km, So that $d = 19\,m$. Consider $N = 5$ and that the lead vechicle momentarily decelerate from

* Use MATLAB Tool Box.

100 Km/hr to 100 (1−k) Km/hr over an interval of $t_s = 1$ Sec according to

$$\boxed{b(t) = k\,t\,e^{(t_s - t)/t_s}}$$

Notice that in this Case the asymptotic value of the perturbation displacement is $v k t_s^2 e$. For $k = 0.2$, $v k t_s^2 e \simeq 15.1$ m or nearly 2.5 Car lengths.

⇒ *Plot the Results for three different reaction time $\tau = 0.5$, 1.5 and 2 Seconds where k takes on the values $k = 0.1$ and 0.2. (*Plot $z_i(t) - (i-1)d$).

⇒ Realize no Collision $z_i < z_{i-1} + d$ and Collision when this Condition is Violated.

⇒ At what position (i.e. which Car) and at what time Collision occurs? (So the above Condition is Violated)

Question 3 Suppose we change the model so that the acceleration is proportional to $\dot{x}_{i-1}(t) - \dot{x}_i(t)$ and inversly proportional to the density. (1) Using the Forward Euler approximation, derive the resulting system of differential-delay equations and perform a numerical simulation. ~~How~~ do

(2) How do your results compare to the situation when the accelration is proportional to the density as modelled in the class?