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MAS710_JN2021_Assignment 1

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Question One

from the discussion in classes, and the problem statement we know that Delay DE is given as:

$$v + \frac{d}{dt} z_i(t+\tau) = v \ln \left\{ \rho_{max} \left[\frac{e}{\rho_{max}} + z_{i-1}(t) - z_i(t) \right] \right\} \Rightarrow \boxed{\text{eqn } 1.1}$$

for $2 \le i \le N$ and $t > 0$.

$z_i(t) = 0$ for $1 \le i \le N$ and $t < 0$.

from the perturbation displacement of the lead car, we know:

$$z_1(t) = \begin{cases} 0 & \text{for } t \le 0 \\ -vB(t) & \text{for } t > 0. \end{cases} \Rightarrow \boxed{\text{eqn } 1.2}$$

where $B(t) = \int b(s) ds$.

~~Modifying~~ since the question asks to substitute $t \to t - \tau$ in intervals $[-\tau, 0]$, ------ $[k\tau, (k+1)\tau]$ we can write eqns 1.1 and 1.2 as:

$$v + \frac{d}{dt} z_i(t) = v \ln \left\{ \rho_{max} \left[\frac{e}{\rho_{max}} + z_{i-1}(t-\tau) - z_i(t-\tau) \right] \right\}$$

modifying this eqn.

$$\cancel{v + \frac{d}{dt} z_i(t) = v \ln \left\{ e + \rho_{max} [z_{i-1}(t) - z_i(t)] \right\}} \quad \rho_{max}[z_{i-1}(t-\tau) -$$

modifying the eqn :

$$v + \frac{d}{dt} z_i(t) = v \ln\left\{ e + s_{max}\left[z_{i-1}(t-\tau) - z_i(t-\tau) \right] \right\}$$

$$v + \frac{d}{dt} z_i(t) = v\ln e + v \ln\left\{ 1 + \frac{s_{max}}{e}\left[z_{i-1}(t-\tau) - z_i(t-\tau) \right] \right\}$$

$$\therefore \frac{d}{dt} z_i(t) = v \ln\left\{ 1 + \frac{s_{max}}{e}\left[z_{i-1}(t-\tau) - z_i(t-\tau) \right] \right\}$$

$\because \ln e = 1$

conditions: $z_i(t-\tau) = 0$ for $t<\tau$.

eqn 1.3

we also know $z_1(t)$ from eqn 1.2 at any values of t.

$$z_1(t-\tau) = \begin{cases} 0 & t \le \tau \\ -vB(t-\tau) & t \ge \tau \end{cases}$$

$\Rightarrow$ eqn 1.4

from the eqn 1.1 itself, for $t \in [-\tau, 0)$

we get $z_i(t) = 0$ for all $1 \le i \le N$

~~Now, let's say we want to find $z_2(t)$ for $t \in [0,\tau]$~~

(I) let's find $z_2(t)$ first.

(i) considering the interval $[0,\tau]$:

$t-\tau \in [-\tau, 0)$

$z_2(t-\tau)$ where $t-\tau \in [-\tau, 0)$ = 0.

we get $$\frac{d}{dt} z_2(t) = v \ln\left\{ 1 + \frac{s_{max}}{e}\left[z_1(t-\tau) \right] \right\}$$

eqn 1.5

in $t \in [0,\tau)$: $t-\tau \in [-\tau; 0)$

$\Rightarrow z_1(t-\tau) = 0$

substituting this, we get:

$$\frac{d}{dt} z_2(t) = v \ln\left\{ 1 + \frac{s_{max}}{e}(0) \right\} = 0$$

$$\frac{d}{dt} z_2(t) = 0 \; ; \; \boxed{z_2(t) = k}$$

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since $z_2(t)$ = continuous and $z_2(t) = 0 = k$ for $t \in [-\tau, 0)$ we get $z_2(t) = 0$ for the range $t \in [0, \tau)$.

$$\therefore \boxed{z_2(t) = 0 \qquad t \in [0, \tau)}$$

(iii) consider interval $[\tau, 2\tau)$:

$$\frac{d}{dt} z_2(t) = v \ln \left\{ 1 + \frac{s_{max}}{e} \left[z_1(t-\tau) - z_2(t-\tau) \right] \right\}$$

$\because t \in (\tau, 2\tau) \Rightarrow t - \tau \in [0, \tau)$

$$z_2(t-\tau) = 0 .$$

$$\therefore \frac{d}{dt} z_2(t) = v \ln \left\{ 1 + \frac{s_{max}}{e} (z_1(t-\tau)) \right\}$$

$z_1(t-\tau)$ when $t \in [\tau, 2\tau) \neq 0$ from eqn 1.4

using ~~z_1 that~~ integration, we can calculate $z_2(t)$ when $t \in [\tau, 2\tau)$.

- we found $z_2(t)$ ~~using~~ for $t \in [\tau, 2\tau)$ using $z_2(t)$ for $t \in [0, \tau)$

Using this we can find subsequently find $z_2(t)$ for intervals $[2\tau, 3\tau)$, ... $[k\tau, (k+1)\tau)$.

~~we use the $z_2 t$~~

for example : for $z_2(t)$ in $[2\tau, 3\tau)$ we can use the result obtained from $z_2(t)$ in $[\tau, 2\tau)$ for $z_2(t-\tau)$.

- This way, by iterating we can find $z_2(t)$ for all $t > 0$.
- Now, knowledge of z_2 on $[0, (k+1)\tau)$ can be used to compute z_3 on $[0, k\tau)$ and so on.

~~To~~ for example to find $z_3(t)$:

$$\frac{d(z_3(t))}{dt} = v/n \left\{ 1 + \frac{s_{max}}{c} \left[z_2(t-\tau) + z_3(t-\tau) \right] \right\}$$

we know that $z_3(t)$ for $t \in [0, \tau) = 0$ and using this fact and since we know $z_2(t)$ in range $t \in [0, (k+1)\tau)$ we can calculate $z_3(t)$ for $t \in [0, (k+1)\tau)$.

same can be done for $z_4(t) \ldots \to z_N(t)$. This can be depicted by a table:

	t			
z	$[-\tau, 0)$	$[0, \tau)$	~~$[0, 2\tau)$~~	$[i\tau, (i+1)\tau)$
z_1	0	given	--	[illegible]
z_2	0	z_2 on $[0, \tau)$		z_2 $[i\tau, (i+1)\tau)$
⋮	0 ⋮		------	
z_k	0	z_k $[0, \tau)$		z_k $[i\tau, (i+1)\tau)$
⋮	⋮			
z_N	0	z_N $[0, \tau)$		z_n $[i\tau, (i+1)\tau)$

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~~The ~~ model was used~~

~~The model was created considering :~~

~~(i)~~

Limitations of the method

~~1) The ~~ increasing recursive complexity of integratio~~
~~to be solved can~~

1) The recursive integrations ~~can~~ will get very complex as we continue this method for more iterations. Extensive computation will be reqd to find the values of $z_i(t)$ when i approaches N.

2) ~~Both~~ Time complexity ~~one~~ will be huge if a program is written as well. It will take both, a lot of time and also a lot of space, since we need to save each iteration's results.

~~Resources used~~

References used:

1) Notes by Prof Sundar
2) Mathemeatical modeling - A case study approach (TB recommended by Prof Sundar).