PROBLEM 6

Find the closest solution to the system of these linear equation:

$$4x_1 - 7x_2 = 2$$
$$-12x_1 + 21x_2 = -6$$

Question 1. Find the nullspace of the coefficient matrix.

Question 2. Find the set of the solutions to this system.

Question 3. Find the basis for the column space of A^T.

Question 4. Find the closest solution to the system of these linear equation. *Answer with 4 decimals.*

Question 5. Is the system of these linear equation consistent?

Question 6. What is the rank of the coefficient matrix?

Question 7. What is the rank of the augmented matrix of the coefficient matrix and the source vector?

$$\begin{bmatrix} 4 & -7 \\ -12 & 21 \end{bmatrix} \rightarrow \begin{bmatrix} 4 & -7 \\ 0 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -7/4 \\ 0 & 0 \end{bmatrix}$$

1) $x_1 = \frac{7}{4} x_2 \rightarrow \alpha \begin{bmatrix} \frac{7}{4} \\ 1 \end{bmatrix}$

$x_2 = \alpha$

$\therefore N(A) = \text{Span}\left(\begin{bmatrix} \frac{7}{4} \\ 1 \end{bmatrix}\right)$

2) $\begin{bmatrix} 4 & 7 & | & 2 \\ -12 & 21 & | & -6 \end{bmatrix} \rightarrow \begin{bmatrix} 4 & -7 & | & 2 \\ 0 & 0 & | & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -7/4 & | & 1/2 \\ 0 & 0 & | & 0 \end{bmatrix}$

$$S = \left\{ X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1/2 \\ 0 \end{bmatrix} + \alpha \begin{bmatrix} 7/4 \\ 1 \end{bmatrix} \text{ for } \alpha \in \mathbb{R} \right\}$$

3) $\begin{bmatrix} 4 & -12 \\ -7 & 21 \end{bmatrix} \rightarrow \begin{bmatrix} 4 & -12 \\ 0 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -3 \\ 0 & 0 \end{bmatrix}$

$C(A^T) = \text{Span} = \text{Span}\left(\begin{bmatrix} 1 \\ -3 \end{bmatrix}\right)$

4) $x'_r = (x \cdot u_{xr}) \cdot u_{xr}$

$$\left[\begin{array}{cc|c} 4 & -7 & 2 \\ -12 & 21 & -6 \end{array}\right] \rightarrow \left[\begin{array}{cc|c} 4 & -7 & 2 \\ 0 & 0 & 0 \end{array}\right] \rightarrow \left[\begin{array}{cc|c} 1 & -\frac{7}{4} & \frac{1}{2} \\ 0 & 0 & 0 \end{array}\right]$$

$$x = \begin{bmatrix} 1/2 \\ 0 \end{bmatrix} \qquad u_{xr} = \begin{bmatrix} 7/4 \\ 1 \end{bmatrix}$$

$$x'_r = \left(\frac{\begin{bmatrix} \frac{1}{2} & 0 \end{bmatrix} \begin{bmatrix} 7/4 \\ 1 \end{bmatrix}}{\left\| \begin{bmatrix} 7/4 \\ 1 \end{bmatrix} \right\|} \right) \cdot \frac{\begin{bmatrix} 7/4 \\ 1 \end{bmatrix}}{\left\| \begin{bmatrix} \frac{7}{4} \\ 1 \end{bmatrix} \right\|} = \left(\frac{7/8}{\frac{\sqrt{65}}{4}} \right) \left(\begin{bmatrix} \frac{7\sqrt{65}}{65} \\ \frac{\sqrt{65}}{4} \end{bmatrix} \right) \quad \therefore r' = \begin{bmatrix} 49/130 \\ 14)\ 69 \end{bmatrix}$$

$$= \begin{bmatrix} 0.37692 \\ 0.92538 \end{bmatrix}$$

5) $\left[\begin{array}{cc|c} 4 & 7 & 2 \\ -12 & 21 & -6 \end{array}\right] \rightarrow \left[\begin{array}{cc|c} 4 & -7 & 2 \\ 0 & 0 & 0 \end{array}\right] \rightsquigarrow \left[\begin{array}{cc|c} 1 & -7/4 & 1/2 \\ 0 & 0 & 0 \end{array}\right]$