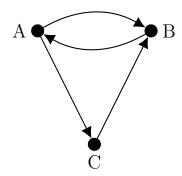
Basic PageRank algorithm

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1 Scenario



This information is represented in the following table.

		From		
		A	В	\mathbf{C}
То	A	0	1	0
	В	$\frac{1}{2}$	0	1
	\mathbf{C}	$\frac{1}{2}$	0	0

Now as a matrix.

$$\mathbf{M} = \begin{bmatrix} 0 & 1 & 0 \\ \frac{1}{2} & 0 & 1 \\ \frac{1}{2} & 0 & 0 \end{bmatrix}$$

2 Basic approach

Imagine everyone starts with $\frac{1}{3}$ of the available vote, and then iterate until the share of vote stabilises.

step 1:
$$\begin{bmatrix} 0 & 1 & 0 \\ \frac{1}{2} & 0 & 1 \\ \frac{1}{2} & 0 & 0 \end{bmatrix} \begin{bmatrix} \frac{1}{3} \\ \frac{1}{3} \\ \frac{1}{3} \end{bmatrix} = \begin{bmatrix} \frac{1}{3} \\ \frac{1}{2} \\ \frac{1}{6} \end{bmatrix}$$
step 2:
$$\begin{bmatrix} 0 & 1 & 0 \\ \frac{1}{2} & 0 & 1 \\ \frac{1}{2} & 0 & 0 \end{bmatrix} \begin{bmatrix} \frac{1}{3} \\ \frac{1}{2} \\ \frac{1}{6} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \\ \frac{1}{3} \\ \frac{1}{6} \end{bmatrix}$$
step 3:
$$\vdots$$

We are looking for a situation where

$$\begin{bmatrix} 0 & 1 & 0 \\ \frac{1}{2} & 0 & 1 \\ \frac{1}{2} & 0 & 0 \end{bmatrix} \begin{bmatrix} A \\ B \\ C \end{bmatrix} = \begin{bmatrix} A \\ B \\ C \end{bmatrix}$$

i.e. where the values of $\begin{bmatrix} A \\ B \\ C \end{bmatrix}$ stabilise.

3 Enter eigenvectors

Notice that this corresponds precisely to $\begin{bmatrix} A \\ B \\ C \end{bmatrix}$ being an eigenvector for eigenvalue $\lambda=1.$

Notes:

- Since A, B and C in this vector are proportions of the available vote, we must have A + B + C = 1.
- A matrix where all columns sum to 1 always has an eigenvalue $\lambda = 1$ though it doesn't hurt to check; if you find yours doesn't have $\lambda = 1$ then maybe you've made a mistake forming the matrix?

For $\lambda=1,$ let $\mathbf{v}=\begin{bmatrix}x\\y\\z\end{bmatrix}$, then we want non-zero x, y and z for

$$\begin{pmatrix}
\begin{bmatrix} 0 & 1 & 0 \\ \frac{1}{2} & 0 & 1 \\ \frac{1}{2} & 0 & 0 \end{bmatrix} - \lambda \mathbf{I} \end{pmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \\
\begin{bmatrix} -1 & 1 & 0 \\ \frac{1}{2} & -1 & 1 \\ \frac{1}{2} & 0 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

From the first row: -x+y=0, i.e. y=x. From the third row: $\frac{1}{2}x-z=0$, i.e. $z=\frac{1}{2}x$. The ratio of x:y:z is therefore 2:2:1 so choose

$$\begin{bmatrix} A \\ B \\ C \end{bmatrix} = \begin{bmatrix} \frac{2}{5} \\ \frac{2}{5} \\ \frac{1}{5} \end{bmatrix}.$$

We choose $\begin{bmatrix} A \\ B \\ C \end{bmatrix}$ to be an eigenvector such that A+B+C=1 since $A,\,B$ and C are shares

of the available vote. So people A and B draw with $\frac{2}{5} ths$ of the available vote.