Tutorial exercise sheet – Linear systems

Peter Rowlett

1. Use the inverse matrix method to solve the following systems of equations.

   (a) $4x - 3y = -3, \quad 2x - 5y = 9;$

   (b) $5r + s - 2t = -1, \quad r + s + t = -3, \quad 7r + 8s = -7;$

   (c) $x + 2y + 3z = 0, \quad 2x + 5y + 3z = 0, \quad x + 8z = 0;$

   (d) $\alpha + 3\beta + \gamma = 4, \quad 2\alpha + 2\beta + \gamma = -1, \quad 2\alpha + 3\beta + \gamma = 3.$

2. Solve the following systems of linear equations using Cramer's rule.

   (a) $3x - 7y = 47, \quad 5x + 2y = 10;$

   (b) $f + 3g + h = 3, \quad 2f + g + 4h = -1, \quad 3f + g - 2h = 6.$

3. Choose an appropriate method to solve, where possible, the following linear systems.

   (a) $i + j + 2k = 8, \quad -i - 2j + 3k = 1, \quad 3i - 7j + 4k = 10;$

   (b) $2u + 2v + 2w = 0, \quad -2u + 5v + 2w = 1, \quad 8u + v + 4w = -1;$

   (c) $-2b + 3c = 1, \quad 3a + 6b - 3c = -2, \quad 6a + 6b + 3c = 5;$

   (d) $w - x + 2y - z = -1, \quad 2w + x - 2y - 2z = -2, \quad -w + 2x - 4y + z = 1, \quad 3w - 3z = -3.$

4. Solve the following systems of linear equations using Cramer's rule.

   (a) $1.985a - 1.358b = 2.212, \quad 0.953a - 0.652b = 1.062;$

   (b) $3\theta + 2\phi + \zeta = 4, \quad \theta - \phi + 2\zeta = -7, \quad 2\theta + 3\phi + 5\zeta = -7.$

5. Use determinants to show that the following pairs of linear equations do not have a unique solution. If the solution to a pair of linear equations represents the point of intersection of two straight lines, give the geometric explanation for a lack of unique solution.

   (a) $2x + 3y = 4, \quad 4x + 6y = 5;$

   (b) $y - 2t = 3, \quad 3y - 6t = 9.$