Brief overview of the history of mathematics

Part 1: Ancient Egypt – Islamic Golden Age

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These talks

- This is a stampede missing much detail!
- And quite a traditional account of mainstream history.
- My intention is to give you an overview, and to pique your interest in some historical topics.

Pre-Greek

Where were we ~5,000 years ago?

- In Egypt: the Pyramid of Djoser (or Zoser), a step pyramid, shows some evidence of geometry in architecture.
- In England: Stonehenge and other sites show:
  - evidence of a standard unit of length;
  - comparatively advanced geometrical knowledge.

Image: The stepped pyramid at Saqqara by Charlesjsharp on Wikimedia Commons.

Egypt

- The classical Greeks believed maths was invented in Egypt.
- However, we have very little evidence of Egyptian maths; only a few papyrus survive.
- The best examples are the Moscow papyrus (c. 1850 BC) and the Rhind papyrus (c. 1650 BC, a copy of an older document probably c. 2000-1800BC).

Image: Rhind Papyrus. © Trustees of the British Museum.

Rhind Papyrus

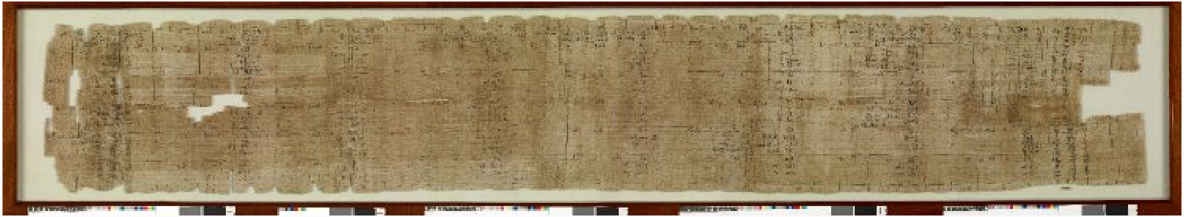


Image: © Trustees of the British Museum.

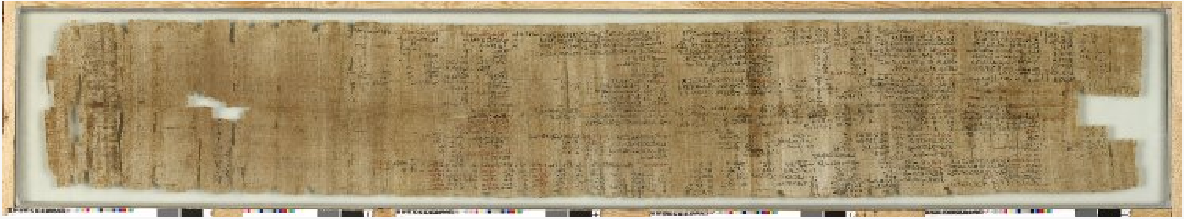


Image: © Trustees of the British Museum.

Rhind Papyrus

Some problems from the Rhind Papyrus:

- "A quantity and its $\frac{1}{2}$ added together become 16. What is the quantity?"
- "Find the volume of a cylindrical granary of diameter 9 and height 10."
- "Example of a triangle of land. Suppose it is said to thee, What is the area of a triangle of side 10 khet and of base 4 khet?"

Fauvel & Gray (1987), pp. 16-19.

Image: © Trustees of the British Museum.

Rhind Papyrus

- Evidence of knowledge of number theory, including composite and prime numbers;
- Shows how to solve first-order linear equations;
- Also arithmetic and geometric series.

Image: © Trustees of the British Museum.

Mesopotamia/Babylon

- Lots of clay tablets with cuniform writing survive;
- Maybe for training scibes;
- Includes solutions of some kinds of quadratic equations;
- Used an early place-value system, the sexagesimal system.

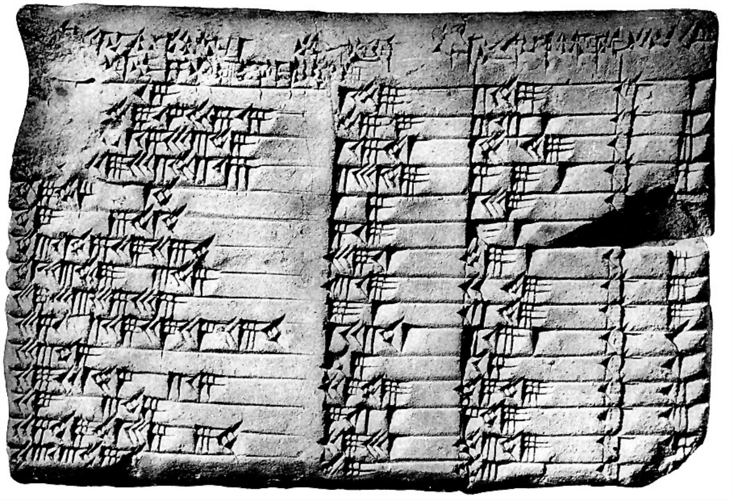
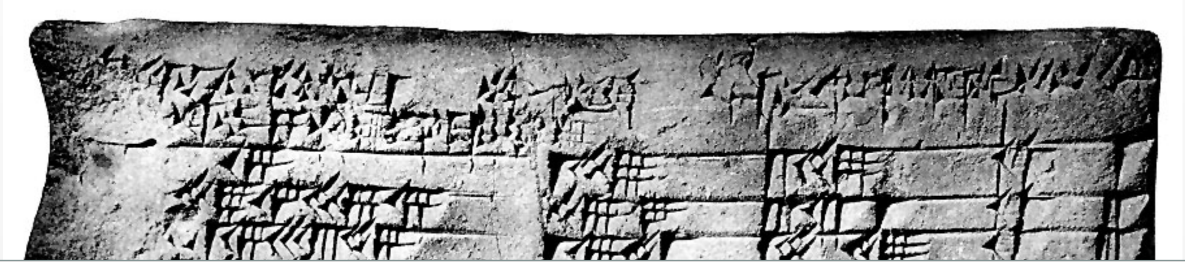


Image: Plimpton 322. Public domain on Wikimedia Commons.

The Sexagesimal system

- Our 'place-value' system is based on 10s, each column has value ten times the prevous.
- The Mesopotamian system:
  - has symbols for 1 and 10;
  - each column has value 60 times the previous.
- Remnants of this system survive in our measurements for time and angles.



Mesopotamia/Babylon tablets

Problems:

- "A cistern was 10 GAR square, 10 GAR deep. I emptied out its water; with its water, how much field did I irrigate to a depth of 1 šu-si?"
- "A triangle 6, 30 is the length, [11,22],30 the area; I did not know its width. 6 brothers divided it. One brother's share exceeded the other's, but how much he exceeded I did not know. How much did one brother exceed the other?"

Fauvel & Gray, pp. 27-28.

Image: Plimpton 322. Public domain on Wikimedia Commons.

Mesopotamia/Babylon tablets

- Tablet YBC 7289 (c. 1800-1600BC) contains a piece of geometry giving an approximation for the square root of 2 that is

$$1+\frac{24}{60}+\frac{51}{3600}+\frac{10}{216000}$$

$$=1.414212\dot{9}\dot{6}.$$

Image: YBC 7289 from the Yale Babylonian Collection. Photo by Bill Casselman www.math.ubc.ca/~cass/Euclid/ybc/ybc.html Reproduced with permission.

Mesopotamia/Babylon tablets

- Squaring this approximation for $\sqrt{2}$, we get:

$$1+\frac{59}{60}+\frac{59}{3600}+\frac{59}{216000}+\frac{38}{12960000}+\frac{1}{777600000}+\frac{40}{46656000000}$$

$$\approx 1.9999983.$$

Image: YBC 7289 from the Yale Babylonian Collection. Photo by Bill Casselman www.math.ubc.ca/~cass/Euclid/ybc/ybc.html Reproduced with permission.

Classical Greece

Classical Greece

- Virtually nothing survives that the Greeks actually wrote (on papyrus),
- so we rely on later versions, commentaries and translations.

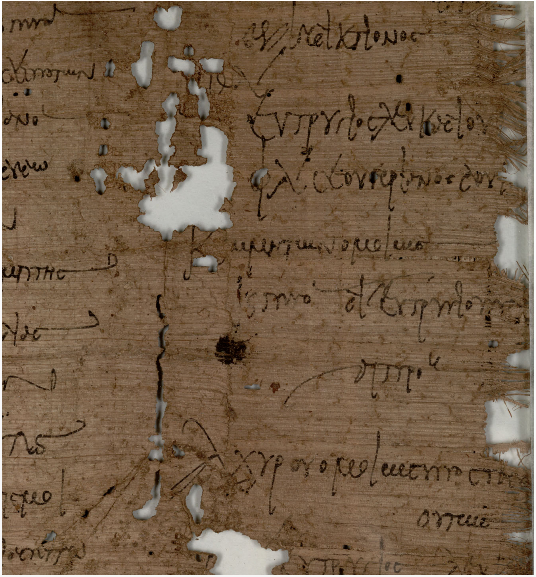


Image: P.Oxy.LXVII 4599 © the Imaging Papyri Project, University of Oxford. Free for educational use.

Greek proof

- Evidence of pre-Greek mathematics uses inductive reasoning; repeated observations of examples used to establish evidence for a general principle.
- The Greeks used deductive reasoning; proceeding from a set of assumptions, called axioms or postulates, deriving a hierarchy of results by mathematical proof.
- Originated several methods of proof, including a proof by contradiction by Thales of Miletus (c.624-c.546BC).

Pythagoras (c. 570-490BC)

- May or may not have actually existed!
- Known for a theorem about a triangle that was certainly in use 1,000 years earlier in Mesopotamia; may have provided the first proof.

Image: Pythagoras Bust Vatican Museum. Public domain on Wikimedia Commons.

Pythagoras (c. 570-490BC)

- Formed a philosophical school, now known as the Pythagoreans, in southern Italy.
  - both men and women;
  - strict regime, having no personal possessions and eating only vegetables (except beans)!
  - studied a four part classification of the mathematical sciences 'the quadrivium' (a Roman word): arithmetic, geometry,

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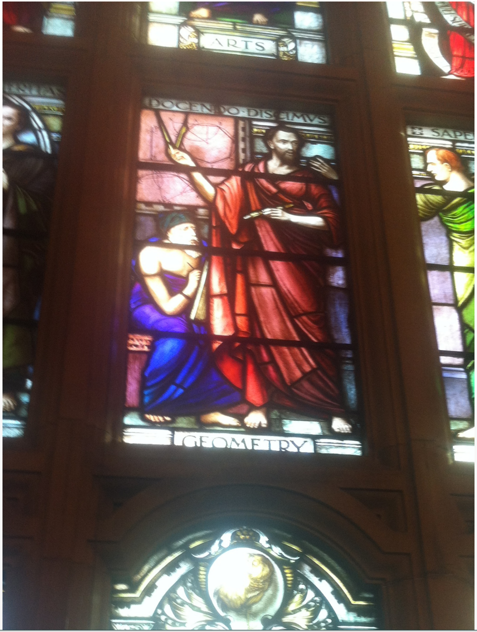
  - ‘Arithmetic’: closer to modern ‘number theory’.
  - ‘Music’: closer to ‘harmony’.

The quadrivium

- The quadrivium (arithmetic, geometry, astronomy and music), together with the trivium (grammar, rhetoric and logic) comprised the 'liberal arts' which formed the curriculum of academies and universities over the following 2,000 years.



Plato (427 BC-347 BC)

- born in Athens;
- travelled in Egypt, Sicily and Italy;
- established an academy in Athens around 387 BC;
- this became a focus for much mathematical and philosophical activity;
- over the entrance appeared the inscription: "Let no-one ignorant of geometry enter here".

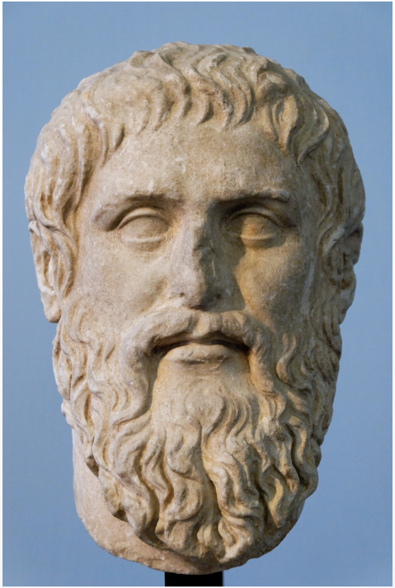


Image: Plato.

Plato (427 BC-347 BC)

- Plato did much in mathematics and philosophy.
- An example: he discussed the five regular solids: tetrahedron, cube, octahedron, dodecahedron and icosahedron.
- In these, the faces are all regular polygons of the same type and they meet at identical vertices.
- We now call the Platonic solids.

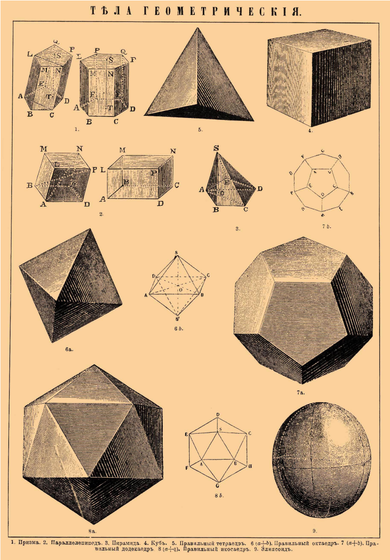


Image: Illustration from Brockhaus and Efron Encyclopedic Dictionary (1890—1907). Public domain on Wikimedia Commons.

Euclid (c.325-c.265BC)

We know very little about Euclid except

- he lived in Alexandria;
- probably around 300BC.

Image: Euclid. The MacTutor History of Mathematics archive.

Euclid's *Elements*

- Believed to be a summary of previous work, rather than a report of Euclid's own work.
- Probably the most printed book apart from the Bible and the most successful text book of all time.

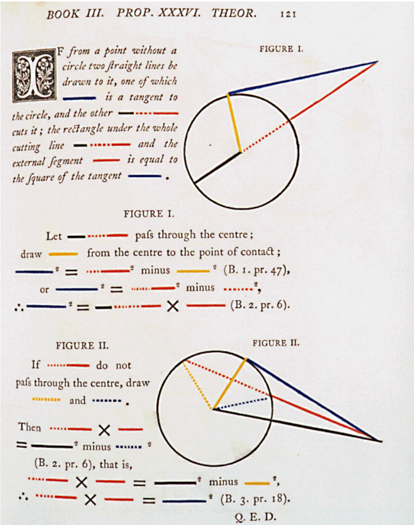


Image: Page 121 from *The First Six Books of the Elements of Euclid in which coloured diagrams and symbols are used instead of letters for the greater ease of learners*, Oliver Byrne (1847). Public domain on Wikimedia Commons.

Euclid's *Elements*

- Starts with definitions: point, line, circle, etc.
- Then a set of axioms (postulates) that allow certain geometric operations using an unmarked ruler and a pair of compasses, including:
  - drawing a straight line from one point to any other;
  - drawing a circle with any given centre and radius.

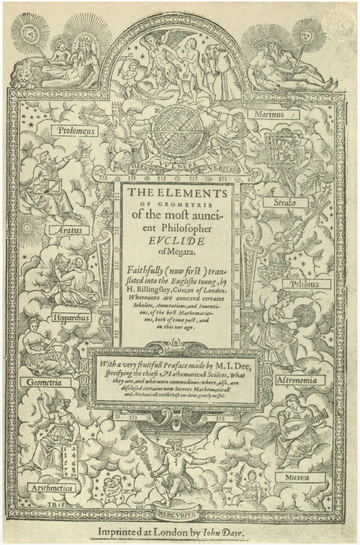
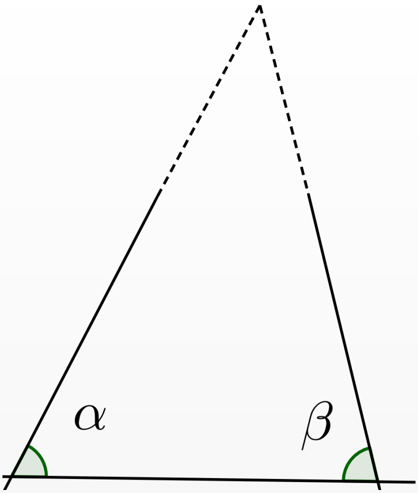


Image: Title page of Sir Henry Billingsley's first English version of Euclid's Elements, 1570. Public domain on Wikimedia Commons.

Euclid's *Elements*

- The fifth postulate, also called the parallel postulate, says that:
  "If a line segment intersects two straight lines forming two interior angles on the same side that sum to less than two right angles, then the two lines, if extended indefinitely, meet on that side on which the angles sum to less than two right angles."



Euclid's *Elements*

- Euclid then presents a series of results (called propositions), each based on the postulates and previous propositions;
- these cover the basics of plane geometry, proportions, number theory and solid geometry;
- ends with the construction of the Platonic solids and the result that no other regular polyhedra can be constructed.

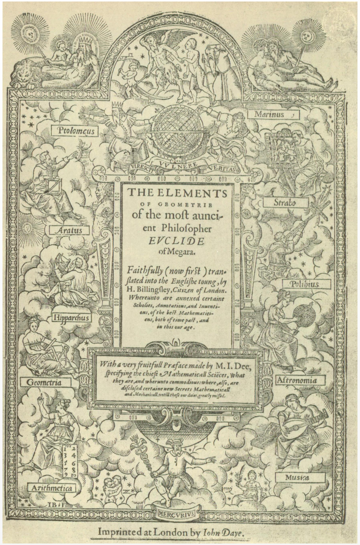


Image: Title page of Sir Henry Billingsley's first English version of Euclid's Elements, 1570. Public domain on Wikimedia Commons.

e.g. Book I, Proposition 6

- “In any triangle, if two angles are equal, the sides opposite to them are also equal.”
- Euclid provides a proof by contradiction.

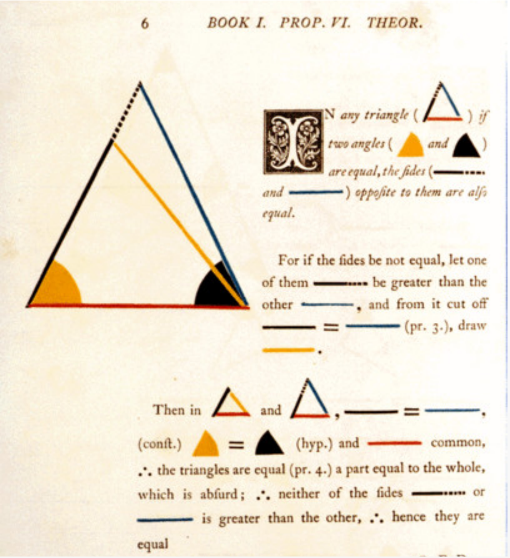


Image: Proposition 6 from Byrne’s version. Public domain on Wikimedia Commons.

Archimedes (c.287-212BC)

- We have more of the writings of Archimedes than any other great mathematician of antiquity, meaning:
  - he was productive;
  - he was later studied and copies and commentaries of his works made.
- Calculated the area under the arc of a parabola with the summation of an infinite series.
- Also a calculation of π using a similar method.

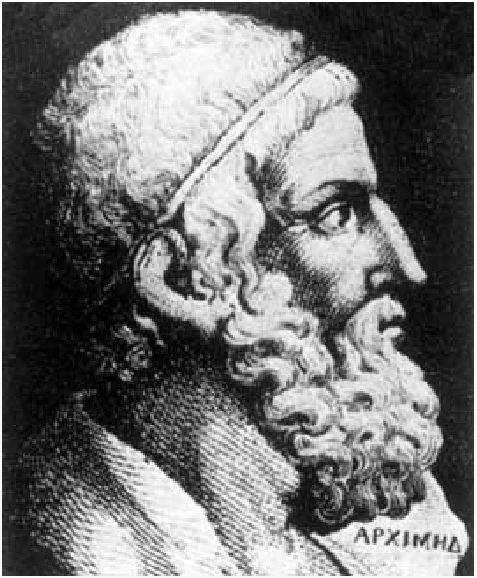


Image: Archimedes. The MacTutor History of Mathematics archive.

Archimedes (c.287-212BC)

- Archimedes also investigated the 'semi-regular' polyhedra, which are composed of two or more types of regular polygons meeting at identical vertices;
- he found that there are only thirteen such solids;
- we call these Archimedean solids.
- Pictured is the truncated icosohedron (or football), made of twelve pentagons and twenty hexagons.

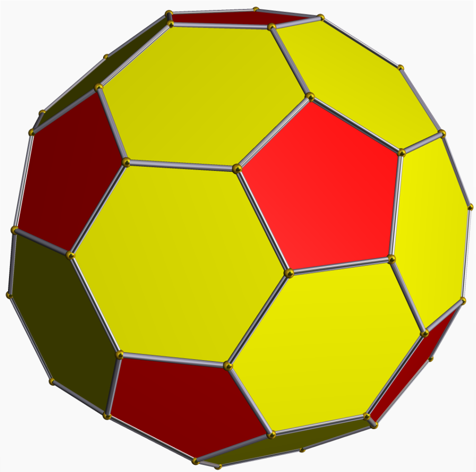


Image: Truncated icosahedron. By Robert Webb's Stella software www.software3d.com/Stella.php

Apollonius (c.262-190BC)

- Showed that all three types of conic sections (parabola, ellipse and hyperbola) can be obtained by varying the angle of a plane intersecting a cone.
- Proved many results relating to conics that were useful to later mathematicians studying planetary motion.

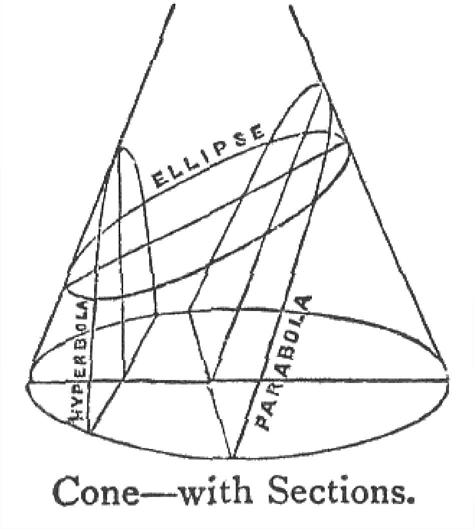


Image: Illustration from 1908 Chambers's Twentieth Century Dictionary. Public domain on Wikimedia Commons.

Other Greek interests besides geometry

- mechanics;
- optics;
- music (proportions, harmonics, etc.);
- number theory.
- late Greek: Diophantus (perhaps c.200-c.284AD) wrote *Arithmetica*, which contains a collection of essentially algebraic problems.

Group work this week

- Please choose a mathematical topic and explore its history, or choose a historical person or culture and explore their connection with mathematics.

References

1. Fauvel, J. and Gray, J. (1987). *The History of Mathematics – A Reader*. Milton Keynes: Open University.
2. Flood, R., and Wilson, R. (2011). *The Great Mathematicians: Unravelling the mysteries of the universe*. London: Arcturus.
3. O'Connor, J.J. and Robertson, E.F. (2016). *The Mactutor History of Mathematics Archive*. Retrieved from `http://www-history.mcs.st-andrews.ac.uk`