

Game theory revision questions

1. Choose one of the following games and analyse it using the mex technique.

- (a) **Officers:** You have a line of soldiers. Players take turns sending a soldier on leave, but a soldier can only go on leave if they have a neighbour who can look after their kit. The player who makes the last possible move wins.

Here is an example starting position.



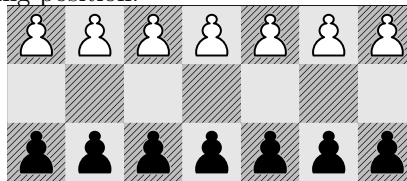
- (b) **Grundy's game:** Starting with a single pile of coins, players take turns to split a single pile of coins into two heaps of different sizes. The player who makes the last possible move wins.

Here is an example starting position.



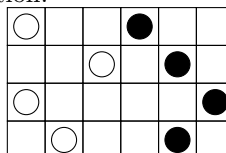
- (c) **Dawson's chess:** Played on a $3 \times n$ chessboard with n pawns on each side. Players take turn to move any piece like pawns in chess. Having moved, players must then complete any captures that are possible. The player who makes the last possible move is the winner.

Here is an example starting position.



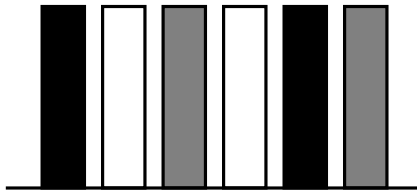
- (d) **Northcott's game.** A rectangular grid has a black token and a white token in each row. Each turn a player must move a piece of their colour any number of spaces along the row without jumping over the other piece. The player who makes the last possible move is the winner.

Here is an example starting position.



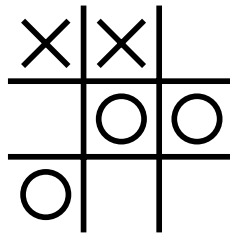
2. Produce a set of three game positions to be played simultaneously, one Single-pile Nim, one Multi-pile Nim, and one from the game you analysed in question 1. Apply the multi-game technique, equating them each to Nim heaps and calculating Nim sum to work out the next optimal move.

3. **Toppling Dominoes** Played on a row of black, white or grey dominoes. Players take turns to choose a domino and topple it either left or right, whereupon every domino in that direction also topples and all toppled dominoes are removed from play. Left chooses either black or grey dominoes, Right chooses either white or grey dominoes. The first player who cannot make a legal move loses. Here is a sample starting position.



Analyse this game, finding some simple positions that are equivalent to numbers.

4. **Play to Lose Noughts and Crosses.** Draw a game tree and determine the value of the following position. One player is Noughts and the other Crosses. Players take turns placing their own symbol on the board. The first to get three in a row of their symbol loses.



5. Consider the following two-player game.

		Player 2	
		C	D
Player 1	A	(5, 10)	(5, 1)
	B	(2, 4)	(9, 8)

Assuming Player 1 plays strategy A with probability p and Player 2 plays strategy C with probability q :

- find the values of p and q that create a Nash equilibrium;
- find the expected outcomes for each player in this situation.