Some structured problems

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These questions come in three parts, which are supposed to get progressively harder. For each, there is much more that can be explored. Try to be curious and ask questions about what you have learned in solving the problems. Is there a more open-ended part (d) you could ask and solve? What else can you do with the ideas covered?

1. To find the home prime of a number, follow this procedure: first, find its prime factors; then concatenate these by writing the prime factors in size order to form a new number using all of their digits; then repeat until the resulting number is prime.

   For example, $9 = 3 \times 3$, so its prime factors are 3 and 3. These concatenate to give 33. Now, find the prime factors of 33. They are 3 and 11, which concatenate to give 311. Since 311 is prime, we stop and say that the home prime of 9 is 311.

   The home prime of a prime number is the number itself. So, for example, the home prime of 2 is 2.

   (a) What is the home prime of 6?

   (b) How about the home prime of 10?

   (c) Which number less than 20 has a home prime of 1129?

2. You must take two steps to get from a given number to make 10. Each step must change the number by adding, subtracting, multiplying by or dividing by a number from 1 to 9. (Multiplying or dividing by 1 isn't allowed as it doesn't change the number.)

   For example, starting with 35, one way would be to first divide by 7 then add 5.

   (a) Can you take two steps to get from 42 to 10?

   (b) Is it possible to get to 10 in two steps from all the numbers 11 to 30?

   (c) And can you find a two-digit number for which there is no way to get to 10 in two steps?

3. (a) Abbie and Bryn are allocated places to sit at random in a row of four seats. What is the probability that they are put next to each other?

   (b) What if there are 20 seats?

   (c) On a different occasion, Abbie and her friend Ana have booked two adjacent spots on the front row, which is 22 seats long. Bryn and his friend Beth also have two adjacent seats among the 22. What is the probability the two groups are next to each other?

4. (a) Fill in the numbers 1-16 once each in a four-by-four grid so that the sum of the numbers on the top row is even and the numbers on the bottom row sum to a different even number.

   For example, if we arrange the square so that the numbers across the top row are 1, 2, 3 and 4, they will sum to an even number, 10.

   (b) Can you find a solution where all four edges have different sums that are all even?

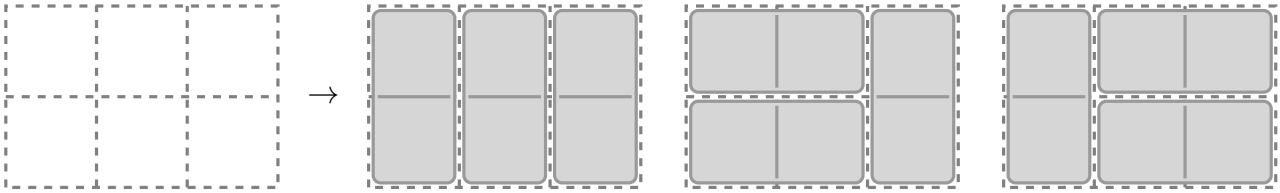
   (c) Can you find a solution where all rows, columns and diagonals sum to different even numbers?

5. (a) Which is bigger, 30×30 or 29×31?

   (b) If we choose a number other than 30 and compare its product with itself to the product of the numbers one above and one below it, which is bigger? Does this always work the same way?

   (c) If you choose any two numbers that are above and below 30 by the same amount, what is the difference between their product and 30×30?

6. There are three ways to cover a 3×2 grid with dominoes.



   (a) How many ways are there to cover a 4×2 grid?

   (b) What about a 6×2 grid?

   (c) Can you find a pattern that would help you work out the number of different ways dominoes can be used to cover any n $\times$ 2 rectangle?

7. We play a game starting with a counter in the centre of a row of squares. Every turn, we each roll a six-sided die. If the difference between the values rolled is 2 or less, the counter moves one place to the left. If the difference is more than 2, it moves one place to the right. I win if the counter reaches the left-hand end of the row and you win if it gets to the right.

   (a) Am I offering you a fair game or does one direction have an advantage?

   (b) The game is adjusted so that if the difference is 2 or less, the counter still moves one place to the left, but if the difference is more than 2, it moves two places to the right. Under these rules, what is the smallest number of rolls after which the counter could return to its starting position?

   (c) Is this adjusted game fair?