Motivations

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Numbers

- The Indian numeral system made its way to Persia, where it was used by al-Khwārizmī and others.
- Eventually it made its way to Christian Europe, where it drove a revolution in accountancy and trade.
- This is part of a story of growing European dominance.
- For example, the British East India Company used it corporate power to dominate and ultimately control India.

(Dalrymple, 2024)

Impact

- Clearly this is something that just happened, and there was no intention in the work of al-Khwārizmī or Fibonacci, say, to see this route to European dominance over India.
- It is worth exploring the motivations behind a piece of mathematics, and its later impact.

Why develop new mathematics?

- Individual motivations vary.
- Some are driven by pure curiosity, others inspired by real world applications.
- Questions:
  - Are the impacts of mathematics always planned?
  - Are the motivations always pure? (Spoiler: some dark stories ahead!)

Unplanned impact

Mathematics

- Some mathematics is inspired by real world applications.
- Some by curiosity and abstraction.
- Sometimes, whether inspired by real world or not, mathematicians push research far beyond what is applicable.
- Far into the abstract.
- Extension and abstraction without apparent direction or purpose – fundamental aspect of mathematics research.

Cédric Villani

- 2010 Fields Medallist, interviewed in *New Scientist* (Aron, 2011):

  Do you like that there are applications for the mathematics you do, or do you prefer abstract puzzles?

  When I started my PhD, I wanted something that had real applications. Now it's a bit different. I like the fact that there are applications, but it's not the most important thing to me. On the other hand, the fact that it is rooted in the real world contributes to the aesthetic of the equation – I find the problem is more beautiful if there is a root in reality.

Unexpectedly useful

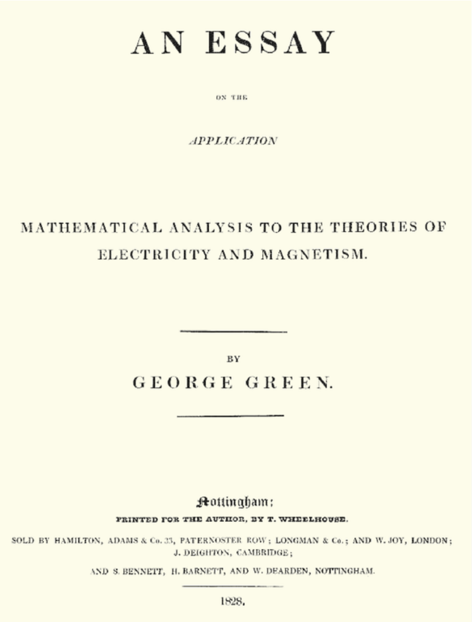
- Trying to solve real-world problems, researchers often discover that the tools they need were developed years, decades or even centuries earlier by mathematicians with no prospect of, or care for, applicability.
- Common examples:
  - Number theory in modern cryptography.
  - Work in logic leading to computing.
  - Imaginary numbers in fluid mechanics (e.g. flight) and electricity.

Example: George Green

- 1793-1841.
- Lived most of his life in Sneinton.
- Early life working his father's mill.
- Self taught in mathematics.

Green's Essay

- "An Essay on the Application of Mathematical Analysis to the Theories of Electricity and Magnetism" by George Green, 1828.
- Introduced Green's theorem, Green's functions and electric potential.
- Later work on elasticity, sound & light.



AN ESSAY

ON THE

APPLICATION

OF

MATHEMATICAL ANALYSIS TO THE THEORIES OF
ELECTRICITY AND MAGNETISM.

BY

GEORGE GREEN.

Nottingham:

PRINTED FOR THE AUTHOR, BY T. WHEELHOUSE.

SOLD BY HAMILTON, ADAMS & Co. 33, PATERNOSTER ROW; LONGMAN & Co.; AND W. JOY, LONDON;
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1828.

Legacy

- Green's work, particularly the Essay, was discovered by William Thompson (later Lord Kelvin), who brought it to greater attention.

  It is thanks to Kelvin that Green's concepts and techniques were integrated into nineteenth century science, referred to and used by the leading scientists and mathematicians. With Kelvin's generation, which included his friend, George Gabriel Stokes, and James Clerk Maxwell, science in England, and in Cambridge in particular, finally emerged from a long period of stagnation.

  (Cannell, 1988, pp. 62–63).

Applied maths

- Green was certainly trying to apply mathematics to the real world in 1828.
  - Title: "An Essay on the Application of Mathematical Analysis to the Theories of Electricity and Magnetism".
  - Introduction: "Should the present Essay tend in any way to facilitate the application of analysis to one of the most interesting of the physical sciences, the author will deem himself amply repaid for any labour he may have bestowed upon it" (Green, 1828).
- Take up in 19th century science might be surprising in extent, but it was not 'unplanned impact'.

Legacy

- In the early 20th century, physicists discovered that the laws describing how things move were totally untrue when they were studied on an atomic scale.
- Over time, the new theory of quantum mechanics was introduced.
- We might expect Green's mathematics to vanish as the physics it is used for is reinvented . . . but in fact it grew in importance!

Legacy

- In an account given in 1993, Julian Schwinger describes his use of Green's work:
  - on microwave radar during World War II;
  - in the development of the microtron and synchrotron particle accelerators;
  - to solve a problem on quantum electrodynamics, the solution of which earned him the 1965 Nobel Prize for Physics.

(Schwinger, 1993)

Unplanned impact

- I do not believe that George Green, in 1828, could have imagined work in radar, particle accelerators and quantum physics to which his work was later applied.

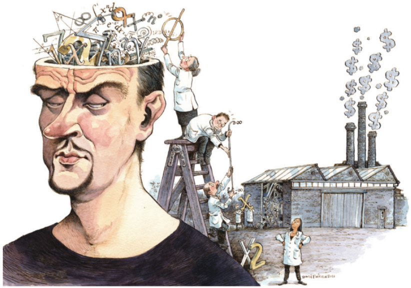
Unplanned impact

- Let's define 'unplanned impact' as either:
  - work in pure mathematics, pursued without any prospect of application, which later finds an application (like number theory in cryptography);
  - work designed to be applied in one area that turns out to be useful in some other, unexpected, application (like work on electricity applied to quantum physics).

Nature

- In 2011 a group of us from the British Society for the History of Mathematics had a paper published in *Nature* on ‘The unplanned impact of mathematics’.

COMMENT



ILLUSTRATIONS BY DAVID PARKINS

The unplanned impact of mathematics

Peter Rowlett introduces seven little-known tales illustrating that theoretical work may lead to practical applications, but it can't be forced and it can take centuries.

As a child, I read a joke about someone who invented the electric plug and had to wait for the invention of a socket to put it in. Who would invent something so useful without knowing what purpose it would serve? Mathematics often displays this astonishing quality. Trying to solve real-world problems, researchers often discover that the tools they need were developed years, decades or even centuries earlier by mathematicians with no prospect of, or care for, applicability. And the toolbox is vast, because, once a mathematical result is proven to the satisfaction of the discipline, it doesn't need to be re-evaluated in the light of new evidence or refuted, unless it contains a mistake. If it was true for Archimedes, then it is true today.

The mathematician develops topics that no one else can see any point in pursuing, or pushes ideas far into the abstract, well beyond where others would stop. Chatting with a colleague over tea about a set of problems that ask for the minimum number of stationary guards needed to keep under observation every point in an art gallery, I outlined the basic mathematics, noting that it only works on a two-dimensional floor plan and breaks down in three-dimensional situations, such as when the art gallery contains a mezzanine. “Ah,” he said, “but if we move to 5D we can adapt …” This extension and abstraction without apparent direction or purpose is fundamental to the discipline. Applicability is not the reason we work, and plenty that is not applicable contributes to the beauty and magnificence of our subject.

There has been pressure in recent years for researchers to predict the impact of their work before it is undertaken. Alan Thorpe, then chair of Research Councils UK, was quoted by *Times Higher Education* (22 October 2009) as saying: “We have to demonstrate to the taxpayer that this is an investment, and we do want researchers to think about what the impact of their work will be.” The US National Science Foundation is similarly focused on broader

Nature article

- Gave seven examples
  - Quaternions in computer vision & video games (Mark McCartney & Tony Mann);
  - Riemann geometry in relativity (Graham Hoare);
  - Stacking oranges in modem channel coding (Edmund Harriss);
  - Parrondo's paradox in epidemiology & finance (Juan Parrondo & Noel-Ann Bradshaw);
  - Law of large numbers in insurance (Peter Rowlett);
  - Topology in everything (Julia Collins);
  - Hilbert spaces in quantum mechanics (Chris Linton).

Not just a historical problem!

- Tim Harford writing in the Financial Times (2011):

  Academics are always being asked to demonstrate the 'impact' of their research. ... But while it is not unreasonable to ask whether a particular piece of academic research is useful, the difficulties in answering the question are extraordinary.

- Gives examples – his own (imaginary numbers in electrical engineering); Caroline Series' (non-Euclidian geometry & special relativity) and several from our *Nature* piece.

Motivations

Motivations

- Sometimes the impact of a mathematical development is not unambiguously positive.
- Sometimes, the impact is unplanned.
- Other times, a development is very much attached to a purpose, and sometimes those motivations are not pure.

19th C. mathematical physics

- Thermodynamics:
  - mathematical models in physics for systems involving heat, work, and temperature;
  - early steam engines powered the coal mines, factories and ships that drove the triangular trade.
- Electromagnetism: development of the telegraph with the aim of enabling better communications over long distances for the British Empire.

Colonisation

- Literally, a process of establishing occupation of or control over foreign territories for the purpose of exploitation, cultivation, trade, settlement.
- For example:
  - Ancient Greeks in Sicily;
  - Romans in Britain;
  - Vikings in Iceland, Greenland and Vinland;
  - the British Empire.

Colonisation

- There are still non-self-governing territories on the UN decolonisation list.
- And there are many former colonies that are still living with the consequences of colonisation.
- There are intersections with other topics such as racism, gender, extraction of fossil fuels and other resources, etc.
- But that is not our topic.

Colonisation

- Colonisation as a more abstract concept is about systematic exploitation and control, treating land or people as resources to be exploited.
- Decolonisation

Examples

- Astronomy: Use of colonies and plantation workers to collect astronomical data.
- Investors in the slave trade, e.g. Newton.
- Worker control: Babbage was interested in tabulation of data on the speed and outputs of workers, and associated discipline measures, focused on recovering the cost of training skilled workers.

Navigation

- Navigation was a big driver of maths for a long time.
- The Longitude Prizes – inducement prizes by the British government to encourage the development of a practical method for the precise determination of a ship's longitude at sea.
- This was to smooth transoceanic voyages, especially between Britain and the West Indies, to enable 'trade', which included the transport of enslaved people and the extraction of natural resources.

Warfare

- Mechanics: Why were people interested in projectiles? Flight?
- Cryptography: Why did people want to send secret messages?
- World War II logistics.
- Early game theory and Mutually Assured Destruction/the Cold War.
- Early computing and the internet driven by military applications.

Data and statistics

- Many early statistical techniques were developed by eugenicists who were looking for evidence of racial superiority, to be used to remove 'defectives' from the population.
- IBM supplied punchcard machines that were used to catalogue and tabulate data used in the classification of Jews in the 1930s, as well as IBM technology being used in the organisation of railways and registration at concentration camps. IBM may have maintained the equipment and trained Nazi officers in its use. (Handby, 2021).

Financial mathematics

- Focus on short-term financial gain vs. broader socio-economic goals.

Weekly task

- Pick a mathematical topic and explore its origins, focused on why the developers did their work.

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