Connectives exercises

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1. Consider the sentence: "Did you see Angharad or Ros earlier?"

   Which type of 'or', OR ($\vee$) or XOR ($\oplus$), is being used?

2. Consider the sentence: "The door is open or closed."

   Which type of 'or', OR ($\vee$) or XOR ($\oplus$), is being used?

3. Consider the following conversation.

   Waiter: Would you like tea or coffee?
   Customer: Tea, please.
   Waiter: Would you like milk or sugar?
   Customer: Both, please.

   Which type of 'or', OR ($\vee$) or XOR ($\oplus$), is intended by the waiter in each of the two questions?

4. In natural language, it is often irritating to communicate in a way that is strictly logical. Consider the following unhelpful dialogue. Is person 2 right?

   Person 1: Is the window open or closed?
   Person 2: Yes.

5. While we are on the subject of natural language, as an aside, can you make sense of the following dialogue?

   Three people meet a friend in a pub.

   Friend: Do you all want a beer?
   Person 1: I don't know.
   Person 2: I don't know.
   Person 3: Yes.

6. Suppose we have the following propositions.

   $$p : \text{"It is cold"}$$
   $$q : \text{"It is raining"}$$
   $$u : \text{"It is cold but not raining"}$$

   The relationship of u to p and q can be described using a combination of the operators discussed above.

   $$u = p \text{ AND } (\text{NOT } q)$$
   $$u = p \wedge \neg q$$

   Complete the table below.

p	q	$\neg q$	$u = p \wedge \neg q$
false	false		
false	true		
true	false		
true	true		

7. Express the following using simpler mathematical notation. The first is done for you.

   (a) $(\pi > 0) \wedge (\pi < 10)$ [Answer: $0 < \pi < 10$.]
   (b) $(p \geq 7) \wedge (p < 12)$.
   (c) $(x > 5) \wedge (x < 7)$.
   (d) $x < 4 \wedge x < 6$.
   (e) $(x \geq 0) \wedge (x \leq 0)$.
   (f) $(x = 0) \vee (x > 0)$.
   (g) $\neg(x > 7)$.
   (h) $\neg(x = 1)$.
   (i) $\neg(\neg(x > 0))$.
   (j) $\neg(x$ is even$)$.

8. Is this statement true or false? "$(3 < 5) \vee (1 = 2)$".

9. What would you need to do to demonstrate that $p \wedge q \wedge r \wedge s \wedge t$ is true?

10. What would you need to do to demonstrate that $p \wedge q \wedge r \wedge s \wedge t$ is false?

11. What would you need to do to demonstrate that $p \vee q \vee r \vee s \vee t$ is true?

12. What would you need to do to demonstrate that $p \vee q \vee r \vee s \vee t$ is false?

13. Is it possible for one of $(p \wedge q) \wedge r$ and $p \wedge (q \wedge r)$ to be true and the other false? Write out a truth table to investigate.

14. Is it possible for one of $(p \vee q) \vee r$ and $p \vee (q \vee r)$ to be true and the other false? Write out a truth table to investigate.

15. Simplify the following statements.

   (a) $\neg(p \vee \neg q)$.
   (b) $\neg(\neg p \wedge q)$.
   (c) $\neg(p \vee q)$.
   (d) $\neg(p \wedge q)$.

16. A *tautology* is a statement such that the truth table has 'true' for all outputs. Draw a truth table to show that '$p \vee \neg p$' is a tautology.

17. A *contradiction* is a statement such that the truth table has 'false' for all outputs. Draw a truth table to show that '$p \wedge \neg p$' is a contradiction.