Switches puzzle, hints and solution

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Switches puzzle

- For complicated reasons that don't matter, a set of switches is wired so that if the following rules are not followed, an alarm will sound:
    1. The switch on the right may be turned on and off at will;
    2. Any other switch may be turned on or off only if the switch to its immediate right is on and all other switches to its right are off.
- Starting with all switches on, what is the smallest number of moves to turn all switches off without activating the alarm if there are 6 switches in the row?

Problem-solving hints

1. Plan

Problem-solving

- Draw a diagram.
- Invent a notation (e.g. a grid of numbered switches with 1 for on and 0 for off).

Sw 1	**Sw 2**	**Sw 3**	**Sw 4**	**Sw 5**	**Sw 6**
1	1	1	1	1	1

Problem-solving

- Draw a diagram.
- Invent a notation (e.g. a grid of numbered switches with 1 for on and 0 for off).

Move	**Sw 1**	**Sw 2**	**Sw 3**	**Sw 4**	**Sw 5**	**Sw 6**
start	1	1	1	1	1	1
1	1	1	1	1	0	1
2	1	1	1	1	0	0
3	1	1	0	1	0	0
⋮	⋮	⋮	⋮	⋮	⋮	⋮

Problem-solving

- Draw a diagram.
- Invent a notation (e.g. a grid of numbered switches with 1 for on and 0 for off).

Move	**Sw 1**	**Sw 2**	**Sw 3**	**Sw 4**	**Sw 5**	**Sw 6**
start	1	1	1	1	1	1
1	1	1	1	1	0	1
2	1	1	1	1	0	0
3	1	1	0	1	0	0
⋮	⋮	⋮	⋮	⋮	⋮	⋮

- Try a simpler problem (in this case, fewer switches).

Solution for 3 switches

3 switches

Move	Sw 1	Sw 2	Sw 3
start	1	1	1
1			
$\vdots$	$\vdots$	$\vdots$	$\vdots$

3 switches

Move	**Sw 1**	**Sw 2**	**Sw 3**
start	1	1	1
1	1	1	0
2			
⋮	⋮	⋮	⋮

3 switches

Move	**Sw 1**	**Sw 2**	**Sw 3**
start	1	1	1
1	1	1	0
2	0	1	0
3			
⋮	⋮	⋮	⋮

3 switches

Move	**Sw 1**	**Sw 2**	**Sw 3**
start	1	1	1
1	1	1	0
2	0	1	0
3	0	1	1
4			
⋮	⋮	⋮	⋮

3 switches

Move	Sw 1	Sw 2	Sw 3
start	1	1	1
1	1	1	0
2	0	1	0
3	0	1	1
4	0	0	1
5			

3 switches

Move	**Sw 1**	**Sw 2**	**Sw 3**
start	1	1	1
1	1	1	0
2	0	1	0
3	0	1	1
4	0	0	1
5	0	0	0

Solution for 4 switches

4 switches

Move	Sw 1	Sw 2	Sw 3	Sw 4
0	1	1	1	1
1				
⋮	⋮	⋮	⋮	⋮

4 switches

Move	**Sw 1**	**Sw 2**	**Sw 3**	**Sw 4**
0	1	1	1	1
1	1	1	0	1
2				
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$

4 switches

Move	Sw 1	Sw 2	Sw 3	Sw 4
0	1	1	1	1
1	1	1	0	1
2	1	1	0	0
3				
⋮	⋮	⋮	⋮	⋮

4 switches

Move	Sw 1	Sw 2	Sw 3	Sw 4
0	1	1	1	1
1	1	1	0	1
2	1	1	0	0
3	0	1	0	0
4				
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$

4 switches

Move	Sw 1	Sw 2	Sw 3	Sw 4
0	1	1	1	1
1	1	1	0	1
2	1	1	0	0
3	0	1	0	0
4	0	1	0	1
5				
⋮	⋮	⋮	⋮	⋮

4 switches

Move	Sw 1	Sw 2	Sw 3	Sw 4
0	1	1	1	1
1	1	1	0	1
2	1	1	0	0
3	0	1	0	0
4	0	1	0	1
5	0	1	1	1
6				
⋮	⋮	⋮	⋮	⋮

4 switches

Move	Sw 1	Sw 2	Sw 3	Sw 4
0	1	1	1	1
1	1	1	0	1
2	1	1	0	0
3	0	1	0	0
4	0	1	0	1
5	0	1	1	1
6	0	1	1	0
7				
⋮	⋮	⋮	⋮	⋮

4 switches

Move	Sw 1	Sw 2	Sw 3	Sw 4
0	1	1	1	1
1	1	1	0	1
2	1	1	0	0
3	0	1	0	0
4	0	1	0	1
5	0	1	1	1
6	0	1	1	0
7	0	0	1	0
8				
⋮	⋮	⋮	⋮	⋮

4 switches

Move	Sw 1	Sw 2	Sw 3	Sw 4
0	1	1	1	1
1	1	1	0	1
2	1	1	0	0
3	0	1	0	0
4	0	1	0	1
5	0	1	1	1
6	0	1	1	0
7	0	0	1	0
8	0	0	1	1
9				
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$

4 switches

Move	Sw 1	Sw 2	Sw 3	Sw 4
0	1	1	1	1
1	1	1	0	1
2	1	1	0	0
3	0	1	0	0
4	0	1	0	1
5	0	1	1	1
6	0	1	1	0
7	0	0	1	0
8	0	0	1	1
9	0	0	0	1
10				

4 switches

Move	Sw 1	Sw 2	Sw 3	Sw 4
0	1	1	1	1
1	1	1	0	1
2	1	1	0	0
3	0	1	0	0
4	0	1	0	1
5	0	1	1	1
6	0	1	1	0
7	0	0	1	0
8	0	0	1	1
9	0	0	0	1
10	0	0	0	0

A hint: notice a pattern

Notice: 3 switches

Move	Sw 1	Sw 2	Sw 3
start	1	1	1
1	1	1	0
2	0	**1**	**0**
3	0	**1**	**1**
4	0	**0**	**1**
5	0	**0**	**0**

- We deal with the first digit, then have to go from **1 0** to **0 0**.

Notice: 4 switches

- We deal with the first digit, then have to go from **1 0 0** to **0 0 0**.

Move	Sw 1	Sw 2	Sw 3	Sw 4
start	1	1	1	1
1	1	1	0	1
2	1	1	0	0
3	0	**1**	**0**	**0**
4	0	**1**	**0**	**1**
5	0	**1**	**1**	**1**
6	0	**1**	**1**	**0**
7	0	**0**	**1**	**0**
8	0	**0**	**1**	**1**
9	0	**0**	**0**	**1**
10	0	**0**	**0**	**0**

Going from 0 0 0 to 1 0 0

$$\begin{array}{c} 0\ 0\ \ldots\ 0 \\ \downarrow \\ 1\ 0\ \ldots\ 0 \end{array}$$

- So think about the number of moves required to convert n swithes from all off to only the leftmost switch on. Call this G_n and express this in terms of G_{n-1}.
- (Try some smaller cases!)
- (Convince yourself that it takes the same number of moves to turn the left-most switch on as off in these circumstances.)

A stronger hint

Notice: 4 switches

- The part before G_3 is the case with 2 fewer switches: 1 1.

Move	Sw 1	Sw 2	Sw 3	Sw 4
start	1	1	1	1
1	1	1	0	1
2	1	1	0	0
3	0	1	0	0
4	0	1	0	1
5	0	1	1	1
6	0	1	1	0
7	0	0	1	0
8	0	0	1	1
9	0	0	0	1
10	0	0	0	0

2. Carry out your plan

A stronger hint

- Let M_n be the number of moves required if there are n swiches in the row. Express M_n in terms of M_{n-2} and G_{n-1}.

3. Review

Answers

Answer

- $G_n = 2G_{n-1} + 1.$
- $M_n = M_{n-2} + G_{n-1} + 1.$

Answer

- $G_n = 2G_{n-1} + 1$.
- $M_n = M_{n-2} + G_{n-1} + 1$.

n	G_n
2	3
3	7
4	15
5	31

n	M_n
3	5
4	10
5	21
6	42

- Answer for 6 switches: 42.

Reflect on what happened

- In groups now, think about the process of solving this problem.
    - What went well?
    - What went wrong?
    - Could you have avoided the dead ends, or were they a necessary part of solving the problem?
    - What do you wish you had known when you first attempted the problem?
- Also think about what you have learned and whether you can use this to write down a different problem you can now solve.