Running costs – solution

The cost of running a machine until $t = n$ is

$$\begin{aligned}\int_0^n 950 + 50t^2 \mathrm{d}t &= \left[950t + \frac{50}{3}t^3\right]_0^n \\ &= 950n + \frac{50}{3}n^3.\end{aligned}$$

The cost of buying the machine is the original £6,000 less the money regained in resale:

$$6000 - 3000e^{-\frac{t}{2}} + 200 = 5800 - 3000e^{-\frac{t}{2}}.$$

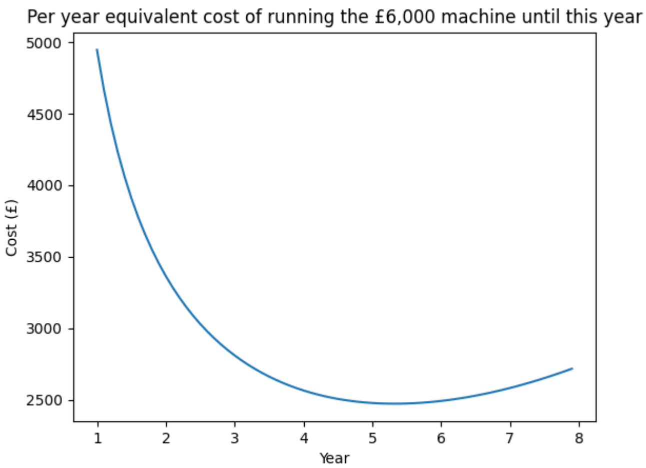
Therefore the cost of buying the machine, running it for n years and then selling it is

$$950n + \frac{50}{3}n^3 + 5800 - 3000e^{-\frac{n}{2}}$$

Running the machine for longer costs more, but this is not necessarily a bad thing, so for comparison we consider a per year equivalent cost, E.

$$\begin{aligned}E &= \frac{950n + \frac{50}{3}n^3 + 5800 - 3000e^{-\frac{n}{2}}}{n} \\ &= 950 + \frac{50}{3}n^2 + 5800n^{-1} - 3000n^{-1}e^{-\frac{n}{2}}.\end{aligned}$$

Plotting this non-linear function, we see the minimum occurs around year 5.



Aside: Not that it matters for our purposes here since we are rounding to 5, but I used Python's `scipy.optimize` to find the minimum of this function and it returned 5.325093947235502.

```
from scipy.optimize import fminbound
from numpy import exp
def f(n):
return 950 + 50/3 * n**2 + n**(-1)*(5800 - 3000*exp(-n/2))
print(fminbound(f, 5, 5.5))
```

Sources

- Sasieni, M., Yaspan, A. and Friedman, L. (1959). Operational Research: Methods and Problems. London: John Wiley & Sons.