Propositional logic summary

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Reminder

NOT	$\neg p$	true only if p is false
AND	$p \wedge q$	true only if p is true and q is true.
OR	$p \vee q$	true if p is true, q is true, or both p and q are true.
XOR	$p \oplus q$	true if p is true or q is true, but not both.

Implication

- We say $p \implies q$ ("p implies q") to mean that if p is true, then q will also be true.
- We are not saying p causes q.
- $p \implies q$ is true unless p is true when q is false.
- If both $p \implies q$ and $q \implies p$ we write $p \iff q$ and say "p is true if and only if q is true".

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- Consider the statement "If $x > 3$, then $x^2 - 2y > 5$."
- There are two propositions:
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- The statement says $p \implies q$.
- If we are to disprove this statement, we must find a situation where p is true and q is false.
- Consider $x = 4$ and $y = 6$. Then

$$x = 4 \wedge y = 6 \implies x^2 - 2y = 4 < 5.$$

- Since p is true and this does not correspond to a true q, we do not have $p \implies q$.

Arguments

- You had a go at some arguments.
- Arguments are a series of premises and a conclusion.
- e.g. if we know that p is true and $p \implies q$ is true, then we know q is true.

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  2. “This dish is a pudding”.
  3. “No nice things are wholesome”.

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- Statement 1 says $p \implies n$.

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- Statement 2 tells us p is true.
- Statement 1 says $p \implies n$.
- Statement 3 says $n \implies \neg w$.

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  - p: "This dish is a pudding";
  - n: "This dish is nice";
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- Statement 2 tells us p is true.
- Statement 1 says $p \implies n$.
- Statement 3 says $n \implies \neg w$.
- Conclusion: $p \implies n \implies \neg w$, this dish is nice but not wholesome.