Counting Up To Symmetry

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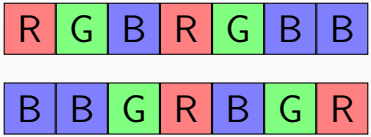
Scarves

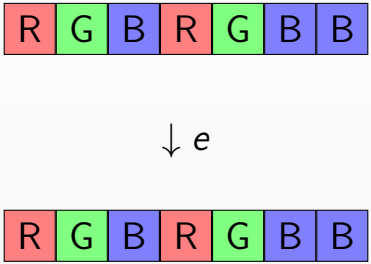
Counting up to symmetry

Suppose that we are making a scarf with coloured knitted squares. How many ways can we make a scarf with 7 patches given that we have 3 different colours of wool?



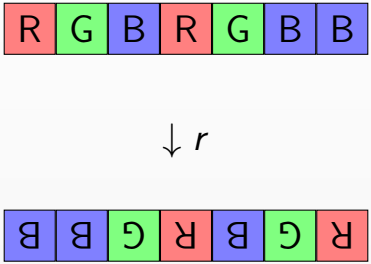
Step 1: List the symmetries

There's the identity (doing nothing):



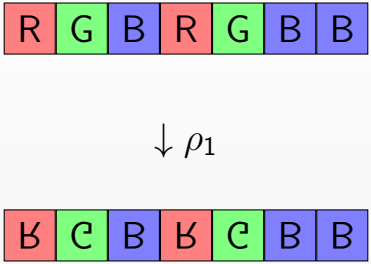
Step 1: List the symmetries

There's rotating it, so the ends swap but the front stays at the front:



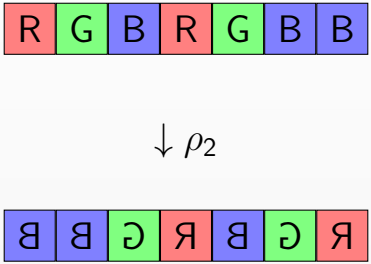
Step 1: List the symmetries

There's flipping it over, so the ends stay where they are but the back and front switch:



Step 1: List the symmetries

There's flipping it round, so the ends swap *and* the front and back switch:



Step 1: List the symmetries

We have four symmetries:

e:

r:

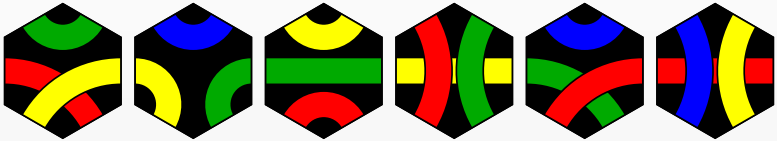
ρ_1:

ρ_2:

Tantrix tiles

What is Tantrix?

- Tantrix is game played with hexagonal tiles
- The game is interesting both to play and for its mathematical aspects
- What we are going to be interested in is not the game or its rules but the game pieces, the tiles.



Tantrix Tiles

To describe the tiles:

- Opaque hexagonal tiles, so the design can't be seen on the reverse of the tile
- Three colours on each tile from a choice of four: blue, green, red, yellow
- Each colour connects two edges of the hexagon
- Colours can't share an edge (i.e., only one colour connected to each edge)
- Colours can cross

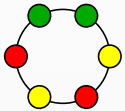
Tantrix Tiles

- Essentially, the crossings are just decorative, so the two tiles below are considered to be the same.
- In fact, only the left-hand tile is in the game.

- There are also banned tiles. So all tiles of the following form are not included in the game.

Graphs and necklaces

- Here we will consider the Tantrix tiles as a particular kind of graph called a **necklace**, because it's easier to deal with and draw.



- Two necklaces are considered the same if one can be rotated (but not flipped over) to form the other.
- There is a similar type of graph called a **bracelet**, where two are considered the same under rotation *and* reflection.