Set theory exercises

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1. The game **Addiction** (1978) included thirteen dice with letters on the faces and numbers used in scoring. A player rolls a die and then must play the letter that is showing on top of the die on a 5 × 5 grid. Once placed, a die cannot be moved. The game proceeds in this way until all thirteen dice are placed on the grid. The player scores points for those dice which form words in a linked intersection formation.

I rolled eight of the **Addiction** dice and obtained this set of letters: {A, U, C, X, E, V, W, T}. Find from this

- (a) A subset that forms a word representing a four-legged family pet.
- (b) Two subsets, one that spells an underground chamber, and a second that you get when it rains, which have a non-empty intersection.
- (c) Two subsets, one that spells a reduction, and a second that spells a product made by bees, which have an empty intersection.
- 2. What is the cardinality of the following sets?

(a) $\{1, 2, 5, 4, 6\}$;	(f) Ø;
(b) $\{3, 4, \text{cat}\};$	(g) \mathbb{N} ;
(c) $\{3, \{4, \text{cat}\}\}\};$	(h) ∅;

- (d) $\{\pi, 6, \{\pi, 5, 8, 10\}\};$ (i) $\{\emptyset\};$
- (e) $\{\pi, 6, \{\pi, 5, 8, 10\}, \{\log, \text{cat}, \{5\}\}\};$ (j) $\{\emptyset, \{\emptyset\}\}.$
- 3. Are the following statements true or false?

(a)
$$1 \in \mathbb{Z}$$
; (b) $0 \in \emptyset$; (c) $\frac{3}{2} \in \mathbb{N}$; (d) $\frac{3}{2} \in \mathbb{R}$; (e) $\pi \notin \mathbb{C}$; (f) $\frac{\pi}{2} \in \mathbb{Q}$.

- 4. Consider $A = \{2, 3, 7, 15\}$. Give rules that define subsets of three elements which exclude each of the members in turn from A. For example, $B = \{x \mid x > 2\}$ would exclude 2, though you could try to be more creative.
- 5. Consider $A=\{x\in\mathbb{N}\mid 1\leq x\leq 20\}$ and $B=\big\{x\mid x,\frac{x}{2}\in\mathbb{N}\wedge 2\leq x\leq 30\big\}.$
 - (a) Write out in words what numbers are in sets A and B.
 - (b) Write out full lists of the members of A and B.
 - (c) Write out the list of elements in these sets:

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i. \{x \mid x \in A \land x \in B\};
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ii.
$$\{x \in B \mid x \text{ is prime}\};$$

iii.
$$\{x \in A \mid \sqrt{x} \in \mathbb{Z}\}.$$

- 6. Write out the list of elements in these sets:
 - (a) $\{x \in \mathbb{Z} \mid x^2 \le 25\};$
 - (b) $\{x \in \mathbb{R} \mid x^2 = 2\};$
 - (c) $\{x \in \mathbb{Z} \mid x^2 = 2\};$
 - (d) $\{x \in \mathbb{R} \mid 6 < x < 3\}.$
- 7. Write definitions of the form $\{x \in \dots \mid \dots \}$ for the following sets.
 - (a) $\{1, 4, 9, 16, 25, 36, 49, \dots\};$
 - (b) $\{1, 2, 4, 8, 16, 32, \dots\};$
 - (c) $\{10, 11, 12, 13, 14, 15, 16, 17, 18, 19\}.$
- 8. Which, if any, of the following sets are equal to each other?
 - (a) $A = \{1, 2, 3\};$
 - (b) $B = \{x \in \mathbb{N} \mid x > 0 \land x^2 < 10\};$
 - (c) $C = \{x \in \mathbb{N} \mid n^2 < 1\};$
 - (d) $D = \emptyset$.
- 9. Which of the following statements are true?
 - (a) $\mathbb{Z} \subseteq \mathbb{N}$;

(h) $\{1\} \in \mathbb{Z}$;

(b) $\mathbb{N} \subseteq \mathbb{Z}$;

(i) $\{1\} \subseteq \mathbb{Z}$;

(c) $\{1,3,7\} \subset \mathbb{N}$;

 $(j) \emptyset \subseteq \mathbb{Z};$

(d) $\{1,3,7\} \subset \{1,3,7\};$

(k) $\{0\} \subseteq \emptyset$;

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 $(1) \emptyset \subseteq \{1, 2\};$

(e) $\{1,3,7\} \subseteq \{1,3,7\};$

(m) $\{\emptyset\} \subseteq \emptyset$;

(f) $1 \in \mathbb{Z}$; (g) $1 \subseteq \mathbb{Z}$;

- (n) $\emptyset \subseteq \{\emptyset\}$.
- 10. Let $X = \{0,1\}$ and $Y = \{1,2,3\}$. What are the elements of $X \times Y$?
- 11. Let $P = \{x \in \mathbb{R} \mid \sin(x) = 0\}$ and $Q = \{n\pi \mid n \in \mathbb{Z}\}$. What is the relationship between P and Q?
- 12. Consider the sets $A = \{x \in \mathbb{Z} \mid 2 \le x\}$ and $B = \{x \in \mathbb{Z} \mid x \le 5\}$. Show that $A \cap B$ is finite. $A \cup B$ has a special name, what is it?
- 13. Find $\mathbb{Z} \cap \mathbb{Z}$, $\mathbb{Z} \cap \emptyset$, and $\mathbb{Z} \cap \mathbb{R}$.
- 14. Considering a proposition p acting within some set D, the truth set of p is the set of elements $x \in D$ for which p is true, i.e. $\{x \in D \mid p(x)\}$.

What is the truth set of the following propositions?

- (a) p: "x is a day you currently have classes".
- (b) q: "x is a logical connective studied in this module".
- (c) r: "x is a Sheffield Hallam lecturer who has taught you".
- (d) s: "x is a real number and $x^2 4x + 3 = 0$ ".