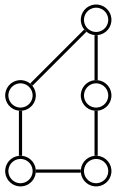
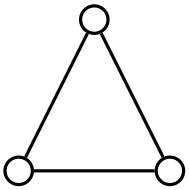
Worksheet 8: Counting up to symmetry

1. Suppose we consider pairs of the 26 letters of the alphabet, e.g., (a, b), (x, z), (b, a), etc.
   (a) How many pairs of letters are there?
   (b) If we consider, for example, (a, b) and (b, a) to be the same pair, then how many fewer pairs of letters are there?

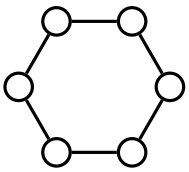
2. How many ways are there to colour the vertices of the figure shown using
   (a) two colours;
   (b) three colours;
   (c) seven colours.



3. How many ways are there to colour the vertices of the figure shown using
   (a) two colours;
   (b) three colours;
   (c) seven colours.



4. How many ways are there to colour the vertices of the figure shown using
   (a) two colours;
   (b) three colours;
   (c) seven colours.



5. A patchwork quilt is made up of different coloured squares, which are only visible on the front of the quilt.
   (a) How many ways can a 2×2 quilt be made using up to 2 colours? Use Polya enumeration to compute what combinations of colours these use and draw all cases.
   (b) How many ways can a 3×3 quilt be made using up to 2 colours? Use Polya enumeration to compute what combinations of colours these use, but do not draw all cases.
   (c) How many ways can a 2×3 quilt be made using up to 2 colours? Use Polya enumeration to compute what combinations of colours these use, but do not draw all cases.