

Connectives exercises

Peter Rowlett

1. Consider the sentence: “Did you see Angharad or Ros earlier?”
Which type of ‘or’, OR (\vee) or XOR (\oplus), is being used?
2. Consider the sentence: “The door is open or closed.”
Which type of ‘or’, OR (\vee) or XOR (\oplus), is being used?
3. Consider the following conversation.
Waiter: Would you like tea or coffee?
Customer: Tea, please.
Waiter: Would you like milk or sugar?
Customer: Both, please.
Which type of ‘or’, OR (\vee) or XOR (\oplus), is intended by the waiter in each of the two questions?
4. In natural language, it is often irritating to communicate in a way that is strictly logical. Consider the following unhelpful dialogue. Is person 2 right?
Person 1: Is the window open or closed?
Person 2: Yes.
5. While we are on the subject of natural language, as an aside, can you make sense of the following dialogue?
Three people meet a friend in a pub.
Friend: Do you all want a beer?
Person 1: I don’t know.
Person 2: I don’t know.
Person 3: Yes.
6. Suppose we have the following propositions.

p : “It is cold”

q : “It is raining”

u : “It is cold but not raining”

The relationship of u to p and q can be described using a combination of the operators discussed above.

$$u = p \text{ AND (NOT } q)$$

$$u = p \wedge \neg q$$

Complete the table below.

p	q	$\neg q$	$u = p \wedge \neg q$
false	false		
false	true		
true	false		
true	true		

7. Express the following using simpler mathematical notation. The first is done for you.

(a) $(\pi > 0) \wedge (\pi < 10)$ [Answer: $0 < \pi < 10$.]

(b) $(p \geq 7) \wedge (p < 12)$.

(c) $(x > 5) \wedge (x < 7)$.

(d) $x < 4 \wedge x < 6$.

(e) $(x \geq 0) \wedge (x \leq 0)$.

(f) $(x = 0) \vee (x > 0)$.

(g) $\neg(x > 7)$.

(h) $\neg(x = 1)$.

(i) $\neg(\neg(x > 0))$.

(j) $\neg(x \text{ is even})$.

8. Is this statement true or false? " $(3 < 5) \vee (1 = 2)$ ".

9. What would you need to do to demonstrate that $p \wedge q \wedge r \wedge s \wedge t$ is true?

10. What would you need to do to demonstrate that $p \wedge q \wedge r \wedge s \wedge t$ is false?

11. What would you need to do to demonstrate that $p \vee q \vee r \vee s \vee t$ is true?

12. What would you need to do to demonstrate that $p \vee q \vee r \vee s \vee t$ is false?

13. Is it possible for one of $(p \wedge q) \wedge r$ and $p \wedge (q \wedge r)$ to be true and the other false? Write out a truth table to investigate.

14. Is it possible for one of $(p \vee q) \vee r$ and $p \vee (q \vee r)$ to be true and the other false? Write out a truth table to investigate.

15. Simplify the following statements.

(a) $\neg(p \vee \neg q)$.

(b) $\neg(\neg p \wedge q)$.

(c) $\neg(p \vee q)$.

(d) $\neg(p \wedge q)$.

16. A *tautology* is a statement such that the truth table has 'true' for all outputs. Draw a truth table to show that ' $p \vee \neg p$ ' is a tautology.

17. A *contradiction* is a statement such that the truth table has 'false' for all outputs. Draw a truth table to show that ' $p \wedge \neg p$ ' is a contradiction.