Game Theory and Recreational Mathematics
/ Week 8: Inclusion-exclusion and generators and enumerators
/ Caterpillar table

Caterpillar table to accompany slides

selection	Fs	Rs	Ls	Ms	length	arrangements
$f^3r^2l^2m$	3	2	2	1	8	$\frac{8!}{3!2!2!1!} = 1680$
f^3rl^2m	3	1	2	1	7	$\frac{7!}{3!1!2!1!} = 420$
$f^3r^2l^2$	3	2	2	0	7	$\frac{7!}{3!2!2!0!} = 210$
f^3r^2lm	3	2	1	1	7	$\frac{7!}{3!2!1!1!} = 420$
$f^2r^2l^2m$	2	2	2	1	7	$\frac{7!}{2!2!2!1!} = 630$
f^3l^2m	3	0	2	1	6	$\frac{6!}{3!0!2!1!} = 60$
f^3rl^2	3	1	2	0	6	$\frac{6!}{3!1!2!0!} = 60$
f^3rlm	3	1	1	1	6	$\frac{6!}{3!1!1!1!} = 120$
f^3r^2l	3	2	1	0	6	$\frac{6!}{3!2!1!0!} = 60$
f^3r^2	3	2	0	0	5	$\frac{5!}{3!2!0!0!} = 10$
f^3rl	3	1	1	0	5	$\frac{5!}{3!1!1!0!} = 20$
f^3lm	3	0	1	1	5	$\frac{5!}{3!0!1!1!} = 20$
f^3l^2	3	0	2	0	5	$\frac{5!}{3!0!2!0!} = 10$
f^3rm	3	1	0	1	5	$\frac{5!}{3!1!0!1!} = 20$
f^3r	3	1	0	0	4	$\frac{4!}{3!1!0!0!} = 4$
f^3l	3	0	1	0	4	$\frac{4!}{3!0!1!0!} = 4$
f^3m	3	0	0	1	4	$\frac{4!}{3!0!0!1!} = 4$
f^3	3	0	0	0	3	$\frac{3!}{3!0!0!0!} = 1$

selection	3s	2s	0s	1s	length	arrangements
f^3r^2m	3	2	0	1	6	$\frac{6!}{3!2!0!1!} = 60$
f^2rl^2m	2	1	2	1	6	$\frac{6!}{2!1!2!1!} = 180$
$f^2r^2l^2$	2	2	2	0	6	$\frac{6!}{2!2!2!0!} = 90$
f^2r^2lm	2	2	1	1	6	$\frac{6!}{2!2!1!1!} = 180$
fr^2l^2m	1	2	2	1	6	$\frac{6!}{1!2!2!1!} = 180$
f^3l^2	3	0	2	0	5	$\frac{5!}{3!0!2!0!} = 10$
f^3lm	3	0	1	1	5	$\frac{5!}{3!0!1!1!} = 20$
f^3rl	3	1	1	0	5	$\frac{5!}{3!1!1!0!} = 20$
f^3rm	3	1	0	1	5	$\frac{5!}{3!1!0!1!} = 20$
f^3r^2	3	2	0	0	5	$\frac{5!}{3!2!0!0!} = 10$
f^2l^2m	2	0	2	1	5	$\frac{5!}{2!0!2!1!} = 30$
f^2rl^2	2	1	2	0	5	$\frac{5!}{2!1!2!0!} = 30$
f^2rlm	2	1	1	1	5	$\frac{5!}{2!1!1!1!} = 60$
f^2r^2l	2	2	1	0	5	$\frac{5!}{2!2!1!0!} = 30$
f^2r^2m	2	2	0	1	5	$\frac{5!}{2!2!0!1!} = 30$
frl^2m	1	1	2	1	5	$\frac{5!}{1!1!2!1!} = 60$
f^2r^2						$\frac{5!}{} =$

selection	1s	2s	2s	0s	length	arrangements
fr^2l^2	1	2	2	0	5	$\frac{5!}{1!2!2!0!} = 30$
fr^2lm	1	2	1	1	5	$\frac{5!}{1!2!1!1!} = 60$
r^2l^2m	0	2	2	1	5	$\frac{5!}{0!2!2!1!} = 30$
f^3l	3	0	1	0	4	$\frac{4!}{3!0!1!0!} = 4$
f^3m	3	0	0	1	4	$\frac{4!}{3!0!0!1!} = 4$
f^3r	3	1	0	0	4	$\frac{4!}{3!1!0!0!} = 4$
f^2l^2	2	0	2	0	4	$\frac{4!}{2!0!2!0!} = 6$
f^2lm	2	0	1	1	4	$\frac{4!}{2!0!1!1!} = 12$
f^2rl	2	1	1	0	4	$\frac{4!}{2!1!1!0!} = 12$
f^2rm	2	1	0	1	4	$\frac{4!}{2!1!0!1!} = 12$
f^2r^2	2	2	0	0	4	$\frac{4!}{2!2!0!0!} = 6$
fl^2m	1	0	2	1	4	$\frac{4!}{1!0!2!1!} = 12$
frl^2	1	1	2	0	4	$\frac{4!}{1!1!2!0!} = 12$
$frlm$	1	1	1	1	4	$\frac{4!}{1!1!1!1!} = 24$
fr^2l	1	2	1	0	4	$\frac{4!}{1!2!1!0!} = 12$
fr^2m	1	2	0	1	4	$\frac{4!}{1!2!0!1!} = 12$
r^2						$\frac{4!}{}$

selection	0s	1s	2s	1s	length	arrangements
rl^2m	0	1	2	1	4	$\frac{4!}{0!1!2!1!} = 12$
r^2l^2	0	2	2	0	4	$\frac{4!}{0!2!2!0!} = 6$
r^2lm	0	2	1	1	4	$\frac{4!}{0!2!1!1!} = 12$
f^3	3	0	0	0	3	$\frac{3!}{3!0!0!0!} = 1$
f^2l	2	0	1	0	3	$\frac{3!}{2!0!1!0!} = 3$
f^2m	2	0	0	1	3	$\frac{3!}{2!0!0!1!} = 3$
f^2r	2	1	0	0	3	$\frac{3!}{2!1!0!0!} = 3$
fl^2	1	0	2	0	3	$\frac{3!}{1!0!2!0!} = 3$
flm	1	0	1	1	3	$\frac{3!}{1!0!1!1!} = 6$
frl	1	1	1	0	3	$\frac{3!}{1!1!1!0!} = 6$
frm	1	1	0	1	3	$\frac{3!}{1!1!0!1!} = 6$
fr^2	1	2	0	0	3	$\frac{3!}{1!2!0!0!} = 3$
l^2m	0	0	2	1	3	$\frac{3!}{0!0!2!1!} = 3$
rl^2	0	1	2	0	3	$\frac{3!}{0!1!2!0!} = 3$
rlm	0	1	1	1	3	$\frac{3!}{0!1!1!1!} = 6$
r^2l	0	2	1	0	3	$\frac{3!}{0!2!1!0!} = 3$
r^3						$\frac{3!}{} =$

selection	0s	2s	0s	1s	length	arrangements
r^2m	0	2	0	1	3	$\frac{3!}{0!2!0!1!} = 3$
f^2	2	0	0	0	2	$\frac{2!}{2!0!0!0!} = 1$
fl	1	0	1	0	2	$\frac{2!}{1!0!1!0!} = 2$
fm	1	0	0	1	2	$\frac{2!}{1!0!0!1!} = 2$
fr	1	1	0	0	2	$\frac{2!}{1!1!0!0!} = 2$
l^2	0	0	2	0	2	$\frac{2!}{0!0!2!0!} = 1$
lm	0	0	1	1	2	$\frac{2!}{0!0!1!1!} = 2$
rl	0	1	1	0	2	$\frac{2!}{0!1!1!0!} = 2$
rm	0	1	0	1	2	$\frac{2!}{0!1!0!1!} = 2$
r^2	0	2	0	0	2	$\frac{2!}{0!2!0!0!} = 1$
f	1	0	0	0	1	$\frac{1!}{1!0!0!0!} = 1$
l	0	0	1	0	1	$\frac{1!}{0!0!1!0!} = 1$
m	0	0	0	1	1	$\frac{1!}{0!0!0!1!} = 1$
r	0	1	0	0	1	$\frac{1!}{0!1!0!0!} = 1$

Table 1: 71 valid selections of segments to form caterpillars, selecting from 8 segments F, F, F, R, R, L, L and M.