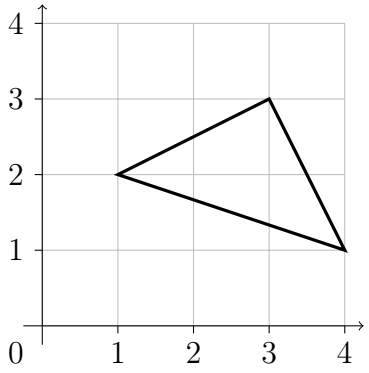
Tutorial exercise sheet – Transformations

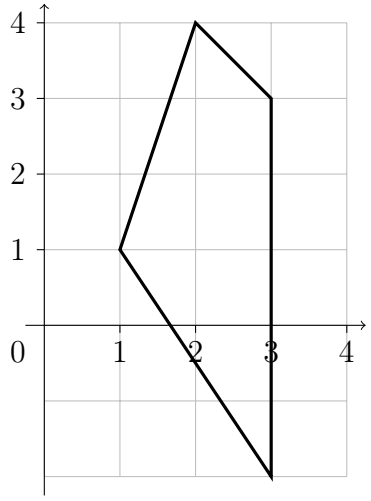
Peter Rowlett

1. Write matrices that represent the shapes shown below.

(a)



(b)



2. Consider three transformation matrices

$$\mathbf{T}_1 = \begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix}, \quad \mathbf{T}_2 = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} \quad \text{and} \quad \mathbf{T}_3 = \begin{bmatrix} 5 & 0 \\ 0 & 3 \end{bmatrix}$$

and the rectangle

$$\mathbf{R} = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}.$$

What are the coordinates of the transformations:

   (a) $\mathbf{T}_1\mathbf{R}$;
   (b) $\mathbf{T}_2\mathbf{R}$;
   (c) $\mathbf{T}_3\mathbf{R}$?

3. Write down the transformation matrix that

   (a) scales horizontal distance by a factor of 3 and vertical distances by a factor of 7;
   (b) scales horizontal distances by a factor of 0.7 and vertical distances by a factor of 1.2;
   (c) leaves horizontal distances unchanged and trebles vertical distances.

4. The point $\begin{bmatrix} 2 \\ 6 \end{bmatrix}$ is rotates anticlockwise about the origin through 40°. Calculate the position of the new point.

5. The triangle defined by

$$\mathbf{T} = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 5 & 1 \end{bmatrix}$$

is rotated clockwise about the origin through 120°. Determine the coordinates of the resulting triangle.

6. The point $\begin{bmatrix}3\\5\end{bmatrix}$ is reflected in the line $y = -x$. Give the coordinates of the resulting point.

7. The point $\begin{bmatrix}-1\\4\end{bmatrix}$ is reflected in the y axis and the resulting point is then reflected in the line $y = x$. Give the coordinates of the resulting point.

8. The point $\begin{bmatrix}a\\b\end{bmatrix}$ is reflected in the x axis. The resulting point is then reflected in the line $y = -x$. Give the coordinates of the resulting point.

9. The square
$$\mathbf{S} = \begin{bmatrix}0 & 0 & 1 & 1\\0 & 1 & 0 & 1\end{bmatrix}$$
is sheared by
$$\mathbf{T} = \begin{bmatrix}1 & 0\\3 & 1\end{bmatrix}.$$
What are the coordinates of the vertices of the shape after shearing?

10. State the effect of the transformation matrix
$$\mathbf{T} = \begin{bmatrix}1 & 2\\0 & 1\end{bmatrix}$$

11. A point is translated from $\begin{bmatrix}1\\6\end{bmatrix}$ to $\begin{bmatrix}4\\8\end{bmatrix}$. State the corresponding translation matrix.

12. A point is translated from $\begin{bmatrix}-3\\-1\end{bmatrix}$ to $\begin{bmatrix}-6\\0\end{bmatrix}$. State the corresponding translation matrix.

13. State the effect of the translation matrix
$$\begin{bmatrix}1 & 0 & 4\\0 & 1 & -3\\0 & 0 & 1\end{bmatrix}.$$

14. Determine a matrix that shears vertically by a factor of 2 and then rotates anticlockwise about the origin by 30°.

15. Determine a matrix that rotates anticlockwise through 40° about the point $\begin{bmatrix}1\\0\end{bmatrix}$.

16. The zoom facility of a computer graphics package has the following options:

    (a) double horizontal lengths only;
    (b) double vertical lengths only;
    (c) double both horizontal and vertical lengths;
    (d) halve both horizontal and vertical lengths.

    Write down the transformation matrix for each option.