

# Logic notes

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## 1 Propositions

A proposition is a sentence that has a truth value, it is unambiguously true or false, for example “Two plus three equals five” and “My apple tree grows purple lemons”.

Saying a sentence is a proposition says nothing about whether it is true.

We might label a proposition with a letter, for example:

- $p$ : “If  $x = 3$ , then  $x^2 = 9$ .”

Using this notation, if we say “ $p$ ” it means we are saying “ $p$  is true”.

## 2 Connectives

Connectives are used to combine propositions to form other propositions.

### 2.1 NOT

NOT is a connective that negates a statement. If  $p$  is true, then NOT  $p$  is false, and vice versa. We will write NOT  $p$  as  $\neg p$ .

We can represent this information in an arrangement called a truth table.

$p$	$\neg p$
true	false
false	true

### 2.2 AND

We can combine two propositions using AND, written as  $\wedge$ . This is only true if both  $p$  and  $q$  are true.

The truth table for AND is as follows.

$p$	$q$	$p \wedge q$
true	true	true
true	false	false
false	true	false
false	false	false

### 2.3 OR

Another way to combine two propositions is using OR, written  $\vee$ . This is true if at least one of  $p$  and  $q$  are true.

The truth table for OR is as follows.

$p$	$q$	$p \vee q$
true	true	true
true	false	true
false	true	true
false	false	false

## 2.4 XOR

The exclusive OR  $p \text{ XOR } q$  or  $p \oplus q$  is used when either  $p$  or  $q$  are true but not both. The following table applies.

$p$	$q$	$p \oplus q$
False	False	False
False	True	True
True	False	True
True	True	False

## 3 Implication

The connective  $\implies$  is called “implies”. A proposition  $p \implies q$  can be read “ $p$  implies  $q$ ” or “if  $p$ , then  $q$ ”.

We can also think about situations where  $q \implies p$ , and if both  $p \implies q$  and  $q \implies p$  are true, then we write  $p \iff q$  and say “ $p$  if and only if  $q$ ”.

The implication  $p \implies q$  is true unless  $p$  is true and  $q$  is false. The truth table is as follows. The  $p \implies q$  column refers to the truth of the implication, not the truth of either  $p$  or  $q$ .

$p$	$q$	$p \implies q$
true	true	true
true	false	false
false	true	true
false	false	true

In the cases where  $p$  is false, this tells us nothing to refute the idea that  $p \implies q$ , so we say the implication is true.

It is also worth noting that if  $q$  is always true, then  $p \implies q$  is true regardless of  $p$ . In fact,  $q$  being true tells us nothing about the truth value of  $p$ .

## 4 Arguments

Within propositional logic we can make arguments. An argument is made up of two parts:

1. a number of propositions, called the premises;
2. a proposition, called the conclusion.

For example, if say we want to establish  $p \implies q$ . If we know  $p \implies r$  and  $r \implies q$ , then we can form an argument  $p \implies r \implies q$ .