Tutorial exercise sheet – Vectors

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1. Which of the following are scalars and which are vectors?

   (a) the volume of a water tank; (b) a length measured in metres; (c) a length measured in miles; (d) the angular velocity of a flywheel; (e) the relative velocity of two aircraft; (f) the work done by a force; (g) the momentum of an atomic particle.

2. A force of $13N$ acts at an angle of $62°$ above the x-axis. Resolve this force into two components, one horizontal and one vertical, and represent this as a matrix.

3. Find the modulus of each of the following vectors:

   (a) $\begin{bmatrix}7\\3\end{bmatrix}$; (b) $\begin{bmatrix}17\\0\end{bmatrix}$; (c) $\begin{bmatrix}0\\-3\end{bmatrix}$; (d) $\begin{bmatrix}2\\3\end{bmatrix}$; (e) $\begin{bmatrix}2\\-3\end{bmatrix}$; (f) $\begin{bmatrix}-2\\-3\end{bmatrix}$.

4. If $\mathbf{a} = \begin{bmatrix}4\\6\end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix}3\\-3\end{bmatrix}$, find $\mathbf{a} \cdot \mathbf{b}$.

5. If $\mathbf{a} = \begin{bmatrix}5\\3\\7\end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix}1\\-2\\-7\end{bmatrix}$, find $\mathbf{a} \cdot \mathbf{b}$.

6. Find the angle between $\mathbf{a} = \begin{bmatrix}2\\1\end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix}5\\11\end{bmatrix}$.

7. Find the component of the vector $\mathbf{a} = \begin{bmatrix}2\\3\end{bmatrix}$ in the direction of the vector $\mathbf{b} = \begin{bmatrix}1\\5\end{bmatrix}$.

8. Evaluate $\mathbf{a} \cdot \mathbf{i}$ where $\mathbf{a} = \begin{bmatrix}4\\8\end{bmatrix}$. Hence find the angle that $\mathbf{a}$ makes with the x-axis.

9. Find the cross product of $\mathbf{a} = \begin{bmatrix}-2\\-3\\0\end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix}4\\7\\0\end{bmatrix}$.

10. Find the cross product of $\mathbf{a} = \begin{bmatrix}3\\-2\\5\end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix}7\\4\\-8\end{bmatrix}$.

11. For the following vectors, calculate $\mathbf{a} \cdot \mathbf{b}$, $\mathbf{b} \cdot \mathbf{a}$, $\mathbf{a} \cdot \mathbf{a}$, $\mathbf{b} \cdot \mathbf{b}$, $\mathbf{a} \times \mathbf{b}$, and $\mathbf{b} \times \mathbf{a}$.

    (a) $\mathbf{a} = \begin{bmatrix}4\\3\\2\end{bmatrix}$, $\mathbf{b} = \begin{bmatrix}2\\-1\\11\end{bmatrix}$;

    (b) $\mathbf{a} = \begin{bmatrix}2\\-3\\-4\end{bmatrix}$, $\mathbf{b} = \begin{bmatrix}-2\\14\\1\end{bmatrix}$.