What is recreational mathematics and when did it arise?

Peter Rowlett

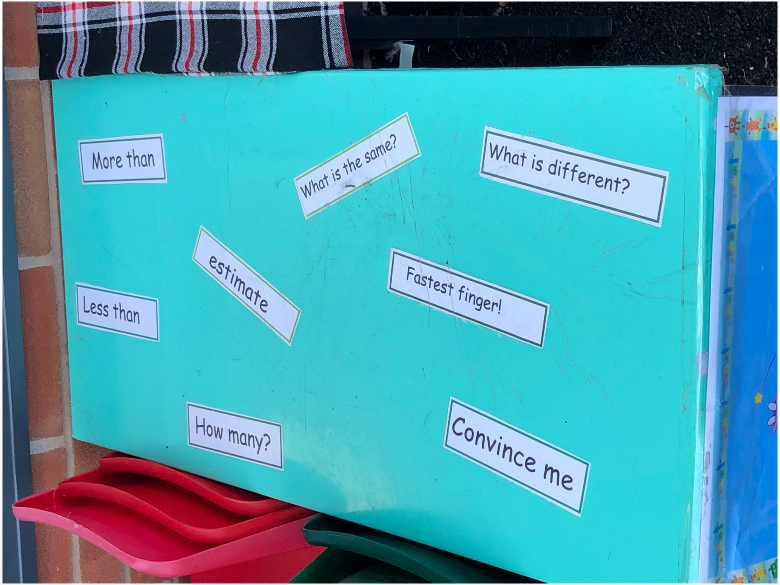
Sheffield Hallam University

p.rowlett@shu.ac.uk

Golden rule for this module

- Introducing his section of hints and solutions, Pólya says "The reader who has earnestly tried to solve the problem has the best chance to profit by the hint and the solution."
- **Please please please**, if you have already seen a problem and know how to solve it, allow others the chance to profit from trying it themselves, rather than simply telling them the solution. Even if you do decide to tell them, try giving hints and asking questions to try to lead them to a solution, rather than giving them the solution wholesale. You and they will benefit from the practice.

Mathematical thinking



What is recreational mathematics?

What is recreational mathematics?

- Games and puzzles? What else?

What is recreational mathematics?

- Games and puzzles? What else?
- "Easier to recognise than define" [Hankin, 2005, p. 48].
- "No classification of particular mathematical topics as recreational or not is likely to gain universal acceptance" [Trigg, 1978, p. 21].

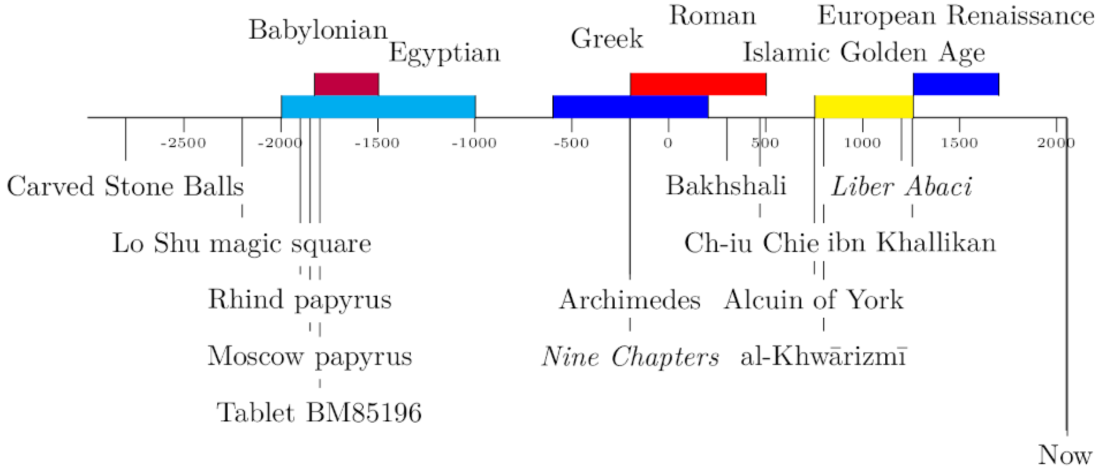
History

- Recreational mathematics has a history as old as formal mathematics, with examples on the oldest mathematical texts [Sumpter, 2015] (problems 1–4).
- *The history of the subject [Mathematical Puzzles] entails nothing short of the actual story of the beginnings and development of exact thinking in man. The historian must start from the time when man first succeeded in counting his ten fingers and in dividing an apple into two approximately equal parts.*

Henry Dudeney, 1917.

(p. v, Preface to *Amusements in Mathematics*)

Timeline (from notes)



Carved stone balls?

By Johnbod - Own work, CC BY-SA 4.0, https://commons.wikimedia.org/w/index.php?curid=37283607

Which came first, recreational maths or serious maths?

> *At the time, graph theory was a mathematical backwater, known for posing fun problems like the four-color conjecture … "I wouldn't call it obscure, but certainly graph theory was not mainstream mathematics because many of the problems or results arose from puzzles or sort of recreational mathematics," said Lovász*[1]*.*

[Hartnett, 2021].

[1] That's 2021 Abel Prize joint-winner László Lovász.

Which came first, recreational maths or serious maths?

- There are many examples of recreational problems acting as stimulus for non-recreational topics [Singmaster, 2016], for example:
  - graph theory has origins in the Königsberg bridges problem (problem 32) and Icosian Game;
  - probability has origins in games of chance (problem 30).

What does the literature say?

- Some give extremely broad definitions – anything fun, or involving play [Moscovich, 2004, Singmaster, 2000, Barcellos, 1979]. Perhaps too broad!
- Play in recreational mathematics might be a puzzle, game, magic trick, paradox, fallacy or curious piece of mathematics [Gardner, 1959].
- Play is positive [Sumpter, 2015] and may be a fundamental aspect of mathematics [Su, 2017].
- Popularity [Singmaster, 2000] - e.g. puzzles in the popular press [Cockcroft, 1982, Dyson, 2011].

Difficult

- Some say recreational mathematics should be elementary, not requiring advanced knowledge [Dyson, 2011, Hankin, 2005, Trigg, 1978].
- However, it is certainly possible to have fun and play with mathematics that is more advanced, depending on an individual's level of mathematics knowledge [Trigg, 1978].

‘Recreational’ and ‘serious’ maths

- Some have recreational maths leading to ‘serious’ maths [Barcellos, 1979, Minor, 2015], though the distinction is not universally accepted [Moscovich, 2004].

Educational value

- Games and puzzles can:
  - help learning and develop logical thinking [Cockcroft, 1982];
  - provoke curiosity, help develop concepts and skills through play and motivate engagement [Moscovich, 2004, Hollis and Felder, 1982, Barcellos, 1979];
  - help take the “drugery” out of practice [Ernest, 1986], with potential to develop creative, problem-solving skills alongside procedural fluency [Foster, 2013, Foster, 2018].

Play

Play

“A free soul ought not to pursue any study slavishly; for while bodily labours performed under constraint do not harm the body, nothing that is learned under compulsion stays with the mind.”

“True,” he said. “Do not, then, my friend, keep children to their studies by compulsion but by play.”

— from Plato’s Republic, c. 375 BC

Play

- [Hollis and Felder, 1982] have recreational mathematics as “a type of play that enables children to acquire concepts and skills useful to the teaching of mathematics” (p. 71).
- Martin Gardner said in an interview: “I’ve always felt that a teacher can introduce recreational math; . . . I don’t know of any better way to hook the interests of the students” [Barcellos, 1979, p. 238].

Educational uses – engagement

- Recreational mathematicatics can be used to enhance the engagement and interest of students [Barcellos, 1979, Böhm, 2017, Devi, 1976, Dyson, 2011, Sumpter, 2015].
- Teaching using games has been shown to increase engagement and attitudes [Ernest, 1986, Oldfield, 1991, Bragg, 2012].

Play – mathematics as an experimental science

Studying the methods of solving problems, we perceive another face of mathematics. Yes, mathematics has two faces; it is the rigorous science of Euclid but it is also something else. Mathematics presented in the Euclidean way appears as a systematic, deductive science; but mathematics in the making appears as an experimental, inductive science. Both aspects are as old as the science of mathematics itself.

– [Pólya, 2004, p. vii]

Understanding

- Recreational mathematics has potential to develop and expand mathematical skills and deepen understanding of mathematical concepts and topics. For example:
  - playing board games is linked to mathematical and strategic thinking and problem-solving skills [Smith and Golding, 2018];
  - recreational mathematics can be used to develop confidence in problem-solving [Eisenberg, 1991].

Formal syllabus and logical thinking

- The Cockcroft report refers to puzzles and games interacting with the formal syllabus [Cockcroft, 1982, p. 67]:

  Whatever the level of attainment of pupils, carefully planned use of mathematical puzzles and 'games' can clarify the ideas in a syllabus and assist the development of logical thinking.

Informal learning

- [Böhm, 2017, p. 12, emphasis original]:
  - "others at my age collected stamps, I collected interesting puzzles ... by solving puzzles I learned quite a lot";
  - "not necessarily *useful* mathematics, in the sense that the acquired knowledge was of much use in our math lessons in school";
  - e.g. "knowing how to construct a magic square will be of no help when dealing with problems from elementary analysis".

A definition

Recreational mathematics is a type of play which is enjoyable and requires mathematical thinking or skills to engage with; typically, it is accessible to a wide range of people and can be effectively used to motivate engagement with and develop understanding of mathematical ideas or concepts.

- (Not perfect, but matches how the term is used in practice.)

Problem solving

What happens when you encounter an unseen problem?

If you can solve it

- Gosh, well done!
- Remember, we are interested in your problem-solving abilities. The fact you can solve this problem is lovely, but may just be a fluke or coincidence.
- Can you do it on different unseen problems, or did you get lucky?
- It's good if you are able to communicate your result clearly and show us you solved this via a sensible problem-solving method.
- Perhaps try a different problem. Better to practice when you can't just see the solution.

If you can't solve it

- Good! Here's where it gets interesting.
- There are problem-solving approaches you can try to get to a solution.
- For this module:
  - it is very important that you try a sensible problem-solving approach which has a good chance at working towards a solution;
  - the aim is to learn about problem solving, not necessarily to solve a particular problem;
  - this is especially true if your solution relies on you getting a hint or sneaking a look at the answer;
  - acknowledge that this is hard, because the emotions in problem solving are all around getting a solution.

Advice for problem solving

1. Plan

- Advice:
  - Stop and really read the problem.
  - Ask yourself: What am I being asked to do? What information have I been given? What information is missing? What would a solution look like?

1. Plan

- Advice:
  - Draw a picture. Introduce suitable notation for the information you have been given and information you are being asked for.

1. Plan

- Advice:
  - Have you seen a similar problem before? How did you solve that one?

1. Plan

- Advice:
  - Separate the problem into smaller parts and examine them separately.

1. Plan

- Advice:
  - Can you write the problem in a different way? Try it. Is what you have written actually the same problem? If not, what is different? If it is, can you solve this version of the problem?
  - Can you solve a related problem? Can you remove part of the restriction and solve a more general problem? Or, can you come up with a specific example and solve that? Do your solutions help you plan to approach the main problem?

2. Carry out your plan

- When your plan is ready, put it into action.
- Advice:
  - Check each step. Is each step correct?
  - You should expect to be stuck quite a lot of the time. Recognise that you are stuck and accept it. Calmly review where you are and try to get unstuck.
  - It is (usually!) okay to wait and mull over the problem for a while.
  - If you are sure your plan cannot work, you may need to return to the *Plan* stage.

3. Review

- First, if you found a solution, check it is correct.
  - Can you check your solution is correct?
  - Does it answer the original problem?
  - Can you get the same solution from a different method?
  - Can you work from your solution and get back to the original problem and, doing so, is the problem you get to the same as the original problem you tried to solve?

3. Review

- Second, reflect on what happened.
  - Remember that the point of solving problems is not just to get marks in assessments. The purpose here is to think about what has happened and see what you can learn.

3. Review

- Second, reflect on what happened.
    - Think about the process you took and particularly any dead ends you went down.
        - What went wrong?
        - Could you have avoided the dead ends, or were they a necessary part of solving the problem?
        - What do you wish you had know when you first attempted the problem?

3. Review

- Second, reflect on what happened.
  - What can you do now that you couldn't before?
    - Can you use the method, or the result, for some other problem?
    - Can you write down a new problem that you are now able to solve?
    - Is the problem you have solved part of a wider family of problems?
    - Can your method be adapted to solve more of them?

What does this mean for an unseen problem?

Attempting a solution

- Make sure you have understood the question and explored this understanding:
  - Rewrite the question in your own words.
  - State the crucial features of the question.
  - Draw a diagram showing the question.
  - Write an explanation of what the question is asking you to do and why this is difficult. Where does the difficulty lay?

Attempting a solution

- Try variants:
  - Try to state and solve a simpler version of the problem.
    - Remove part of the restrictions.
    - Add more restrictions.
    - Try it for a smaller number.
  - Make a conjecture and test it.

If all else fails and you can't find a solution

- Make a list of what you have tried and why you are sure that it doesn't work.
- Critique the problem – what is making it hard to solve?
- Sometimes this will help you get unstuck.

If you do find a solution

- Check your answer is correct.
  - You should be mostly in a position to be able to check the answer against the original puzzle, or at least to verify your method.
  - Perhaps having solved it, you can see a different method that would work? If you get the same answer via different methods, this is good evidence you may be correct.
- Reflect on what happened.
- Play around with the puzzle.
  - Can you rephrase the puzzle into a different context?
  - Can you make a variant of the puzzle?

References I

- Barcellos, A. (1979).
  A conversation with Martin Gardner.
  The Two-Year College Mathematics Journal, 10(4):223–244.
- Böhm, W. (2017).
  Interesting Topics for Bachelor Theses.
  Vienna University of Economics, Vienna.
- Bragg, L. A. (2012).
  The effect of mathematical games on on-task behaviours in the primary classroom.
  Mathematics Education Research Journal, 24(4):385–401.

References II

Cockcroft, W. H. (1982).
Mathematics counts: Report of the Committee of Inquiry into the Teaching of Mathematics in Schools.
Her Majesty's Stationery Office, London.

Devi, S. (1976).
Puzzles to Puzzle You.
Orient Paperbacks, Delhi.

References III

Dyson, F. (2011).
Foreword: Recreational mathematics.
In Pitici, M., editor, *The Best Writing on Mathematics 2011*, pages xi–xvi. Princeton University Press, Princeton.

Eisenberg, T. (1991).
On building self-confidence in mathematics.
Teaching Mathematics and its Applications: An International Journal of the IMA, 10(4):154–158.

References IV

Ernest, P. (1986).
Games. a rationale for their use in the teaching of mathematics in school.
Mathematics in School, 15(1):2–5.

Foster, C. (2013).
Mathematical études: embedding opportunities for developing procedural fluency within rich mathematical contexts.
International Journal of Mathematical Education in Science and Technology, 44(5):765–774.

References V

Foster, C. (2018).
Developing mathematical fluency: comparing exercises and rich tasks.
Educational Studies in Mathematics, 97(2):121–141.

Gardner, M. (1959).
The Scientific American Book of Mathematical Puzzles & Diversions.
Simon and Schuster, New York.

Hankin, R. K. S. (2005).
Recreational mathematics with R: introducing the "magic" package.
R News, 5(1):48–51.

References VI

Hartnett, K. (2021).
Pioneers linking math and computer science win the abel prize.
Quanta Magazine.

Hollis, L. Y. and Felder, B. D. (1982).
Recreational mathematics for young children.
School Science and Mathematics, 82(1):71–75.

Minor, D. (2015).
Summing squares and cubes of integers.
Ohio Journal of School Mathematics, 72:13–17.

References VII

- Moscovich, I. (2004).
  The Hinged Square & Other Puzzles.
  Sterling Publishing, New York.
- Oldfield, B. J. (1991).
  Games in the learning of mathematics: 1: A classification.
  Mathematics in School, 20(1):41–43.
- Pólya, G. (2004).
  How to Solve It: A New Aspect of Mathematical Method.
  Princeton Science Library, Princeton, New Jersey, second edition.

References VIII

Singmaster, D. (2000).
The utility of recreational mathematics.
In Booth, D. J., editor, *Proceedings of the Undergraduate Mathematics Teaching Conference, Sheffield, 6th–9th September 1999*, pages 13–19. Sheffield Hallam University Press, Sheffield.

Singmaster, D. (2016).
The utility of recreational mathematics.
The UMAP Journal, 37(4):339–379.

References IX

- Smith, E. and Golding, L. (2018).
  Use of board games in higher education literature review.
  MSOR Connections, 16(2):24–29.

- Su, F. E. (2017).
  Mathematics for human flourishing.
  The American Mathematical Monthly, 124(6):483–493.

- Sumpter, L. (2015).
  Recreational mathematics – only for fun?
  Journal of Humanistic Mathematics, 5(1):121–138.

References X

- Trigg, C. W. (1978).
  What is recreational mathematics? Definition by example: paradigms of topics, people and publications.
  Mathematics Magazine, 51(1):18–21.