Eigenvalues and eigenvectors answers

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1. (a) non-trivial solutions because equation 3 is a linear combination of the other two;
   (b) only trivial solutions.

2. Remember you may have the eigenvalues and eigenvectors the other way around to me, and that you may have chosen other, equally-correct eigenvectors.

   (a)
      i. Say $\lambda_1 = 3$ & $\lambda_2 = 1$;
      ii. Say $\mathbf{v}_1 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ & $\mathbf{v}_2 = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$
      iii. Say
      $$\begin{bmatrix} 1 & -2 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -1 & -2 \\ -1 & -1 \end{bmatrix}.$$

   (b)
      i. Say $\lambda_1 = 9$ & $\lambda_2 = -8$;
      ii. Say $\mathbf{v}_1 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$ & $\mathbf{v}_2 = \begin{bmatrix} 5 \\ 12 \end{bmatrix}$
      iii. Say
      $$\begin{bmatrix} -1 & 5 \\ 1 & 12 \end{bmatrix} \begin{bmatrix} 9 & 0 \\ 0 & -8 \end{bmatrix} \begin{bmatrix} -\frac{12}{17} & \frac{5}{17} \\ \frac{1}{17} & \frac{1}{17} \end{bmatrix}.$$

   (c)
      i. Say $\lambda_1 = 2$ & $\lambda_2 = -9$;
      ii. Say $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ & $\mathbf{v}_2 = \begin{bmatrix} 4 \\ -7 \end{bmatrix}$
      iii. Say
      $$\begin{bmatrix} 1 & 4 \\ 1 & -7 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 0 & -9 \end{bmatrix} \begin{bmatrix} \frac{7}{11} & \frac{4}{11} \\ \frac{1}{11} & -\frac{1}{11} \end{bmatrix}.$$

   (d)
      i. Say $\lambda_1 = 6$ & $\lambda_2 = -8$;
      ii. Say $\mathbf{v}_1 = \begin{bmatrix} 3 \\ 11 \end{bmatrix}$ & $\mathbf{v}_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

iii. Say

$$\begin{bmatrix} 3 & 1 \\ 11 & -1 \end{bmatrix} \begin{bmatrix} 6 & 0 \\ 0 & -8 \end{bmatrix} \begin{bmatrix} \frac{1}{14} & \frac{1}{14} \\ \frac{11}{14} & -\frac{3}{14} \end{bmatrix}.$$

(e)

i. Say $\lambda_1 = 3$ & $\lambda_2 = 9$;

ii. Say $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ & $\mathbf{v}_2 = \begin{bmatrix} 11 \\ 5 \end{bmatrix}$

iii. Say

$$\begin{bmatrix} 1 & 11 \\ 1 & 5 \end{bmatrix} \begin{bmatrix} 3 & 0 \\ 0 & 9 \end{bmatrix} \begin{bmatrix} -\frac{5}{6} & \frac{11}{6} \\ \frac{1}{6} & -\frac{1}{6} \end{bmatrix}.$$

(f)

i. Say $\lambda_1 = -2$ & $\lambda_2 = -4$;

ii. Say $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ & $\mathbf{v}_2 = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$

iii. Say

$$\begin{bmatrix} 1 & 3 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} -2 & 0 \\ 0 & -4 \end{bmatrix} \begin{bmatrix} -2 & 3 \\ 1 & -1 \end{bmatrix}.$$

3. Remember you may have the eigenvalues and eigenvectors the other way around to me, and that you may have chosen other, equally-correct eigenvectors.

(a)

i. Say $\lambda_1 = -2$, $\lambda_2 = -1$ & $\lambda_3 = 3$;

ii. Say $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 5 \\ 1 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} 1 \\ 12 \\ 5 \end{bmatrix}$ & $\mathbf{v}_3 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$

iii. Say

$$\begin{bmatrix} 1 & 1 & 1 \\ 5 & 12 & 0 \\ 1 & 5 & 1 \end{bmatrix} \begin{bmatrix} -2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} \frac{3}{5} & \frac{1}{5} & -\frac{3}{5} \\ -\frac{1}{4} & 0 & \frac{1}{4} \\ \frac{13}{20} & -\frac{1}{5} & \frac{7}{20} \end{bmatrix}.$$

(b)

i. Say $\lambda_1 = -3$, $\lambda_2 = 2$ & $\lambda_3 = 4$;

ii. Say $\mathbf{v}_1 = \begin{bmatrix} -3 \\ 1 \\ 7 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$ & $\mathbf{v}_3 = \begin{bmatrix} 5 \\ -4 \\ 7 \end{bmatrix}$

iii. Say

$$\begin{bmatrix} -3 & 1 & 5 \\ 1 & -2 & -4 \\ 7 & 1 & 7 \end{bmatrix} \begin{bmatrix} -3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 4 \end{bmatrix} \begin{bmatrix} -\frac{1}{7} & -\frac{1}{35} & \frac{3}{35} \\ -\frac{1}{2} & -\frac{4}{5} & -\frac{1}{10} \\ \frac{3}{14} & \frac{1}{7} & \frac{1}{14} \end{bmatrix}.$$

(c)

i. Say $\lambda_1 = 0$, $\lambda_2 = 1$ & $\lambda_3 = 5$;

ii. Say $\mathbf{v}_1 = \begin{bmatrix} -9 \\ -4 \\ 3 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} -10 \\ -6 \\ 3 \end{bmatrix}$ & $\mathbf{v}_3 = \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}$

iii. Say

$$\begin{bmatrix} -9 & -10 & 2 \\ -4 & -6 & 2 \\ 3 & 3 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 5 \end{bmatrix} \begin{bmatrix} -\frac{3}{5} & \frac{4}{5} & -\frac{2}{5} \\ \frac{1}{2} & -\frac{3}{4} & \frac{1}{2} \\ \frac{3}{10} & -\frac{3}{20} & \frac{7}{10} \end{bmatrix}.$$

(d)

i. Say $\lambda_1 = -4$, $\lambda_2 = 1$ & $\lambda_3 = 3$;

ii. Say $\mathbf{v}_1 = \begin{bmatrix} -3 \\ -\frac{19}{2} \\ 2 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} 1 \\ 4 \\ 6 \end{bmatrix}$ & $\mathbf{v}_3 = \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}$

iii. Say

$$\begin{bmatrix} -3 & 1 & 1 \\ -\frac{19}{2} & 4 & 2 \\ 2 & 6 & 4 \end{bmatrix} \begin{bmatrix} -4 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} -\frac{4}{35} & -\frac{2}{35} & \frac{2}{35} \\ -\frac{6}{5} & \frac{2}{5} & \frac{1}{10} \\ \frac{13}{7} & -\frac{4}{7} & \frac{1}{14} \end{bmatrix}.$$