Number bases and modular arithmetic – exercises

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1. Write down all the numbers in base 5 up to and including 40_5.
2. Give the next 20 numbers after 101_4 in base 4.
3. Convert the following to base 10:
   (a) 2351_7;
   (b) 10010111_2;
   (c) 10101_2;
   (d) $AC1_{16}$.
4. Convert 89_{10} to each of the following bases:
   (a) 2;
   (b) 9.
5. Convert 46_{10} to each of the following bases:
   (a) 2;
   (b) 16.
6. Calculate:
   (a) $10110_2 + 111100111_2$;
   (b) $1C_{16} + 239_{16}$.
7. Convert 110000001101_2 to hexadecimal.
8. Convert $3A27_{16}$ to binary.
9. Convert the following binary fractions to decimal fractions.
   (a) 0.1000;
   (b) 0.1010;
   (c) 0.1111.
10. Convert the following decimal fractions to binary fractions?
    (a) $\frac{3}{4}$;
    (b) $\frac{1}{8}$;
    (c) $\frac{2}{3}$.
11. What is the biggest binary number you can write with five bits?
12. What is the biggest binary number you can write with n bits?
13. If a number has k digits in hexadecimal, how many digits (bits) does it have in binary?
14. Calculate the following:
    (a) $16 \pmod 5$;
    (b) $22 \pmod 4$;
    (c) $-33 \pmod{22}$;
    (d) $7 \pmod 7$;
    (e) $545 \pmod{12}$.
15. Starting from the information that $17^2 = 289$, find the last digit of 17^{23}.
16. Calculate $100 \pmod{24}$ and $1000 \pmod{24}$. What time of day will be in 3000 hours from now?
17. Find $5^{19} \pmod 6$.
    (Hint: $6 = -1 \pmod 7$.)
18. Find the value of $7^{137} \pmod{11}$.
19. Find the value of $7^{137} \pmod 8$. (Hint: $7 = -1 \pmod 8$.)
20. Find the last two digits of 3^{124}.
21. Find the last digit of $1!+2!+3!+\cdots+10!$.