# Markov chains – worksheet

1. The village of Chai Hollow has a population of 1000 who frequent the three cafes in the village. The cafes are called Alchemy Tea Room, Bean & Brew, and Coffee Corner. The residents of the village move between the pubs according to certain known traits. If a person visited Alchemy Tea Room one Saturday then there is a 60% chance they will return the next weekend, whilst there is a 25% chance they will go to Bean & Brew, and 15% chance they will go to Coffee Corner. Similar patterns apply for the visits to Bean & Brew and Coffee Corner. The resultant pattern is shown in the table below.

	Alchemy	Bean & Brew	Coffee Corner
Alchemy	0.6	0.22	0.38
Bean & Brew	0.25	0.55	0.05
Coffee Corner	0.15	0.23	0.57

We can also draw this arrangement as a transition diagram.
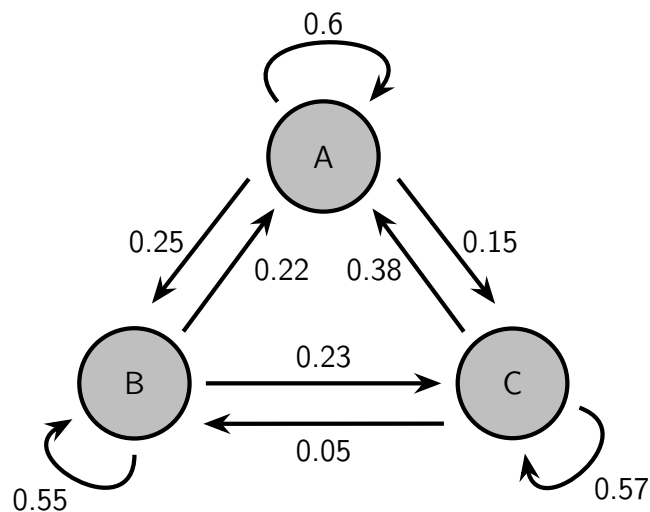
On a particular Saturday, 400 people visited Alchemy, 350 visited Bean & Brew, and 250 visited Coffee Corner.

(a) How many will visit each cafe next Saturday?

(b) What is the steady state for each cafe?

(c) What assumptions are we making with this model? What factors are missing from this model?

2. Consider two brands of teabags, Leafly and Whispering Leaves, for which the following transition probabilities apply.

$$\mathbf{P} = \begin{bmatrix} 0.88 & 0.15 \\ 0.12 & 0.85 \end{bmatrix}.$$

This matrix reveals a high degree of brand loyalty – people are most likely to stick with what they bought last time.

(a) Using the transition probability matrix above, calculate the long-term market share of these brands of teabags for 1000 customers.

(b) Leafly is cheaper than Whispering Leaves. Through an advertising campaign, Whispering Leaves is able to increase the number of people switching to try its teabags. As a result of the increase in new people giving it a go, the number switching back after trying it increases slightly also.

$$\mathbf{P} = \begin{bmatrix} 0.8 & 0.18 \\ 0.2 & 0.82 \end{bmatrix},$$

Starting with the long-term market share values, calculate the effect of this change over the first six weeks of the advertising campaign. Is the campaign a success?

3. The town of Ironford has an active working population of 654 people. They are employed in three sectors of the economy: Manufacturing, Retail and Service Sector. A consultant has developed the following model to show the changing employment patterns every quarter, where M, R and S represent the employment sectors and U represents the unemployed population.

$$M_{j+1} = 0.57M_j + 0.23R_j + 0.12S_j + 0.05U_j;$$
$$R_{j+1} = 0.23M_j + 0.6R_j + 0.16S_j + 0.11U_j;$$
$$S_{j+1} = 0.18M_j + 0.14R_j + 0.67S_j + 0.09U_j;$$
$$U_{j+1} = 0.02M_j + 0.03R_j + 0.05S_j + 0.75U_j.$$

A census was taken in October 2024 of the actual numbers employed in each area. The results were: Manufacturing 286, Retail 95, Service Sector 185.

(a) Represent this information as a Markov chain using matrices.

(b) What will be the predicted number working in each sector in January 2027?

(c) A new company is planning to start a new telephone banking service in April 2025 in the village. Forty new jobs in the service sector will be created. Re-model the situation with this extra piece of information and ascertain the new steady state conditions.

(d) What are the limitations of these models?