Logic notes

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1 Propositions

A proposition is a sentence that has a truth value, it is unambiguously true or false, for example "Two plus three equals five" and "My apple tree grows purple lemons".

Saying a sentence is a proposition says nothing about whether it is true.

We might label a proposition with a letter, for example:

- p: "If $x = 3$, then $x^2 = 9$."

Using this notation, if we say "p" it means we are saying "p is true".

2 Connectives

Connectives are used to combine propositions to form other propositions.

2.1 NOT

NOT is a connective that negates a statement. If p is true, then NOT p is false, and vice versa. We will write NOT p as $\neg p$.

We can represent this information in an arrangement called a truth table.

p	$\neg p$
true	false
false	true

2.2 AND

We can combine two propositions using AND, written as $\wedge$. This is only true if both p and q are true.

The truth table for AND is as follows.

p	q	$p \wedge q$
true	true	true
true	false	false
false	true	false
false	false	false

2.3 OR

Another way to combine two propositions is using OR, written $\vee$. This is true if at least one of p and q are true.

The truth table for OR is as follows.

p	q	$p \vee q$
true	true	true
true	false	true
false	true	true
false	false	false

2.4 XOR

The exclusive OR p XOR q or $p \oplus q$ is used when either p or q are true but not both. The following table applies.

p	q	$p \oplus q$
False	False	False
False	True	True
True	False	True
True	True	False

3 Implication

The connective $\implies$ is called "implies". A proposition $p \implies q$ can be read "p implies q" or "if p, then q".

We can also think about situations where $q \implies p$, and if both $p \implies q$ and $q \implies p$ are true, then we write $p \iff q$ and say "p if and only if q".

The implication $p \implies q$ is true unless p is true and q is false. The truth table is as follows. The $p \implies q$ column refers to the truth of the implication, not the truth of either p or q.

p	q	$p \implies q$
true	true	true
true	false	false
false	true	true
false	false	true

In the cases where p is false, this tells us nothing to refute the idea that $p \implies q$, so we say the implication is true.

It is also worth nothing that if q is always true, then $p \implies q$ is true regardless of p. In fact, q being true tells us nothing about the truth value of p.

4 Arguments

Within propositional logic we can make arguments. An argument is made up of two parts:

1. a number of propositions, called the premises;
2. a proposition, called the conclusion.

For example, if say we want to establish $p \implies q$. If we know $p \implies r$ and $r \implies q$, then we can form an argument $p \implies r \implies q$.