

Brief overview of the history of mathematics

Part 2: Islamic Golden Age, Renaissance Europe, and more

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These talks

- ▶ This is a stampede missing much detail!
- ▶ And quite a traditional account of mainstream history.
- ▶ My intention is to give you an overview, and to pique your interest in some historical topics.

China and India

Chinese mathematics

- ▶ Goes back 3,000 years or more on bamboo or paper;
- ▶ may have been the first to develop a decimal place-value system;
- ▶ constructed sundials;
- ▶ early users of the abacus;
- ▶ a value of π to seven decimal places; the most accurate value for almost 1,000 years;
- ▶ some algebra, work on simultaneous equations.

古法七乘方圖

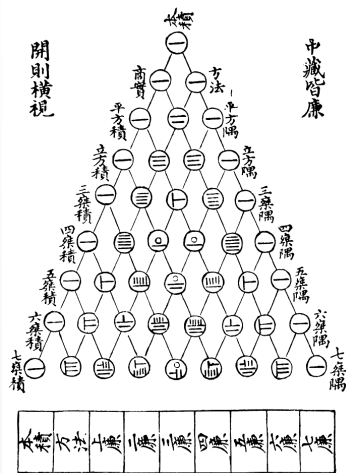


Image: Yanghui triangle. Public domain on Wikimedia Commons.

Indian mathematics

- ▶ Dates perhaps from around 600BC;
- ▶ First appearance of the decimal place-value system we use today, called Hindu-Arabic numerals, and rules for its use;

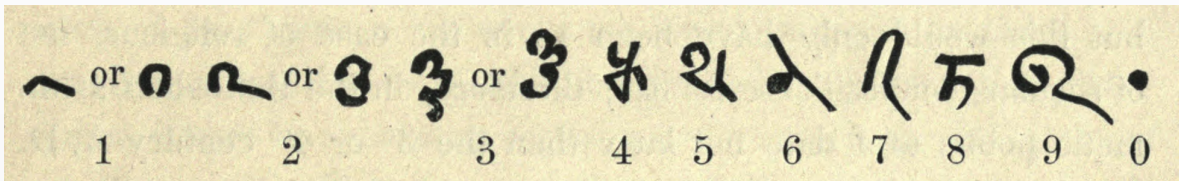


Image: Bakhshali numerals, public domain on Wikimedia Commons

Indian mathematics

- ▶ Used zero from around 400AD; Brahmagupta (598-670AD) developed zero (called *nought* or *cipher*) as a number itself (rather than a place-holder), and developed the idea to negative numbers.
 - ▶ “The sum of cipher and negative is negative; of positive and nought, positive; of two ciphers, cipher”;
 - ▶ “Negative taken from cipher becomes positive, and positive from cipher is negative; cipher taken from cipher is nought”;
 - ▶ “The product of cipher and positive, or of cipher and negative, is nought; of two ciphers is cipher...”
(Flood & Wilson, p.43).

Islamic Golden Age

The 'Dark Ages'

- ▶ From 500 to 1000 in Europe.
- ▶ A few writings on the calendar, finger reckoning and arithmetical problems by
 - ▶ the Venerable Bede;
 - ▶ Alcuin of York;
 - ▶ and others;
- ▶ Otherwise, mathematical activity was sparse.



Image: Alcuin of York. The MacTutor History of Mathematics archive.

Islamic mathematics

- ▶ Golden age from approx. 750-1258AD (Abbasid Caliphate).
- ▶ Important for:
 - ▶ keeping Greek mathematics alive (in translation);
 - ▶ translating also Indian mathematics;
 - ▶ own contributions, including
 - ▶ significant improvements in geometry;
 - ▶ some claims Greek geometry contained flaws, including the parallel postulate;
 - ▶ added the decimal point notation to the Hindu-Arabic numerals;
 - ▶ systematised the study of algebra and began to consider the relationship between algebra and geometry.
- ▶ All six main trigonometric functions (\sin , \cos , \tan , \cot , \sec & \csc) were known, either developed here or known from Greek or Indian translations.

al-Khwārizmī (c.783-850)

- ▶ Discussed the cancellation of like terms on opposite sides of an equation, which he described as *al-jabr*, from which we get 'algebra'.
- ▶ His algebra was not concerned with a series of problems to be resolved, but a general topic from which an infinite class of problems may be defined.
- ▶ General formula for solving quadratic equations.



Image: al-Khwārizmī. The MacTutor History of Mathematics archive.

al-Khwārizmī (c.783-850)

- ▶ His book *Arithmetic* was important for introducing the Indian numeral system to the Islamic world and later to Europe.
- ▶ We derive 'algorithm' from his name, to mean a step-by-step procedure for solving problems.

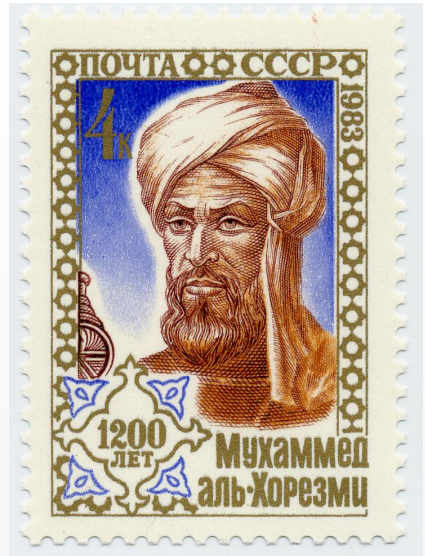


Image: al-Khwārizmī. The MacTutor History of Mathematics archive.

Early Renaissance Europe

Transmission to Europe

► e.g.

- Gerbert of Aurillac (Pope Sylvester II) (c.940-1003), trained in Islamic Spain and may have been the first to introduce the Hindu-Arabic numerals to Christian Europe.
- Adelard of Bath (1075-1160) travelled a lot and became an expert in Arabic, possibly in Spain or Sicily, made a wholesale conversion of Arabic texts into Latin.

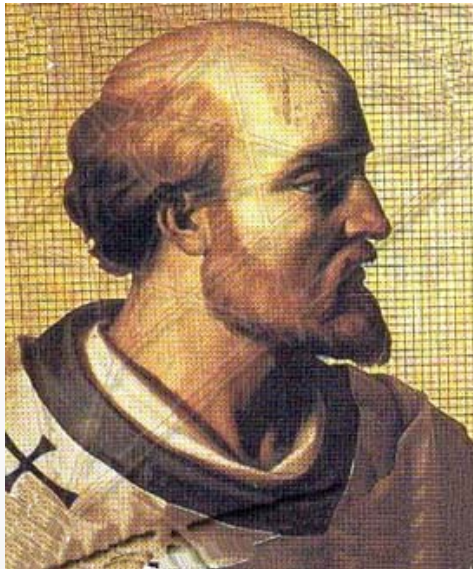


Image: Gerbert. The MacTutor History of Mathematics archive.

Leonardo of Pisa (c. 1170-1240)

- ▶ later known as Fibonacci;
- ▶ the son of a merchant who had travelled widely and studied under a Muslim teacher;
- ▶ wrote *Liber Abaci* (1202), the first significant mathematical work in Europe for a thousand years.



Image: Fibonacci. The MacTutor History of Mathematics archive.

Liber Abaci (1202)

- ▶ used Hindu-Arabic numerals;
- ▶ in part concerned with the use of arithmetic in business applications;
- ▶ included The Rabbits Problem, the Fibonacci sequence.
- ▶ A problem from *Liber Abaci*:
“If a lion can eat a sheep in 4 hours, a leopard can eat it in 5 hours, and a bear can eat it in 6 hours, how long would they take eating it together?”
Flood & Wilson (2011, p. 54).

Perspective

- ▶ Fifteenth century work concerned with depicting 3D objects in a realistic way;
- ▶ e.g. by della Francesca (c.1415-1492), Dürer (1471-1528), da Vinci (1452-1519) and others.
- ▶ Alberti (1404-1472) wrote in his *Della Pittura* (1436) that “the first duty of a painter is to know geometry.”



Image: Adoration of the Magi, Dürer (1504). Web Gallery of Art.

The Sixteenth Century

Cubics and quartics

- ▶ 1520s: del Ferro (1465-1526) found a general method for solving cubic equations of the form “A cube and things equal to numbers”
($x^3 + ax = b$).
- ▶ Around the same time: Tartaglia (c.1500-1557) found a method for equations of the form “A cube and squares equal to numbers”
($x^3 + ax^2 = b$).
- ▶ del Ferro's student Fior challenged Tartaglia with a month to solve thirty cubics of the first type;
- ▶ Tartaglia gave Fior thirty cubics of the second type in return.

Cubics and quartics

- ▶ Fior lost the contest – he could not solve Tartaglia's cubics, while Tartaglia found a method to solve Fior's.
- ▶ Cardano published methods for solving cubics and quartics in his *Ars Magna* (1545), giving credit to Tartaglia.
- ▶ This left open the question of solving equations involving x^5 , x^6 , etc.



Image: Cardano. The MacTutor History of Mathematics archive.

Complex numbers

- When solving the quadratic equation $x(10 - x) = 40$, Cardano observed:

Dismissing mental tortures, and multiplying $5 + \sqrt{-15}$ by $5 - \sqrt{-15}$, we obtain $25 - (-15)$. Therefore the product is 40. ... and thus far does arithmetical subtlety go, of which this, the extreme, is, as I have said, so subtle that it is useless.

i.e.

$$\begin{aligned} & (5 + \sqrt{-15})(5 - \sqrt{-15}) \\ &= 5^2 + \cancel{(5\sqrt{-15})} - \cancel{(5\sqrt{-15})} - (\sqrt{-15})^2 \\ &= 25 - (-15) = 40. \end{aligned}$$

Complex numbers

- ▶ Later, Bombelli (c.1526-1572) was the first to show how to add and subtract complex numbers, and gave rules for multiplying them, to find real solutions when Tartaglia's method gave answers with $\sqrt{-1}$.

The Seventeenth Century

Kepler (1571-1630)

- ▶ Building on the work of Copernicus (1473-1543) and Galileo (1564-1642);
- ▶ Kepler used observations left to him by Tycho Brahe (1546-1601) when he died to investigate orbits of planets;
- ▶ developed three laws based on elliptical orbits.
- ▶ Kepler was also interested in geometry;
- ▶ by dividing a volume into very thin discs, Kepler determined the volumes of over ninety solids by rotating conics and other curves around an axis.

Logarithms

- ▶ Developed by Napier (1550-1617) as a method to replace lengthy computations involving multiplications and divisions with simpler ones using addition and subtraction.
- ▶ Napier's logarithms were awkward to use, and were refined by Briggs (1561-1630).

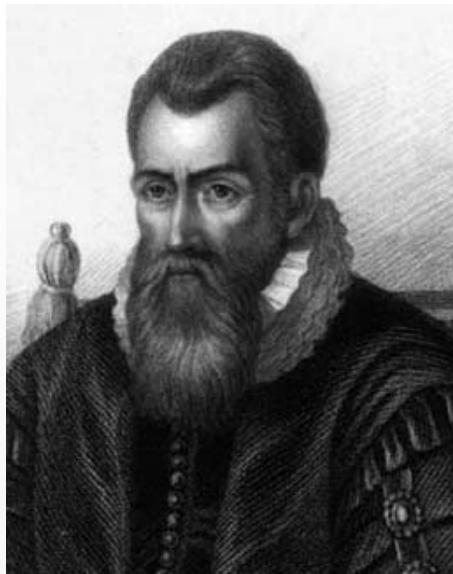


Image: Napier. The MacTutor History of Mathematics archive.

Logarithms

e.g. If A and B are big numbers and you want to find AB , do

$$\log (AB) = \log (A) + \log (B)$$

and you have turned a hard problem of multiplying two big numbers into one of adding the logs of those numbers, which is easier (provided you know the logarithm values).

Logarithms

- ▶ Briggs printed tables of logarithms in base 10, starting with *Logarithmorum Chilias Prima* (The First Thousand Logarithms) in 1617;
- ▶ and, in 1624, tables of logarithms to base 10 of integers from 1 to 20,000 and 90,000 to 100,000, all calculated by hand to fourteen decimal places;
- ▶ the gap from 20,000 to 90,000 was filled by Vlacq (1628).
- ▶ The slide rule, an instrument based on logarithms, first appeared around 1630;
- ▶ versions of this were used until the invention of the pocket calculator in the 1970s.

Fermat (1601-1665)

- ▶ Considered mathematics as a hobby, published little – but did much!
- ▶ We know his work from notes he left and letters to other mathematicians.
- ▶ Resurrected number theory and helped to introduce analytic geometry.



Image: Fermat. The MacTutor History of Mathematics archive.

Fermat's Last Theorem

- ▶ A conjecture that

For any integer n (greater than 2), there do not exist positive integers x , y and z for which $x^n + y^n = z^n$.

- ▶ left as a comment in the margin of his copy of *Arithmetica* by Diophantus, that he had “an admirable proof which this margin is too narrow to contain”.
Flood & Wilson (p. 91).

Pascal (1623-1662)

- ▶ Invented a calculating machine that could add and subtract, The Pascaline, in 1642.
- ▶ The modern theory of probability arose from a correspondence between Pascal and Fermat in 1654 (earlier work by Cardano had not yet been published).
- ▶ Carried out the first systematic investigation of Pascal's triangle.



Image: Pascal. The MacTutor History of Mathematics archive.

Calculus

- ▶ Many people developed methods for finding tangents to curves and areas under them, including Cavalieri (1598-1647), Roberval (1602-1675), Kepler, Fermat, Descartes (1596-1650), Pascal, Saint-Vincent (1584-1667), Wallis (1616-1703), Torricelli (1608-1647), Barrow (1630-1677).
- ▶ Newton (1642-1727) and Leibniz (1646-1716) independently went beyond this to develop the calculus, namely:
 - ▶ differentiation;
 - ▶ integration;
 - ▶ the inverse relationship between them.

Group work this week

- ▶ Please choose a mathematical topic and explore its history, or choose a historical person or culture and explore their connection with mathematics.

References

1. Fauvel, J. and Gray, J. (1987). *The History of Mathematics – A Reader*. Milton Keynes: Open University.
2. Flood, R., and Wilson, R. (2011). *The Great Mathematicians: Unravelling the mysteries of the universe*. London: Arcturus.
3. O'Connor, J.J. and Robertson, E.F. (2016). *The MacTutor History of Mathematics Archive*. Retrieved from <http://www-history.mcs.st-andrews.ac.uk>