# Brief overview of the history of mathematics

# Part 2: Early Renaissance Europe – present

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# These talks

- This is a stampede missing much detail!
- And quite a traditional account of mainstream history.
- My intention is to give you an overview, and to pique your interest in some historical topics.

# China and India

# Chinese mathematics

- Goes back 3,000 years or more on bamboo or paper;
- may have been the first to develop a decimal place-value system;
- constructed sundials;
- early users of the abacus;
- a value of $\pi$ to seven decimal places; the most accurate value for almost 1,000 years;
- some algebra, work on simultaneous equations.

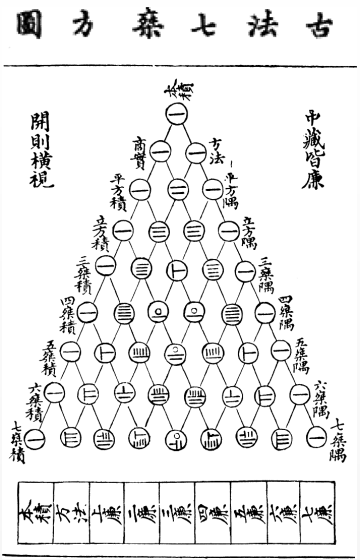


**Image**: Yanghui triangle. Public domain on Wikimedia Commons.

# Indian mathematics

- Dates perhaps from around 600BC;
- First appearance of the decimal place-value system we use today, called Hindu-Arabic numerals, and rules for its use;

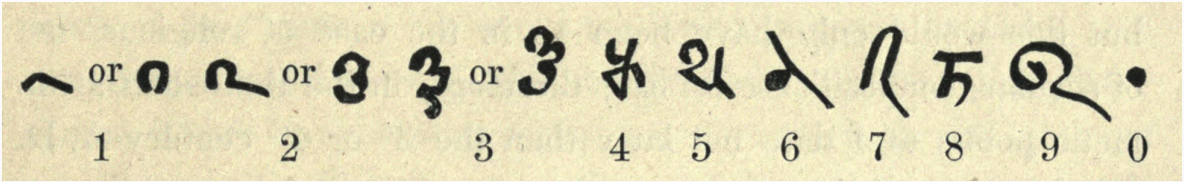


**Image**: Bakhshali numerals, public domain on Wikimedia Commons

# Indian mathematics

- Used zero from around 400AD; Brahmagupta (598-670AD) developed zero (called *nought* or *cipher*) as a number itself (rather than a place-holder), and developed the idea to negative numbers.
  - "The sum of cipher and negative is negative; of positive and nought, positive; of two ciphers, cipher";
  - "Negative taken from cipher becomes positive, and positive from cipher is negative; cipher taken from cipher is nought";
  - "The product of cipher and positive, or of cipher and negative, is nought; of two ciphers is cipher..."
    (Flood & Wilson, p.43).

# Islamic Golden Age

# The ‘Dark Ages’

- From 500 to 1000 in Europe.
- A few writings on the calendar, finger reckoning and arithmetical problems by
  - the Venerable Bede;
  - Alcuin of York;
  - and others;
- Otherwise, mathematical activity was sparse.

**Image**: Alcuin of York. The MacTutor History of Mathematics archive.

# Islamic mathematics

- Golden age from approx. 750-1258AD (Abbasid Caliphate).
- Important for:
  - keeping Greek mathematics alive (in translation);
  - translating also Indian mathematics;
  - own contributions, including
    - significant improvements in geometry;
    - some claims Greek geometry contained flaws, including the parallel postulate;
    - added the decimal point notation to the Hindu-Arabic numerals;
    - systematised the study of algebra and began to consider the relationship between algebra and geometry.
- All six main trigonometric functions (sin, cos, tan, cot, sec & cosec) were known, either developed here or known from Greek or Indian translations.

# al-Khwārizmī (c.783-850)

- Discussed the cancellation of like terms on opposite sides of an equation, which he described as *al-jabr*, from which we get ‘algebra’.
- His algebra was not concerned with a series of problems to be resolved, but a general topic from which an infinite class of problems may be defined.
- General formula for solving quadratic equations.


**Image**: al-Khwārizmī. The MacTutor History of Mathematics archive.

# al-Khwārizmī (c.783-850)

- His book *Arithmetic* was important for introducing the Indian numeral system to the Islamic world and later to Europe.
- We derive 'algorithm' from his name, to mean a step-by-step procedure for solving problems.


**Image**: al-Khwārizmī. The MacTutor History of Mathematics archive.

# Early Renaissance Europe

# Transmission to Europe

- e.g.
  - Gerbert of Aurillac (Pope Sylvester II) (c.940-1003), trained in Islamic Spain and may have been the first to introduce the Hindu-Arabic numerals to Christian Europe.
  - Adelard of Bath (1075-1160) travelled a lot and became an expert in Arabic, possibly in Spain or Sicily, made a wholesale conversion of Arabic texts into Latin.

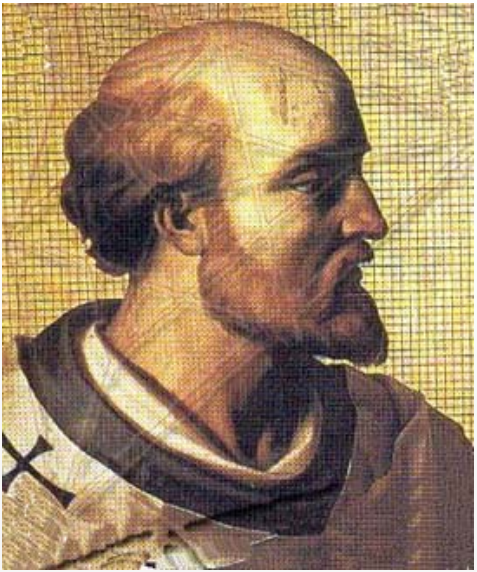

**Image**: Gerbert. The MacTutor History of Mathematics archive.

# Leonardo of Pisa (c. 1170-1240)

- later known as Fibonacci;
- the son of a merchant who had travelled widely and studied under a Muslim teacher;
- wrote *Liber Abaci* (1202), the first significant mathematical work in Europe for a thousand years.

**Image**: Fibonacci. The MacTutor History of Mathematics archive.

# *Liber Abaci* (1202)

- used Hindu-Arabic numerals;
- in part concerned with the use of arithmetic in business applications;
- included The Rabbits Problem, the Fibonacci sequence.
- A problem from *Liber Abaci*:
  "If a lion can eat a sheep in 4 hours, a leopard can eat it in 5 hours, and a bear can eat it in 6 hours, how long would they take eating it together?"
  Flood & Wilson (2011, p. 54).

# Perspective

- Fifteenth century work concerned with depicting 3D objects in a realistic way;
- e.g. by della Francesca (c.1415-1492), Dürer (1471-1528), da Vinci (1452-1519) and others.
- Alberti (1404-1472) wrote in his *Della Pittura* (1436) that "the first duty of a painter is to know geometry."

**Image**: Adoration of the Magi, Dürer (1504). Web Gallery of Art.

# The Sixteenth Century

# Cubics and quartics

- 1520s: del Ferro (1465-1526) found a general method for solving cubic equations of the form "A cube and things equal to numbers" ($x^3 + ax = b$).
- Around the same time: Tartaglia (c.1500-1557) found a method for equations of the form "A cube and squares equal to numbers" ($x^3 + ax^2 = b$).
- del Ferro's student Fior challenged Tartaglia with a month to solve thirty cubics of the first type;
- Tartaglia gave Fior thirty cubics of the second type in return.

# Cubics and quartics

- Fior lost the contest – he could not solve Tartaglia's cubics, while Tartaglia found a method to solve Fior's.
- Cardano published methods for solving cubics and quartics in his *Ars Magna* (1545), giving credit to Tartaglia.
- This left open the question of solving equations involving $x^5$, $x^6$, etc.

**Image**: Cardano. The MacTutor History of Mathematics archive.

# Complex numbers

- When solving the quadratic equation $x(10 - x) = 40$, Cardano observed:

  *Dismissing mental tortures, and multiplying* $5 + \sqrt{-15}$ *by* $5 - \sqrt{-15}$*, we obtain* $25 - (-15)$*. Therefore the product is* $40$*. ... and thus far does arithmetical subtlety go, of which this, the extreme, is, as I have said, so subtle that it is useless.*

i.e.

$$\begin{aligned}&(5 + \sqrt{-15})(5 - \sqrt{-15})\\ =&5^2 + \cancel{(5\sqrt{-15})} - \cancel{(5\sqrt{-15})} - (\sqrt{-15})^2\\ =&25 - (-15) = 40.\end{aligned}$$

# Complex numbers

- Later, Bombelli (c.1526-1572) was the first to show how to add and subtract complex numbers, and gave rules for multiplying them, to find real solutions when Tartaglia's method gave answers with $\sqrt{-1}$.

# The Seventeenth Century

# Kepler (1571-1630)

- Building on the work of Copernicus (1473-1543) and Galileo (1564-1642);
- Kepler used observations left to him by Tycho Brahe (1546-1601) when he died to investigate orbits of planets;
- developed three laws based on elliptical orbits.
- Kepler was also interested in geometry;
- by dividing a volume into very thin discs, Kepler determined the volumes of over ninety solids by rotating conics and other curves around an axis.

# Logarithms

- Developed by Napier (1550-1617) as a method to replace lengthy computations involving multiplications and divisions with simpler ones using addition and subtraction.
- Napier's logarithms were awkward to use, and were refined by Briggs (1561-1630).

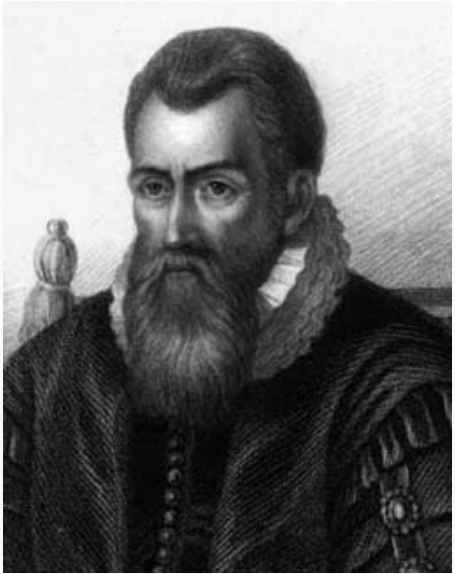

**Image**: Napier. The MacTutor History of Mathematics archive.

# Logarithms

e.g. If $A$ and $B$ are big numbers and you want to find $AB$, do

$$\log(AB) = \log(A) + \log(B)$$

and you have turned a hard problem of multiplying two big numbers into one of adding the logs of those numbers, which is easier (provided you know the logarithm values).

# Logarithms

- Briggs printed tables of logarithms in base 10, starting with *Logarithmorum Chilias Prima* (The First Thousand Logarithms) in 1617;
- and, in 1624, tables of logarithms to base 10 of integers from 1 to 20,000 and 90,000 to 100,000, all calculated by hand to fourteen decimal places;
- the gap from 20,000 to 90,000 was filled by Vlacq (1628).
- The slide rule, an instrument based on logarithms, first appeared around 1630;
- versions of this were used until the invention of the pocket calculator in the 1970s.

# Fermat (1601-1665)

- Considered mathematics as a hobby, published little – but did much!
- We know his work from notes he left and letters to other mathematicians.
- Resurrected number theory and helped to introduce analytic geometry.

**Image**: Fermat. The MacTutor History of Mathematics archive.

# Fermat's Last Theorem

- A conjecture that

  *For any integer $n$ (greater than 2), there do not exist positive integers $x$, $y$ and $z$ for which $x^n + y^n = z^n$.*

- left as a comment in the margin of his copy of *Arithmetica* by Diophantus, that he had "an admirable proof which this margin is too narrow to contain".
  Flood & Wilson (p. 91).

# Pascal (1623-1662)

- Invented a calculating machine that could add and subtract, The Pascaline, in 1642.
- The modern theory of probability arose from a correspondance between Pascal and Fermat in 1654 (earlier work by Cardano had not yet been published).
- Carried out the first systematic investigation of Pascal's triangle.

**Image**: Pascal. The MacTutor History of Mathematics archive.

# Calculus

- Many people developed methods for finding tangents to curves and areas under them, including Cavalieri (1598-1647), Roberval (1602-1675), Kepler, Fermat, Descartes (1596-1650), Pascal, Saint-Vincent (1584-1667), Wallis (1616-1703), Torricelli (1608-1647), Barrow (1630-1677).
- Newton (1642-1727) and Leibniz (1646-1716) independently went beyond this to develop the calculus, namely:
  - differentiation;
  - integration;
  - the inverse relationship between them.

# Group work this week

- Please choose a mathematical topic and explore its history, or choose a historical person or culture and explore their connection with mathematics.

# References


1. Fauvel, J. and Gray, J. (1987). *The History of Mathematics – A Reader*. Milton Keynes: Open University.
2. Flood, R., and Wilson, R. (2011). *The Great Mathematicians: Unravelling the mysteries of the universe*. London: Arcturus.
3. O'Connor, J.J. and Robertson, E.F. (2016). *The Mactutor History of Mathematics Archive*. Retrieved from `http://www-history.mcs.st-andrews.ac.uk`