Worksheet 8: Counting up to symmetry

- 1. Suppose we consider pairs of the 26 letters of the alphabet, e.g., (a, b), (x, z), (b, a), etc.
 - (a) How many pairs of letters are there?
 - (b) If we consider, for example, (a, b) and (b, a) to be the same pair, then how many fewer pairs of letters are there?
- 2. How many ways are there to colour the vertices of the figure shown using



- (a) two colours;
- (b) three colours;
- (c) seven colours.
- 3. How many ways are there to colour the vertices of the figure shown using



- (a) two colours;
- (b) three colours;
- (c) seven colours.
- 4. How many ways are there to colour the vertices of the figure shown using



- (a) two colours;
- (b) three colours;
- (c) seven colours.
- 5. A patchwork quilt is made up of different coloured squares, which are only visible on the front of the quilt.
 - (a) How many ways can a 2×2 quilt be made using up to 2 colours? Use Polya enumeration to compute what combinations of colours these use and draw all cases.
 - (b) How many ways can a 3×3 quilt be made using up to 2 colours? Use Polya enumeration to compute what combinations of colours these use, but do not draw all cases.
 - (c) How many ways can a 2×3 quilt be made using up to 2 colours? Use Polya enumeration to compute what combinations of colours these use, but do not draw all cases.