Labelling and colouring – exercises

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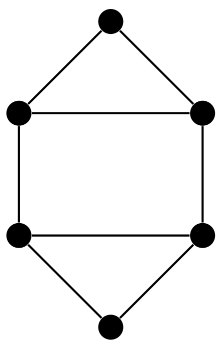
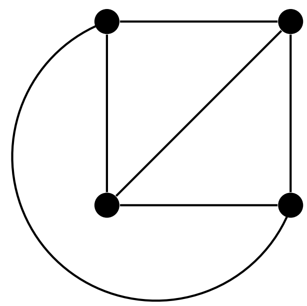
1 Vertex-magic labelling

Find a vertex-magic labelling for these graphs (a labelling of the edges such that the sum of labels at each vertex is a constant).



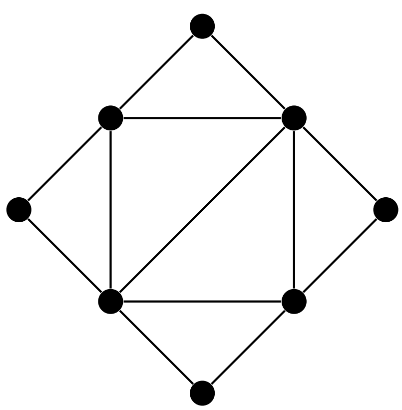
1.



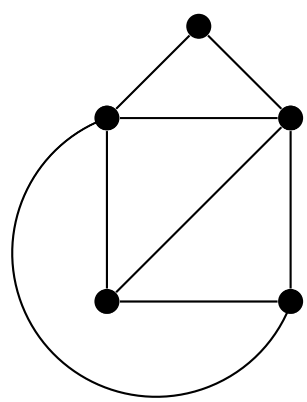
2.

2 Zero-sum magic graphs

Find a zero-sum vertex-magic labelling for these graphs (a labelling of the edges such that the sum of labels at each vertex is 0).



1.



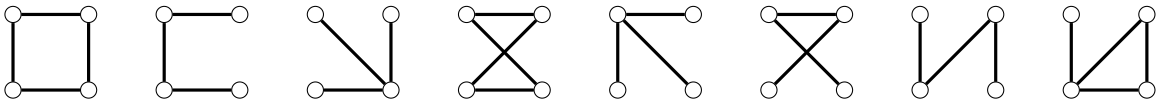
2.

3 Something to think about

- Can you make some other graphs that allow vertex-magic labelling, edge-magic labelling, total-magic labelling and zero-sum magic labelling? For some, you may need to fill in some values to give a hint.
- Can you make a graph that cannot be vertex-magic labelled? How about the other labellings?

4 Colouring

(For colouring, you might have coloured pens, or you might just label the vertices with a set of labels representing the colours.)

1. 

   (a) Which of these graphs are different?

   (b) Colour these graphs using as few colours as possible.

2. Give examples of graphs with chromatic numbers 3, 4, 5 and 6.

3. What is the chromatic number of an arbitrary tree? (Recall that a tree is a graph that contains no cycles.)

4. The director of a play is trying to work out the minimum number of actors they need. An actor can play multiple characters provided their characters are not together in a scene. The play has 8 characters. Here is a table of which characters appear in which scenes.

Scene	**Characters**
Scene 1	Ashley and Bryn
Scene 2	Bryn, Corin and Dale
Scene 3	Ashley, Corin and Edith
Scene 4	Corin, Edith and Finn
Scene 5	Edith, Finn and Gwen
Scene 6	Corin, Gwen and Howard

   Represent each character with a vertex and join two vertices if and only if those characters appear in a scene together. By colouring the vertices, obtain the minimum number of actors necessary for this play.

5. A chemist wants to store five chemicals a, b, c, d, and e in a warehouse. But some of the chemicals react violently with each other, so must be kept in separate areas. In the table below, a tick shows which pairs of chemicals must be separated. Can you use some ideas about graph colouring to find out how many areas are needed?

	a	b	c	d	e
a	-	✓	✓	✓	-
b	✓	-	✓	✓	✓
c	✓	✓	-	✓	-
d	✓	✓	✓	-	✓
e	-	✓	-	✓	-

6. Nearby radio masts need to operate on different frequencies. But those which are not in range of each other are able to operate on the same frequency without worrying about interference. The dots below represent radio masts and the dotted circles indicate their range. Can you use some ideas about graph colouring to choose a minimum number of different frequencies for these masts?