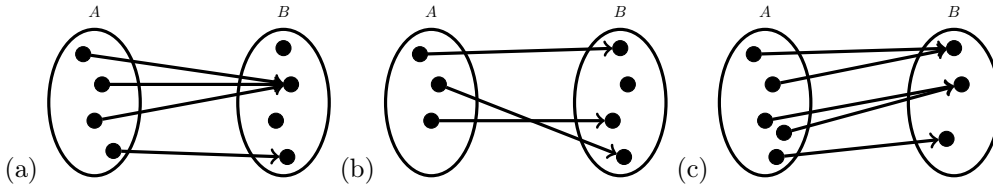


Maps – exercises

Peter Rowlett

1. Are the following maps injective or surjective?



2. Write down all the maps of the set $A = \{a, b\}$ into $B = \{1, 2, 3\}$. How many of these are injective?

3. Which of the following maps are (i) injective; (ii) surjective?

(a) $\theta : \mathbb{Z} \rightarrow \mathbb{Z}$, $\theta(x) = x^2$ for all $x \in \mathbb{Z}$;

(b) $\theta : \mathbb{Z} \rightarrow \mathbb{Z}$, $\theta(x) = -x$ for all $x \in \mathbb{Z}$;

(c) $\theta : \mathbb{Z} \rightarrow \mathbb{Z}$, for all $x \in \mathbb{Z}$, $\theta(x) = \begin{cases} \frac{1}{2}x & \text{if } x \text{ is even;} \\ 0 & \text{if } x \text{ is odd.} \end{cases}$

(d) $\theta : \mathbb{Z} \rightarrow \mathbb{Z}$, $\theta(x) = 2x + 1$ for all $x \in \mathbb{Z}$;

(e) $\theta : \mathbb{R} \rightarrow \mathbb{R}$, $\theta(x) = 5x - 3$ for all $x \in \mathbb{R}$.

(f) $\theta : \mathbb{R} \rightarrow \mathbb{R}$, $\theta(x) = x^2 + 2x + 3$ for all $x \in \mathbb{R}$.

4. The formula $f(x) = \frac{1}{x-3}$ does not define a map from $\mathbb{R} \rightarrow \mathbb{R}$.

(a) Why not?

(b) Can you write down a set X for which $f : X \rightarrow \mathbb{R}$?

5. Show that \mathbb{N} has the same cardinality as

(a) square numbers;

(b) the negative odd numbers;

(c) integers greater than 100;

(d) the set $\{1, 1/2, 1/3, \dots, 1/n, \dots\}$.