Problem-solving advice and what to do if you get stuck

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Problem-solving advice

Problem-solving is a valuable skill. Being able to calmly and logically analyse some information and process this to solve a problem is useful in mathematics, of course, but also in most other subjects, careers and many areas of everyday life.

Sometimes you can look at a problem and instantly know how to solve it. This is good, but you aren't really learning a problem-solving process. It is important that you sometimes try problems or puzzles that you don't find easy because at some point you will start to encounter hard problems and before then you need to practice your approach to problem-solving. Below is some advice on three stages of problem-solving. Each stage is equally important.

1. Plan

Many people expect to read the problem and immediately attempt a solution. This is rarely possible, and when the first attempt fails, they may give up. You can avoid this trap by taking the time to understand the problem and plan your approach.

Advice:

- Stop and really read the problem.
- Ask yourself: What am I being asked to do? What information have I been given? What information is missing? What would a solution look like?
- Draw a picture. Introduce suitable notation for the information you have been given and information you are being asked for.
- Have you seen a similar problem before? How did you solve that one?
- Separate the problem into smaller parts and examine them separately.
- Can you write the problem in a different way? Try it. Is what you have written actually the same problem? If not, what is different? If it is, can you solve this version of the problem?
- Can you solve a related problem? Can you remove part of the restriction and solve a more general problem? Or, can you come up with a specific example and solve that? Do your solutions help you plan to approach the main problem?

2. Carry out your plan

When your plan is ready, put it into action.

Advice:

- Check each step. Is each step correct?
- You should expect to be stuck quite a lot of the time. Recognise that you are stuck and accept it. Calmly review where you are and try to get unstuck.
- It is (usually!) okay to wait and mull over the problem for a while.
- If you are sure your plan cannot work, you may need to return to the *Plan* stage.

3. Review

If you find a solution or are about to give up, move on to the review stage. It is important to check your solution is correct and think about what you have learned about this type of problem and about the problem-solving process.

Advice:

- First, if you found a solution, check it is correct.
  - Can you check your solution is correct?
  - Does it answer the original problem?
  - Can you get the same solution from a different method?
  - Can you work from your solution and get back to the original problem and, doing so, is the problem you get to the same as the original problem you tried to solve?
- Second, reflect on what happened.
  - Remember that the point of solving problems is not just to get marks in assessments. The purpose here is to think about what has happened and see what you can learn.
  - Think about the process you took and particularly any dead ends you went down.
    - What went wrong?
    - Could you have avoided the dead ends, or were they a necessary part of solving the problem?
    - What do you wish you had know when you first attempted the problem?
  - What can you do now that you couldn't before?
    - Can you use the method, or the result, for some other problem?
    - Can you write down a new problem that you are now able to solve?
    - Is the problem you have solved part of a wider family of problems?
    - Can your method be adapted to solve more of them?

Further information

The advice here is heavily inspired by George Pólya's book *How to Solve It* and John Mason, Leone Burton and Kaye Stacey's book *Thinking Mathematically*, both of which contain a lot more detail and example problems, and are available in the SHU library.