Quantifiers – exercises

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1. Rewrite the following using $\forall$ and $\exists$.
   (a) For all the integers x, x is odd or even.
   (b) There exist two prime numbers such that their sum is prime.
   (c) There exists a rational number greater than $\sqrt{2}$.
   (d) If x is a real number, then x^2 is greater than x.
   (e) For all $n \in \mathbb{N}$ there exists a prime p such that $p > n$.
2. Decide whether the following are true or false. Explain your answers.
   (a) $\forall x \in \mathbb{R}\ \exists y \in \mathbb{R}\ (x^2 = y)$.
   (b) $\forall y \in \mathbb{R}\ \exists x \in \mathbb{R}\ (x^2 = y)$.
   (c) $\forall x \in \mathbb{Z}\ \exists y \in \mathbb{Z}\ (x^2 = y)$.
   (d) $\forall y \in \mathbb{Z}\ \exists x \in \mathbb{Z}\ (x^2 = y)$.
   (e) $\forall x \in \mathbb{R}\ \exists y \in \mathbb{R}\ (x + y = 0)$.
   (f) $\exists x \in \mathbb{R}\ \forall y \in \mathbb{R}\ (x + y = 1)$.
   (g) $\forall x\ p \implies \exists x\ p$, where p is some proposition.
   (h) $\exists x\ p \implies \forall x\ p$, where p is some proposition.
   (i) $\exists n \in \mathbb{N}$ such that $n^2 \leq n$.
3. Rewrite the following using $\forall$ and $\exists$.
   (a) If a and b are real numbers with $a \neq 0$, then $ax + b = 0$ has a solution.
   (b) If a and b are real numbers with $a \neq 0$, then $ax + b = 0$ has a unique solution.
4. Negate the following.
   (a) There exists a grey cat.
   (b) For all cats there exists an owner.
   (c) There exists a grey cat for all owners.
   (d) Every fire engine is red and and every ambulance is white.
5. Negate the following.
   (a) Some of the students in the class are not here today.
   (b) Let $x, y, z \in \mathbb{N}$. For all x there exists y such that $x = y + z$.
   (c) There exists unique x such that p is true.
   (d) All mathematics students are hardworking.
   (e) Only some of the students in the class are here today.
   (f) The number $\sqrt{x}$ is rational if x is an integer.
6. Show the following are true.
   (a) $\exists N \in \mathbb{N}$ such that $\forall n \geq N$, $\frac{1}{n} < \frac{25}{37}$.
   (b) $\exists N \in \mathbb{N}$ such that $\forall n \geq N$, $\frac{5n^2+2}{n^2} - 5 < \frac{1}{1000}$.