

Combinations + permutations.

1. a) $4! = 24$

b) $6! = 720$

c) i. $\binom{28}{2} = 378$ ii. $\binom{28}{3} = 3276$ iii. $\binom{28}{4} = 20475$ iv. $\binom{28}{5} = 98280$

2.

Tassel of 10 colours

	1	2	3	4	5	6	7	8	9	10
" 9 "									1	9
" 8 "								1	8	36
" 7 "							1	7	28	84
" 6 "						1	6	21	56	126
" 5 "					1	5	15	35	70	126
" 4 "				1	4	10	20	35	56	84
" 3 "			1	3	6	10	15	21	28	36
" 2 "		1	2	3	4	5	6	7	8	9
" 1 "	1	1	1	1	1	1	1	1	1	1

(Do you see how this is Pascal's triangle?)

3. a) $\boxed{6} \times \boxed{5} \times \boxed{4} = \frac{6 \times 5 \times 4}{3!} = 20.$

b) $\underbrace{\boxed{} \boxed{}}_6 \times \boxed{5} = 6 \times 5 = 30.$

c) $\underbrace{\boxed{} \boxed{} \boxed{}}_6 = 20 + 30 + 6 = 56.$

d)

1	1	2
1	2	3
1	3	4
2	2	4
1	4	5
2	3	5
1	5	6
2	4	6
3	3	6

4. a) $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 5 & 1 & 4 & 6 & 2 \\ 4 & 5 & 2 & 6 & 3 & 1 \end{pmatrix}$

b) $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 1 & 4 & 6 & 5 & 3 \\ 5 & 3 & 4 & 2 & 6 & 1 \end{pmatrix}$

c) $\begin{pmatrix} 3 & 5 & 1 & 4 & 6 & 2 \\ 1 & 2 & 3 & 4 & 5 & 6 \end{pmatrix} = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 6 & 1 & 4 & 2 & 5 \end{pmatrix}$

d) $\begin{pmatrix} 2 & 1 & 4 & 6 & 5 & 3 \\ 1 & 2 & 3 & 4 & 5 & 6 \end{pmatrix} = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 1 & 6 & 3 & 5 & 4 \end{pmatrix}$

e) $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 5 & 1 & 4 & 6 & 2 \\ 1 & 6 & 3 & 4 & 2 & 5 \\ 3 & 2 & 1 & 4 & 5 & 6 \\ 1 & 5 & 3 & 4 & 6 & 2 \\ 3 & 6 & 1 & 4 & 2 & 5 \\ 1 & 2 & 3 & 4 & 5 & 6 \end{pmatrix}$

Order of G_3 is 6.

$$5 \text{ a) } \binom{30}{9} = 14\,307\,150$$

$$b) \text{ i. } \frac{4}{52} \quad \text{ii. } \frac{4}{5} \quad \text{iii. } \frac{1}{5} \quad \text{iv. } \frac{1}{365} \quad \text{v. } \frac{364}{365}$$

$$6. \text{ a) i) } \frac{1}{\binom{49}{6}} = \frac{1}{13\,989\,816}$$

$$\text{ii) For } n \text{ balls choosing } r, \frac{1}{\binom{n}{r}}.$$

$$b) \text{ i) } \frac{\binom{6}{5} \binom{43}{1}}{\binom{49}{6}} = \frac{258}{13\,989\,816}$$

ii) Draw r balls from n and match t of them:

$$\frac{\binom{r}{t} \binom{n-r}{r-t}}{\binom{n}{r}}$$

$$7. \text{ 10 stars and 5 bars: } \binom{15}{5} = 3003.$$