Exercise answers

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From Town 1 we obtained the transition matrix:

$$\mathbf{P_1} = \begin{bmatrix} 0.5 & 0.481 \\ 0.5 & 0.519 \end{bmatrix}$$

The Town 2 data give the following transition counts:

		Now	
		Sunny	Rainy
Next	Sunny	16	7
	Rainy	7	21

Resultant transition matrix for Town 2:

$$\mathbf{P_2} = \begin{bmatrix} 0.696 & 0.25 \\ 0.304 & 0.75 \end{bmatrix}.$$

1. It seems likely that the probabilities used to generate the data for Town 2 are not the same as the probabilities used to generate the data for Town 1, because the transition probabilities based on the data differ substantially.

2. The eigenvector corresponding to eigenvalue $\lambda = 1$ for Town 1 is

$$\begin{bmatrix} 0.4903 \\ 0.5097 \end{bmatrix}.$$

   The eigenvector corresponding to eigenvalue $\lambda = 1$ for Town 2 is

$$\begin{bmatrix} 0.4513 \\ 0.5487 \end{bmatrix}.$$

   So it appears that in the long-term, these towns are not equally likely to experience rain – we expect rain at Town 1 51% of the time and rain at Town 2 55% of the time.

3. The transition probability matrix is:

$$\mathbf{P} = \begin{bmatrix} 0.5 & 0.24 & 0.2 \\ 0.25 & 0.6 & 0.5 \\ 0.25 & 0.16 & 0.3 \end{bmatrix}.$$

   The eigenvector corresponding to eigenvalue $\lambda = 1$ is

$$\begin{bmatrix} 0.3125 \\ 0.46875 \\ 0.21875 \end{bmatrix},$$

   so in the long-run, you might expect 31% of days to have strong wind, 47% to have light wind and 22% to have no wind, based on these data.