Graph theory – exercises

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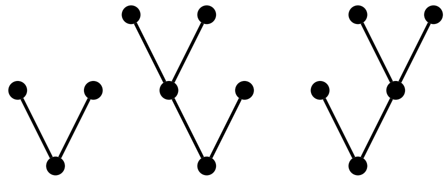
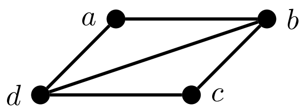
1. Draw the graph $G = (V, E)$ formed from the set of vertices $V = \{1, 2, 3, 4, 5, 6, 7\}$ and the set of edges
$$E = \{(1, 2), (2, 3), (2, 4), (3, 5), (4, 6), (5, 7), (6, 7)\}.$$

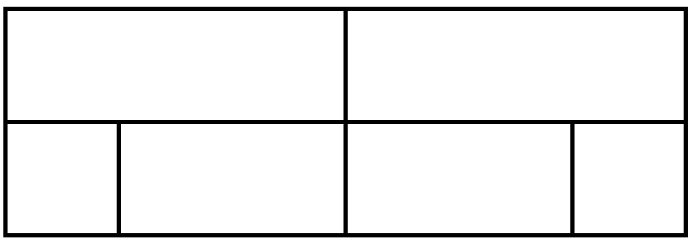
2. In a binary tree every branch can split upwards into exactly two more branches. The ends of the branches are called leaves. The diagram shows all binary trees with two leaves and all binary trees with three leaves. How many binary trees are there which have five leaves?



3. Represent the following graph as an adjacency matrix, $\mathbf{G}$. Calculate $\mathbf{G}^3$ and check that this gives the number of paths of length 3 between each of the different vertices.

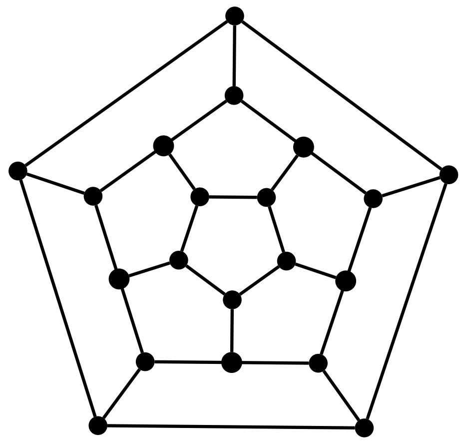


4. The floor plan of a building is shown below. There is a door between every pair of rooms and every room has a door to the outside. Can you go through each door exactly once and get back to where you started? If not, can you go through each door exactly once but end up somewhere different to where you started?



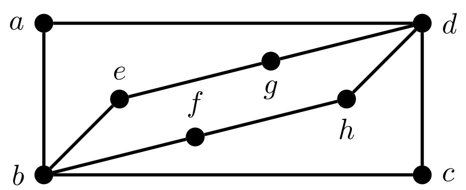
5. Find a Hamiltonian cycle through the vertices of a cube. You may find it useful to draw a 'flattened' version of the shape.

6. Can you find a Hamiltonian cycle in the following graph?

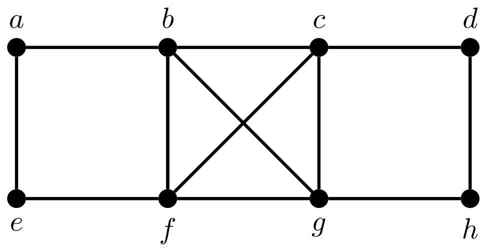


7. For each of the following graphs, decide if it is

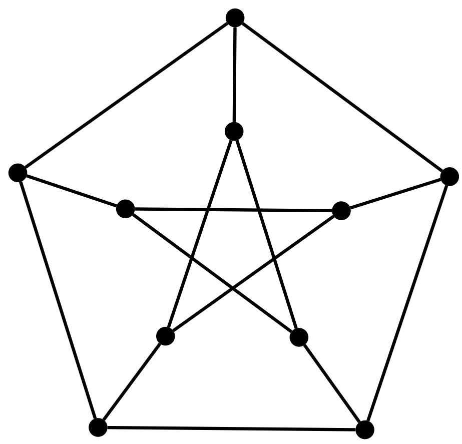
- Eulerian, semi-Eulerian, or non-Eulerian;
- Hamiltonian, semi-Hamiltonian, or non-Hamiltonian.



(a)



(b)



(c)

8. Draw nine different graphs. Each must be connected and must have exactly eight vertices. Your graphs must have the following properties:

   (a) Eulerian and Hamiltonian;
   (b) Eulerian and semi-Hamiltonian;
   (c) Eulerian and non-Hamiltonian;
   (d) Semi-Eulerian and Hamiltonian;
   (e) Semi-Eulerian and semi-Hamiltonian;
   (f) Semi-Eulerian and non-Hamiltonian;
   (g) non-Eulerian and Hamiltonian;
   (h) non-Eulerian and semi-Hamiltonian;
   (i) non-Eulerian and non-Hamiltonian.