

# Switches puzzle, hints and solution

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# Switches puzzle

- ▶ For complicated reasons that don't matter, a set of switches is wired so that if the following rules are not followed, an alarm will sound:
  1. The switch on the right may be turned on and off at will;
  2. Any other switch may be turned on or off only if the switch to its immediate right is on and all other switches to its right are off.
- ▶ Starting with all switches on, what is the smallest number of moves to turn all switches off without activating the alarm if there are 6 switches in the row?

# Problem-solving hints

# 1. Plan

# Problem-solving

- ▶ Draw a diagram.
- ▶ Invent a notation (e.g. a grid of numbered switches with 1 for on and 0 for off).

Sw 1	Sw 2	Sw 3	Sw 4	Sw 5	Sw 6
1	1	1	1	1	1

# Problem-solving

- ▶ Draw a diagram.
- ▶ Invent a notation (e.g. a grid of numbered switches with 1 for on and 0 for off).

Move	Sw 1	Sw 2	Sw 3	Sw 4	Sw 5	Sw 6
start	1	1	1	1	1	1
1	1	1	1	1	0	1
2	1	1	1	1	0	0
3	1	1	0	1	0	0
⋮	⋮	⋮	⋮	⋮	⋮	⋮

# Problem-solving

- ▶ Draw a diagram.
- ▶ Invent a notation (e.g. a grid of numbered switches with 1 for on and 0 for off).

Move	Sw 1	Sw 2	Sw 3	Sw 4	Sw 5	Sw 6
start	1	1	1	1	1	1
1	1	1	1	1	0	1
2	1	1	1	1	0	0
3	1	1	0	1	0	0
⋮	⋮	⋮	⋮	⋮	⋮	⋮

- ▶ Try a simpler problem (in this case, fewer switches).

# Solution for 3 switches



# 3 switches

Move	Sw 1	Sw 2	Sw 3
start	1	1	1
1			
⋮	⋮	⋮	⋮

# 3 switches

Move	Sw 1	Sw 2	Sw 3
start	1	1	1
1	1	1	0
2			
⋮	⋮	⋮	⋮

# 3 switches

Move	Sw 1	Sw 2	Sw 3
start	1	1	1
1	1	1	0
2	0	1	0
3			
⋮	⋮	⋮	⋮

# 3 switches

Move	Sw 1	Sw 2	Sw 3
start	1	1	1
1	1	1	0
2	0	1	0
3	0	1	1
4			
⋮	⋮	⋮	⋮

# 3 switches

Move	Sw 1	Sw 2	Sw 3
start	1	1	1
1	1	1	0
2	0	1	0
3	0	1	1
4	0	0	1
5			

# 3 switches

Move	Sw 1	Sw 2	Sw 3
start	1	1	1
1	1	1	0
2	0	1	0
3	0	1	1
4	0	0	1
5	0	0	0

# Solution for 4 switches

# 4 switches

Move	Sw 1	Sw 2	Sw 3	Sw 4
0	1	1	1	1
1				
⋮	⋮	⋮	⋮	⋮



## 4 switches

Move	Sw 1	Sw 2	Sw 3	Sw 4
0	1	1	1	1
1	1	1	0	1
2				
⋮	⋮	⋮	⋮	⋮

# 4 switches

Move	Sw 1	Sw 2	Sw 3	Sw 4
0	1	1	1	1
1	1	1	0	1
2	1	1	0	0
3				
⋮	⋮	⋮	⋮	⋮

# 4 switches

Move	Sw 1	Sw 2	Sw 3	Sw 4
0	1	1	1	1
1	1	1	0	1
2	1	1	0	0
3	0	1	0	0
4				
⋮	⋮	⋮	⋮	⋮

# 4 switches

Move	Sw 1	Sw 2	Sw 3	Sw 4
0	1	1	1	1
1	1	1	0	1
2	1	1	0	0
3	0	1	0	0
4	0	1	0	1
5				
⋮	⋮	⋮	⋮	⋮

# 4 switches

Move	Sw 1	Sw 2	Sw 3	Sw 4
0	1	1	1	1
1	1	1	0	1
2	1	1	0	0
3	0	1	0	0
4	0	1	0	1
5	0	1	1	1
6				
⋮	⋮	⋮	⋮	⋮

# 4 switches

Move	Sw 1	Sw 2	Sw 3	Sw 4
0	1	1	1	1
1	1	1	0	1
2	1	1	0	0
3	0	1	0	0
4	0	1	0	1
5	0	1	1	1
6	0	1	1	0
7				
⋮	⋮	⋮	⋮	⋮

# 4 switches

Move	Sw 1	Sw 2	Sw 3	Sw 4
0	1	1	1	1
1	1	1	0	1
2	1	1	0	0
3	0	1	0	0
4	0	1	0	1
5	0	1	1	1
6	0	1	1	0
7	0	0	1	0
8				
⋮	⋮	⋮	⋮	⋮

# 4 switches

Move	Sw 1	Sw 2	Sw 3	Sw 4
0	1	1	1	1
1	1	1	0	1
2	1	1	0	0
3	0	1	0	0
4	0	1	0	1
5	0	1	1	1
6	0	1	1	0
7	0	0	1	0
8	0	0	1	1
9				
⋮	⋮	⋮	⋮	⋮



# 4 switches

Move	Sw 1	Sw 2	Sw 3	Sw 4
0	1	1	1	1
1	1	1	0	1
2	1	1	0	0
3	0	1	0	0
4	0	1	0	1
5	0	1	1	1
6	0	1	1	0
7	0	0	1	0
8	0	0	1	1
9	0	0	0	1
10				

# 4 switches

Move	Sw 1	Sw 2	Sw 3	Sw 4
0	1	1	1	1
1	1	1	0	1
2	1	1	0	0
3	0	1	0	0
4	0	1	0	1
5	0	1	1	1
6	0	1	1	0
7	0	0	1	0
8	0	0	1	1
9	0	0	0	1
10	0	0	0	0

A hint: notice a pattern

## Notice: 3 switches

Move	Sw 1	Sw 2	Sw 3
start	1	1	1
1	1	1	0
2	0	1	0
3	0	1	1
4	0	0	1
5	0	0	0

- We deal with the first digit, then have to go from **1 0** to **0 0**.

# Notice:

## 4 switches

- We deal with the first digit, then have to go from  
**1 0 0**  
to  
**0 0 0.**

Move	Sw 1	Sw 2	Sw 3	Sw 4
start	1	1	1	1
1	1	1	0	1
2	1	1	0	0
3	0	<b>1</b>	<b>0</b>	<b>0</b>
4	0	<b>1</b>	<b>0</b>	<b>1</b>
5	0	<b>1</b>	<b>1</b>	<b>1</b>
6	0	<b>1</b>	<b>1</b>	<b>0</b>
7	0	<b>0</b>	<b>1</b>	<b>0</b>
8	0	<b>0</b>	<b>1</b>	<b>1</b>
9	0	<b>0</b>	<b>0</b>	<b>1</b>
10	0	<b>0</b>	<b>0</b>	<b>0</b>

Going from 0 0 0 to 1 0 0

$$\begin{array}{cccc} 0 & 0 & \dots & 0 \\ & \downarrow & & \\ 1 & 0 & \dots & 0 \end{array}$$

- So think about the number of moves required to convert  $n$  switches from all off to only the leftmost switch on. Call this  $G_n$  and express this in terms of  $G_{n-1}$ .
- (Try some smaller cases!)
- (Convince yourself that it takes the same number of moves to turn the left-most switch on as off in these circumstances.)

# A stronger hint

# Notice: 4 switches

- The part before  $G_3$  is the case with 2 fewer switches:  
**1 1.**

Move	Sw 1	Sw 2	Sw 3	Sw 4
start	1	1	<b>1</b>	<b>1</b>
1	1	1	<b>0</b>	<b>1</b>
2	1	1	<b>0</b>	<b>0</b>
3	0	<b>1</b>	<b>0</b>	<b>0</b>
4	0	<b>1</b>	<b>0</b>	<b>1</b>
5	0	<b>1</b>	<b>1</b>	<b>1</b>
6	0	<b>1</b>	<b>1</b>	<b>0</b>
7	0	<b>0</b>	<b>1</b>	<b>0</b>
8	0	<b>0</b>	<b>1</b>	<b>1</b>
9	0	<b>0</b>	<b>0</b>	<b>1</b>
10	0	<b>0</b>	<b>0</b>	<b>0</b>



## 2. Carry out your plan

## A stronger hint

- Let  $M_n$  be the number of moves required if there are  $n$  switches in the row. Express  $M_n$  in terms of  $M_{n-2}$  and  $G_{n-1}$ .

# 3. Review

# Answers

# Answer

- ▶  $G_n = 2G_{n-1} + 1.$
- ▶  $M_n = M_{n-2} + G_{n-1} + 1.$

# Answer

- ▶  $G_n = 2G_{n-1} + 1.$
- ▶  $M_n = M_{n-2} + G_{n-1} + 1.$

$n$	$G_n$
2	3
3	7
4	15
5	31

$n$	$M_n$
3	5
4	10
5	21
6	42

- ▶ Answer for 6 switches: 42.

# Reflect on what happened

- ▶ In groups now, think about the process of solving this problem.
  - ▶ What went well?
  - ▶ What went wrong?
  - ▶ Could you have avoided the dead ends, or were they a necessary part of solving the problem?
  - ▶ What do you wish you had known when you first attempted the problem?
- ▶ Also think about what you have learned and whether you can use this to write down a different problem you can now solve.