Introduction to mathematical modelling

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What is mathematical modelling?

- In semester 1, you learned some ways of thinking about and solving problems:
  - problem solving;
  - logic and proof;
  - programming.
- Now we add another: mathematical modelling.
- Mathematical modelling is the process of taking some real world question or phenomenon and analysing it using mathematics, in order to better understand its behaviour and / or make predictions about it.

Mathematical modelling

- This deals with how mathematics can be used to investigate the world around us.
- We use models to build theory, test hypotheses, and make decisions.
- This is not just about mathematical examples in application contexts: it is about how we build models.
- You will need to think about how realistic the models that you have developed are, what their limitations are, and whether you can refine them to better reflect reality.

Mathematical modelling

- In some cases the laws governing the situation being modelled are well understood, and mathematical models can give very accurate predictions as to how something will behave.
  - e.g. rise and fall of tides, movement of an object fired into the air.
- Other situations cannot be modelled nearly as accurately, perhaps because there is a random element to the situation
  - e.g. modelling population growth or economic issues

  or perhaps because the situation is extremely complex and depends on a large number of inter-related variables
  - e.g. the weather.
- Despite the limitations of models in situations like these, they may nonetheless provide useful insights and offer useful predictions.

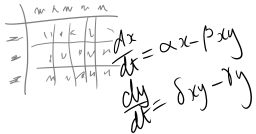
Building models

- Modelling is not like learning mathematical techniques then answering questions on those techniques.
- The world is messy, and problems don't come neatly packaged.
- Often the first step is sorting through information to work out what you have and what question to ask.
- It is often not clear what mathematical techniques to use.
- Often there are several different approaches that are equally valid, and no definite correct answer.

The Modelling Cycle

Scenario

A problem arises that we'd like to understand.



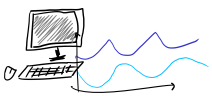
Mathematical model

A simple version of the scenario we can ask precise questions about.

refine?

Solve it

Using various mathematical methods.



Check & communicate

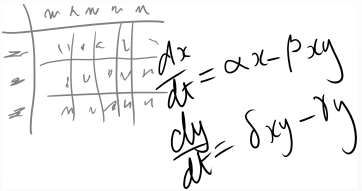
Does the solution make sense? Interpret and talk using the original context.

Scenario / Mathematising the world

- The first step in modelling is often thinking about the scenario and turing this into something we can ask precise mathematical questions about.
- Sometimes this involves finding or approximating missing information.
- Sometimes this involves making assumptions. Assumptions are not necessarily things we think are true, but are simplifications made to make the mathematical analysis possible.
  "All models are wrong, but some are useful".

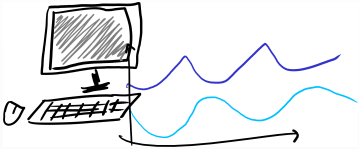
Mathematical model

- Once the scenario has been turned into a mathematical problem, we can ask precise questions about it.
- We consider the range of techniques available to us and come up with a plan to solve the problem.



Solve it

- Hopefully this is the part you are most familiar with: solving the problem.
- Essentially, this is putting your plan into action.



Check & communicate

- First, we check to see whether our analysis makes sense in the context of the original scenario.
  - Does your model make sense?
  - Can it make useful predictions about the scenario, or explain its features?
- Second, we must communicate our findings.
  - The people affected by the original scenario are interested in solutions to their problem, they likely don't want to be given solutions from mathematical analysis.
  - This is a job of effective communication.