Tutorial exercise sheet – Matrices

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1. Given the matrices

$$\mathbf{A} = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 2 & 4 \\ 0 & 1 \end{bmatrix}, \quad \mathbf{C} = \begin{bmatrix} 3 & -1 & 2 \\ 1 & -2 & 0 \end{bmatrix},$$

$$\mathbf{D} = \begin{bmatrix} 1 & -1 \\ 4 & 0 \\ 2 & 5 \end{bmatrix}, \quad \mathbf{E} = \begin{bmatrix} 2 & 1 & 3 \\ 1 & 1 & 1 \\ 3 & 0 & 1 \end{bmatrix} \quad \text{and} \quad \mathbf{F} = \begin{bmatrix} 0 & 2 & 6 \\ 2 & 1 & 2 \\ 6 & 0 & 3 \end{bmatrix},$$

determine, if possible, each of the following.

(a) $\mathbf{A}+\mathbf{B}$; (b) $3\mathbf{A}-2\mathbf{B}$; (c) $\mathbf{B}+\mathbf{C}$; (d) $2\mathbf{E}-\mathbf{F}$; (e) $\mathbf{AB}$; (f) $\mathbf{BA}$; (g) $\mathbf{AC}$; (h) $\mathbf{BD}$; (i) $\mathbf{CD}$; (j) $\mathbf{FE}$; (k) $\mathbf{A}^2-\mathbf{B}^2$; (l) $\mathbf{A}^3$; (m) $\mathbf{E}-\mathbf{DAC}$; (n) $2\mathbf{B}^\mathsf{T}-3\mathbf{A}^\mathsf{T}$; (o) $(\mathbf{B}^\mathsf{T}-(\mathbf{AB})^\mathsf{T})^\mathsf{T}$; (p) $\mathbf{C}^\mathsf{T}\mathbf{A}^\mathsf{T}\mathbf{D}^\mathsf{T}$.

2. Given

$$\mathbf{A} = \begin{bmatrix} 1 & 1 \\ 2 & -1 \\ 0 & 3 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 0 & -3 \\ -1 & 2 \end{bmatrix} \quad \text{and} \quad \mathbf{C} = \begin{bmatrix} 0 & -5 \\ 3 & 2 \end{bmatrix},$$

   (a) evaluate $(\mathbf{AB})\mathbf{C}$ and $\mathbf{A}(\mathbf{BC})$;

   (b) verify that $\mathbf{A}(\mathbf{B}+\mathbf{C}) = \mathbf{AB}+\mathbf{AC}$.

3. If $\mathbf{A} = \begin{bmatrix} 3 & 2 \\ -2 & -1 \end{bmatrix}$, show that $(\mathbf{A}-\mathbf{I})^2 = 0$.

4. In each case, determine whether the statement is true or false, and justify your answer.

   (a) An $m \times n$ matrix has m columns and n rows.

   (b) For every matrix $\mathbf{A}$, it is true that $(\mathbf{A}^\mathsf{T})^\mathsf{T} = \mathbf{A}$.

   (c) If $\mathbf{A}$ has a column of zeros, then so does $\mathbf{AB}$ if this product is defined.

   (d) If $\mathbf{A}$ has a column of zeros, then so does $\mathbf{BA}$ if this product is defined.

   (e) If $\mathbf{A}$ and $\mathbf{B}$ are square matrices of the same order, then $(\mathbf{AB})^\mathsf{T} = \mathbf{A}^\mathsf{T}\mathbf{B}^\mathsf{T}$.

   (f) If $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$ are square matrices of the same order such that $\mathbf{AC} = \mathbf{BC}$, then $\mathbf{A} = \mathbf{B}$.