Brief overview of the history of mathematics

Part 3: Eighteenth Century – present

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These talks

- This is a stampede missing much detail!
- And quite a traditional account of mainstream history.
- My intention is to give you an overview, and to pique your interest in some historical topics.

The Eighteenth Century

Euler (1707-1783)

- The most prolific mathematician of all time.
- Too many contributions to fit here!
- He introduced use of e for the exponential function, f for a function and i for the square root of -1.
- As well as contributing to many active areas of his day, his work is considered the start of topology and graph theory.

Image: Euler. The MacTutor History of Mathematics archive.

Mechanics

- Advances beyond Newton through the work of various Bernoullis, Euler, Clairaut and d'Alembert.
- Lagrange (1736-1813) showed how to generally answer problems about the motion of points by reducing them to problems in ordinary and partial differential equations.
- Later, work extended by Laplace (1749-1827).

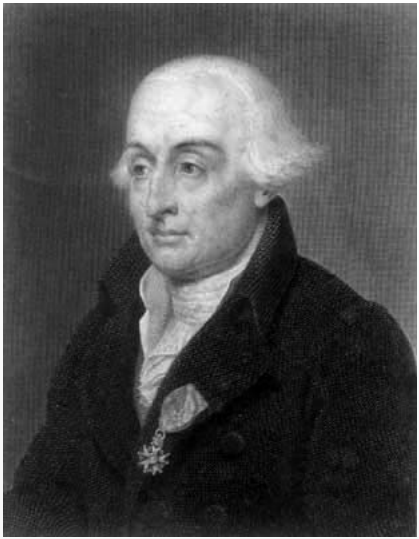

Image: Lagrange. The MacTutor History of Mathematics archive.

The Nineteenth Century

Gauss (1777-1855)

- One of the greatest mathematicians of all time;
- worked in many areas of mathematics, astronomy, physics, etc.;
- introduced modular arithmetic;
- proved in his doctoral thesis The Fundamental Theorem of Algebra, that any polynomial of degree n has n complex roots.

Image: Gauss. The MacTutor History of Mathematics archive.

Cauchy (1789-1857)

- formalised the concepts of limit, continuity, derivative and integral;
- putting the calculus on a more rigorous footing.
- He also led development of complex analysis.

Image: Cauchy. The MacTutor History of Mathematics archive.

Much work on applications of mathematics

- Fourier (1768-1830) on heat and Fourier series;
- Electricity, magnetism and elasticity, by e.g. Poisson (1781-1840), Stokes (1819-1903), Thomson (Kelvin) (1824-1907), Tait (1831-1901), Maxwell (1831-1879).

Algebra

- Abel (1802-1829) proved that no general solution can exist for polynomial equations of degree 5 or above;
- and proved results on other topics;
- before dying of tuberculosis aged 26.

Image: Abel. The MacTutor History of Mathematics archive.

Algebra

- Galois (1811-1832) determined when such equations *can* be solved, work that led to new areas of group theory and Galois theory.
- Galois was a radical Republican during the reign of Louis Philippe I, the last king to rule France.
- He also died young – aged 20 – when he got involved in a duel, possibly over a matter of love, and lost.
- He spent the night before the duel writing letters explaining his mathematical developments to Chevalier.

Image: Galois. The MacTutor History of Mathematics archive.

Algebra

- Vector spaces - Grassmann (1809-1877);
- Non-commutative algebra - Hamilton (1805-1865);
- Boolean algebra - Boole (1815-1864).

Non-Euclidean Geometry

- Formed by removing Euclid's fifth postulate, about non-parallel lines meeting;
- Hyperbolic geometry – Bolyai (1802-1860) and Lobachevsky (1792-1856);
- Elliptic geometry – Riemann (1826-1866);
- then Riemann geometry, which generalises the three types of geometry.

Topology

- Möbius did early work in topology and other areas;
- he is remembered particularly for the Möbius strip.
- Klein is also remembered for the Klein bottle.
- Both are non-orientable surfaces.
- Both men did much else besides!

Babbage and Lovelace

- Inspired by the error-prone process of producing tables of logarithms and trigonometric values, Babbage (1791-1871) sought to design a calculating machine that would do the job.

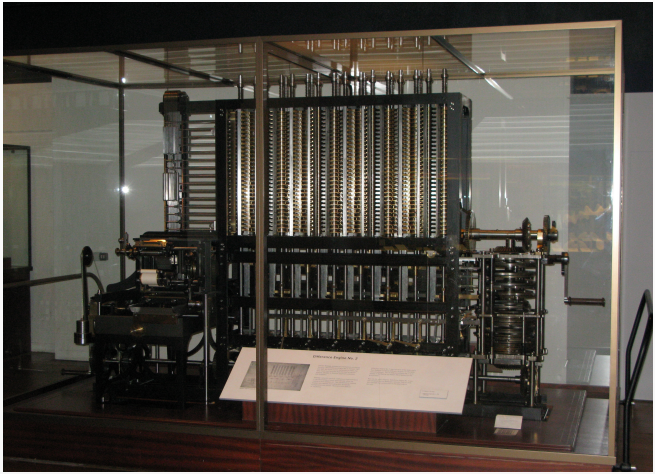

Image: Babbage Difference Engine, by User:geni at Wikimedia Commons.

Babbage and Lovelace

- The Difference Engine, which would calculate polynomial approximations to functions like sin, cos and log, ran into engineering, financial and political difficulties, and was not constructed.
- Babbage designed a new engine, the Analytical Engine, that would store and print numbers based on control instructions input via punch cards.
- It was also never built!
- Ada, Countess of Lovelace (1816-1852) wrote a commentary on the Analytical Engine, in which she outlined what is considered the world's first computer program.

Set theory

- Cantor (1845-1918) established the importance of one-to-one correspondence between sets;
- founded the theory of transfinite numbers;
- and showed that infinities can have different sizes.

Image: Cantor. The MacTutor History of Mathematics archive.

Statistics

- analysis of variance: Fisher (1890-1962);
- t-test: Gosset (under his pen name ‘Student’) (1876-1937);
- Florence Nightingale (1820-1910) was an early pioneer in the use of graphical data visualisation.

The Twentieth Century

Hilbert (1862-1943)

- Hilbert worked to find axiomatic systems that were
  - consistent (do not lead to contradictions);
  - independent (no axiom can be deduced from the others); and,
  - complete (any statement within the system can be proved to be either true or false).
- Euclid's axioms did not meet these requirements.

Image: Hilbert. The MacTutor History of Mathematics archive.

Russell's paradox

- Russell (1872-1970) came up with a paradox that showed set theory to be incomplete.
- The following is almost a version of Russell's paradox:

  Consider a village (in which everyone shaves) in which the (male) barber
  - *shaves all those who do not shave themselves;*
  - *does not shave those who shave themselves.*

  The question arises: Who shaves the barber?
- (Russell actually considered the set of all sets that are not members of themselves.)

Gödel (1906-1978)

- Gödel proved his incompleteness theorem, which shows that

  In any axiomatic system that includes the integers, there are true results that cannot be proved, and there are 'undecidable' results that cannot be proved either true or false.

Image: Gödel. The MacTutor History of Mathematics archive.

Turing (1912-1954)

- Turing came up with the abstract concept of a computing machine to investigate whether a given proposition is true or false.
- Some mathematical propositions are undecidable – no algorithm can decide whether they are true or false.
- This and other work on computation – long before the computer was invented – led to the modern discipline of computer science.

Image: Turing. The MacTutor History of Mathematics archive.

Work on applications continued

- Einstein (1879-1955) and Minkowski's (1864-1909) spacetime, essential to general relativity, and quantum mechanics all rely on non-Euclidean geometry;
- More practical work built on that of Navier (1785-1836) and Stokes (1819-1903) on fluids (particularly for flight), including by Lighthill (1924-1998).

Fractals

- The Sierpinski triangle (1915) (an early stage is pictured) has zero area but its boundary has infinite length.

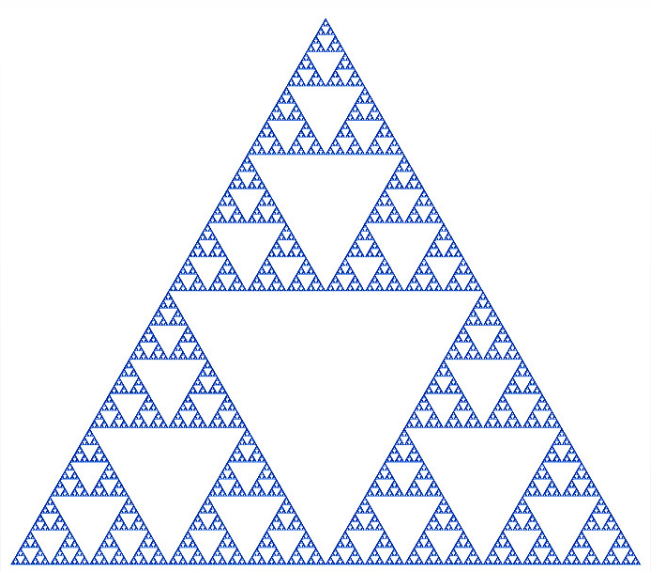

Image: Sierpinski triangle. Public domain on Wikimedia Commons.

Chaos

- Systems that are deterministic but extremely sentitive to changes in initial conditions.
- Important in, e.g., weather and air turbulance (pictured).
- Work by, e.g., Lorenz (1917-2008).

Image: Airplane vortex. Public domain on Wikimedia Commons.

The Four Colour Theorem

- Posed by De Morgan in 1852: "if a figure be any how divided and the compartments differently coloured so that figures with any portion of common boundary line are differently coloured – four colours may be wanted, but not more."
- Proved by Appel and Haken, 1976.
- Building on work by many others they showed that every possible map is reducible to one that can be four-coloured.
- Important as first major theorem to be proved using a computer.
- Used twelve hundred hours of computer time.

The most recent piece of well-known mathematics taught in an undergraduate mathematics degree?

- Black–Scholes (1973)?
- RSA (1977)?
- Actually, probably some of the computational methods used in applied maths are more recent.

Recent results

- In 1995, Wiles (1953–) published a proof of Fermat's Last Theorem.
- In 1998, Hales (1958–) proved the Kepler conjecture, which says that no arrangement of equally sized spheres filling space has a greater average density than that pictured.

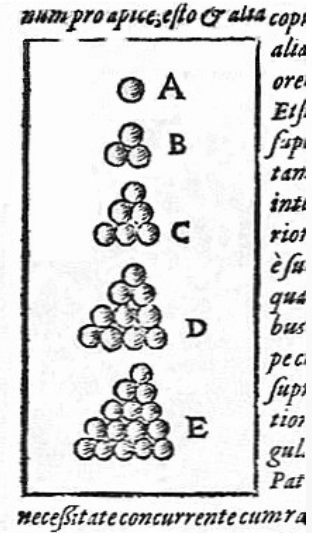

Image: A diagram from Johannes Kepler's 1611 *Strena Seu de Nive Sexangula*. Public domain on Wikimedia Commons.

The Twenty-first Century

Clay Problems

- Seven Millennium Prize Problems, with a $1m prize for each.

1. P versus NP;
2. The Hodge conjecture;
3. The Poincaré conjecture;
4. The Riemann hypothesis;
5. Yang-Mills existence and mass gap;
6. Navier-Stokes existence and smoothness;
7. The Birch and Swinnerton-Dyer conjecture.

Perelman (1966–)

- Proved the Poincaré conjecture in 2003.
- He turned down the Fields Medal and the Clay Institute prize.

Image: Perelman. The MacTutor History of Mathematics Archive.

Recent results

- Many active areas of development, though many quite complicated!
- Common to see high-profile results in number theory, including in recent years on
  - twin prime conjecture by Yitang Zhang (stated in 1849);
  - Goldbach conjecture by Harald Helfgott (open since 1742);
  - the *abc* conjecture by Shinichi Mochizuki (open since 1985) – maybe.
- Maths is a vibrant and active discipline.

  e.g. `https://arxiv.org/catchup?group=grp_math&%2Fcatchup=Catchup`

Group work this week

- Please choose a mathematical topic and explore its history, or choose a historical person or culture and explore their connection with mathematics.

References

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