Puzzles through history

Puzzles

In groups, have a look through the following problems. Pick some to have a think about and discuss. How would you resolve them?[1]

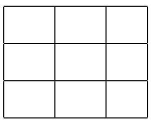
1. There are seven houses each containing seven cats. Each cat kills seven mice and each mouse would have eaten seven ears of spelt. Each ear of spelt would have produced seven hekats of grain. Houses, cats, mice, spelt and grain, what is the total of all these?
   (Egyptian, Problem 79, Rhind papyrus, c. 1650 BCE; PBCIP #1.)

2. I think of a number, and add to it two-thirds of the number. I then subtract one-third of the sum. My answer is now 10. What number did I think of?
   (Egyptian, Problem 29, Rhind papyrus, c. 1650 BCE; PBCIP #7.)

3. An area A, consisting of the sum of two squares, is 1000. The side of one square is 10 less than two-thirds of the other square. What are the sides of the square?
   (Bablyonian, c. 1800 BCE; PBCIP #13.)

4. A spear of length 30 is standing upright against a wall. If the upper end slides down the wall a distance of 6, how far will the lower end move out from the wall?
   (Babylonian, Problem 9 on tablet BM85196, 1800 BCE; PBCIP #14.)

5. How can the numbers 1 to 9 be arranged in the cells of this square so that the sums of every row and column and both diagonals are equal?
   (Chinese, *Lo Shu*, possibly c. 650 BCE; PBCIP #59.)



6. The sun god had a herd of cattle consisting of bulls and cows, one part of which was white, a second black, a third spotted, and a fourth brown. Among the bulls, the number of white ones was one half plus one third the number of the black greater than the brown; the number of the black, one quarter plus one fifth the number of the spotted greater than the brown; the number of the spotted, one sixth and one seventh the number of the white greater than the brown. Among the cows, the number of white ones was one third plus one quarter of the total black cattle; the number of the black, one quarter plus one fifth the total of the spotted cattle; the number of spotted, one fifth plus one sixth the total of the brown cattle; the number of the brown, one sixth plus one seventh the total of the white cattle. What was the composition of the herd?
   (Greek, This is Archimedes' Cattle Problem, c. 250 BCE; PBCIP #16.)

7. Suppose there are a number of rabbits and pheasants confined in a cage, in all thirty-five heads and ninety-four feet. How many were there of each?
   (Chinese, *The Nine Chapters of Mathematical Art*, c. 200 BCE; PBCIP #60.)

[1] PBCIP refers to the number of the puzzle in the *Penguin Book of Curious and Interesting Puzzles*, 1992.

8. Of two water weeds, one grows 3 feet and the other 1 foot on the first day. The growth of the first becomes every day half of that of the preceeding day, while the other grows twice as much as on the day before. On which day will the two have grown to equal heights?
(Chinese, *The Nine Chapters of Mathematical Art*, c. 200 BCE; PBCIP #68.)

9. Find two rectangles, with integer sides, such that the area of the first is three times the area of the second, and the perimeter of the second is three times the perimeter of the first.
(Greek, Heron of Alexandria, c. 75; PBCIP #21.)

10. What number must be added to 100 and to 20 (the same number to each) so that the sums are in the ratio 3:1?
(Greek, Book I of Diophantos' *Arithmetica*, c. 250; PBCIP #23.)

11. Twenty men, women and children earn twenty coins between them. Each man earns 3 coins, each woman $1\frac{1}{2}$ coins and each child $\frac{1}{2}$ coin. How many men women and children are there?
(Indian, the Bakhshali manuscript, c. 300; PBCIP #47.)

12. 100 fowls are sold for 100 shillings, the cocks being sold for 5 shillings each, the hens for 3 shillings and the chicks for $\frac{1}{3}$ shilling each. How many of each were sold?
(Chinese, *The Arithmetical Classic of Ch-iu Chien*, c. 470; PBCIP #74.)

13. A woman dies, leaving her husband, a son and three daughters. She also leaves $\frac{1}{8} + \frac{1}{7}$ of her estate to a stranger. According to law, the husband receives one quarter of the estate and the son receives double the share of a daughter, but this division is made only after the legacy to the stranger has been paid. How must the inheritance be divided?
(Arabic, al-Khwārizmī, c. 825; PBCIP #37.)

14. Three merchants saw in the road a purse containing money. One said, "If I secure this purse, I shall become twice as rich as both of you together."

    Then the second said, "I shall become three times as rich."

    Then the third said, "I shall become five times as rich."

    What is the value of the money in the purse, as also the money held by each of the three merchants?
(Indian, Mahavira's *Ganita-Sara-Sangraha*, c. 850; PBCIP #52.)

15. A king ordered his servant to collect an army from thirty manors, in such a way that from each manor he would take the same number of men as he had collected up to then. The servant went to the first manor alone; to the second he went with one other... How many men were collected in all
(English/German, Problem XIII in *Propositions to Sharpen Up the Young*, written c. 1000 in a German monestry, possibly attributed to Alucin of York (c. 732-804); PBCIP #78.)

16. A stairway consists of 100 steps. On the first step stands a pigeon; on the second, two pigeons; on the third, three; on the fourth, four; on the fifth, five; and so on every step up to the hundredth. How many pigeons are there altogether?
(English/German, Problem XLII in *Propositions to Sharpen Up the Young*, written c. 1000 in a German monestry, possibly attributed to Alucin of York (c. 732-804); PBCIP #85.)

17. In an expedition to size his enemy's elephants, a king marched 2 yojanas in the first day. With what increasing rate of daily march did he proceed, since he reached his foe's city, a distance of 80 yojanas, in a week?
(Indian, Bhaskara's *Līlāvatī*, c. 1150; PBCIP #55.)

18. A certain man put a pair of rabbits in a place surrounded on all sides by a wall. How many pairs of rabbits can be produced from that pair in a year if it is supposed that every month

each pair begets a new pair which from the second month on becomes productive?
(Italian, Fibonacci's *Liber Abaci*, 1202; PBCIP #88.)

19. A lion would take four hours to eat one sheep; a leopard would take five hours; and a bear would take six. If a single sheep were to be thrown to them, how many hours would they take to devour it?
(Italian, Fibonacci's *Liber Abaci*, 1202; PBCIP #89.)

20. Sissa ben Dahir was asked by the Indian King Shirham what he desired as a reward for inventing the game of chess.

    "Majesty, give me a grain of wheat to place on the first square, and two grains of wheat to place on the second square, and four grains of wheat to place on the third, and eight grains of wheat to place on the fourth, and so, Oh King, let me cover each of the sixty-four squares on the board."

    "And is this all you wish, Sissa, you fool?" exclaimed the astonished King.

    How many grains of wheat did Sissa request?
(Arabic, Ibn Kallikan, c. 1256; PBCIP #46.)

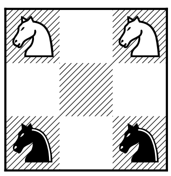
21. A serpent lies at the bottom of a well whose depth is 30. It starts to climb, rising up $\frac{2}{3}$ every day and falling back $\frac{1}{5}$ at night. How long does it take to climb out of the well?
(Italian, Dell'Abaco, c. 1370; PBCIP #92.)

22. The Holy Father sent a courier from Rome to Venice, commanding him that he reach Venice in seven days. And the most illustrious Signoria of Venice also sent another courier to Rome, who should reach Rome in nine days. And from Rome to Venice is 250 miles. It happened that by order of these lords the couriers started on their journeys at the same time. In how many days will they meet?
(Italian, The *Treviso Arithmetic*, 1478; PBCIP #94.)

23. You have two jars holding 5 and 3 pints respectively, neither jar being marked in any way. How can you measure exactly 4 pints from a cask, given that you are allowed to pour liquid back into the cask?
(French, Chuquet, c. 1480; PBCIP #97.)

24. Two black knights and two white knights are placed at the opposite corners on a 3×3 chessboard. How can the white knights take the place of the black knights, and vice versa, moving according to the rules of Chess?
(Italy, Guarini di Forli, 1512; PBCIP #101.)

    (Knights move in an L-shape, either one square across and then two squares up or down, or two squares across and one square up or down.)



25. A dishonest servant removes 3 pints of wine from a barrel, replacing them with water. He repeats his theft twice more, removing in total 9 pints and replacing them with water. As a result the wine remaining in the barrel is of half its former strength. How much wine did the barrel originally hold?
(Italy, Tartaglia, c. 1550; PBCIP #104.)

26. What is the least number of weights that can be used on a scale pan to weigh any integral number of pounds from 1 to 40 inclusive, if the weights can be placed in either of the scale pans?
(French, Bachet's *Problèms plaisans et délectables*, 1612; PBCIP #108.)

27. Where can a man look south in all directions?
(French, Henry van Etten's *Mathematical Recreations, Or a Collection of sundrie excellent Problemes out of ancient and moderne Phylosophers Both usefull and Recreative*, 1633; PBCIP #119.)

28. Find the point whose sum of distances from the vertices of a given triangle is a minimum.
(French, Pierre de Fermat, letter to Torricelli, c. 1650; PBCIP #125.)

29. Which is most likely, to throw at least 1 six with 6 dice, at least 2 sixes with 12 dice, or at least 3 sixes with 18 dice?
(English, Samuel Pepys, letter to Isaac Newton, 1693; PBCIP #129.)

30. Gamblers were used to betting on the event of getting at least one 1 in four rolls of a dice. As a more trying variation, two die were rolled 24 times with a bet on having at least one double ace. According to the reasoning of Chevalier de Méré, two 1s in two rolls are $\frac{1}{6}$ as likely as one 1 in one roll. (Which is correct.) To compensate, de Méré thought, the two die should be rolled 6 times. And to achieve the probability of one 1 in four rolls, the number of the rolls should be increased four fold – to 24. Thus reasoned de Méré, who expected a couple of 1s to turn up in 24 double rolls with the frequency of a single 1 in 4 single rolls. However, he lost consistently. Why?
(French, Blaise Pascal, letter to Pierre de Fermat, c. 1654.)

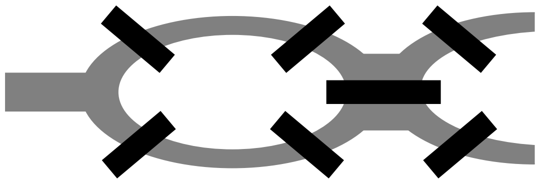
31. If a cows graze b fields bare in c days,

    and a' cows graze b' fields bare in c' days,

    and a'' cows graze b'' fields bare in c'' days,

    what is the relationship between the nine magnitudes a to c''?
(English, Isaac Newton's *Arithmetica Universalis*, 1707; PBCIP #128.)

32. In the town of Königsberg there were seven bridges across the river Pregel. Is it possible to go for a walk, crossing each bridge once, but not crossing any bridge twice?
(Russian, Leonhard Euler, 1736; PBCIP #133.)



33. How can a knight make a complete tour of an 8×8 chessboard, visiting each square once and only once, and ending up a knight's move from its starting square – so that the circuit is continuous?
(German, Leonhard Euler, c. 1750; PBCIP #132.)

34. How can twenty-five officers, comprising a colonel, lieutenant-colonel, major, captain and lieutenant from each of five regiments be arranged in a square array so that no rank or regiment will be repeated in any row or column?
(Russian, Leonhard Euler, 1782.)