Curiosity

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- Francis Su writes about "the ability to ask questions like **Why?** and **How?** and **What happens if. . . ?**" saying

> "All children do this, yet somewhere along the way, some stop asking questions — perhaps because they are told to memorize things and not to understand things. They are taught to follow procedures rather than to explore why those procedures work. They begin to think that there is just one right way to solve a problem rather than developing their own path to a solution. At every opportunity we need to counter the idea that math is memorization, and replace it with the idea that math is exploration. A math memorizer doesn't know how to react in unfamiliar situations, but a math explorer can flexibly adapt to changing conditions, because she has learned to ask questions that will prepare her for many scenarios."

Mathematics for Human Flourising, page 26 (emphasis added).

Notice/Wonder

- A prompt some teachers like to use is
  - What do you notice?
  - What do you wonder?

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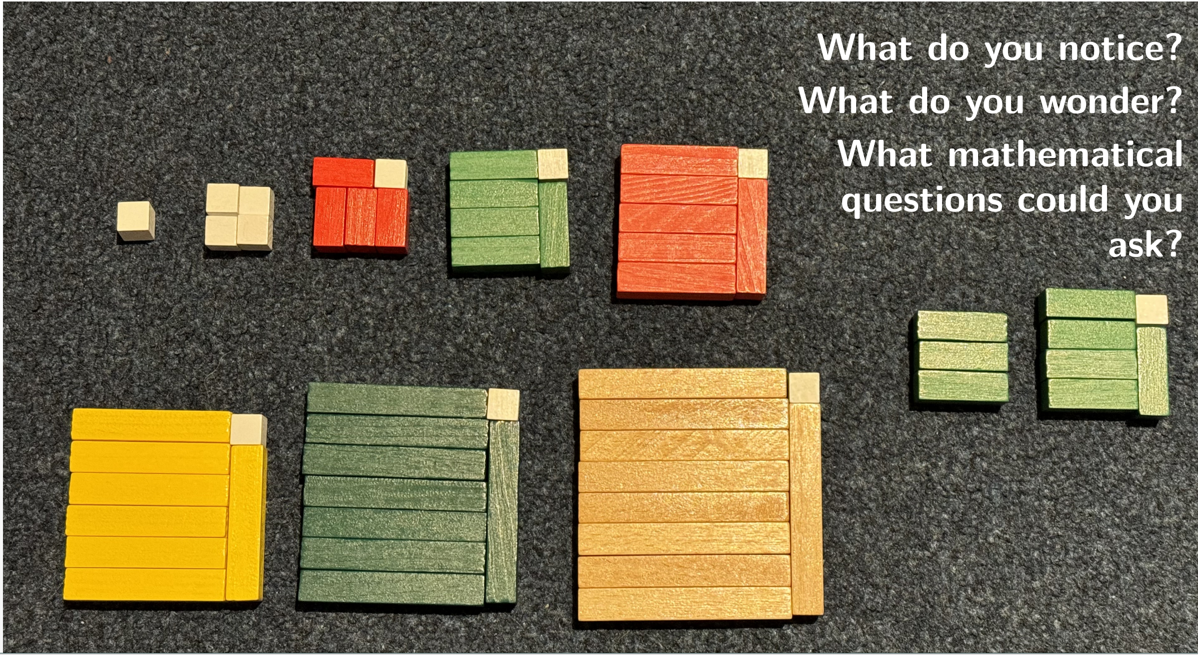
What do you wonder?

What mathematical questions could you ask?

What do you notice?
What do you wonder?
What mathematical questions could you ask?



$$
\begin{aligned}
2^2 &= 1^2 + 2 \times 1 + 1 \\
3^2 &= 2^2 + 2 \times 2 + 1 \\
4^2 &= 3^2 + 2 \times 3 + 1 \\
5^2 &= 4^2 + 2 \times 4 + 1 \\
6^2 &= 5^2 + 2 \times 5 + 1 \\
7^2 &= 6^2 + 2 \times 6 + 1 \\
8^2 &= 7^2 + 2 \times 7 + 1 \\
&\vdots \\
(n+1)^2 &= n^2 + 2n + 1
\end{aligned}
$$

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- Try a few

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$$\begin{aligned} 15^2 &= 225 \\ 25^2 &= 625 \\ 35^2 &= 1225 \\ 45^2 &= 2025 \end{aligned}$$

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- How does this work?

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- Trick is: to square $x5$, put $x^2 + x$ in the hundreds column and add 25.
- How does this work? $x5$ can be expressed $10x + 5$. Then

$$(10x + 5)^2 = 100x^2 + 100x + 25 = 100(x^2 + x) + 25.$$

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- Call an ending stubborn if it does this.

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- Four digit? Five digit?
- Are there any patterns? What do you notice? What do you wonder?