Limits

Limits using SymPy

To find a limit such as

$$\lim_{x \to 2} \frac{x^2 - 4}{x^2 - 5x + 6}$$

we use the command `limit`. This takes three items, first the expression being analysed, second the variable of interest, third the value of interest.

For example, to solve the limit above we import SymPy and define a variable `x`:

```
from sympy import *
from sympy.abc import x
```

Now we can ask Python for

```
limit((x**2-4)/(x**2-5*x+6),x,2)
```

We can ask for the limit as $x \to \infty$ where ∞ is written `oo`. For example, for

```
limit((1+1/x)^x,x,oo)
```

SymPy will return `E`, which is its representation of $e = 2.7182\ldots$

If a limit is ∞, SymPy will return `oo`, for example try this

```
limit(1/(x-3),x,3)
```

And of course, the limit could be $-\infty$, for example

```
limit(1/(5-x),x,5)
```

Derivative using limits in SymPy

We can think of the gradient as

$$\frac{\text{change in } f(x)}{\text{change in } x}$$

for some function $f(x)$.

Between a point $(x, f(x))$ and a nearby point $(x + h, f(x + h))$, where $h \in \mathbb{R}$ is a small number, the gradient is

$$\frac{f(x+h) - f(h)}{(x+h) - x} = \frac{f(x+h) - f(h)}{h}$$

The derivative is the limit of this as $h \to 0$, which we can write

$$\lim_{h \to 0} \frac{f(x+h) - f(h)}{h}$$

In Python, first we import SymPy and define `x` and `h`.

```
from sympy import *
from sympy.abc import x,h
```

Now we can define a function, say $f(x) = x^2$:

```
f = x**2
```

We can substitute other values into this function using `.subs()` to which we pass the variable we want to substitute and what we want to substitute it for. For example $f(x+h)$ is substituting `x` for `x+h`.

```
f.subs(x,x+h)
```

Now we can ask Python for the derivative using

```
limit((f.subs(x,x+h) - f)/h,h,0)
```

Running this, you should obtain the answer `2*x`.

You can perform this operation on more complicated functions. For example, defining a function

```
f = 3 * cos(4 * x + pi) + exp(-2*x)
```

we can find the derivative using limits by asking for

```
limit((f.subs(x,x+h) - f)/h,h,0)
```

Of course, we can find the derivative more directly using

```
diff(f,x)
```

For this function, Python gives different output for these two. You should verify that these are equivalent. One way could be to use `simplify` on the limit:

```
simplify(limit((f.subs(x,x+h) - f)/h,h,0))
```

Limits in LaTeX

To typset a limit we use the command `\lim_{}`, where the condition is placed inside the braces. For example, the limit at the top of this page is typeset using

```
\[ \lim_{x \to 2} \frac{x^2-4}{x^2-5x+6} \]
```

The derivative is typset using

```
\[ \lim_{h \to 0} \frac{f(x+h) - f(h)}{h} \]
```