

Combinatorics – exercises

1. If we deal six cards from a standard pack of playing cards, what is the probability that exactly one of them is a King?
2. My son tells me seven of the children in his nursery class are his “best friend”. For his fourth birthday, we invite the whole class of 20 children, and 12 attend. What is the probability that exactly three of his best friends attend the party?*
3. A crisp company ran an advert claiming that eating several of their crisps at the same time is a good idea. This claims putting crisps together in this way can create 318,000 possibilities[†].

The advert shows a man stacking three crisps together and eating them.

- (a) An accompanying website allows you to choose flavours and shows them being stacked together. This requires that precisely three crisps are stacked from a choice of 18 flavours, and you are permitted to stack the same flavour multiple times.

How many possibilities does this allow?

- (b) You ask the company to explain their reasoning. They reply saying:

“There are 25 flavours, you are allowed 2, 3 or 4 different-flavoured crisps in a stack, and we do not count different orderings as different.”

How many possibilities does this allow?

- (c) You wonder if the company has itself confused, and attempt to correct the statement thus:

“There are 25 flavours, you are allowed 2, 3 or 4 ~~different-flavoured~~ crisps in a stack, and we do not count different orderings as different.”

How many possibilities does this allow?

- (d) You wonder if the company has itself confused, and attempt to correct the statement thus:

“There are 25 flavours, you are allowed 2, 3 or 4 different-flavoured crisps in a stack, and we do ~~not~~ count different orderings as different.”

How many possibilities does this allow?

4. An article in the *Daily Mail* in 2011 has the title ‘Whist players astonished after each receives full suit in one hand.. at odds of 2,235,197,406,895,366, 368,301,559,999-to-one.’

The rules of Whist aren’t important here, what is important is that the game is set up by dealing all 52 playing cards equally between the player. The surprise here is that one player was dealt all the Hearts, another all the Clubs, a third all the Diamonds, and the fourth all the Spades.

The article says “Probability expert says perfect hand is probably the first time it has happened in history of the game ... Mathematicians say the odds of this happening are a jaw-dropping 2,235,197,406,895,366, 368,301,559,999 to one.”

- (a) Is this number correct?

- (b) Similar headlines appeared in, for example, 1929, 1935, 1949, 1954, 1963, 1978, and 1998. What might be happening?

*Based on a true story.

[†]Also based on a true story, including the website and statement from the company.

5. I have a pile of letters and wish to make 3-digit codes from them. I have four A 's, two B 's and one C . How many ways can I do this?

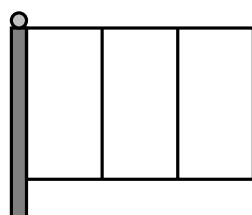
6. I have a pile of digits and wish to make 3-digit codes from them. I have one each of the digits 1–3, two 0's, three 4's and four 5's. How many ways can I do this?

7. You are arranging a game that requires players to have unique flags.

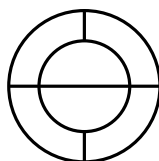
You have enough material to give each player two blue strips, one red strip and two yellow strips, from which they must construct a flag of three strips.

How many different flags can be constructed?

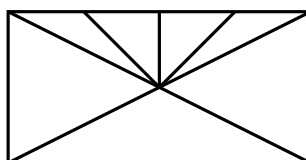
(Note that each colour's position in relation to the flag pole is important, so a flag that is reflected left-to-right is considered a different flag.)



8. How many ways are there to colour the sections of this game tile using 3 colours, if it can be rotated but not flipped?



9. How many ways are there to colour this stained glass window pattern using 6 colours, if it can be rotated and reflected?



10. How many ways are there to colour this stained glass window pattern using 6 colours, if it can be rotated and reflected?

