Eigenvalues and eigenvectors exercises

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1. Determine whether the following systems have non-trivial solutions. If they do have non-trivial solutions, why is this?

   (a)
   $$\begin{aligned} 3x - y + z &= 0 \\ x + 2y + 2x &= 0 \\ 4x + y + 3z &= 0 \end{aligned}$$

   (b)
   $$\begin{aligned} 3x - y + z &= 0 \\ x + 2y + 2z &= 0 \\ 5x - y + 3z &= 0 \end{aligned}$$

2. For each of the matrices:

   i. find the eigenvalues;
   ii. find the eigenvectors;
   iii. diagonalise the matrix.

   (a) $\begin{bmatrix} -1 & -4 \\ 2 & 5 \end{bmatrix}$ (c) $\begin{bmatrix} -2 & 4 \\ 7 & -5 \end{bmatrix}$ (e) $\begin{bmatrix} 14 & -11 \\ 5 & -2 \end{bmatrix}$

   (b) $\begin{bmatrix} 4 & -5 \\ -12 & -3 \end{bmatrix}$ (d) $\begin{bmatrix} -5 & 3 \\ 11 & 3 \end{bmatrix}$ (f) $\begin{bmatrix} -8 & 6 \\ -4 & 2 \end{bmatrix}$

3. For each of the matrices:

   i. find the eigenvalues;
   ii. find the eigenvectors;
   iii. diagonalise the matrix.

   (a) $\begin{bmatrix} 1 & -1 & 2 \\ -3 & -2 & 3 \\ 2 & -1 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 2 & 1 & 2 \\ -1 & 1 & -1 \\ 8 & 3 & 0 \end{bmatrix}$ (c) $\begin{bmatrix} -2 & 6 & 2 \\ 0 & 3 & 4 \\ 3 & -3 & 5 \end{bmatrix}$ (d) $\begin{bmatrix} 3 & -2 & 1 \\ 2 & -4 & 3 \\ 16 & -4 & 1 \end{bmatrix}$