## Proof exploration

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Investigate the following theorems. What are they saying? Can you rewrite them in your own words? Try out some special cases. What might a proof look like?

- 1. Suppose that  $m, n \in \mathbb{N}$ . Then mn is even if and only if m and n are even.
- 2. An integer n is divisible by 9 if and only if the sum of its digits is equivalent to 0 (mod 9).
- 3. If A and B are finite sets, then  $|A \times B| = |A||B|$ .
- 4. A train goes 500 miles along a straight track, without stopping, completing the trip with an average speed of exactly 50 miles per hour. It travels, however, at different speeds along the way. There must be a segment of 50 miles that the train traverses in precisely one hour.
- 5. Let  $n \in \mathbb{N}$  with  $n \geq 3$ . For n distinct points on a circle connect consecutive points by a straight line. The sum of the interior angles of the resulting shape is  $(n-2) \times 180^{\circ}$ .
- 6. If  $n \in \mathbb{N}$  and  $n \geq 7$ , then

$$\frac{n}{n^2 - 8n + 12} \ge \frac{1}{n}.$$

- 7. Let A be a finite set. Let S be the set of all subsets of A. Then  $|S| = 2^{|A|}$ . (Note: The set S is called the *power set* of A.)
- 8. For sets A, B and C we have
  - (a)  $A (B \cup C) = (A B) \cap (A C);$
  - (b)  $A (B \cap C) = (A B) \cup (A C)$ ;
  - (c)  $A \neq B$  if and only if  $(A B) \cup (A C)$ ;
  - (d)  $A \cup B \subseteq C$  if and only if  $A \subseteq C$  and  $B \subseteq C$ .

What happens in the extreme case(s) where some (or all) sets are empty?