Number bases

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1 Place value

Regular numbers are written in decimal format, a place value system where each place is ten times the previous.

...					
...	$10^4 = 10000$	$10^3 = 1000$	$10^2 = 100$	$10^1 = 10$	$10^0 = 1$

We can work in other number bases in the same way. For example, a base 2 system would see each place representing twice the previous. Such a system is called binary.

...					
...	$2^4 = 16$	$2^3 = 8$	$2^2 = 4$	$2^1 = 2$	$2^0 = 1$

A base 3 system is called ternary.

...					
...	$3^4 = 81$	$3^3 = 27$	$3^2 = 9$	$3^1 = 3$	$3^0 = 1$

In general, a base n system has each place representing n times the previous.

...					
...	n^4	n^3	n^2	n^1	$n^0 = 1$

Decimals in other bases can also be defined. For example, in base n

...								...
...	n^2	n^1	$n^0 = 1$	$\cdot$	$n^{-1} = \frac{1}{n}$	$n^{-2} = \frac{1}{n^2}$	$n^{-3} = \frac{1}{n^3}$	...

2 Representing numbers

Numbers in decimal are represented in each place by a digit drawn from $\{x \in \mathbb{Z} \mid 0 \leq x < 10\}$. In base n, the 'digits'[1] take values from $\{x \in \mathbb{Z} \mid 0 \leq x < n\}$. So for example, in binary bits are values from $\{0, 1\}$.

In bases > 10, we run out of single digit symbols. One common approach is to use letters, so the hexadecimal (base 16) system uses symbols 0–9 to represent the numbers 0–9 and the symbols A–F to represent the numbers 10–15 (with 16 being represented 10).

2.1 Example

In binary, the first few numbers are

0	0
1	1
2	10
3	11
4	100
5	101
6	110
7	111
8	1000

2.2 Example

27 in hexadecimal is one 16 and eleven ones, written $1B$.

3 Distinguishing bases

When it is necessary to specify which base we are working in, the base can be added as a subscript to the number. For example, if 1011 is a binary number, we could write 1011_2.

[1] The word 'digits' is in quotes here because 'digit' refers to the number ten (we have ten digits). In binary, these are called 'bits'. Informally, people tend to say 'digit' even when talking about other bases.