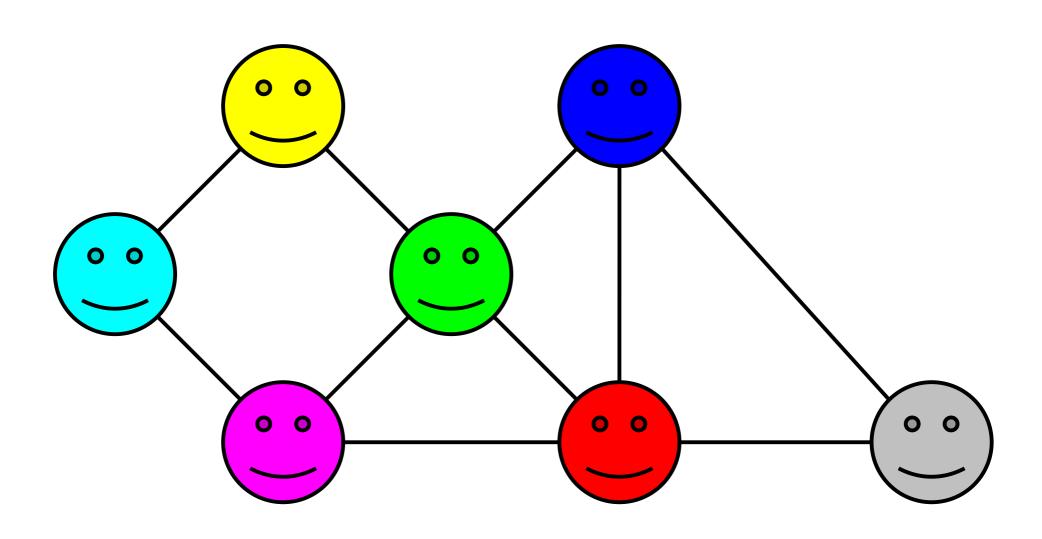
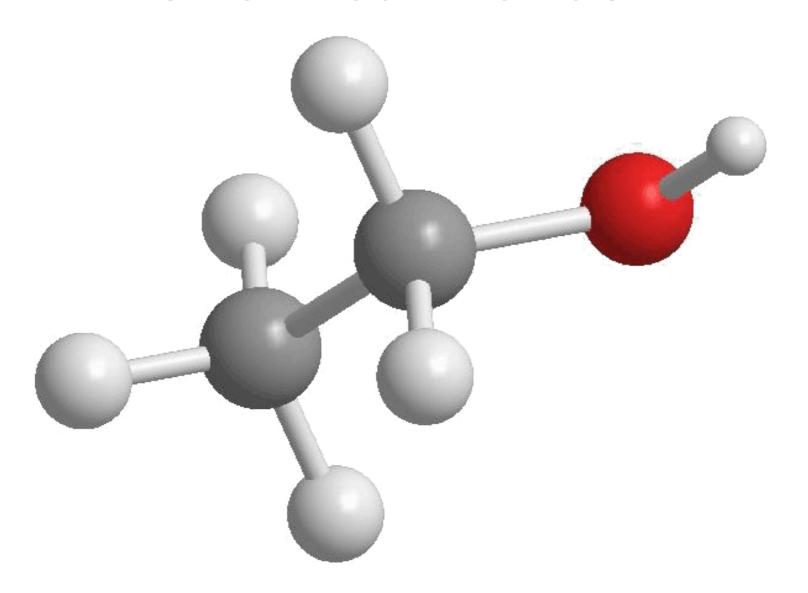
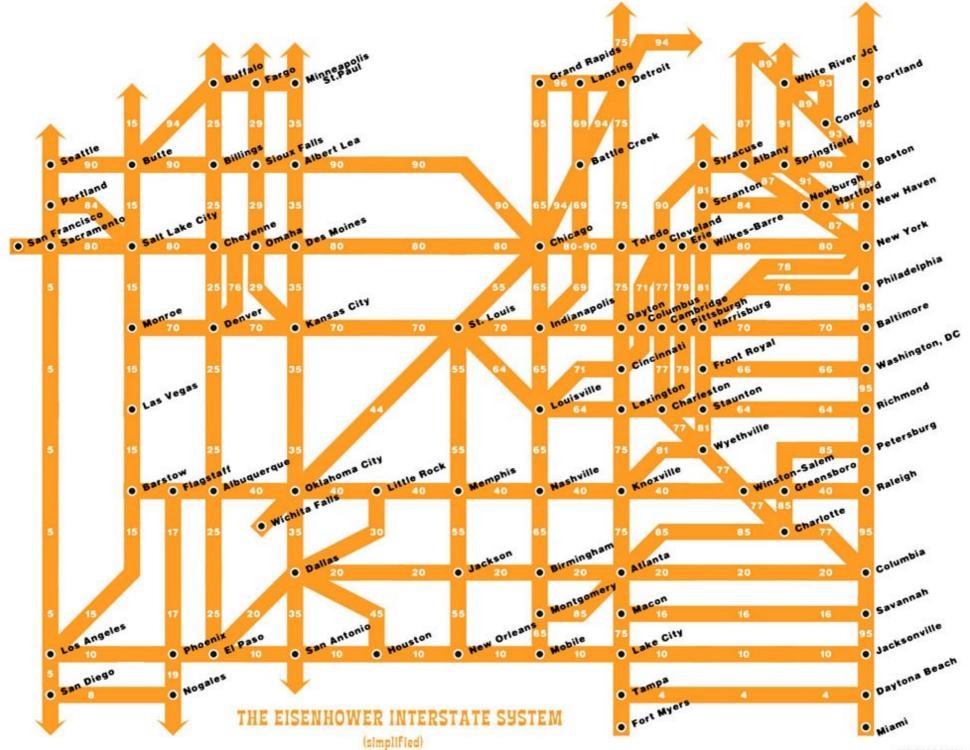
Graphs

A Social Network

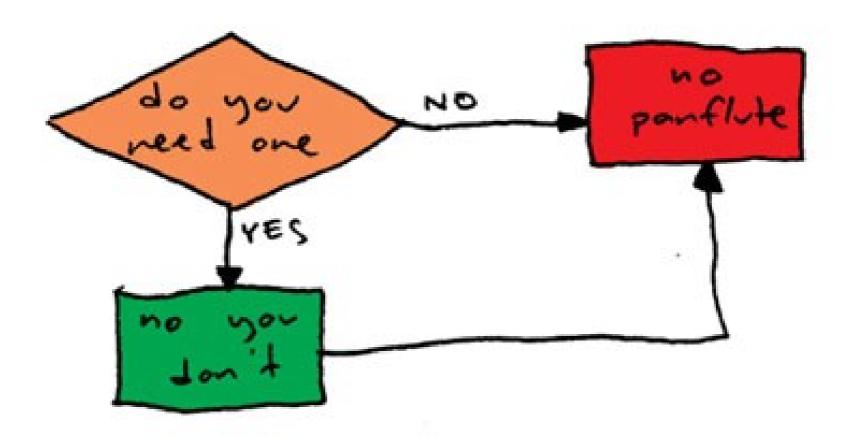


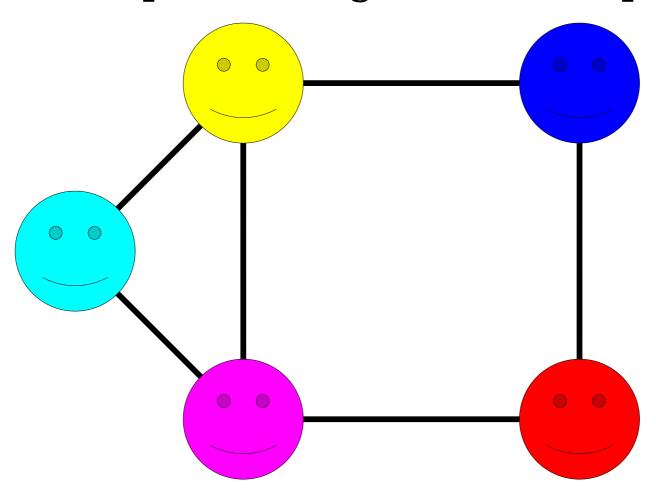
Chemical Bonds

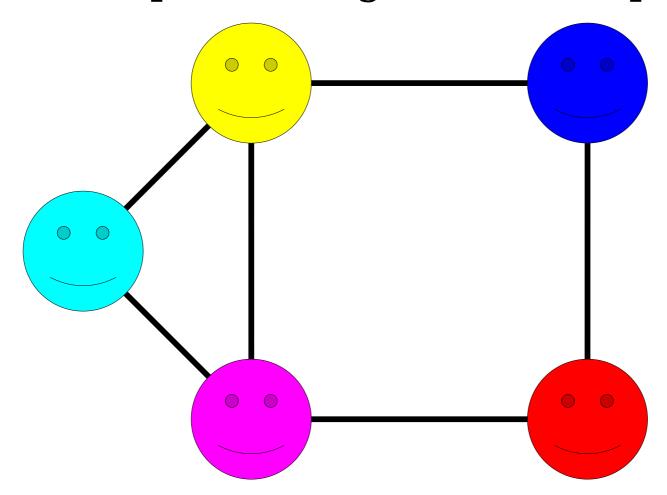




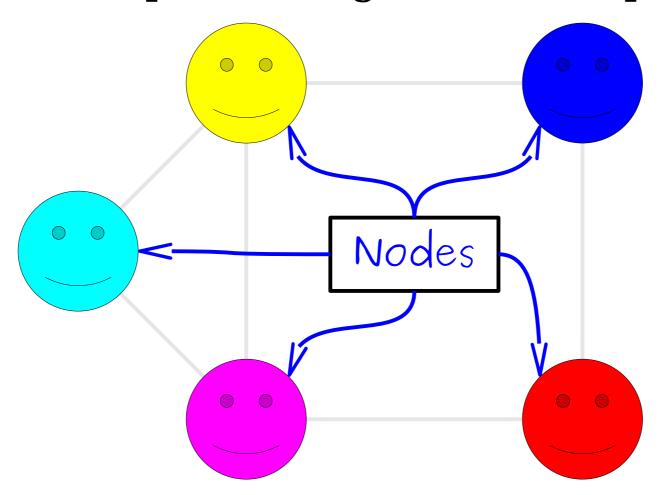
PANFLUTE FLOWCHART



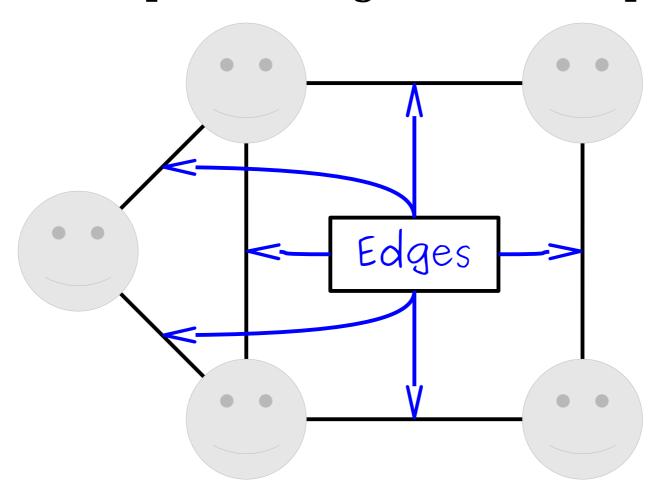




A graph consists of a set of **nodes** connected by **edges**.

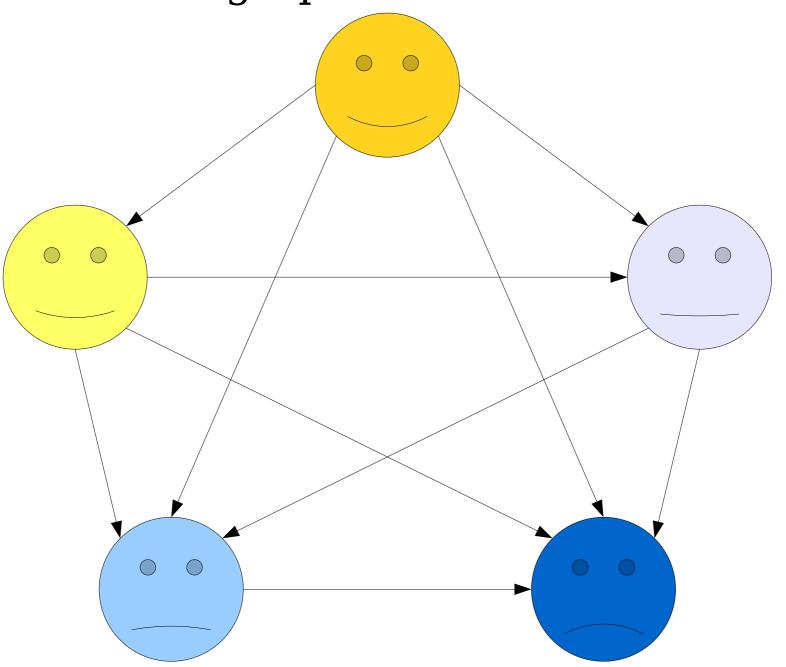


A graph consists of a set of **nodes** connected by **edges**.

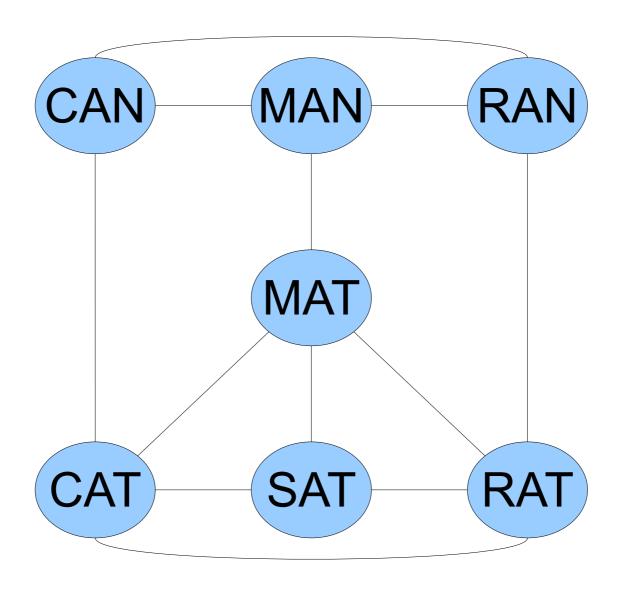


A graph consists of a set of **nodes** connected by **edges**.

Some graphs are *directed*.



Some graphs are *undirected*.



How can we represent graphs in C++?

Representing Graphs

Node

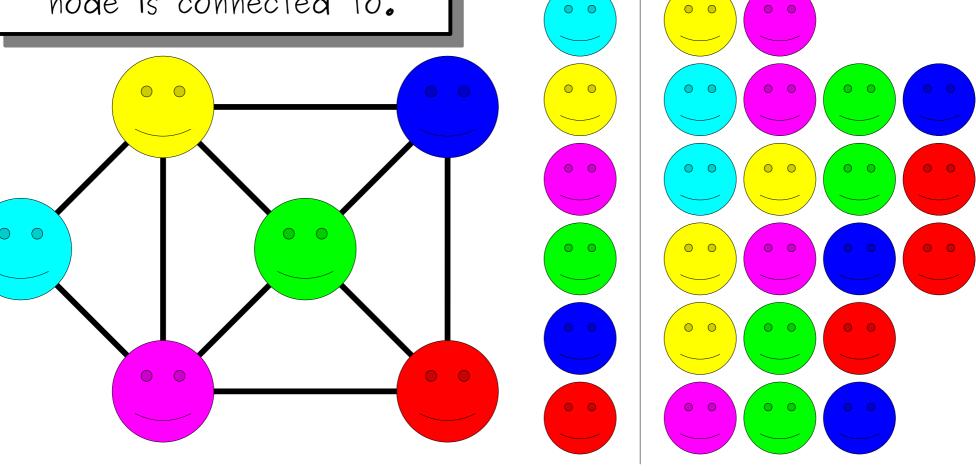
Node

Map<Node, Vector<Node>>

Vector<Node>

Adjacent To

We can represent a graph as a map from nodes to the list of nodes each node is connected to.



Representing Graphs

 The approach we just saw is called an adjacency list in comes in a number of different forms:

Map<string, Vector<string>>
 Map<string, Set<string>>
 Vector<Vector<int>>

 The core idea is that we have some kind of mapping associating each node with its outgoing edges.

Representing Graphs

The approach we just saw is called an *adjacency list* in comes in a number of different forms:

Map<string, Vector<string>>

Map<string, Set<string>>

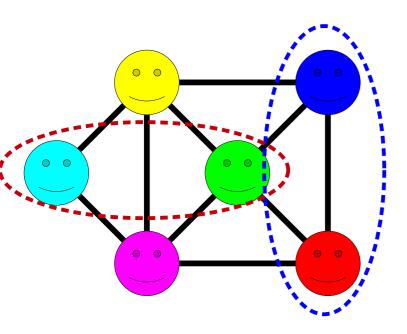
Vector/Vector/int>>

The core idea of mapping a its outgoing e

Where have you seen this before?

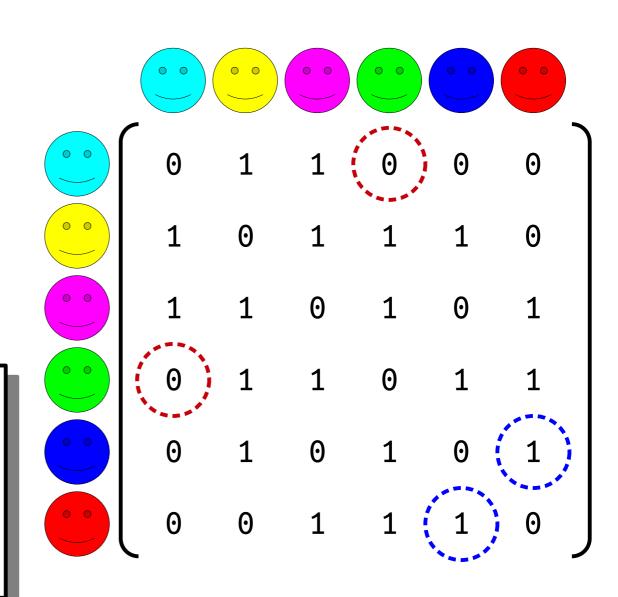
Answer at https://pollev.com/cs106bwin23

Other Graph Representations

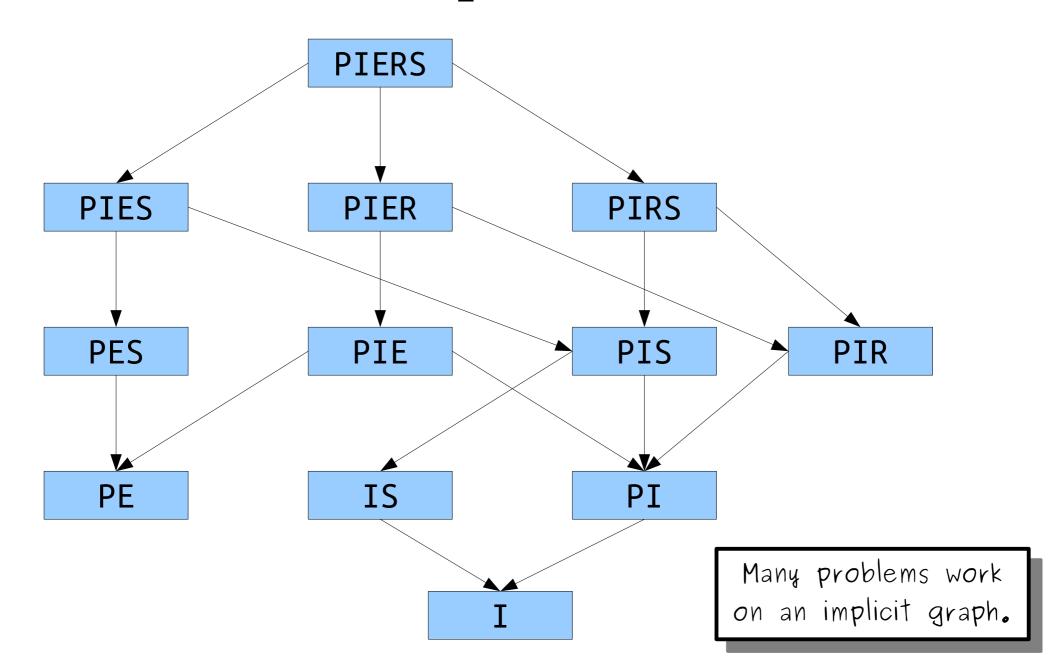


This representation is called an adjacency matrix.

For those of you in Math 51: if A is an adjacency matrix for a graph G, what is the significance of the matrix A²?



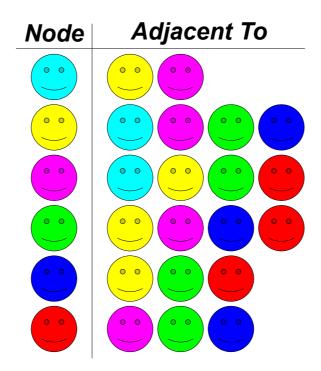
Other Representations



You'll find graphs just about everywhere you look.

They're an *extremely* versatile and powerful abstraction to be aware of.

Going forward, unless stated otherwise, assume we're using an *adjacency list*.

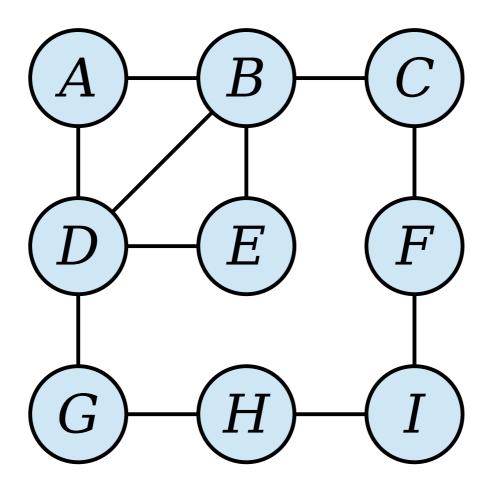


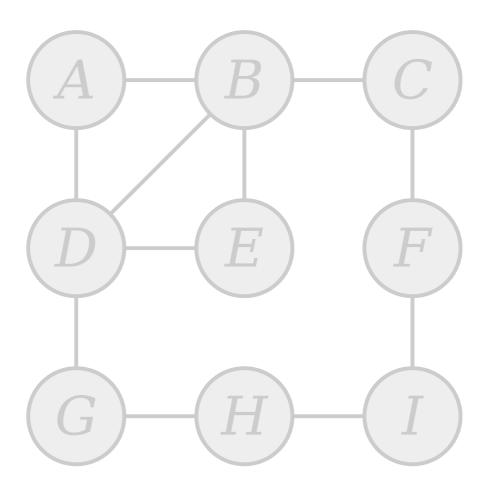
Traversing Graphs

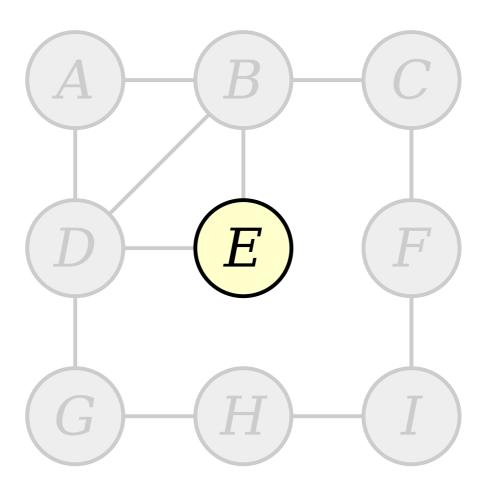
Iterating over a Graph

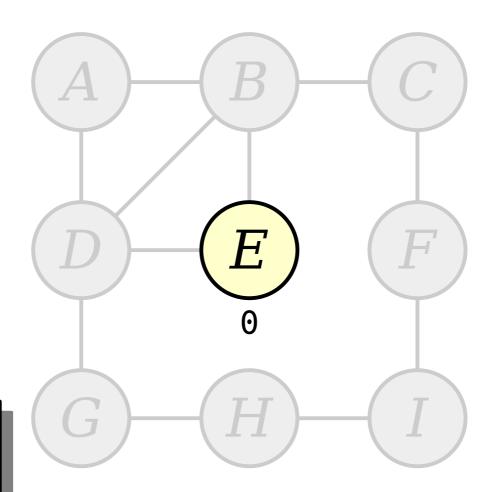
- In a singly-linked list, there's pretty much one way to iterate over the list: start at the front and go forward!
- In a binary search tree, there are many traversal strategies:
 - An *inorder traversal* that produces all the elements in sorted order.
 - A **postorder traversal** used to delete all the nodes in the BST.
- There are *many* ways to iterate over a graph, each of which have different properties.

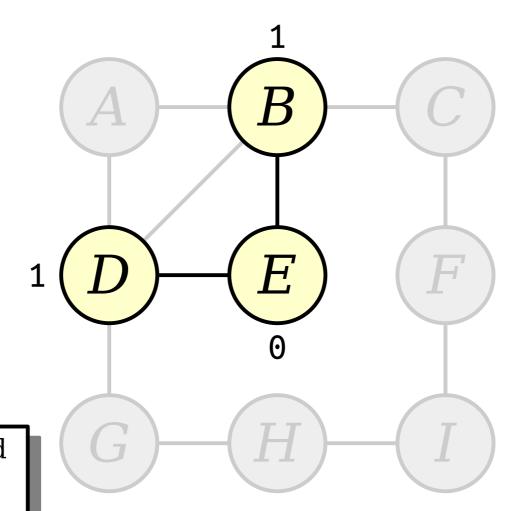
One Search Strategy

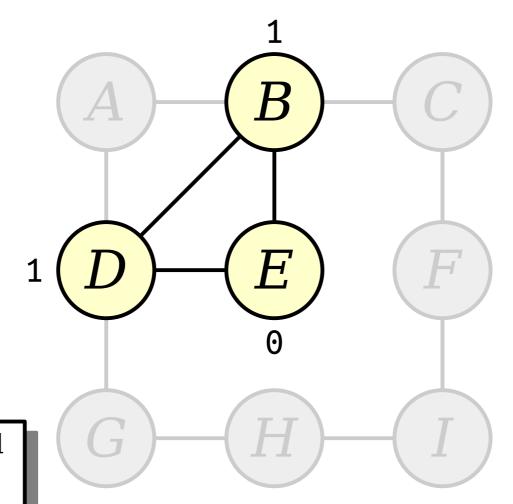


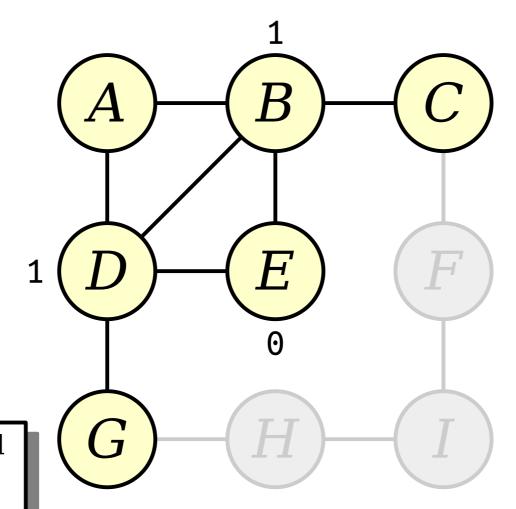


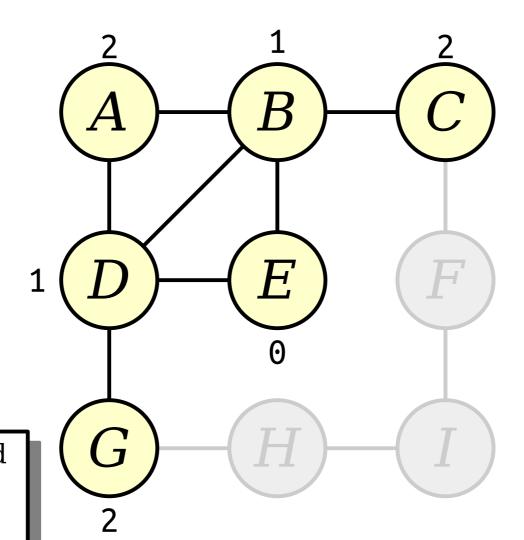


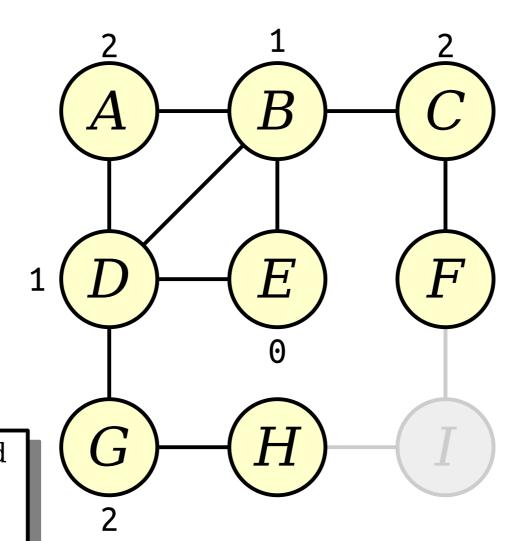


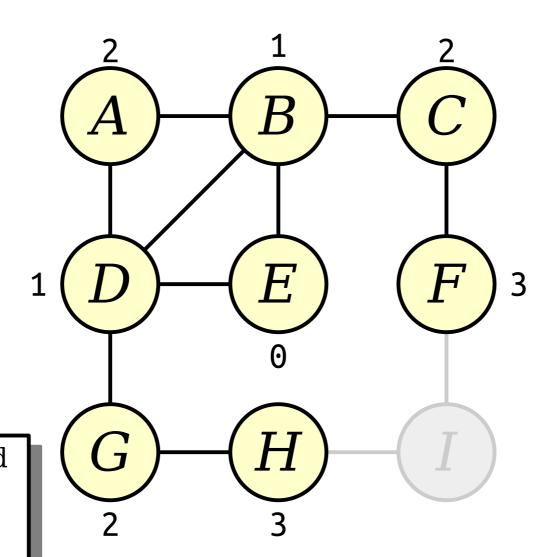


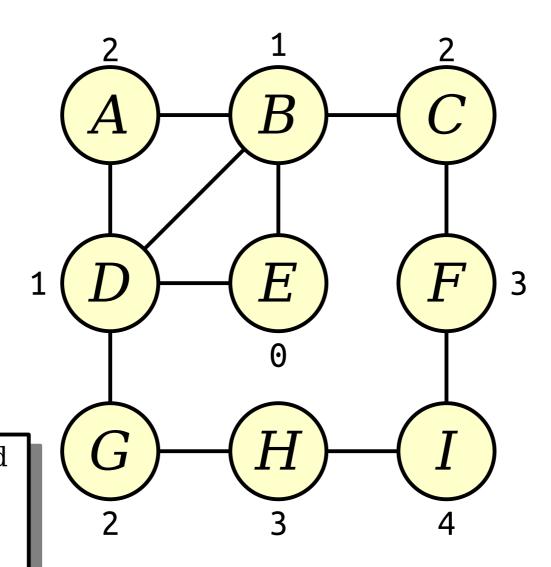




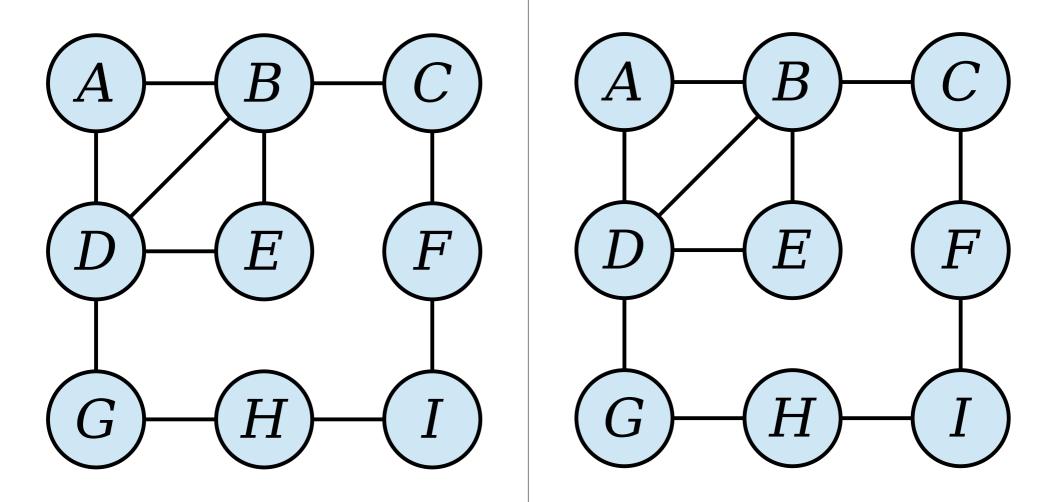


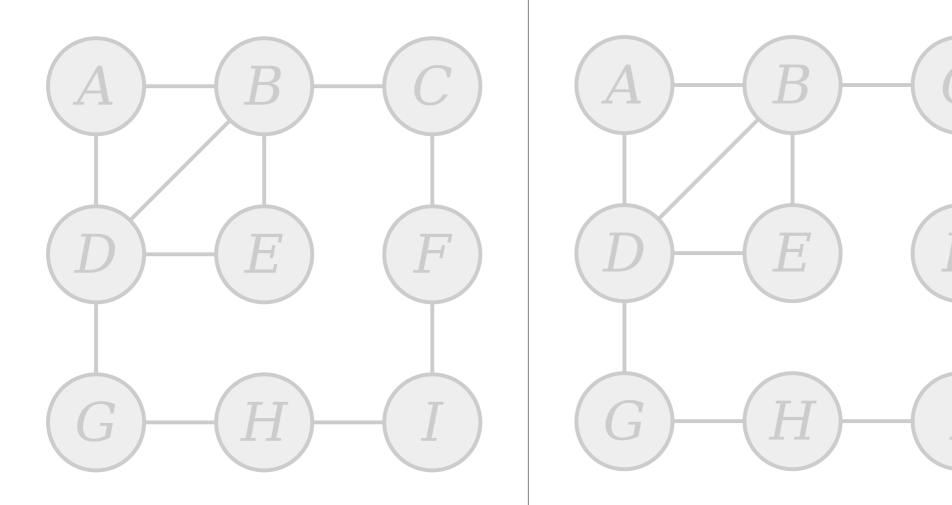


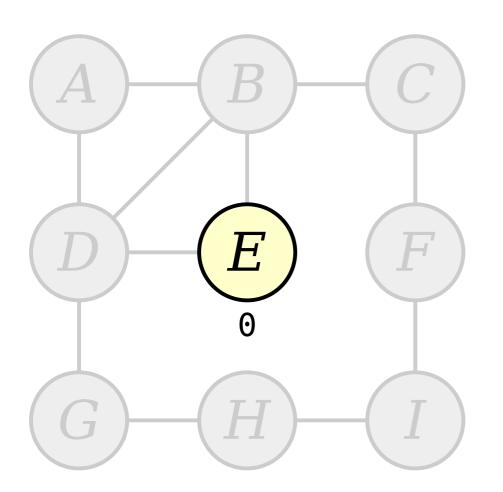


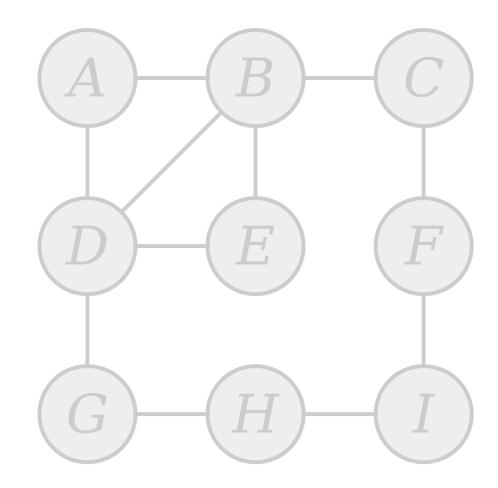


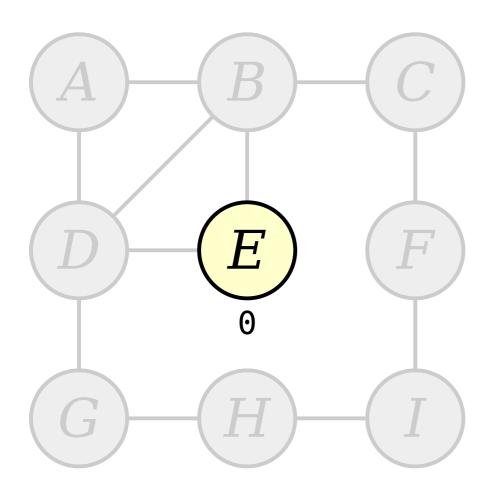
Implementing this Idea

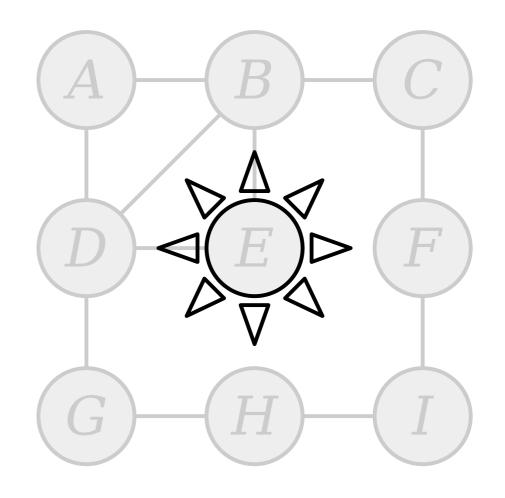






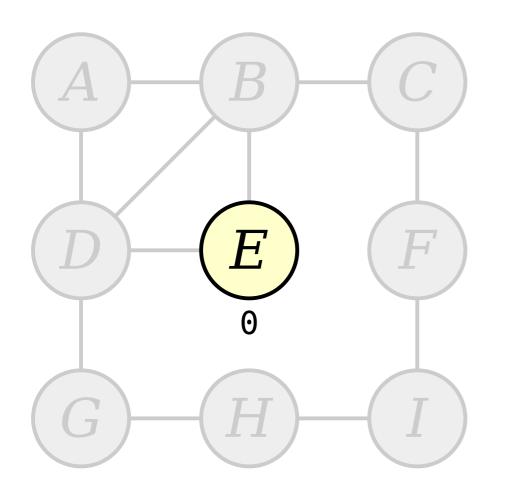


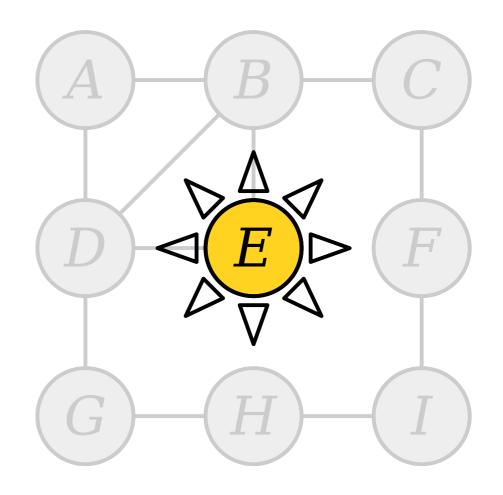




Queue: E

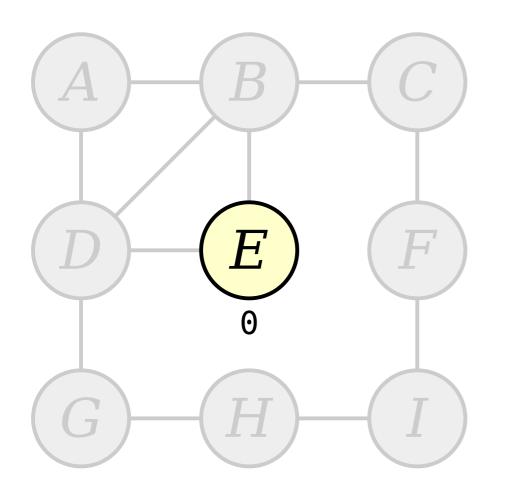
Visit nodes in ascending order of distance from the start node E.

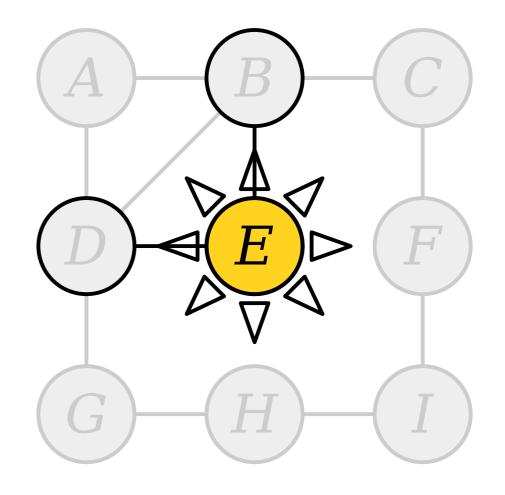




Queue:

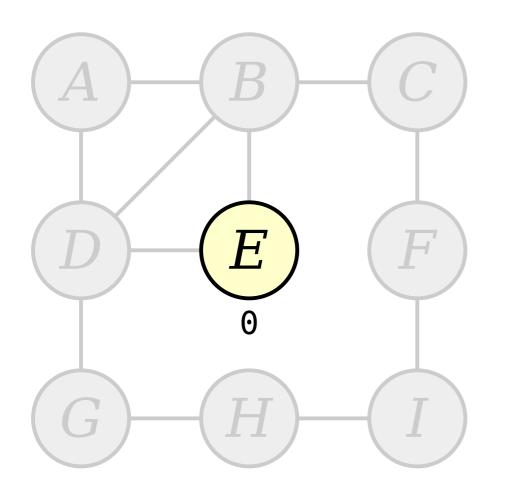
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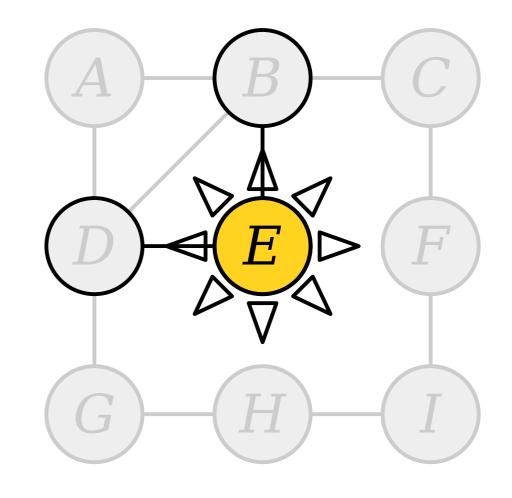




Queue:

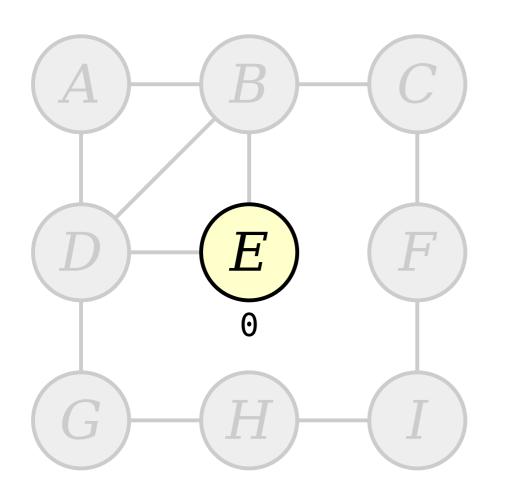
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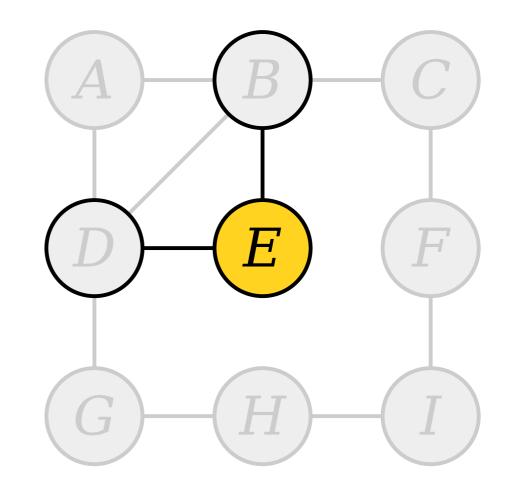




Queue: $\bigcirc B$

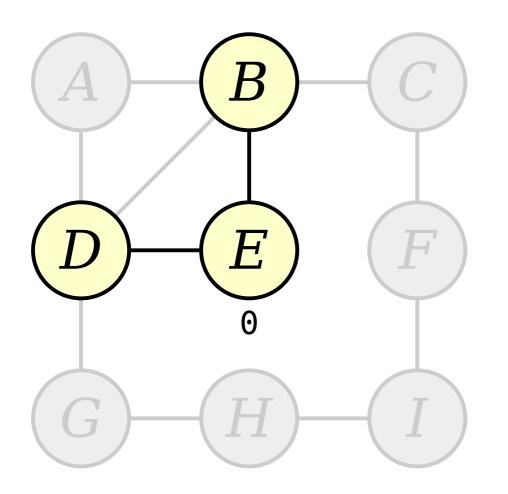
Visit nodes in ascending order of distance from the start node E.

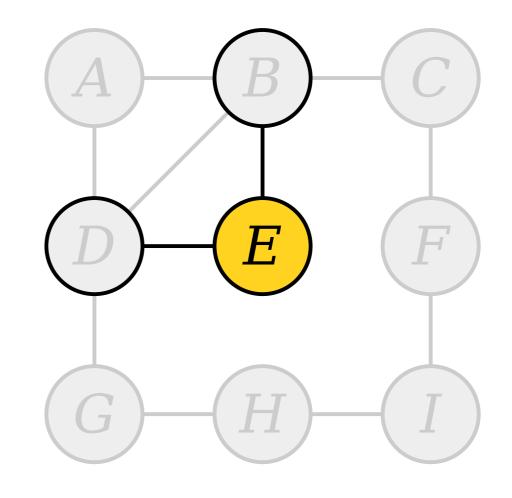




Load newly-discovered nodes into a queue.

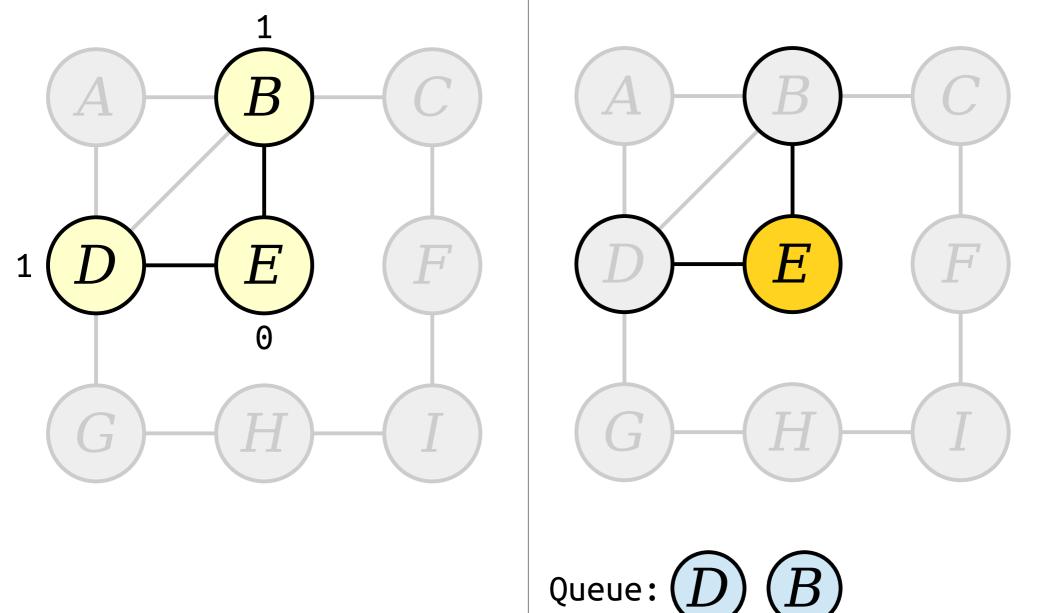
Queue:

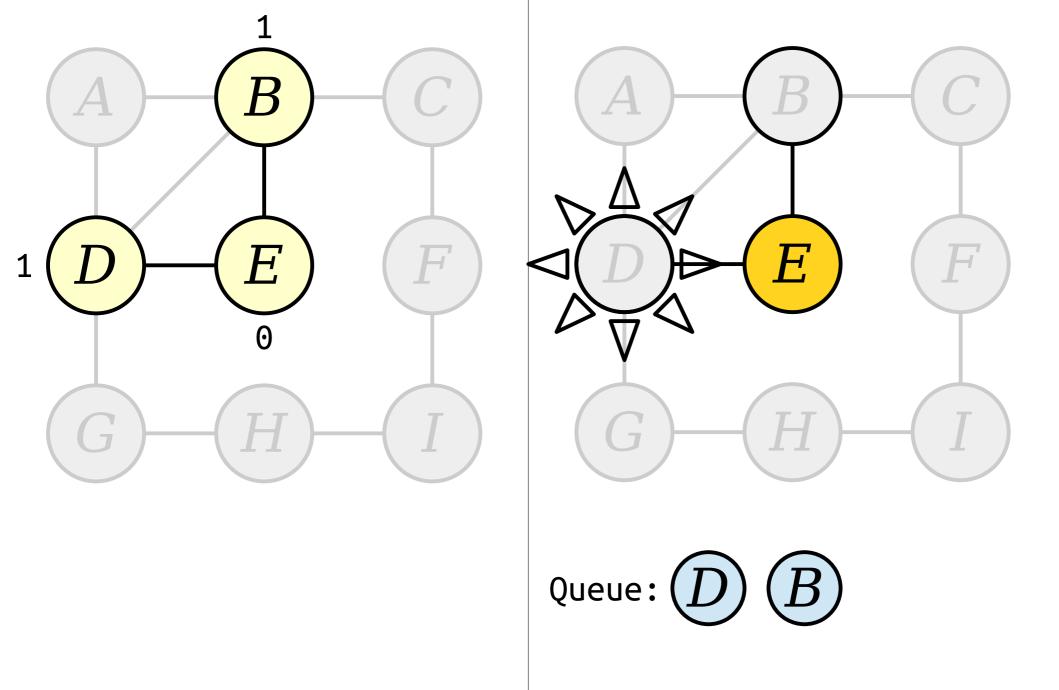


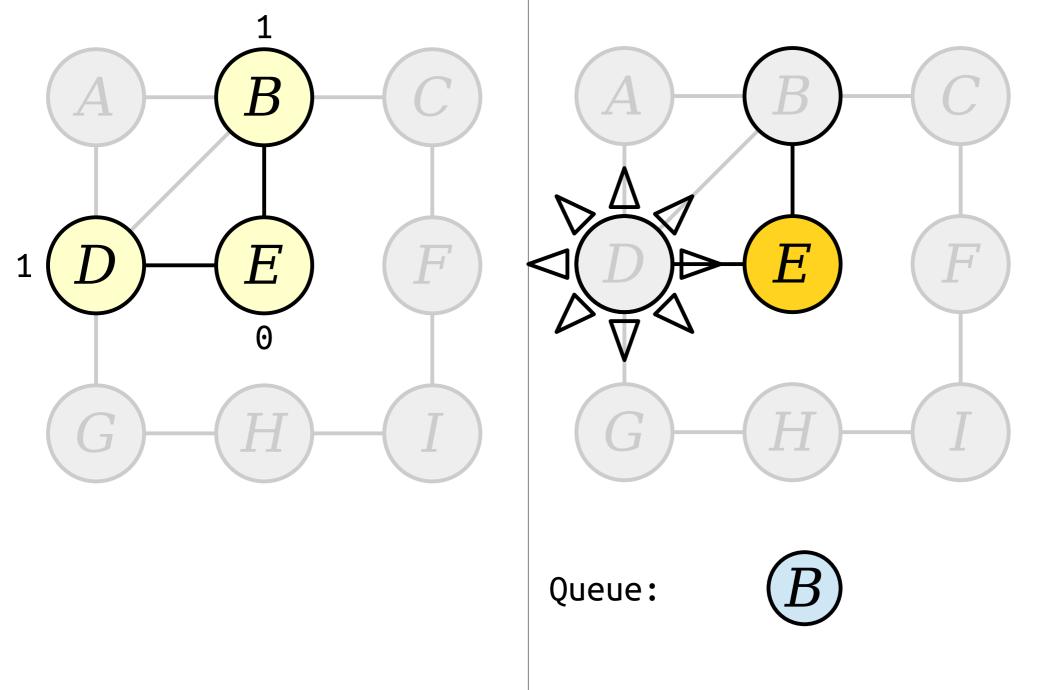


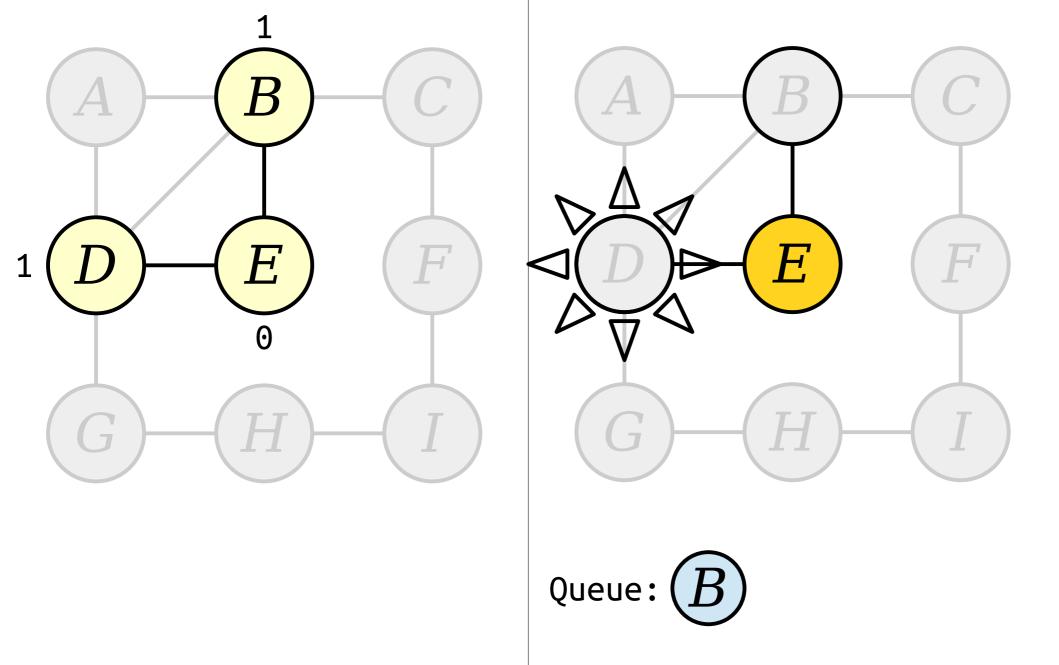
Queue: $\bigcirc B$

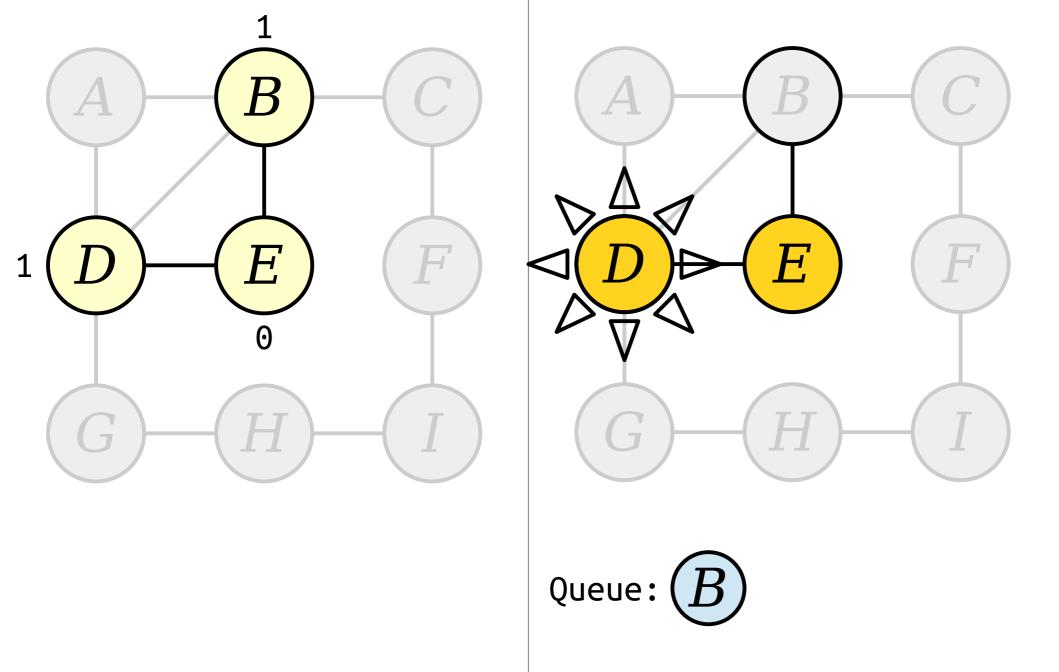
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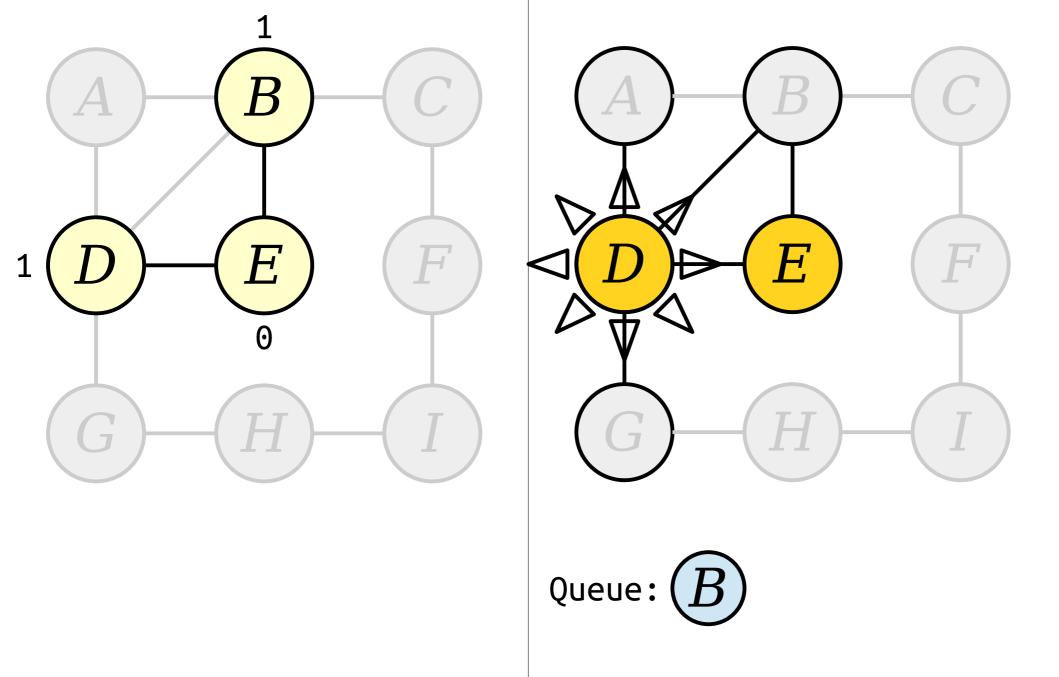


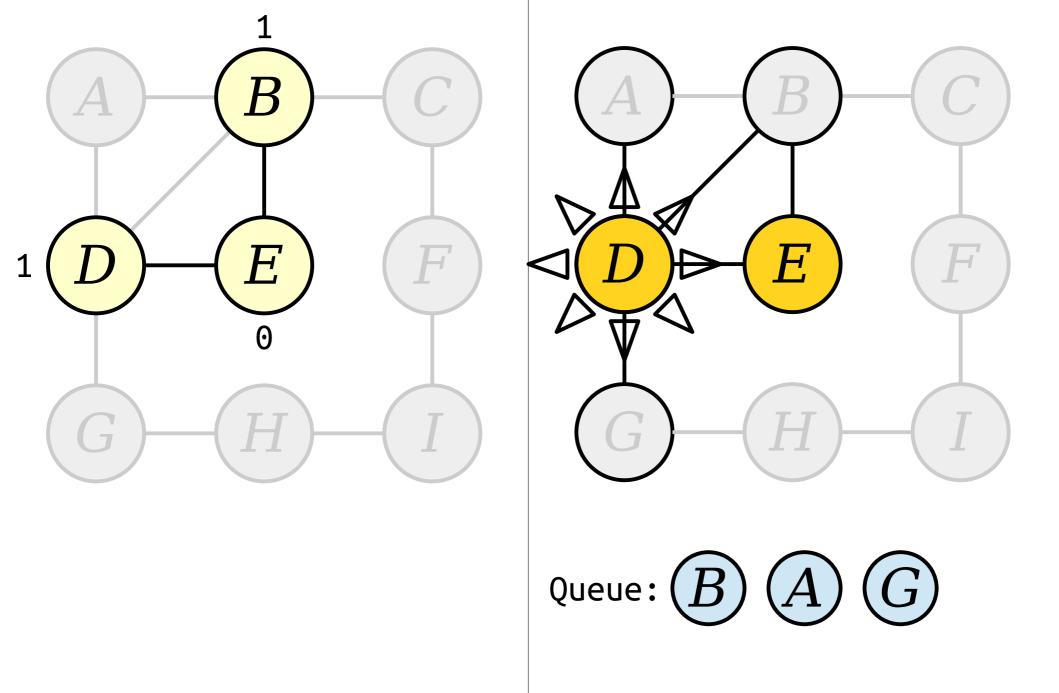


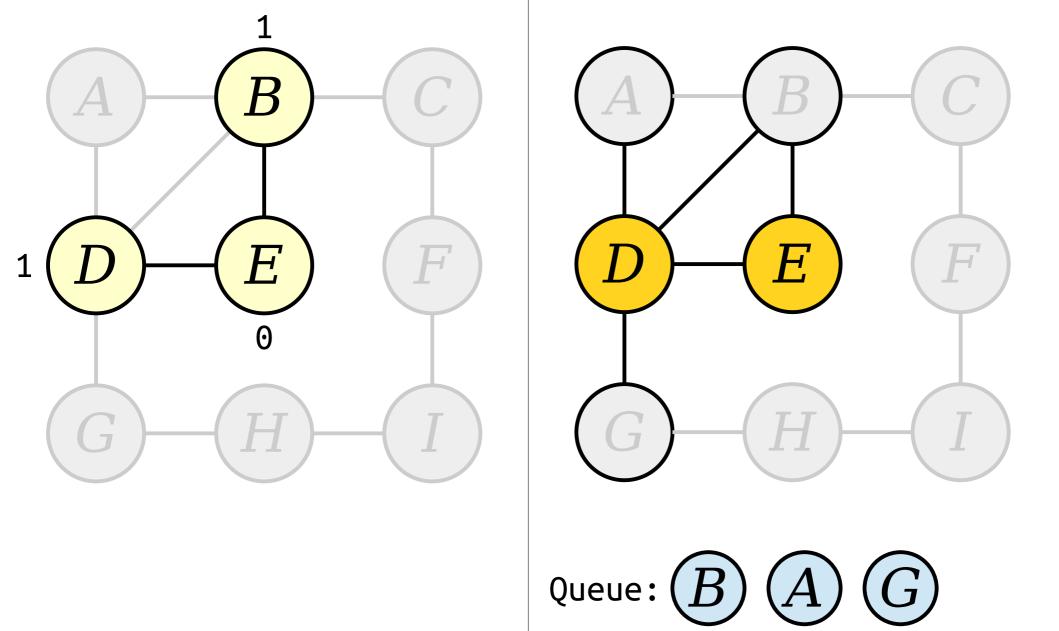


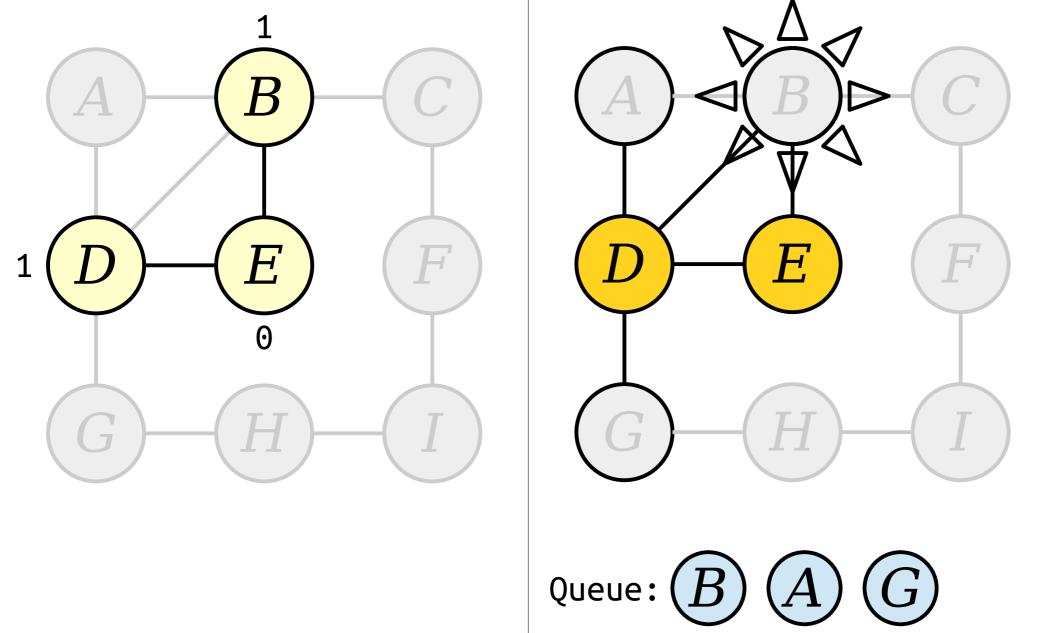


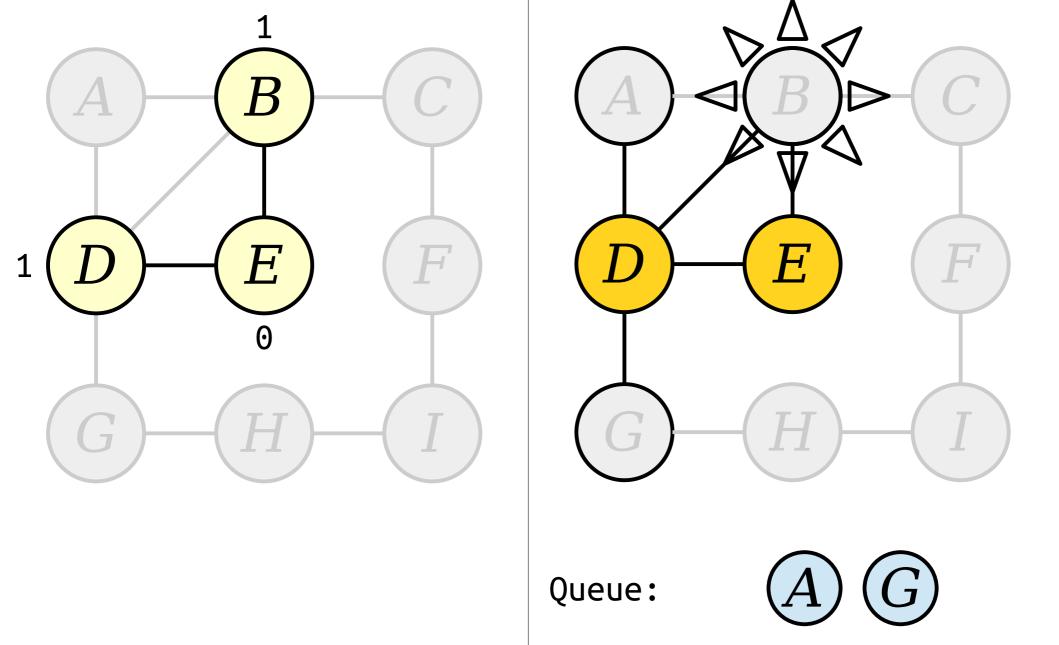


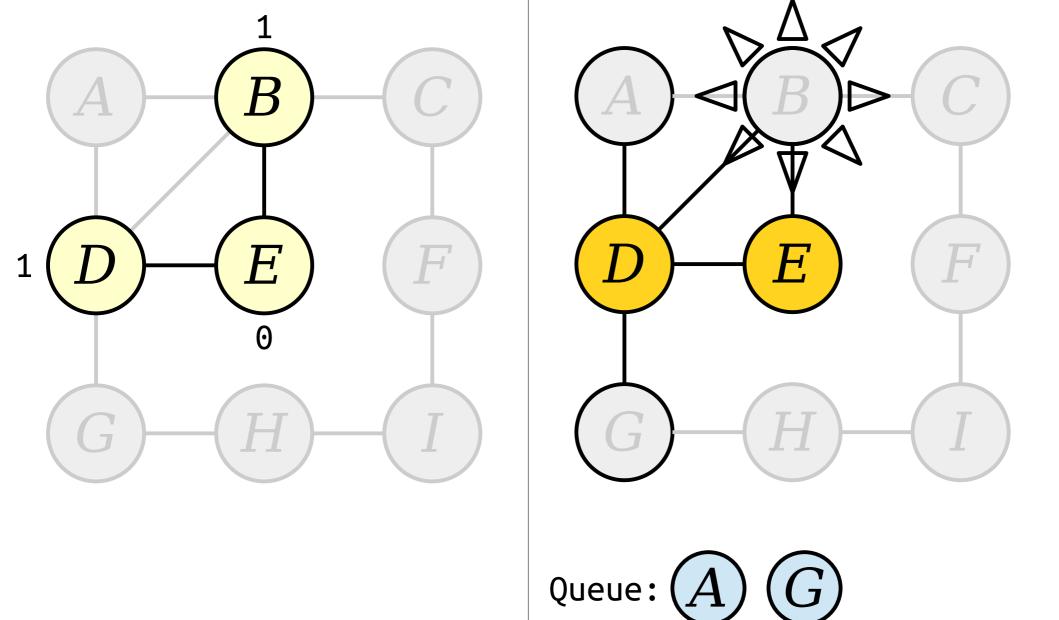


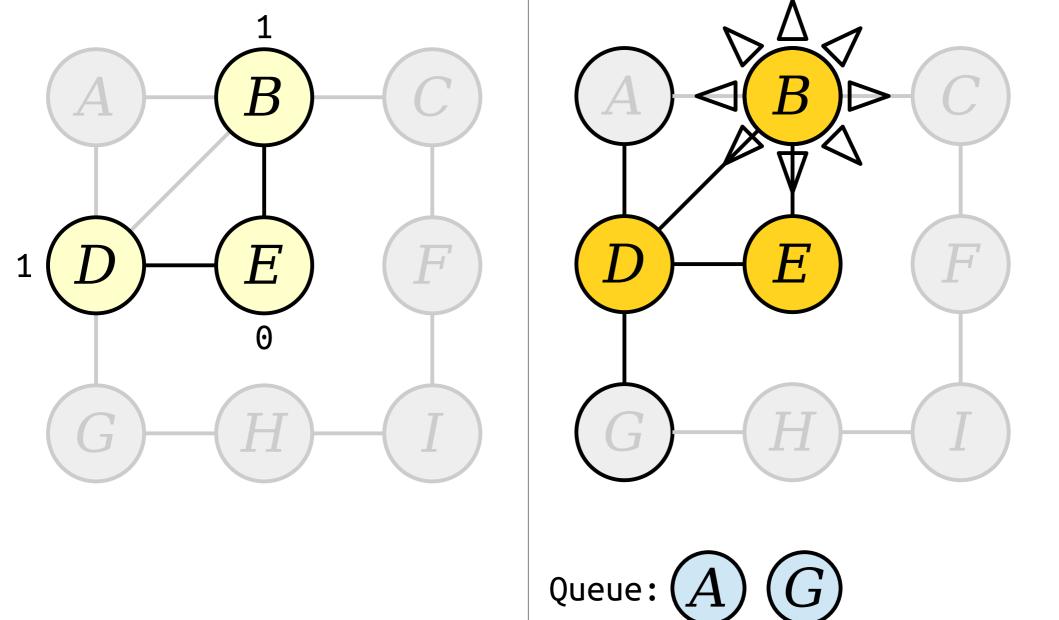


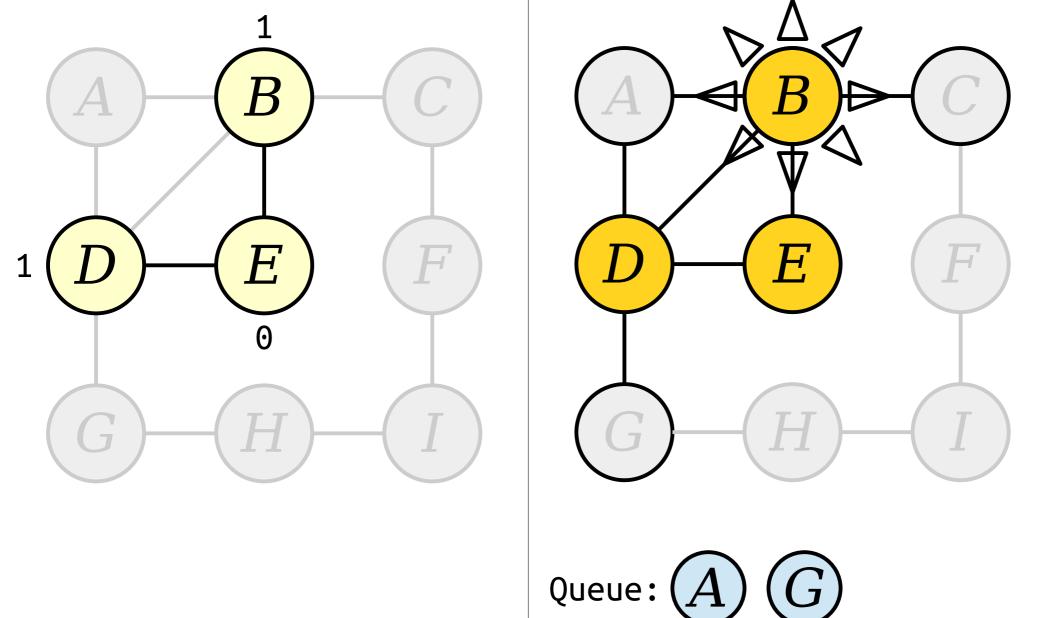


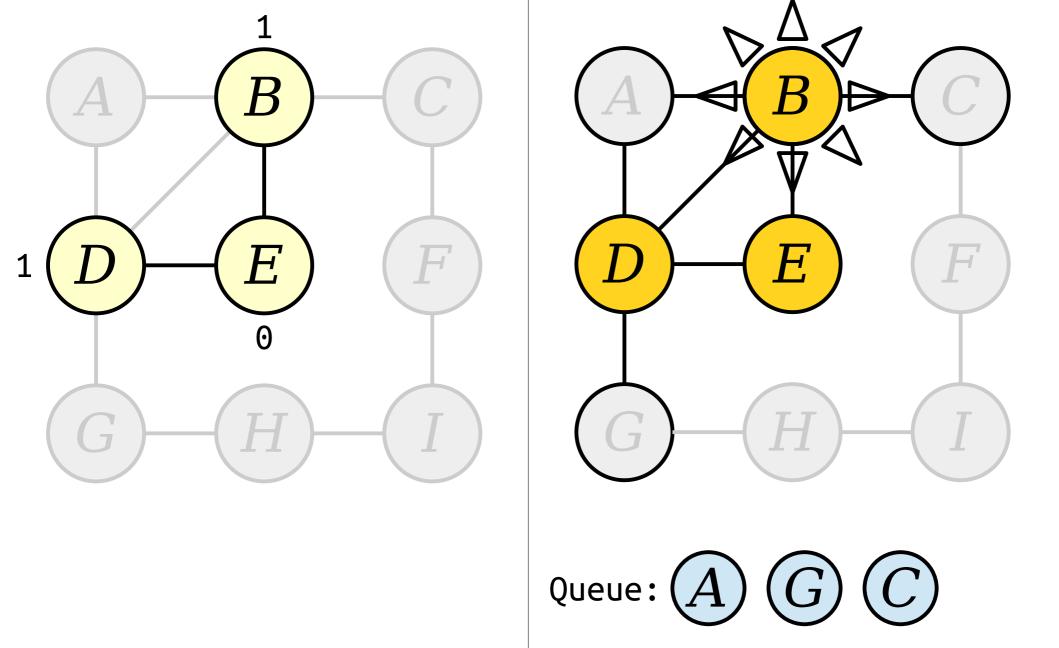


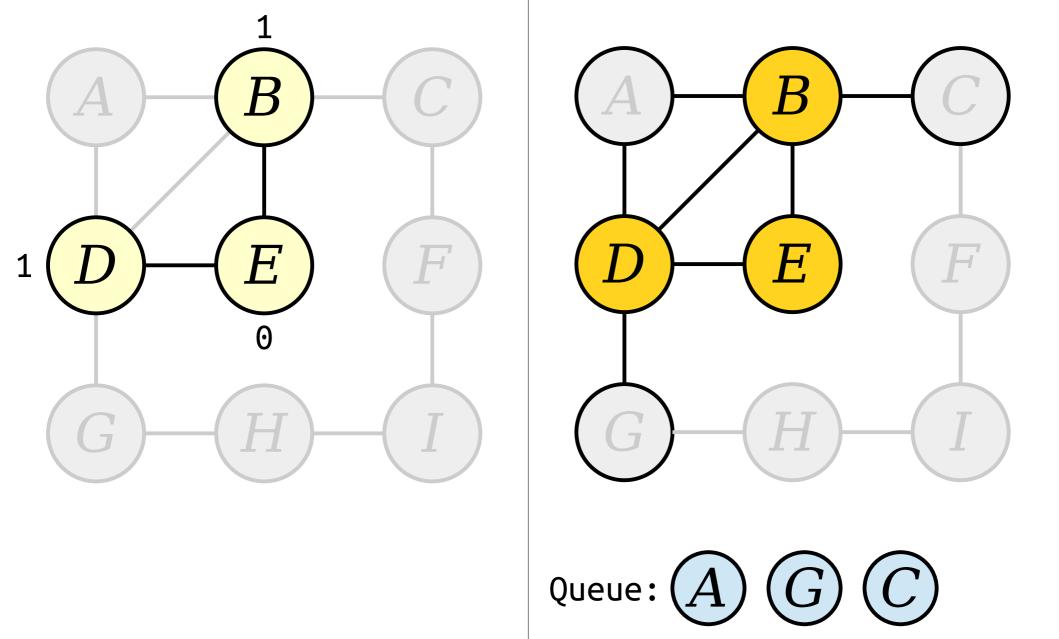


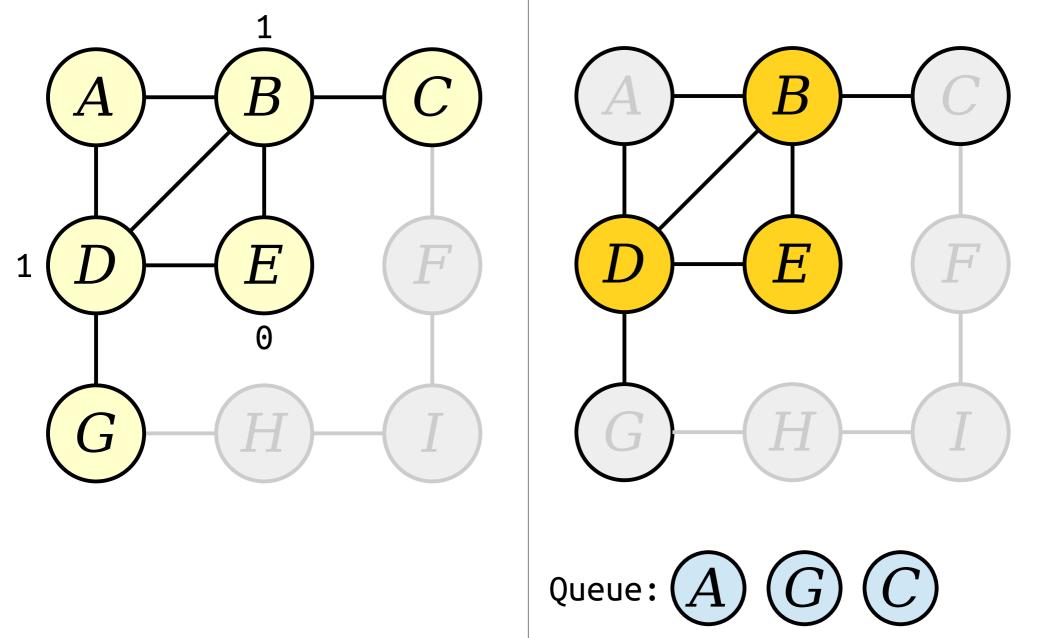


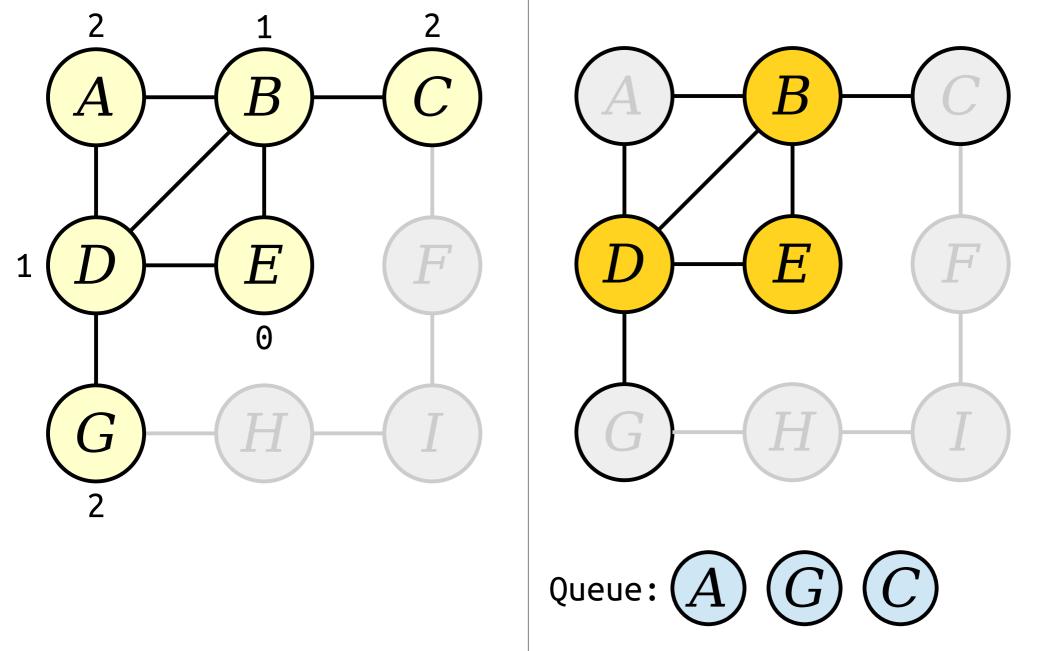


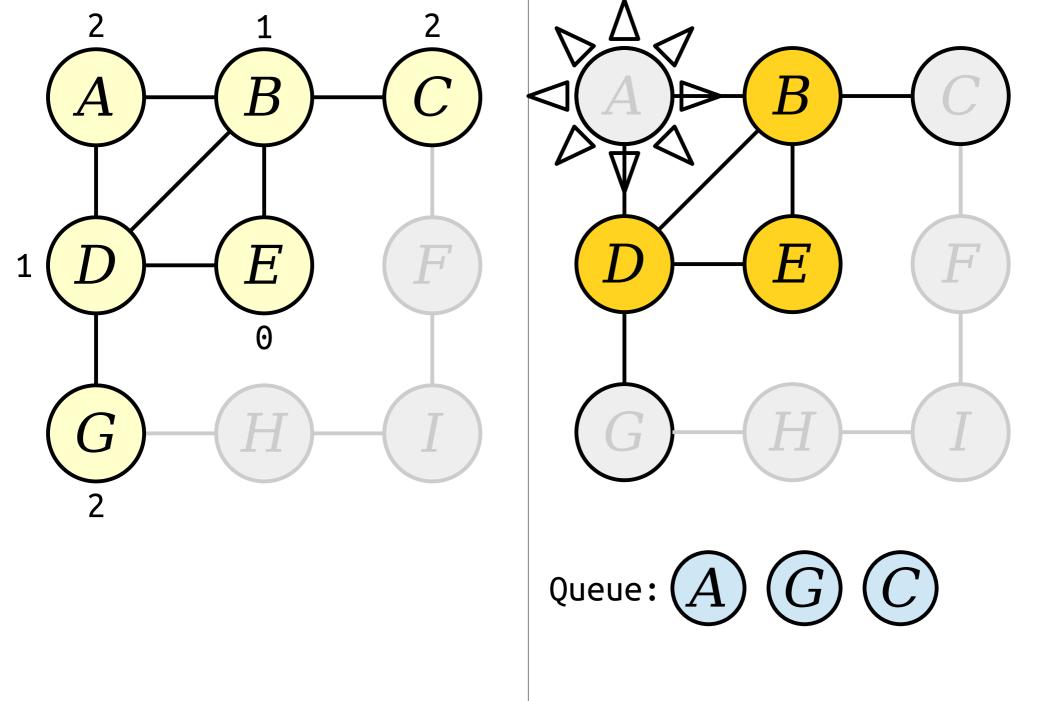


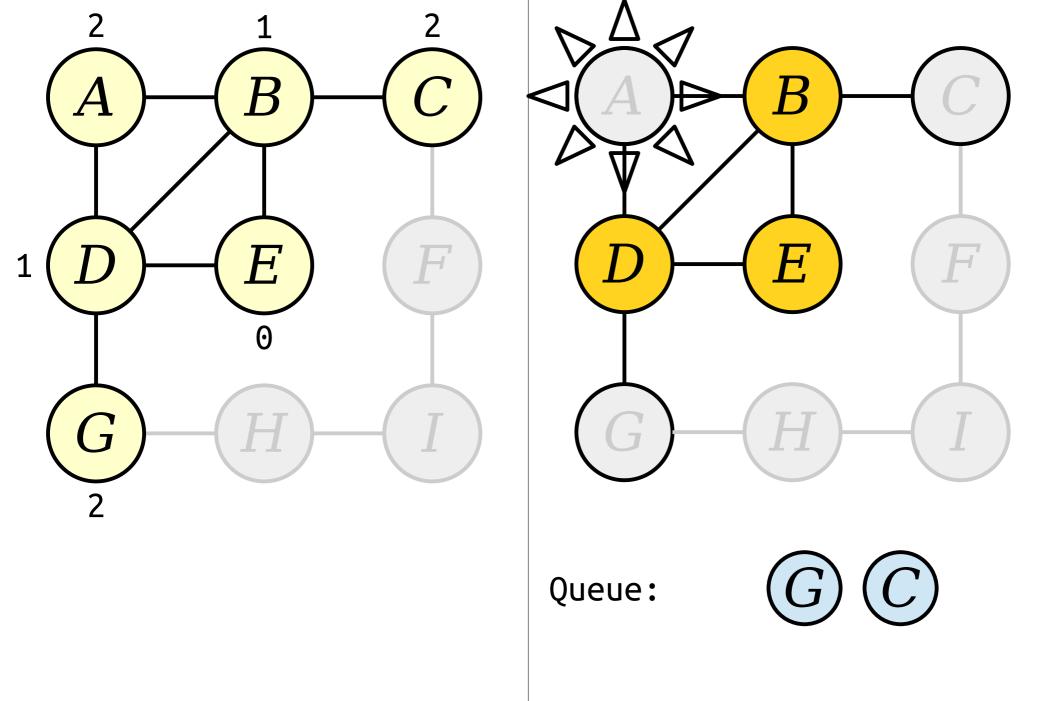


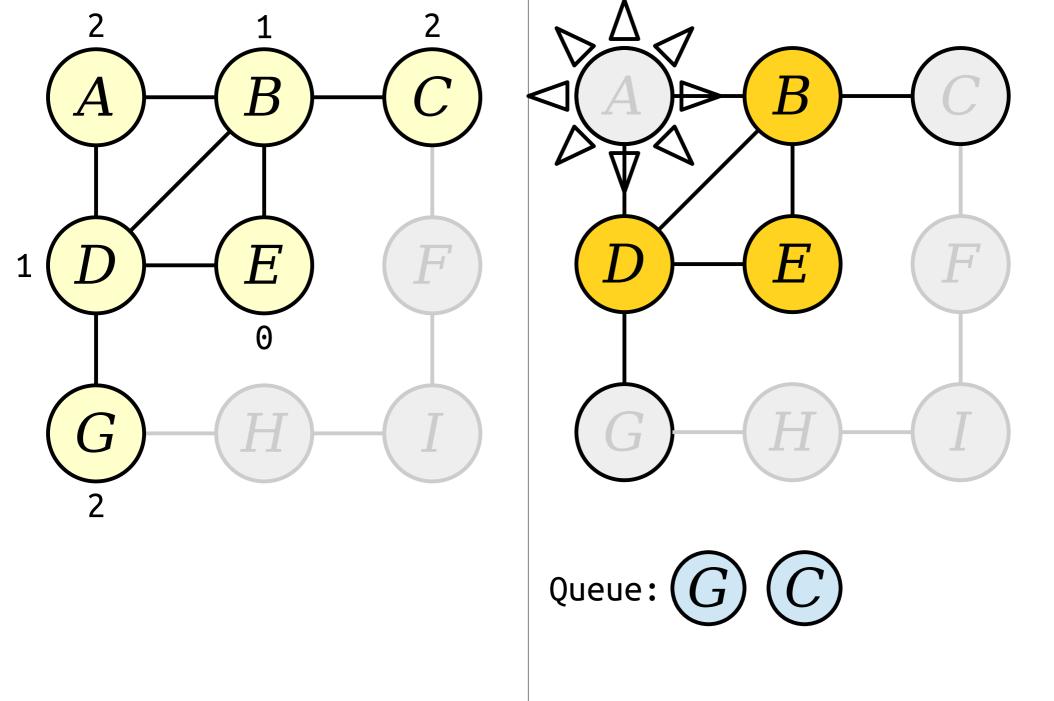


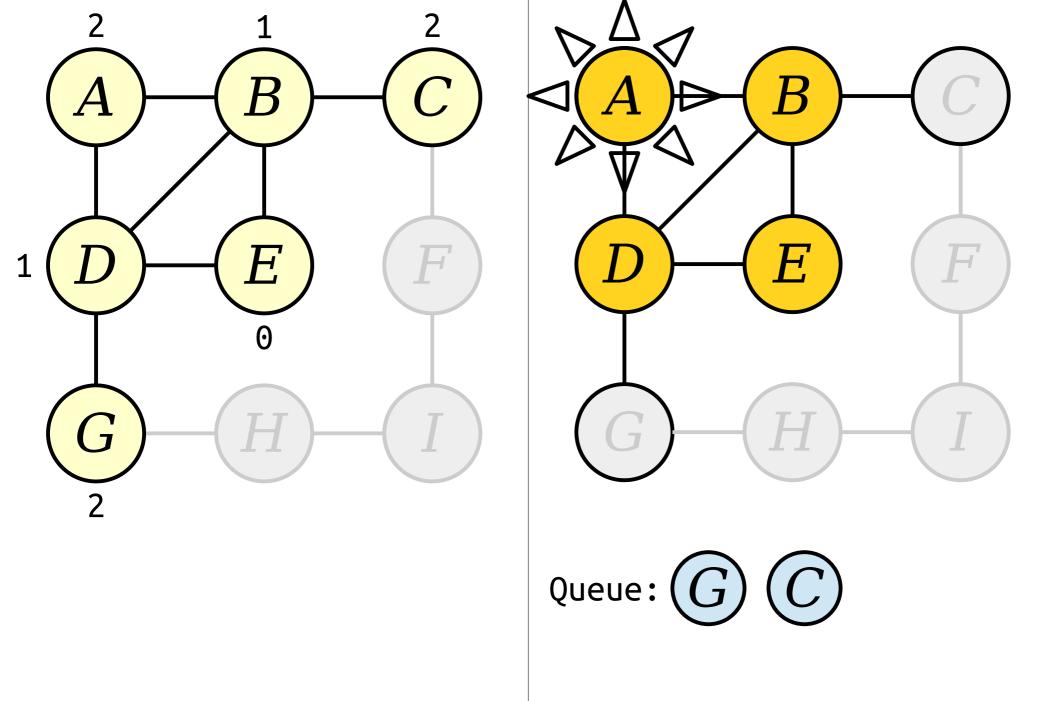


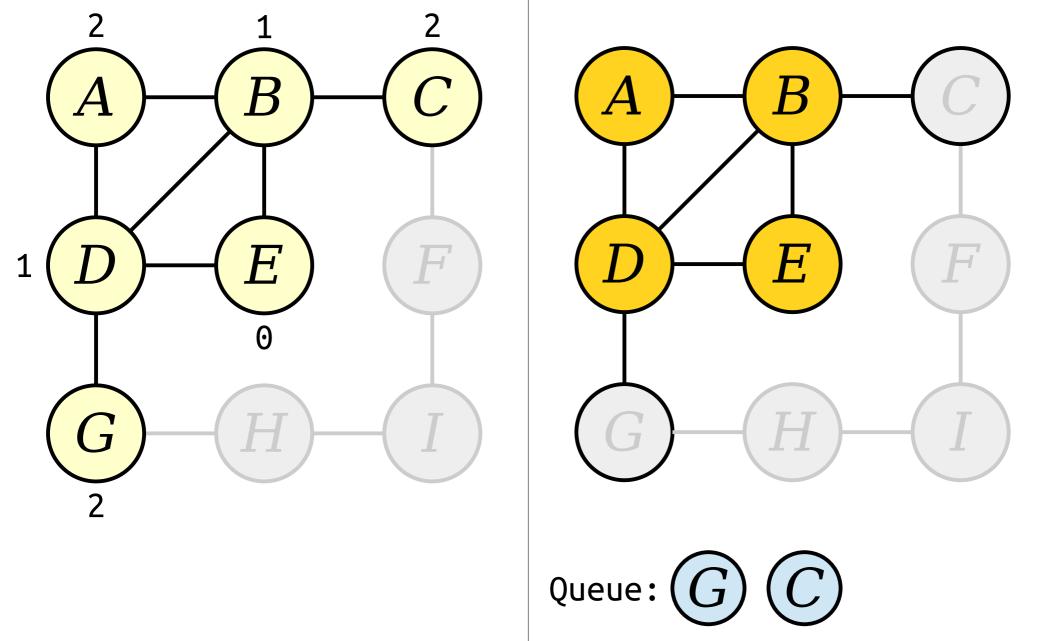


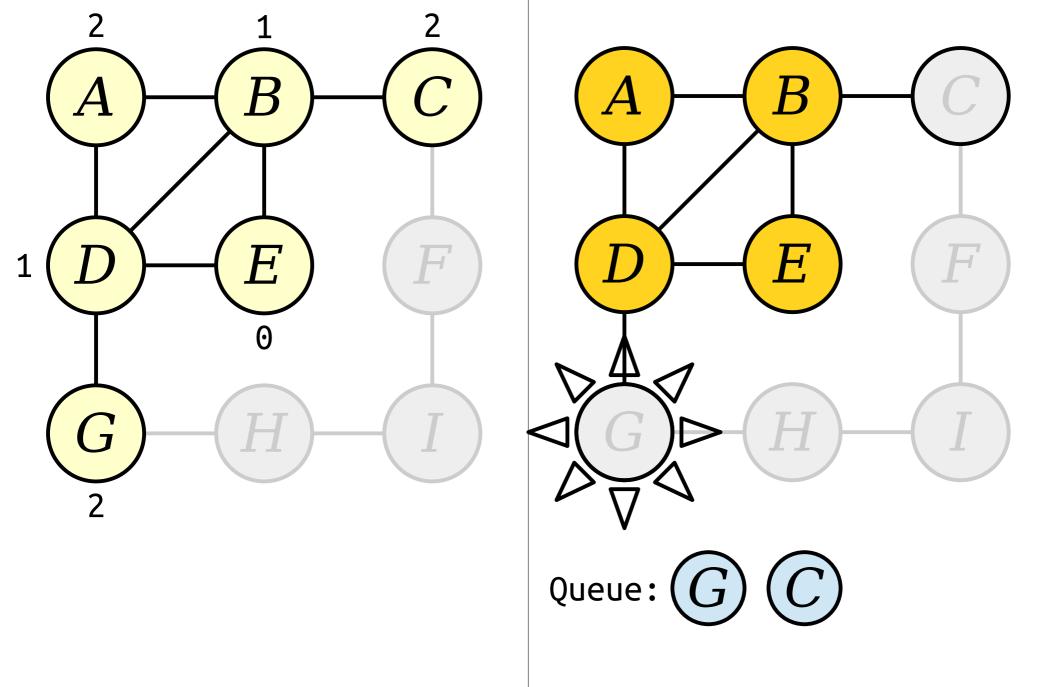


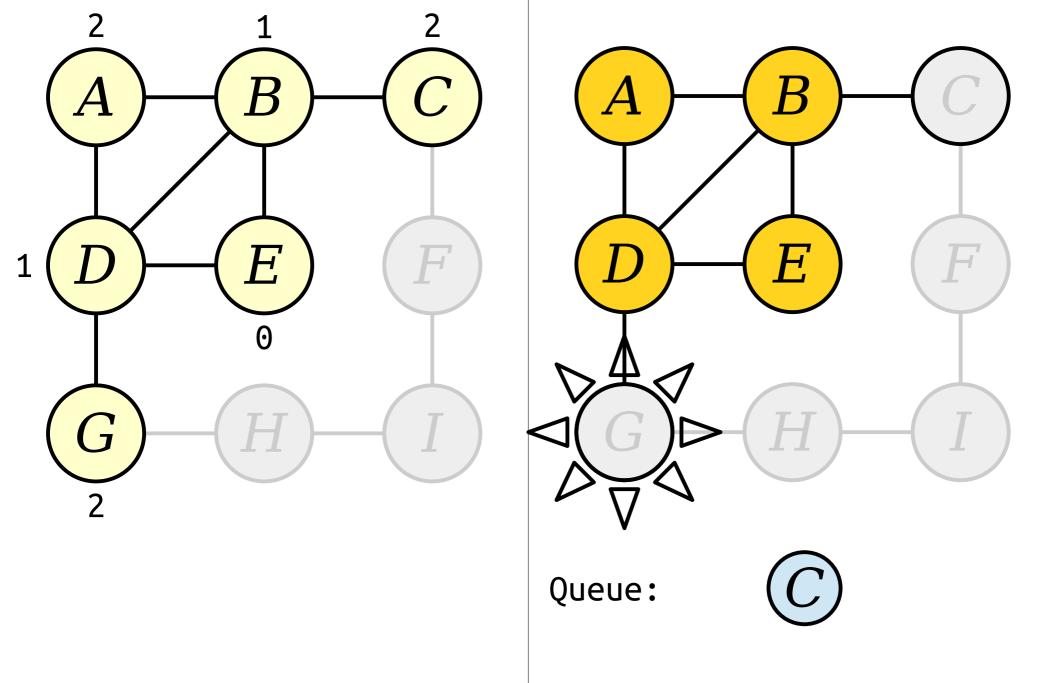


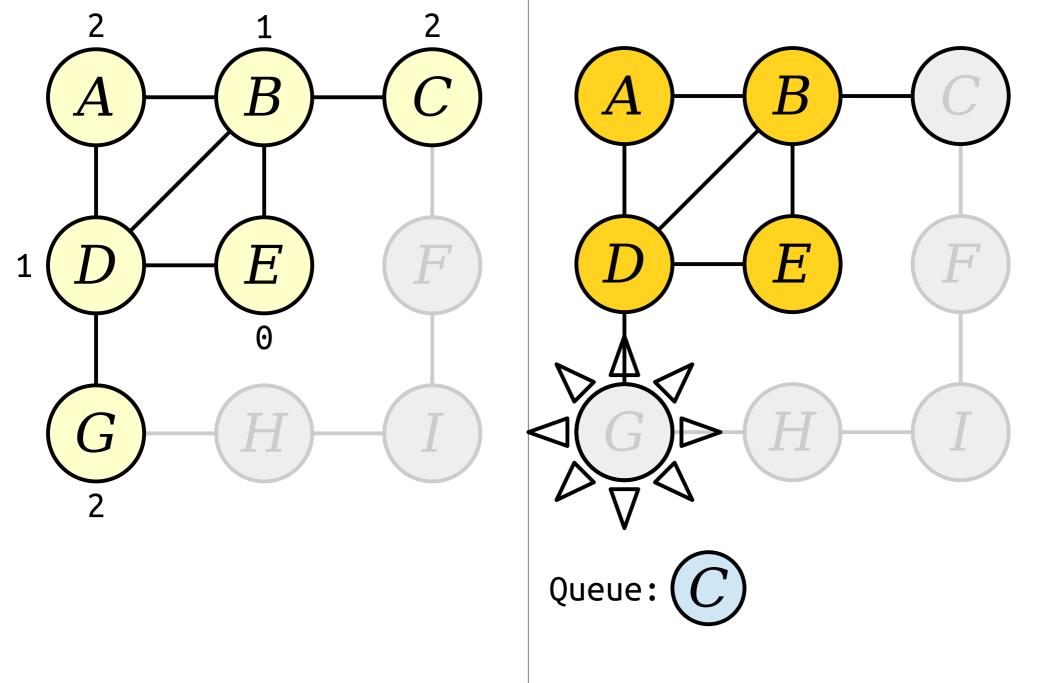


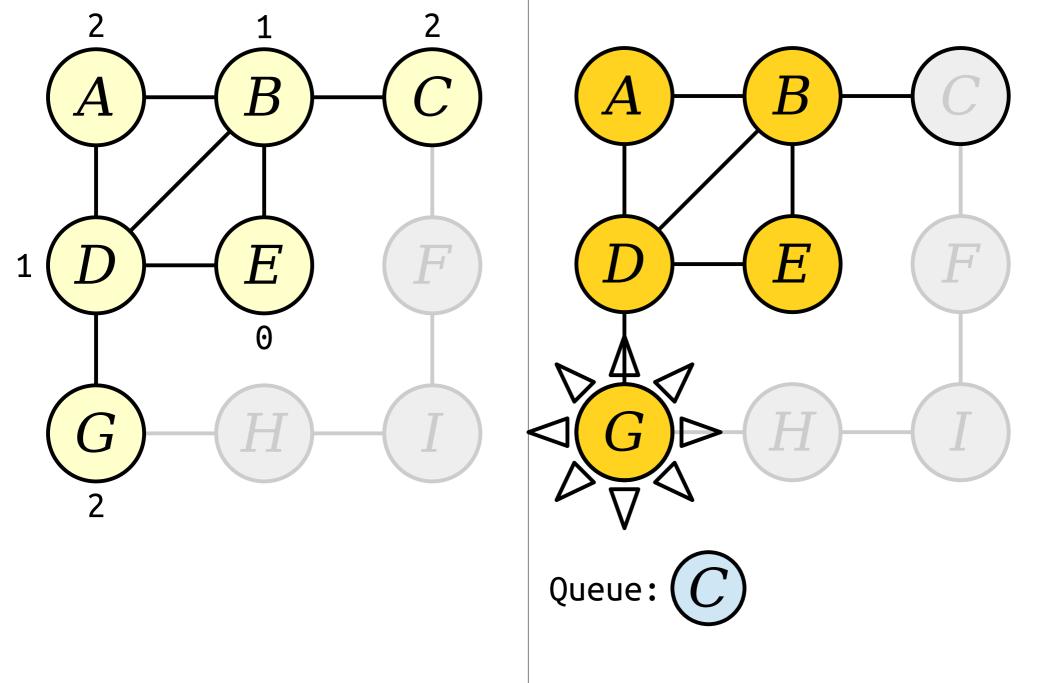


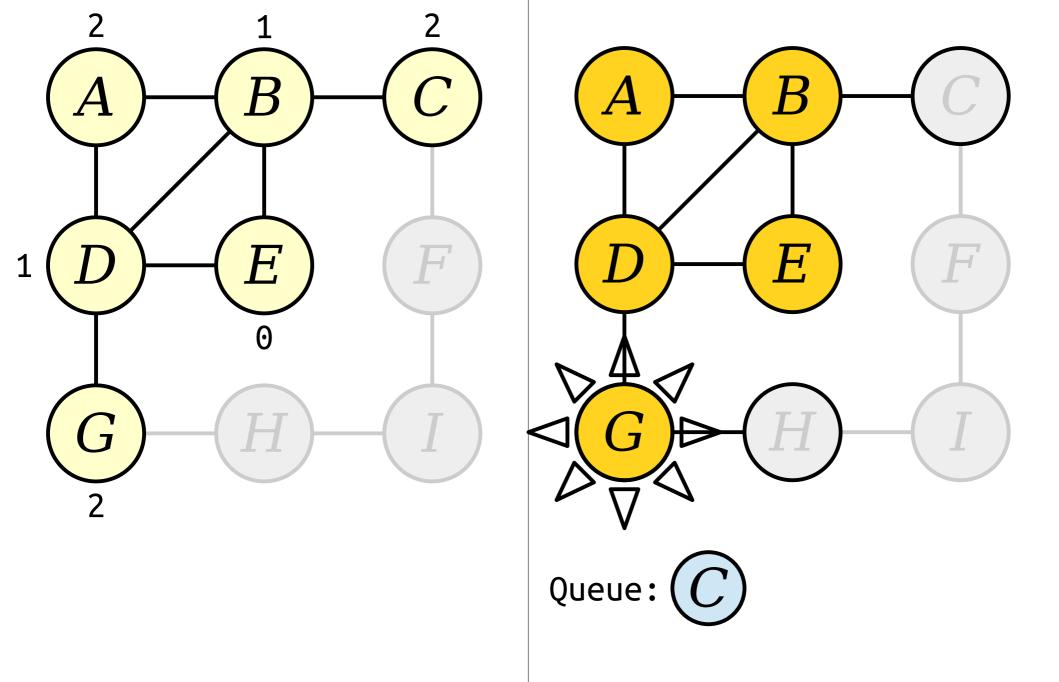


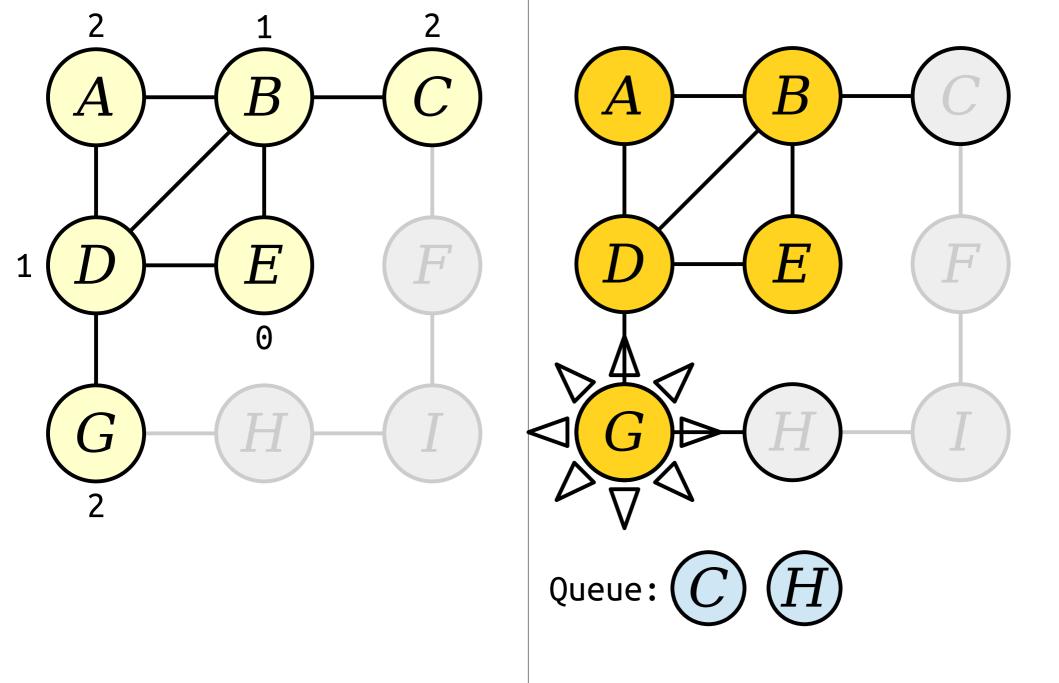


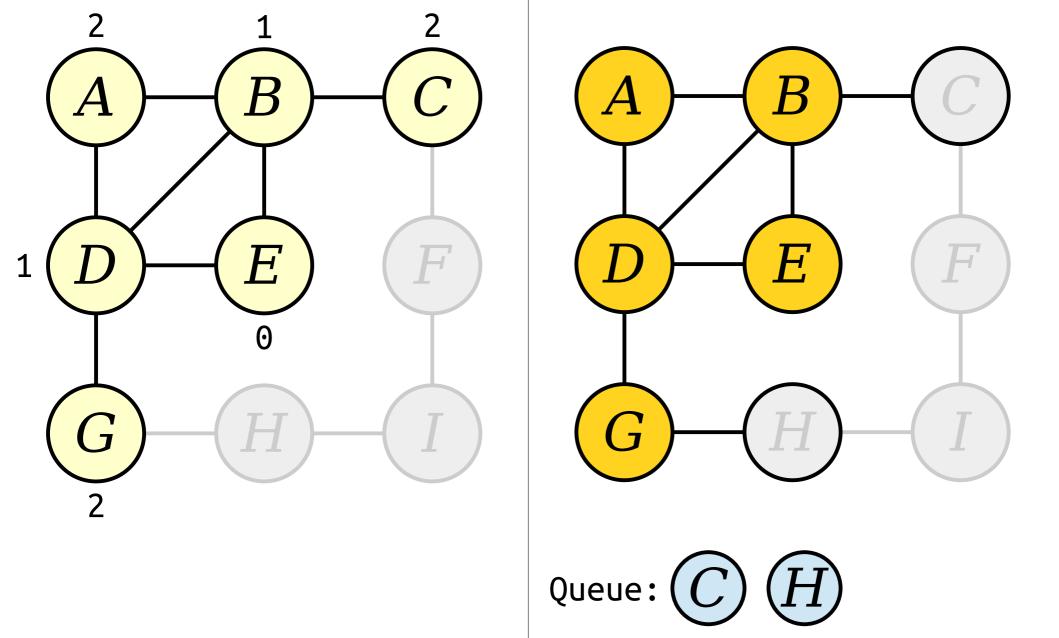


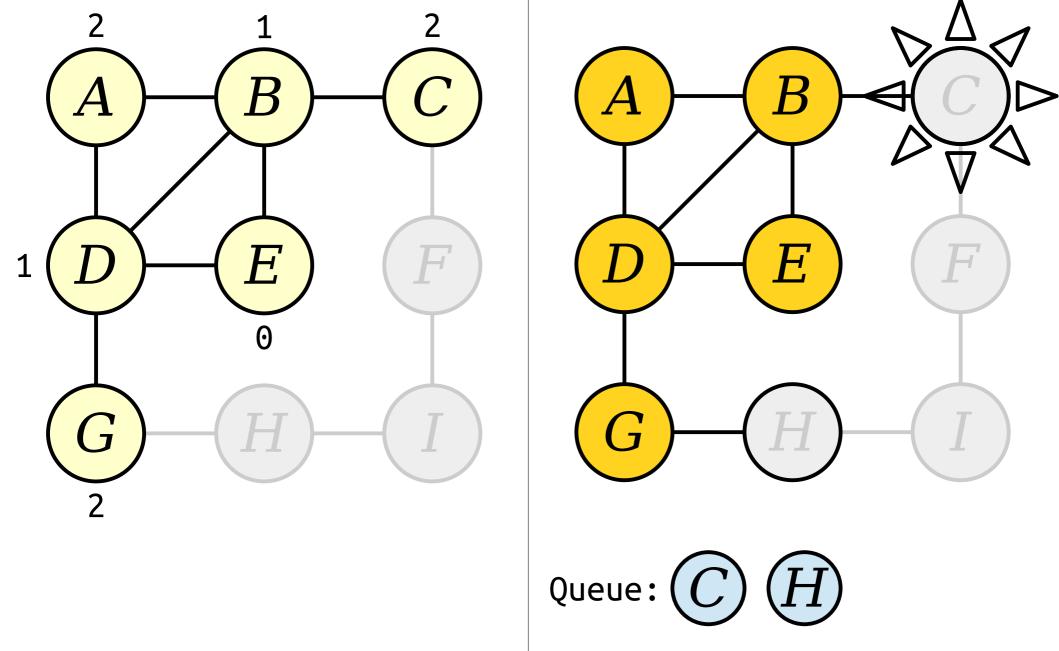


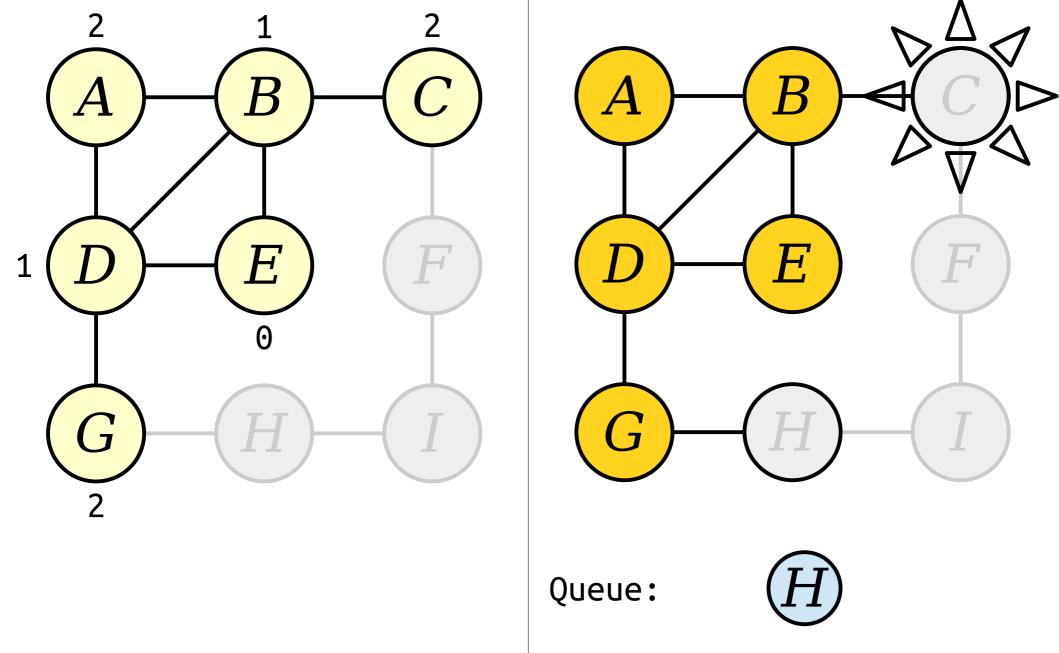


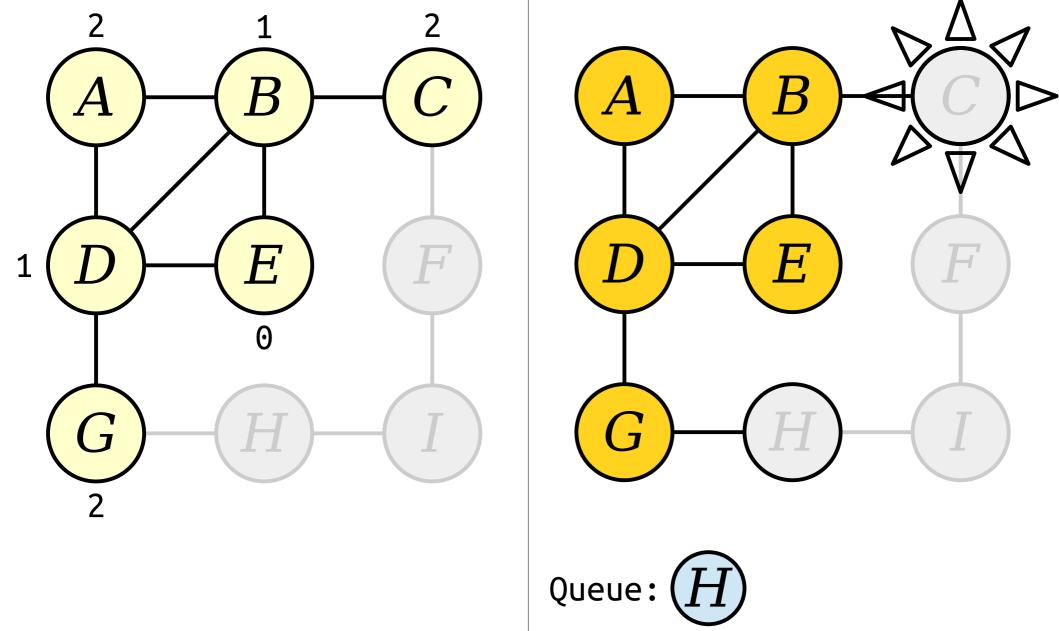


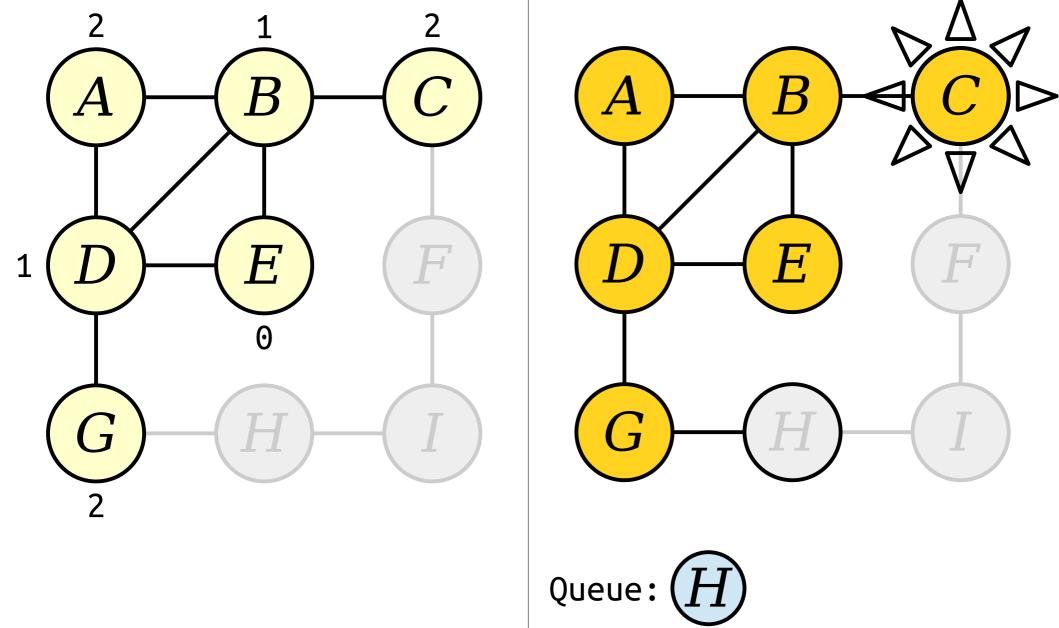


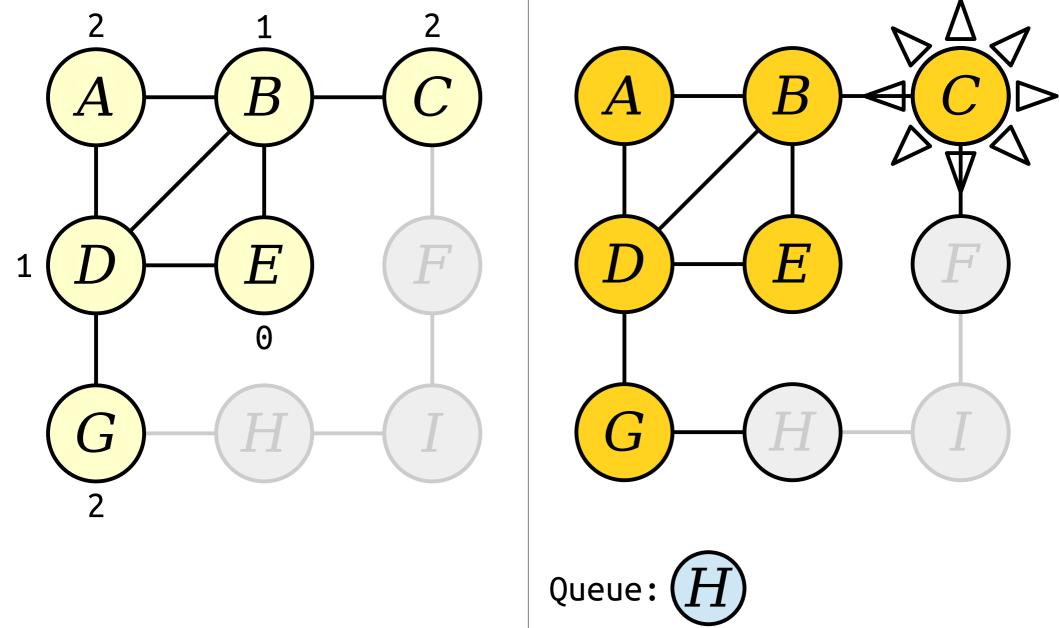


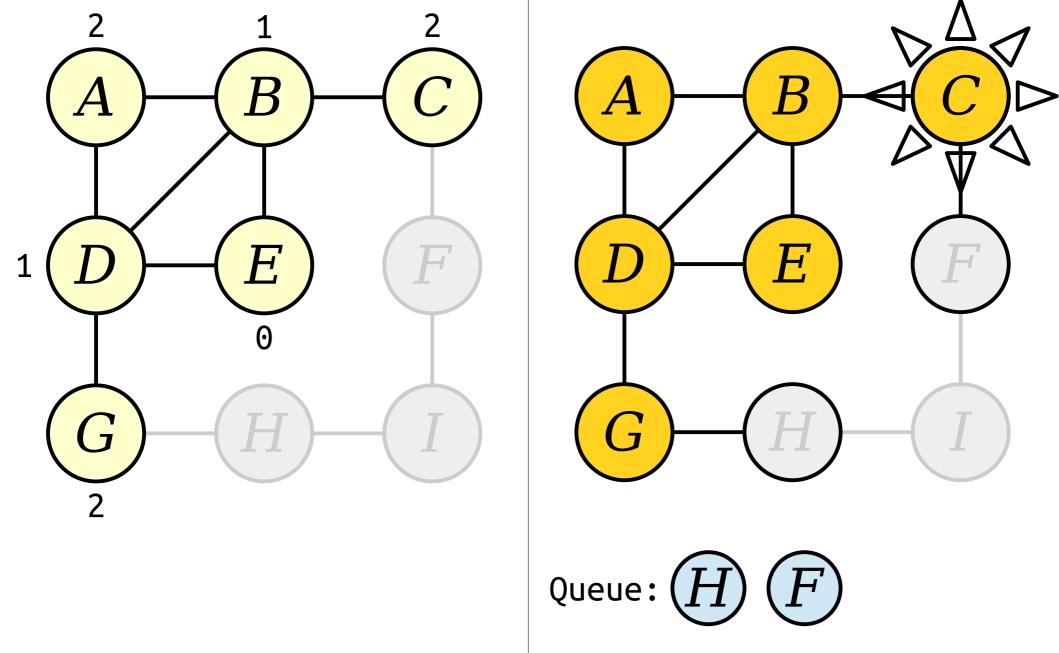


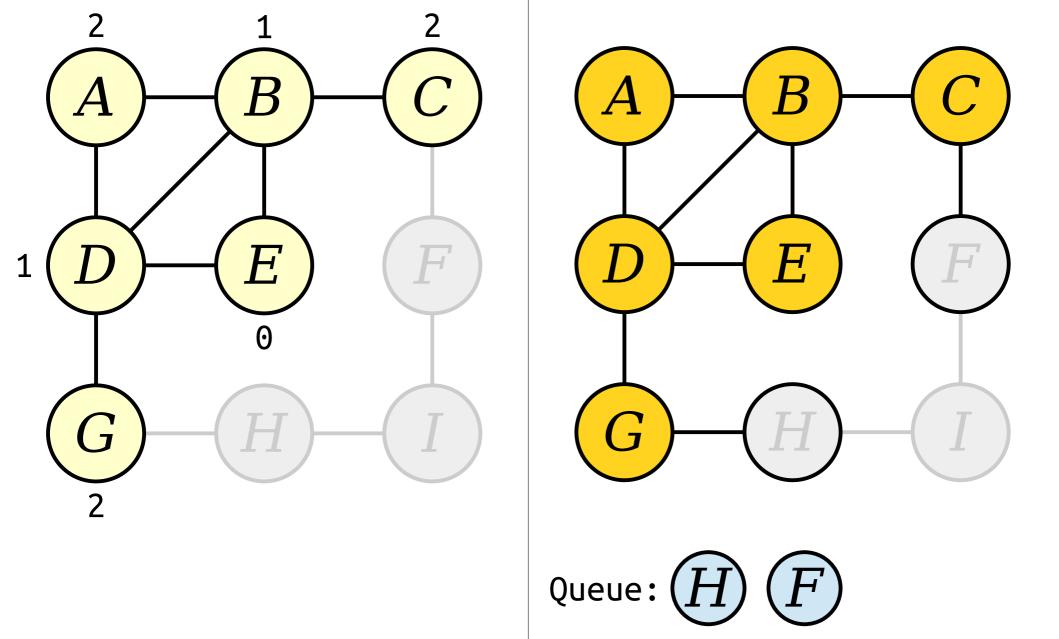


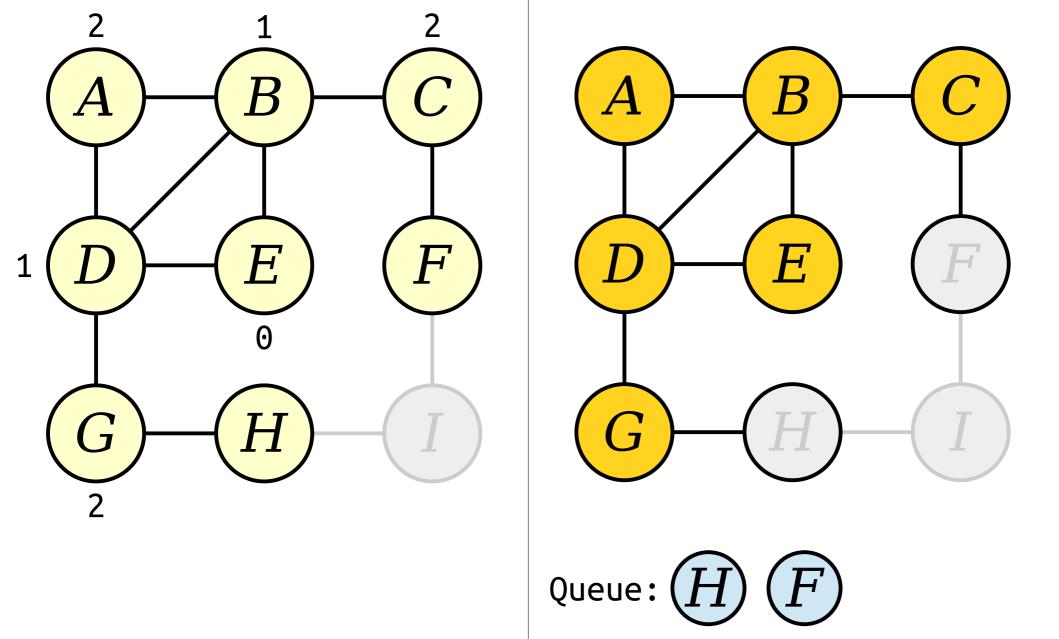


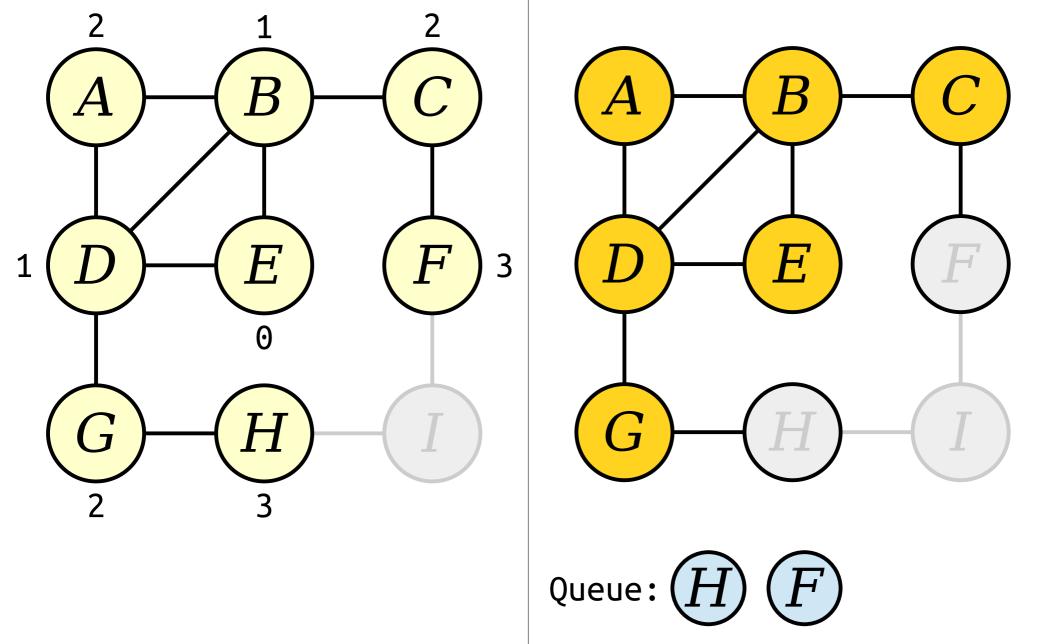


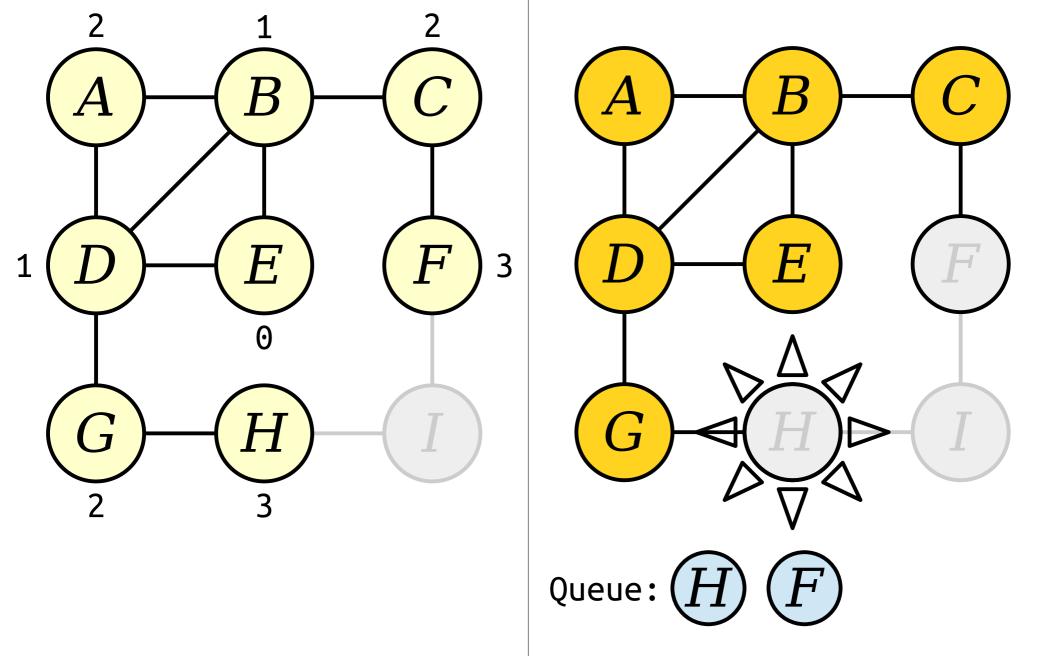


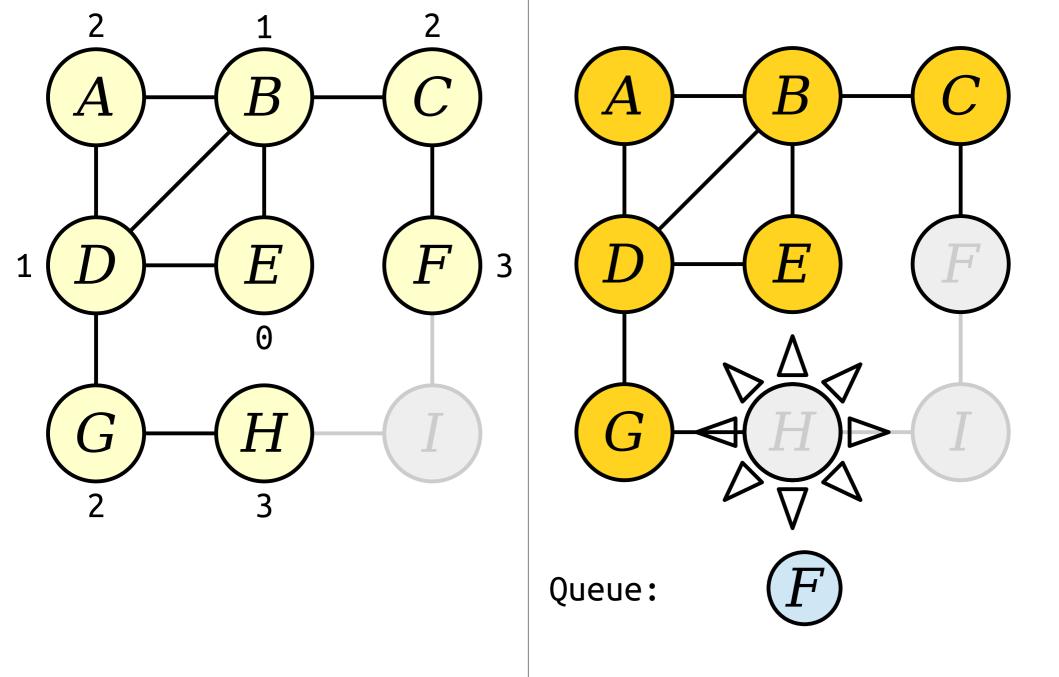


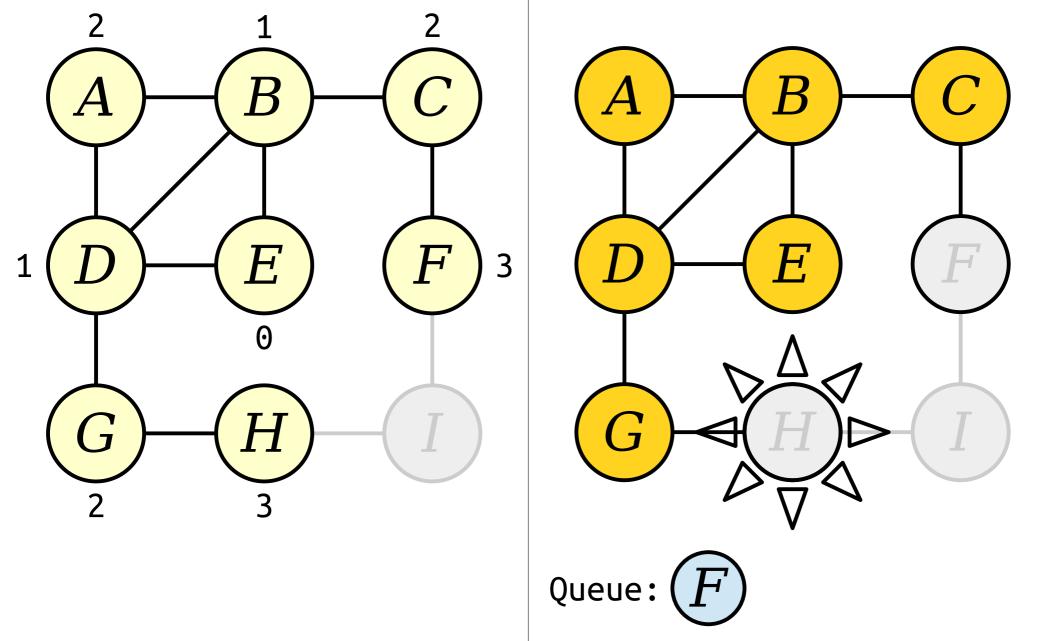


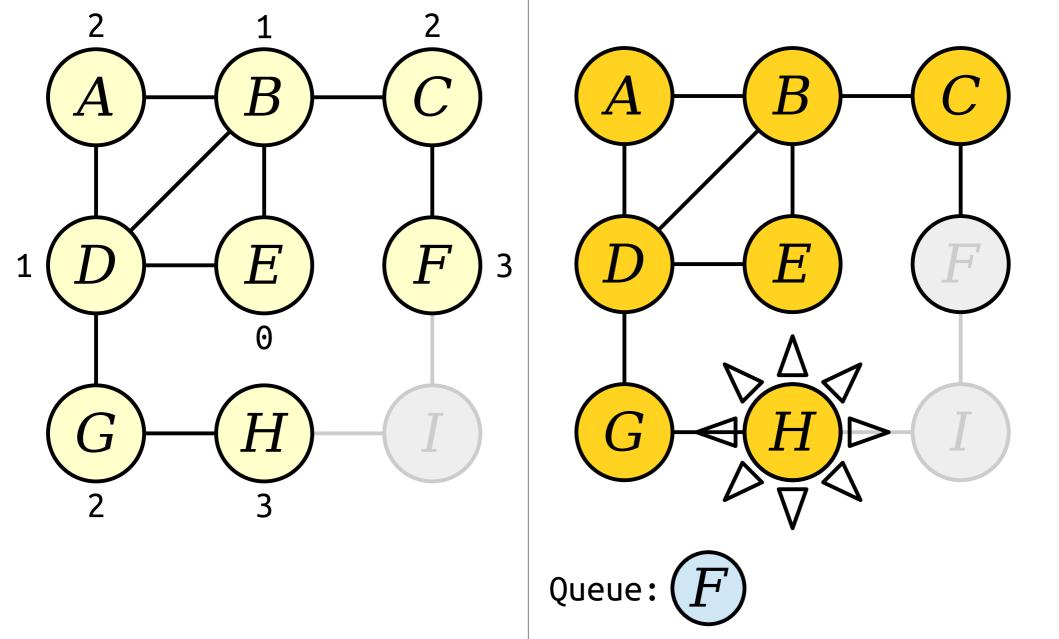


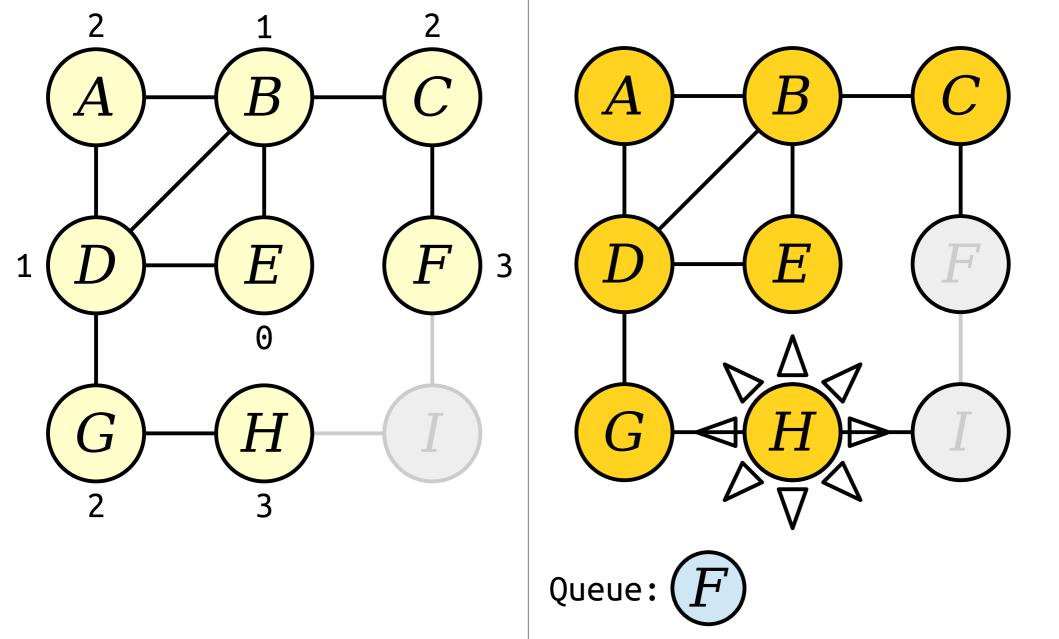


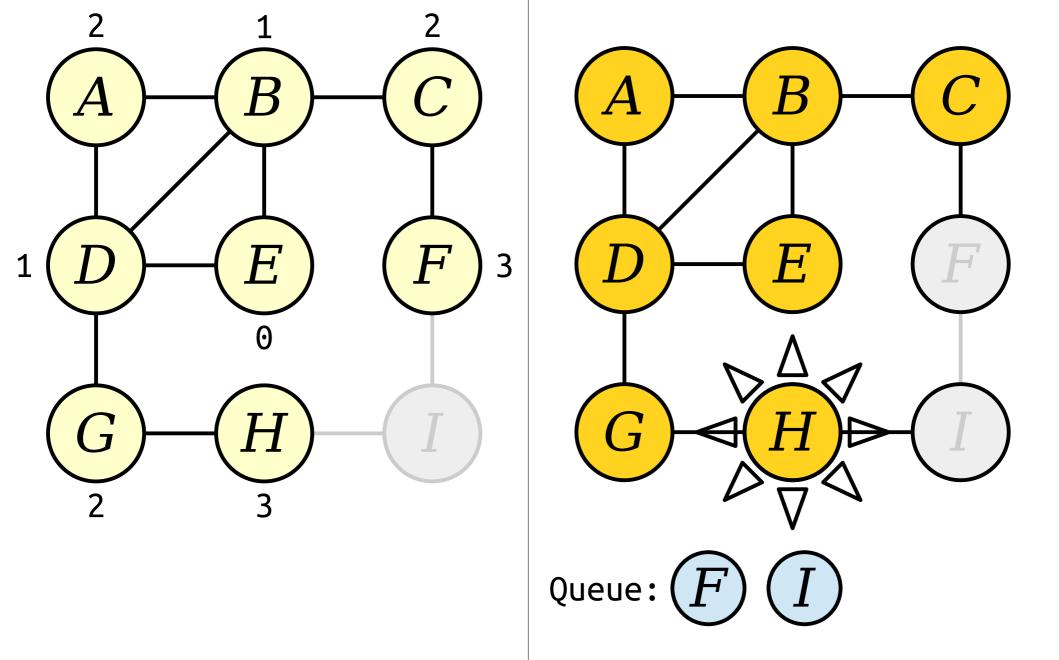


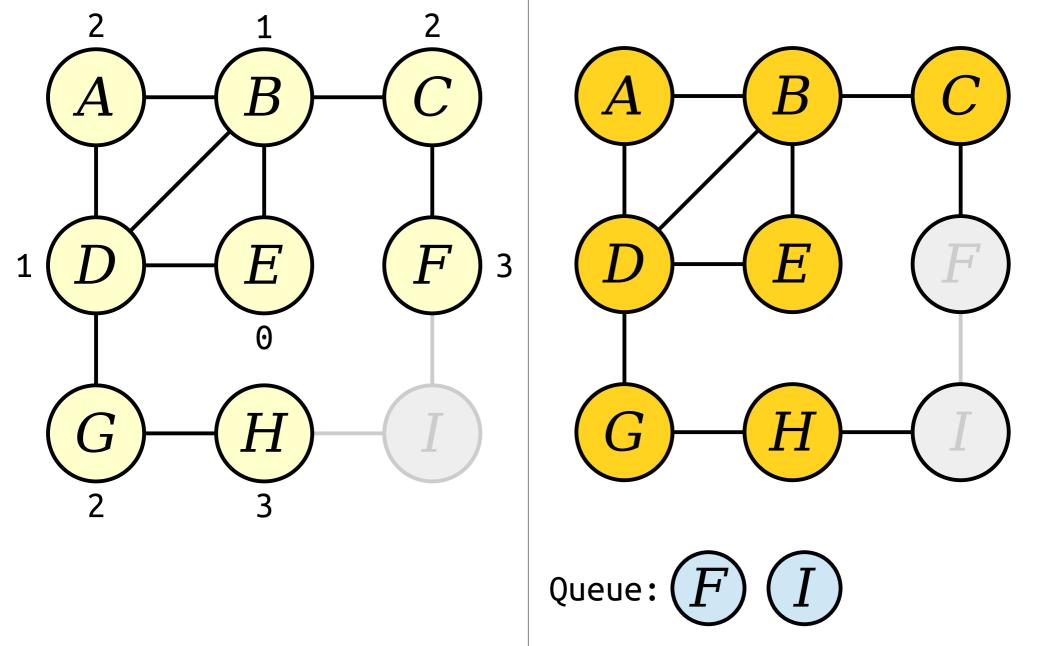


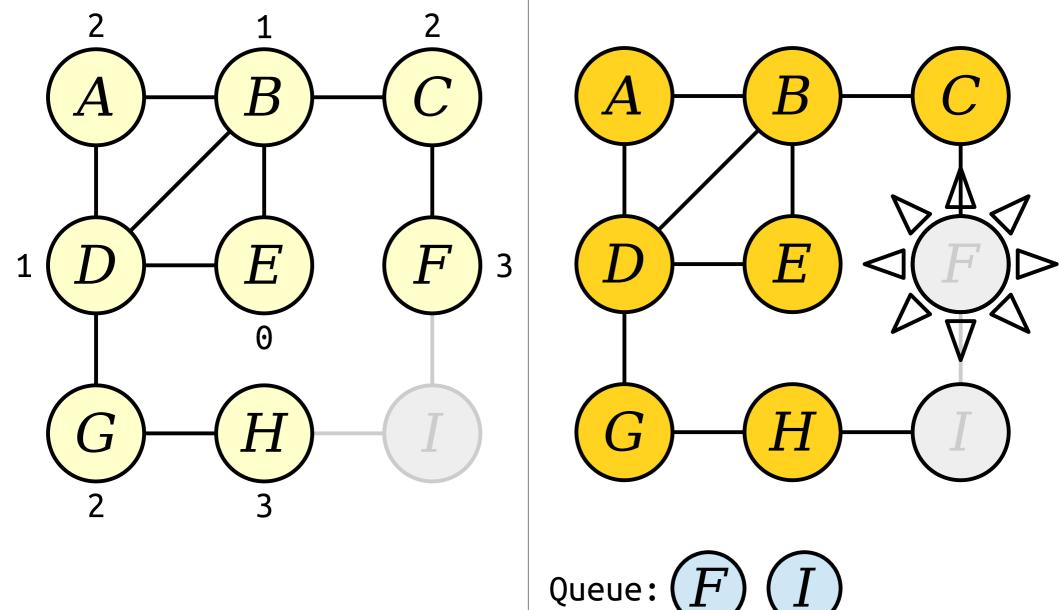


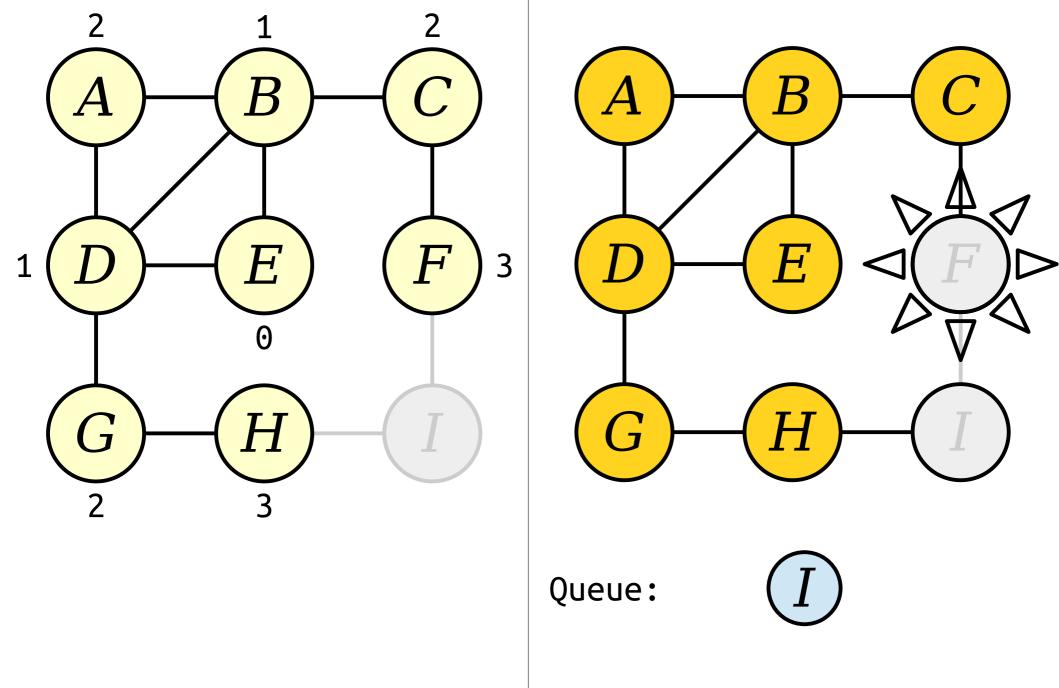


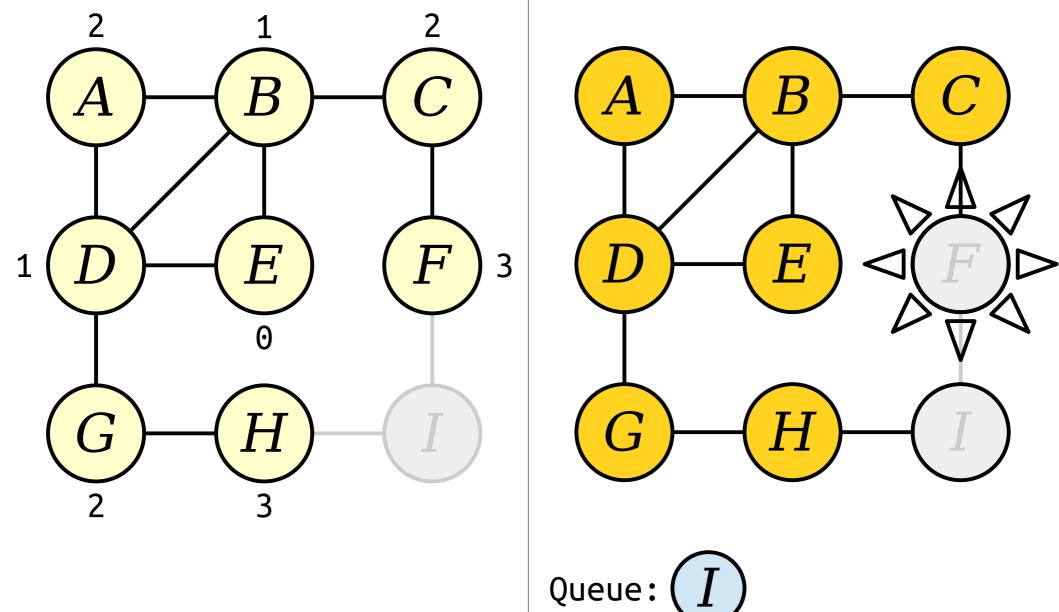


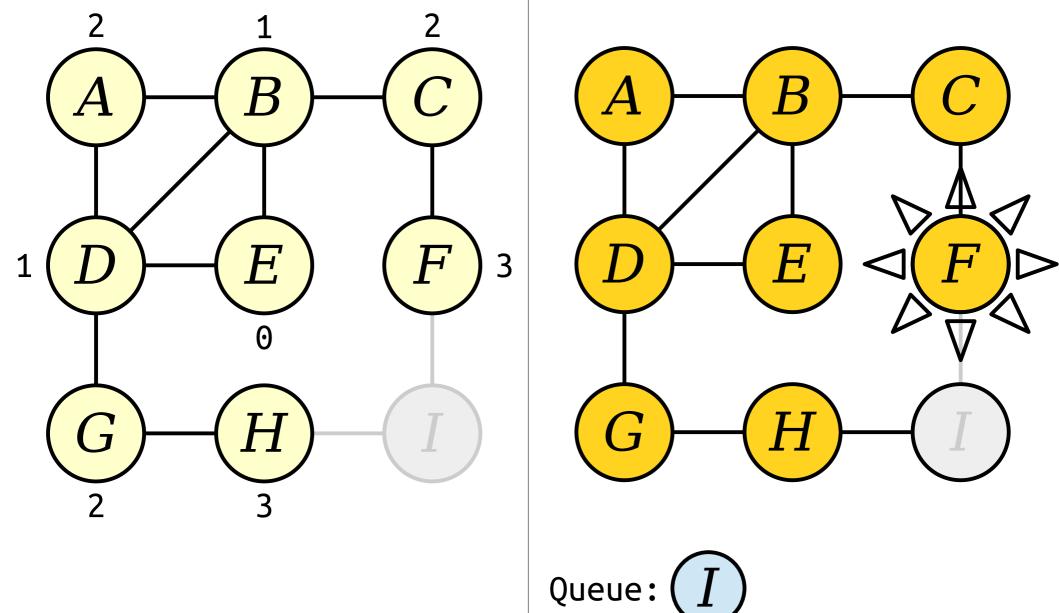


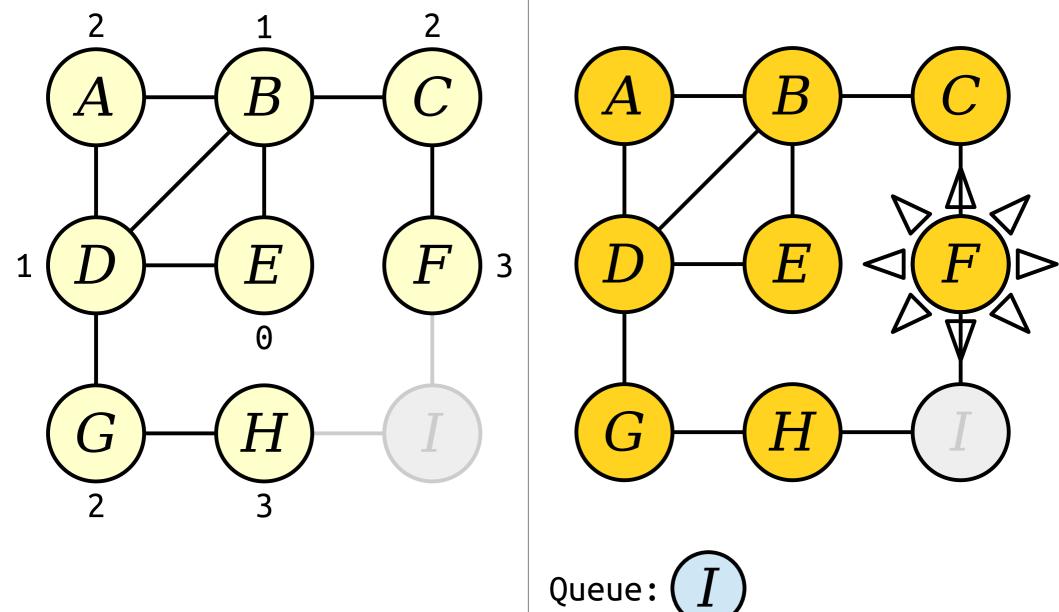


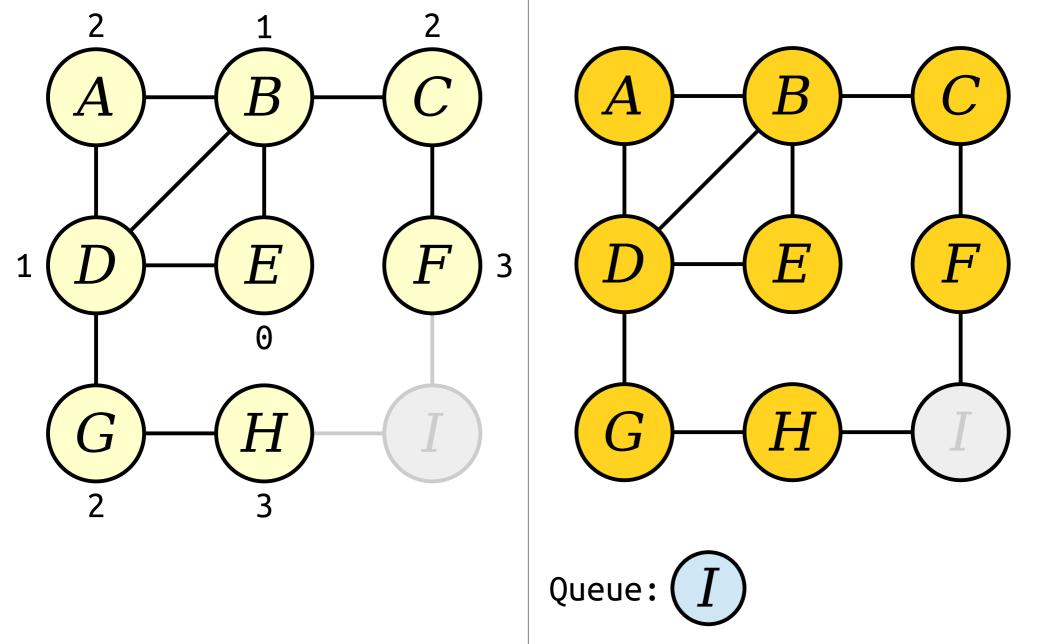


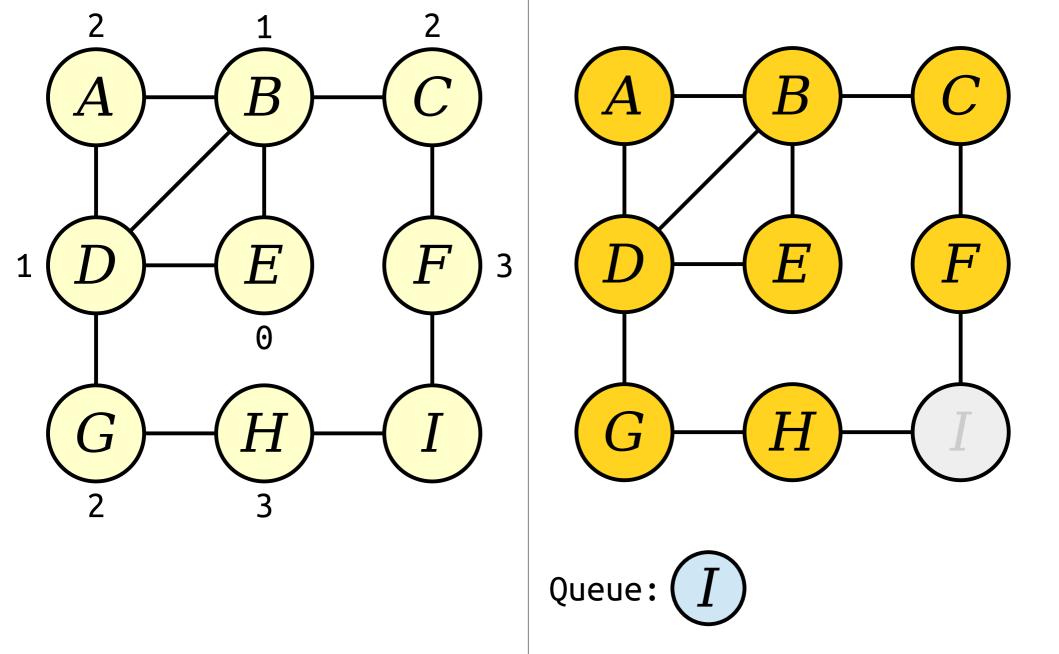


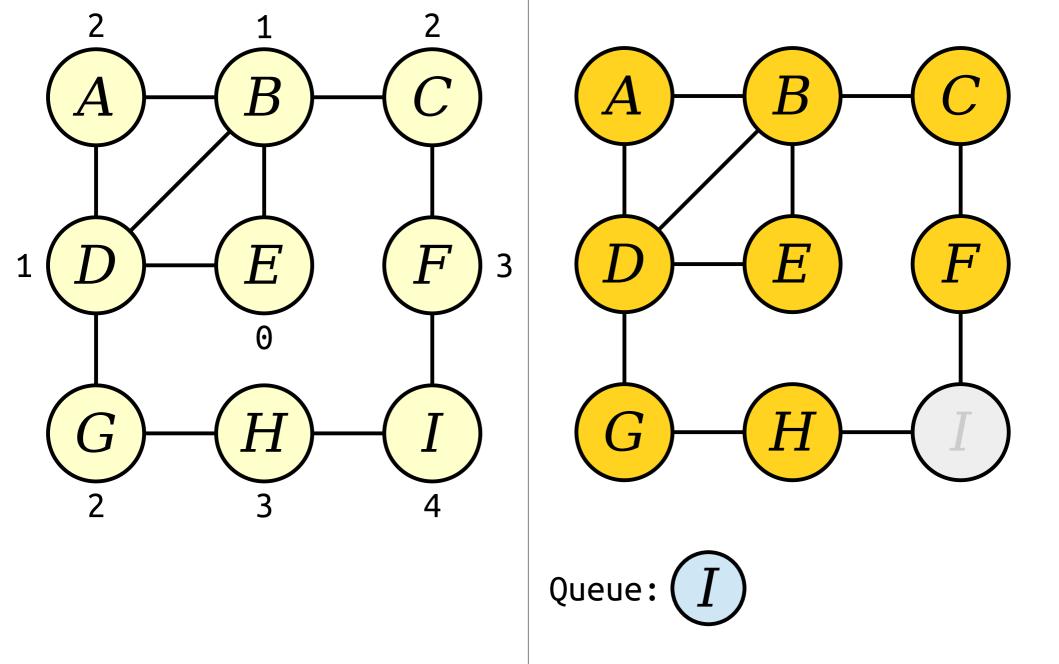


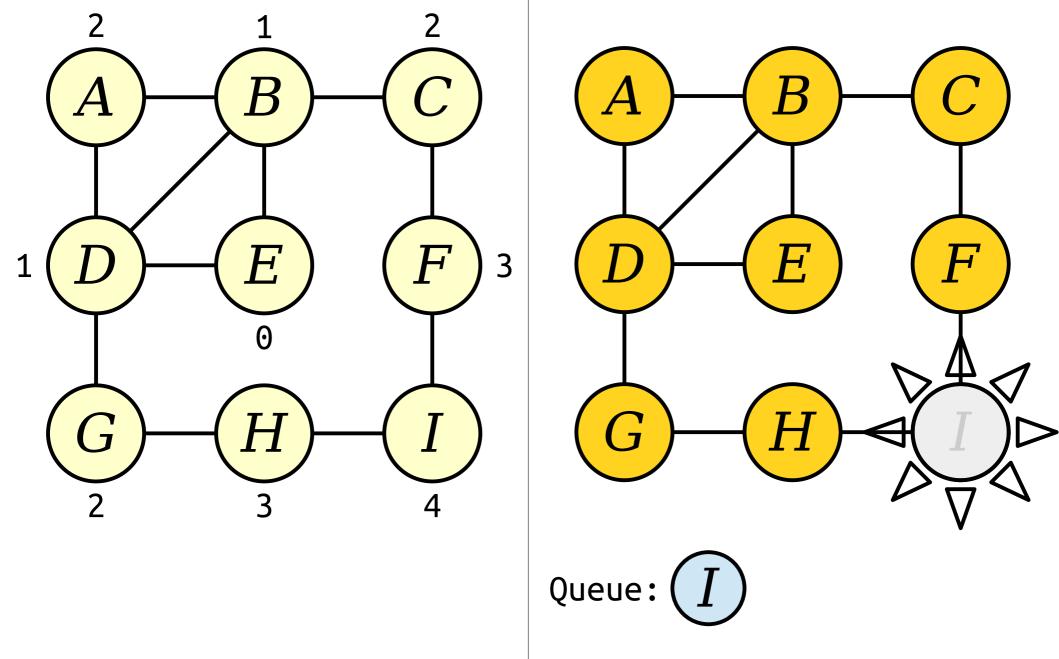


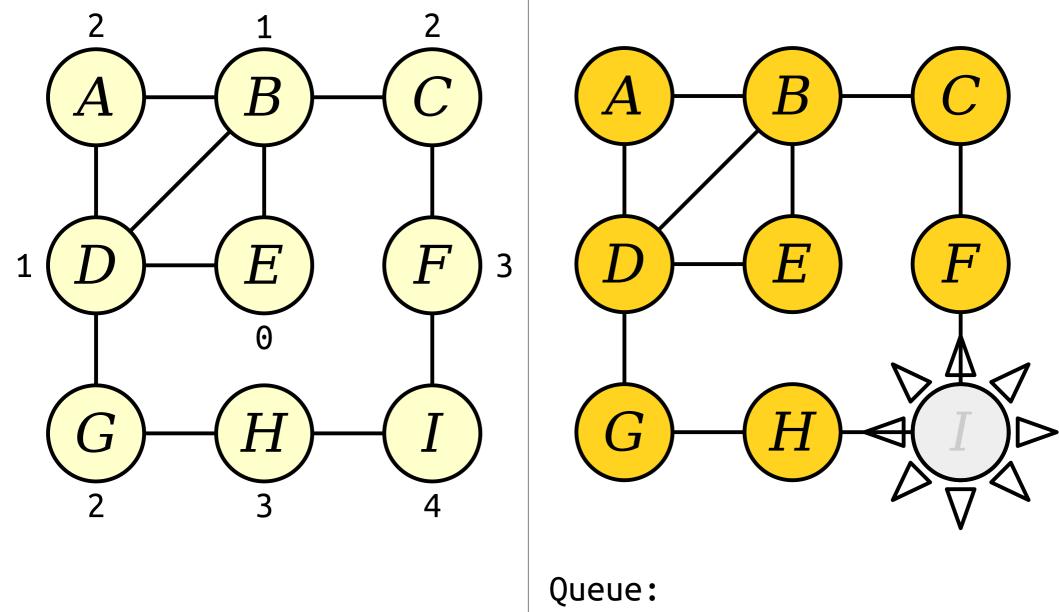


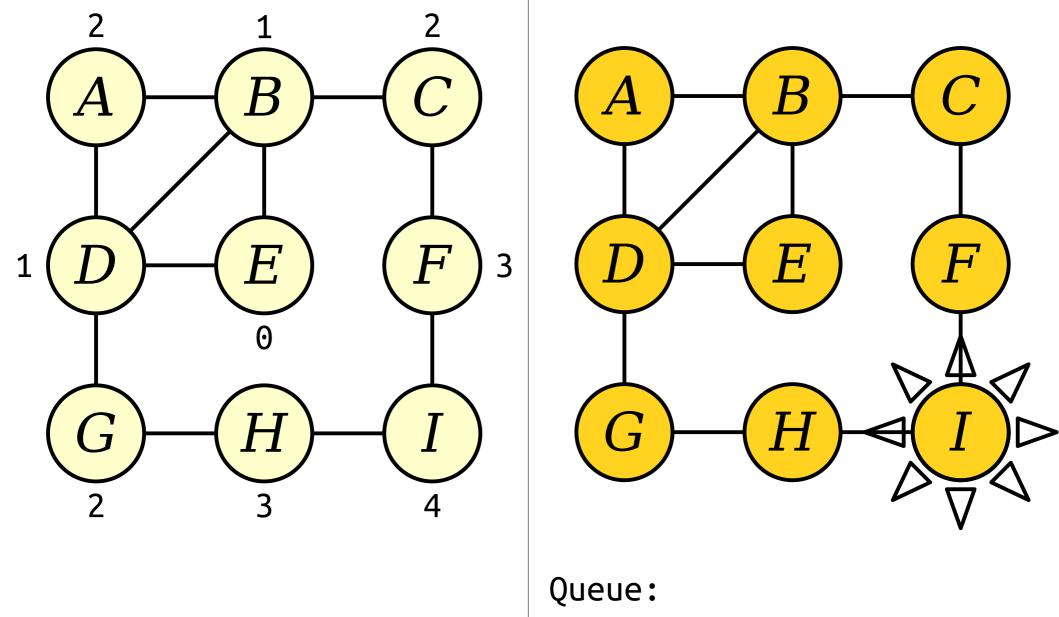


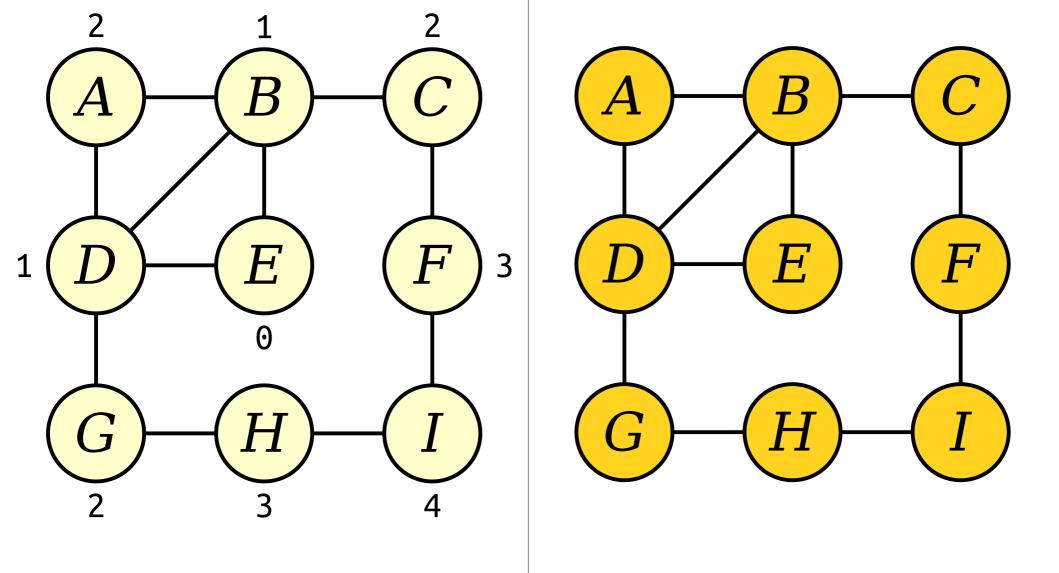






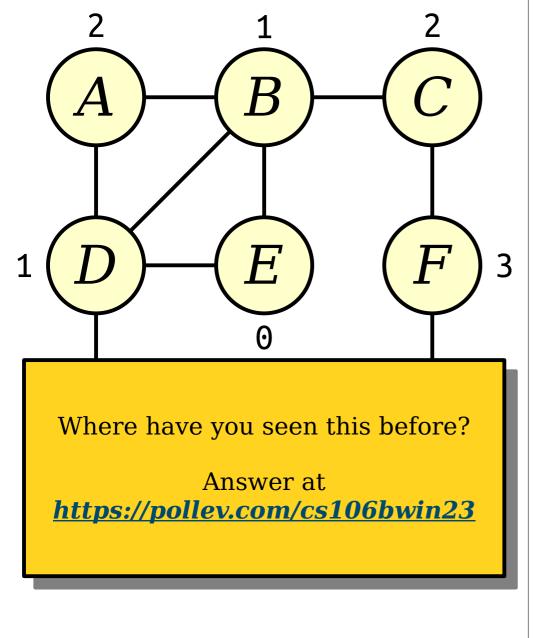


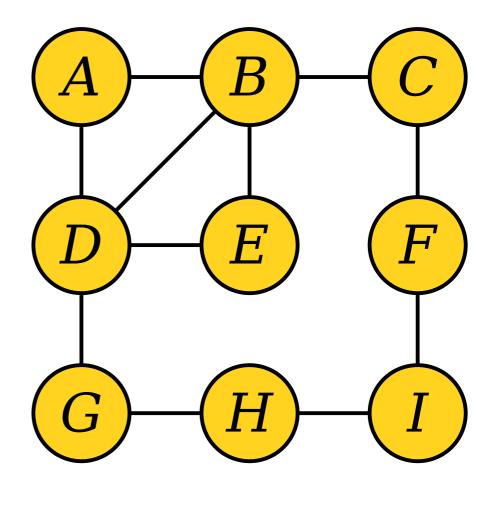




Queue:

Visit nodes in ascending order of distance from the start node E.





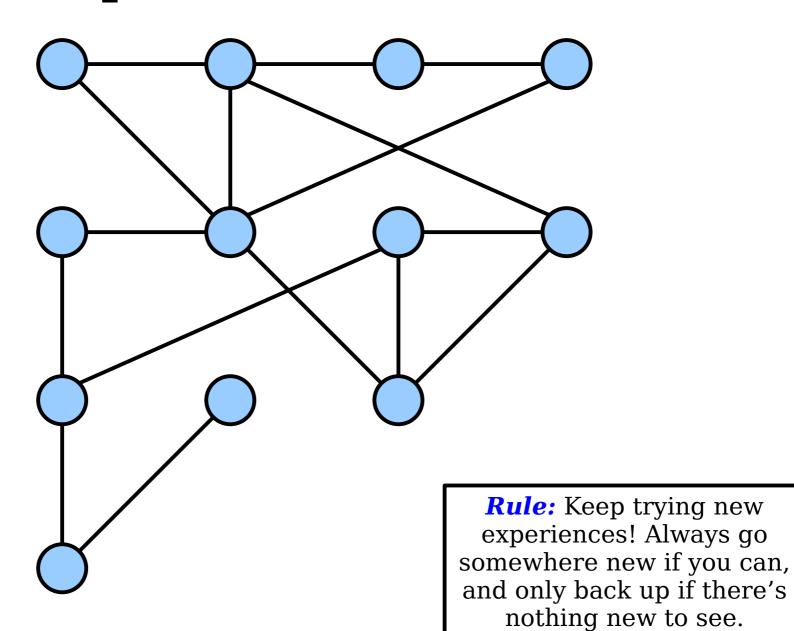
Queue:

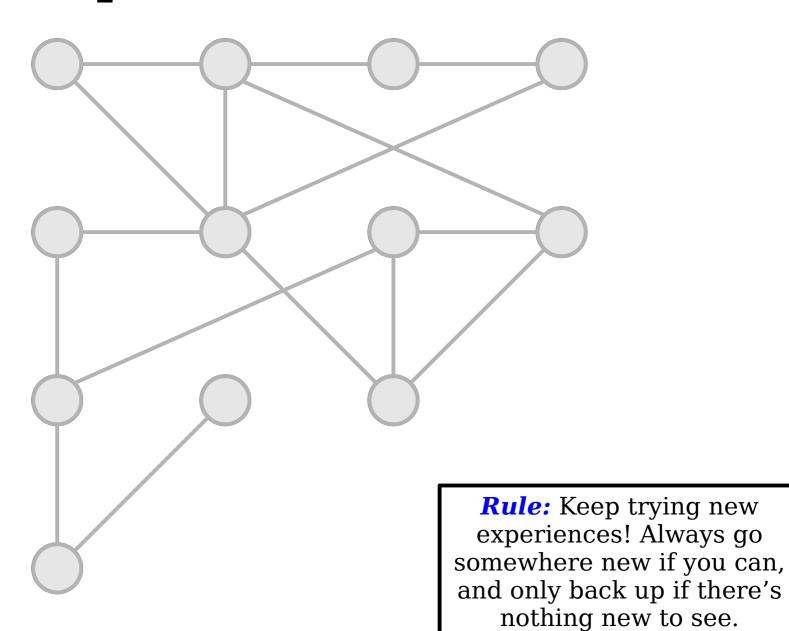
Visit nodes in ascending order of distance from the start node E.

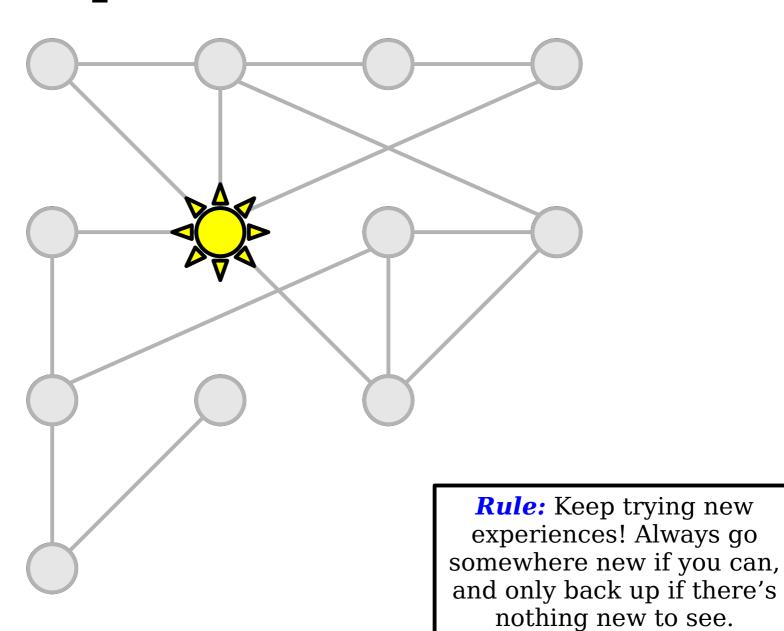
Breadth-First Search

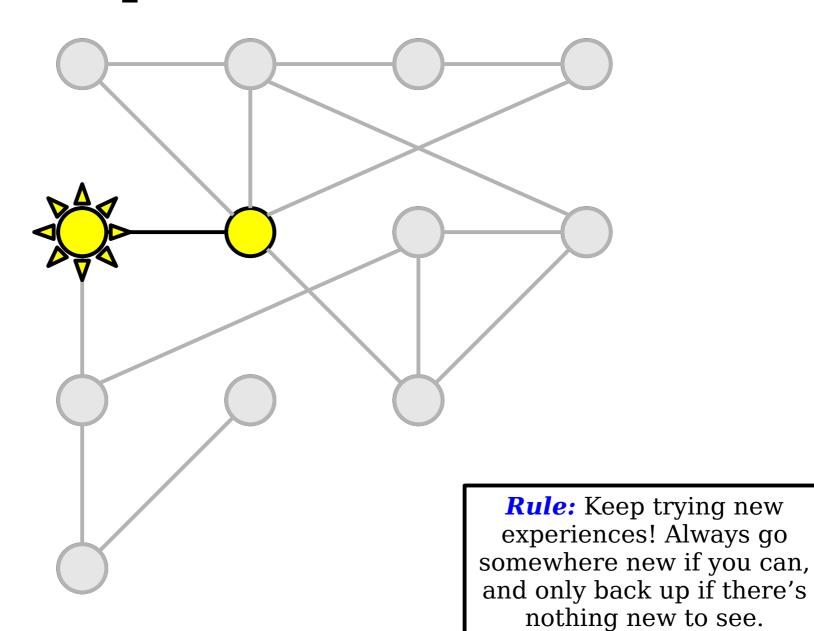
- The Queue-based search strategy we just saw is called *breadth-first search* (or just *BFS* for short).
- In pseudocode:

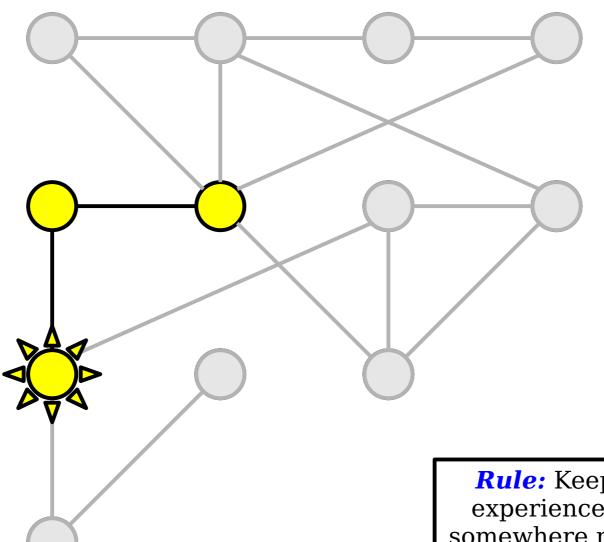
```
bfs-from(node v) {
    make a queue of nodes, initially seeded with v.
    while the queue isn't empty:
        dequeue a node curr.
        process the node curr.
        for each node adjacent to curr:
            if that node has never been enqueued:
                 enqueue that node.
}
```



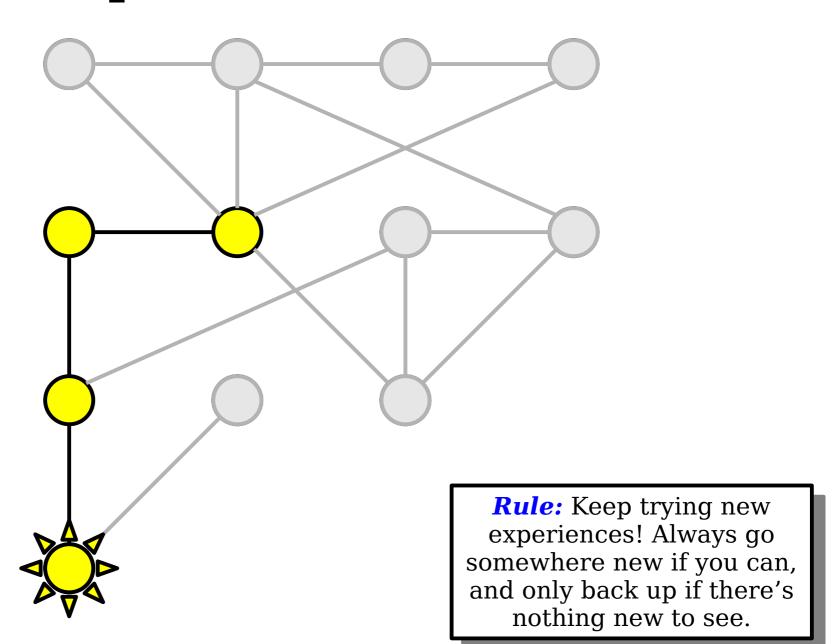


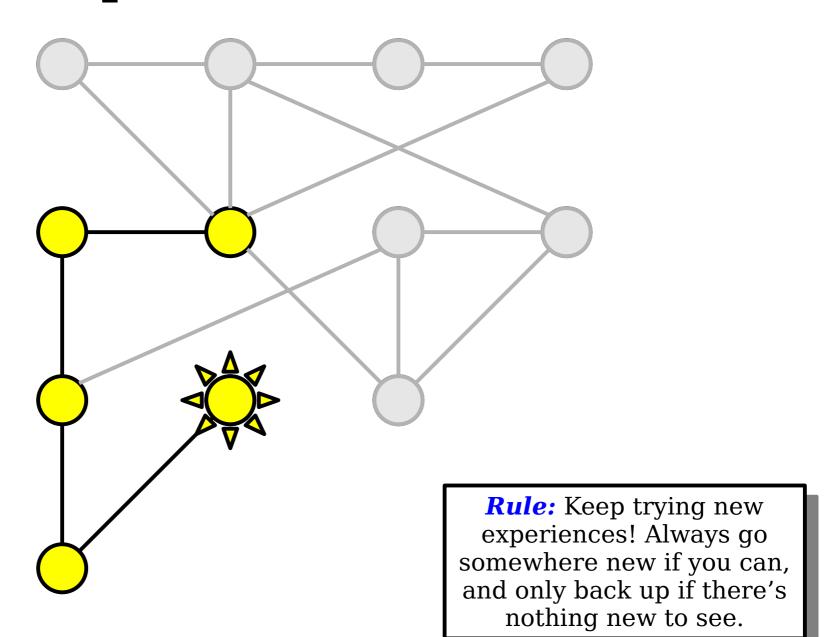


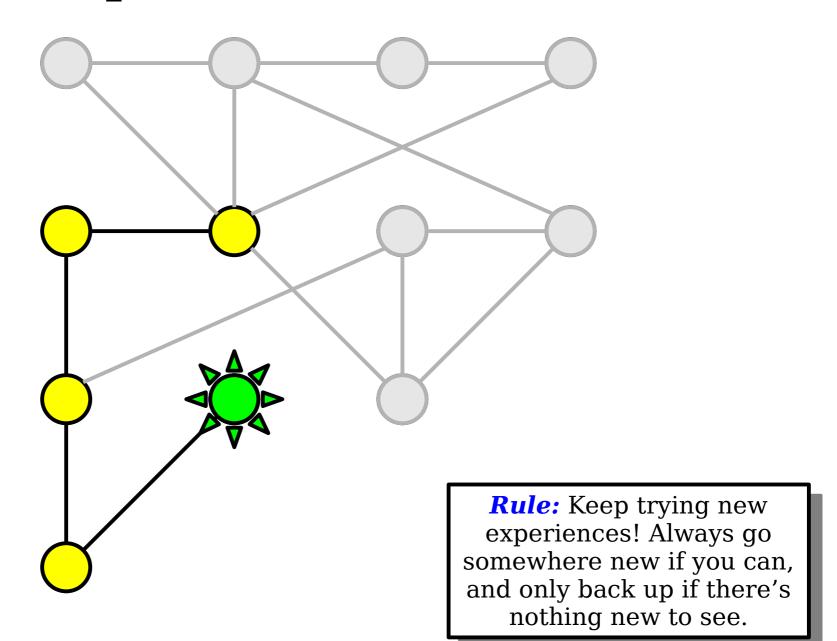


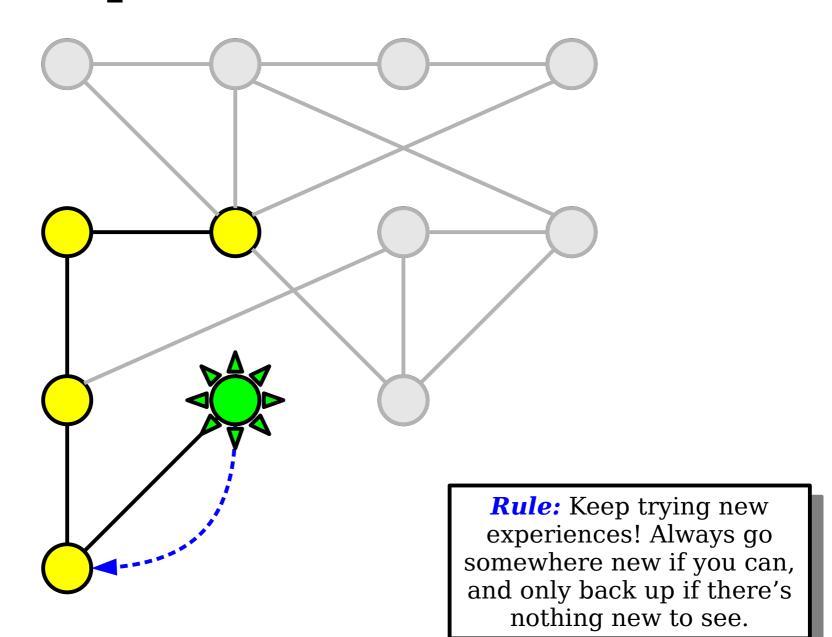


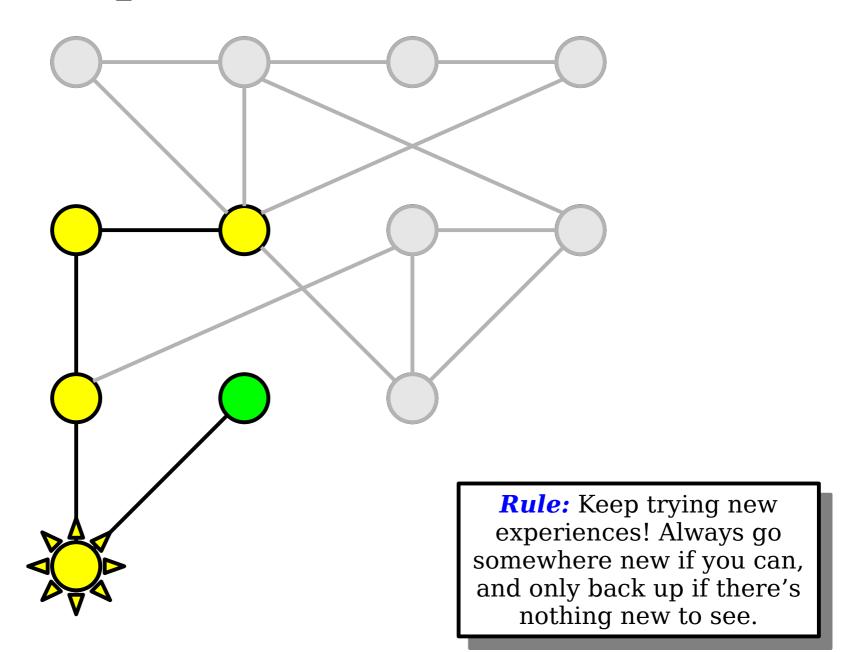
Rule: Keep trying new experiences! Always go somewhere new if you can, and only back up if there's nothing new to see.

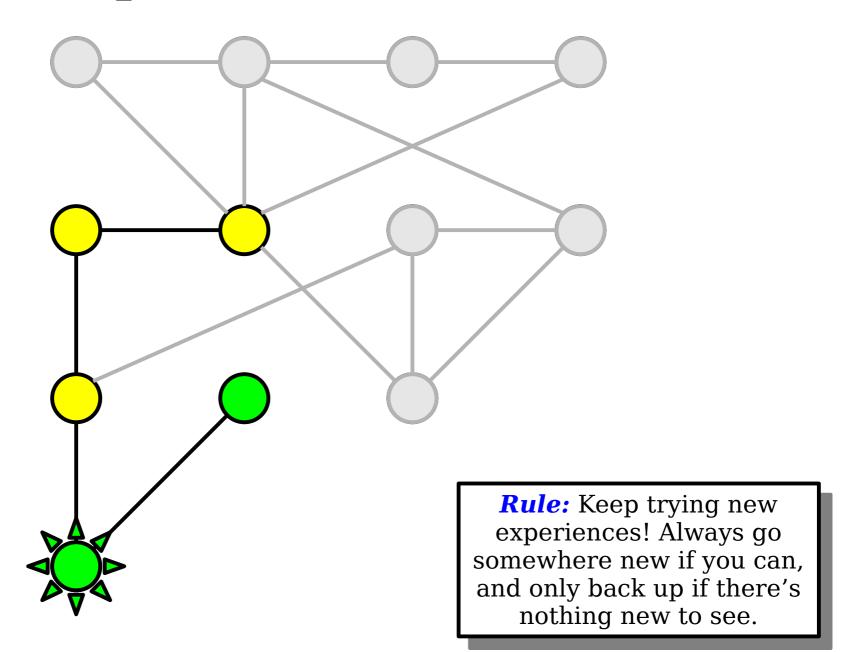


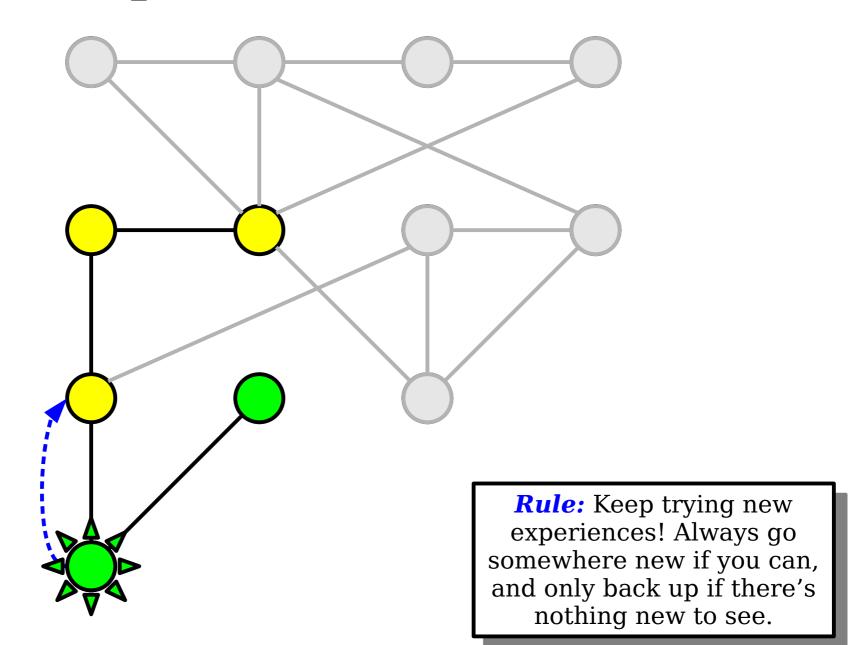


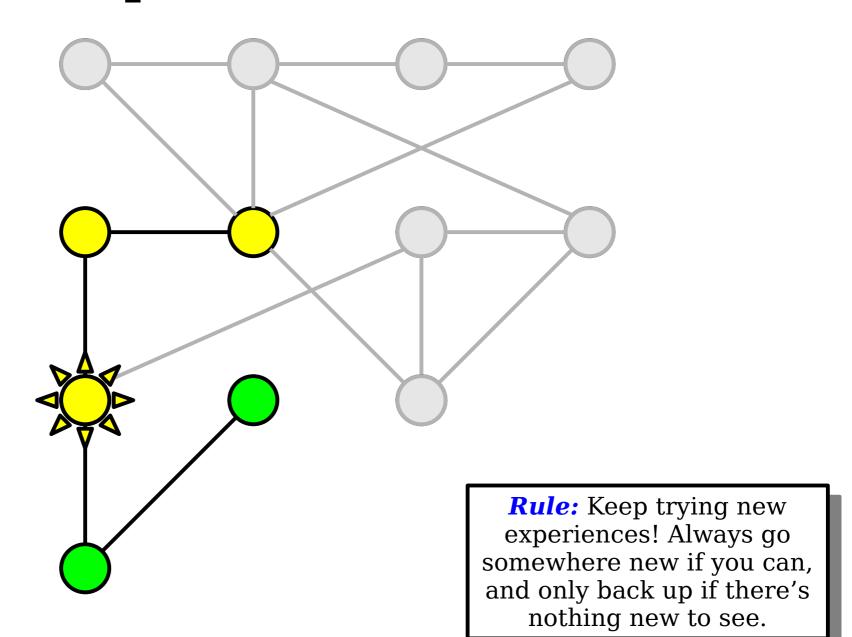


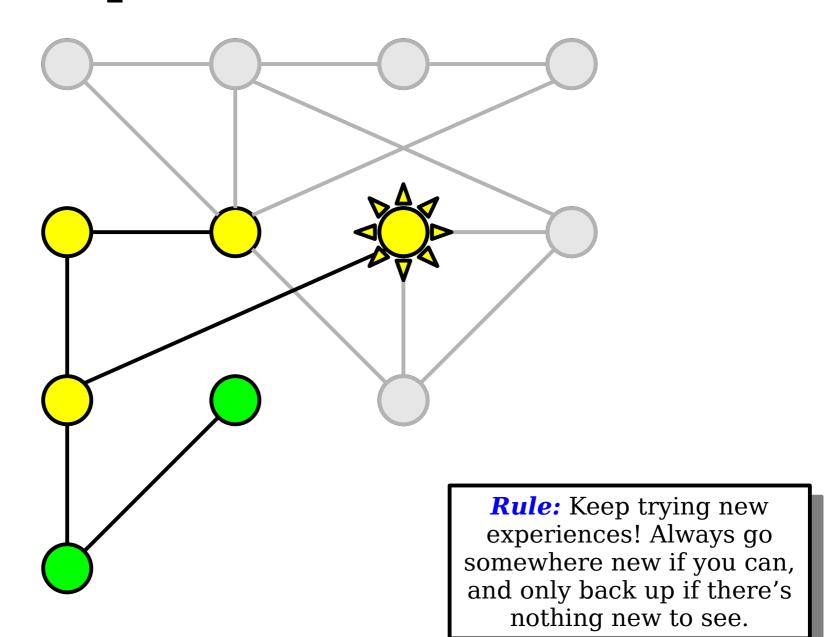


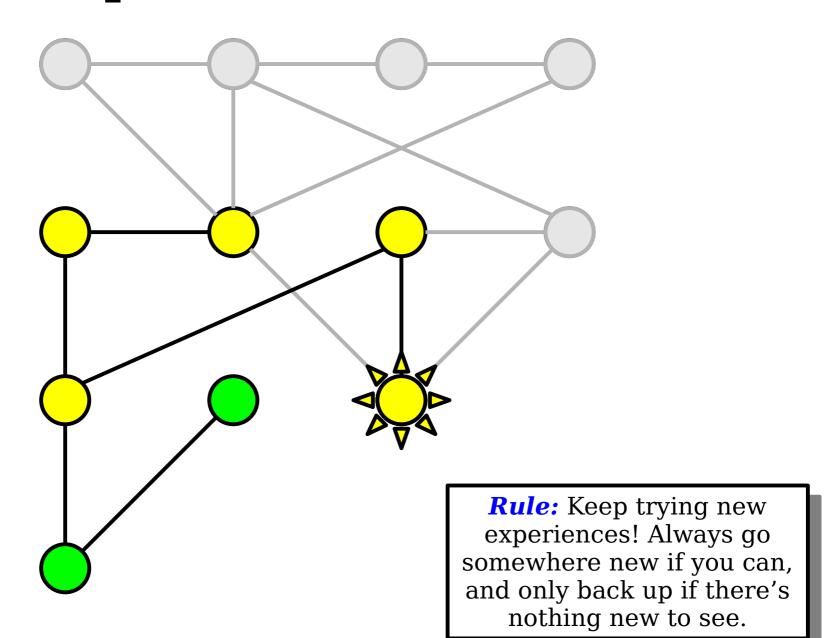


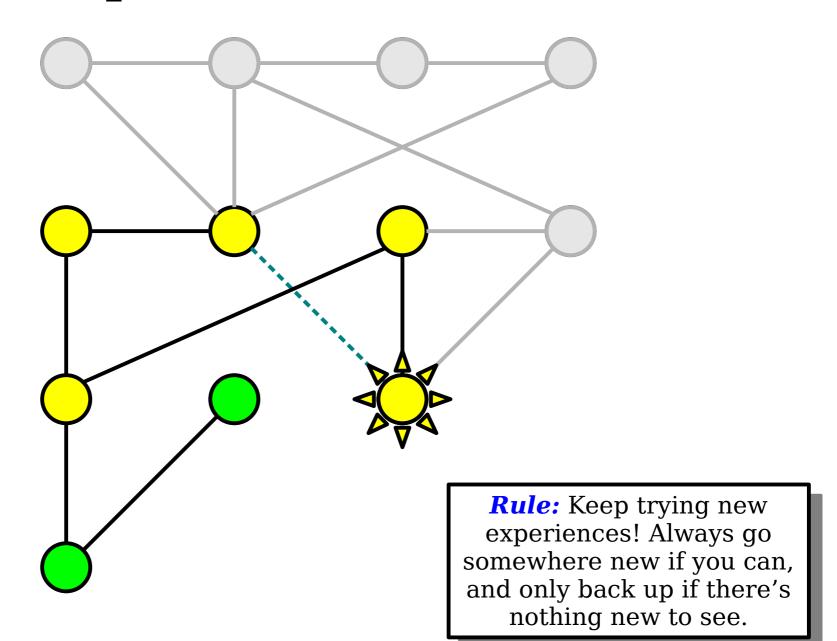


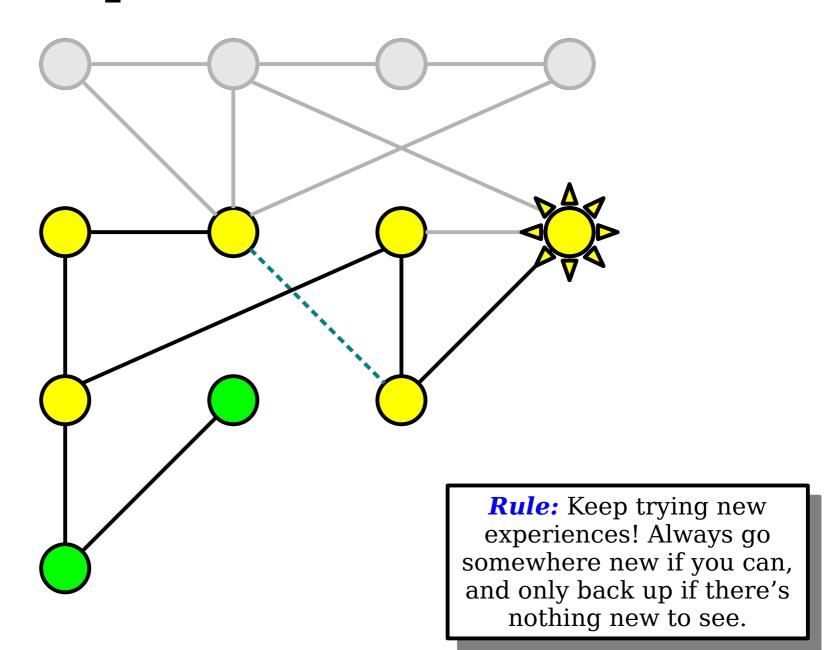


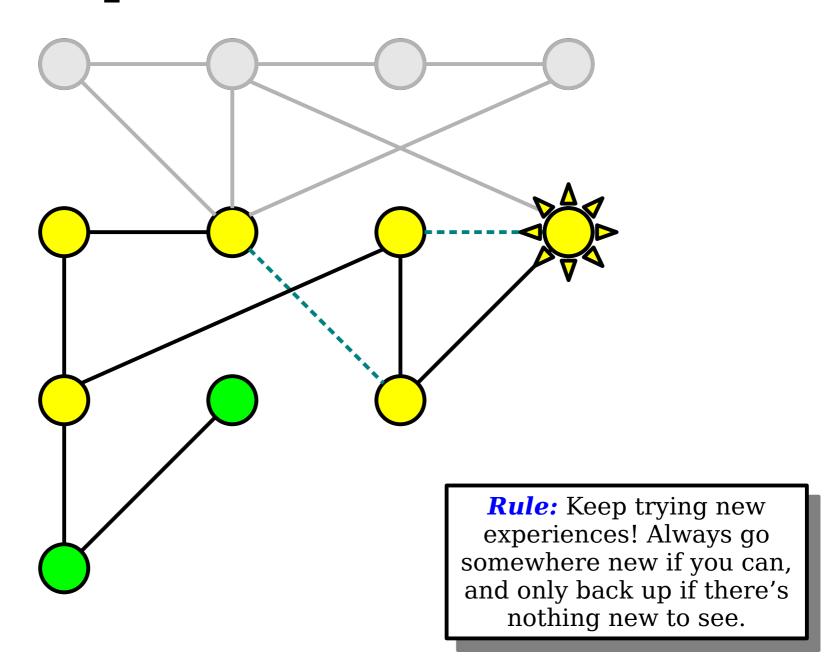


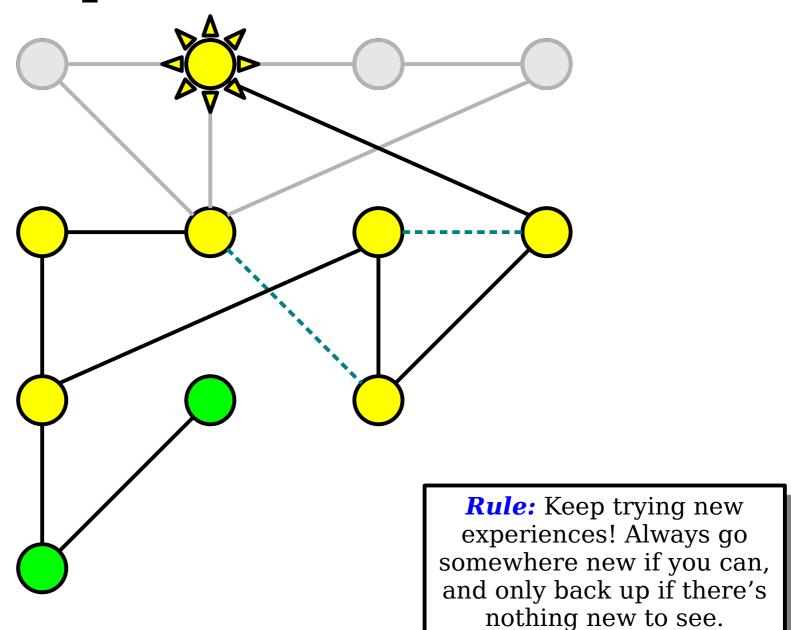


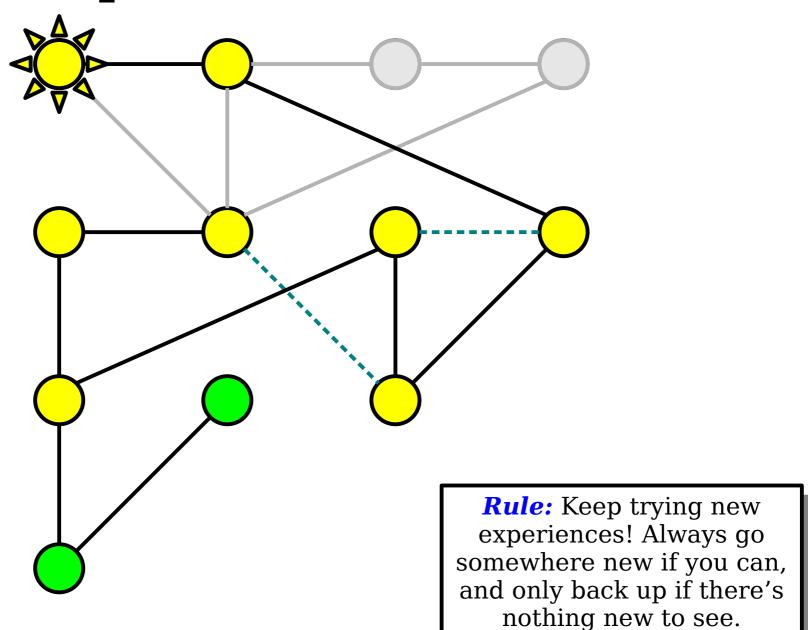


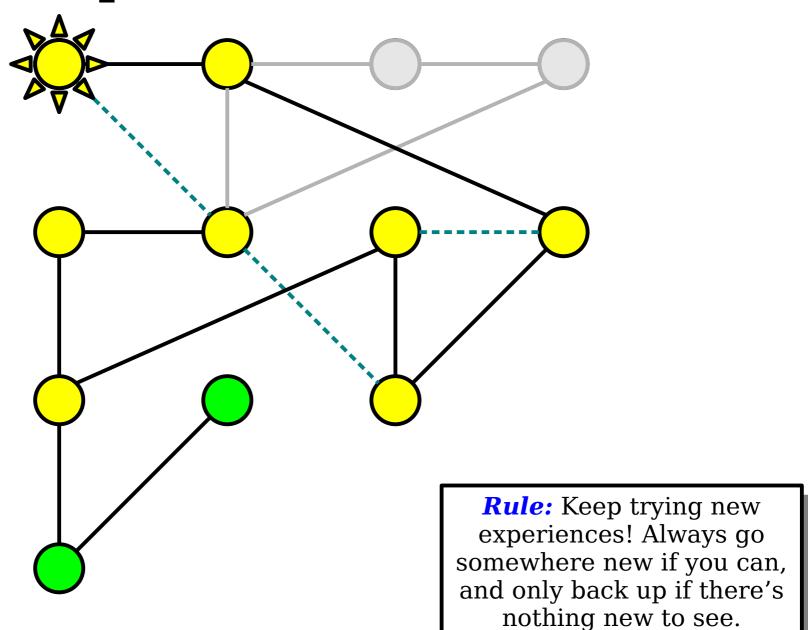


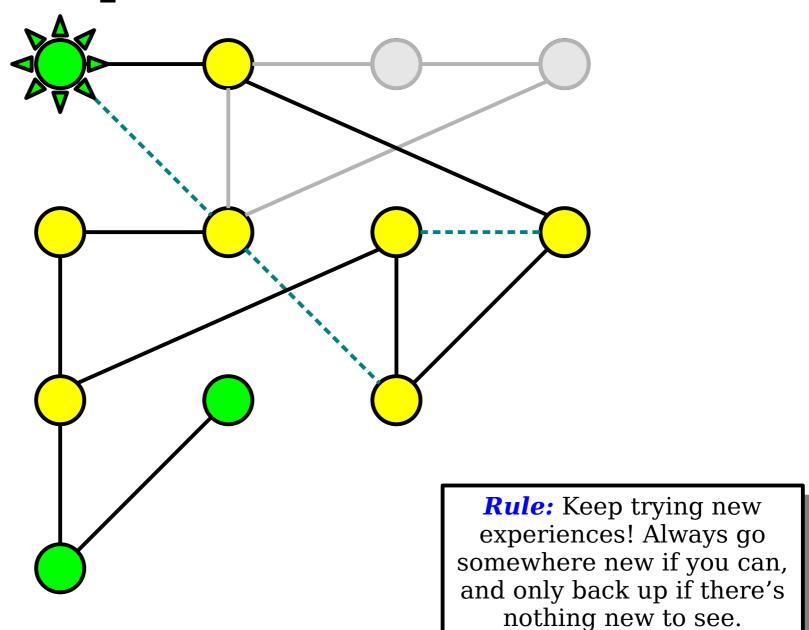


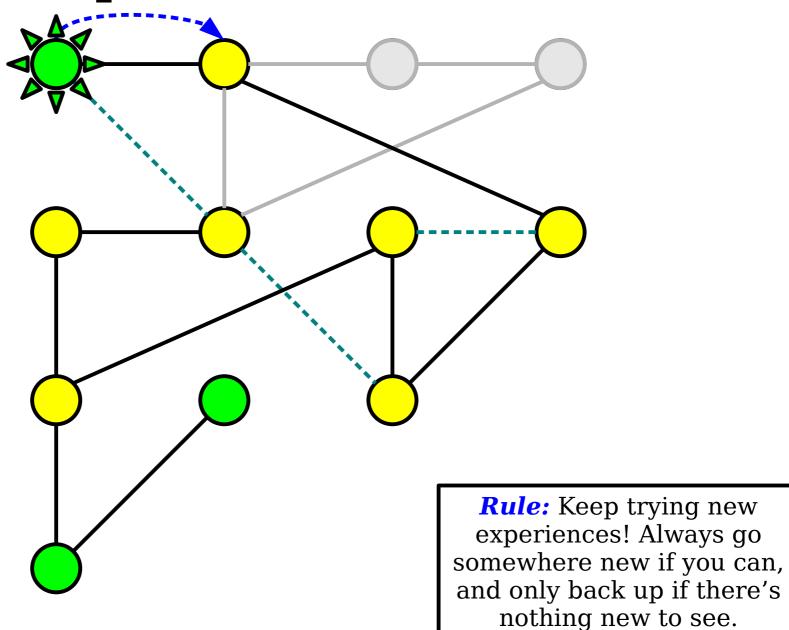


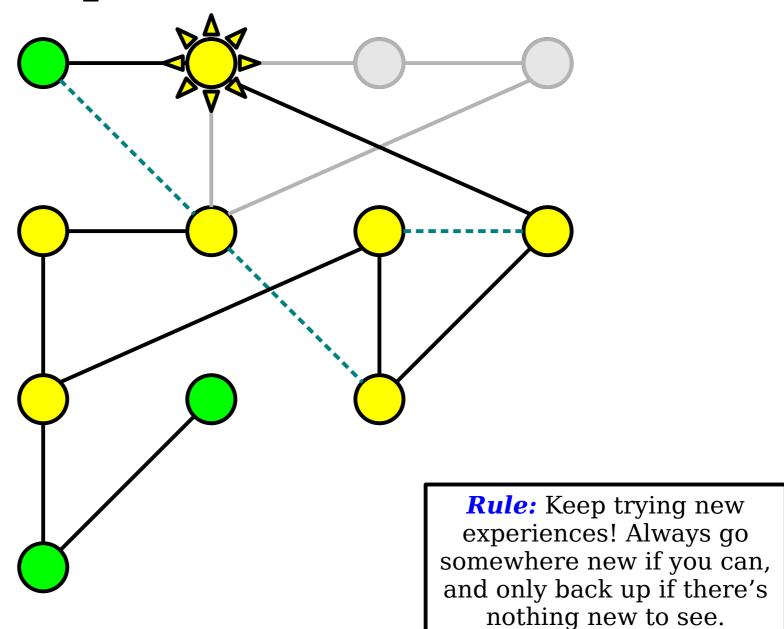


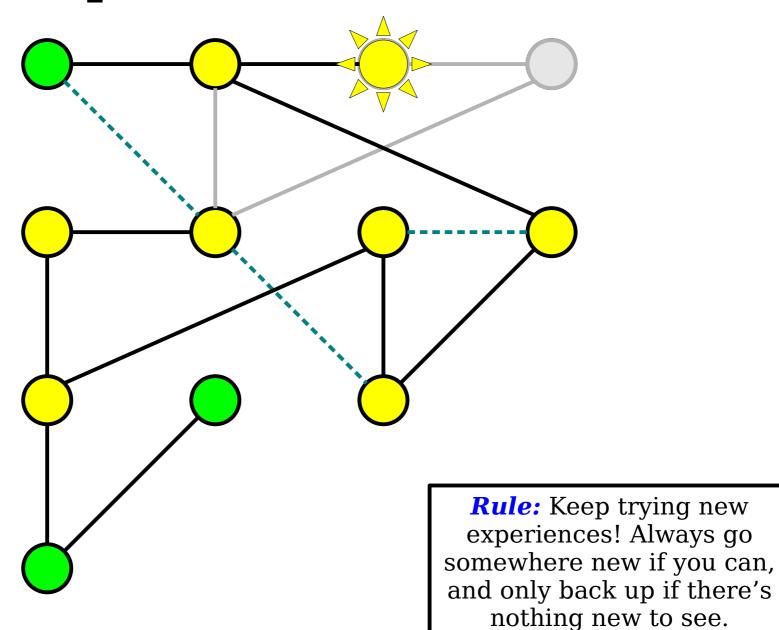


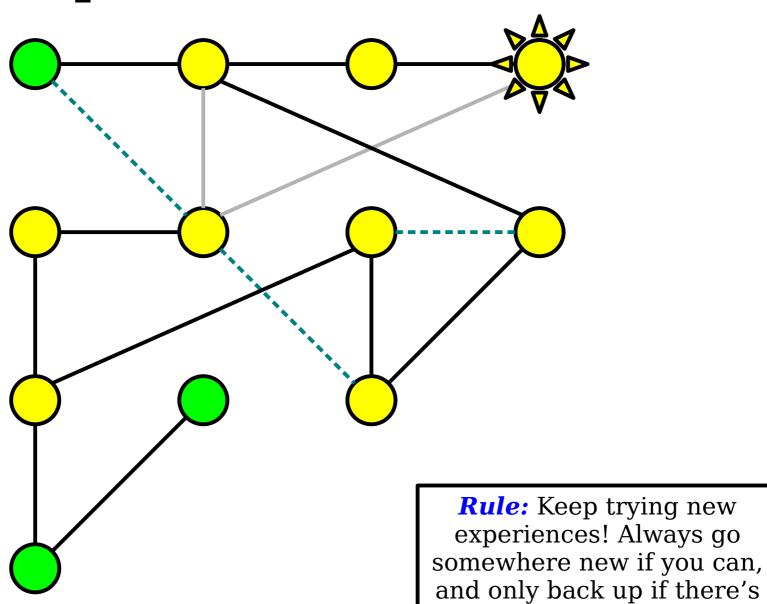


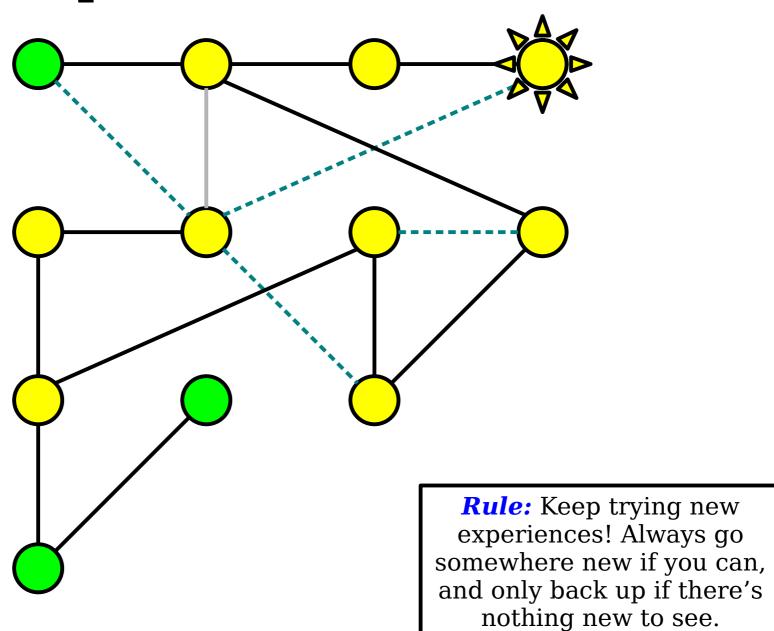


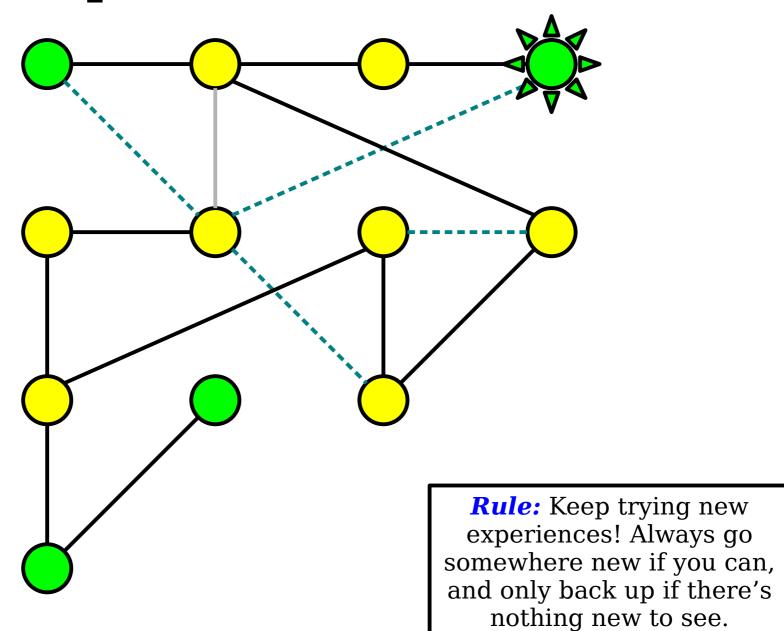


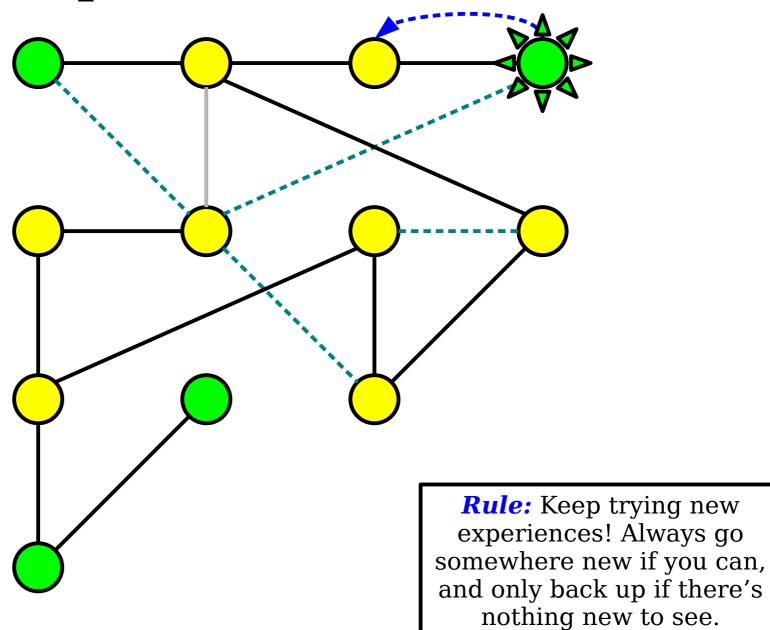


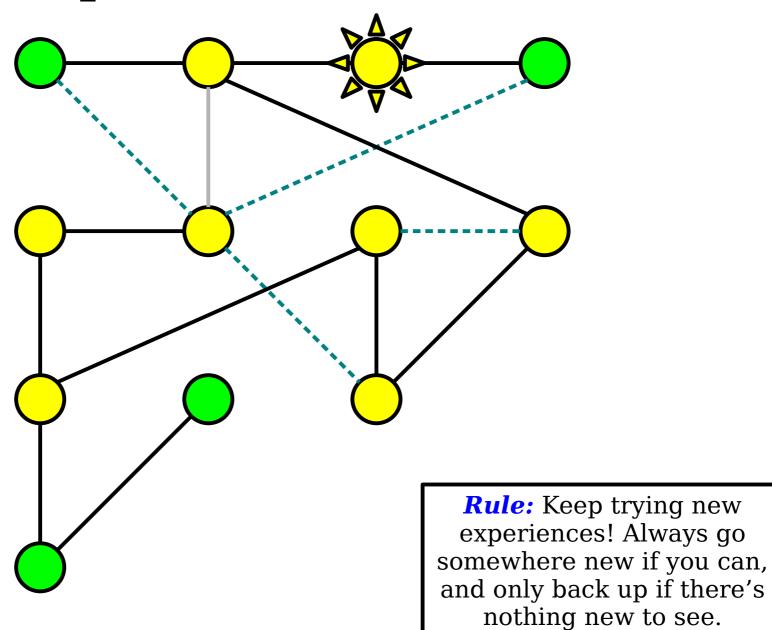


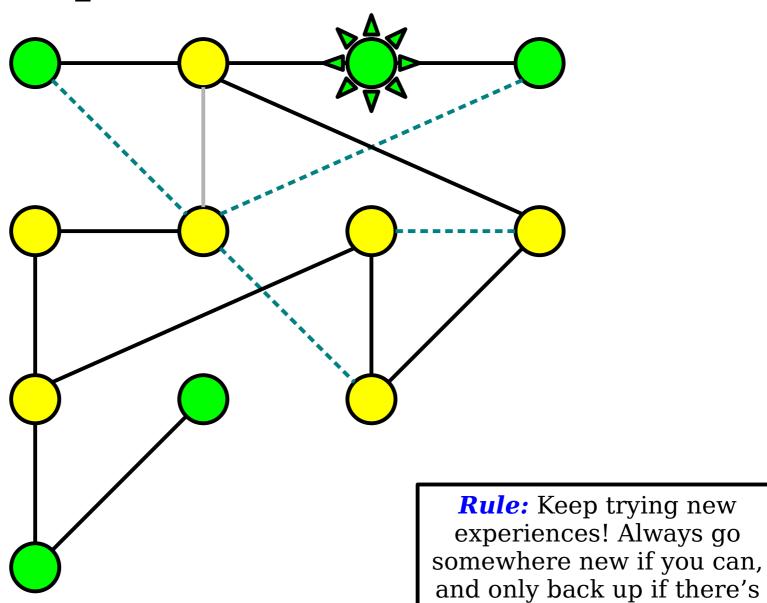


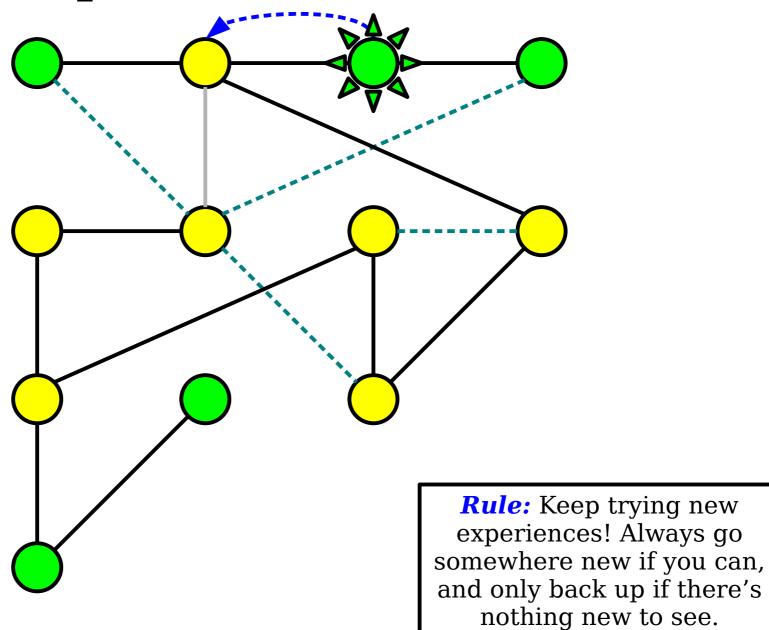


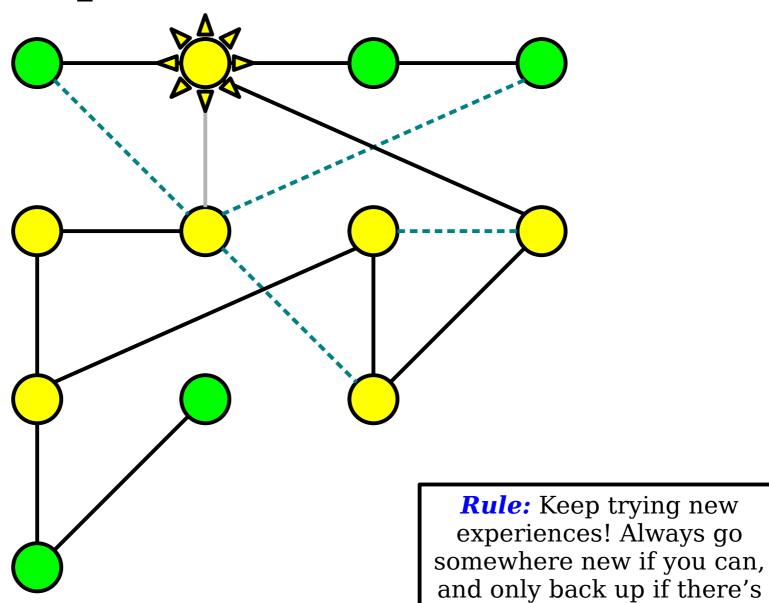


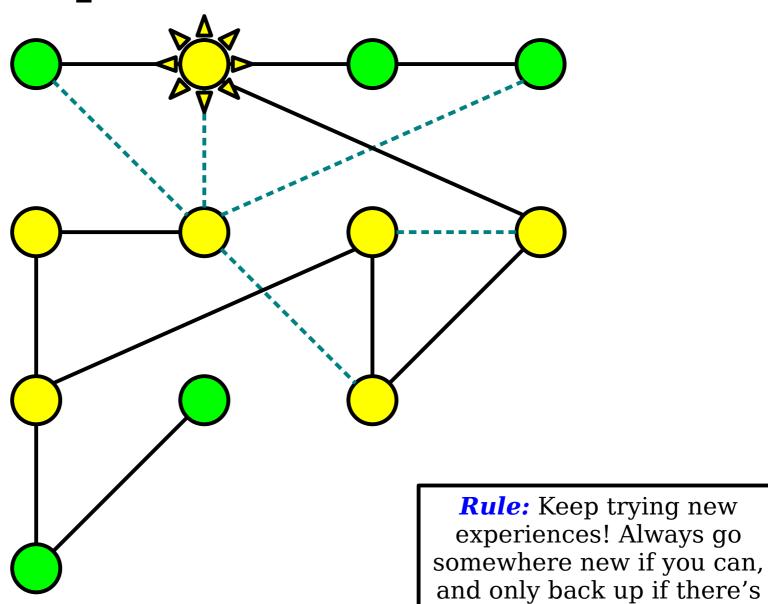


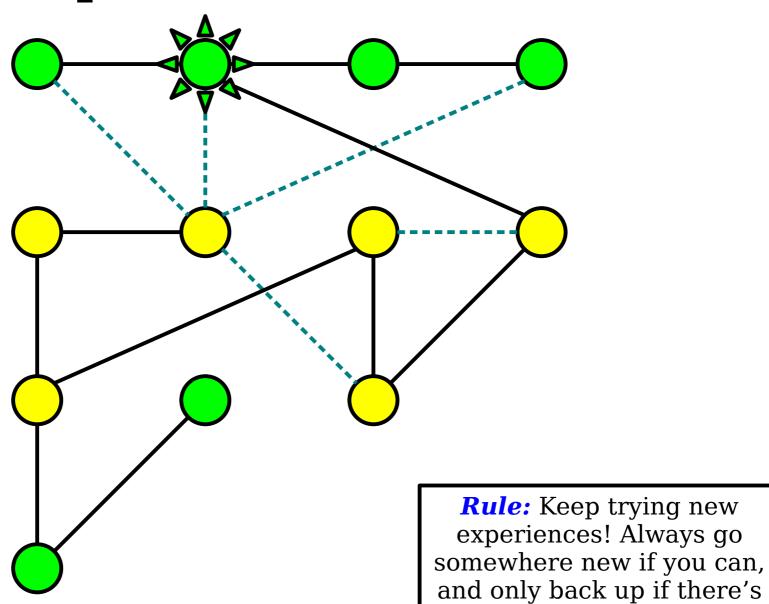


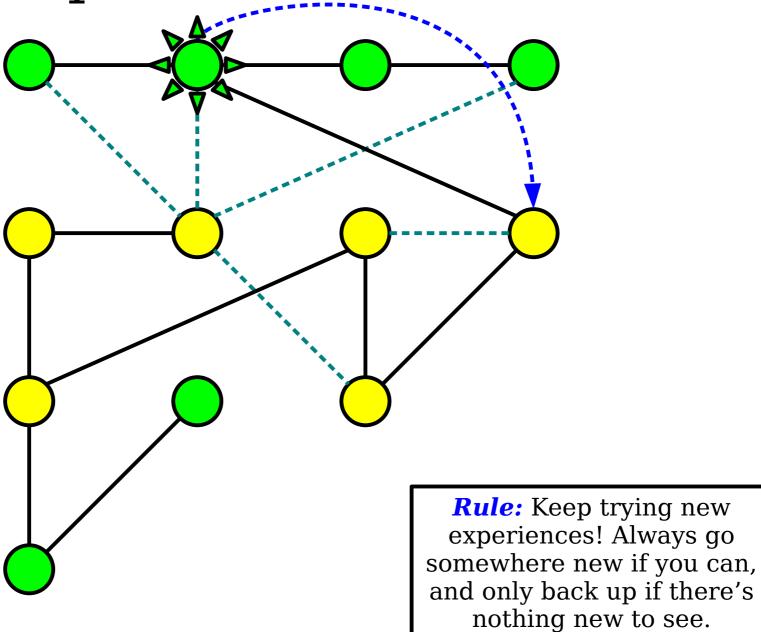


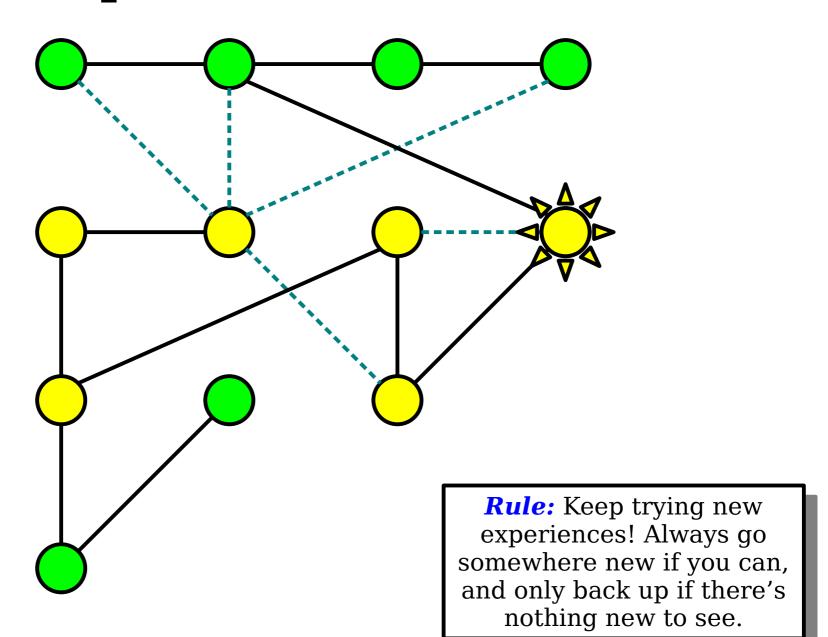


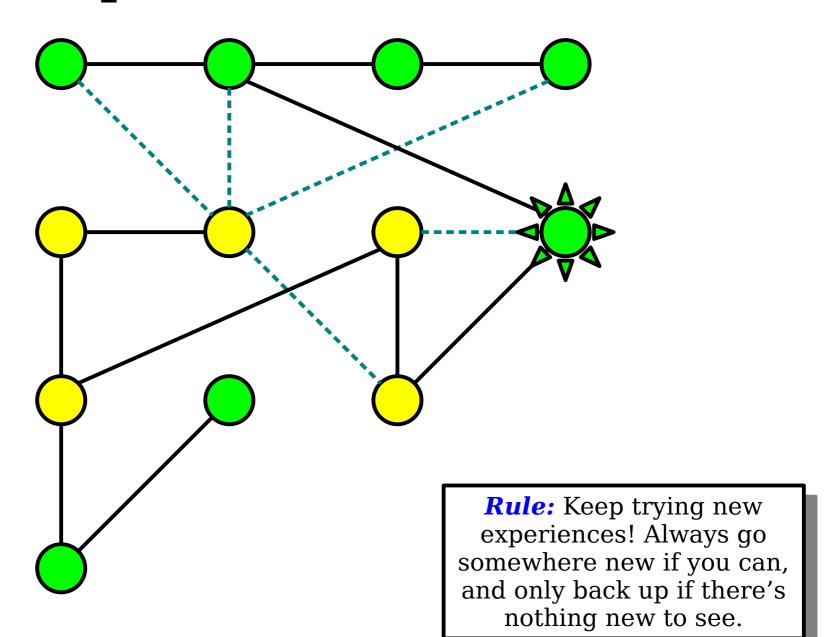


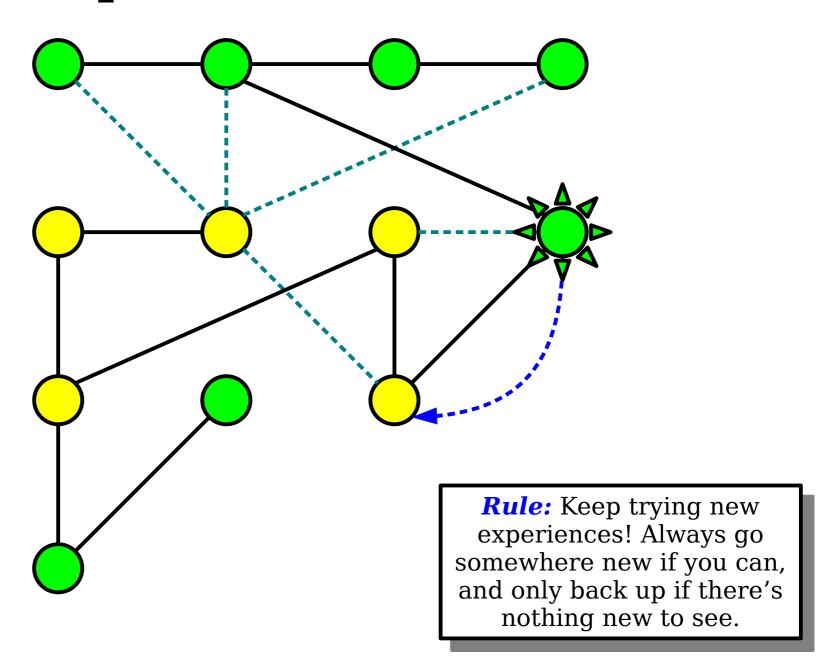


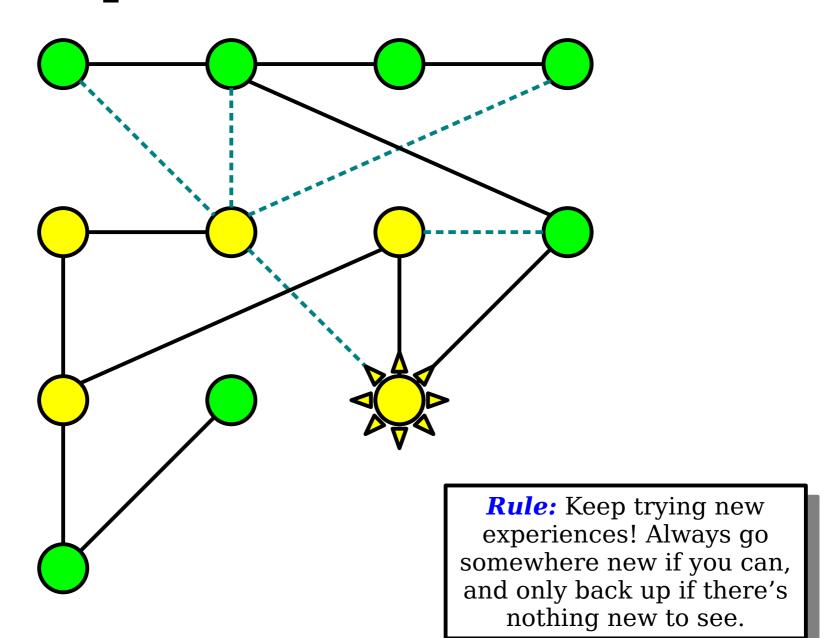


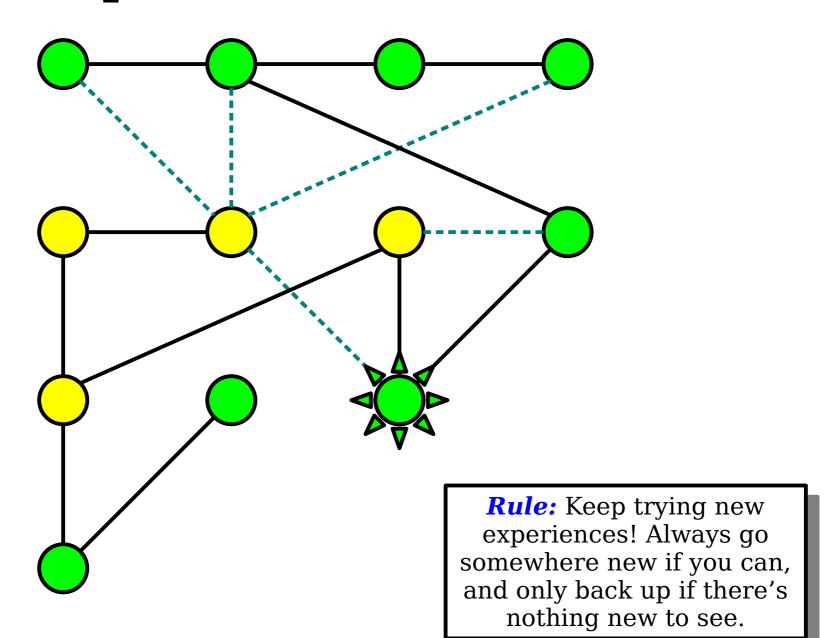


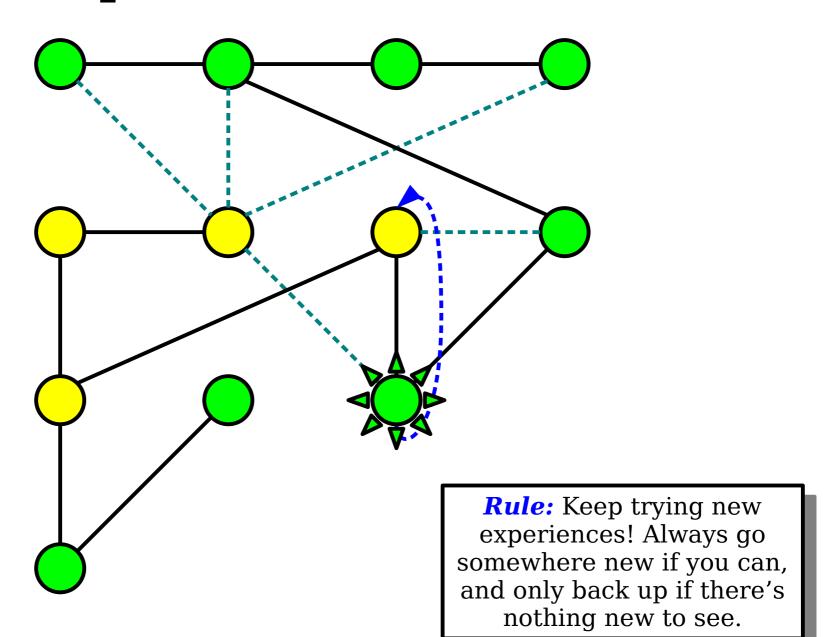


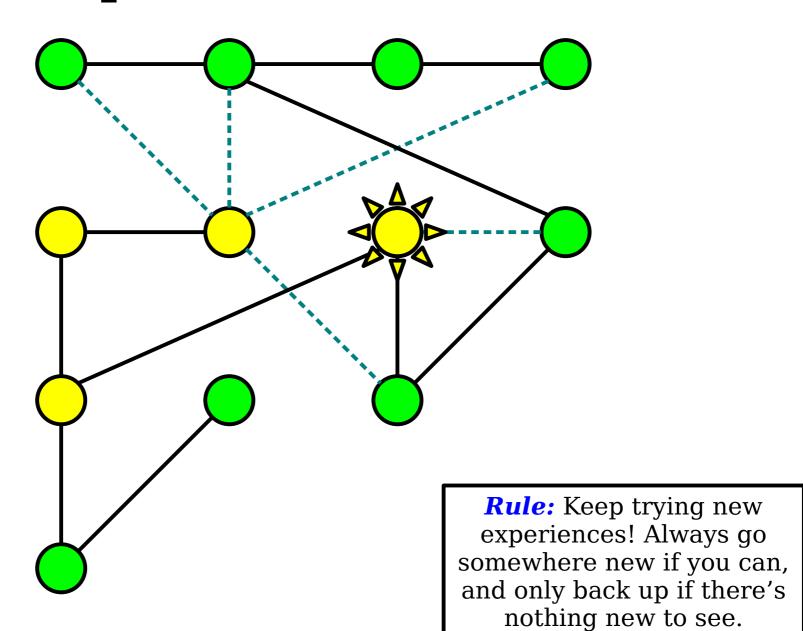


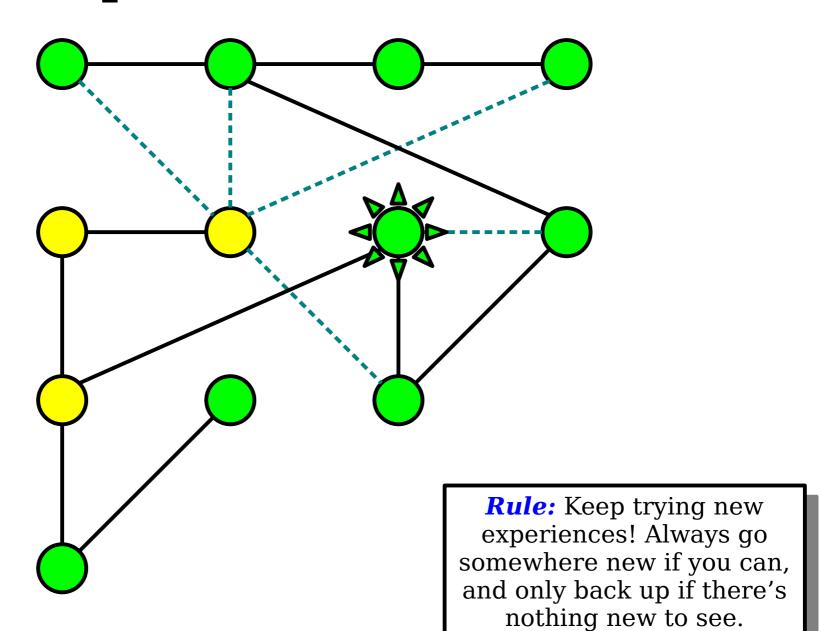


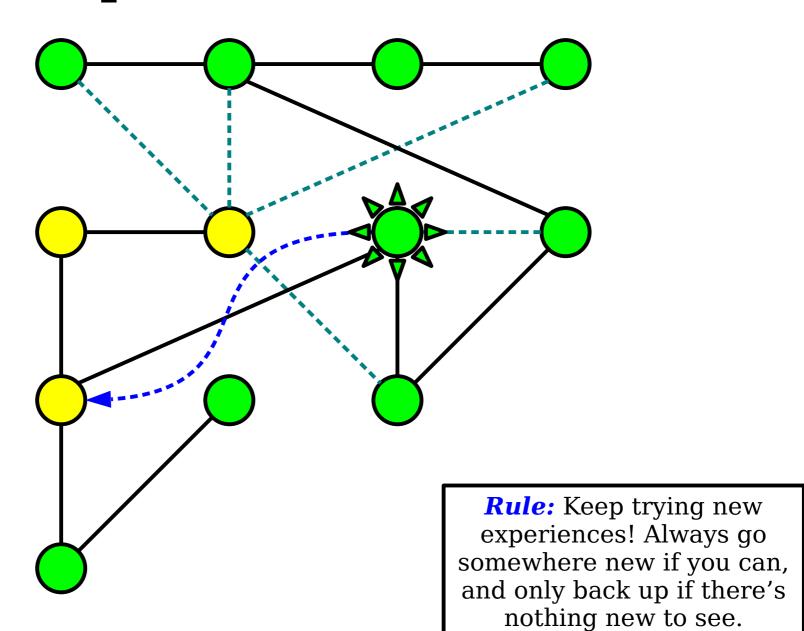


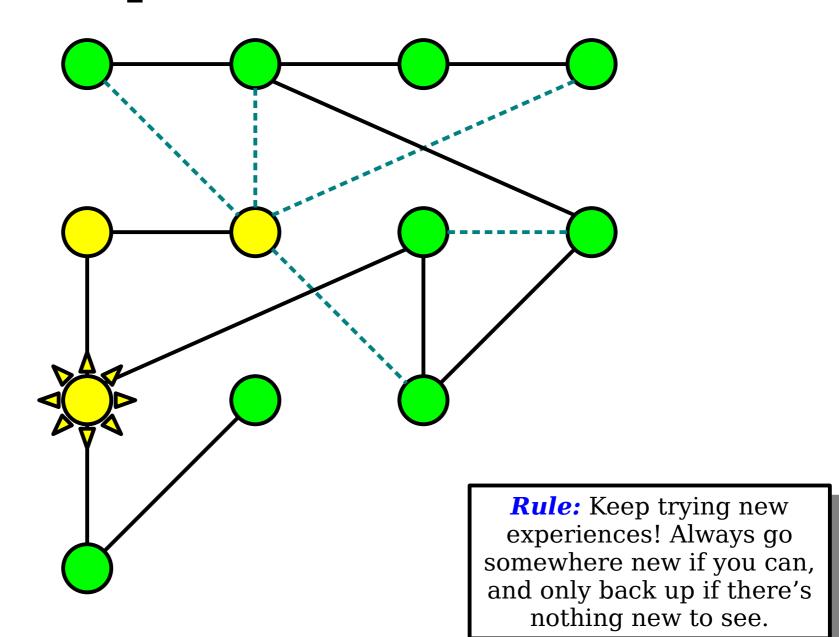


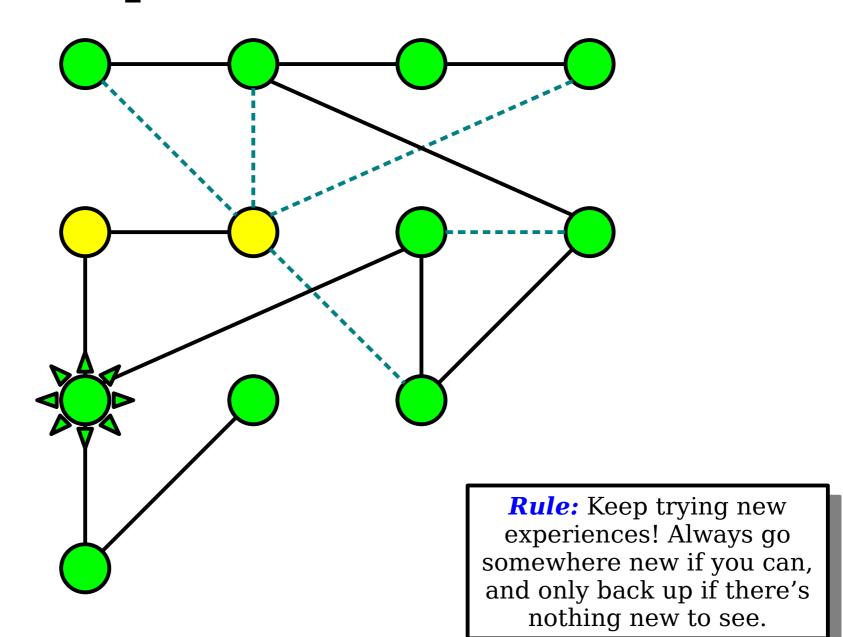


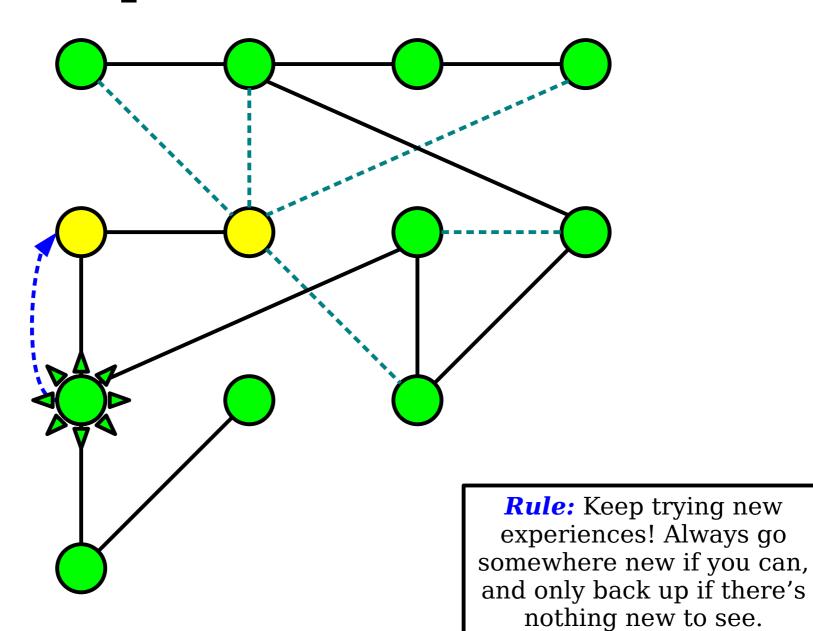


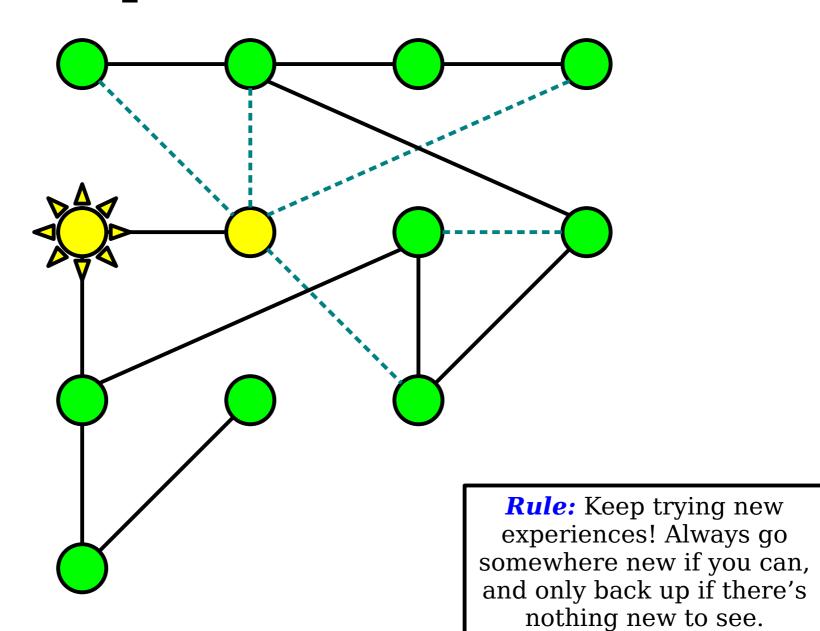


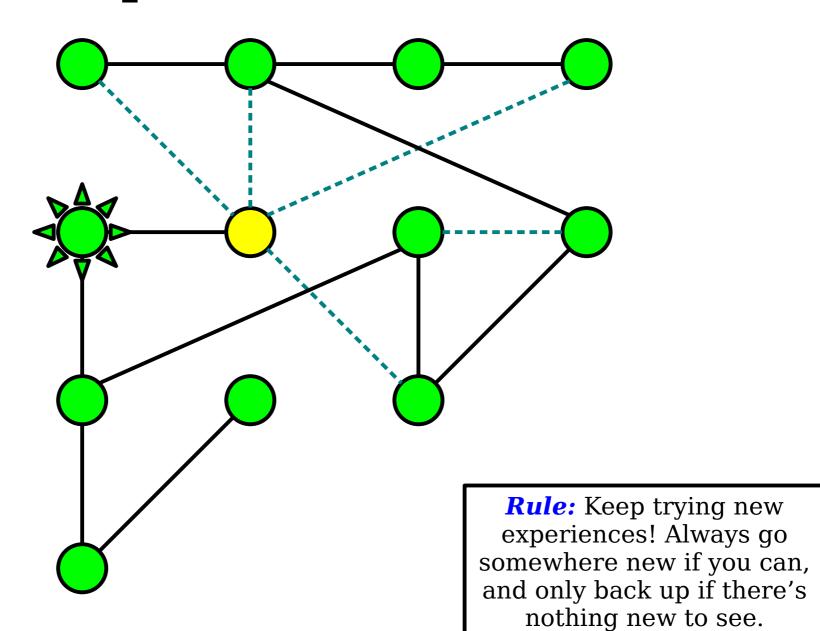


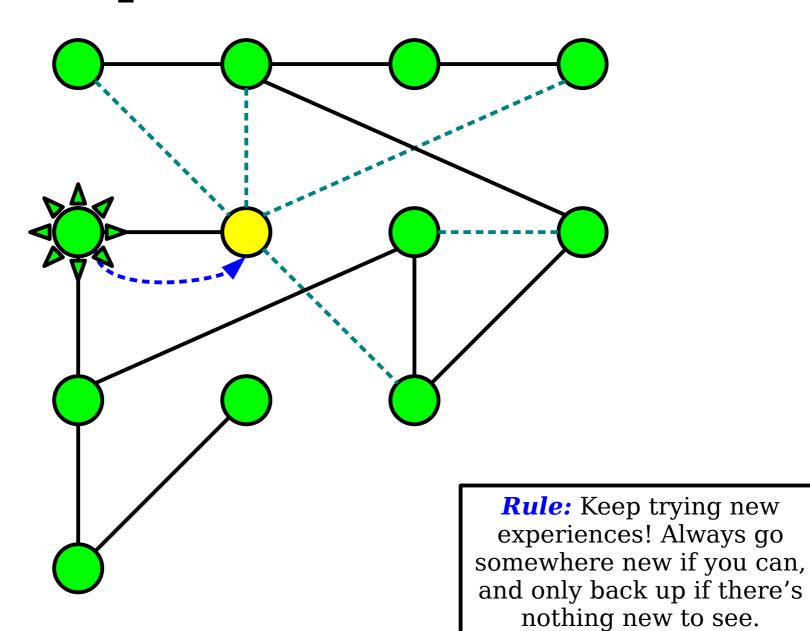


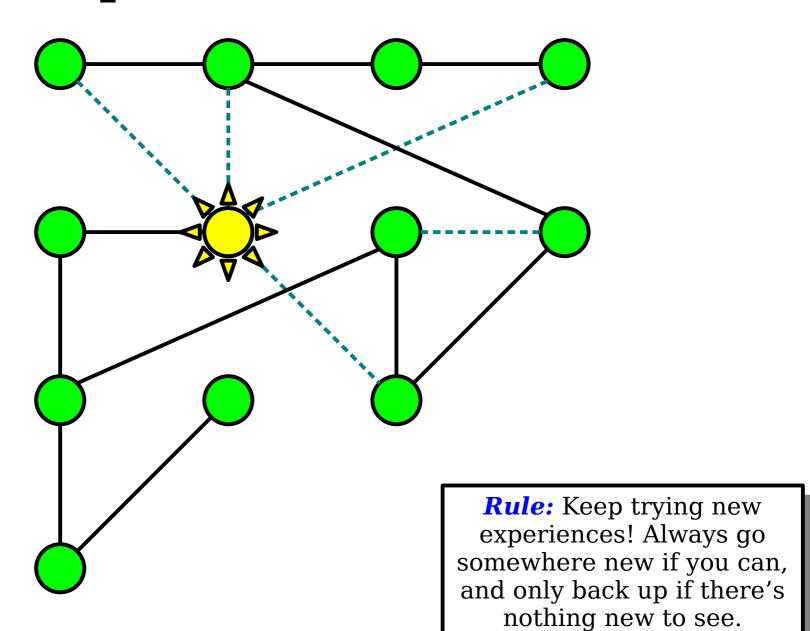


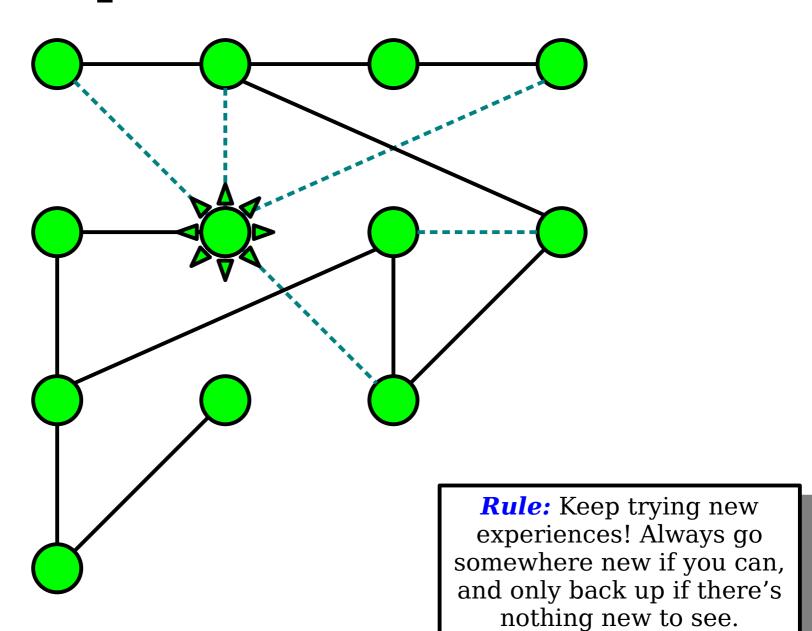


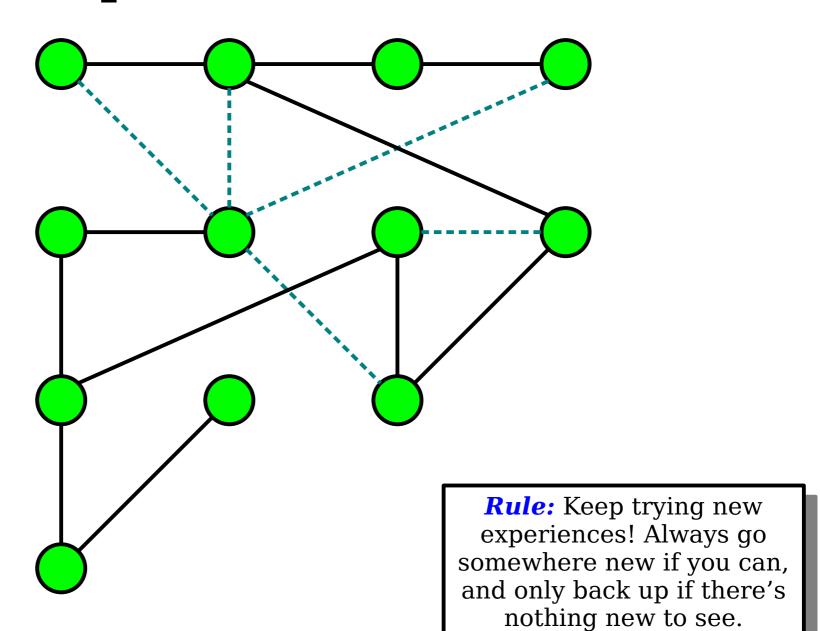












How can we implement this?

Breadth-First Search

Queue: X Q V A L

I've just been discovered! Pay attention to me!

These nodes got here first, so they get processed first.

```
bfs-from(node v) {
    make a queue of nodes, initially seeded with v.
    while the queue isn't empty:
        dequeue a node curr.
        process the node curr.

    for each node adjacent to curr:
        if that node has never been enqueued:
        enqueue that node.
}
```

Stack: X Q V A L

I've just been discovered! Pay attention to me!

Oooh! A shiny new node! Who cares about these ones?

For Comparison

For Comparison

```
dfs-from(node v) {
    if this is first time we've called dfs-from(v):
        process node v
        for each node adjacent to v:
        call dfs-from on that node
}
```

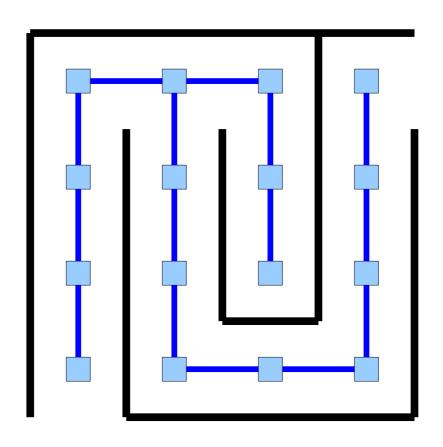
When you see a stack-based algorithm, think recursion!

BFS and DFS

- Running BFS or DFS from a node in a graph will visit the same set of nodes, but probably in a different order.
- BFS will visit nodes in increasing order of distance.
- DFS does visit nodes in *some* interesting order, but not order of distance.
 - Take CS161 for more details!

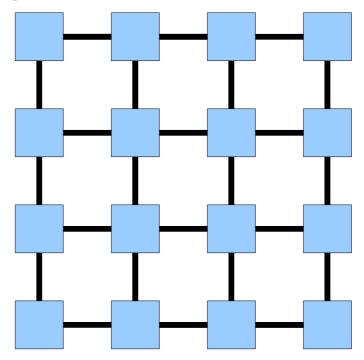
A Whimsical Application

Mazes as Graphs



Creating a Maze with DFS

• Create a *grid graph* of the appropriate size.



- Starting at any node, run a depth-first search, choosing neighbor orderings at random.
- The resulting DFS tree is a maze with one solution.

Next Time

- Minimum Spanning Trees
 - How to wire an electrical grid cheaply.
- Applications of MSTs
 - Data clustering, computational biology, and more!