331 – Intro to Intelligent Systems
Week 05
Game Theory II
Mixed Strategies
R&N Chapter 17.5, 17.6

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Mixed Strategies

- Mixed strategies allow us to deal with unpredictability (chance events)
- Unpredictability can be the result of:
  - Uncertainty about the outcome of an event
  - Uncertainty about the structure of the game
  - Uncertainty about the pure strategy of a player

Mixed Strategies

- Many games do not have a pure strategy Nash equilibrium
  - In these cases, using a pure strategy will not lead to an optimal strategy for both players
  - In 1928 von Neumann showed that you can always find an optimal strategy for each player in a two-player zero-sum game if each player is allowed to use a *mixed strategy* – (sometimes you do one thing, and sometimes you do something else)
    - Nash later extended this result to non-zero sum games and non-cooperative games with more than two players

The Even-Odd Game

- Two players, Even and Odd
- Each player picks a number, 1, 2, or 3
- If the sum of the two numbers picked is even then Odd has to pay Even that amount
- If the sum of the two numbers picked is odd then Even has to pay Odd that amount
- For example, if Even picks 2 and Odd picks 3 then Even pays Odd $5 (Even’s payoff is -5 and Odd’s payoff is 5)

The Even-Odd Game

		Odd player		
		1	2	3
Even player	1	2, -2	-3, 3	4, -4
	2	-3, 3	4, -4	-5, 5
	3	4, -4	-5, 5	6, -6

This game has no pure Nash equilibrium. If the total is even, then Odd will want to change. If the total is odd, then Even will want to change.

The Even-Odd Game

- Assume the players do not know what their best mixed strategies are, so they arbitrarily choose to play each number a certain percentage of the time
  - For example, suppose Odd chooses 1 30% of the time, 2 60% of the time, and 3 10% of the time

		Odd player		
		1 (0.3)	2 (0.6)	3 (0.1)
Even player	1	2, -2	-3, 3	4, -4
	2	-3, 3	4, -4	-5, 5
	3	4, -4	-5, 5	6, -6

The Even-Odd Game

- What should Even do?
- What is the average payoff for Even, assuming she knows what Odd's mixed strategy is?
  - The average payoff is computed as the sum of every

		Odd player			Even's expected payoff
		1 (0.3)	2 (0.6)	3 (0.1)	
Even player	1	2, -2	-3, 3	4, -4	0.3(2) + 0.6(-3) + 0.1(4) = -0.8
	2	-3, 3	4, -4	-5, 5	0.3(-3) + 0.6(4) + 0.1(-5) = 1.0
	3	4, -4	-5, 5	6, -6	0.3(4) + 0.6(-5) + 0.1(6) = -1.2

The Even-Odd Game

- If Even always plays a fixed strategy of 2, then Odd will want to change his strategy to playing 3 100% of the time, so he can win \$5 each round

		Odd player			Even's expected payoff
		1 (0.3)	2 (0.6)	3 (0.1)	
Even player	1 (0.4)	2, -2	-3, 3	4, -4	0.3(2) + 0.6(-3) + 0.1(4) = -0.8
	2 (0.5)	-3, 3	4, -4	-5, 5	0.3(-3) + 0.6(4) + 0.1(-5) = 1.0
	3 (0.1)	4, -4	-5, 5	6, -6	0.3(4) + 0.6(-5) + 0.1(6) = -1.2
	Odd's expected payoff	0.4(-2) + 0.5(3) + 0.1(-4) = 0.3	0.4(3) + 0.5(-4) + 0.1(5) = -0.3	0.4(-4) + 0.5(5) + 0.1(-6) = 0.3	

The Even-Odd Game

- But what is the <u>best</u> mixed strategy for each player?
  - What are the optimal percentages for each option?
  - The best mixed strategy for both players is the mixed Nash equilibrium
  - If your opponent plays his or her best strategy, then you cannot do any worse
  - If your opponent makes a mistake, then you can do better

Optimal Even-Odd Game

- Consider a simpler version of the Even-Odd game, where each player can say only 1 or 2

	Odd says 1	Odd says 2
Even says 1	2, -2	-3, 3
Even says 2	-3, 3	4, -4

Optimal Even-Odd Game

- Both Odd and Even should play “1” 7/12th of the time, and “2” 5/12th of the time

			Subtract Even's payoffs in column 2	Subtract Even's payoffs in column 1
			4 - (-3) = 7 7/12	2- (-3) = 5 5/12
			Odd says 1	Odd says 2
Subtract Odd's payoffs in row 2	3 - (-4) = 7 7/12	Even says 1	2, -2	-3, 3
Subtract Odd's payoffs in row 1	3 - (-2) = 5 5/12	Even says 2	-3, 3	4, -4

Optimal Even-Odd Game

- The expected payoff for Even, playing a “1” in the optimal game is 7/12(2) + 5/12(-3) = -1/12
- The expected payoff for Even, playing a “2” in the optimal game is 7/12(-3) + 5/12(4) = -1/12
  - Even loses 1/12 of a dollar on average each game
- Similarly, the expected payoff for Odd, playing a “1” or a “2” in the optimal game is +1/12
- Optimal game does not mean both players win. It means that this it the *best both players can do under the circumstances* (it’s not a fair game!)

Non-Zero-Sum Games

- In a zero-sum game, each player’s payoff is inversely linked to the other – if one player does well, then the other player does equally badly
- In a non-zero-sum game, a player’s payoff is not necessarily affected by how well the other player does
  - “I don’t mind if you do well, as long as I am not worse off”

Non-Zero-Sum Games

- Consider working with a colleague on a project. You both get a bonus of \$6000 if the project is excellent, a bonus of \$3000 if the project is good, and a bonus of \$2000 if the project is fair.
- In order to get an excellent project, you both have to put in \$2000 of extra work. If only one of you does the extra work, you end up with a good project. If neither of you put in extra work the project will be fair

	Colleague works extra	**Colleague slacks**
You work extra	**4, 4**	**1, 3**
You slack	**3, 1**	**2, 2**

Non-Zero-Sum Games

- The best overall strategy (the payoff dominant one) if for both of you to work extra
- But if you think your colleague might slack, then it is better for you to slack also
- There is nothing wrong with this line of reasoning – it is logically consistent
- However, *communication*, coupled with a *binding agreement* (a cooperative strategy) will lead to a better outcome for all involved