4/5

1

Alex Jacob
Prof. Barlow

MATH 231-S4
Exam 2

1) $\frac{dy}{dt} + \frac{2}{t} y = \frac{e^t}{t^2}$, $y(1) = 2$

(y_h) $\frac{dy_h}{dt} + \frac{2}{t} y_h = 0 \Rightarrow \frac{dy_h}{dt} = -\frac{2}{t} y_h \Rightarrow \int \frac{1}{y_h} dy_h = \int -2/t \, dt$

$\Rightarrow \ln|y_h| = -2\ln|t| + \tilde{c} \Rightarrow e^{\ln|y_h|} = e^{-2\ln|t| + \tilde{c}} \Rightarrow y_h = e^{\tilde{c}} \cdot e^{-2\ln|t|}$

$\Rightarrow y_h = \tilde{c} \cdot e^{\ln|t^{-2}|} \Rightarrow y_h = \tilde{c} \cdot t^{-2}$

(y_p) $y_p = v(t) \cdot t^{-2} \Rightarrow y_p' = v'(t) \cdot t^{-2} + v(t) \cdot (-2t^{-3})$

$\frac{dy_p}{dt} + \frac{2}{t} y_p = \frac{e^t}{t^2} \Rightarrow \frac{v'(t)}{t^2} + \cancel{\frac{v(t) \cdot -2}{t^3}} + \cancel{\frac{2v(t)}{t \cdot t^2}} = \frac{e^t}{t^2}$

$\Rightarrow \frac{v'(t)}{t^2} = \frac{e^t}{t^2} \Rightarrow v'(t) = e^t \Rightarrow v(t) = e^t$

$y_p = \frac{e^t}{t^2} \Rightarrow y = y_h + y_p \Rightarrow y = \frac{\tilde{c}}{t^2} + \frac{e^t}{t^2} \Rightarrow y = \frac{e^t + \tilde{c}}{t^2}$

$y(1) = 2 \Rightarrow 2 = \frac{e^1 + \tilde{c}}{1^2} \Rightarrow 2 = e + \tilde{c} \Rightarrow 2 - e = \tilde{c}$

$\Rightarrow \boxed{y = \frac{e^t + 2 - e}{t^2}}$

2) $\frac{dy}{dt} + \cos(t)\, y = \cos(t)$

(y_h) $\frac{dy_h}{dt} + \cos(t)\, y_h = 0 \Rightarrow \frac{dy_h}{dt} = -\cos(t)\, y_h \Rightarrow \int \frac{dy_h}{dy} = \int -\cos(t)\, dt$

$\Rightarrow \ln|y_h| = -\sin(t) + C \Rightarrow e^{\ln|y_h|} = e^{-\sin(t)+C} \Rightarrow y_h = e^{C} \cdot e^{-\sin(t)}$

$\Rightarrow y_h = \tilde{C} \cdot e^{-\sin(t)}$

(y_p) $y_p = v(t) \cdot e^{-\sin(t)} \Rightarrow y_p' = v'(t) \cdot e^{-\sin(t)} + v(t) \cdot (-\cos(t) \cdot e^{-\sin(t)})$

$\frac{dy_p}{dt} + \cos(t)\, y_p = \cos(t)$

$\Rightarrow \frac{v'(t)}{e^{\sin(t)}} + \cancel{\frac{-v(t)\cos(t)}{e^{\sin(t)}}} + \cancel{\frac{\cos(t)\, v(t)}{e^{\sin(t)}}} = \cos(t)$

$\Rightarrow \frac{v'(t)}{e^{\sin(t)}} = \cos(t) \Rightarrow v'(t) = e^{\sin(t)} \cdot \cos(t)$

$v(t) = e^{\sin(t)}$

$y_p = e^{\sin(t)} \cdot e^{-\sin(t)} = 1$

$y = y_h + y_p \Rightarrow \tilde{C} \cdot e^{-\sin(t)} + 1 \Rightarrow \boxed{y = \tilde{C} e^{-\sin(t)} + 1}$

3) $y'' + by' = 0$

$y = C_1 + C_2 e^{-bt}$ * C_1 is alone therefore a root is 0 *

a) $r^2 C e^{rt} + brCe^{rt} \Rightarrow r^2 + br = 0 \Rightarrow r(r+b) = 0$ let $b = 5$

$r_1 = 0,\ r_2 = -b$

b) $y'' + 5y' = 0$

$r^2 Ce^{rt} + 5rCe^{rt} = 0 \Rightarrow r^2 + 5r = 0 \Rightarrow r(r+5) = 0$

$r_1 = 0,\ r_2 = -5$

$y = C_1 e^{0 \cdot t} + C_2 e^{-5 \cdot t} \Rightarrow y = C_1 + C_2 e^{-5t}$

c) $y = C_1 + C_2 e^{-5t}$ $y(0) = 0,\ y'(0) = 1$

(y) $0 = C_1 + C_2 e^{-5 \cdot (0)}$
$0 = C_1 + C_2$

$y' = -5C_2 e^{-5t}$
(y') $1 = -5C_2 e^{-5 \cdot (0)}$
$1 = -5C_2$
$\boxed{-1/5 = C_2}$

$0 = C_1 + C_2 \Rightarrow 0 = C_1 - 1/5 \Rightarrow \boxed{C_1 = 1/5}$

$y = (1/5) + (-1/5)e^{-5t}$

$\boxed{y = (1/5) - (1/5)e^{-5t}}$

4) a) $b = 0$ * there is no $e^{(x)}$ in front of the equation

$c = 4$ * we must get $\pm 2i$ from quadratic formula *

$$\pm 2i = \frac{-(0) \pm \sqrt{0^2 - 4(1)(c)}}{2}$$

$$\pm 2i = \frac{\pm\sqrt{-4c}}{2} \Rightarrow \pm 4i = i\sqrt{4c} \Rightarrow \pm 4i = \pm 2i\sqrt{c} \Rightarrow$$

$$\Rightarrow 2 = \sqrt{c}$$

$$\Rightarrow \boxed{4 = c}$$

b) $y'' + 0y' + 4y = 0$

$$y'' + 4y = 0$$

$$r^2(e^{rt}) + 4(e^{rt}) = 0$$

$$r^2 + 4 = 0$$

$$x = \frac{-0 \pm \sqrt{0^2 - 4(1)(4)}}{2(1)} \Rightarrow x = \frac{\pm\sqrt{-16}}{2} \Rightarrow x = \frac{\pm 4i}{2} \Rightarrow x = \pm 2i$$

$r_1 = +2i$, $r_2 = -2i$

$$y = C_1 e^{2it} + C_2 e^{-2it}$$

$$\boxed{y = C_1 \cos(2t) + C_2 \sin(2t)}$$

5

5) $\frac{d^7y}{dt^7} + \frac{2d^6y}{dt^6} + \frac{2d^5y}{dt^5} = 0$

$r^7 Ce^{rt} + 2r^6 Ce^{rt} + 2r^5 Ce^{rt} = 0$

$r^7 + 2r^6 + 2r^5 = 0$

$r^5(r^2 + 2r + 2) = 0$

$x = \frac{-2 \pm \sqrt{2^2 - 4(1)(2)}}{2(1)} \Rightarrow x = \frac{-2 \pm \sqrt{4-8}}{2} \Rightarrow x = \frac{-2 \pm \sqrt{-4}}{2} \Rightarrow x = \frac{-2 \pm 2i}{2}$

$x = -1 \pm i$

$r_1 = r_2 = r_3 = r_4 = r_5 = 0 \qquad r_6 = -1+i, \; r_7 = -1-i$

$y = C_1 e^{r_1 t} + tC_2 e^{r_2 t} + t^2 C_3 e^{r_3 t} + t^3 C_4 e^{r_4 t} + t^4 C_5 e^{r_5 t} + C_6 e^{(-1+i)t} + C_7 e^{(-1-i)t}$

$y = C_1 e^{0t} + tC_2 e^{0t} + t^2 C_3 e^{0t} + t^3 C_4 e^{0t} + t^4 C_5 e^{0t} + e^{-t} C_6 \cos(t) + e^{-t} C_7 \sin(t)$

$\boxed{y = C_1 + tC_2 + t^2 C_3 + t^3 C_4 + t^4 C_5 + e^{-t}\left[C_6 \cos(t) + C_7 \sin(t)\right]}$