HW #3

MATH-23754
HW#4

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Prof Barlow

1) $\frac{dx}{dt} \Rightarrow \frac{x_{n+1} - x_n}{\Delta t} = 40(y_n - x_n) + 0.16 x_n z_n$

$$\frac{y_{n+1} - y_n}{\Delta t} = 55 + 20 y_n - x_n z_n$$

$$\frac{z_{n+1} - z_n}{\Delta t} = -0.65 x_n^2 + x_n y_n + \frac{11}{6} z_n$$

$$x_{n+1} = \Delta t \left[40(y_n - x_n) + 0.16 x_n z_n \right] + x_n$$

$$y_{n+1} = \Delta t \left[55 + 20 y_n - x_n z_n \right] + y_n$$

$$z_{n+1} = \Delta t \left[-0.65 x_n^2 + x_n y_n + \frac{11}{6} z_n \right] + z_n$$

2) $\frac{d^2\theta}{dt^2} = \frac{-g}{L} \sin(\theta)$

$\frac{d\theta}{dt} = w$ $\qquad \frac{\theta_{n+1} - \theta_n}{\Delta t} = w_n$

$\frac{dw}{dt} = \frac{-g}{L} \sin(\theta)$ $\qquad \theta_{n+1} = [w_n \cdot \Delta t] + \theta_n$

$g = 9.81\ m/s^2$ $\qquad \frac{w_{n+1} - w_n}{\Delta t} = \frac{-g}{L} \sin(\theta_n) \cdot \Delta t$

$$w_{n+1} = \left[\frac{-g}{L} \sin(\theta_n) \cdot \Delta t \right] + w_n$$

- The smaller Δt becomes, the smaller in difference becomes. This is the essence of numerical convergence

3) $$\frac{d^2x}{dt^2} = \frac{-xmG}{(x^2+y^2)^{3/2}} \Rightarrow \frac{dx}{dt} = x \Rightarrow \frac{x_{n+1}-x_n}{\Delta t} = (Vx)_n = (Vx)_{n+1} = [Vx_n \cdot \Delta t] + x_n$$

$$\Rightarrow \frac{Vx_{n+1} - Vx_n}{\Delta t} = \frac{-x_n mG}{(x_n^2+y_n^2)^{3/2}} \Rightarrow Vx_{n+1} = \left[\frac{-x_n mG \cdot \Delta t}{(x_n^2+y_n^2)^{3/2}}\right] + Vx_n$$

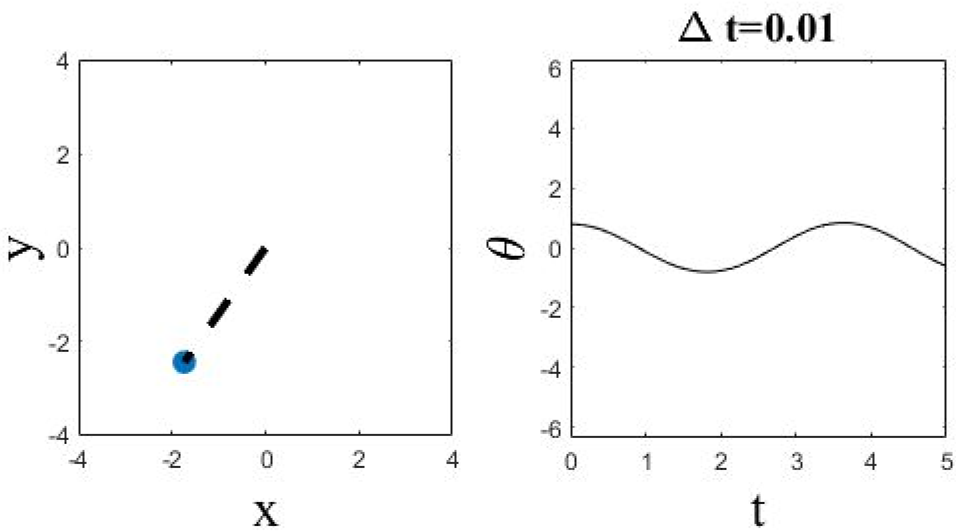
$$\frac{d^2y}{dt^2} = \frac{-ymG}{(x^2+y^2)^{3/2}}$$

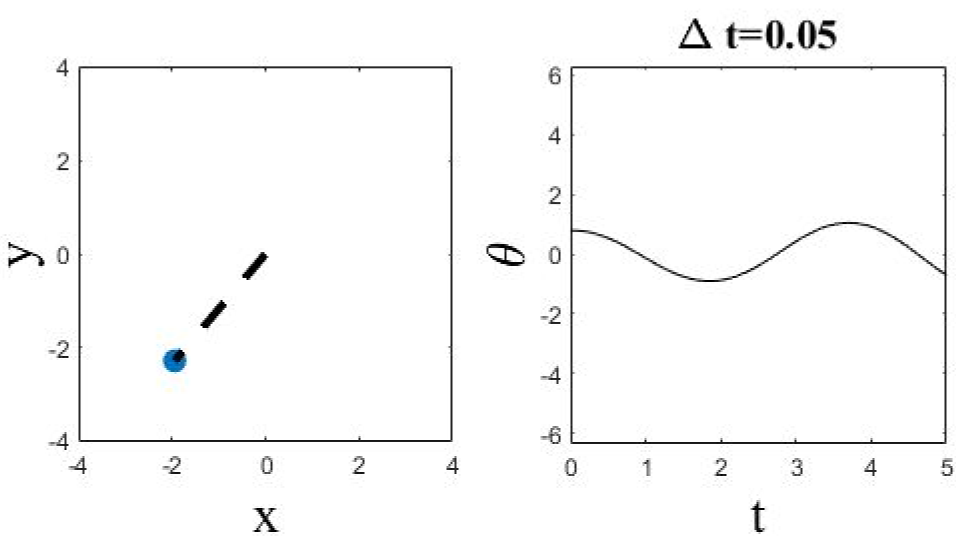
$$\Rightarrow \frac{dy}{dt} = y \Rightarrow y_{n+1} = [V_{y_n} \cdot \Delta t] + y_n$$

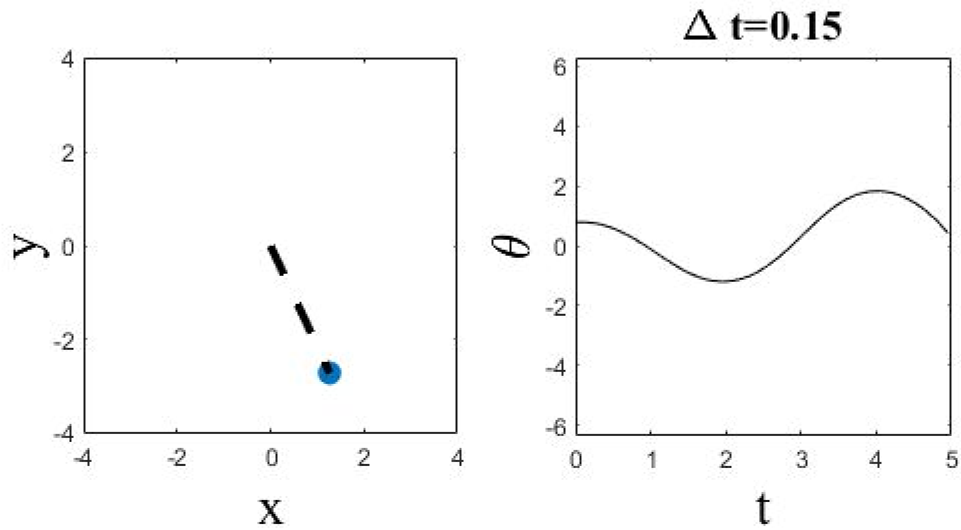
$$\Rightarrow \frac{Vy_{n+1} - V_{y_n}}{\Delta t} = \frac{-y_n mG}{(x_n^2+y_n^2)^{3/2}} = (Vy)_{n+1} = \left[\frac{-y_n mG \cdot \Delta t}{(x_n^2+y_n^2)^{3/2}}\right] + V_{y_n}$$

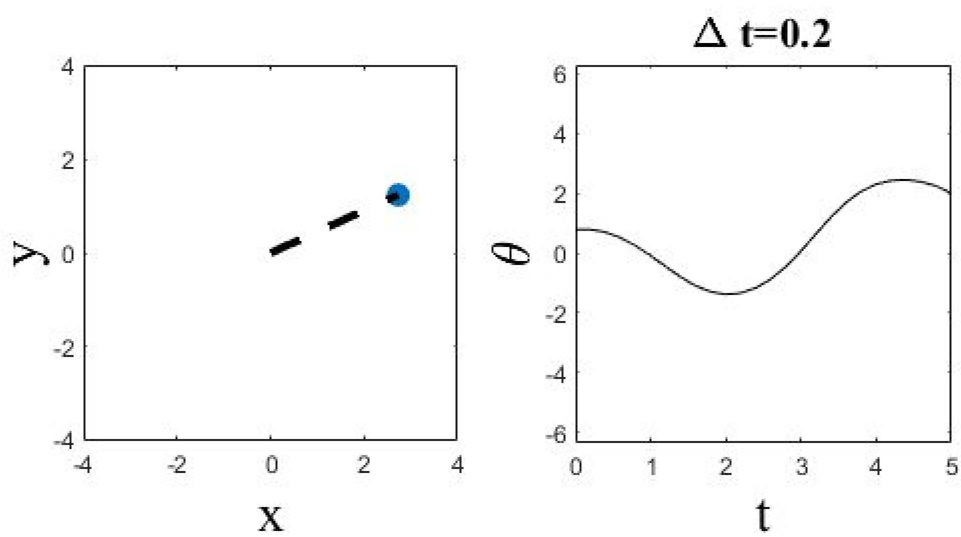
- The smaller Δt becomes, the difference between the graphs decreases until it becomes a perfect ellipse

Pendulum plots:

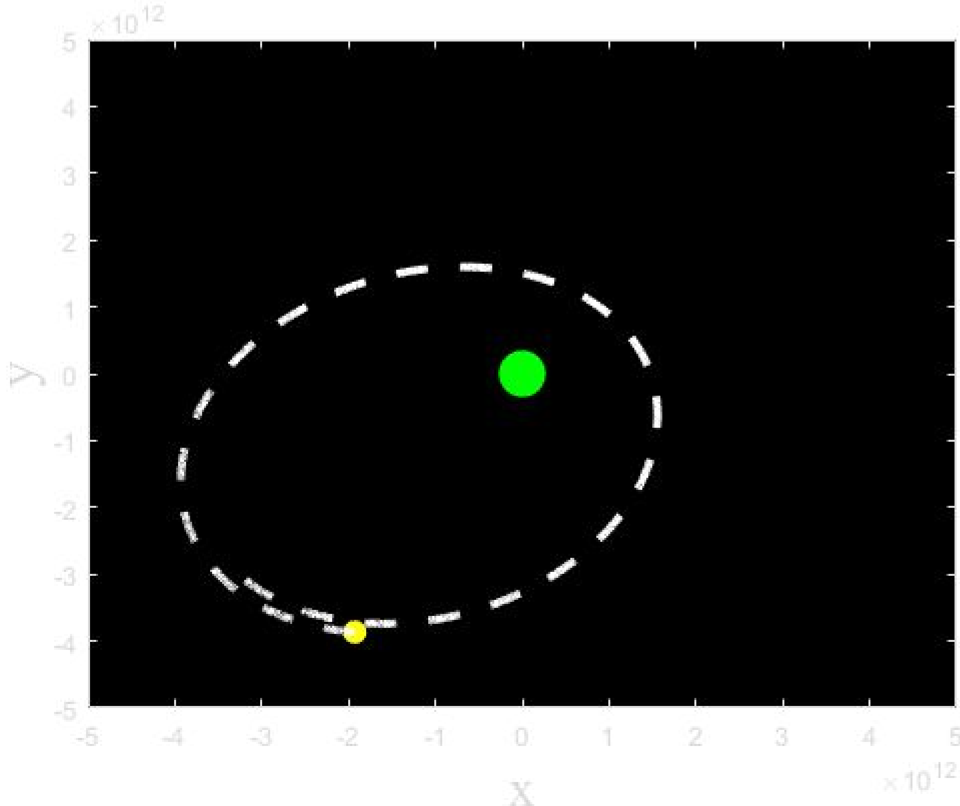


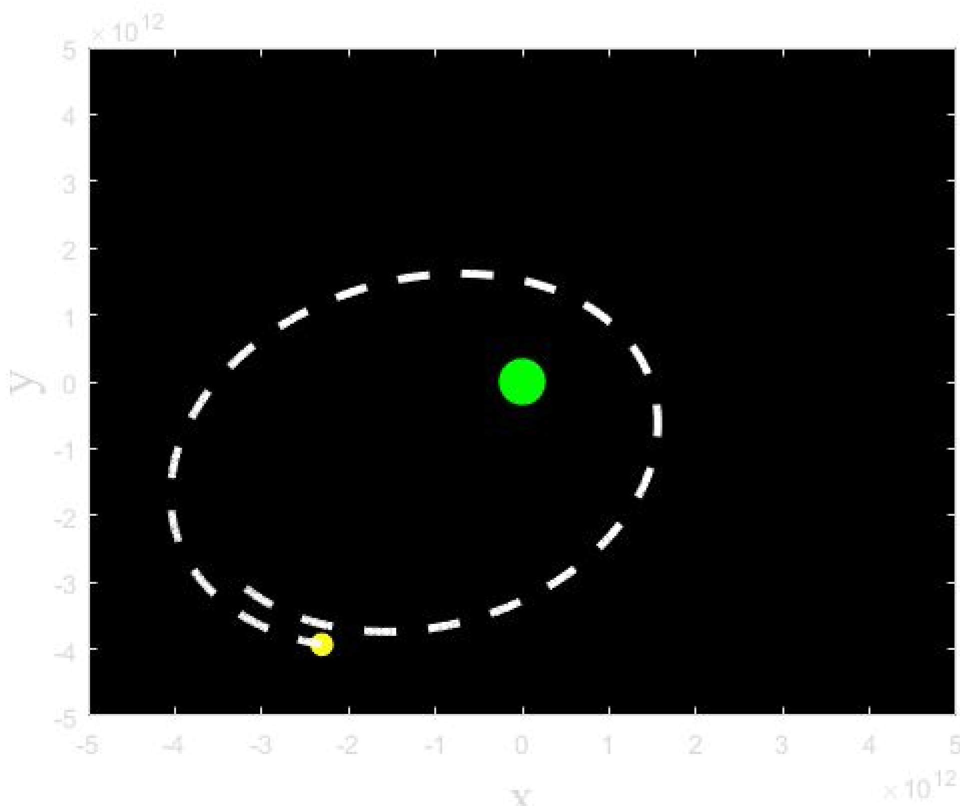


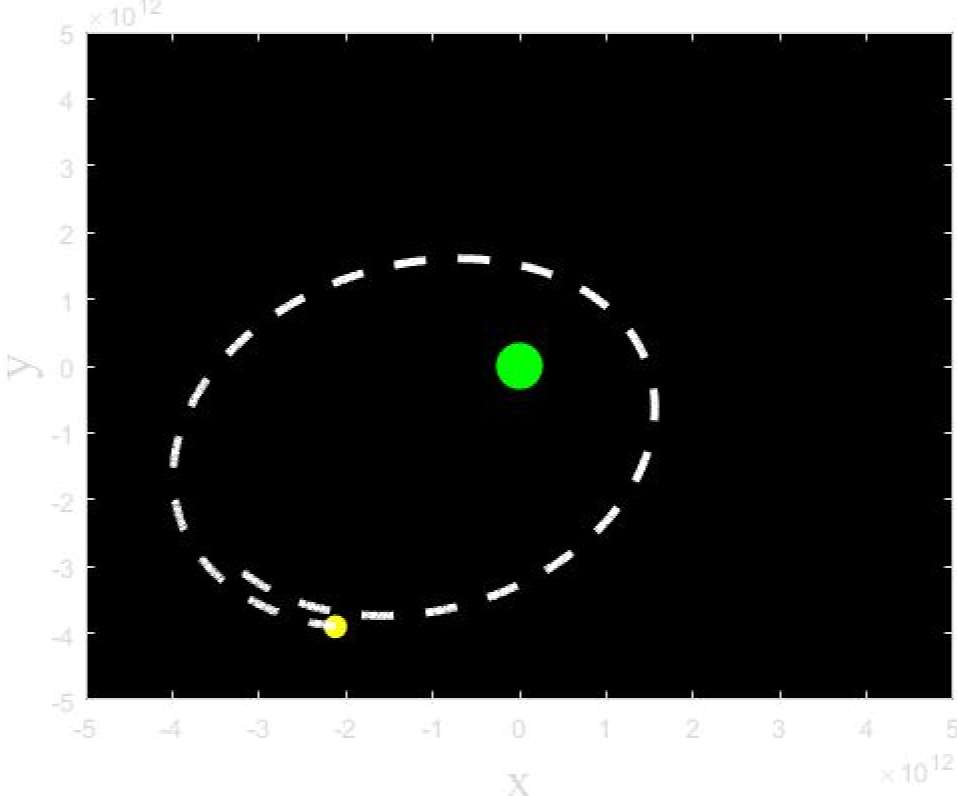

Δ t=0.15
4
2
0
-2
-4
y
-4
-2
0
2
4
x
6
4
2
0
-2
-4
-6
θ
0
1
2
3
4
5
t


Δ t=0.2
4
2
0
-2
-4
y
-4
-2
0
2
4
x
6
4
2
0
-2
-4
-6
θ
0
1
2
3
4
5
t

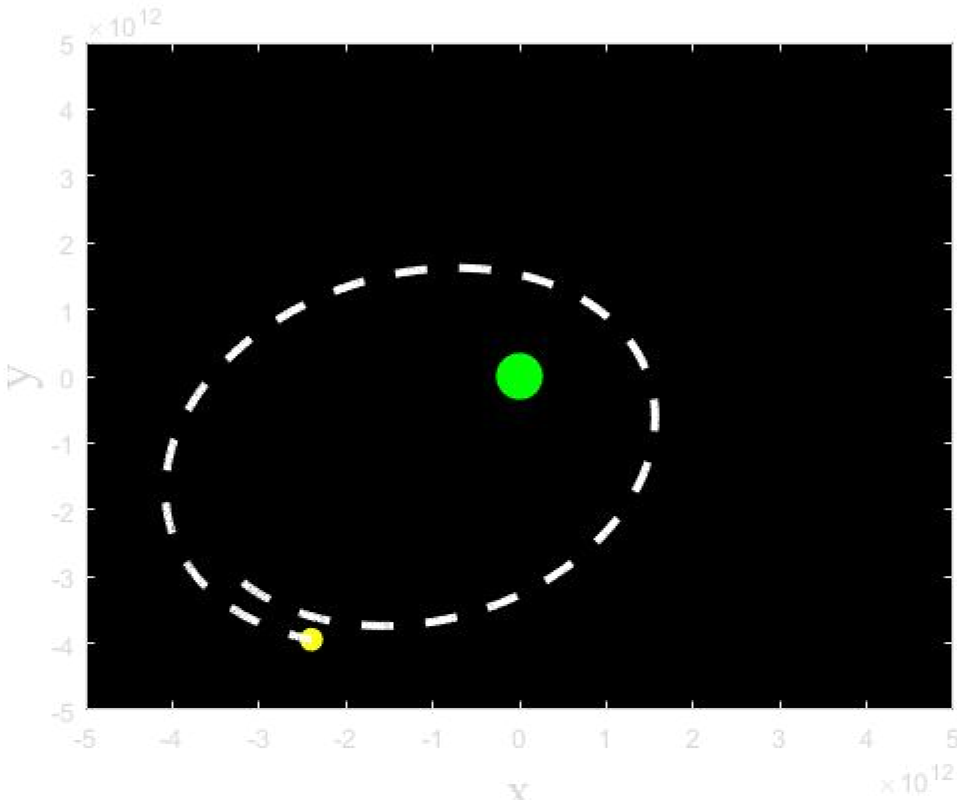
Pluto Sun plots: where dt = 5, 7. 9, 10 respectively







×10^12
5
4
3
2
1
0
-1
-2
-3
-4
-5
y
-5 -4 -3 -2 -1 0 1 2 3 4 5
x
×10^12



×10^12
5
4
3
2
1
0
-1
-2
-3
-4
-5
y
-5 -4 -3 -2 -1 0 1 2 3 4 5
x
×10^12