MATH 351–004 – Assignment #5

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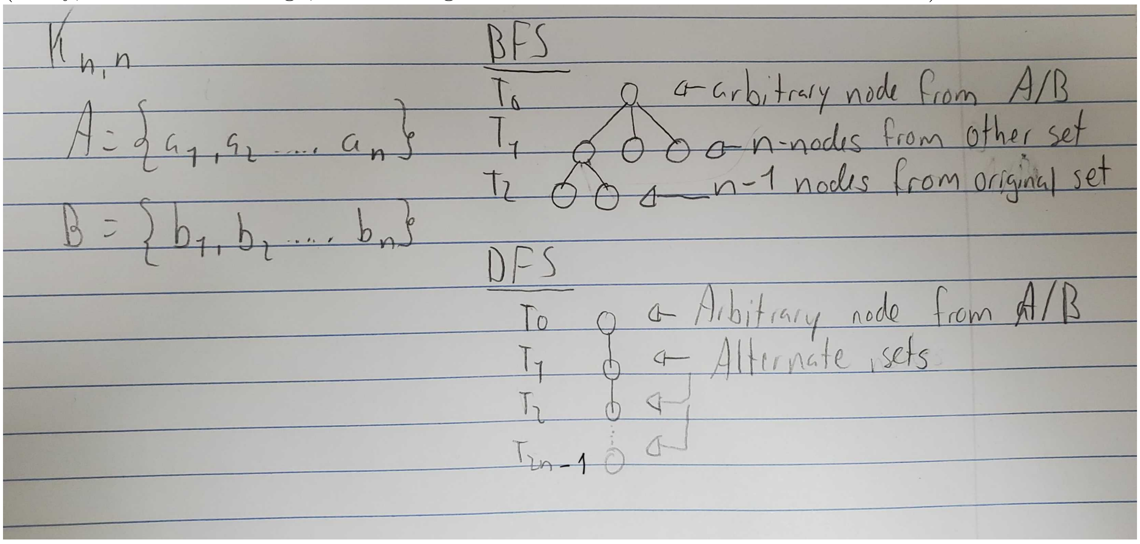
Problem 1 - Find a BFS tree and DFS tree for $K_{n,n}$

(Sorry, not the best image, but writing this out in LaTeX seemed more difficult)



Problem 2 - Prove that an edge e of a connected graph is a bridge if and only if e belongs to every spanning tree of G

Forward: A connected graph's edge is a bridge if it belongs to every spanning tree of the graph.
Contrapositive: Suppose that e does not belong to every spanning tree of G. There exists spanning tree T that does not contain edge e, therefore, tree T is spanning subgraph $G - e$. Using Theorem 4.2, there are two arbitrary vertices, $uandv$ that have a unique uv-path in T and $G - e$. This means that e is not a bridge.

Backward: An edge belongs to every spanning tree then it is a bridge in a connected graph.
Contrapositive: Suppose that e is not a bridge, then we know that $G - e$ is connected and has spanning tree T. Since $G - e$ has the same vertices as G, then T does not contain edge e.

Problem 3 - Apply both Kruskal's and Prim's algorithms to find a minimum spanning tree in the weighted graph in figure 4.12. In each case, show how this tree is constructed, as in Figures 4.5 and 4.9

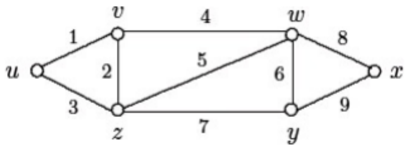
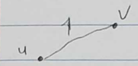
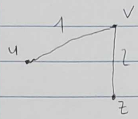


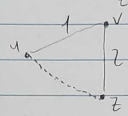
Figure 4.12: The weighted graph in Exercise 4.28

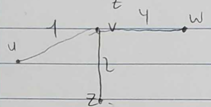
Kruskal's Algorithm

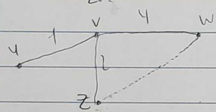
E(G) = {uv, vz, uz, vw, zw, wy, zy, wx, yz}

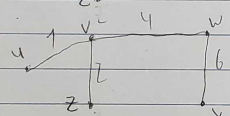
1)  Add edge uv, no cycle

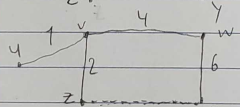
2)  Add edge vz, no cycle

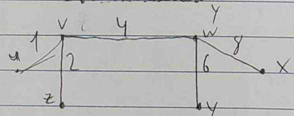
3)  Don't add edge uz, cycle is created

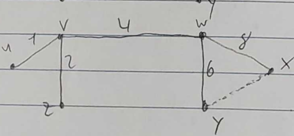
4)  Add edge vw, no cycle

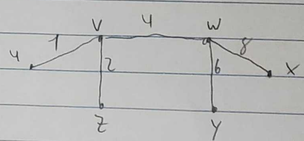
5)  Don't add edge zw, cycle is created

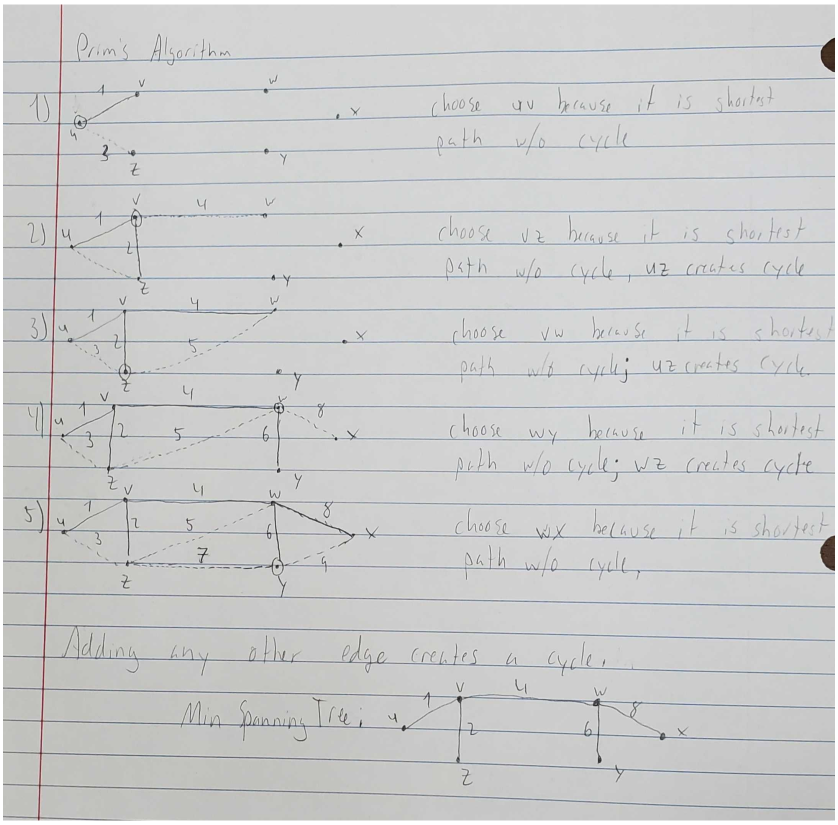
6)  Add edge wy, no cycle

7)  Don't add edge zy, cycle is created

8)  Add edge wx, no cycle

9)  Don't add edge xy, cycle is c

Min Spanning Tree: 



Problem 4 - Let G be a connected weighted graph and T a minimum spanning tree of G. Show that T is a unique minimum spanning tree of G if and only if the weight of each edge e of G that is not in T exceeds the weight of every other edge on the cycle in $T + e$

Suppose that T is a unique minimum spanning tree of G. Let e be an edge which is not in T, which means that $T + e$ contains a cycle C.
Suppose that there exists another edge d, such that its weight is greater than e, $w(e) < w(d)$. We can create another spanning tree $T\prime = (T - e) - d$. Since we know what $w(e) < w(d)$, this means that $W(T) > W(T\prime)$ Contradiction