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MATH-237-54
HW#2

8) $\frac{dx}{dt} = 3xt^2 \Rightarrow \frac{dx}{dt}\cdot\frac{1}{3x} = t^2 \Rightarrow \int\frac{dx}{3x} = \int t^2 dt$

$\Rightarrow \frac{1}{3}\int\frac{1}{x}dx = \frac{1}{3}t^3 + C \Rightarrow \frac{1}{3}\ln|x| = \frac{1}{3}t^3 + C \Rightarrow \ln|x| = t^3 + C$

$\Rightarrow e^{\ln|x|} = e^{t^3+C} \Rightarrow x = e^{t^3}\cdot e^C \Rightarrow \boxed{x = \tilde{C}\cdot e^{t^3}}$

11) $x\cdot\frac{dv}{dx} = \frac{1-4v^2}{3v} \Rightarrow \int\frac{3v}{1-4v^2}dv = \int\frac{dx}{x} \Rightarrow -\frac{3\ln|1-4v^2|}{8} = \ln|x| + C$

$\Rightarrow \ln|1-4v^2| = \ln|x^{-8/3}| + C \Rightarrow e^{\ln|1-4v^2|} = e^{\ln|x^{-8/3}|+C} \Rightarrow 1-4v^2 = x^{-8/3}\cdot\tilde{C}$

$\Rightarrow v^2 = \frac{\tilde{C}x^{-8/3}-1}{-4} \Rightarrow v = \pm\sqrt{\frac{\tilde{C}x^{-8/3}-1}{-4}}$ $\quad v(1) = -1 \Rightarrow -1 = -\sqrt{\frac{\tilde{C}-1}{-4}}$ negative works positive doesn't

$(-1)^2 = \frac{\tilde{C}-1}{-4}$

$-4 = \tilde{C}-1 \Rightarrow -3 = \tilde{C}$

$\boxed{v = -\sqrt{\frac{-3x^{-8/3}-1}{-4}}}$

18) $y' = x^3(1-y)$, $y(0) = 3$

$\frac{dy}{dx} = x^3(1-y) \Rightarrow \int\frac{dy}{(1-y)} = \int x^3 dx \Rightarrow -\ln|1-y| = \frac{1}{4}x^4 + C$

$\Rightarrow e^{\ln|1-y|} = e^{-\frac{1}{4}x^4+C} \Rightarrow 1-y = e^{-\frac{1}{4}x^4}\cdot e^C \Rightarrow -y = \tilde{C}e^{-\frac{1}{4}x^4} - 1$

$y = -\tilde{C}e^{-\frac{1}{4}x^4} + 1$

$3 = -\tilde{C}e^{-\frac{1}{4}\cdot 0^4} + 1 \Rightarrow 3 = \tilde{C}\cdot 1 + 1 \Rightarrow 2 = \tilde{C}$

$\boxed{y = 2e^{-\frac{1}{4}x^4} + 1}$

23) $\frac{dy}{dt} = 2t\cos^2 y$, $y(0) = \pi/4$

$\Rightarrow \int\frac{1}{2}\sec^2 y\,dy = \int t\,dt \Rightarrow \frac{1}{2}\int\sec^2 y\,dy = \frac{t^2}{2} \Rightarrow \tan y = t^2 + C$

$\tan(\frac{\pi}{4}) = 0^2 + C \Rightarrow 1 = C$

$\tan(y) = t^2 + 1 \Rightarrow \boxed{y = \arctan(t^2+1)}$

24) $\frac{dy}{dx} = 8x^3 e^{-2y}$, $y(1)=0$

$\Rightarrow \int \frac{1}{e^{-2y}} dy = \int 8x^3 dx \Rightarrow \frac{e^{2y}}{2} = 2x^4 + C$

$y(1)=0 \Rightarrow \frac{e^{2(0)}}{2} = 2(1)^4 + C \Rightarrow 1/2 = 2 + C \Rightarrow -3/2 = C$

$\Rightarrow \frac{e^{2y}}{2} = 2x^4 - \frac{3}{2} \Rightarrow e^{2y} = 4x^4 - 3 \Rightarrow \ln(e^{2y}) = \ln(4x^4 - 3)$

$\Rightarrow 2y = \ln(4x^4 - 3) \Rightarrow \boxed{y = 1/2 \ln(4x^4 - 3)}$

VB) $\frac{dV}{dt} = g - kv \Rightarrow v = \frac{e^{-kt} \cdot \tilde{c} - g}{-k} \Rightarrow \int dz = \int \frac{\tilde{c} e^{-kt} - g}{-k} dt$

$\Rightarrow z = \frac{-(g \cdot e^{-kt} - g)}{k} \Rightarrow \int dz = \int \frac{-(g \cdot e^{-kt} - g)}{k} dt$

$z = \int \frac{-g e^{-kt}}{k} dt - \int \frac{g}{k} dt \Rightarrow z = \frac{-g}{k}\left(\frac{-e^{-kt}}{k}\right) + \frac{gt}{k} + C$

$\Rightarrow z(0) = z_0 \Rightarrow z_0 = \frac{g e^{-k(0)}}{k^2} + \frac{g(0)}{k} + C \Rightarrow z_0 = \frac{g}{k^2} \Rightarrow z_0 = g/k^2 + C$

$C = z_0 - g/k^2$

$\Rightarrow \frac{g e^{-kt}}{k^2} + \frac{gt}{k} + z_0 - \frac{g}{k^2} = z \Rightarrow \boxed{\frac{g(e^{-kt} - 1)}{k^2} + \frac{gt}{k} + z_0 = z}$

snow.m

```
function snow(N)
clf; colordef black %make it night time
if nargin==0,N=50; end %if not specified, make 50 snowflakes

%make some random snow info
for n=1:N
    xv(n)=(-1)^randi(2)*rand; yv(n)=(-1)^randi(2)*rand; %random (x,y)
    z0(n)=(-1)^randi(2)*rand; %random initial heights
end
g=9.81; %acceleation of gravity in m/s^2
k=5; %drag parameter to play with
% increasing k incrases the size of the snow particles at the end of run
for t=0:0.01:1000
    %%YOUR MISSION: EDIT LINE BELOW TO INCORPORATE DRAG
    %%z=z0+1/2*g*t^2; %here, z is positive down
    z = z0 + (g.*t./k) + g.*(exp(-k .* t) - 1)./(k.^2);
    %%%%%%%%%%%%%%%%
    z(z>1.2)=1.2; % snow collects on ground
    plot3(xv,yv,-z,'w.','markersize',t*20+0.1); %snow gets bigger as it falls
    axis equal;axis([-1.2 1.2 -1.2 1.2 -1.2 1.2]); box on; view(45,30); %make it look nice
    drawnow
    if max(-z)<=-1.2
        break %stop when all snow has fallen
    end
end
```