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Homework #1

Chapter 1.1

1.1: *In every instance of the Stable Matching Problem, there is a stable matching containing a pair (m, w) such that m is ranked first on the preference list of w and w is ranked first on the preference list of m.*

This statement is false. Consider the following stable matching problem:

m1 = [w2, w1]

m2 = [w1, w2]

w1 = [m1, m2]

w2 = [m2, m1]

In this case, if the men propose first, m1 will propose to w2, and she accepts because this is her only proposal. Simultaneously, m2 will propose to w1, and she will accept because this is her only proposal. At the moment, there are no instabilities because both men are with their first preference; therefore, they have no incentive to leave. However, there is no pair of men and women in which the women have their first preference.

1.2: *Consider an instance of the Stable Matching Problem in which there exists a man m and a woman w such that m is ranked first on the preference list of w and w is ranked first on the preference list of m. Then in every stable matching S for this instance, the pair (m, w) belongs to S.*

This statement is true. If a man1 is ranked first on woman1's preference list and woman1 is ranked first on man1's preference lists, then the pair of man1 and woman2 must be in every instance of a stable matching. If man1 and woman1 are matched with any other man or woman, there would be an instability in the matching.

Chapter 2

2.1

a) n^2

i) n doubles: $2(n^2)$ -> 4n

slows down by factor of *4*

ii) n increases by 1: $(n + 1)^2$ -> $n^2 + 2n + 1$

slows down by additional term of *2n + 1*

b) n^3

i) n doubles: $2(n^3)$ -> 8n

Slows down by factor of *8*

ii) n increases by 1: $(n + 1)^3$ -> $n^3 + 3n^2 + 3n + 1$

Slows down by additional term of $3n^2 + 3n + 1$

c) $100n^2$

i) n doubles -100 * $(2n)^2$ -> 400n -> 400 / 200 = 4

Slows down by factor of *4*

ii) n increases by 1 - $100(n + 1)^2$ -> $100n^2 + 200n + 100$

Slows down by additional term of *200n + 100*

d) n * log(n)

i) n doubles: (2n)log(2n) -> n log($(2n)^2$) -> n log (4n * n) ->

n log(4n) + n log(n) -> n(log(4) + log(n)) + n log(n) ->

n log(4) + n log(n) + n log(n) = 2n log(n) + n log (4) ->

2n (log(n) + 1) -> n log(n) * [2 + (2 / log(n))]

Slows down by factor of *[2 + (2 / log(n))] (assuming base of 2)*

ii) n increases by 1: (n + 1) * log(n + 1) -> (n + 1) * log[n (1 + 1/n)] ->

(n + 1) * [log(n) + log((1 + n) / n)] ->

n log(n) + n log((1 + n) / n) + log(n) + log((1 + n) / n) ->

n log(n) + n log((1 + n) / n) + log(1 + n)

Slows down by additional term of *n log((1 + n) / n) + log(1 + n)*

e) 2^n

i) n doubles: 2^{2n} -> $2^n * 2^n$

Slows down by factor of *2^n*

ii) n increases by 1: 2^{n+1} -> $2^n + 2$

Slows down by additional term of *2*

2.2

a) $n^2 = 3.6 * 10^{13}$

$n = 6.0 * 10^6$

b) $n^3 = 3.6 * 10^{13}$

$n = 33019.3$ -> $n = 3.3 * 10^4$ (rounded)

c) $100n^2 = 3.6 * 10^{13}$

$n^2 = 3.6 * 10^{11}$

$n = 6 * 10^5$

d) $n\log(n) = 3.6 * 10^{13}$

$\log(n)^n = 3.6 * 10^{13}$

$2^{\log(n)\wedge n} = 2^{3.6 * 10\wedge 13}$

$2^n * 2^{\log(n)} = 2^{3.6 * 10\wedge 13}$

$n = 9 * 10^{11}$

e) $2^n = 3.6 * 10^{13}$

$(\log(2) / \log(2))\ n = (\log(2) / \log(2))\ 3.6 * 10^{13}$

$n = \log(3.6 * 10^{13}) / \log(2)$

$n = 45.033$ -> $n = 45$ (rounded)

f) $2^{2\wedge n} = 3.6 * 10^{13}$

$(\log(2) / \log(2))\ 2^{2\wedge n} = (\log(2) / \log(2))\ 3.6 * 10^{13}$

$2^n = \log(3.6 * 10^{13}) / \log(2)$

$(\log(2) / \log(2))\ 2^n = (\log(2) / \log(2)) \log(3.6 * 10^{13}) / \log(2)$

n = [log(log(3.6 * 10^13) / log(2)] / log(2) -> *$n = 5.4929$ -> $n = 5$ (rounded)*

2.3: *Take the following list of functions and arrange them in ascending order of growth rate.*

$f_1(n) = n^{2.5}$

$f_2(n) = sqrt(2n)$

$f_3(n) = n + 10$

$f_4(n) = 10^n$

$f_5(n) = 100^n$

$f_6(n) = n^2 log(n)$

(grows the slowest) sqrt(2n) < n + 10 < n^2log(n) < $n^{2.5}$ < 10^n < 100^n (grows the fastest)

E1) *An investor places $30,000 into a stock fund. 10 years later, the account has a value of $69,000. Using logarithms and anti-logarithms, present a formula. Show work.*

$A = P(1 + (r / 100))^t$

A = Final = $69K

P = Initial = $30K

r = rate of growth

t = Time = 10 years

Assuming logarithm base of 10

$$69{,}000 = 30{,}000 \left(1 + (r / 100) \right)^{10}$$

$$2.3 = \left(1 + (r / 100) \right)^{10}$$

$$\log (2.3) = \log \left(\left(1 + (r / 100) \right)^{10} \right)$$

$$\log(2.3) = 10 \ \log \left(\left(1 + (r / 100) \right) \right)$$

$$10 \wedge \left(\log(2.3) / 10 \right) = 10 \wedge \log \left(\left(1 + (r / 100) \right) \right)$$

$$10 \wedge \left(\log(2.3) / 10 \right) = 1 + (r / 100)$$

$$[10 \wedge (\log(2.3) / 10)] - 1 = r / 100$$

$$0.0869 = r / 100$$

$$8.69 = r$$

$$A = 30{,}000 \left(1 + (8.69 / 100) \right)^{10}$$