Language SET

In this version of our source language, we allow for the assignment of values to variables. Languages that allow for the mutation of variables are called *side-effecting*; such languages are inherently more difficult to reason about, which accounts for why functional programming has received so much attention and also for why it is so difficult to produce high-quality software in most side-effecting programming languages.

So far, Languages V1 through V6 have treated denoted values (the things that variables are bound to) as being the same as expressed values (the values that expressions can have). For example, a variable `x` in one of these languages always evaluates to the same thing no matter where it appears in its scope.

When we add variable mutation (also called "assignment"), such as with

```
set x = add1(x)
```

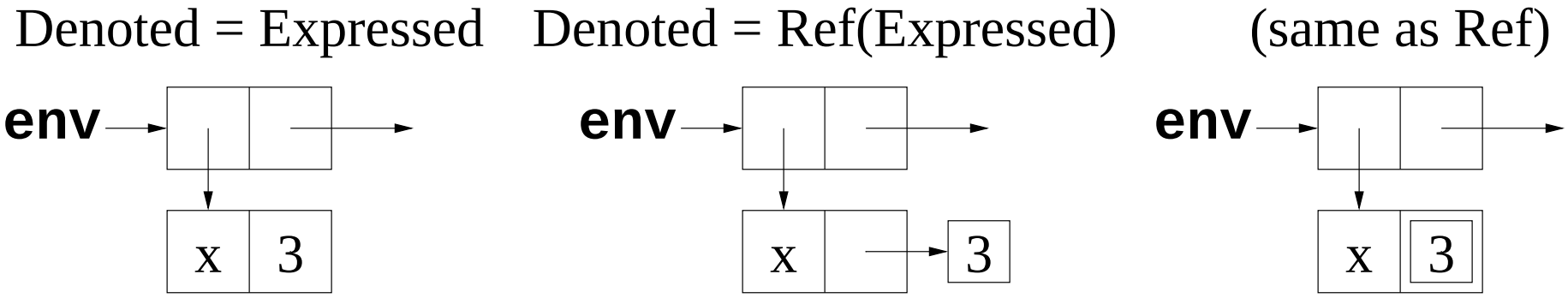
the meaning of `x` on the LHS is different from its meaning on the RHS. The expression `x` in the RHS of this "assignment" represents an expressed value, whereas `x` on the LHS represents a denoted value that can be modified. In order to implement variable assignment, we need to find a way to disconnect denoted values from expressed values.

Language SET (continued)

We introduce the notion of a *reference*, something that *refers to* a mutable location in memory. Instead of binding a variable directly to an expressed value, we bind the variable to a reference which itself contains an expressed value.

Expressed value = Val = IntVal+ProcVal
Denoted value = Ref(Expressed)

To mutate a variable bound to a reference, we change the *contents* of the reference; the variable is still bound to the same reference.



The two right-hand diagrams depict the same environment. The rightmost one uses a more compact representation.

Language SET (continued)

References will also be used to implement various parameter-passing mechanisms as described later in these notes.

We choose the following concrete and abstract syntax for variable mutation:

```
<exp>:SetExp ::= SET <VAR> EQUALS <exp>
                 SetExp(Token var, Exp exp)
```

We can now write the following program in our newly extended source language:

```
let
  x = 42
in
  { set x = add1(x) ; x }
```

This evaluates to 43.

Language SET (continued)

The ability to modify the value bound to a variable allows us to "capture" an environment in a procedure and use the procedure to modify its captured environment. For example, consider:

```
define g = let
             count = 0
           in
             proc() set count = add1(count)
.g() % => 1
.g() % => 2
.g() % => 3
```

The value of `count` is captured in the local `let` bindings that defines the `proc()`. Each time we evaluate `.g()`, the procedure increments the value of `count` and returns this newly incremented value. The variable `count` persists from one invocation to the other because the `proc` captures the environment in which it is defined, namely the one with the variable `count`.

In this example, the `count` variable is unbound in the top-level environment, so an attempt to evaluate it throws an exception:

```
count % unbound variable
```

For our purposes, we want a reference to be a Java object whose contents can be mutated. When we bind a variable to a reference (its denoted value), this binding does not change, but the contents of the reference itself – the thing it refers to – can change.

The `Ref` abstract class embodies our notion of a reference – the thing that a variable can be bound to. For now, the only subclass of the `Ref` class is the `ValRef` class.

```
ValRef(Val val)
```

The contents of a `ValRef` object is a `Val`, and we say that such an object is a *reference to a value*. (Recall that a `Val` object is either an `IntVal` or a `ProcVal` – the only two `Val` types that we currently have.)

A `Ref` object has two methods:

```
public abstract Val deRef();
public abstract Val setRef(Val v);
```

In the `ValRef` class, The `deRef` (dereference) method simply returns the `Val` object stored in the object's `val` field, and the `setRef` (set reference) method modifies the `val` field by changing it to the `Val` parameter `v` (and returning the new `Val` object as well).

Language SET (continued)


```
Ref
%%%
public abstract class Ref {

    public abstract Val deRef();
    public abstract Val setRef(Val v);

}
%%%
```

Language SET (continued)


```
ValRef
%%%
public class ValRef extends Ref {

    public Val val;

    public ValRef(Val val) {
        this.val = val;
    }

    public Val deRef() {
        return val;
    }

    public Val setRef(Val v) {
        return val = v;
    }
}
%%%
```

Language SET (continued)

Our denoted values (the things that variables are bound to) are now references instead of values, so we need to change our `Binding` objects to bind an identifier to a reference. (Notice that we use the terms "variable", "identifier", and "symbol" interchangeably.)

```
Binding(String id, Ref ref)
```

In the `Env` class, we want `applyEnv` to continue to return a `Val` object, whereas the bindings now associate identifiers with references, so we split up the responsibilities as follows:

```
// returns the reference bound to sym
public abstract Ref applyEnvRef(String sym);

public Val applyEnv(String sym) {
    return applyEnvRef(sym).deRef();
}
```

The `applyEnvRef` method behaves exactly like the previous `applyEnv` method (but returns a `Ref` instead) and throws an exception if there is no reference bound to the given symbol. The `applyEnv` method simply gets the `Ref` object using `applyEnvRef` and dereferences it to return the corresponding value.

Language SET (continued)

In our semantics code, we need to modify all of the instances of `Binding` or `Bindings` objects so that they use references instead of values. To create a "binding" of a variable to a value, first wrap the value into a new reference and then bind the variable to the newly created reference. Here's an example of how to create a binding of the variable named `x` to (a reference to) an integer `10`:

```
String var = "x";
Val val = new IntVal(10);
Binding b = new Binding(var, new ValRef(val));
```

The `valsToRefs` static method in the `Ref` class takes a list of `Vals` and returns a corresponding list of `Refs`. This is used, for example, in the code for `AppExp` objects (which need to bind formal parameter symbols to references to their actual parameter values) and for `LetExp` objects (which need to bind their LHS variable symbols to reference to their RHS expression values).

```
public static List<Ref> valsToRefs(List<Val> valList) {
    List<Ref> refList = new ArrayList<Ref>(valList.size());
    for (Val v : valList)
        refList.add(new ValRef(v));
    return refList;
}
```

Language SET (continued)


```
<exp>:SetExp ::= SET <VAR> EQUALS <exp>
                 SetExp(Token var, Exp exp)
```

So far, we have dealt only with the implementation details of environments. How do we implement the semantics of `set` expressions? Coding this is now simple:

```
SetExp
%%%
    public Val eval(Env env) {
        Val val = exp.eval(env); // the RHS expression value
        Ref ref = env.applyEnvRef(var); // the LHS reference
        return ref.setRef(val);  // sets the ref and returns val
    }
%%%
```

Notice that a `set` expression evaluates to the value of the RHS of the assignment. This means that multiple `set` operations can appear in one expression.

Language SET (continued)

For example, the following expression evaluates to 12:

```
let
  t = 3
  u = 42
  v = 0
in
  { set v = set u = set t = add1(t) ; +(t,+(u,v)) }
```

The first expression in the body of this `let` gets evaluated like this:

```
    set v = { set u = { set t = add1(t) } }
```

What happens if you try to mutate the value of an identifier that is one of the formal parameters to a procedure? For example, what value is returned by the following program?

```
let
  x = 3
  p = proc(t) set t = add1(t)
in
  { .p(x) ; x }
```

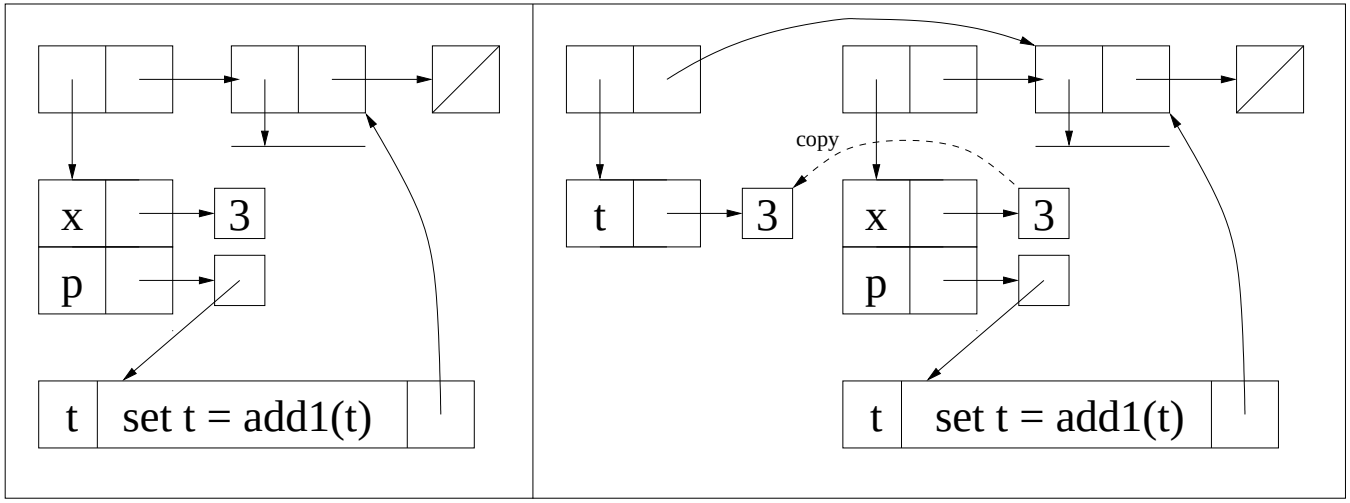
In our procedure application semantics (see the `AppExp` code), the formal parameters are bound to (references to) the *values* of the actual parameters. Since the value of the actual parameter `x` in the expression `.p(x)` is 3, this means that the variable `t` in the body of the procedure is bound to (a reference to) the value 3, and evaluating the body of the procedure modifies this binding to the value 4, but it's the variable `t`, not the variable `x`, that gets modified. Thus the value of this entire expression is 3.

Language SET (continued)

```
let
  x = 3
  p = proc(t) set t = add1(t)
in
  { .p(x) ; x }
```

The following illustration shows

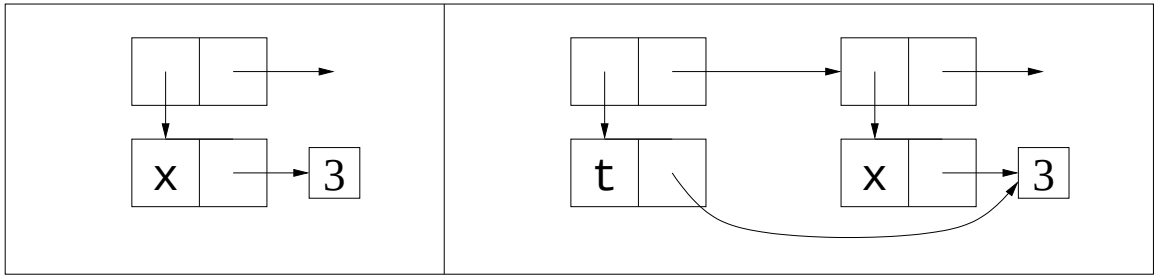
- the environment immediately before the procedure application `.p(x)` – in particular, the binding of `x` to a reference to the value 3,
- and the environment during the procedure application `.p(x)`, binding the formal parameter `t` to a *new* reference containing a copy of the value of `x` (as shown by the dashed line).



Language REF

A parameter passing semantics that evaluates actual parameters and that binds the formal parameters to these actual parameter values is called *call-by-value*. This is what is used in languages `V1` to `V6`. In the language `SET`, where bindings are to references instead of values, the actual parameter values are turned into *new* references, and these references are bound to the formal parameters.

Considering the illustration in the previous slide, Suppose we *want* a behavior that binds the formal parameter `t` to the *same* reference that is bound to `x` instead of a new reference containing a copy. The following diagram shows how the bindings in the previous diagram change when `t` is bound to the same reference as verb'x':



Such a parameter passing semantics is called *call-by-reference*. We explore call-by-reference next, along with variants on this theme.

Language REF (continued)

To repeat:

- The parameter passing semantics that we have been using up to now is called *call-by-value*. In call-by-value semantics – also referred to as simply *value semantics*, when an actual parameter expression in a procedure application is a variable, the procedure's corresponding formal parameter denotes a new reference to the expressed value of the actual parameter.
- In *call-by-reference* semantics – also referred to as simply *reference semantics*, when an actual parameter expression in a procedure application is a variable, the procedure's corresponding formal parameter denotes the *same reference* as the actual parameter.

The differences between value and reference semantics only apply when the actual parameter expression is a variable. When the actual parameter expression is not a variable, the corresponding formal parameter always denotes a new reference to the expressed value of the actual parameter.

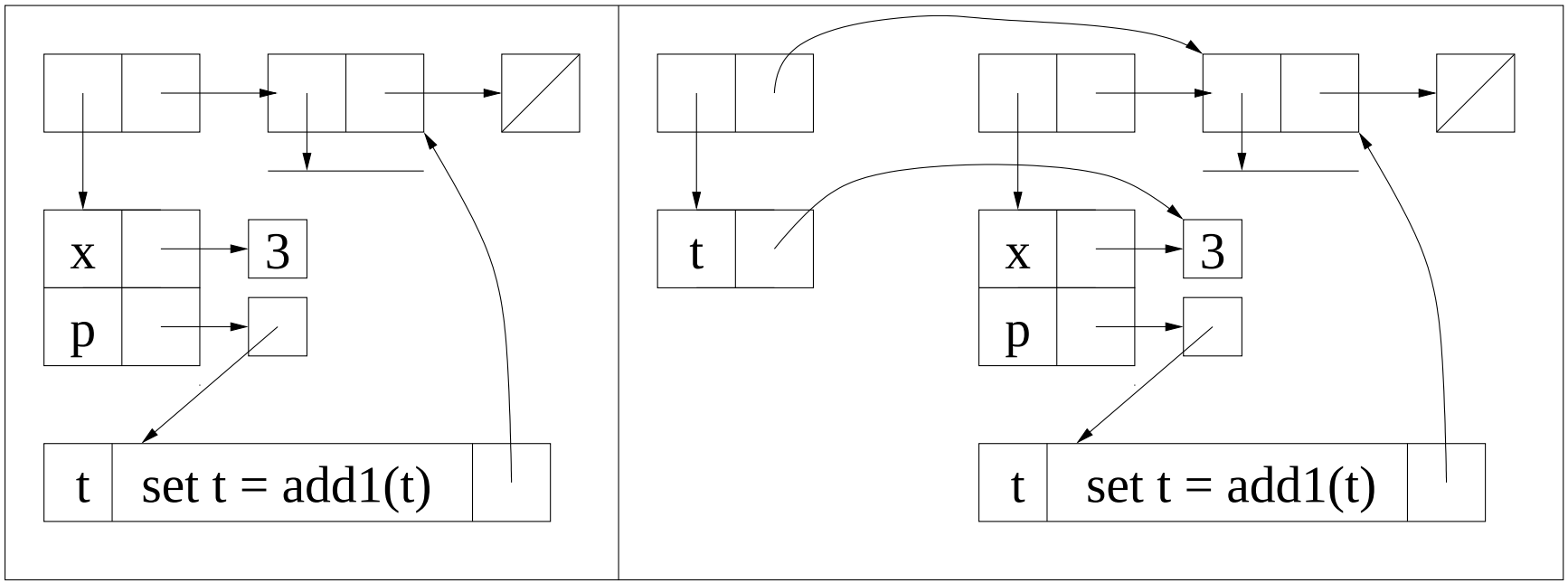
Observe that in `let` and `letrec` expressions, we always use value semantics for the variable bindings. This means that each LHS variable in a `let/letrec` expression always denotes a new reference to the expressed value of its corresponding RHS expression.

Language REF (continued)

Using call-by-reference semantics, the program

```
let
  x = 3
  p = proc(t) set t = add1(t)
in
  { .p(x) ; x }
```

returns the value 4, since `t` denotes the same reference as `x`. The bindings created by the `let` and then during evaluation of the application `.p(x)` (just prior to evaluating the procedure body) are illustrated in the following figure:



When actual parameter expressions are not themselves variables, we use value semantics. To illustrate this, consider the value returned by the following program:

```
let
  x = 3
  p = proc(t) set t = add1(t)
in
  { .p(+(x,0)) ; x }
```

Clearly the expressed value of the actual parameter `+(x,0)` is the same as that of `x`, but the expression `+(x,0)` is not a variable, so value semantics apply to this actual parameter. This means that when we apply the procedure `p`, the formal parameter `t` denotes a *new* reference to the value of this expression: the variables `t` and `x` have the same expressed values, but they have different denoted values, so modifying `t` does not affect the value of `x`. This expression evaluates to 3.

Language REF (continued)

The term *L-value* refers to an expression that can be interpreted as a reference. (It's called an L-value because it is the sort of expression that can appear to the *left* of the = in a `set`.) While a variable `x` can always be considered as an L-value, the expression `+(x,0)` can only be interpreted as a value, never a reference. In Language REF, only variable expressions are L-values.

In summary, **if an actual parameter is a L-value in Language REF, (and therefore can be interpreted as a reference), then the corresponding formal parameter is bound to the same reference. If an actual parameter is something other than an L-value, then the corresponding formal parameter is bound to a *new* temporary reference containing the value of the actual parameter.**

Language REF (continued)

Our `REF` language has exactly the same grammar rules as our `SET` language. The *only* differences are in the bindings of formal parameters during procedure application. As the discussion on the previous slides show, we need to handle actual parameters that are variables differently from actual paramemeters that are expressions. The idea here is to define an `evalRef` method for instances of the `Exp` classes that takes care of how to translate themselves into a reference: for anything but a `VarExp`, `evalRef` evaluates the expression and returns a new reference to the value. For a `VarExp`, `evalRef` returns the same reference that the actual parameter denotes.

So in the `Exp` class, the `evalRef` method has the following *default* behavior:

```
public Ref evalRef(Env env) {
    return new ValRef(eval(env));
}
```

For the `VarExp` subclass – *and only for this class*, `evalRef` is implemented as:

```
public Ref evalRef(Env env) {
    return env.applyEnvRef(var);
}
```

The `evalRef` method in the `VarExp` class overrides the `evalRef` method in the `Exp` class. In all other classes that extend the `Exp` class, the default definition in the parent `Exp` class is used.

Language REF (continued)

The other change is in the `Rands` code. In the `SET` language, the `evalRands` method was used in the implementation of `eval` for both a `LetExp` object and an `AppExp` object, since both created new bindings to values. In the `REF` langauge, an `AppExp` object needs new bindings to values except for actual parameters which are variables – a situation that is described in the previous slide. Therefore, to implement the correct `eval` semantics for an `AppExp` object, we need to collect `evalRef` references instead of `eval` values to bind them to the formal parameters. The method `evalRandsRef` in the `Rands` class does this work for us. The `eval` method in the `AppExp` class uses the `evalRandsRef` method to create the bindings of the formal parameters to their appropriate references. The definition for `evalRandsRef` follows:

```
public List<Ref> evalRandsRef(Env env) {
    List<Ref> refList = new ArrayList<Ref>(expList.size());
    for (Exp exp : expList)
        refList.add(exp.evalRef(env));
    return refList;
}
```

Language REF (continued)

Remember that we always use value semantics for `let` bindings. This means that a Language REF program such as

```
let
  x = 3
in
  let
    y = x
  in
    { set y = add1(y) ; x }
```

evaluates to 3.

Our observation (see Slide 3.79) that any `let` can be re-written as an equivalent procedure application no longer applies with languages that implement call-by-reference semantics. Specifically, if we attempt to re-write the inner `let` in the above Language REF program as a procedure application using the algorithm given on Slide 3.79, we get

```
let
  x = 3
in
  .proc(y) {set y = add1(y) ; x } (x)
```

which evaluates to 4.

Language NAME

We now turn to a different parameter passing mechanism, *call-by-name*. In call-by-name procedure application, we bind each procedure's formal parameter to its corresponding *un-evaluated actual parameter expression*. Each time we reference a formal parameter in the procedure body, we evaluate its corresponding actual parameter expression *in the environment where the procedure was called* – the *calling environment*, and this value becomes the expressed value of the formal parameter.

Call-by-name has behaviors that differ from call-by-reference: (1) if we never reference the formal parameter in the procedure body, we never evaluate the actual parameter expression; and (2) every time we evaluate the formal parameter in the procedure body, we re-evaluate the actual parameter expression.

In the presence of side-effects, call-by-name has interesting properties that make it very powerful but often difficult to reason about. The language ALGOL 60 had call-by-value and call-by-name as its parameter passing mechanisms. ALGOL 60 had its greatest influence on languages such as Pascal, C/C++, and Java. Although call-by-name has been all but abandoned by modern imperative (side-effecting) programming languages – mostly because of its inefficiency, it still plays a role in functional programming: Scheme supports a variant, *call-by-need*, by means of `promise/force`; Haskell also supports call-by-need. We proceed to implement both call-by-name and call-by-need.

Two actual parameter expressions that can appear in procedure applications have *constant behavior*: They are `LitExp` and `ProcExp`. Evaluating these expressions do not produce any side-effects, and their expressed values can never change. (Don't confuse *evaluting* a procedure expression with *applying* such an expression.) Because of this, in call-by-name we can evaluate these actual parameter expressions directly during procedure application and can bind their corresponding formal parameters to new references to their values directly. In other words, we use value semantics for these actual parameter expressions.

Evaluating variables that appear as actual parameter expressions (`VarExp`) also do not produce any side-effects, but such variables may have denoted values that can change. We therefore use reference semantics for actual parameter expressions that are variables.

In the presence of side-effects, call-by-reference and call-by-name may give different results. Consider evaluating the following expression:

```
let
  x = 1
  f = proc(t,u)
        {
          set t = add1(t) ;
          u
        }
in
  .f(x, +(x,5))
```

In call-by-reference, when we evaluate the application `.f(x, +(x,5))`, the formal parameter `t` in the definition of `f` denotes the same reference that `x` denotes (initially containing 1), whereas the formal parameter `u` denotes a new reference to the value 6. Modifying `t` in the body of `f` changes the expressed value of `x` (because `t` and `x` denote the same reference) but does not change the expressed value of `u`. Thus this expression evaluates to 6.

Language NAME (continued)

```
let
  x = 1
  f = proc(t,u)
        {
          set t = add1(t) ;
          u
        }
in
  .f(x, +(x,5))
```

Consider now what happens when we evaluate `.f(x, +(x,5))` using call-by-name: The formal parameter `t` still denotes the same reference that `x` denotes (initially containing 1), but the formal parameter `u` denotes the (un-evaluated) expression `+(x,5)`.

The `set` operation in the body of this procedure increments the formal parameter `t`; but since `t` denotes the same reference as `x`, the value of `x` changes too, to two. When we then evaluate the formal parameter `u` at the end of the `proc` body, we evaluate the expression `+(x,5)` denoted by `u` *in the environment of the caller*. Since this expression gets evaluated after the `set`, and the value of `x` is now 2, the value of the expression `+(x,5)` (and thus the value returned by the procedure application) is `+(2,5)` or 7. Thus the entire expression evaluates to 7.

Language NAME (continued)

Consider the following definition:

```
define while = proc(test?, do, ans)
  letrec loop = proc()
    if test? then {do ; .loop()} else ans
  in .loop()
```

Using call-by-name, the expression

```
let x = 0 sum = 0 in
  .while(
    <=?(x,10),
    { set sum=+(sum,*(x,x)) ; set x = add1(x) },
    sum
  )
```

returns the sum

$$\sum_{x=1}^{10} x^2 = 385$$

Using call-by-reference as in the language `REF`, the expression never terminates because the actual parameter expression `<=?(x,10)` is evaluated only once, to 1 (true) when `x` is initially 0, and so the `test?` parameter is bound permanently to (a reference to) 1. Evaluating `test?` repeatedly always returns 1 (true), so the "loop" never terminates.

We proceed to implement call-by-name. We take our call-by-reference implementation as a starting point.

If an actual parameter is a literal expression (such as `4`), we bind the formal parameter to (a reference to) the literal value. If an actual parameter is a procedure, we bind the formal parameter to (a reference to) the procedure's closure in the calling environment. If an actual parameter is an identifier, we bind the formal parameter to the same reference as the actual parameter, using reference semantics.

If an actual parameter is any other kind of expression, we bind the formal parameter to a `Ref` object that captures the expression in the environment in which it was called and that can be evaluated, when needed, by the called procedure. We call such an object a *thunk*.

A thunk amounts to a parameterless procedure that consists of an expression and an environment in which the expression is to be evaluated. It looks just like a closure, except that there is no formal parameter list.

```
ThunkRef(Exp exp, Env env)
```

A `ThunkRef` is a `Ref`, since we want to de-reference (`deRef`) it whenever we refer to the corresponding actual parameter. We bind a formal parameter to a thunk reference only during procedure application. Thunks will otherwise not play a role in expression semantics.

To change from call-by-reference to call-by-name, we need to change the default `evalRef` behavior of the `Exp` objects so that `evalRef` returns a thunk for most expressions *except for* `LitExp`, `ProcExp`, and `VarExp`. This means that we define `evalRef` in the `Exp` class with its default behavior as follows:

```
public Ref evalRef(Env env) {
    return new ThunkRef(this, env);
}
```

For a `LitExp` and a `ProcExp`, a thunk is not necessary, so we return an ordinary `ValRef` as in the `REF` language:

```
public Ref evalRef(Env env) {
    return new ValRef(eval(env));
}
```

Finally, for a `VarExp`, we simply use reference semantics as in the `REF` language:

```
public Ref evalRef(Env env) {
    return env.applyEnvRef(var);
}
```

Language NAME (continued)

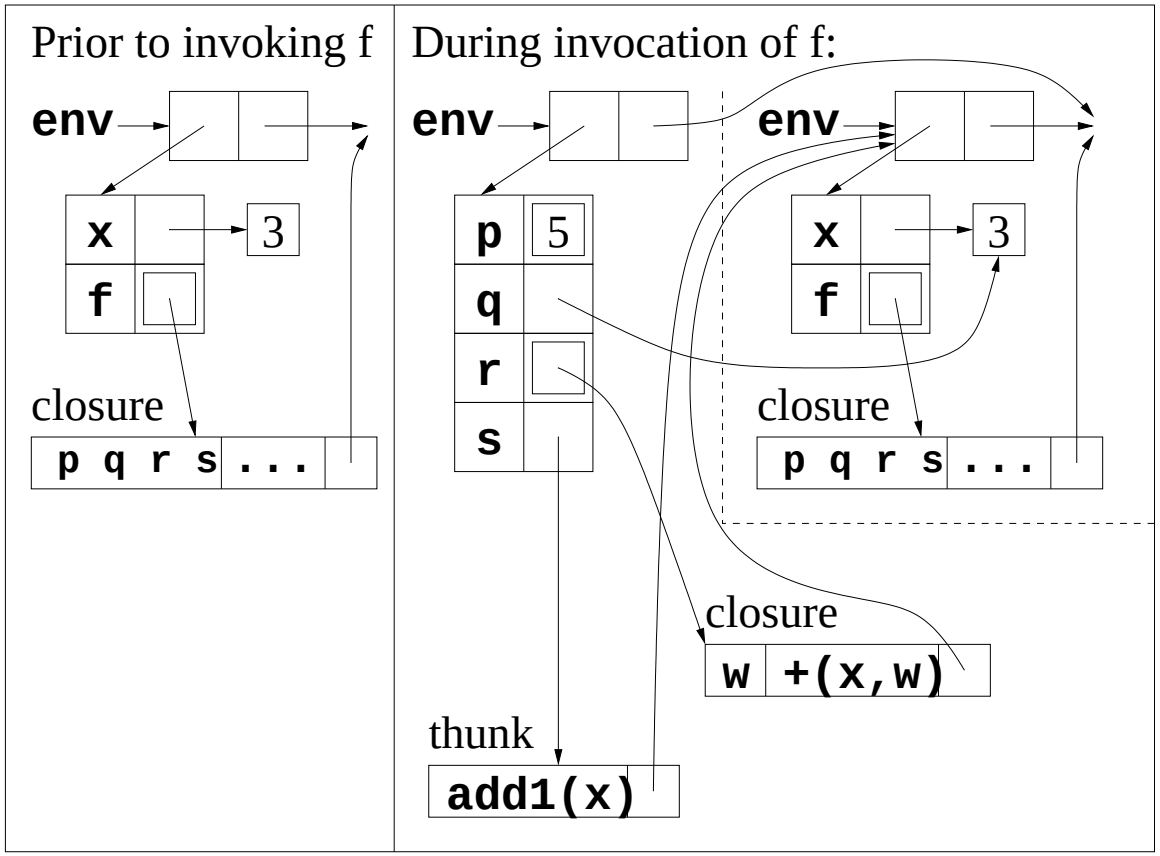
The `ThunkRef` class is straight-forward:

```
ThunkRef
%%%
public class ThunkRef extends Ref {

    public Exp exp;
    public Env env;

    public ThunkRef(Exp exp, Env env) {
        this.exp = exp;
        this.env = env;
    }

    public Val deRef() {
        return exp.eval(env);
    }

    public Val setRef(Val v) {
        throw new RuntimeException("cannot modify a read-only expression");
    }
}
%%%
```

Observe that the `setRef` method throws an exception. This method is only used to evaluate `set` expressions, and it doesn't make sense to have something other than a `VarExp` on the LHS of a `set`.

The following illustration may help you to understand how these bindings work. The example shows all four possible cases of actual parameter expressions: literal, variable, procedure, and other:

```
let x = 3
    f = proc(p,q,r,s) ...
in .f(5, x, proc(w) +(x,w), add1(x))
```



Language NEED

The call-by-need parameter passing mechanism is the same as call-by-name, except that a thunk is called at most once, and its value is remembered (*memoized*).

Suppose a procedure with formal parameter `x` is invoked with actual parameter `set z = add1(z)`, using call-by-need semantics. As with call-by-name, the formal parameter `x` is bound to the thunk with `set z = add1(z)` as its body, in an enclosing environment that we will assume has `z` bound to (a reference to) the value 8. When `x` is referenced in the body of calling procedure, its corresponding thunk is dereferenced, producing a result of 9 for `set z = add1(z)`. The thunk now remembers (*memoizes*) the value 9, and any further references to the formal parameter `x` in the body of the procedure will continue to evaluate to 9 without making any further changes to the variable `z`.

If call-by-name had been used in the above example, additional evaluations of `x` in the body of the procedure would result in evaluating the body of the thunk for each such evaluation, further modifying `z` and yielding values 10, 11, 12, and so forth.

In both call-by-need and call-by-name (and unlike call-by-reference), if the formal parameter is never referenced in the body of the procedure, the thunk is never evaluated. Compared to call-by-name, call-by-need reduces the overhead of repeatedly evaluating a thunk when evaluating the corresponding formal parameter.

Language NEED (continued)

Implementing call-by-need is easy, starting from the call-by-name interpreter. The principal change is to have a `Val` field named `val` in the `ThunkRef` class that is used to memoize the value of the body of the thunk. This field is initialized to `null` when the thunk is created. When the thunk's `deRef` method is invoked, it checks to see if the `val` field has been memoized (*i.e.*, is non-null). If so, the `deRef` method simply returns the memoized value. Otherwise, it evaluates the body of the thunk, saves the value in the `val` field (thereby memoizing it), and then returns that value; subsequent `deRef` calls simply use the resulting memoized value.

In the NEED language, the `ThunkRef` constructor initializes the `val` field to `null`, indicating that the thunk has not been memoized. This field is modified when the thunk's `deRef` method is called.

Language NEED (continued)

Here are the appropriate changes to `ThunkRef` ...

```
ThunkRef
%%%
public class ThunkRef extends Ref {

    public Exp exp;
    public Env env;
    public Val val;

    public ThunkRef(Exp exp, Env env) {
        this.exp = exp;
        this.env = env;
        this.val = null;
    }

    public Val deRef() {
        if (val == null)
            val = exp.eval(env);
        return val;
    }

...

}
%%%
```

Language NEED (continued)

You might have noticed in the `code` file that an instance of the class `ValRORef` is constructed by the `evalRef` methods in the `LitExp` and `ProcExp` classes. This is a slight change from the `NAME` language, where the instances were `ValRefs`. The `RO` part stands for "Read Only". We do this because it doesn't make sense for a literal or procedure to be modified.

Consider, for example, the following code:

```
let
  f = proc(x) set x=add1(x)
in
  .f(3)
```

One's intuition would be to think of the procedure application `.f(3)` as saying:

```
set 3=add1(3)
```

But of course this doesn't make any sense.

We have already seen that the `setRef` method in the `ThunkRef` class throws an exception. What we are now doing is to have this same behavior for *any* actual parameter expression except for a variable (where call-by-reference is the default).

Language NEED (continued)

The following example illustrates the difference between call-by-name and call-by-need:

```
let
  x = 3
  p = proc(t) {t;t;t}
in
  .p(set x=add1(x))
```

With call-by-name, when we apply the procedure `p`, its formal parameter `t` is bound to a thunk containing the expression `set x=add1(x)`. Each time we evaluate the formal parameter `t` in the body of the procedure `p`, its thunk is dereferenced, resulting in evaluation of the expression `set x=add1(x)`. So since we evaluate `t` three times in the body of `p`, the expression `set x=add1(x)` gets evaluated three times, incrementing `x` from 3 to 6. Consequently, the entire expression evaluates to 6.

With call-by-need, the first time we evaluate `t` in the body of `p`, its corresponding actual parameter expression `set x=add1(x)` is evaluated, which has the side-effect of incrementing the value of `x` to 4 and evaluates to 4. However, the thunk memoizes the expressed value of 4, so any further references we make to `t` evaluate to 4. Consequently, the entire expression evaluates to 4.

Language NEED (continued)

Here's another example illustrating the difference between call-by-reference and call-by-name/need. Examine the definition of `seq`, which seems to recurse infinitely but doesn't with call-by-name (why?).

```
define pair = proc(x,y)
  proc(t) if t then y else x
define first = proc(p) .p(0)
define rest = proc(p) .p(1)
define nth = proc(n,lst)    % zero-based
  if n then .nth(sub1(n),.rest(lst)) else .first(lst)
define seq = proc(n) .pair(n,.seq(add1(n)))
define natno = .seq(0)      % all the natural numbers!!
%% The above never terminates with call-by-reference.
%% With call-by-name or call-by-need, we get:
.first(natno)               % => 0
.first(.rest(natno))        % => 1
.first(.rest(.rest(natno))) % => 2, and so forth ...
.nth(100,natno)             % => 100
```

Order of evaluation

Let's examine the following example:

```
let
  x = 3
in
  let
    y = {set x = add1(x)}
    z = {set x = add1(x)}
  in
    z
```

Consider the inner `let`. We know that the right-hand side expressions (here written inside curley braces for clarity) are evaluated before their values are bound to the left-hand variables. Our language specifies the order in which the right-hand side expressions are evaluated, namely left-to-right.

In the absence of side-effects, *i.e.* in our early interpreters without `set`, the order of evaluation of the RHS expressions wouldn't matter. However, when side-effects are possible, as in our interpreters such as `SET` and `REF`, the order of evaluation does matter.

In the above example, if the second `set` were to be evaluated first, then `z` becomes 4 and `y` becomes 5, so the entire expression evaluates to 4 – the value of `z`. If the order of evaluation is reversed, the entire expression evaluates to 5.

Order of evaluation (continued)

A similar situation exists when evaluating actual parameter expressions, as shown by this example, assuming call-by-value semantics.

```
let
  x = 3
  p = proc(t,u) t
in
  .p(set x = add1(x), set x = add1(x))
```

When the actual parameters are evaluated left-to-right as specified in our languages, `t` would be bound to 4 and `u` to 5, so the entire expression evaluates to 4. If the evaluation order had been right-to-left, the entire expression would evaluate to 5.

Order of evaluation (continued)

You can see that both `evalRands` and `evalRandsRef` use `for-each` loops (also called enhanced `for` loops) to traverse and evaluate the expressions in the list of actual parameters. The traversal is guaranteed by the Java API specification to be "natural", in the sense that the elements of the list are visited in ascending item number order. Here is the code for `evalRands` in the `Rands` class:

```
public List<Val> evalRands(Env env) {
    List<Val> valList = new ArrayList<Val>();
    for (Exp e : expList)
        valList.add(e.eval(env));
    return valList;
}
```

Our order of evaluation semantics depends on the behavior of Java's `for-each` order of evaluation mechanism, which is guaranteed to be left-to-right. If we had chosen right-to-left semantics, we could, instead, explicitly traverse the `expList` from last to first if we wished.

The point is that, to make any language semantics well-defined and unambiguous, it is necessary to specify the order of evaluation. Unless the language specification clearly addresses the issue of order of evaluation, the language implementor can choose any evaluation order. *Let the buyer beware!*

Order of evaluation (continued)

Both Java and Python specify that actual parameter expressions are evaluated left-to-right. However, the C language specification explicitly states that the order in which actual parameter expressions are evaluated is *undefined* – which means that the evaluation order is implementation dependent. In the following C program, the output is 3 (using the GCC compiler on the date this file was last modified), which shows that the the operand expressions `foo(3)` and `foo(5)` are evaluated right-to-left. Your mileage may vary!

```
#include <stdio.h>
#include <stdlib.h>
static int xx = 0;
void foo2(int x1, int x2) {
    return;
}
int foo(int x) {
    xx = x;
    return x;
}
int main(int argc, char ** argv) {
    foo2(foo(3), foo(5));
    printf("xx=%d\n", xx);
    return 0;
}
```

Order of evaluation (continued)

Order of evaluation does not matter in languages without side-effects, which makes functional languages immune to order of evaluation issues. See `http://en.wikipedia.org/wiki/Evaluation_strategy` for more information about order of evaluation.

Another way to avoid order of evaluation problems is to require that all procedures have at most one formal parameter. In languages that use this approach, coupled with with call-by-need, there is never an "order of evaluation" issue because there is never more than one actual parameter to evaluate.

While you may think that a language with procedures having only one formal parameter might be limited, it's possible for such a language to behave like having multiple formal parameters using an approach called "Currying", as employed in the Haskell programming language – named after Haskell Curry. The following slide gives an example.

Order of evaluation (continued)

Here is an example without currying:

```
let
  x = 3
  y = 5
  p = proc(t,u) +(t,u)
in
  .p(x,y) % => 8
```

Here is semantically equivalent code that has exactly one formal parameter per `proc`:

```
let
  x = 3
  y = 5
  p = proc(t) proc(u) +(t,u)
in
  ..p(x)(y)
```

In the second example above, `x` must be evaluated first, so that the procedure `.p(x)` can then be applied to the value of `y`.

Aliasing

Side-effecting languages that use call-by-reference suffer from another danger. Consider, for example, the following program using the REF language semantics:

```
let
  addplus1 = proc(x,y) {set x = add1(x) ; +(x,y)}
in
  .addplus1(3,3)
```

It's clear that this program returns 7. But what about the following program?

```
let
  a = 3
  addplus1 = proc(x,y) {set x = add1(x) ; +(x,y)}
in
  .addplus1(a,a)
```

Aliasing (continued)


```
let
  a = 3
  addplus1 = proc(x,y) {set x = add1(x) ; +(x,y)}
in
  .addplus1(a,a)
```

Using call-by-reference, when `addplus1` is applied to the actual parameters `a` and `a`, both formal parameters `x` and `y` of `addplus1` refer to the *same cell* as `a`. Therefore the `set x = add1(x)` expression is equivalent to the expression `set a = add1(a)` which increments `a` to 4, and the next expression `+(x,y)` is essentially equivalent to the expression `+(a,a)` which now evaluates to 8. Thus the value of the program is 8.

Aliasing occurs when two different formal parameters refer to the same actual parameter. As this example shows, aliasing can lead to unexpected side-effects and should be avoided. **Of course, the best way to avoid problems such as order of evaluation ambiguities and aliasing is to avoid using langauges with side-effects!**