# 331 – Intro to Intelligent Systems
# Week07b
# First-Order Predicate Logic
# R&N Chapter 8

T.J. Borrelli

# First-Order (Predicate) Logic

- Propositional logic assumes the world contains facts
- Predicate logic (like natural language) assumes the world contains:
  - Objects: people, houses, numbers, colors, baseball games, wars, …
  - Relations: red, round, prime, brother of, bigger than, part of, comes between, …
  - Functions: father of, best friend, one more than, plus, …

# Why FOL?

- Why do we need First-order logic?
- Propositional logic is too weak a language to represent knowledge of complex environments in a concise way
- We already saw this to some degree with the Wumpus world example – we would need many Rules ($R_n$) in order to make valid conclusions
- FOL has sufficient expressive power to deal with partial information

# FOL

- We adopt the foundation of propositional logic:
- Declarative – knowledge and inference are separate, and inference is entirely domain independent
- Compositionality – the meaning of a sentence is a function of the meaning of its parts.
- Context-independent and unambiguous
- FOL can easily express facts about **some** or **all** objects in the universe

# Logics

- **Ontological commitment – what the logic assumes about the nature of reality

- **Epistemological commitments – the possible states of knowledge that it allows with respect to each fact
- ** more on this if you put on exam.

# Propositional Logic VS FOL

| Logic | Ontological commitment (what exists in the world) | Epistemological Commitment (what an agent believes about facts) |
|---|---|---|
| Propositional Logic | facts | True/false/unknown |
| First-order Logic | Facts, objects, relations | True/false/unknown |

# Syntax of Predicate Calculus

Symbols represent constants, variables, functions, or predicates:

- Constants begin with a lower-case letter
  - bill, john, etc.
  - The constants ‘true’ and ‘false’ are reserved as truth symbols
- Variables begin with an upper-case letter
  - For example, X, is a variable that can represent *any* constant
- Functions have input and produce an output
  - plus(5,4)
  - The *arity* of a function is the number of arguments
  - For example, ‘plus’ has an arity of 2
- Predicates are similar to functions, but return true
  - A predicate names a relationship between objects in the world
  - For example, father(bill, john) means that “bill is the father of john”
  - father(bill, X) refers to any child of bill

# Semantics of Predicate Calculus

- Quantification of variables is important
- When a variable appears in a sentence, that variable serves as a *placeholder*
  - For example, the "X" in likes(george, X)
  - Any constant allowed under the interpretation can be substituted for it in the expression
  - Substituting "kate" or "susie" for "X" forms the statement likes(george, kate) or likes(george, susie)

# Sentences in Predicate Calculus

- Sentences are created by combining predicates using logical operators (the same as used for propositional calculus) and *quantifiers*
- Examples:

  likes(george, kate) ∧ friend(X, george)

  ¬helps(X, X)

  has_children(ben, plus(2, 3))

  friend(father_of(david, X), father_of(andrew, Y))
- *likes*, *friend*, *helps* and *father_of* and *has_children* are predicates
- *plus* is a function

# Sentences in Predicate Calculus

- When a variable appears in a sentence, it refers to unspecified objects in the domain. First order predicate calculus includes two additional symbols, the variable quantifiers ∀ and ∃, that constrain the meaning of a sentence:

  $\exists$Y friend(Y, peter)

  $\forall$X likes(X, ice_cream)

# Quantifiers ∀ and ∃

- ∀ The universal quantifier

  $\forall X\ p(X)$ is read “For all X, p(X) is true”

- ∃ The existential quantifier

  $\exists X\ p(X)$ is read “There exists an X such that p(X) is true”

- Relationship between the quantifiers:

  $\exists X\ p(X) \equiv \neg(\forall X)\neg p(X)$

  “There exists an X for which p(X) is true” is equivalent to “it is not true that for all X p(X) does not hold”

# Universal Quantification

- Typically, $\rightarrow$ is the main connective with $\forall$

  Example: "Everyone at RIT is smart":

  $$\forall X\ at(X, rit) \rightarrow smart(X)$$

- Common mistake: using $\wedge$ as the main connective with $\forall$:

  $$\forall X\ at(X, rit) \wedge smart(X)$$

  means "Everyone is at RIT and everyone is smart"

# Existential Quantification

- Typically, $\wedge$ is the main connective with $\exists$

  Example: “Someone at RIT is smart”:

  $$\exists X \; at(X, rit) \wedge smart(X)$$

- Common mistake: using $\rightarrow$ as the main connective with $\exists$:

  $$\exists X \; at(X, rit) \rightarrow smart(X)$$

This means:

“If RIT has a student then that student is smart.”

# Properties of Quantifiers

- $\forall X\ \forall Y$ is the same as $\forall Y\ \forall X$
- $\exists X\ \exists Y$ is the same as $\exists Y\ \exists X$
- $\exists X\ \forall Y$ is not the same as $\forall Y\ \exists X$
- $\exists X\ \forall Y$ loves(X,Y)
  - "There is a person who loves everyone"
- $\forall Y\ \exists X$ loves(X,Y)
  - "Everyone is loved by someone"
- Quantifier duality: each can be expressed using the other:
  - $\forall X$ likes(X, iceCream)= $\neg \exists X \neg$likes(X, iceCream)
  - $\exists X$ likes(X, broccoli) = $\neg \forall X \neg$likes(X, broccoli)

# Equality

- $term_1 = term_2$ is true under a given interpretation if and only if $term_1$ and $term_2$ refer to the same object

- For example, the definition of *sibling* in terms of *parent* is:

  $$\forall X,Y\ (\neg(X = Y) \land \exists M,F\ \neg(M = F) \land parent(M,X) \land parent(F,X) \land parent(M,Y) \land parent(F,Y) \rightarrow sibling(X,Y))$$

# Using First-Order Logic

First-order rules for the kinship domain:

- A brother is a male sibling
- $\forall X,Y\ (male(X) \wedge sibling(X,Y) \rightarrow brother(X,Y))$

- “sibling” is symmetric
- $\forall X,Y\ (sibling(Y,X) \rightarrow sibling(X,Y))$

- One's mother is one's female parent
- $\forall M,C\ (female(M) \wedge parent(M,C) \rightarrow mother(M,C))$

# Using First-Order Logic

For example, given the following facts:

mother(eve,abel)
mother(eve,cain)
father(adam,abel)
father(adam,cain)

And the following rules:

$\forall X,Y\ (father(X,Y) \lor mother(X,Y) \rightarrow parent(X,Y))$
$\forall X,Y,Z\ (parent(X,Y) \land parent(X,Z) \rightarrow sibling(Y,Z))$

One can conclude that cain and abel are siblings (or half-siblings)

# Examples of First-Order Predicates

- “If it doesn’t rain on Monday, Tom will go to the mountains.”

  $\neg$weather(rain, monday) $\rightarrow$ go(tom, mountains)
- “Emma is a Doberman pinscher and a good dog.”

  is_a(emma,doberman) $\wedge$ good_dog(emma)
- “All basketball players are tall.”

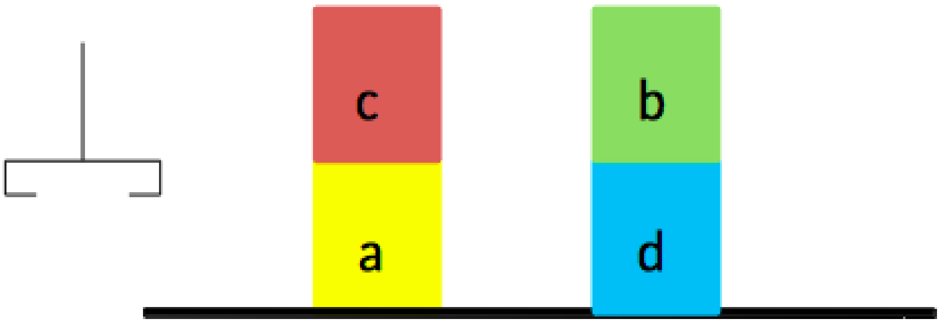
  $\forall$ X (basketball_player(X) $\rightarrow$ tall(X))
- “Some people like anchovies.”

  $\exists$ X (person(X) $\wedge$ likes(X, anchovies))
- “If wishes were horses, beggars would ride.”

  equal(wishes, horses) $\rightarrow$ ride(beggars)
- “Nobody likes taxes.”

  $\neg\exists$ X likes(X, taxes)

# Blocks World Example




A collection of logical clauses describes the important properties and relationships in a Blocks World:

on(c,a)
on(b,d)
onTable(a)
onTable(d)
clear(b)
clear(c)

Suppose you want to define a test to determine whether **all** blocks are clear (have nothing stacked on top of them):

$\forall$ X ($\neg$ $\exists$ Y on (Y, X)) $\rightarrow$ clear(X).

"For all X, if there does not exist a Y such that Y is on X, then X is clear."