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CSCI 261 Section 2

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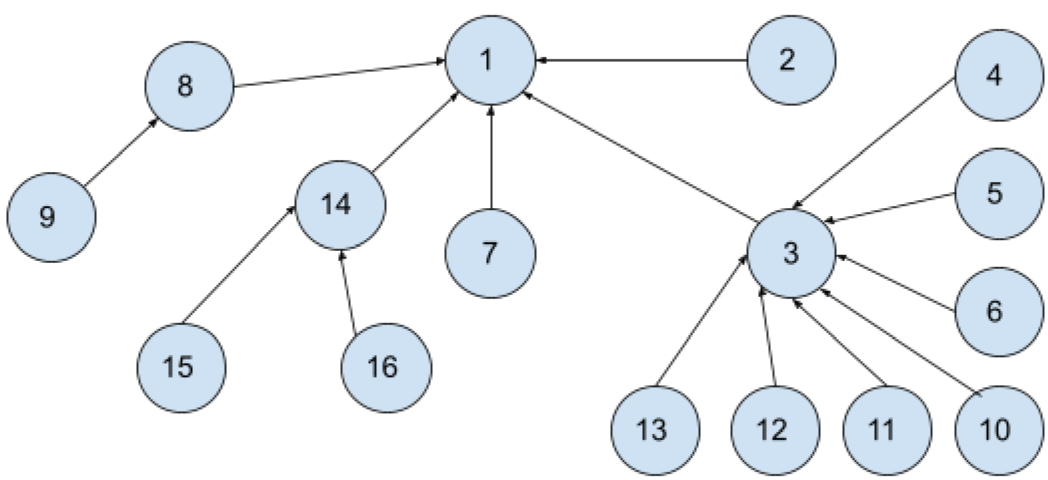
Homework #4

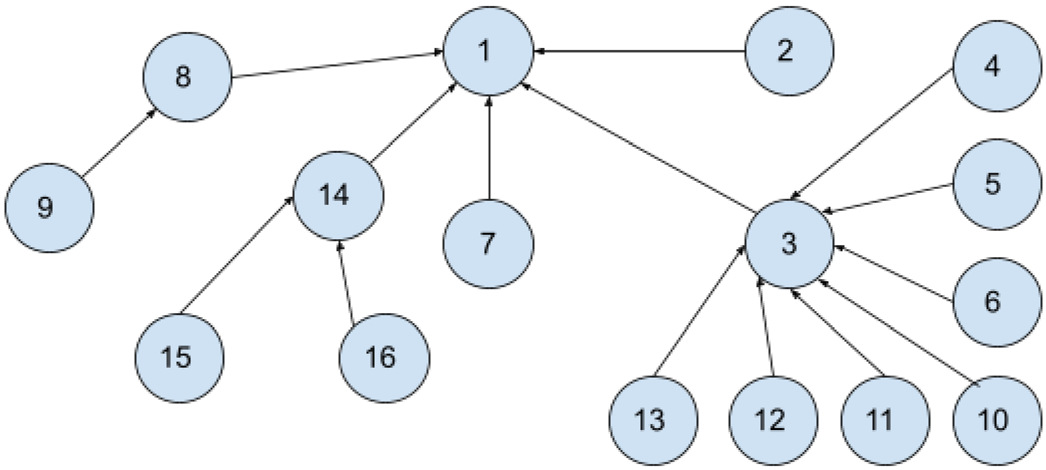
E.1:*Show the result of the following sequence of instructions. For arbitrary unions, union the second parameter to the first. Assume that the union occurs on the roots of the arguments, that is union(a, b) = union(find(a), find(b)). Draw your resulting sets as trees.*

union(1,2), union(3,4), union(3,5), union(1,7), union(3,6), union(8,9), union(1,8), union(3,10), union(3,11), union(3,12), union(3,13), union(14,15), union(16,0), union(14,16), union(1,3), union(1,14).

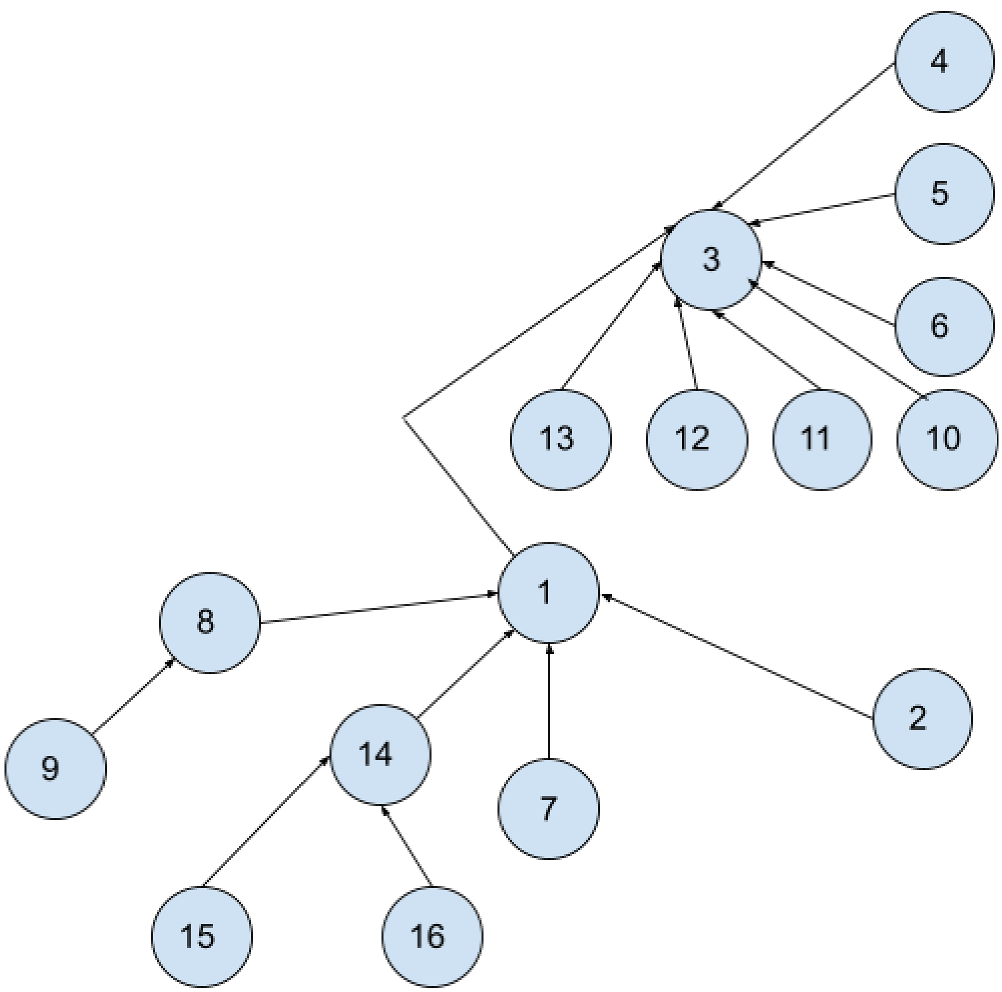
a) perform unions arbitrarily



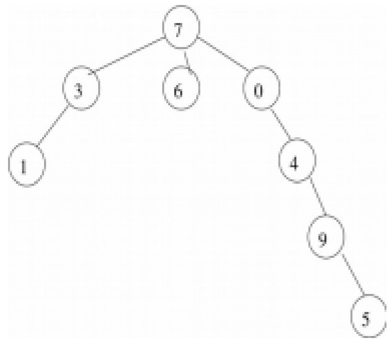
b) perform unions by tree height



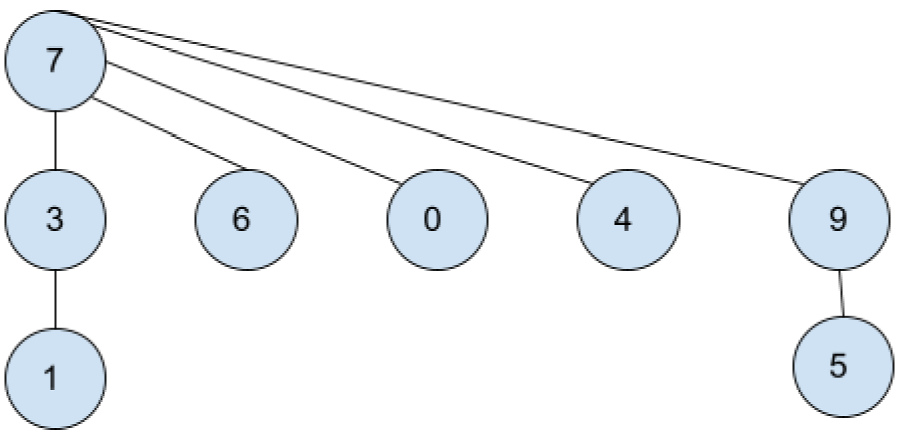
c) perform unions by tree size



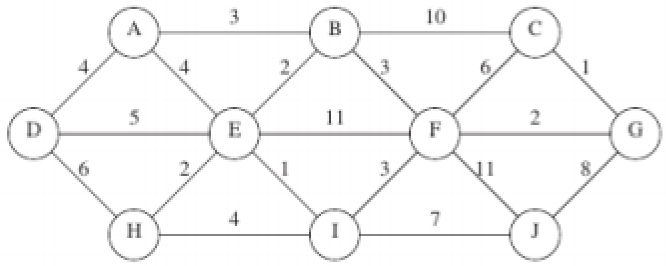
E.2: *Consider the following tree: Show the tree after performing find(9) with path compression.*



find(9) = 7



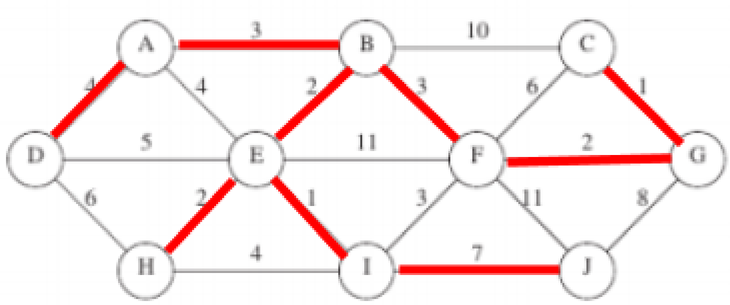
E.3: *Find a minimum spanning tree for the following graph using both Prim's and Kruskal's algorithm. Show work.*



Prim's Algorithm:

Vertex	*Known*	*Cost*	*Path*
A	*F -> **T***	*4*	*D*
B	*F -> **T***	*3*	*A*
C	*F -> **T***	*1*	*G*
D	*F -> **T***	*0*	*0*
E	*F -> **T***	*2*	*B*
F	*F -> **T***	*3*	*B*
G	*F -> **T***	*2*	*F*
H	*F -> **T***	*2*	*E*
I	*F -> **T***	*1*	*E*
J	*F -> **T***	*7*	*I*

MST is all red paths:

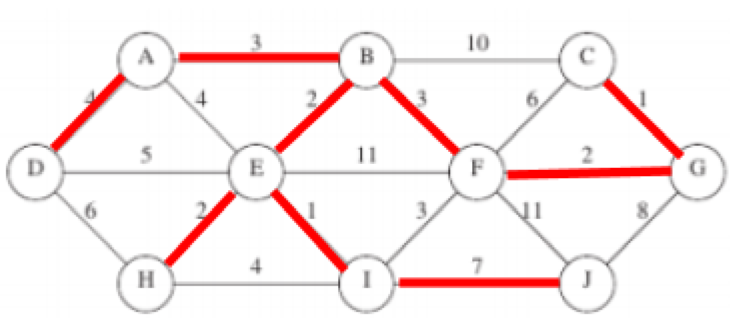


Kruskal's Algorithm:

Edge	*Weight*	*Action*
(A, D)	*4*	*Accepted*
(A, B)	*3*	*Accepted*
(B, E)	*2*	*Accepted*
(B, F)	*3*	*Accepted*
(C, G)	*1*	*Accepted*
(E, H)	*2*	*Accepted*
(E, I)	*1*	*Accepted*
(I, J)	*7*	*Accepted*

Rejected paths are: (D, E), (D, H), (H, I), (A, E), (E, F), (B, C), (I, F), (F, J), and (G, J)

MST is all red paths:



Text 4.8: *Suppose you are given a connected graph G, with edge costs that are all distinct. Prove that G has a unique minimum spanning tree.*

If we suppose that there are two distinct MSTs(Minimum Spanning trees) from graph G, T, and T' (initiating proof by contradiction). Both of these trees have the same number of edges. However, they are not equal. There is the same edge $e \in$ T and $e \notin$ T'. Now, if we add e to T, we get a cycle. If we let e be the most expensive edge on this cycle, we know that by the Cycle property, $e \notin$ any MST. We get a contradiction, which proves the statement.