MATH 351–004 – Assignment #3

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Problem 1 - For which integers x ($0 \leq x \leq 7$), if any, is the sequence s = 7, 6, 5, 4, 3, 2, 1, x graphical?)

Using Corollary 2.3, we know that every graph must have an even number of odd vertices, currently in s, there are 4 and adding another odd vertex would make the number odd, which we cannot have, therefore x must be even.
Using Theorem 2.10:

x = 0
s = 7, 6, 5, 4, 3, 2, 1, 0
$s\prime$ = 5, 4, 3, 2, 1, 0, -1
Not graphical because there exists a negative number in $s\prime$.

x = 2
s = 7, 6, 5, 4, 3, 2, 1, 2
s = 7, 6, 5, 4, 3, 2, 2, 1
$s\prime$ = 5, 4, 3, 2, 1, 1, 0
$s\prime\prime$ = 3, 2, 1, 0, 0, 0
$s\prime\prime\prime$ = 1, 0, 0, 0, -1
Not graphical because there exists a negative number in $s\prime\prime\prime$.

x = 4
s = 7, 6, 5, 4, 3, 2, 1, 4
s = 7, 6, 5, 4, 4, 3, 2, 1
$s\prime$ = 5, 4, 3, 3, 2, 1, 0
$s\prime\prime$ = 3, 2, 2, 1, 0, 0
$s\prime\prime\prime$ = 1, 1, 0, 0, 0
Graphical because $s\prime\prime\prime$ is just 5 points with an edge between two vertices

x = 6
s = 7, 6, 5, 4, 3, 2, 1, 6
s = 7, 6, 6, 5, 4, 3, 2, 1
$s\prime$ = 5, 5, 4, 3, 2, 1, 0
$s\prime\prime$ = 4, 3, 2, 1, 0, 0
$s\prime\prime\prime$ = 2, 1, 0, 0, -1
Not graphical because there exists a negative number in $s\prime\prime\prime$.

Problem 2 - Which pairs in Figure 3.12 are isomorphic? Explain your answer.

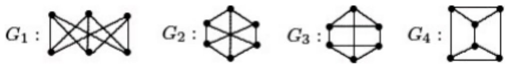
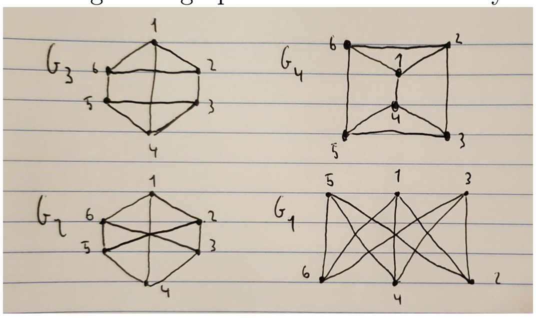


Figure 3.12: The graphs in Exercise 3.4

Two graphs, G and H, can be labeled as isomorphic if there exists a correspondence between $V(G)$ and $V(H)$ such that $uv \in E(G) iff \phi(u)\phi(v) \in E(H)$.

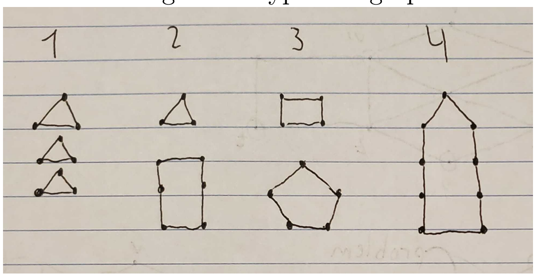
Graph G_3 is isomorphic to G_4.
Graph G_1 is isomorphic to G_2.

Labeling these graphs' vertices in such way allows their structures to be the same.



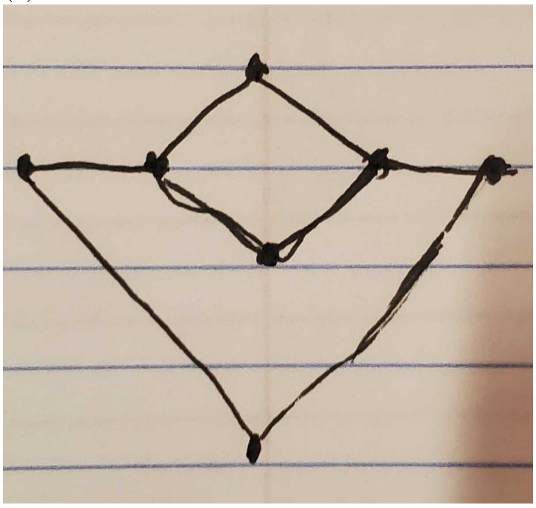
Problem 3 - How many (non-isomorphic) graphs have the degree sequence s: 6, 6, 6, 6, 6, 6, 6, 6, 6. (Nine 6's)

Using Theorem 3.1, we can describe this graph's degree sequence as s^c: 2, 2, 2, 2, 2, 2, 2, 2, 2, 2.
There are 4 general types of graphs that fall under this degree sequence, which are shown below.



Problem 4 - For the deck D of cards given in Figure 3.42, where card i contains the subgraph $G_i = G \ v_i, v_i \in V(G)$, for some graph G, answer the following with explanation.

(a) What is the order n of G?
The order is 7.
(b) What is the size m of G?
The size is 8.
(c) What are the degrees of vertices in G?
$s = 3, 3, 2, 2, 2, 2, 2$
(d) Is G connected?
G is connected.
(e) What are the solutions of D



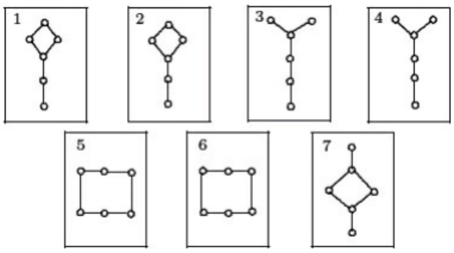


Figure 3.42: The deck of cards for Exercise 3.34