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MATH-237-54
Exam 3

1) $y' = 8e^t\cos(t)$

$y_h = C_1$

$y_p = Ae^t\sin(t) + Be^t\cos(t)$
$y_p' = Ae^t\sin(t) - Be^t\sin(t) + Ae^t\cos(t) + Be^t\cos(t)$

$y_p' = 8e^t\cos(t)$

$\Rightarrow Ae^t\sin(t) - Be^t\sin(t) + Ae^t\cos(t) + Be^t\cos(t) = 8e^t\cos(t)$
$(A-B)e^t\sin(t) + (A+B)e^t\cos(t) = 8e^t\cos(t)$

$A - B = 0 \quad : \quad A + B = 8$
$A = B \qquad\qquad A + A = 8$
$\boxed{4 = B} \qquad\qquad 2A = 8$
$\boxed{A = 4}$

$\Rightarrow y_p = 4e^t\sin(t) + 4e^t\cos(t)$

$\boxed{y = y_h + y_p = C_1 + 4e^t\sin(t) + 4e^t\cos(t)}$

2) $y'' = 4 + e^t$, $y(0) = 0$, $y'(0) = 0$

$y_h = C_1 + C_2 t$

$y_p = A + Be^t$ (A offends y_h)
$y_p = At + Be^t$ (At offends y_h)
$y_p = At^2 + Be^t$ (y_p does not offend y_h)
$y_{p_1} = 2At + Be^t$
$y_{p_{11}} = 2A + Be^t$

$y_p'' = 4 + e^t$

$\Rightarrow 2A + Be^t = 4 + e^t$

$2A = 4 \quad : \quad B = 1$
$A = 2$

$\Rightarrow y_p = 2t^2 + 1e^t$

$\Rightarrow y = y_h + y_p \Rightarrow C_1 + C_2 t + 2t^2 + e^t$

$y(0): \; 0 = C_1 + C_2(0) + 2(0)^2 + e^0$
$0 = C_1 + 1$
$\boxed{-1 = C_1}$

$y' = C_2 + 4t + e^t$

$y'(0): \; 0 = C_2 + 4(0) + e^0$
$0 = C_2 + 1$
$\boxed{-1 = C_2}$

$y = -1 + (-1)t + 2t^2 + e^t$

$\boxed{y = -1 - t + 2t + e^t}$

3) $y'' - \left(\frac{1+t}{t}\right)y' + \left(\frac{1}{t}\right)y = te^t$

$y_h = C_1 e^t + C_2(1+t)$ $\qquad f(t) = te^t$

$y_1 = e^t \qquad y_2 = 1+t$

$y_1' = e^t \qquad y_2' = 1$

$W = y_1 y_2' - y_1' y_2 \Rightarrow W = (e^t)(1) - (e^t)(1+t)$

$W = e^t + (-e^t - te^t)$

$W = -te^t$

$v_1 = -\int \frac{f(t) y_2}{W} dt \Rightarrow -\int \frac{(te^t)(1+t)}{-te^t} dt \Rightarrow -\int \frac{1+t}{-1} dt \Rightarrow t + t^2/2$

$v_2 = \int \frac{f(t) y_1}{W} dt \Rightarrow \int \frac{(te^t)(e^t)}{-te^t} dt \Rightarrow -\int e^t dt = -e^t$

$y_p = y_1 v_1 + y_2 v_2$

$y_p = (e^t)(t + t^2/2) + (1+t)(-e^t)$

$y_p = \left(te^t + \frac{e^t t^2}{2}\right) + (-e^t - te^t)$

$y_p = \frac{e^t t^2}{2} - e^t$

$y = y_h + y_p \Rightarrow C_1 e^t + C_2(1+t) + \frac{e^t t^2}{2} - e^t$

4) $y''' = 4/t$

$f(t) = 4/t$

$y_h = C_1 + C_2 t + C_3 t^2$

$y_1 = 1$	$y_2 = t$	$y_3 = t^2$
$y_1' = 0$	$y_2' = 1$	$y_3' = 2t$
$y_1'' = 0$	$y_2'' = 0$	$y_3'' = 2$

eqn 1: $y_1 v_1' + y_2 v_2' + y_3 v_3' = 0$

eqn 2: $y_1' v_1' + y_2' v_2' + y_3' v_3' = 0$

eqn 3: $y_1'' v_1' + y_2'' v_2' + y_3'' v_3' = f(t)$

eqn 1:

$(1)v_1' + (t)v_2' + (t^2)v_3' = 0$

$v_1' + (t)(-4) + (t^2)(2/t) = 0$

$v_1' + (-4t) + (2t) = 0$

$v_1' - 2t = 0$

$\boxed{v_1' = 2t}$

$\boxed{v_1 = t^2}$

eqn 2:

$(0)v_1' + (1)v_2' + (2t)v_3' = 0$

$v_2' + (2t)(2/t) = 0$

$\boxed{v_2' = -4}$

$\boxed{v_2 = -4t}$

eqn 3:

$(0)v_1' + (0)v_2' + 2(v_3') = 4/t$

$2v_3' = 4/t$

$\boxed{v_3' = 2/t}$

$\boxed{v_3 = 2\ln|t|}$

$y_p = y_1 v_1 + y_2 v_2 + y_3 v_3$

$y_p = (1)(t^2) + (t)(-4t) + (t^2)(2\ln|t|)$

$y_p = t^2 - 4t^2 + 2t^2 \ln|t|$

$y_p = -3t^2 + 2t^2 \ln|t|$

$\boxed{y = y_h + y_p = C_1 + C_2 t + C_3 t^2 - 3t^2 + 2t^2 \ln|t|}$