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MATH-231 54
HW#6

17) $y'' - 2y' - 3y = 3t^2 - 5$

$\Rightarrow r^2 - 2r - 3 = 0$
$(r-3)(r+1)$
$r_1 = 3,\ r_2 = -1$

$y_p(t) = at^2 + bt + c$
$y_p'(t) = 2at + b$
$y_p''(t) = 2a$

$y_h = C_1 e^{3t} + C_2 e^{-t}$

$y_p''(t) - 2(y_p'(t)) - 3(y_p(t)) = 3t^2 - 5$

$2a + 2(-2at - b) + 3(-at^2 - bt - c) = 3t^2 - 5$

$2a - 4at - 2b - 3at^2 - 3bt - 3c = 3t^2 - 5$

$\underline{(-3a)t^2} - \underline{(4a - 3b)t} + \underline{(2a - 2b - 3c)} = \underline{3t^2} + \underline{0t} - \underline{5}$

$-3at^2 = 3t^2$
$-3a = 3$
$a = -1$

$: (-4a - 3b)t = 0t$
$(-4a - 3b) = 0$
$(+4 - 3b) = 0$
$-3b = +4$
$b = 4/3$

$: 2a - 2b - 3c = -5$
$-2 - 8/3 - 3c = -5$
$-8/3 - 3c = -9/3$
$-3c = -1/3$
$c = 1/9$

$y_p(t) = at^2 + bt + c \Rightarrow y_p(t) = -t^2 + 4/3\,t + 1/9$

$y(t) = y_h(t) + y_p(t) = \boxed{C_1 e^{3t} + C_2 e^{-t} - t^2 + 4/3\,t + 1/9}$

14) $y''(x) - 3y'(x) + 2y(x) = e^x \sin(x)$

$\Rightarrow r^2 - 3r + 2 = 0$

$(r-2)(r-1)$

$r_1 = 2, \; r_2 = 1$

$y_p(t) = ae^x \sin(x) + be^x \cos(x)$

$y_p'(t) = (a-b)e^x \sin(x) + (a+b)e^x \cos(x)$

$y_p''(t) = 2ae^x \cos(x) - 2be^x \sin(x)$

$y_h = C_1 e^{2t} + C_2 e^t$

$y_p''(t) - 3y_p'(t) + 2y_p(t) = e^x \sin(x)$

$2ae^x \cos(x) - 2be^x \sin(x) - 3\big((a-b)e^x \sin(x) + (a+b)e^x \cos(x)\big) + 2\big(ae^x \sin(x) + be^x \cos(x)\big)$

$2ae^x \cos(x) - 2be^x \sin(x) - 3\big(ae^x \sin(x) - be^x \sin(x) + ae^x \cos(x) + be^x \cos(x)\big) + 2ae^x \sin(x) + 2be^x \cos(x)$

$\underline{2ae^x \cos(x)} \; \underline{- 2be^x \sin(x)} \; \underline{- 3ae^x \sin(x)} + \underline{3be^x \sin(x)} \; \underline{- 3ae^x \cos(x)} \; \underline{- 3be^x \cos(x)} + \underline{2ae^x \sin(x)} + \underline{2be^x \cos(x)}$

$-ae^x \cos(x) + be^x \cos(x) - ae^x \sin(x) + be^x \sin(x) = e^x \sin(x)$

$-(a+b)e^x \cos(x) + (a+b)e^x \sin(x) = e^x \sin(x)$

when $x=0$: $-(a+b)e^0 \cos(0) + (-a+b)e^0 \sin(0) = e^0 \sin(0)$

$-(a+b) \cdot 1 \cdot 1 + (-a+b) \cdot 1 \cdot 0 = 0$

$-(a+b) = 0$

$b = -a$

when $x = \pi/2$: $(-a+b)e^{\pi/2} \cos(\pi/2) + (a+b)e^{\pi/2} \sin(\pi/2) = e^{\pi/2} \sin(\pi/2)$

$(-a+b)e^{\pi/2} \cdot 0 + (-a+b)e^{\pi/2} \cdot 1 = e^{\pi/2}$

$+(-a+b) \cdot e^{\pi/2} = e^{\pi/2}$

$-a + b = 1$

$b + b = 1$

$b = 1/2, \quad a = -1/2$

$y = y_h + y_p \Rightarrow \boxed{C_1 e^{2t} + C_2 e^t - \tfrac{1}{2} e^x \sin(x) + \tfrac{1}{2} e^x \cos(x)}$

③

29) $y'' - y = \sin(\theta) - e^{2\theta}$, $y(0) = 1$, $y'(0) = -1$

$\Rightarrow r^2 - 1 = 0$

$(r+1)(r-1)$

$r_1 = -1$, $r_2 = 1$

$y_h = C_1 e^{-t} + C_2 e^{t}$

$y_p = a\sin(\theta) + b\cos(\theta) + ce^{2\theta}$

$y_{p1} = a\sin(\theta) + b\cos(\theta)$

$y_{p1}' = a\cos(\theta) - b\sin(\theta)$

$y_{p1}'' = -a\sin(\theta) - b\cos(\theta)$

$y_{p2} = ce^{2\theta}$

$y_{p2}' = 2ce^{2\theta}$

$y_{p2}'' = 4ce^{2\theta}$

$y_{p1}'' - y_{p1} = \sin(\theta)$

$(-a\sin(\theta) - b\cos(\theta)) - (a\sin(\theta) + b\cos(\theta)) = \sin(\theta)$

$-a\sin(\theta) - b\cos(\theta) - a\sin(\theta) - b\cos(\theta) = \sin(\theta)$

$-2a\sin(0) - 2b\cos(\theta) = \sin(\theta)$

$-2a = 1$; $-2b = 0$

$a = -1/2$ $\quad b = 0$

$y_{p1} = -\frac{1}{2}\sin(\theta)$

$y_{p2}'' - y_{p2} = -e^{2\theta}$

$(4ce^{2\theta}) - (ce^{2\theta}) = e^{-2\theta}$

$3ce^{2\theta} = -e^{2\theta}$

$3c = -1$

$c = -1/3$

$y_{p2} = -\frac{1}{3}e^{2\theta}$

$y = y_h + y_{p1} + y_{p2} \Rightarrow C_1 e^{-t} + C_2 e^{t} - \frac{1}{2}\sin(\theta) - \frac{1}{3}e^{2\theta}$

$y' = -C_1 e^{-t} + C_2 e^{t} - \frac{1}{2}\cos(\theta) - \frac{2}{3}e^{2\theta}$

$y(0)$

$1 = C_1 e^{-0} + C_2 e^{0} - \frac{1}{2}\sin(0) - \frac{1}{3}e^{2\cdot 0}$

$1 = C_1 + C_2 - \frac{1}{3}$

$\frac{4}{3} = C_1 + C_2$

$y'(0)$

$-1 = -C_1 e^{-0} + C_2 e^{0} - \frac{1}{2}\cos(0) - \frac{2}{3}e^{2\cdot 0}$

$-1 = -C_1 + C_2 - \frac{1}{2} - \frac{2}{3}$

$-1 = -C_1 + C_2 - \frac{7}{6}$

$\frac{1}{6} = -C_1 + C_2$

$\frac{8}{6} = C_1 + C_2$:

$\frac{8}{6} - C_1 = C_2$

$\frac{1}{6} = -C_1 + C_2$

$\frac{1}{6} = -C_1 + (\frac{8}{6} - C_1)$

$\frac{1}{6} = \frac{8}{6} - 2C_1$

$-\frac{7}{6} = -2C_1$

$\frac{7}{6} = 2C_1$

$\frac{7}{12} = C_1$

: $\frac{16}{12} = C_1 + C_2$

$\frac{16}{12} = \frac{7}{12} + C_2$

$\frac{9}{12} = C_2$

$$\boxed{y = (\tfrac{7}{12})e^{-\theta} + (\tfrac{3}{4})e^{\theta} - (\tfrac{1}{2})\sin(\theta) - (\tfrac{1}{3})e^{2\theta}}$$

5) $y''' - 2y'' - 5y' + 6y = e^x + x^2$

$r^3 - 2r^2 - 5r + 6 = 0$

$(r-1)(r-3)(r+2) = 0$

$r_1 = 1,\ r_2 = 3,\ r_3 = -2$

$y_h = C_1 e^t + C_2 e^{3t} + C_3 e^{-2t}$

$y_p = axe^x + b + cx + dx^2$

$y_p' = ae^x + axe^x + c + 2dx$

$y_p'' = 2ae^x + axe^x + 2d$

$y_p''' = 3ae^x + axe^x$

$y_p''' - 2y_p'' - 5y_p' + 6y_p = e^x + x^2$

$(3ae^x + axe^x) - 2(2ae^x + axe^x + 2d) - 5(ae^x + axe^x + c + 2dx) + 6(axe^x + b + cx + dx^2) = e^x + x^2$

$\underline{3ae^x} + \underline{axe^x} - \underline{4ae^x} - \underline{2axe^x} - \underline{4d} - \underline{5ae^x} - \underline{5axe^x} - \underline{5c} - \underline{10dx} + \underline{6axe^x} + \underline{6b} + \underline{6cx} + \underline{6dx^2} = e^x + x^2$

$-6ae^x + 6b - 5c - 4d + 6cx - 10dx + 6dx^2 = e^x + x^2$

$-6ae^x + (6b - 5c - 4d) + (6c - 10d)x + (6d)x^2 = e^x + x^2$

$-6a = 1$: $\boxed{a = -1/6}$

$6b - 5c - 4d = 0$

$6b - 5(5/18) - 4(1/6) = 0$

$6b - 25/18 - 4/6 = 0$

$6b = 37/18$

$\boxed{b = 37/108}$

$6c - 10d = 0$

$6c - 10(1/6) = 0$

$6c = 5/3$

$\boxed{c = 5/18}$

$6d = 1$: $\boxed{d = 1/6}$

$y = y_h + y_p \Rightarrow \boxed{C_1 e^t + C_2 e^{3t} + C_3 e^{-2t} + (-1/6)xe^x + 37/108 + (5/18)x + (1/6)x^2}$