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13) $\mathcal{L}\{6e^{-3t} - t^2 + 2t - 8\}$

$\Rightarrow \mathcal{L}\{6e^{-3t}\} - \mathcal{L}\{t^2\} + \mathcal{L}\{2t\} - \mathcal{L}\{8\}$

$\Rightarrow 6\mathcal{L}\{e^{-3t}\} - \mathcal{L}\{t^2\} + 2\mathcal{L}\{t\} - 8\mathcal{L}\{1\}$

$\Rightarrow 6\cdot\left(\frac{1}{s+3}\right) - \left(\frac{2!}{s^3}\right) + \left(\frac{2\cdot 1!}{s^2}\right) - \left(\frac{7}{8}\right)$

$\Rightarrow \boxed{6/s+3 - 2/s^3 + 2/s^2 - 1/8}$

20) $\mathcal{L}\{e^{-2t}\cos\sqrt{3}t - t^2 e^{-2t}\}$

$\Rightarrow \mathcal{L}\{e^{-2t}\cos\sqrt{3}t\} - \mathcal{L}\{t^2 e^{-2t}\}$

$\Rightarrow \frac{s+2}{(s+2)^4 + (\sqrt{3})^2} - \frac{2!}{(s+2)^3}$

$\Rightarrow \boxed{\frac{s+2}{s^2+4s+7} - \frac{2}{s^3+6s^2+12s+8}}$

1) $6/(s+7)^4 \Rightarrow t^n e^{at} = \frac{n!}{(s-a)^{n+1}} \Rightarrow n=3, a=7 \Rightarrow \boxed{t^3 e^{7t}}$

4) $4/(s^2+9) \Rightarrow (\sin(at)) = \frac{a}{s^2+a^2} \Rightarrow a=3 \Rightarrow (c)\frac{3}{s^2+9} \Rightarrow c=4/3 \Rightarrow \boxed{4/3\sin(3t)}$

11) $\frac{s^2-26s-47}{(s-1)(s+2)(s+5)} \Rightarrow \frac{A}{(s-1)} + \frac{B}{(s+2)} + \frac{C}{(s+5)} \Rightarrow \boxed{\frac{-4}{(s-1)} + \frac{-1}{(s+2)} + \frac{6}{(s+5)}}$

$s^2-26s-47 = A(s+2)(s+5) + B(s-1)(s+5) + C(s-1)(s+2)$

let $s=-2 \Rightarrow (-2)^2 - 26(-2) - 47 = B(-2-1)(-2+5)$

$\qquad\qquad\qquad 9 = B(-3)(3)$

$\qquad\qquad\qquad \boxed{-1 = B}$

let $s=-5 \Rightarrow (-5)^2 - 26(-5) - 47 = C(-5-1)(-5+2)$

$\qquad\qquad\qquad 108 = C(-6)(-3)$

$\qquad\qquad\qquad \boxed{6 = C}$

let $s=1 \Rightarrow (1)^2 - 26(1) - 47 = A(1+2)(1+5)$

$\qquad\qquad\qquad -72 = A(3)(6)$

$\qquad\qquad\qquad \boxed{-4 = A}$

13) $\dfrac{-2s^2-3s-2}{s(s+1)^2} \to \dfrac{A}{s} + \dfrac{B}{(s+1)} + \dfrac{C}{(s+1)^2} \Rightarrow \boxed{\dfrac{-2}{s} + \dfrac{0}{(s+1)} + \dfrac{1}{(s+1)^2}}$

$\Rightarrow -2s^2-3s-2 = A(s+1)^2 + B(s)(s+1) + C(s)$

$-2s^2-3s-2 = A(s^2+2s+1) + B(s^2+s) + Cs$

$-2s^2-3s-2 = As^2 + A2s + A + Bs^2 + Bs + Cs$

$-2s^2-3s-2 = (A+B)s^2 + (A2+B+C)s + A$

$-2 = A+B \quad : \quad -3 = A2+B+C \quad : \quad \boxed{-2 = A}$

$-2 = -2 + B \qquad -3 = (-2)2 + 0 + C$

$\boxed{0 = B} \qquad\qquad -3 = -4 + C$

$\qquad\qquad\qquad \boxed{1 = C}$

16) $\dfrac{-5s-36}{(s+2)(s^2+9)} \to \dfrac{A}{(s+2)} + \dfrac{Bs+C}{(s^2+9)} \Rightarrow \boxed{\dfrac{-2}{(s+2)} + \dfrac{2s-9}{s^2+9}}$

$\Rightarrow -5s-36 = A(s^2+9) + (Bs+C)(s+2)$

$-5s-36 = As^2 + A9 + Bs^2 + B2s + Cs + C2$

$-5s-36 = (A+B)s^2 + (B2+C)s + (A9+C2)$

$0 = A+B \quad : \quad -5 = B2+C \quad : \quad -36 = A9 + C2$

$-B = A \qquad -5 - B2 = C \qquad -36 = A9 + 2(-5-B2)$

$-B = -2 \qquad -5 + (2)(2) = C \qquad -36 = A9 - 10 - B4$

$\boxed{B = 2} \qquad -5 - 4 = C \qquad -36 = A9 - 10 + A4$

$\qquad\qquad \boxed{-9 = C} \qquad -26 = A13$

$\qquad\qquad\qquad\qquad \boxed{-2 = A}$

21) $\dfrac{6s^2-13s+2}{s(s-1)(s-6)} \to \dfrac{A}{(s)} + \dfrac{B}{(s-1)} + \dfrac{C}{(s-6)} \Rightarrow \boxed{\dfrac{1/3}{s} + \dfrac{1}{(s-1)} + \dfrac{14/3}{(s-6)}}$

$\Rightarrow 6s^2-13s+2 \Rightarrow A(s-1)(s-6) + B(s)(s-6) + C(s)(s-1)$

let $s=1 \Rightarrow 6(1)^2 - 13(1) + 2 \doteq B(1)(1-6)$

$-5 = B(-5)$

$\boxed{1 = B}$

let s=6 => $6(6)^2 - 13(6) + 2 = ((6)(6-1)$

$$140 = (30$$

$$\boxed{14/3 = C}$$

let s=0 => $6(0)^2 - 13(0) + 2 = A(0-1)(0-6)$

$$2 = A6$$

$$\boxed{1/3 = A}$$

$$\mathcal{L}^{-1}\left\{\frac{1}{3s} + \frac{1}{s-1} + \frac{14}{3(s-6)}\right\} \Rightarrow \mathcal{L}^{-1}\left\{\frac{1}{3s}\right\} + \mathcal{L}^{-1}\left\{\frac{1}{s-1}\right\} + \mathcal{L}^{-1}\left\{\frac{14}{3(s-6)}\right\}$$

$$\Rightarrow \frac{1}{3}\mathcal{L}^{-1}\left\{\frac{1}{s}\right\} + \mathcal{L}^{-1}\left\{\frac{1}{s-1}\right\} + \frac{14}{3}\mathcal{L}^{-1}\left\{\frac{1}{(s-6)}\right\}$$

$$\Rightarrow \frac{1}{3}(1) + e^{1t} + \frac{14}{3}e^{6t} \Rightarrow \boxed{\frac{1}{3} + e^t + \frac{14}{3}e^{6t}}$$

2) $y'' - y' - 2y = 0 \qquad y(0) = -2 \qquad y'(0) = 5$

$$\Rightarrow s^2 y(s) - sy(0) - y'(0) - sy(s) + y(0) - 2y(s) = 0$$

$$\Rightarrow s^2 y(s) - (-2)s - (5) - sy(s) + (-2) - 2y(s) = 0$$

$$(s^2 - s - 2)y(s) + (2)s + (-7) = 0$$

$$(s^2 - s - 2)y(s) = 7 - 2s$$

$$y(s) = \frac{7-2s}{s^2-s-2} \Rightarrow \frac{7-2s}{(s-2)(s+1)}$$

$$\frac{7-2s}{(s-2)(s+1)} = \frac{A}{(s-2)} + \frac{B}{(s+1)} \Rightarrow 7-2s = A(s+1) + B(s+2)$$

let s=-1 : $7 - 2(-1) = B(-1+2)$

$9 = B(-3)$

$$\boxed{-3 = B}$$

let s=2 : $7 - 2(2) = A(2+1)$

$3 = A(3)$

$$\boxed{1 = A}$$

$$y(s) = \frac{1}{(s-2)} + \frac{-3}{s+1}$$

$$y(t) = \mathcal{L}^{-1}\left\{\frac{1}{(s-2)} - \frac{3}{(s+1)}\right\}$$

$$\mathcal{L}^{-1}\left\{\frac{1}{(s-2)}\right\} - 3\mathcal{L}^{-1}\left\{\frac{1}{s+1}\right\} \Rightarrow e^{2t} - 3(e^{-1t}) \Rightarrow \boxed{e^{2t} - \frac{3}{e^t}}$$

8) $y'' + 4y = 4t^2 - 4t + 10$ $y(0) = 0$ $y'(0) = 3$

$\mathcal{L}\{y'' + 4y\} = \mathcal{L}\{4t^2 - 4t + 10\}$

$\mathcal{L}\{y''\} + 4\mathcal{L}\{y\} = 4\mathcal{L}\{t^2\} - 4\mathcal{L}\{t\} + 10\mathcal{L}\{1\}$

$\Rightarrow s^2 y(s) - sy(0) - y'(0) + 4y(s) = \left(4 \cdot \frac{2!}{s^3}\right) - \left(4 \cdot \frac{1!}{s^2}\right) + \left(10 \cdot \frac{1}{s}\right)$

$(s^2 + 4) y(s) - 3 = 8/s^3 - 4/s^2 + 10/s$

$(s^2 + 4) y(s) = 8/s^3 - 4/s^2 + 10/s - 3$

$(s^2 + 4) y(s) = \dfrac{-3s^3 + 10s^2 - 4s + 8}{s^3}$

$y(s) = \dfrac{-3s^3 + 10s^2 - 4s + 8}{s^3(s^2 + 4)} = \dfrac{A}{s^3} + \dfrac{B}{s^2} + \dfrac{C}{s} + \dfrac{Ds + E}{s^2 + 4}$

$\Rightarrow -3s^3 + 10s^2 - 4s + 8 = A(s^2 + 4) + B(s)(s^2 + 4) + C(s^2)(s^2 + 4) + (Ds + E)(s^3)$

$-3s^3 + 10s^2 - 4s + 8 = As^2 + A4 + Bs^3 + B4s + Cs^4 + C4s^2 + Ds^4 + Es^3$

$-3s^3 + 10s^2 - 4s + 8 = (C + D)s^4 + (B + E)s^3 + (A + C4)s^2 + (B4)s + (A4)$

$C + D = 0$: $B + E = 3$: $A + C4 = 10$: $B4 = -4$: $A4 = 8$

$2 + D = 0$ $-1 + E = 3$ $2 + C4 = 10$ $\boxed{B = -1}$ $\boxed{A = 2}$

$\boxed{D = -2}$ $\boxed{E = 4}$ $C4 = 8$

$\boxed{C = 2}$

$\Rightarrow y(s) = (2)/s^3 + (-1)/s^2 + (2)/s + \dfrac{(-2)s + 4}{(s^2 + 4)} \Rightarrow 2/s^3 - 1/s^2 + 2/s + \dfrac{-2s + 4}{(s^2 + 4)}$

$y(t) = \mathcal{L}^{-1}\left\{\dfrac{2}{(s^3)} - \dfrac{1}{(s^2)} + \dfrac{2}{(s)} + \dfrac{4 - 2s}{(s^2 + 4)}\right\} \Rightarrow \mathcal{L}^{-1}\left\{\dfrac{2}{s^3}\right\} - \mathcal{L}^{-1}\left\{\dfrac{1}{s^2}\right\} + \mathcal{L}^{-1}\left\{\dfrac{2}{s}\right\} + \mathcal{L}^{-1}\left\{\dfrac{4 - 2s}{s^2 + 4}\right\}$

$\Rightarrow t^2 - t + 2 + \mathcal{L}\left\{\dfrac{4}{s^2 + 4}\right\} - \mathcal{L}\left\{\dfrac{2s}{s^2 + 4}\right\} \Rightarrow \boxed{t^2 - t + 2 + 2\sin(2t) - 2\cos(2t)}$