Alex Iacob

Prof. Borrelli

CSCI 331 Section 1

June 30th, 2021

Homework 4

1) *It's February 1943, the Island of New Guinea during World War II. The Allies control the southern coast of the island, and the Japanese control the northern coast. The Japanese are bringing in reinforcements – lots of them – from China and Japan. They've already reached the city of Rabaul. Rabaul is on the eastern tip of an island called New Britain. The troops massed in Rabaul are intended for the city of Lae in New Guinea. They'll be convoyed by ship, and everybody knows it. What's not clear is what route they'll take. New Britain runs east-west, so the trip will be either along the north coast of New Britain or the southern coast. Either way, it's a three-day trip. The significant difference is the expected weather. To the south, the weather is clear; to the north, the weather is stormy. The Japanese fleet is fine with either kind of weather.*

   It's the Allies who care about the weather. They're going to send recon planes to find the fleet, then bombers to bomb it. In clear weather, both of these things can be done on the same day. In bad weather, the bombers go out a day after the enemy fleet is sighted. That cuts down on bombing days, and of course, if you don't even find the fleet, you can't bomb it at all. The recon planes can't search both routes on the same day. All of this information is known to both sides. Which route (north or south) should the Japanese commanders sail, and which route (north or south) should the Allied Forces commanders search?

	Allies search north while stormy	*Allies search south while clear*
Japanese sail north while stormy	Japanese found day 1, bombed days 2 and 3	Japanese found day 2, bombed on day 3
Japanese sail south while clear	Japanese found day 2, bombed on days 2 and 3	Japanese found day 1, bombed on days 1, 2, and 3

Both parties should go north; the Japanese sail north and the Allies search north. The Japanese are guaranteed one day of safety while the Allies can bomb them the next day.

2) *A dominant strategy is a Nash equilibrium. Is a Nash equilibrium necessarily a dominant strategy? (If your answer is "yes", prove this. If your answer is "no" provide a counter-example.)*

No, a Nash equilibrium is a not always dominant strategy. There are cases in which the resulting payoffs for each individual party could not be the max that they could get if they were playing individually. Like question 3, if the sandwiches could get a max profit, they would sell for $5 but the danishes would not get its max payout.

3) *Two vendors are selling breakfast items. The price depends on demand. The Fixed cost for making a breakfast sandwich is $2; the fixed cost for a danish is $1. Payoffs are represented as profits for Danish and Sandwich sellers respectively.*

	Sandwich: $3	Sandwich: $4	Sandwich: $5	Sandwich: $6
Danish: $3	$D = 40, S = 12$	$D = 44, S = 22$	$D = 48, S = 28$	$D = 39, S = 18$
Danish: $4	$D = 36, S = 14$	$D = 42, S = 24$	$D = 46, S = 32$	$D = 38, S = 24$
Danish: $5	$D = 16, S = 17$	$D = 24, S = 26$	$D = 32, S = 36$	$D = 40, S = 16$
Danish: $6	$D = 32, S = 18$	$D = 48, S = 25$	$D = 50, S = 29$	$D = 41, S = 32$
Danish: $7	$D = 42, S = 19$	$D = 36, S = 18$	$D = 49, S = 17$	$D = 42, S = 28$

a) *Find the Nash Equilibrium using Iterated Elimination of Dominated Strategies (IEDS). Show your work with each iteration by specifying which row/column is dominated and eliminated.*

First iteration: the $4 danish is eliminated because the $3 danish dominated

	Sandwich: $3	*Sandwich: $4*	*Sandwich: $5*	*Sandwich: $6*
Danish: $3	D = 40, S = 12	D = 44, S = 22	D = 48, S = 28	D = 39, S = 18
Danish: $5	D = 16, S = 17	D = 24, S = 26	D = 32, S = 36	D = 40, S = 16
Danish: $6	D = 32, S = 18	D = 48, S = 25	D = 50, S = 29	D = 41, S = 32
Danish: $7	D = 42, S = 19	D = 36, S = 18	D = 49, S = 17	D = 42, S = 28

Next iteration: the $5 danish is eliminated because the $6 danish dominated

	Sandwich: $3	*Sandwich: $4*	*Sandwich: $5*	*Sandwich: $6*
Danish: $3	D = 40, S = 12	D = 44, S = 22	D = 48, S = 28	D = 39, S = 18
Danish: $6	D = 32, S = 18	D = 48, S = 25	D = 50, S = 29	D = 41, S = 32
Danish: $7	D = 42, S = 19	D = 36, S = 18	D = 49, S = 17	D = 42, S = 28

Next iteration: the $3 sandwich is eliminated because the $6 sandwich dominated

	Sandwich: $4	*Sandwich: $5*	*Sandwich: $6*
Danish: $3	D = 44, S = 22	D = 48, S = 28	D = 39, S = 18
Danish: $6	D = 48, S = 25	D = 50, S = 29	D = 41, S = 32
Danish: $7	D = 36, S = 18	D = 49, S = 17	D = 42, S = 28

Next iteration: the $3 danish is eliminated because the $6 danish dominated

	Sandwich: $4	*Sandwich: $5*	*Sandwich: $6*
Danish: $6	D = 48, S = 25	D = 50, S = 29	D = 41, S = 32
Danish: $7	D = 36, S = 18	D = 49, S = 17	D = 42, S = 28

Next iteration: the $4 sandwich is eliminated because the $5 sandwich dominated

	Sandwich: $5	*Sandwich: $6*
Danish: $6	D = 50, S = 29	D = 41, S = 32
Danish: $7	D = 49, S = 17	D = 42, S = 28

Next iteration: the $5 sandwich is eliminated because the $6 sandwich dominated

	Sandwich: $6
Danish: $6	D = 41, S = 32
Danish: $7	D = 42, S = 28

Last iteration: the $6 sandwich is eliminated because the $7 sandwich dominated

	Sandwich: $6
Danish: $7	D = 42, S = 28

b) *Show any/all cells that strongly Pareto Dominate the Nash Equilibrium value.*

$5 sandwich with $4 danish and $5 sandwich and $6 danish. These two combos have a larger payoff than the Nash Equilibrium value.

4) *The game "Rock-Paper-Scissors-Lizard-Spock" is an extension to the popular "Rock-Paper-Scissors." In addition to the usual rules of Rock smashes Scissors, Scissors cuts Paper and Paper covers Rock, we also have:*

Rock crushes Lizard • Lizard poisons Spock • Spock smashes Scissors • Scissors decapitates Lizard • Lizard eats Paper • Paper disproves Spock • Spock vaporizes Rock

a) *Fill in the payoff matrix for this game using values of 0 for a tie, −1 for a loss, and +1 for a win.*

Player A is on the columns and Player B is on the rows

	Rock	Paper	Scissors	Lizard	Spock
Rock	0, 0	-1, 1	1, -1	1, -1	-1, 1
Paper	1, -1	0, 0	-1, 1	-1, 1	1, -1
Scissors	-1, 1	1, -1	0, 0	1, -1	-1, 1
Lizard	-1, 1	1, -1	-1, 1	0, 0	1, -1
Spock	1, -1	-1, 1	1, -1	-1, 1	0, 0

b) *If Player A is utilizing the following mixed strategy: Rock = 20% Paper = 15% Scissors = 18% Lizard = 22% Spock = 25%*

What are the expected Payoff values for each option for Player B?

If Player B plays:

Rock: 0 - 0.15 + 0.18 + 0.22 - 0.25 = **0**

Paper: 0.2 + 0 - 0.18 - 0.22 + 0.25 = **0.05**

Scissors: -0.2 + 0.15 + 0 + 0.22 - 0.25 = **-0.08**

Lizard: -0.2 + 0.15 - 0.18 + 0 + 0.25 = **0.02**

Spock: 0.2 - 0.15 + 0.18 - 0.22 + 0 = **0.01**

c) *Based on the expected Payoff values for Player B, what is the best mixed strategy for Player B?*

The best mixed strategy for Play B is to favor playing Paper(25%), Lizard(22%), or Spock(21%) while staying away from Scissors(12%) and Rock(20%).