331 – Intro to Intelligent Systems
Week 04
Game Theory I
Pure strategies in Adversarial search
R&N Chapter 17.5, 17.6

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Algorithmic Game Theory

- Game theory is the study of *strategic interactive decision-making among rational agents*
- There are three major components of any game:
  1. Players – the agents who play the game
  2. Strategies – what the agents do, how they will respond in *every possible* situation
  3. Payoffs – how much each player likes the result

Algorithmic Game Theory

- Includes:
  - Sequential games
  - Simultaneous games
  - Threats, promises, commitments
  - Credibility, deterrence, compellence
  - Signaling and screening
  - Incentives
  - Voting, auctions, bargaining

Algorithmic Game Theory

- Modern game theory began in 1944 with the publication of *Theory of Games and Economic Behavior* by John von Neumann and Oskar Morgenstern
- Their goal was to allow economics to be studied as a science, similar to physics
- Many Nobel Prizes in economics have been awarded to people for their work in game theory

Some Applications of Game Theory

- Consumer behavior
- Elections
- War
- Terrorism
- Dating
- Global warming
- Traffic congestion – human and network
- Computer games
- Interactions among multi-agent robotic systems
- Machine learning
- Many, many, more

Strategies

- Pure strategies
  - Do not involve any element of chance
    - In other words, you are not going to flip a coin to decide what to do
- Mixed strategies
  - Involve chance
  - The strategy must be kept a secret from your opponent
  - You must keep your opponent guessing by “mixing it up”

Mixed strategy

- Mixed strategies
  - Chooses some action *a*
  - With probability *p*
  - If there are only two actions then the probability of *b* is *1-p*
  - Some games only have solutions with a mixed strategy

Payoffs

- Once the strategies interact and play themselves out, the game is over and the players receive a payoff
- The payoff represents how much each player likes the outcome of the game
- The bigger the payoff, the more the player likes the outcome (i.e., an outcome of 4 is better than an outcome of 2)
- Negative values are also sometimes used to show utility/disutility

Rational Decision-Making

- Example: Let's say that I would like to sell you a vase for $11 that I had previously bought for $8. You believe the vase is worth $18
- If the deal does not go through then both of us have a profit of $0 (ignoring the residual value of the vase)
- If it does go through, then I have a profit of $3 and you have a profit of $7

Rational Decision-Making

- Payoffs represent what ***each player*** cares about, not what another player thinks the other player should care about
  - Therefore, payoffs for different players cannot always be directly compared
  - In the previous example, if all each player cares about is money, then the payoffs are $3 and $7 respectively
  - If one player feels that he or she is getting “ripped off” then the payoff changes to reflect the real value of the transaction

Rational Decision-Making

- Being rational means that each player makes decisions based on what that player believes will lead to the best expected payoff *for them!*
- Example – The Ultimatum Game: Take it or leave it?
- Even when considering $$, one dollar may not be equivalent to another

Finite vs. Infinite Games

- Finite games
  - Must be guaranteed to eventually end
  - Must have a finite number of choices for each player
  - Played with the goal of winning
  - Debates, sports, receiving a degree, etc.
- Infinite (non-finite) games
  - No definite beginning or ending
  - Played with the goal of continuing to play
  - Beginning to play does not require volunteering or conscious thought, continuing to play does
  - Life

Ordinal vs. Cardinal Payoffs

- Ordinal payoffs
  - You only need to know the ordering, or preferences of the outcomes, i.e., first choice, second choice, third choice, etc.
  - Consider an ice cream payoff: first choice = vanilla, second choice = chocolate, third choice = horseradish ripple
- Cardinal payoffs
  - Are on an interval scale, i.e., the difference between 10 and 20 is the same as the difference between 30 and 40
  - You must know more than just the ordering of the outcomes
  - The ice cream payoffs shown above are ordinal, not cardinal

Common Knowledge

- Assume that the rules of the game and the rationality of the players is common knowledge
  - In other words, everyone knows the rules of the game
- In addition, everyone knows that everyone knows the rules of the game
- And everyone knows that everyone knows that everyone knows the rules of the game, etc.

Sequential Games

- Sequential games represent events unfolding over time
  - Also called “dynamic games”
  - Players have full knowledge of other players’ moves
  - Chess, monopoly, open auctions, etc.
- Simultaneous games represent events occurring at the same time
  - Also called “static games”
  - Players do not know what other players are doing
  - Silent auctions, clicker game, etc.

First-Mover Advantage?

- Do first movers always have an advantage in sequential games?
  - Not necessarily
  - Going first means *committing to a course of action*
  - Not going first means *flexibility of response*
- Does order make a difference in the payoffs to the individual players?
  - In general, yes – going 2nd allows the player to weigh options against a fixed decision from the other player

Non-Cooperative Games

- Cooperative games imply that binding agreements between the players are possible

- Non-cooperative games imply that binding agreements are not possible

Dominant Strategy

- A strategy that **strongly** dominates does so if the outcome of the strategy for the player is better than any other outcome
- A strategy **weakly** dominate if it is no worse than any other
- A dominant strategy is a strategy that dominates all others
- It is irrational to play a dominat**ed** strategy
- It is irrational not to play a dominant strategy if one exists

Pareto

- An outcome is **Pareto optimal** - If there is no other outcome that **all** players would prefer

- An outcome is **Pareto dominated** by another outcome if all players would prefer the other outcome

The Roll-Back Approach

- In order for the roll-back approach to work, the game must be finite (no randomness), non-cooperative, sequential, and must have *perfect information*
  - Perfect information means that all players know the potential payoffs for each player before any moves are made, and all decisions are made public (no secrets)
  - Chess is an example of a game of perfect information, poker is a game of imperfect information

Nash Equilibrium

- If a game outcome is an equilibrium, then no player can gain from unilaterally changing his or her strategy
  - No regrets! Even if you don’t end up with exactly what you wanted.
  - John Forbes Nash invented the idea of the “Nash Equilibrium”
  - If every player is playing the Nash equilibrium, then you might as well also because you will not gain anything by changing your strategy

Nash Equilibrium

- Nash equilibrium is essentially a local optimum
- Nash (the mathematician) proved that a game has at least one **mixed strategy** equilibrium
- But not necessarily a pure strategy equilibrium

Simultaneous Games

- In simultaneous games all of the players make their decisions at essentially the same time
- Players do not know what other players are doing at the time they make their decisions
- No one "goes first"
- Use a payoff matrix instead of a game tree
  - 2 x 2 matrix
  - Each element in the matrix contains the cardinal value of each player's preferences (the payoff)

Simultaneous Games

- The Coordination Game
- The Battle of the Sexes
- The Game of Chicken
- The Prisoner’s Dilemma

Simultaneous Games

- Label the rows of the payoff matrix with the choices of player 1, and the columns with the choices of player 2 (book uses slightly different convention)

	Player 2 : Choice 1	Player 2 : Choice 2
Player 1 : Choice 1	P1=payoff, P2=payoff	P1=payoff, P2=payoff
Player 1 : Choice 2	P1=payoff, P2=payoff	P1=payoff, P2=payoff

The Coordination Game

- Consider the example of two firms choosing whether to use standard X or standard Y for their joint software project
- They both prefer standard X over standard Y, but the least favorite option is to disagree with the other firm (one chooses X and the other chooses Y, or vice-versa)

	Firm B : standard X	Firm B : standard Y
Firm A : standard X	A=2, B=2	A=0,B=0
Firm A : standard Y	A=0, B=0	A=1,B=1

The Coordination Game

- Assume common knowledge – both players know all of the game matrix payoffs
- Assume both players are rational – their stated preferences are their true preferences
- Then choosing the “standard X/standard X” option is best

The Battle of the Sexes

- What if we change the situation slightly – Player A prefers Opera over Football, and Player B prefers Football over Opera, but both still want to work together (coordinate)

	Player B : Opera	Player B : Football
Player A : Opera	A=2, B=1	A=0,B=0
Player A : Football	A=0, B=0	A=1,B=2

- No good solution – unless you can find a Schelling (focal) point (i.e. a point that players will tend to choose in absence of communication)

The Game of Chicken (anti-coordination)

- Suppose Player A and B are fierce competitors and the success of their products rely on using different standards; however, the standard X action is clearly superior to any other on the market (there are several others)

	Player B : standard X	Player B : other standard
Player A : standard X	A=0, B=0	A=3,B=1
Player A : other standard	A=1, B=3	A=2,B=2

The Game of Chicken

- What is the best thing to do?
- It may seem most fair for both firms to choose another standard, but if Player A (or B) knew that the other Player was going with the other standard, then Player A (B) would prefer to stay with standard X to get the payoff
- Therefore (2,2) is <u>NOT</u> a Nash equilibrium
- Solution again is to create a Schelling point

The Prisoner’s Dilemma

- Two criminals committed a crime together, and are being interrogated in separate cells. If neither one confesses, they’ll each get a year in prison. If one confesses, that one goes free but the other one gets five years. If they both confess, they both get three years.
- Assign values to these outcomes (free = 5, 1 year = 3, 3 years = 1, 5 years = 0)

The Prisoner's Dilemma

- What if both players refuse to defect if the other player gets to use the better choice (for spite)?

	Prisoner 2 : confess	Player 2 : keep silent
Prisoner 1 : confess	P1=1, P2=1	P1=5,P2=0
Prisoner 1: keep silent	P1=0, P2=5	P1=3,P2=3

  – No matter what Prisoner 2 does, it is better for Prisoner 1 to choose action "confess"
  – Therefore, both firms will choose "confess" (payoff = 1,1)
  – Notice that this payoff is *lower for both players (!)* than if both chose to keep silent (payoff = 3,3)

The Prisoner’s Dilemma

- Both will confess and both will get three years, whereas they would have been better off if they had both kept silent!
- The *dilemma* in the prisoner's dilemma is that the equilibrium outcome is worse for both players than the outcome they would get if they both refused
- (1, 1) outcome for (confess, confess) is Pareto dominated by (3, 3) outcome of (silent, silent)

The Prisoner’s Dilemma

Consider 1964 when the federal government banned cigarette advertising on television. Before the ban:

	Company B: advertise	Company B: don't advertise
Company A : advertise	A=1150, B=1150	A=2020, B=630
Company A: don't advertise	A=630, B=2020	A=1500, B=1500

The Prisoner’s Dilemma

- The Soviet Union exploded its atomic bomb in 1949
  - The Prisoner’s Dilemma was discovered in 1950
- Preferences (in order)?
  1. We nuke them
  2. No one nukes anyone
  3. Everyone nukes everyone
  4. They nuke us
- If this is the order of preferences, then one equilibrium is nuclear war – both sides launch missiles
- In the years following 1950 many people (including von Neumann) thought that nuclear war with the U.S.S.R. was inevitable

No Dominant strategy

- Here is a game where there is no dominant strategy
- However, there are *two* Nash equilibria: (bluray, bluray) and (dvd,dvd).
- These are both Nash equilibria because if either player unilaterally moves to a different strategy, that player will be worse off

	Best: bluray	Best: dvd
Acme: bluray	A=9, B=9	A=-4, B=-1
Acme: dvd	A=-3, B=-1	A=5, B=5

Tragedy of the Commons

- Let's consider another game where countries set their policy on controlling air pollution.
- Each country has a choice:
  - they can *reduce* pollution at a cost of -10 for implementing the changes
  - They can *continue* to pollute which gives them a net utility of -5 (in added health costs etc.) AND contributes -1 to every other country because air shared

Tragedy of the Commons

- Clearly, the dominant strategy for each country is “continue to pollute”
- This is bad enough if there are two countries (similar to the prisoner's dilemma) however, let's assume we have 100 countries
- If everyone follows this policy, then each country gets a total utility value of -104
- Whereas if every country reduced pollution, they would each have a utility of -10
- This is the tragedy of the commons: if nobody has to pay for a common resource, it tends to be exploited in a way that leads to a lower total utility for all

The Tragedy of the Commons

- From the individual game-theoretic perspective, shirking is the right thing to do because it is the dominant strategy
- From a social good perspective, it is a tragedy
- Self interest, in this case does not maximize the common good (contrary to Adam Smith’s philosophy) because the cost that the shirkers create have to be borne by others