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4) $y'' - y' - 2y = 0$

$r^2(e^{rt} - r(e^{rt} - 2(e^{rt} = 0$

$r^2 - r - 2 = 0$

$(r-2)(r+1) = 0$

$r_1 = 2, \; r_2 = -1$

$y = C_1 e^{r_1 t} + C_2 e^{r_2 t}$

$\boxed{y = C_1 e^{2t} + C_2 e^{-t}}$

5) $y'' + 8y' + 16y = 0$

$r^2(e^{rt} + 8r(e^{rt} + 16(e^{rt} = 0$

$r^2 + 8 + 16 = 0$

$(r+4)(r+4) = 0$

$r_1 = r_2 = -4$

$y = C_1 e^{r_1 t} + t C_2 e^{r_2 t}$

$\boxed{y = C_1 e^{-4t} + t C_2 e^{-4t}}$

15) $y'' - 4y' + 3y = 0$, $\quad y(0) = 1, \quad y'(0) = 1/3$

$r^2(e^{rt} - 4r(e^{rt} + 3(e^{rt} = 0$

$r^2 - 4r + 3 = 0$

$(r-3)(r-1) = 0$

$r_1 = 3, \; r_2 = 1$

$y = C_1 e^{r_1 t} + C_2 e^{r_2 t}$

$y = C_1 e^{3t} + C_2 e^{t}$

$1 = C_1 e^{3 \cdot 0} + C_2 e^{0}$

$1 = C_1 \cdot 1 + C_2 \cdot 1$

$1 = C_1 + C_2$

$y' = 3 C_1 e^{3t} + C_2 e^{t}$

$1/3 = 3 C_1 e^{3 \cdot 0} + C_2 e^{0}$

$1/3 = 3 C_1 + C_2$

$1 = C_1 + C_2$

$1/3 = 3 C_1 + C_2$

$2/3 = -2 C_1$

$-1/3 = C_1$

$1 = -1/3 + C_2$

$4/3 = C_2$

$\boxed{y = \left(-\tfrac{1}{3}\right) e^{3t} + \left(\tfrac{4}{3}\right) e^{t}}$

43) $y''' - y' = 0$, $y(0) = 2$, $y'(0) = 3$, $y''(0) = -1$

$r^3 \ell e^{rt} - r \ell e^{rt} = 0$

$r^3 - r = 0$

$r(r^2 - 1) = 0$

$r_1 = 0, r_2 = -1, r_3 = 1$

$y = c_1 e^{0t} + c_2 e^{-1t} + c_3 e^{1t}$

$y = c_1 + c_2 e^{-t} + c_3 e^{t}$

$y' = -c_2 e^{-t} + c_3 e^{t}$

$y'' = c_2 e^{t} + c_3 e^{t}$

(y) $2 = c_1 + c_2 + c_3$

(y') $3 = -c_2 + c_3$

(y'') $-1 = c_2 + c_3$

$3 = -c_2 + c_3$

$-1 = c_2 + c_3$

$4 = -2c_2$

$-2 = c_2$

$3 = -(-2) + c_3$

$3 = 2 + c_3$

$1 = c_3$

$c_1 = 3$, $c_2 = -2$, $c_3 = 1$

$2 = c_1 + (-2) + 1$

$2 = c_1 - 1$

$3 = c_1$

$\boxed{y = 3 + -2e^{-t} + e^{t}}$

10) $v'' + 11v = 0$

$r^2 \ell e^{rt} + 11 \ell e^{rt} = 0$

$r^2 + 11 = 0$

$x = \dfrac{-(0) \pm \sqrt{(0)^2 - 4(1)(11)}}{2(1)} = x = \dfrac{\pm\sqrt{-44}}{2} = \dfrac{\pm 2i\sqrt{11}}{2} = \pm i\sqrt{11}$

$r_1 = i\sqrt{11}$, $r_2 = -i\sqrt{11}$

$y = c_1 e^{i\sqrt{11}t} + c_2 e^{-i\sqrt{11}t}$

$\boxed{y = c_1 \cos(\sqrt{11}t) + c_2 \sin(\sqrt{11}t)}$

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29) $y'' + 4y' + 7y = 0$, $y(0) = 1$, $y'(0) = -2$

$r^2(e^{rt} + 4r(e^{rt} + 7(e^{rt} = 0$

$r^2 + 4y + 7 = 0$

$x = \dfrac{-(4) \pm \sqrt{(4)^2 - 4(7)(7)}}{2(7)} = \dfrac{-4 \pm \sqrt{16 - 28}}{2} = \dfrac{-4 \pm \sqrt{-12}}{2} = \dfrac{-4 \pm 2i\sqrt{3}}{2} = -2 \pm i\sqrt{3}$

$r_1 = -2 + i\sqrt{3} \qquad r_2 = -2 - i\sqrt{3}$

$y = C_1 e^{(-2+i\sqrt{3})} + C_2 e^{(-2-i\sqrt{3})}$

$y = e^{-2t}\left[C_1 \cos(\sqrt{3}t) + C_2 \sin(\sqrt{3}t)\right]$

$y' = \left[-2e^{-2t}\left[C_1\cos(\sqrt{3}t) + C_2\sin(\sqrt{3}t)\right]\right] + \left[e^{-2t}\left[-\sqrt{3}C_1\sin(\sqrt{3}t) + \sqrt{3}\cos(\sqrt{3}t)\right]\right]$

(y) $1 = e^{-2\cdot 0}\left[C_1\cos(\sqrt{3}\cdot 0) + C_2\sin(\sqrt{3}\cdot 0)\right]$

$1 = 1[C_1 + 0]$

$1 = C_1$

(y') $-2 = \left[-2e^{-2\cdot 0}\left[1\cdot\cos(\sqrt{3}\cdot 0) + C_2\sin(\sqrt{3}\cdot 0)\right]\right] + \left[e^{-2\cdot 0}\left[-\sqrt{3}\cdot 1\cdot\sin(\sqrt{3}\cdot 0) + \sqrt{3}\cos(\sqrt{3}\cdot 0)\right]\right]$

$-2 = \left[-2\left[1 + 0\right]\right] + \left[\sqrt{3}\cdot C_2\right]$

$-2 = -2 + \sqrt{3}C_2$

$0 = \sqrt{3}C_2$

$0 = C_2$

$y = e^{-2t}\left[(1\cdot\cos(\sqrt{3}t)) + (0\cdot\sin(\sqrt{3}t))\right]$

$\boxed{y = e^{-2t}\cos(\sqrt{3}t)}$

36) $9y'' + 12y' + 4y = 0 \qquad y(0) = -3, \quad y'(0) = 3$

$9r^2(e^{rt} + 12r(e^{rt} + 4(e^{rt} = 0$

$9r^2 + 12r + 4 = 0$

$x = \dfrac{-12 \pm \sqrt{(12)^2 - 4(9)(4)}}{2(9)} = \dfrac{-12 \pm \sqrt{144 - 144}}{18} = \dfrac{-12 \pm 0}{18} = \quad x = -2/3$

$y = c_1 e^{-2/3 t} + c_2 t e^{-2/3 t}$

$y' = (-\tfrac{2}{3}(c_1 + c_2)\left[e^{-2/3 t} - \tfrac{2}{3} c_2 t e^{-2/3 t} \right]$

$(y) \quad -3 = c_1 e^{-2/3 \cdot 0} + c_2 \cdot 0 \cdot e^{-2/3 \cdot 0}$

$-3 = c_1$

$(y') \quad 3 = \left(-\tfrac{2}{3}(-3) + c_2\right)\left[e^{-2/3 \cdot 0} - \tfrac{2}{3} c_2 \cdot 0 \cdot e^{-2/3 \cdot 0} \right]$

$3 = (2 + c_2)[1 - 0]$

$3 = 2 + c_2 \qquad\qquad y = -3 e^{-2/3 t} + t e^{-2/3 t}$

$1 = c_2 \qquad\qquad \boxed{y = e^{(-2/3)t}(-3 + t)}$

NB1) $y'''' - 7y'' - 18y = 0$

$r^4 - 7r^2 - 18 = 0$

$r_1 = \sqrt{2}, \; r_2 = -\sqrt{2}, \; r_3 = 3i, \; r_4 = -3i$

$y = c_1 e^{\sqrt{2}t} + c_2 e^{-\sqrt{2}t} + c_3 e^{3it} + c_4 e^{-3it}$

$\boxed{y = c_1 e^{\sqrt{2}t} + c_2 e^{-\sqrt{2}t} + e^t[c_3 \cos(3t) + c_4 \sin(3t)]}$