MATH 351–004 – Assignment #1

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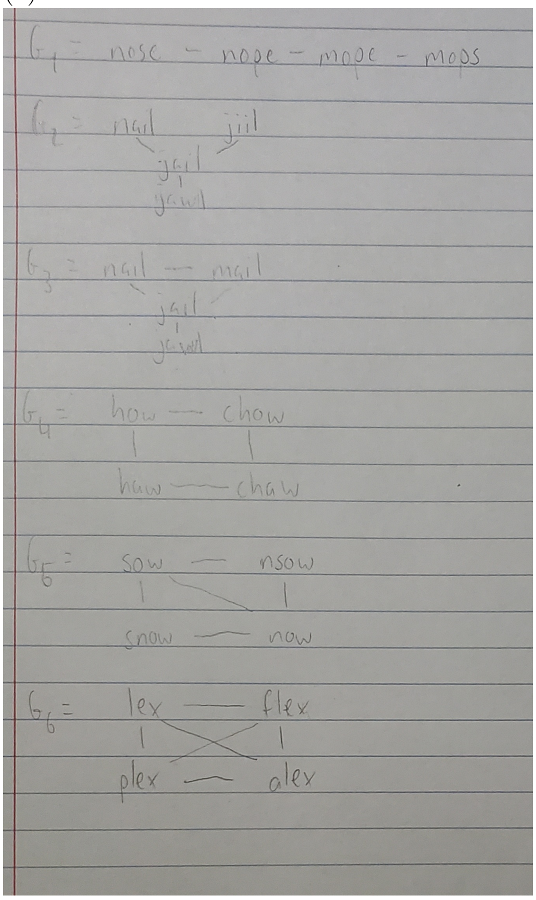
September 2, 2021

Part 2

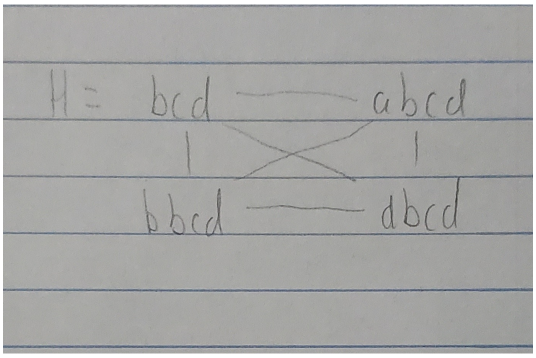
Problem 1.8 **Let S be a finite set of 3-letter and/or 4-letter words. In this case, the word graph $G(S)$ of S is that graph whose vertex set is S and such that two vertices (words) w_1 and w_2 are adjacent if either (1) or (2) below occurs:**
(1) one of the words can be obtained from the other by replacing one letter by another letter,
(2) w_1 is a 3-letter word and w_2 is a 4-letter word and w_2 can be obtained from w_1 by the insertion of a single letter (anywhere, including the beginning or the end) into w_1.

(a) Find six sets S_1, S_2, . . . , S_6 of 3-letter and/or 4-letter words so that for each integer i $(1 <= i <= 6)$ the graph G_i of Figure 1.13 is the word graph of S_i.
(b) For another graph H (of your choice), determine whether H is a word graph of some set.

(a)



(b)



Problem 1.16 Let $P = (u = v_0, v_1, ..., v_k = v)$, $k >= 1$, be a $u - v$ geodesic in a connected graph G. Prove that $d(u, v_i) = i$ for each integer i with $1 <= i <= k$.

In order for a graph to be connected, there is a path from any point to any other point on the graph. Because of this connectivity, the shortest path between two points must be 1. The largest path between any two points is the amount of nodes, k. Anything in between these two values is the path length from any two points.

Problem 1.19 Theorem 1.10 states that a graph G of order 3 or more is connected if and only if G contains two distinct vertices u and v such that $G - u$ and $G - v$ are connected. Based on this, one might suspect that the following statement is true. Every connected graph G of order 4 or more contains three distinct vertices u, v and w such that G - u, G - v and G - w are connected. Is it?

This is not true. Adding a third distinct vertex does not further prove that the given graph is connected, rather the opposite.

Problem 1.20 (a)Let u and v be distinct vertices in a connected graph G. There may be several connected subgraphs of G containing u and v. What is the minimum size of a connected subgraph of G containing u and v? Explain your answer.
(b)Does the question in (a) suggest another question to you?

(a) Both subgraphs can be single vertices, so the minimum size of a connected subgraph is 1.
(b) The suggested question is "What is the minimum possible size for a connected subgraph?"