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MATH231-54
HW#3

7) $\frac{dy}{dx} - y - e^{3x} = 0$

$y' - y = e^{3x}$

$y_h' - y_h = 0 \Rightarrow \frac{dy_h}{dx_h} = y_h \Rightarrow \int \frac{dy_h}{y_h} = \int dx_h \Rightarrow \ln|y_h| = x + C$

$\Rightarrow y_h = e^{x+C}$

$y_h = \tilde{C}e^x$

$y_p = v(x)e^x$

$y_p' = v'(x)e^x + v(x)e^x$

$y_p' - y_p = e^{3x}$

$v'(x)e^x + \cancel{v(x)e^x} - \cancel{v(x)e^x} = e^{3x}$

$v'(x)e^x = e^{3x}$

$v'(x) = e^{2x}$

$v(x) = \frac{1}{2}e^{2x}$

$y = \frac{e^{2x}}{2} \cdot e^x + \tilde{C}e^x$

$\boxed{y = \frac{e^{3x}}{2} + \tilde{C}e^x}$

12 $\frac{dy}{dx} = x^2 e^{-4x} - 4y$

$$y' + 4y = x^2 e^{-4x}$$

$$y_h' + 4y_h = 0$$

$$\frac{dy_h}{dx_h} = -4y_h \Rightarrow \int \frac{dy_h}{y_h} = \int -4\,dx_h \Rightarrow \ln|y_h| = -4x + C$$

$$\Rightarrow y_h = e^{-4x+C}$$

$$\Rightarrow y_h = e^{-4x}\tilde{C}$$

$$y_p = v(x)e^{-4x}$$

$$y_p' = v'(x)e^{-4x} + (-4)e^{-4x}v(x)$$

$$v'(x)e^{-4x} \cancel{- 4e^{-4x}v(x) + 4v(x)e^{-4x}} = x^2 e^{-4x}$$

$$v'(x)e^{-4x} = x^2 e^{-4x}$$

$$v'(x) = x^2$$

goes into y_p $\quad v(x) = x^3/3$

$$y_p = \frac{x^3}{3} \cdot e^{-4x}$$

y is always equal to $y_p + y_h$

$$y_h = e^{-4x}\tilde{C}$$

$$y = \tilde{C}e^{-4x} + \frac{x^3 e^{-4x}}{3}$$

22) $y' \sin(x) + y\cos(x) = x\sin(x) \qquad y\left(\frac{\pi}{2}\right) = 2$

$$y_h' \sin(x) + y_h \cos(x) = 0$$

$$\frac{dy_h}{dx_h} \sin(x) + y_h \cos(x) = 0$$

$$\frac{dy_h}{dx_h} \sin(x) = -y_h \cos(x)$$

$$\int \frac{dy_h}{-y_h} = \int \cot(x)\, dx_h \Rightarrow -\ln|y_h| = \ln|\sin(x)| + C$$

$$\ln|y_h| = -\ln|\sin(x)| + C$$

$$y_h = e^{-\ln|\sin(x)| + C}$$

$$y_h = \tilde{C}(\sin(x)^{-1})$$

$$y_h = \tilde{C}\csc(x)$$

$$y_p = v(x)\csc(x)$$

$$y_p' = v'(x)\csc(x) + v(x)(-\cot(x))(\csc(x))$$

$$y_p' = v'(x)\frac{1}{\sin(x)} - v(x)\left(\frac{\cos(x)}{\sin(x)}\right)\cdot\left(\frac{1}{\sin(x)}\right) = \csc(x)\left[v'(x) - v(x)\cot(x)\right]$$

$$\cancel{\sin(x)} \cdot \left(1/\cancel{\sin(x)}\right) \cdot \left[v'(x) - v(x)\cot(x)\right] + v(x)\left(1/\sin(x)\right)\cdot\cos(x) = x\sin(x)$$

$$v'(x) \cancel{- v(x)\cot(x)} + \cancel{v(x)\cot(x)} = x\sin(x)$$

$$v'(x) = x\sin(x)$$

$$v(x) = \sin(x) - x\cos(x)$$

$$y_p = \left[\sin(x) - x\cos(x)\right] \cdot 1/\sin(x) = 1 - x\cot(x) \quad , y\left(\frac{\pi}{2}\right) = 2$$

$$y = \frac{\tilde{C}}{\sin(x)} + 1 - \frac{x\cos(x)}{\sin(x)} \qquad \boxed{y = \frac{1 - x\cos(x)}{\sin(x)} + 1}$$

$$2 = \frac{\tilde{C} - \left(\frac{\pi}{2}\right)\cos\left(\frac{\pi}{2}\right)}{\sin\left(\frac{\pi}{2}\right)} + 1$$

$$1 = \frac{\tilde{C} - \left(\frac{\pi}{2}\right)\left(\cos\left(\frac{\pi}{2}\right)\right)}{\sin\left(\frac{\pi}{2}\right)} \Rightarrow 1 = \frac{\tilde{C}}{1} \Rightarrow \tilde{C} = 1$$

34) $y' - \frac{2y}{x} = x^2\cos(x)$, $y(\pi) = 2$

$y' - \left(\frac{2}{x}\right)y = x^2\cos(x)$

$y_h' - \left(\frac{2}{x}\right)y_h = 0$

$\frac{dy_h'}{dx_h} = \frac{2}{x}y_h \Rightarrow \int \frac{dy_h}{y_h} = \int \frac{2}{x}dx \Rightarrow \ln|y_h| = 2\ln|x| + C$

$y_h = e^{\ln|x^2| + C}$

$y_h = \tilde{C}e^{\ln|x^2|}$

$y_h = \tilde{C}x^2$

$y_p = x^2 v(x)$

$y_p' = 2x\,v(x) + v'(x)x^2$

$y_p' - \left(\frac{2}{x}\right)y_p = x^2\cos(x)$

$2x\,v(x) + v'(x)x^2 - \left(\frac{2}{x}\right)x^2 v(x) = x^2\cos(x)$

$v'(x)x^2 + \cancel{2x\,v(x)} - \cancel{2x\,v(x)} = x^2\cos(x)$

$v'(x)x^2 = x^2\cos(x)$

$v'(x) = \cos(x)$

$v(x) = \sin(x)$

$y_p = x^2 v(x) \Rightarrow x^2\sin(x)$ $\quad y(\pi) = 2$

$y = x^2\sin(x) + \tilde{C}x^2$

$2 = \pi^2\sin(\pi) + \tilde{C}\pi^2$

$2 = \tilde{C}\pi^2$

$2/\pi^2 = \tilde{C} \Rightarrow \boxed{y = x^2\sin(x) + \frac{2x^2}{\pi^2}}$