(1)

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MATH 231-54
Exam 1

1) a) $\frac{\partial^2 f}{\partial t^2} + \frac{\partial f}{\partial x} + e^t f = 0$

Classification:
I : indep : t, x
II : dep : f
III : order : 2nd
IV : Partial Diff. eqn
V : linear
VI : Constant coefficient
VII : Homogeneous

b) $y^3 \frac{d^2 y}{dx^2} = -1$

Classification:
I : indep : x
II : dep : y
III : order : 2nd
IV : Ordinary Diff eqn
V : nonlinear

2) $y' = dy/dx$ $\qquad y^3 y'' = -1$

a) $y = \sqrt{x^2-2x}$ $\qquad y' = \frac{(2x-2)}{2\sqrt{x^2-2x}} \Rightarrow y'' = \frac{-1}{\sqrt[3]{(x^2-2x)^2}}$

(Correct)

$y^3 = (\sqrt{x^2-2x})^3 = \sqrt[3]{(x^2-2x)^2}$

$y^3 y'' = \cancel{\sqrt[3]{(x^2-2x)^2}} \cdot \frac{-1}{\cancel{\sqrt[3]{(x^2-2x)^2}}} \Rightarrow -1 = -1$ ✓

b) $y = \sqrt{x^2+1}$ $\qquad y' = \frac{x}{\sqrt{x^2+1}} \Rightarrow y'' = \frac{1}{\sqrt[3]{(x^2+1)^2}}$

(Doesn't work)

$y^3 = (\sqrt{x^2+1})^3 = (x^2+1)^{3/2}$

$y^3 y'' = \cancel{(x^2+1)^{3/2}} \cdot \frac{1}{\cancel{(x^2+1)^{3/2}}} = 1 \neq -1$ ✗

②

3) $y' = at^3 - by^2 \Rightarrow$ Graph (B)
$y' = e^{(-ay^2)} \Rightarrow$ Graph (A)
$y' = at^2 + bt + c \Rightarrow$ Graph (C)

4) $\frac{dy}{dt} - e^{t-y} = 0 \Rightarrow \frac{dy}{dt} - \frac{e^t}{e^y} = 0 \Rightarrow \frac{dy}{dt} = \frac{e^t}{e^y} = e^y dy = e^t dt$

$\Rightarrow \int e^y dy = \int e^t dt \Rightarrow e^y = e^t + C \Rightarrow \ln(e^y) = \ln(e^t) + \ln(C)$

$\Rightarrow y = t + \ln(C)$, $y(0) = 0 \Rightarrow 0 = 0 + \ln(C)$
$0 = \ln(C)$
$1 = C$

$y = t + \ln(1) \Rightarrow y = t + 0 \Rightarrow \boxed{y = t}$

5) $\frac{dy}{dt} + \cos(t)y = f(t) \Rightarrow \frac{dy}{dt} = f(t) - \cos(t)y$

let $f(t) = k\cos(t)$, k can be any number

$\frac{dy}{dt} = k\cos(t) - \cos(t)y \Rightarrow \frac{dy}{dt} = \cos(t)(k - y)$

$\Rightarrow \int \frac{dy}{(k-y)} = \int \cos(t)\,dt \Rightarrow -\ln|k-y| = \sin(t) + C$
$\ln|k-y| = -\sin(t) - C$
$k - y = e^{(-\sin(t) - c)}$
$y = k - e^{(-\sin(t) - c)}$
$y = k + e^{-\sin(t)} \cdot e^{-\tilde{c}}$

$\boxed{y = e^{-\tilde{c}} \cdot e^{-\sin(t)} + k}$ $\leftarrow$

6) $\frac{dy}{dt} + p(t)\sqrt{y} = 0 \Rightarrow \frac{dy}{dt} = -p(t)\sqrt{y} \Rightarrow \int 1/\sqrt{y}\,dy = \int -p(t)\,dt$

$\Rightarrow 2\sqrt{y} = -\int p(t)dt + C \Rightarrow \sqrt{y} = \frac{-\int p(t)dt}{2} \Rightarrow \boxed{y = \frac{\left[\int p(t)dt\right]^2}{4}}$