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CSCI 331 Section 1

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Homework 5

1) *Given the Wumpus world example from the notes. Suppose the agent has progressed to the point shown in Figure 7.4(a) on Page 239, having perceived nothing at [1,1], a breeze in [2,1], and a stench in [1,2], and is now concerned with the contents of [1,3], [2,2], [3,1]. Each of these can contain a pit, and at most one can contain a wumpus. Following the example of Figure 7.5, construct the set of possible worlds (Hint: there are 32 of them). Mark the worlds in which KB is true and those in which each of the following sentences is true:*

   $\alpha 2$ = "There is not pit in [2,2]"

   $\alpha 3$ = "There is a wumpus in [1,3]"

   Hence show that KB |= $\alpha 2$ and KB |= $\alpha 3$.

2) *Use a truth table to show that* $\{p \rightarrow q, (m \rightarrow p \lor q), m\} \models q$

p	q	m	$p \rightarrow q$	$m \rightarrow p \lor q$
T	T	T	T	T
T	T	F	T	T
T	F	T	F	T
T	F	F	F	T
F	T	T	T	T
F	T	F	T	T
F	F	T	T	F
F	F	F	T	T

3) *Use a direct proof (not proof by contradiction) to show the following.*

$p \rightarrow q$

$q \rightarrow r$

$\vdash p \rightarrow r$

For each step of the proof, indicate the premise and the logic rule used. Use only the rules from the notes.

p -> q	Given
(p -> q) $\land$ (q -> r)	And-Introduction
p -> r	Hypothetical Syllogism

4) *Which of the following are correct? If they are incorrect, show the truth assignments that show it. (Hint: Look at page 249 in R&N.)*

   a) *False |= True*

      Correct because F -> T is a tautology.

   b) *True |= False*

      Incorrect because T -> F is not a tautology.

   c) *(A ∧ B) |= (A ⇔ B)*

A	B	$A \wedge B$	$A \Leftrightarrow B$	$A \wedge B$ -> $A \Leftrightarrow B$
T	T	T	T	T
T	F	F	F	T
F	T	F	F	T
F	F	F	T	T

Correct because $A \wedge B$ -> $A \Leftrightarrow B$ is a tautology.

   d) *(A ⇔ B) |= A ∨ B*

A	B	$A \Leftrightarrow B$	$A \wedge B$	$A \Leftrightarrow B$ -> $A \wedge B$
T	T	T	T	T
T	F	F	F	T
F	T	F	F	T
F	F	T	F	F

Incorrect because $A \Leftrightarrow B$ -> $A \wedge B$ is not a tautology.

e) $(A \wedge B) \rightarrow C \models (A \rightarrow C) \vee (B \rightarrow C)$

A	*B*	*A ∧ B*	*C*	*(A ∧ B) -> C*	*A -> C*	*B -> C*	*(A -> C) ∨ (B -> C)*
T	T	T	T	T	T	T	T
T	T	T	F	F	F	F	F
T	F	F	T	T	T	T	T
T	F	F	F	T	F	T	T
F	T	F	T	T	T	T	T
F	T	F	F	T	T	F	T
F	F	F	T	T	T	T	T
F	F	F	F	T	T	T	T

((A ∧ B) -> C) -> ((A -> C) ∨ (B -> C))
T
T
T
T
T
T
T
T

Correct because ((A ∧ B) -> C) -> ((A -> C) ∨ (B -> C)) is a tautology.

5) *Given the following, prove the deduction by (a) a direct proof and (b) a Reductio Ad Absurdum (proof by contradiction). For each step of the proof, indicate the premise and the logic rule used.*

$H \rightarrow I \wedge J \rightarrow K$

$(I \vee K) \rightarrow L$

$\neg L$

|- $\neg(H \vee J)$

I am more accustomed to using "~" as the negation operator from MATH-190

a)

(H -> I) ∧ (J -> K)	Given
(~I -> ~H)	Contraposition on H-> I
(~K -> ~J)	Contraposition on J -> K
~I -> ~H ∧ ~K -> ~J	Putting back into original
(I ∨ K) -> L	Given
~L -> ~(I ∨ K)	Contraposition on (I ∨ K) -> L
~(I ∨ K)	Modus Ponens on ~L -> ~(I ∨ K)
~I ∧ ~K	De Morgan's Law on ~(I ∨ K)
~K	And Elimination on ~I ∧ ~K
~I	And Elimination on ~I ∧ ~K
~H	Modus Ponens on ~I -> ~H, ~I
~J	Modus Ponens on ~K -> ~J, ~K
(~H ∧ ~J)	Putting back into original
~(H ∨ J)	De Morgan's Law

b)

(I ∨ K) -> L	Given
~L -> ~(I ∨ K)	Contraposition on (I ∨ K) -> L
~(I ∨ K)	Modus Ponens on ~L -> ~(I ∨ K)
~I ∧ ~K	De Morgan's Law on ~(I ∨ K)
~K	And Elimination on ~I ∧ ~K
K -> ⊥	
(H -> I) ∧ (J -> K)	Given
(~I -> ~H)	Contraposition on H-> I
~H	Modus Ponens on (~I -> ~H)
J -> K	And Elimination on (H -> I) ∧ (J -> K)
J	Disjunctive Syllogism
K	Modus Ponens on J -> K, J
⊥	Modus Ponens on K -> ⊥, K
~(H ∨ J)	

6) *Convert the following to CNF notation: Hint: implication has a higher precedence than AND or OR.*

   a) $C \wedge F \rightarrow \neg B$

      ~(C ∧ F) ∨ ~B

      ~C ∨ ~F ∨ ~B

   b) $\neg B \rightarrow (C \wedge D \wedge E)$

      ~(~B) ∨ (C ∧ D ∧ E)

      B ∨ (C ∧ D ∧ E)

      (B ∨ C) ∧ (B ∨ D) ∧ (B ∨ E)

   c) $(A \vee B) \Leftrightarrow (C \wedge D)$

      (A ∨ B) -> (C ∧ D) ∧ (C ∧ D) -> (A ∨ B)

      (~(A ∨ B) ∧ (C ∧ D)) ∧ (~(C ∧ D) ∧ (A ∨ B))

      (~A ∧ ~B) ∧ (C ∧ D)) ∧ (~C ∨ ~D) ∧ (A ∨ B))