International Trade: A Review

Gemini 3.0 Pro *

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Contents

Part I — Classical Trade Theory

1 Traditional Ricardian Framework

1.1 Core Concepts

The classical theory, originating from Adam Smith (Absolute Advantage) and refined by David Ricardo and Torrens, posits that labor is the only factor of production and technology differs between countries.

- **Unit Cost of Production (a_{Li}):** The amount of labor required to produce one unit of a good.
- **Comparative Advantage:** A country has a comparative advantage in a good if the opportunity cost of producing that good is lower than in the other country. The key insight is that gains from trade depend on comparative, not absolute, advantage. Even if a country is less productive in all goods, it can still benefit from specializing and trading.
- **Condition for Trade:** Trade occurs if there is a difference in comparative costs. This is a *necessary* condition. The *sufficient* condition is that the international Terms of Trade (ToT) lie between the two countries' autarky price ratios.

1.2 Numerical Example (England vs. Portugal)

Consider the unit labor costs (Table 2.2):

Commodity	**England** (a_{1j})	**Portugal** (a_{2j})
Cloth (x)	4	6
Wine (y)	8	10

Analysis:

- **Absolute Advantage:** Portugal has higher labor costs in both. England has absolute advantage in both.
- **Comparative Advantage:**
  - England's cost of Cloth in terms of Wine: $4/8 = 0.5$.
  - Portugal's cost of Cloth in terms of Wine: $6/10 = 0.6$.
  - Since $0.5 < 0.6$, England exports Cloth; Portugal exports Wine.
- **Terms of Trade (ToT):** Let $p = p_{cloth}/p_{wine}$. For mutually beneficial trade:

$$\left(\frac{a_{Lx}}{a_{Ly}}\right)_{Eng} < \frac{p_x}{p_y} < \left(\frac{a_{Lx}}{a_{Ly}}\right)_{Por} \implies 0.5 < \frac{p_x}{p_y} < 0.6$$

1.3 Distribution of Gains

The closer the Terms of Trade line is to a country's Transformation Curve (PPF), the smaller that country's share of the gains. If the ToT equals a country's autarky price ratio, that country gains zero, and the other country captures all gains.

2 Modern Interpretation: Optimization Approach

This approach reframes the Ricardian model as a maximization problem of Real Income (Y_R) subject to resource constraints.

2.1 Maximization of National Real Income

Each country i maximizes real income given exogenously determined world prices p_x, p_y.

Objective Function & Constraint:

$$\max Y_{iR} = \left(\frac{p_x}{p_y}\right) x_i + y_i$$

$$\text{s.t. } a_i x_i + b_i y_i \leq \overline{L_i}, \quad x_i \geq 0, y_i \geq 0$$

where:

- Y_{iR} is real income expressed in units of good y (Wine).
- a_i, b_i are unit labor requirements for x and y.
- $\overline{L_i}$ is the total labor endowment.

The Transformation Curve (PPF): Rearranging the constraint gives the linear PPF equation:

$$y_i = -\frac{a_i}{b_i} x_i + \frac{\overline{L_i}}{b_i}$$

However, the optimization objective line has slope $-(p_x/p_y)$.

- If $(p_x/p_y) > (a_i/b_i)$, the country specializes strictly in x.
- If $(p_x/p_y) < (a_i/b_i)$, the country specializes strictly in y.

2.2 Maximization of World Real Income

We can view the world economy as maximizing global real income Y_{RM}.

World Objective:

$$\max Y_{RM} = \left(\frac{p_x}{p_y}\right)(x_1 + x_2) + (y_1 + y_2) = \left(\frac{p_x}{p_y}\right) x_M + y_M$$

World Transformation Curve (World PPF): The World PPF is constructed by summing the individual PPFs. Because comparative costs differ ($a_1/b_1 \neq a_2/b_2$), the World PPF is **bowed out** (concave to origin) consisting of linear segments. This shape reflects efficiency: the world allocates production to the lowest-cost country first.

2.3 The Ricardo Point

The **Ricardo Point** (Point R in Fig 2.4) represents the efficient allocation where complete specialization occurs.

- At this point, Country 1 produces only x (Cloth) and Country 2 produces only y (Wine).
- **Intuition:** This point maximizes the intercept of the world iso-value line (World Real Income) when the ToT lies strictly between the two countries' opportunity costs.
- At the Ricardo Point, the world is utilizing the "cheapest" producer for every good.

3 Negishi's Interpretation: Subsistence Wage

Classical theory is often criticized for being "supply-side only" and unable to determine the exact Terms of Trade without demand (Mill/Marshall). Takashi Negishi (1982) argued that the classical system *can* determine ToT using only cost-price relations if we incorporate the **Subsistence Wage**.

3.1 The Setup

- **Subsistence Wage (w):** Wages are driven by the cost of a fixed basket of goods required for survival.

$$w = c_A p_A + c_B p_B$$

  where c_A, c_B are the bare-bone needs for Cloth (A) and Wine (B).
- **Production Technology:** Capital is strictly **circulating capital** (the wage bill advanced to workers).
- **Pricing Condition:** Price = (Wage Bill) × (1 + Rate of Profit).

3.2 Pre-Trade Equilibrium (Autarky)

In autarky (e.g., Country 1), prices are determined by labor input ($a_1 = 100, b_1 = 120$) and the domestic profit rate r_1:

$$p_{1A} = (1 + r_1)100w_1$$
$$p_{1B} = (1 + r_1)120w_1$$

The relative price is fixed by technology: $p_{1A}/p_{1B} = 100/120$.

3.3 Post-Trade Equilibrium

Assume England (1) specializes in A, and Portugal (2) specializes in B. Let r_1^c and r_2^c be the profit rates in England and Portugal under international trade.

1. Active Production Equations: Since England produces A and Portugal produces B, these equality conditions hold:

$$p_A = (1 + r_1^c)100(c_A p_A + c_B p_B) \tag{3.1}$$
$$p_B = (1 + r_2^c)80(c_A p_A + c_B p_B) \tag{3.2}$$

Note: w has been substituted with $(c_A p_A + c_B p_B)$.

2. Inactive Production Constraints (The "Missing Link"): For specialization to be sustainable, it must be unprofitable (or at least not more profitable) to produce the other good. This is the crucial no-arbitrage condition for production patterns. England should *not* produce B; Portugal should *not* produce A. This implies the hypothetical cost of producing the imported good domestically is higher than (or equal to) the import price:

$$p_A \leq (1 + r_2^c)90(c_A p_A + c_B p_B) \quad \text{(Port. producing A is too expensive)} \tag{3.3}$$
$$p_B \leq (1 + r_1^c)120(c_A p_A + c_B p_B) \quad \text{(Eng. producing B is too expensive)} \tag{3.4}$$

3.4 Deriving the Terms of Trade Limits

Negishi defines a ratio of discount factors $\alpha = \frac{1+r_1^c}{1+r_2^c}$. Dividing the active production equations (3.1) and (3.2):

$$\frac{p_A}{p_B} = \frac{1+r_1^c}{1+r_2^c}\frac{100}{80} = \alpha\frac{100}{80}$$

By using the inequality constraints, we can bound α:

$$\frac{80}{120} \leq \alpha \leq \frac{90}{100}$$

Substituting α back into the price ratio equation gives the limits of the Terms of Trade:

$$\frac{100}{120} \leq \frac{p_A}{p_B} \leq \frac{90}{80}$$

Conclusion: The subsistence wage assumption allows us to define the limits of the Terms of Trade strictly through cost/profit relations (supply side), recovering the classical result without needing explicit utility functions or demand curves.

Part II — Heckscher-Ohlin and Related

4 The Heckscher-Ohlin (H-O) Framework

The Neoclassical theory extends the classical model by introducing two factors of production (Capital K and Labor L). It focuses on differing factor endowments as the source of comparative advantage.

4.1 Core Assumptions (2x2x2 Model)

The standard model assumes 2 countries (Home, Foreign), 2 goods (Cloth A, Food B), and 2 factors (K, L).

1. **Perfect Competition:** Zero profits in equilibrium; prices equal unit costs.
2. **Constant Returns to Scale (CRS):** Production functions are homogeneous of degree one.
3. **Identical Technology:** Both countries share the same production functions $A = F_A(K, L)$ and $B = F_B(K, L)$.
4. **Identical Homothetic Preferences:** Tastes are the same across countries and do not vary with income levels (Unitary income elasticity).
5. **Factor Mobility:** Factors are perfectly mobile across sectors within a country but perfectly immobile between countries.
6. **No Transport Costs or Tariffs.**

4.2 The Four Core Theorems

1. **Heckscher-Ohlin Theorem:** A country exports the good that intensively uses its abundant factor.
2. **Factor Price Equalization (FPE) Theorem:** Free trade in goods equalizes relative and absolute factor prices across countries, provided strictly incomplete specialization occurs.
3. **Stolper-Samuelson Theorem:** An increase in the relative price of a good increases the real return to the factor used intensively in that good and decreases the real return to the other factor. This theorem identifies the winners and losers from trade.
4. **Rybczynski Theorem:** An increase in the endowment of one factor (holding prices constant) leads to a more than proportionate increase in the output of the good intensive in that factor and a decrease in the output of the other good.

5 Factor Intensity and Reversal (FIR)

5.1 Factor Intensity Definition

Factor intensity is defined by the capital-labor ratio ($k_i = K_i/L_i$) used in production at a given factor price ratio $\omega = w/r$.

- Good A is **Capital Intensive** if $k_A(\omega) > k_B(\omega)$ for all relevant ω.
- Good B is **Labor Intensive** if $k_B(\omega) < k_A(\omega)$ for all relevant ω.

5.2 Factor Intensity Reversal (FIR)

FIR occurs when the factor intensity ordering of two goods flips at different relative factor prices. Geometrically, this means the isoquants of the two goods cross more than once in the Lerner-Pearce diagram.

Condition for Absence of FIR: The likelihood of FIR depends on the **Elasticity of Substitution** (σ) in each sector.

$$\sigma_i = \frac{d\ln(K_i/L_i)}{d\ln(w/r)}$$

From the lecture notes (Week 6-1.2), the relationship between factor intensity changes and relative factor prices is governed by:

$$\frac{d\rho_i}{d(p_L/p_K)} = \sigma_i \frac{\rho_i}{(p_L/p_K)}$$

where $\rho_i = K_i/L_i$.

Theorem on FIR: If the elasticity of substitution is constant in each sector (CES functions):

1. If $\sigma_A = \sigma_B$, no FIR occurs (intensity curves are parallel in log space).
2. If $\sigma_A \neq \sigma_B$, FIR *must* occur at some factor price ratio (single crossing of intensity curves).

Intuition: If one sector (e.g., Agriculture) can easily substitute labor for machinery (high σ) while another (e.g., Energy) cannot (low σ), a massive change in wages might make Agriculture switch from being labor-intensive to capital-intensive relative to Energy.

6 The Heckscher-Ohlin (H-O) Theorem

6.1 Statement

A country abundant in a factor exports the good whose production is intensive in that factor.

- **Physical Definition of Abundance:** Country 1 is Capital Abundant if $(K/L)_1 > (K/L)_2$.
- **Price Definition of Abundance:** Country 1 is Capital Abundant if $(r/w)_1 < (r/w)_2$ in autarky.

6.2 Proof: The Production Bias (Algebraic Derivation)

We verify the link between endowment (K/L) and output ratio (A/B).

1. Full Employment Conditions:

$$a_{KA}A + a_{KB}B = K$$
$$a_{LA}A + a_{LB}B = L$$

Dividing by L, and letting $x_A = A/L$, $x_B = B/L$, and $\rho = K/L$:

$$a_{KA}x_A + a_{KB}x_B = \rho$$
$$a_{LA}x_A + a_{LB}x_B = 1$$

2. Solving for Outputs (Cramer's Rule): The determinant of the coefficient matrix is $|D| = a_{KA}a_{LB} - a_{LA}a_{KB}$. Assuming A is K-intensive ($a_{KA}/a_{LA} > a_{KB}/a_{LB}$), then $|D| > 0$. The solutions are:

$$x_A = \frac{a_{LB}\rho - a_{KB}}{|D|}, \quad x_B = \frac{a_{KA} - a_{LA}\rho}{|D|}$$

3. The Output Ratio:

$$\frac{A}{B} = \frac{x_A}{x_B} = \frac{a_{LB}\rho - a_{KB}}{a_{KA} - a_{LA}\rho}$$

4. Effect of Endowment Change (Differentiation): Differentiating A/B with respect to ρ (K/L):

$$\frac{d(A/B)}{d\rho} = \frac{a_{LB}(a_{KA} - a_{LA}\rho) - (-a_{LA})(a_{LB}\rho - a_{KB})}{(a_{KA} - a_{LA}\rho)^2}$$

Simplifying the numerator:

$$Num = a_{LB}a_{KA} - a_{LB}a_{LA}\rho + a_{LA}a_{LB}\rho - a_{LA}a_{KB} = a_{LB}a_{KA} - a_{LA}a_{KB} = |D|$$

Since A is K-intensive, $|D| > 0$. Thus:

$$\frac{d(A/B)}{d(K/L)} > 0$$

Conclusion: An increase in the capital-labor ratio (K/L) leads to an increase in the relative output of the capital-intensive good (A/B). Since Country 1 is K-abundant $(K_1/L_1 > K_2/L_2)$, it produces relatively more A. Given identical homothetic demand (consumption ratios are identical), Country 1 must export the excess supply of A.

7 Factor Price Equalization (FPE) Theorem

7.1 Statement

International trade in commodities, under H-O assumptions and incomplete specialization, equalizes relative and absolute factor prices across countries, even though factors are immobile.

$$p_A^1 = p_A^2 \implies w_1 = w_2 \text{ and } r_1 = r_2$$

7.2 Conditions for FPE

1. **No Factor Intensity Reversal (FIR):** There must be a one-to-one correspondence between good prices and factor prices.
2. **Incomplete Specialization:** Both countries must continue to produce both goods (which requires endowments to be sufficiently similar, inside the "Cone of Diversification").

7.3 Proof: Zero Profit Conditions (Unit Cost Approach)

In a competitive equilibrium, price equals unit cost.

$$p_A = c_A(w, r)$$
$$p_B = c_B(w, r)$$

Since technology is identical, the cost functions c_A and c_B are the same for both countries.

The Inversion Problem: Can we uniquely solve for (w, r) given (p_A, p_B)? Consider the system of equations. Differentiating totally (using Jones' Algebra notation where $\hat{x} = dx/x$):

$$\hat{p}_A = \theta_{LA}\hat{w} + \theta_{KA}\hat{r}$$
$$\hat{p}_B = \theta_{LB}\hat{w} + \theta_{KB}\hat{r}$$

where θ_{ij} is the distributive share of factor i in good j (e.g., $\theta_{LA} = wa_{LA}/p_A$). Note that $\theta_{LA} + \theta_{KA} = 1$.

We can write this in matrix form:

$$\begin{pmatrix} \hat{p}_A \\ \hat{p}_B \end{pmatrix} = \begin{pmatrix} \theta_{LA} & \theta_{KA} \\ \theta_{LB} & \theta_{KB} \end{pmatrix} \begin{pmatrix} \hat{w} \\ \hat{r} \end{pmatrix}$$

For a unique solution to exist, the determinant of the share matrix $|\theta|$ must be non-zero.

$$|\theta| = \theta_{LA}\theta_{KB} - \theta_{KA}\theta_{LB} = \theta_{LA}(1 - \theta_{LB}) - (1 - \theta_{LA})\theta_{LB} = \theta_{LA} - \theta_{LB}$$

Since θ_{LA} represents labor intensity in value terms:

- If Good A is K-intensive, $\theta_{KA} > \theta_{KB} \implies \theta_{LA} < \theta_{LB}$.
- Thus, $|\theta| \neq 0$.

Global Univalence (Gale-Nikaido Condition): Mathematically, we need the mapping from (w, r) to (p_A, p_B) to be globally invertible. Week 6-1.2 Notes derive the Jacobian of the system:

$$J = \det \begin{pmatrix} \frac{\partial c_A}{\partial w} & \frac{\partial c_A}{\partial r} \\ \frac{\partial c_B}{\partial w} & \frac{\partial c_B}{\partial r} \end{pmatrix} = \det \begin{pmatrix} a_{LA} & a_{KA} \\ a_{LB} & a_{KB} \end{pmatrix}$$

Using the fact that unit factor requirements (a_{ij}) depend on factor prices, but assuming no FIR ensures the determinant never changes sign. Therefore, if $p_A^1 = p_A^2$ and $p_B^1 = p_B^2$ (due to free trade), the unique solution is $(w_1, r_1) = (w_2, r_2)$.

7.4 Intuition

Trade in goods is a ”substitute” for trade in factors. When Country 1 exports the capital-intensive good, it is effectively exporting ”bundled” capital services. This increases derived demand for capital in Country 1 (raising r) and increases the supply of bundled capital in Country 2 (lowering r), until $r_1 = r_2$.

8 Heckscher-Ohlin-Vanek (HOV) Theorem

8.1 Context

The HOV theorem generalizes the H-O model to many goods, many factors, and many countries. It shifts focus from the direction of trade in goods (which good is exported?) to the **factor content** of trade (which factor services are exported?).

8.2 Statement

A country will be a net exporter of the services of its abundant factors and a net importer of the services of its scarce factors.

$$F_k^i > 0 \iff \frac{V_k^i}{V_k^w} > s^i$$

where:

- F_k^i is the net export of factor k by country i.
- V_k^i is country i's endowment of factor k.
- V_k^w is the world endowment of factor k.
- s^i is country i's share of world consumption (and world GDP with balanced trade).

8.3 Assumption Requirements

1. Identical technology (A matrix is same everywhere).
2. Identical homothetic preferences (Consumption vectors are proportional to world output).
3. Free trade in goods and factor price equalization (FPE) holds (ensuring techniques a_{ij} are identical).

8.4 Proof / Derivation

1. Definition of Factor Content: Let T^i be the vector of net exports of goods for country i. The factor content of trade F^i is the amount of factors embedded in T^i:

$$F^i = AT^i$$

where A is the matrix of unit factor requirements (assumed identical due to FPE).

2. Identity of Production and Consumption: Net exports = Production - Consumption.

$$T^i = Y^i - C^i$$

Multiply by the technology matrix A:

$$AT^i = AY^i - AC^i$$

Since AY^i is the demand for factors to produce output Y^i, and full employment implies $AY^i = V^i$ (country i's endowment), we have:

$$F^i = V^i - AC^i$$

3. The Consumption Assumption: With identical homothetic preferences and free trade (same prices), country i consumes a fixed share s^i of the world production Y^w.

$$C^i = s^i Y^w$$

Thus, the factors embedded in consumption are:

$$AC^i = s^i(AY^w) = s^i V^w$$

(since world factor demand equals world factor endowment V^w).

4. The HOV Equation: Substituting back into the factor content equation:

$$F^i = V^i - s^i V^w$$

For a specific factor k:

$$F_k^i = V_k^i - s^i V_k^w$$

5. Interpretation of Abundance: Dividing by V_k^w:

$$\frac{F_k^i}{V_k^w} = \frac{V_k^i}{V_k^w} - s^i$$

If country i is abundant in factor k relative to the world (i.e., its share of the world's factor k exceeds its share of world GDP/consumption s^i), then $\frac{V_k^i}{V_k^w} > s^i$, which implies $F_k^i > 0$. The country exports the services of factor k.

9 Additional Derivations: Stolper-Samuelson & Rybczynski

9.1 The "Magnification Effect" (Jones Algebra)

Both theorems can be proved elegantly using the equations of change (hat calculus).

System of Equations:

$$\theta_{LA}\hat{w} + \theta_{KA}\hat{r} = \hat{p}_A \tag{9.1}$$
$$\theta_{LB}\hat{w} + \theta_{KB}\hat{r} = \hat{p}_B \tag{9.2}$$

$$\lambda_{LA}\hat{A} + \lambda_{LB}\hat{B} = \hat{L} \tag{9.3}$$
$$\lambda_{KA}\hat{A} + \lambda_{KB}\hat{B} = \hat{K} \tag{9.4}$$

where λ_{ij} is the fraction of factor i employed in sector j.

9.2 Stolper-Samuelson Theorem (S-S)

Assume Good A is K-intensive ($\theta_{KA} > \theta_{KB}$) and its price rises ($\hat{p}_A > \hat{p}_B = 0$). Subtracting the price equations yields:

$$\hat{p}_A - \hat{p}_B = (\theta_{LA} - \theta_{LB})\hat{w} + (\theta_{KA} - \theta_{KB})\hat{r}$$

Since $\theta_{Li} = 1 - \theta_{Ki}$, we have $\theta_{LA} - \theta_{LB} = -(\theta_{KA} - \theta_{KB})$.

$$\hat{p}_A = (\theta_{KA} - \theta_{KB})(\hat{r} - \hat{w})$$

Since A is K-intensive, $\theta_{KA} > \theta_{KB}$, so $\hat{r} > \hat{w}$. From the original equation $\hat{p}_A = \theta_{LA}\hat{w} + \theta_{KA}\hat{r}$, since $\hat{p}_A > 0$ and $\hat{r} > \hat{w}$, it must be that $\hat{r} > \hat{p}_A > \hat{w}$.

Result (Magnification Effect):

$$\hat{r} > \hat{p}_A > \hat{p}_B > \hat{w}$$

The real return to Capital (r/p_A and r/p_B) rises. The real return to Labor falls.

9.3 Rybczynski Theorem

Assume Good A is K-intensive ($\lambda_{KA}/\lambda_{LA} > \lambda_{KB}/\lambda_{LB}$). Assume Capital endowment grows ($\hat{K} > 0$) while $\hat{L} = 0$, holding prices constant ($\hat{p} = 0 \implies \hat{w} = \hat{r} = 0$, so a_{ij} are constant).

Solving the endowment system for output changes:

$$\hat{A} = \frac{\lambda_{LB}\hat{K} - \lambda_{KB}\hat{L}}{|\lambda|}$$

$$\hat{B} = \frac{\lambda_{KA}\hat{L} - \lambda_{LA}\hat{K}}{|\lambda|}$$

where determinant $|\lambda| = \lambda_{LA}\lambda_{KB} - \lambda_{LB}\lambda_{KA}$. If A is K-intensive, A uses K relatively more than L, so A is "L-saving" relative to B? No, A is K-intensive implies $|\lambda| < 0$ (careful with sign convention of determinant defined as L row first). Let's use the condition: $\frac{K_A}{L_A} > \frac{K_B}{L_B} \implies \frac{\lambda_{KA}}{\lambda_{LA}} > \frac{\lambda_{KB}}{\lambda_{LB}} \implies \lambda_{KA}\lambda_{LB} > \lambda_{KB}\lambda_{LA}$. Determinant definition in standard texts (Jones): $|\lambda| = \lambda_{LA}\lambda_{KB} - \lambda_{LB}\lambda_{KA} < 0$.

If $|\lambda| < 0$:

$$\hat{A} = \frac{\lambda_{LB}\hat{K}}{-|\text{pos}|}$$

Wait, standard Jones algebra derivation: $\hat{A} = \frac{\lambda_{KB}\hat{L} - \lambda_{LB}\hat{K}}{|\lambda|}$... Let's restate the Magnification Effect for quantities: If A is K-intensive and $\hat{K} > \hat{L}$:

$$\hat{A} > \hat{K} > \hat{L} > \hat{B}$$

Intuition: To absorb the new capital at constant factor prices (constant K/L ratios in production), sector A (K-intensive) must expand. However, expanding A requires Labor. It must draw Labor from sector B. Thus, sector B contracts.

Part III — New Trade Theory

10 New Trade Theory: Monopolistic Competition

10.1 Overview

Unlike the Classical or Neoclassical models which rely on constant returns to scale (CRS) and perfect competition, New Trade Theory (Krugman, 1979) introduces:

- **Internal Economies of Scale:** Average costs fall as output increases ($AC > MC$).
- **Product Differentiation:** Consumers love variety.
- **Monopolistic Competition:** Many firms, free entry, zero long-run profit, but each firm has some market power over its unique variety.

10.2 The Demand Side: Preference for Variety

We model preferences using the **Spence-Dixit-Stiglitz (S-D-S)** utility function. Consider a utility function with two goods: a homogeneous good Y and a differentiated good aggregate D.

1. Utility Function:

$$U = Y + A\theta^{-1}D^{\theta}, \quad 0 < \theta < 1$$

where D is a CES aggregator of n varieties:

$$D = \left(\sum_{i=1}^{n} D_i^{\alpha}\right)^{1/\alpha}, \quad 0 < \alpha < 1$$

The parameter $\sigma = \frac{1}{1-\alpha} > 1$ represents the **elasticity of substitution** between varieties.

2. Optimization (Two-Stage Budgeting): *Stage 1: Choice between Y and Aggregate D.* Maximize U subject to budget $I = Y + PD$, where P is the price index for the differentiated good. This yields the total demand for the differentiated sector:

$$D = BP^{-\epsilon}, \quad \text{where } \epsilon = \frac{1}{1-\theta}$$

Stage 2: Choice among varieties D_i. Maximize the sub-utility D subject to the spending allocation on varieties $E_D = \sum p_i D_i$. Using the Lagrangian method, we derive the demand for an individual variety i:

$$D_i = \left(\frac{p_i}{P}\right)^{-\sigma} D$$

Substituting the aggregate demand D:

$$D_i = Bp_i^{-\sigma}P^{\sigma-\epsilon}$$

3. The Price Index (P): The exact price index that corresponds to the expenditure required to buy one unit of utility D is:

$$P = \left(\sum_{i=1}^{n} p_i^{1-\sigma}\right)^{\frac{1}{1-\sigma}}$$

Intuition: As n increases, P decreases (even if individual prices p_i stay constant). This reflects the **love for variety** effect—consumers are better off just by having more choices.

10.3 The Supply Side: Increasing Returns

Firms use labor (L) as the only input.

- **Cost Function:** $l_i = g(x_i)$ with $g'(x_i) > 0$ and $g''(x_i) < 0$ (or simply fixed cost + constant marginal cost).
- **Average Cost (AC):** Decreasing in x (Economies of Scale).
- **Marginal Cost (MC):** $wg'(x_i)$.

10.4 Market Equilibrium

1. Profit Maximization (PP Curve): Each firm faces a downward-sloping demand curve with constant elasticity σ. It sets Marginal Revenue equal to Marginal Cost.

$$MR = p_i\left(1 - \frac{1}{\sigma}\right) = MC = wg'(x_i)$$

Rearranging gives the pricing rule (Markup Pricing):

$$\frac{p_i}{w} = \frac{\sigma}{\sigma - 1} g'(x_i) \quad \text{(PP Relation)}$$

Note: Since $\sigma > 1$, Price $> MC$.

2. Free Entry / Zero Profit (ZZ Curve): In the long run, firms enter until profits are zero. Price must equal Average Cost.

$$p_i x_i = wg(x_i) \implies \frac{p_i}{w} = \frac{g(x_i)}{x_i} \quad \text{(ZZ Relation)}$$

3. Equilibrium Output (x^*): Equating the PP and ZZ relations:

$$\frac{\sigma}{\sigma - 1} g'(x^*) = \frac{g(x^*)}{x^*}$$

This equation uniquely determines the equilibrium output per firm x^*. Notice that x^* depends only on technology (g) and preferences (σ), **not on market size (L)**.

4. Equilibrium Number of Firms (n^*): Using the full employment condition $L = n \cdot g(x^*)$:

$$n^* = \frac{L}{g(x^*)}$$

Intuition: A larger market ($L \uparrow$) does *not* lead to larger firms (since x^* is fixed by σ), but rather to *more* firms ($n \uparrow$).

10.5 The Gains from Trade

When two identical countries trade (market integration):

- **Integrated Market Size:** $L_{world} = L_1 + L_2$.
- **Output per firm:** Remains x^* (firms do not slide down the AC curve in this specific CES specification, though they do in other variants).
- **Total Varieties:** $n_{world} = \frac{L_1 + L_2}{g(x^*)} = n_1 + n_2$.
- **Welfare Gain:** Consumers in both countries now have access to n_{world} varieties instead of just n_1. Since utility increases in n, everyone is better off.
- **No Comparison of Costs Needed:** Trade is driven by the desire for variety and economies of scale, not comparative advantage.

10.6 Trade with Transport Costs (Iceberg Model)

If transport costs exist (only a fraction $1/(1+\varphi)$ arrives), the market is not perfectly integrated.

- **Home Market Equilibrium:** Production for domestic use (x_{11}) + Production for export (x_{12}).
- **Price Discrimination:** Firms may perceive different elasticities or costs, but in the CES case, the mill price is the same, so consumers pay the transport cost.
- **Home Bias:** Due to transport costs, domestic consumers will consume relatively more domestic varieties than imported ones.

11 Reciprocal Dumping Model (Brander-Krugman)

11.1 Overview

This model explains two-way trade in identical goods (intra-industry trade) between two countries with identical costs. It relies on **Oligopoly (Cournot Competition)** and **Segmented Markets**.

11.2 Setup

- Two countries: Home and Foreign.
- One firm in each country (Duopoly).
- **Identical Marginal Cost:** c. Fixed Cost F.
- **Segmented Markets:** Firms can set different prices in Home and Foreign markets (Price Discrimination).
- **Iceberg Transport Cost:** To sell 1 unit abroad, a firm must ship $g > 1$ units (or $1/\tau$ where $\tau < 1$ arrives). Let's use marginal cost of export = $c + t$ or c/τ. The slides use f as marginal cost and f/g as export marginal cost.

11.3 The Home Market Game

Let Q_1^1 be Home firm's sales in Home. Let Q_2^1 be Foreign firm's sales in Home. Total Quantity $Q^1 = Q_1^1 + Q_2^1$. Inverse Demand: $P^1 = a - b(Q_1^1 + Q_2^1)$.

1. Home Firm's Maximization:

$$\max_{Q_1^1} \pi_1 = [a - b(Q_1^1 + Q_2^1)]Q_1^1 - fQ_1^1 - F$$

FOC (Marginal Revenue = Marginal Cost):

$$a - 2bQ_1^1 - bQ_2^1 = f$$

Home Reaction Function (R_1):

$$Q_1^1 = \frac{a-f}{2b} - \frac{1}{2}Q_2^1$$

2. Foreign Firm's Maximization (Exporting): Foreign firm faces marginal cost f/g (where $g < 1$ is the fraction arriving, effectively raising MC).

$$\max_{Q_2^1} \pi_2 = [a - b(Q_1^1 + Q_2^1)]Q_2^1 - (f/g)Q_2^1 - F$$

FOC:

$$a - bQ_1^1 - 2bQ_2^1 = f/g$$

Foreign Reaction Function (R_2):

$$Q_2^1 = \frac{a - f/g}{2b} - \frac{1}{2}Q_1^1$$

11.4 Equilibrium and Dumping

Solving the system of reaction functions:

- Because $f/g > f$ (transport costs make exporting expensive), the Foreign firm has a lower market share in the Home market ($Q_2^1 < Q_1^1$).
- Similarly, in the Foreign market, the Home firm has a smaller share than the Foreign firm.

Why is it "Dumping"? Dumping is defined as setting a lower price for export (FOB) than for domestic sales.

- **Domestic Price (P_{dom}):** Determined by local intersection of reaction curves.
- **Export Price (Net of Transport):** The firm receives the market price P_{for} but pays transport costs.
- Due to the "effective" marginal cost being higher for exports, the firm restricts export quantity more, essentially absorbing some of the freight.
- The **FOB price** (Price received at factory gate for exports) is often lower than the factory gate price for domestic sales because the firm "absorbs" part of the transport cost to stay competitive abroad and accepts a lower markup on exports to gain market share.

11.5 Welfare Implications

1. Pro-competitive Effect: The entry of the foreign firm breaks the domestic monopoly. Total quantity Q^1 increases, Price falls. This increases Consumer Surplus. **2. Waste Effect:** Transporting goods consumes resources ($f/g - f$ per unit). This is a deadweight loss ("shipping coal to Newcastle").

Net Welfare: Ambiguous. If transport costs are low, the pro-competitive effect dominates. If transport costs are high (near prohibitive), the waste dominates.

12 The Falvey Model (New Heckscher-Ohlin)

12.1 Overview

The Falvey model reconciles **intra-industry trade** with traditional **comparative advantage**. It focuses on **Vertical Differentiation** (Quality differences) rather than Horizontal Differentiation (Variety).

- Countries export products of different *qualities* within the same industry.
- Trade is driven by endowment differences (Capital vs Labor), consistent with H-O logic.

12.2 Setup

- **Goods:** One industry (e.g., Cloth) produces a continuum of qualities q.
- **Cost Structure:** Higher quality requires relatively more Capital (K).
- **Production Function:** Fixed coefficients for a given quality q.
- **Unit Cost Functions:**
  - Home: $c(q) = w + qr$
  - Foreign: $c'(q) = w' + qr'$

  Here, w is the wage (Labor return) and r is the rental rate (Capital return). Note that "Quality" (q) essentially acts as the capital intensity parameter.

12.3 Factor Endowments and Prices

Assume Home is **Capital Abundant** and Foreign is **Labor Abundant**. In autarky (or incomplete specialization trading equilibrium):

- Capital is cheaper in Home: $r < r'$
- Labor is cheaper in Foreign: $w > w'$ (Assuming w' is low due to labor abundance).

12.4 The Threshold Quality (q_1)

We seek the quality level where production costs are identical in both countries.

$$c(q_1) = c'(q_1)$$

$$w + q_1 r = w' + q_1 r'$$

Rearranging to solve for q_1:

$$w - w' = q_1(r' - r)$$

$$q_1 = \frac{w - w'}{r' - r}$$

This q_1 is the **marginal quality**.

12.5 Pattern of Trade

We analyze the cost difference for any quality q:

$$c'(q) - c(q) = (w' - w) + q(r' - r)$$

Using the definition of q_1, we can rewrite this as:

$$c'(q) - c(q) = (r' - r)(q - q_1)$$

Since $r' > r$ (Foreign has expensive capital):

Case 1: High Quality ($q > q_1$) If $q > q_1$, then $(q - q_1) > 0$. Therefore, $c'(q) - c(q) > 0 \implies c'(q) > c(q)$. **Result:** Home has a lower cost for high-quality goods. Home exports high-quality cloth.

Case 2: Low Quality ($q < q_1$) If $q < q_1$, then $(q - q_1) < 0$. Therefore, $c'(q) - c(q) < 0 \implies c'(q) < c(q)$. **Result:** Foreign has a lower cost for low-quality goods. Foreign exports low-quality cloth.

12.6 Conclusion

- **Intra-Industry Trade:** We observe two-way trade in "Cloth".
- **Factor Content:** Home (Capital Rich) exports K-intensive high-quality cloth. Foreign (Labor Rich) exports L-intensive low-quality cloth.
- This confirms the H-O theorem prediction *within* a vertically differentiated industry. The model essentially applies the H-O logic to a continuum of goods defined by their capital intensity (q).

Part IV — Foreign Direct Investment

13 Economics of FDI: Fundamentals

13.1 Definitions and Scope

Foreign Direct Investment (FDI) represents a category of cross-border investment associated with a resident in one economy having control or a significant degree of influence on the management of an enterprise resident in another economy.

- **The 10% Threshold (IMF BPM6):** An investor is considered a direct investor if they own **10% or more** of the voting power (equity) in the direct investment enterprise.
- **Key Characteristic:** Unlike portfolio investment (passive, speculative), FDI involves an **effective voice in management**.

Feature	FDI	Portfolio Investment
Intent	Control/Management influence	Passive financial return
Horizon	Long-term	Short-term / Speculative
Assets	Physical/Intangible assets (factories, tech)	Financial assets (stocks, bonds)
Volatility	Relatively stable	Highly volatile (Hot Money)

Table 1: FDI vs. Portfolio Investment

13.2 Composition of FDI Funds

FDI capital flows consist of three main components:

1. **Equity Capital:** Purchase of shares of an enterprise in a foreign country.
2. **Reinvested Earnings:** The direct investor's share of earnings *not* distributed as dividends. These retained profits are reinvested into the affiliate.
3. **Intra-company Loans:** Short- or long-term borrowing and lending of funds between the parent and affiliate enterprises.

13.3 Stocks vs. Flows

- **Flow:** The amount of capital moving across borders within a specific period (e.g., annual Inflows/Outflows).
- **Stock:** The cumulative value of foreign investments held at a specific point in time.

13.4 Modes of Entry

- **Greenfield Investment:**
  - **Definition:** Establishing a new venture from the ground up (building new factories).
  - **Impact:** Creates new production capacity and new jobs immediately. Common in developing countries.
- **Mergers and Acquisitions (M&A):**
  - **Definition:** Purchasing existing production facilities.

  - **Impact:** Transfer of ownership; does not immediately increase capital stock. Common in developed countries.
- **Brownfield Investment:** A hybrid approach (e.g., leasing existing facilities and upgrading them). Common in transition economies.

14 FDI Strategy and Motives

14.1 Structural Classification (Horizontal vs. Vertical)

1. Horizontal FDI

- **Definition:** The MNE undertakes the *same* production activities in multiple countries.
- **Driver:** Market access, avoiding trade barriers (tariff jumping), saving transport costs.
- **Intuition:** "Replicating" the home factory in the host country.

2. Vertical FDI The MNE fragments the production process geographically.

- **Backward Vertical FDI:**
  - The affiliate provides **inputs** for the parent's domestic production.
  - *Example:* A US car maker mining lithium in Chile.
- **Forward Vertical FDI:**
  - The affiliate sells/distributes the **outputs** of the parent's production.
  - *Example:* A Chinese electronics firm setting up a sales network in Europe.

14.2 Economic Motives (OLI Framework context)

1. **Resource-Seeking:** Acquiring specific resources (natural resources, cheap unskilled labor) more efficiently than in the home country.
2. **Market-Seeking:** Penetrating new markets or maintaining existing ones (often driven by market size and growth).
3. **Efficiency-Seeking:** Rationalizing production to exploit economies of scale/scope and differences in factor endowments (e.g., offshoring to low-cost locations).
4. **Strategic-Asset-Seeking:** Tactical investments to acquire specific proprietary assets (brands, R&D, technology) to prevent competitors from getting them.

14.3 The "Dark Side": Transfer Pricing

MNEs may manipulate **Intra-company transfer prices** to minimize global tax liabilities.

- **Mechanism:** Overprice imports from low-tax affiliates or underprice exports to low-tax affiliates.
- **Result:** Profits are shifted to low-tax jurisdictions. This explains phenomena where FDI firms report losses in high-tax host countries but continue to invest.

15 FDI Spillover Effects: The Econometric Model

15.1 The Core Estimation Equation

We estimate the impact of FDI presence on the Total Factor Productivity (TFP) of local firms using the following regression model:

$$\ln TFP_{ijt} = \gamma_0 + \gamma_1 FS_{ijt} + \gamma_2 Horizontal_{jt} + \gamma_3 Backward_{jt} + \gamma_4 Forward_{jt} + \epsilon_{ijt}$$

where:

- i: Firm index.
- j: Industry index.
- t: Time (Year) index.

15.2 Defining the Variables

1. Own-Plant Effect (FS_{ijt})

- **Definition:** The share of foreign equity in firm i.
- **Interpretation:** The direct effect of foreign ownership on the firm's own productivity.

2. Horizontal Spillover ($Horizontal_{jt}$)

- **Definition:** Weighted average of foreign share in the *same* industry j.

$$Horizontal_{jt} = \frac{\sum_{i \in j} \text{Foreign Share}_{it} \times Y_{it}}{\sum_{i \in j} Y_{it}}$$

- **Intuition:** How the presence of foreign competitors affects local firms in the **same sector**.
- **Expected Sign:** Ambiguous. (Positive via demonstration; Negative via market stealing).

3. Backward Spillover ($Backward_{jt}$)

- **Definition:** FDI presence in **downstream** industries (industries that buy from industry j).

$$Backward_{jt} = \sum_{k \neq j} \theta_{jk} Horizontal_{kt}$$

  where θ_{jk} is the share of industry j's output sold to industry k (Input-Output coefficient).

- **Intuition:** Local suppliers (Upstream) benefit from selling to efficient Foreign buyers (Downstream).
- **Note:** Usually the strongest positive spillover channel (Javorcik, AER).

4. Forward Spillover ($Forward_{jt}$)

- **Definition:** FDI presence in **upstream** industries (industries that sell inputs to industry j).

$$Forward_{jt} = \sum_{m \neq j} \theta_{jm} \frac{\sum_{i \in m} \text{Foreign Share}_{it} \times (Y_{it} - EX_{it})}{\sum_{i \in m} (Y_{it} - EX_{it})}$$

- **Intuition:** Local buyers (Downstream) benefit from buying higher quality/cheaper inputs from Foreign suppliers (Upstream). Note: Exports (EX) are excluded because local firms don't use exported inputs.

16 Mechanisms and Measurement

16.1 Transmission Mechanisms of Spillovers

How does productivity actually transfer from FDI to local firms?

1. **Demonstration Effect (Learning by Observing):**
   - Local firms adopt superior technologies or management practices simply by observing MNEs.
   - **Cost:** Often low (imitation), but requires absorptive capacity.
2. **Competition Effect (Market Stealing vs. Efficiency):**
   - **Positive:** MNE entry forces local firms to reduce X-inefficiency and innovate to survive.
   - **Negative (Market Stealing):** MNEs with lower marginal costs steal market share. Local firms push up their Average Cost curves as output falls ($Q \downarrow \implies AC \uparrow$), potentially crowding them out.
3. **Linkage Effect (Vertical):**
   - **Backward Linkage (Demand side):** MNEs demand high-quality inputs, providing technical assistance to local suppliers to ensure quality control.
   - **Forward Linkage (Supply side):** MNEs provide high-quality intermediate goods (machinery, chemicals) that boost the productivity of downstream local firms.
4. **Labor Mobility (Training Effect):**
   - MNEs invest in training local employees.
   - When these employees leave MNEs to join local firms or start their own, they carry human capital and tacit knowledge with them.

16.2 Measurement of Productivity (TFP)

To estimate the model, we need a robust measure of TFP.

- **Crude Measures:** Labor Productivity (Y/L), or OLS residuals from a Cobb-Douglas function.
- **Problem with OLS:** Simultaneity bias (firms observe their own productivity shock and adjust inputs, but the econometrician doesn't see the shock).
- **Advanced Methods:**
  - **OP (Olley & Pakes, 1996):** Uses investment as a proxy for unobserved productivity.
  - **LP (Levinsohn & Petrin, 2003):** Uses intermediate inputs (materials/energy) as a proxy (avoids the zero-investment problem of OP).
  - **ACF / De Loecker:** Refinements to correct for collinearity issues.

16.3 Heterogeneity of Effects

Spillovers are not uniform. They depend on:

- **FDI Characteristics:** Joint Ventures (higher potential for leakage) vs. Wholly Foreign Owned Enterprises (WFOE). Source country (HMT vs. Non-HMT).

- **Local Firm Characteristics:** State-Owned Enterprises (SOE) vs. Private. Export-oriented firms (may already be competitive) vs. Domestic-oriented.
- **Geography:** Special Economic Zones (SEZ) vs. Non-SEZ; Coastal vs. Inland.

Part V — Trade Policy and Welfare

17 Welfare Analysis of Trade Policy

This section analyzes the general equilibrium effects of trade policy (tariffs) on prices, terms of trade, and welfare, distinguishing between "Large Country" and "Small Country" cases.

17.1 The Metzler Paradox

17.1.1 Definition

The **Metzler Paradox** (Metzler, 1949) describes a situation where the imposition of a tariff on an imported good lowers the *domestic* relative price of that good, contrary to the standard expectation that a tariff should raise the domestic price.

17.1.2 Intuition

In a standard large country case, a tariff has two opposing effects on the domestic price of imports:

1. **Direct Tax Effect:** The tariff adds a wedge, tending to raise the domestic price ($P_{dom} = P_{world} + t$).
2. **Terms of Trade (ToT) Effect:** By restricting demand for imports, a large country forces the world price (P_{world}) down.

The paradox occurs when the **ToT effect dominates the Direct Tax effect**. This happens if the foreign country's export supply is highly inelastic (the foreign offer curve is backward-bending or very steep). The large country forces the world price down so drastically that even after adding the tariff, the final domestic price is lower than the free-trade price.

Condition: This typically requires the foreign demand for the home country's exports to be inelastic.

17.2 The Lerner Paradox

17.2.1 Definition

The **Lerner Paradox** (Lerner, 1936) describes a possibility where a tariff might **worsen** a country's Terms of Trade (ToT), rather than improve them.

17.2.2 Intuition

Standard theory suggests an import tariff improves a large country's ToT by reducing import demand. However, the paradox can arise due to the **Income/Revenue Effect**.

- The government collects tariff revenue.
- If the government (or the recipients of the revenue) spends a **disproportionately large fraction** of this income on the imported good.
- This increased demand from government spending outweighs the reduction in private demand caused by the price increase.
- Net demand for the imported good rises, pushing the world price up (worsening ToT).

18 Mathematical Derivation of Trade Paradoxes

We analyze the effects of a tariff using a two-country general equilibrium framework.

18.1 Model Setup

- Countries: Country 1 (Home), Country 2 (Foreign).
- Goods: A and B.
- Trade Pattern:
  - Country 1 exports A, imports B.
  - Country 2 exports B, imports A.
- Prices: Let A be the numéraire. Let $p = P_B/P_A$ be the relative price (Terms of Trade).
- Tariff: Country 1 levies an ad valorem tariff d on imports of B.
- Domestic Price (p_d): $p_d = p(1+d)$.
- Revenue Spending: The government spends a fraction φ of tariff revenue on the imported good B, and $(1-\varphi)$ on A.

18.2 Equilibrium Condition

In equilibrium, the value of excess demand for B in Country 2 must equal the value of excess demand (import demand) for B in Country 1. Taking into account the tariff revenue expenditure:

$$(1+\varphi d)E_{2B}(p_d) + E_{1B}(p) = 0 \tag{18.1}$$

Where E_{iB} represents the excess demand (import demand) function. Note that for Country 2 (exporter of B), E_{2B} is negative (export supply). The equation balances global supply and demand. Substituting $p_d = p(1+d)$:

$$(1+\varphi d)E_{2B}(p(1+d)) + E_{1B}(p) = 0 \tag{18.2}$$

18.3 Comparative Statics (The Jacobian)

We use the Implicit Function Theorem to find $\partial p_d/\partial d$ and $\partial p/\partial d$. The system is defined by:

$$(1+\varphi d)E_{2B}(p_d) + E_{1B}(p) = 0 \tag{18.3}$$

$$p_d - p(1+d) = 0 \tag{18.4}$$

The Jacobian determinant $|J|$ is:

$$|J| = \begin{vmatrix} (1+\varphi d)E'_{2B} & E'_{1B} \\ 1 & -(1+d) \end{vmatrix} \tag{18.5}$$

Evaluating at free trade equilibrium ($d = 0$, implying $p_d = p$ and $E_{2B} = -E_{1B}$):

$$|J| = -(E'_{2B} + E'_{1B}) \tag{18.6}$$

Converting to elasticities (ξ_1 for import demand of Country 1, ξ_2 for import demand of Country 2): Using $\xi = \frac{p}{E}\frac{dE}{dp}$:

$$|J| = -\frac{E_{2B}}{p}(1+\xi_1+\xi_2) \tag{18.7}$$

Stability Condition (Marshall-Lerner): For the market to be stable, $1+\xi_1+\xi_2 < 0$. This implies $|J| > 0$.

18.4 Deriving Price Changes

Differentiating the equilibrium conditions with respect to d and solving via Cramer's Rule yields:

18.4.1 1. Change in Domestic Price (Metzler Paradox Check)

$$\frac{\partial p_d}{\partial d} = p \frac{\varphi + \epsilon_1}{-(1 + \xi_1 + \xi_2)} \tag{18.8}$$

Where $\epsilon_1 = -(1 + \xi_1)$ is the elasticity of export supply of Country 1 (derived via Slutsky relation/Walras law). Since the denominator is positive (stability), the sign depends on the numerator $\varphi + \epsilon_1$. Substituting back elasticities:

$$\text{Sign depends on: } \varphi - \xi_1 - 1 \tag{18.9}$$

Metzler's Condition: For $\frac{\partial p_d}{\partial d} < 0$ (Domestic price falls):

$$\varphi - \xi_1 < 1 \quad \text{or} \quad \varphi + \epsilon_1 < 0 \tag{18.10}$$

This requires ξ_1 (import demand elasticity) to be small (inelastic) or ϵ_1 to be very negative. *Note: The lecture notes define ξ_1 as export supply for Country 1 in some contexts, be careful with notation. Based on formula 24.15 in slides: $\varphi - \xi_1 - 1 < 0$ implies Metzler Paradox.*

18.4.2 2. Change in World Price (Lerner Paradox Check)

$$\frac{\partial p}{\partial d} = p \frac{\varphi + \xi_2}{-(1 + \xi_1 + \xi_2)} \tag{18.11}$$

The sign depends on $\varphi + \xi_2$. Normally, we expect a tariff to lower world prices ($\frac{\partial p}{\partial d} < 0$). **Lerner's Condition:** For $\frac{\partial p}{\partial d} > 0$ (World price rises, ToT worsens):

$$\varphi + \xi_2 > 0 \tag{18.12}$$

This occurs if φ (marginal propensity to spend on imports) is very large, or ξ_2 (foreign import demand elasticity) is very inelastic (close to 0).

19 Lerner Symmetry Theorem

19.1 Definition

Lerner Symmetry (Lerner, 1936) states that an ad valorem import tariff has the exact same effects on real variables (resource allocation, relative prices, welfare) as an equivalent ad valorem export tax, provided the tax revenue is spent in the same way.

19.2 Intuition

Trade depends on relative prices.

- An **Import Tariff** raises the domestic price of imports relative to exports.
- An **Export Tax** lowers the domestic price of exports relative to imports.

Both policies reduce the domestic relative price of exports vs. imports. If P_X/P_M falls, producers shift away from exports towards import-competing sectors, and consumers shift away from imports. The distortion is identical.

19.3 Proof

Consider a small country with world prices P_W^X and P_W^M.

Scenario 1: Import Tariff (t) Domestic prices are:

$$P_{dom}^M = P_W^M(1+t) \tag{19.1}$$
$$P_{dom}^X = P_W^X \tag{19.2}$$

The domestic relative price of the export good is:

$$\left(\frac{P^X}{P^M}\right)_{tariff} = \frac{P_W^X}{P_W^M(1+t)} = \left(\frac{P_W^X}{P_W^M}\right)\frac{1}{1+t} \tag{19.3}$$

Scenario 2: Export Tax (τ) Domestic prices are:

$$P_{dom}^M = P_W^M \tag{19.4}$$
$$P_{dom}^X = \frac{P_W^X}{1+\tau} \quad \text{(Producers receive less)} \tag{19.5}$$

The domestic relative price of the export good is:

$$\left(\frac{P^X}{P^M}\right)_{tax} = \frac{\frac{P_W^X}{1+\tau}}{P_W^M} = \left(\frac{P_W^X}{P_W^M}\right)\frac{1}{1+\tau} \tag{19.6}$$

Conclusion: If $t = \tau$, the distortion to relative prices is identical. Since real economic decisions depend only on relative prices, the equilibrium outcomes are identical.

20 Optimal Tariff Theory

20.1 Concept

A large country possesses **monopsony power** in the world market. By imposing a tariff, it can restrict demand and lower the world price of the imported good (improve its Terms of Trade). The **Optimal Tariff** is the rate that maximizes national welfare by balancing the **ToT gain** against the **efficiency loss** (distortion of consumption and production).

20.2 Maximization Problem

Let Country 2 be the country imposing the tariff on Good B. Maximize the Social Welfare Function V:

$$\max_{d} V = v(A_2^D, B_2^D) = v(A_2 + E_{2A}, B_2 + E_{2B}) \tag{20.1}$$

Subject to the production possibility frontier $A_2 = \psi(B_2)$ and the trade balance constraint $E_{2A} = pE_{1B}$ (where Country 2 imports B, so $E_{2B} = -E_{1B}$). The reduced form objective function becomes:

$$V = v(\psi(B_2) + pE_{1B}(p), B_2 - E_{1B}(p)) \tag{20.2}$$

20.3 Derivation of First Order Conditions

Differentiating V with respect to consumption/trade choices involves equating the Marginal Rate of Substitution (MRS) and Marginal Rate of Transformation (MRT) to the domestic relative price ratio.

From the derivations in the slides (Equations 25.3 - 25.6):

1. The ratio of marginal utilities (Domestic Price ratio) is: $\frac{v_B}{v_A} = -\psi' = p(1+d)$.
2. From the welfare maximization FOC: $\frac{v_A}{v_B} = p\frac{1+\epsilon_1}{\epsilon_1}$.

Here, ϵ_1 is the **Elasticity of Export Supply** of the rest of the world (Country 1) for good B.

20.4 The Optimal Tariff Formula

Equating the domestic price conditions:

$$p(1+d) = p\left(\frac{1+\epsilon_1}{\epsilon_1}\right) \tag{20.3}$$

$$1 + d = 1 + \frac{1}{\epsilon_1} \tag{20.4}$$

$$d^* = \frac{1}{\epsilon_1} \tag{20.5}$$

20.5 Interpretation

- The optimal tariff is the **inverse of the foreign export supply elasticity**.
- **Small Country Case:** If the country is small, it faces an infinite foreign export supply elasticity ($\epsilon_1 \to \infty$). Thus, $d^* = 1/\infty = 0$. Free trade is optimal.
- **Large Country Case:** Since ϵ_1 is finite, $d^* > 0$. The country should exploit its market power.

21 Economics of Dumping

21.1 Definition

Dumping is a form of **international price discrimination** where a firm charges a lower price for exported goods than for the same goods sold domestically ($P_{for} < P_{dom}$).

21.2 Conditions for Dumping

For dumping to be profitable and sustainable, three conditions must hold:

1. **Imperfect Competition:** The firm must be a price setter (Monopoly or Oligopoly) in the domestic market.
2. **Segmented Markets:** Markets must be separated (e.g., by transport costs or tariffs) so that domestic consumers cannot buy the cheaper exports (no arbitrage/re-importation).
3. **Different Elasticities:** The price elasticity of demand must be different in the two markets (specifically, higher elasticity in the foreign market).

21.3 Mathematical Derivation

21.3.1 The Monopolist's Problem

A monopolist sells quantity q_1 in the domestic market and q_2 in the foreign market. Total Cost: $C(q_1 + q_2)$. Total Revenue: $R = R_1(q_1) + R_2(q_2) = p_1(q_1)q_1 + p_2(q_2)q_2$.

Objective:

$$\max_{q_1,q_2} \Pi = R_1(q_1) + R_2(q_2) - C(q_1 + q_2) \tag{21.1}$$

21.3.2 First Order Conditions (FOC)

Differentiating with respect to q_1 and q_2:

$$\frac{\partial \Pi}{\partial q_1} = R_1'(q_1) - C'(q_1 + q_2) = 0 \implies MR_1 = MC \tag{21.2}$$

$$\frac{\partial \Pi}{\partial q_2} = R_2'(q_2) - C'(q_1 + q_2) = 0 \implies MR_2 = MC \tag{21.3}$$

Equilibrium Condition:

$$MR_1 = MR_2 = MC \tag{21.4}$$

Marginal Revenue must be equalized across markets and equal to Marginal Cost.

21.3.3 Elasticity and Pricing

Recall the relationship between Marginal Revenue (MR), Price (P), and the Price Elasticity of Demand ($\eta > 0$):

$$MR = P\left(1 - \frac{1}{\eta}\right) \tag{21.5}$$

Since $MR_1 = MR_2$, we have:

$$P_1 \left(1 - \frac{1}{\eta_1}\right) = P_2 \left(1 - \frac{1}{\eta_2}\right) \tag{21.6}$$

Rearranging for the price ratio:

$$\frac{P_1}{P_2} = \frac{1 - 1/\eta_2}{1 - 1/\eta_1} \tag{21.7}$$

21.3.4 Conclusion

If the foreign market is more competitive (more substitutes available), demand is more elastic ($\eta_2 > \eta_1$). If $\eta_2 > \eta_1$, then $(1 - 1/\eta_2) > (1 - 1/\eta_1)$. Consequently, $P_1 > P_2$. **Intuition:** The firm charges a higher price in the domestic market (inelastic demand) and a lower price in the foreign market (elastic demand) to maximize global profits. This pricing strategy results in dumping.

Part VI — Trade and Technological Progress

22 Economic Growth and Trade: Technical Progress

Technical progress shifts the production function, allowing more output with the same inputs. In the Heckscher-Ohlin framework, we analyze this using the Dual Approach (Unit Cost Functions) and the Rybczynski Theorem analogy.

22.1 Modeling Technical Progress

Consider the production function for Sector j:

$$Y_j = F_j(K_j, L_j, t) \tag{22.1}$$

where t represents the state of technology. Technical progress reduces the unit costs of production. Let $c_j(w, r, t)$ be the unit cost function. The condition for perfect competition (Price = Unit Cost) is:

$$p_j = c_j(w, r, t) \tag{22.2}$$

Differentiating with respect to time (using "hat" notation $\hat{x} = dx/x$):

$$\hat{p}_j = \theta_{Kj}\hat{r} + \theta_{Lj}\hat{w} - \Pi_j \tag{22.3}$$

where $\Pi_j = -\frac{1}{c_j}\frac{\partial c_j}{\partial t}$ is the rate of technical progress (cost reduction) in sector j.

23 Neutral Technical Progress (Hicks Neutrality)

23.1 Definition

Technical progress is Hicks Neutral if, at a constant capital-labor ratio ($k = K/L$), the ratio of marginal products (MP_K/MP_L) remains unchanged.

- **Intuition:** The isoquants shift inward radially toward the origin. The firm has no incentive to change its factor proportions at constant relative factor prices.
- **Functional Form:** $Y_j = A(t)F_j(K_j, L_j)$. The productivity parameter $A(t)$ scales the entire function.

23.2 Effects on Output (Small Country Case)

Assume prices (p_A, p_B) are fixed (Small Country). Technical progress occurs in Sector A ($\Pi_A > 0, \Pi_B = 0$).

1. The Factor Price Effect: Since p_A is constant and efficiency rises, the real return to factors used intensively in A must rise (Stolper-Samuelson logic).

$$\hat{p}_A + \Pi_A = \theta_{KA}\hat{r} + \theta_{LA}\hat{w}$$

If A is K-intensive, the "effective price" increase leads to $\hat{r} > \hat{w}$.

2. The Output Effect: Hicks neutral progress in Sector A is equivalent to a simultaneous increase in both factors (K and L) allocated to Sector A. However, a more powerful intuition is derived from the **Effective Endowment** concept:

- Neutral progress in Sector A is equivalent to an increase in the economy's effective endowments of K and L, weighted by the sector's usage.
- **Result:** The output of Sector A increases ($Y_A \uparrow$). The effect on Sector B is ambiguous without specific parameters, but typically, resources are drawn into the innovating sector, so Y_B may fall or grow slower.

23.3 Effects on Terms of Trade (Large Country Case)

For a large country, prices are endogenous.

- **Export-Biased Growth:** If technical progress occurs in the Export sector, the export supply curve shifts out. The world price of exports falls.
- **Result:** Terms of Trade (ToT) **deteriorate** ($P_{ex}/P_{im} \downarrow$).
- **Import-Biased Growth:** If technical progress occurs in the Import-competing sector, import demand falls (domestic supply rises). The world price of imports falls relative to exports.
- **Result:** Terms of Trade (ToT) **improve** ($P_{ex}/P_{im} \uparrow$).

24 Biased Technical Progress

Biased progress changes the optimal capital-labor ratio at constant factor prices. This acts "as if" the endowment of one factor has increased relative to the other.

24.1 Hicksian Classifications

Defined by how the progress affects the Marginal Rate of Technical Substitution ($MRTS = MP_L/MP_K$) at a constant K/L ratio.

1. **Labor-Saving (Capital-Using):**
   - At constant K/L, MP_K increases more than MP_L.
   - Firms want to substitute Capital for Labor (K/L rises at constant w/r).
   - **Equivalent Factor Effect:** It saves Labor, so it acts like an **increase in the Labor endowment** ($\hat{L} > 0$).
2. **Capital-Saving (Labor-Using):**
   - At constant K/L, MP_L increases more than MP_K.
   - Firms want to substitute Labor for Capital (K/L falls at constant w/r).
   - **Equivalent Factor Effect:** It saves Capital, so it acts like an **increase in the Capital endowment** ($\hat{K} > 0$).

24.2 Analysis of Specific Cases (Small Country)

We assume Sector A is **Capital-Intensive** and Sector B is **Labor-Intensive**. Prices are constant.

24.2.1 Case 1: Capital-Saving Progress in K-Intensive Sector (A)

Intuition:

1. Sector A becomes more efficient. It needs less Capital (K) to produce the same output.
2. This "releases" Capital into the economy.
3. **Rybczynski Effect:** This is equivalent to an increase in the endowment of Capital ($\hat{K} > 0$).
4. Since Sector A is K-Intensive, an increase in K disproportionately increases the output of A and reduces the output of B.

Mathematical Logic:

$$\text{Effective } \hat{K} > 0 \implies \hat{Y}_A > \hat{K} > 0 > \hat{Y}_B$$

Outcome: Y_A increases significantly; Y_B decreases.

24.2.2 Case 2: Labor-Saving Progress in K-Intensive Sector (A)

Intuition:

1. Sector A becomes more efficient, but specifically saves Labor.

2. This "releases" Labor into the economy.
3. **Rybczynski Effect:** This is equivalent to an increase in the endowment of Labor ($\hat{L} > 0$).
4. Since Sector B is L-Intensive (and A is K-Intensive), an increase in L boosts B and hurts A.
5. **Conflict:** The *direct* effect of better tech in A wants to raise Y_A. The *biased* effect (releasing L) wants to lower Y_A via Rybczynski.

Outcome:

- Y_B definitely increases (Rybczynski effect helps L-intensive sector).
- Y_A is ambiguous. It may rise (if direct effect dominates) or fall (if Rybczynski effect dominates).

24.2.3 Case 3: Neutral Progress in K-Intensive Sector (A)

Intuition: Equivalent to $\hat{K} > 0$ AND $\hat{L} > 0$ allocated to A. **Outcome:** Y_A rises. Y_B usually falls (drawing resources away).

25 Summary Table of Technical Progress Effects

Assume:

- **Sector A:** Capital-Intensive (Export Good for this table).
- **Sector B:** Labor-Intensive (Import Good for this table).
- **Analysis:** Combined effects of Direct Productivity + Rybczynski Factor Release.

Type of Progress in Sector A (K-Int)	Equivalent Endowment Shock	Output A (K-Int, Export)	Output B (L-Int, Import)	Terms of Trade (Large Country)
Neutral	$\hat{K} > 0$ and $\hat{L} > 0$	Increases (↑↑)	Decreases (↓)	Worsens (↓)
Capital-Saving	$\hat{K} > 0$ (Saves intensive factor)	Increases strongly (↑↑↑)	Decreases strongly (↓↓)	Worsens strongly (↓↓)
Labor-Saving	$\hat{L} > 0$ (Saves scarce factor)	Ambiguous (?); Rises less or falls	Increases (↑)	Ambiguous; may improve

Table 2: Impact of Technical Progress in the Capital-Intensive Sector

General Rule of Thumb:

1. If progress saves the factor used **intensively** in that sector (e.g., K-saving in K-intensive A): The output of that sector explodes. ToT worsens significantly.
2. If progress saves the factor used **intensively in the OTHER** sector (e.g., L-saving in K-intensive A): The output of the *other* sector (B) rises. The innovating sector (A) might shrink. ToT might improve.

26 Immiserizing Growth (Bhagwati)

26.1 Definition

A situation where economic growth (factor accumulation or technical progress) leads to a sufficiently large deterioration in the Terms of Trade that the country's national welfare actually **declines**.

26.2 Necessary Conditions

For immiserizing growth to occur, three conditions must typically coincide:

1. **Large Country:** The country must be able to influence world prices.
2. **Export-Biased Growth:** Growth must significantly increase the supply of the export good (e.g., K-saving progress in the K-intensive export sector).
3. **Inelastic Foreign Demand:** The foreign Price Elasticity of Demand for the country's exports must be very low (inelastic). This causes the price (P_{ex}) to crash when supply increases.

26.3 Graphical Intuition (Offer Curves)

- The Home Offer Curve shifts outward (willing to export more A for B).
- Because the Foreign Offer Curve is steep (inelastic), the equilibrium Terms of Trade (P_A/P_B) falls drastically.
- The new consumption point lies on a lower social indifference curve than the pre-growth point.

26.4 Welfare Decomposition

The change in welfare (dW) can be decomposed into:

$$dW = \underbrace{dy}_{\text{Volume Effect (+)}} + \underbrace{(M \cdot dp)}_{\text{ToT Effect (-)}} \tag{26.1}$$

where dy is the change in output volume (positive) and $M \cdot dp$ is the loss from import purchasing power. Immiserizing growth occurs when the negative ToT effect dominates the positive Volume effect.