Everything About the Constant Elasticity of Substitution (CES) Function

Gemini 3 and GPT-5*

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CES Cheat Sheet: Key Formulas and Intuition

- **Aggregator (discrete):** $Y = A\left[\sum_{i=1}^{n} \alpha_i x_i^{\rho}\right]^{1/\rho}$, with $\sigma = \dfrac{1}{1-\rho}$.
- **Limits:**
  - Cobb–Douglas ($\sigma = 1$): $Y = A\prod_i x_i^{\alpha_i}$.
  - Leontief ($\sigma \to 0$): $Y = \min\{x_1, \ldots, x_n\}$.
  - Linear ($\sigma \to \infty$): $Y = \sum_i \alpha_i x_i$.
- **Marshallian demand:** $x_j = \alpha_j^{\sigma}\left(\dfrac{p_j}{P}\right)^{-\sigma}\dfrac{I}{P}$, where $P = \left[\sum_i \alpha_i^{\sigma} p_i^{1-\sigma}\right]^{1/(1-\sigma)}$.
- **Relative demand (two goods):** $\dfrac{x_j}{x_k} = \left(\dfrac{\alpha_j}{\alpha_k}\right)^{\sigma}\left(\dfrac{p_j}{p_k}\right)^{-\sigma}$; on log-log axes, the slope is $-\sigma$.
- **Expenditure shares:** $E_j = p_j x_j = \alpha_j^{\sigma} p_j^{1-\sigma} P^{\sigma} Y$. For $\sigma > 1$, expenditure falls with price; for $\sigma = 1$, unchanged; for $\sigma < 1$, rises with price.
- **Intuition on σ:**
  - High σ: goods are close substitutes; small price changes cause large quantity shifts.
  - Low σ: goods are complements; quantities barely adjust to price changes; expenditure may rise with price.
  - $\sigma = 1$: expenditure shares stay constant as prices move.
- **Dixit–Stiglitz (continuum):** $C = \left[\int_0^N c(i)^{(\sigma-1)/\sigma}\, di\right]^{\sigma/(\sigma-1)}$, price index $P = \left[\int_0^N p(i)^{1-\sigma}\, di\right]^{1/(1-\sigma)}$
- **Love for variety (symmetric):** If all p and c equal, $C = N^{1/(\sigma-1)} \cdot \dfrac{E}{p}$.

Rule of thumb: The larger σ, the more price-sensitive relative quantities are; the smaller σ, the more rigid the composition.

Contents

1 Introduction

The Constant Elasticity of Substitution (CES) function is arguably the most versatile tool in modern economic modeling. Whether in consumer theory (utility), production theory (technology), or international trade (aggregation of varieties), the CES form allows economists to move beyond the restrictive assumptions of Cobb-Douglas (where expenditure shares are fixed) and Leontief (where substitution is impossible).

The general discrete form for aggregating n inputs $(x_1, \ldots, x_n)$ into an aggregate Y is given by:

$$Y = A \left[\sum_{i=1}^{n} \alpha_i x_i^{\rho} \right]^{\frac{1}{\rho}} \tag{1}$$

Where:

- Y is the aggregate output or utility.
- A is a total factor productivity or utility shifter.
- α_i are distribution parameters (often summing to 1).
- ρ is the substitution parameter, where $\rho \leq 1$.

2 The Elasticity of Substitution (σ)

2.1 Defining the Relationship

The parameter ρ is mathematical, but the economic behavior is governed by σ (Sigma), the elasticity of substitution. σ measures the percentage change in the ratio of two inputs used in response to a percentage change in their marginal rate of substitution (or relative prices).

The mapping between the mathematical parameter ρ and the economic parameter σ is:

$$\sigma = \frac{1}{1-\rho} \quad \Longleftrightarrow \quad \rho = \frac{\sigma - 1}{\sigma} \tag{2}$$

2.2 The Spectrum of Sigma

The value of σ dictates the curvature of the isoquants (indifference curves) and the "flexibility" of the agent.

2.2.1 Case 1: The Cobb-Douglas Limit ($\sigma \to 1, \rho \to 0$)

As $\sigma \to 1$, $\rho \to 0$. The exponent $1/\rho$ approaches infinity while the internal sum approaches a constant. We must use L'Hôpital's rule (on the log of the function) to show that:

$$\lim_{\rho \to 0} \ln(Y) = \sum \alpha_i \ln(x_i) \implies Y = A \prod_{i=1}^{n} x_i^{\alpha_i} \tag{3}$$

Intuition: This represents "Unit Elasticity." If the price of a good rises by 10%, the quantity demanded falls by exactly 10%. Expenditure shares remain constant.

2.2.2 Case 2: The Leontief Limit ($\sigma \to 0, \rho \to -\infty$)

As ρ becomes a large negative number, the smallest x_i in the summation dominates the value of the function (similar to how the smallest link determines the strength of a chain).

$$Y = \min\{x_1, x_2, \dots, x_n\} \tag{4}$$

Intuition: Perfect Complements. The inputs must be used in fixed proportions. The isoquants are L-shaped. Price changes cause *no* substitution effect; agents only adjust due to income effects.

2.2.3 Case 3: The Linear Limit ($\sigma \to \infty, \rho \to 1$)

When $\rho = 1$:

$$Y = \sum \alpha_i x_i \tag{5}$$

Intuition: Perfect Substitutes. The goods are indistinguishable aside from the weights α_i. The isoquants are straight lines. Consumers will corner solution into the cheapest good entirely.

3 Derivation of Demand and the Price Index

This section derives the Marshallian demand function and the Ideal Price Index. This is the "Dual" problem: minimizing expenditure for a target utility level implies the same behavior as maximizing utility for a fixed budget.

3.1 The Optimization Problem

Consider a consumer maximizing CES utility subject to a budget constraint I.

$$\max_{x_i} \quad U = \left[\sum_{i=1}^{n} \alpha_i x_i^{\rho} \right]^{\frac{1}{\rho}}$$
$$\text{s.t.} \quad \sum_{i=1}^{n} p_i x_i = I$$

3.2 Step 1: The Lagrangian

We set up the Lagrangian $\mathcal{L}$:

$$\mathcal{L} = \left[\sum_{i=1}^{n} \alpha_i x_i^{\rho} \right]^{\frac{1}{\rho}} - \lambda \left(\sum_{i=1}^{n} p_i x_i - I \right) \tag{6}$$

3.3 Step 2: First Order Conditions (FOC)

Take the derivative with respect to any generic good x_j:

$$\frac{\partial \mathcal{L}}{\partial x_j} = \frac{1}{\rho} \left[\sum \alpha_i x_i^{\rho} \right]^{\frac{1}{\rho}-1} \cdot \rho \alpha_j x_j^{\rho-1} - \lambda p_j = 0 \tag{7}$$

Note that the term in the brackets is simply U. Also, $\frac{1}{\rho} - 1 = \frac{1-\rho}{\rho}$. Simplifying:

$$U^{1-\rho} \cdot \alpha_j x_j^{\rho-1} = \lambda p_j \tag{8}$$

3.4 Step 3: Relative Demand

The trick to solving CES is to look at the ratio of FOCs for two goods, j and k, to cancel out λ and U:

$$\frac{U^{1-\rho}\alpha_j x_j^{\rho-1}}{U^{1-\rho}\alpha_k x_k^{\rho-1}} = \frac{\lambda p_j}{\lambda p_k} \tag{9}$$

$$\frac{\alpha_j}{\alpha_k}\left(\frac{x_j}{x_k}\right)^{\rho-1} = \frac{p_j}{p_k} \tag{10}$$

Rearranging to solve for x_j in terms of x_k:

$$\frac{x_j}{x_k} = \left(\frac{\alpha_j}{\alpha_k}\frac{p_k}{p_j}\right)^{\frac{1}{1-\rho}} \tag{11}$$

Recall that $\sigma = \frac{1}{1-\rho}$. Thus:

$$x_j = x_k\left(\frac{\alpha_j}{\alpha_k}\right)^{\sigma}\left(\frac{p_j}{p_k}\right)^{-\sigma} \tag{12}$$

Economic Insight: This equation defines the relative demand. The ratio of quantities depends on the ratio of prices raised to the power of $-\sigma$. If σ is high, a small price increase in j leads to a massive drop in relative quantity x_j.

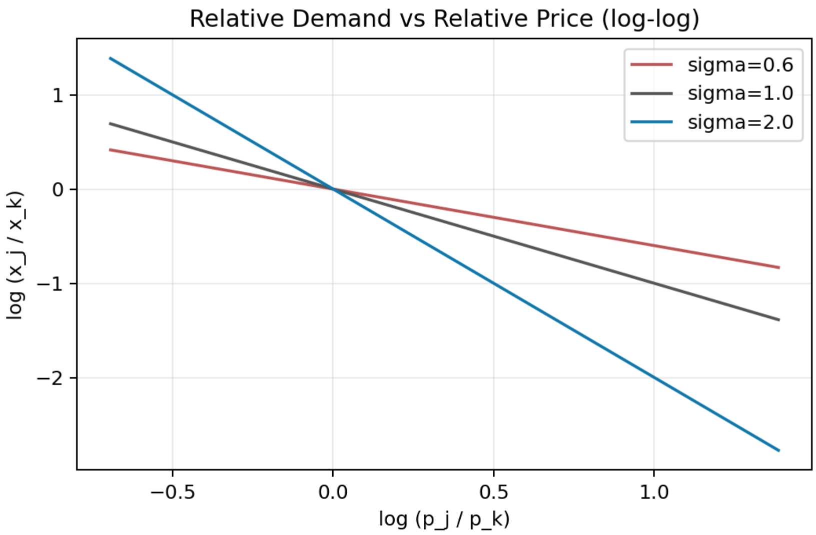

Figure 1: Log-log relative demand vs relative price: the slope of $\log(x_j/x_k)$ against $\log(p_j/p_k)$ equals $-\sigma$.

3.5 Step 4: The Ideal Price Index

To find the absolute demand, we substitute the expression for x_j back into the budget constraint. First, a definition. We define the **Ideal Price Index** P such that $I = P \cdot U$. P represents the minimum cost to buy one unit of utility.

The mathematical derivation (omitted for brevity, involves substituting the relative demand into the definition of U) leads to the dual CES aggregator:

$$P = \left[\sum_{i=1}^{n} \alpha_i^{\sigma} p_i^{1-\sigma} \right]^{\frac{1}{1-\sigma}} \tag{13}$$

Note: If weights α_i are normalized differently, this formula varies slightly, but the exponent $1/(1-\sigma)$ is universal.

3.6 Step 5: Final Demand Function

We can now write the demand for good j simply as:

$$x_j = \alpha_j^{\sigma} \left(\frac{p_j}{P} \right)^{-\sigma} \frac{I}{P} \quad \text{or} \quad x_j = \alpha_j^{\sigma} \left(\frac{p_j}{P} \right)^{-\sigma} Y \tag{14}$$

Where Y is total real output (or utility).

Interpretation:

- Demand depends on aggregate income Y.
- Demand depends on the **relative price** of the good compared to the aggregate price index (p_j/P).
- The sensitivity to this relative price is exactly σ.

4 Economic Intuition: Why σ Matters

The Elasticity of Substitution is not just a math artifact; it tells a story about the market structure.

4.1 Expenditure Shares

Let $E_j = p_j x_j$ be the expenditure on good j. Using the demand function:

$$E_j = p_j \cdot \left[\alpha_j^\sigma p_j^{-\sigma} P^\sigma Y \right] = \alpha_j^\sigma p_j^{1-\sigma} P^\sigma Y \tag{15}$$

How does expenditure change when price p_j rises? It depends on the exponent $(1 - \sigma)$.

1. **Substitutes ($\sigma > 1$):** The exponent $(1 - \sigma)$ is negative.
   - If $p_j \uparrow$, then Expenditure $\downarrow$.
   - **Intuition:** The quantity drops so much that you actually spend *less* total money on the good, even though it costs more. You abandon the product.
2. **Cobb-Douglas ($\sigma = 1$):** The exponent is 0.
   - If $p_j \uparrow$, Expenditure is constant.
   - **Intuition:** The price increase is perfectly offset by the quantity decrease.
3. **Complements ($\sigma < 1$):** The exponent $(1 - \sigma)$ is positive.
   - If $p_j \uparrow$, then Expenditure $\uparrow$.
   - **Intuition:** You are "held hostage" by the good. You cannot easily switch away, so you are forced to pay the higher price, increasing your total spend on it (e.g., Oil, Healthcare).

4.2 Graphical Intuition

Think in terms of isoquants (or indifference curves) in (x_j, x_k) space:

- When σ is **high**, isoquants are flatter. A small price change induces a large movement along the isoquant: agents swap between inputs easily.

Figure 2: Expenditure E_j vs price p_j for different σ. When $\sigma > 1$, expenditure falls as price rises; when $\sigma = 1$, expenditure stays constant; when $\sigma < 1$, expenditure rises with price.

- When σ is **low**, isoquants are kinked and steep. Price changes barely move the bundle: agents are stuck with near-fixed proportions.
- Cobb–Douglas ($\sigma = 1$) sits in the middle: curvature is moderate; substitution exactly offsets price changes in expenditure shares.

4.3 Numerical Examples

extbfExample A (Consumer choice). Two goods j and k with prices $(p_j, p_k) = (2, 1)$ and $\alpha_j = \alpha_k = 1/2$.

- If $\sigma = 2$, then relative demand is $\frac{x_j}{x_k} = \left(\frac{1/2}{1/2}\frac{1}{2}\right)^2 = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$. A price doubling cuts x_j sharply.
- If $\sigma = 0.5$, then $\frac{x_j}{x_k} = \left(\frac{1}{2}\right)^{0.5} \approx 0.707$. Quantity falls little; complements blunt substitution.

extbfExample B (Production). Firm uses capital K and labor L with CES technology and faces a wage increase.

- High σ (automation-ready): wage hikes lead the firm to substitute towards K aggressively.
- Low σ (task complementarity): wage hikes raise costs with limited scope for automation.

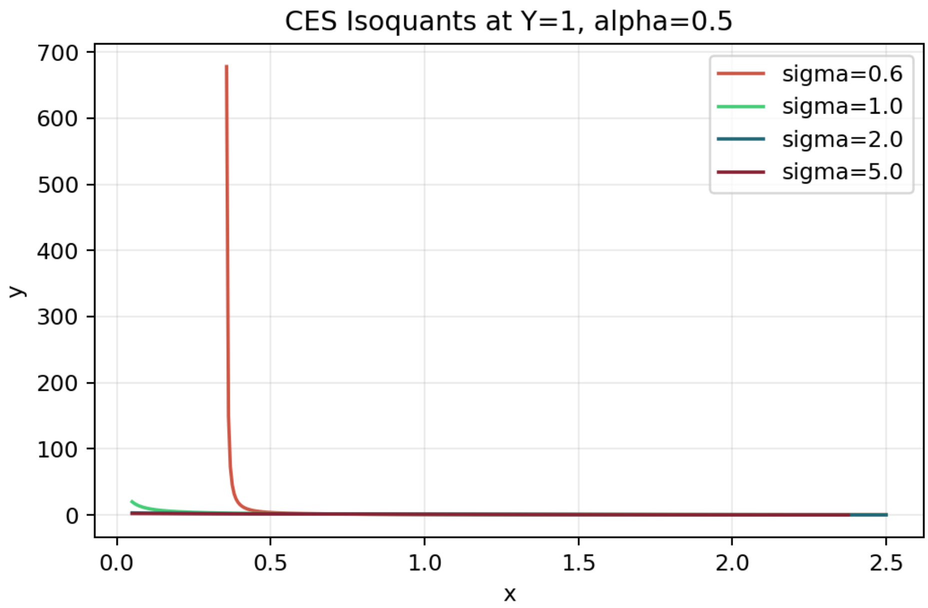

Figure 3: CES isoquants ($Y = 1,\ \alpha = 0.5$) under different σ. High σ yields flatter curves; low σ yields steeper/kinked shapes.

4.4 Interpretation in Markets

- **Retail products**: Many close substitutes (high σ), prices matter a lot; brands compete on price and features.
- **Essential inputs**: Energy, healthcare (low σ), prices matter less for quantities; expenditure rises with price.
- **Digital vs. physical**: In software, features can be replicated cheaply (higher σ); in specialized capital goods, complementarity lowers σ.

5 Key Properties of CES

5.1 Homogeneity and Returns to Scale

The standard CES function is **Homogeneous of Degree 1**. Proof: Let us scale all inputs by a factor λ.

$$\begin{aligned} Y(\lambda x_1, \dots, \lambda x_n) &= \left[\sum \alpha_i (\lambda x_i)^\rho\right]^{1/\rho} \\ &= \left[\sum \alpha_i \lambda^\rho x_i^\rho\right]^{1/\rho} \\ &= \left[\lambda^\rho \sum \alpha_i x_i^\rho\right]^{1/\rho} \\ &= (\lambda^\rho)^{1/\rho} \left[\sum \alpha_i x_i^\rho\right]^{1/\rho} \\ &= \lambda Y(x_1, \dots, x_n) \end{aligned}$$

This implies **Constant Returns to Scale (CRS)**. If you double all inputs, you double the output. This is crucial for general equilibrium models because it allows for zero-profit conditions to hold in perfect competition.

5.2 Inada Conditions (And Failure Thereof)

Standard neoclassical growth models assume Inada conditions:

$$\lim_{x \to 0} \mathrm{MP}_x = \infty, \quad \lim_{x \to \infty} \mathrm{MP}_x = 0 \tag{16}$$

CES functions with $\sigma > 1$ **violate** the first condition.

$$\mathrm{MP}_j = \alpha_j \left(\frac{Y}{x_j}\right)^{1-\rho} \tag{17}$$

If inputs are substitutes, $\rho > 0$, so $1 - \rho < 1$. The marginal product is bounded at zero. **Implication:** You can produce output using *only* capital or *only* labor. In a Cobb-Douglas world, if Labor is 0, Output is 0. In a CES ($\sigma > 1$) world, automation can fully replace labor.

5.3 Calibration Tips for Empiricists

- **Recovering σ from data**: Use price and quantity variation across goods or time; regress log relative quantities on log relative prices to estimate $-\sigma$.
- **Normalization**: Ensure $\sum_i \alpha_i = 1$ or scale A accordingly; identifiability improves with consistent normalizations.
- **Units and scaling**: CES is homogeneous of degree 1; scaling inputs rescales output linearly. Keep units consistent to avoid spurious elasticities.
- **Price index**: Use the dual index $P = \left[\sum_i \alpha_i^\sigma p_i^{1-\sigma}\right]^{\frac{1}{1-\sigma}}$ when computing real quantities.

5.4 Common Pitfalls

- **Misinterpreting ρ**: ρ is a mathematical convenience; *economic* elasticity is $\sigma = \frac{1}{1-\rho}$.
- **Extreme σ**: As $\sigma \to \infty$, corner solutions become common; as $\sigma \to 0$, numerical optimization can be unstable.
- **Ignoring budget consistency**: Always check $\sum_i p_i x_i = I$ with the derived demands; errors often come from index mismeasurement.
- **Mixing aggregators**: Do not combine Leontief at one stage with CES at another without clear economic justification; implied elasticities differ.

6 The Dixit-Stiglitz Aggregator

In modern macroeconomics (e.g., New Keynesian models) and International Trade (e.g., Melitz 2003), we assume that the number of goods is not a discrete list $\{1, ..., n\}$, but a **continuum** of varieties indexed by $i \in [0, N]$. This requires us to adapt the CES function from a summation form to an integral form.

6.1 Model Setup

Let C be the aggregate consumption bundle (often called Y in production contexts). The aggregator is defined as:

$$C = \left[\int_0^N c(i)^{\frac{\sigma-1}{\sigma}} \, di \right]^{\frac{\sigma}{\sigma-1}} \tag{18}$$

Where:

- $i \in [0, N]$ identifies a specific variety.
- $c(i)$ is the quantity consumed of variety i.
- $\sigma > 1$ is the elasticity of substitution.
- For simplified notation, let $\rho = \frac{\sigma-1}{\sigma}$. Note that $0 < \rho < 1$.

The consumer maximizes C subject to the budget constraint:

$$\int_0^N p(i)c(i) \, di = E \tag{19}$$

Where E is total nominal expenditure and $p(i)$ is the price of variety i.

6.2 Mathematical Technique: Differentiating the Integral

The core difficulty in solving this model is taking the derivative of an integral with respect to a specific function value $c(j)$. This is formally a problem in the *Calculus of Variations*, but economists use a specific heuristic.

6.2.1 The Lagrangian

We set up the Lagrangian $\mathcal{L}$:

$$\mathcal{L} = \left[\int_0^N c(i)^\rho \, di\right]^{\frac{1}{\rho}} - \lambda\left(\int_0^N p(i)c(i)\, di - E\right) \tag{20}$$

6.2.2 The Derivative Step (Crucial)

We want to find the First Order Condition (FOC) with respect to the consumption of a specific variety j, denoted as $c(j)$.

Mathematical Intuition: How do we compute $\frac{\partial}{\partial c(j)} \int_0^N c(i)^\rho di$?

1. Think of the integral as an infinite sum: $\lim_{N\to\infty} \sum c(i)^\rho \Delta i$.
2. When we differentiate with respect to $c(j)$, the derivative of $c(i)$ for all $i \neq j$ is **zero**.
3. The only term that survives is the term containing $c(j)$.
4. Therefore, the derivative ”picks out” the specific function at j and applies the power rule.

Applying the Chain Rule to the Utility term:

$$\frac{\partial U}{\partial c(j)} = \underbrace{\frac{1}{\rho}\left[\int_0^N c(i)^\rho \, di\right]^{\frac{1}{\rho}-1}}_{\text{Outer Derivative}} \times \underbrace{\frac{\partial}{\partial c(j)}\left(\int_0^N c(i)^\rho \, di\right)}_{\text{Inner Derivative}} \tag{21}$$

Based on the logic above, the Inner Derivative is simply $\rho c(j)^{\rho-1}$. *(Note: Strictly speaking, in measure theory, changing a function at a single point j doesn't change the integral. However, in economic modeling, we treat $c(j)$ as having a non-zero measure or as a discrete approximation. The result holds.)*

6.2.3 First Order Conditions

Substituting the derivatives back into the FOC equation:

$$\frac{1}{\rho} C^{1-\rho} \cdot \rho c(j)^{\rho-1} - \lambda p(j) = 0 \tag{22}$$

The ρ terms cancel out. We rearrange to isolate $c(j)$:

$$C^{1-\rho} c(j)^{\rho-1} = \lambda p(j) \tag{23}$$

$$c(j)^{\rho-1} = \lambda p(j) C^{\rho-1} \tag{24}$$

Raise both sides to the power of $\frac{1}{\rho-1}$ (which equals $-\sigma$):

$$c(j) = (\lambda p(j))^{-\sigma} (C^{\rho-1})^{-\sigma} \tag{25}$$

Recall that $\rho - 1 = -\frac{1}{\sigma}$, so $(\rho - 1)(-\sigma) = 1$. This simplifies to:

$$c(j) = \lambda^{-\sigma} p(j)^{-\sigma} C \tag{26}$$

6.3 Derivation of the Price Index (The Aggregator)

We still have the unknown Lagrange multiplier λ. To eliminate it, we use the "Dual" approach. We substitute the demand function (26) back into the budget constraint.

$$\int_0^N p(j)c(j)\,dj = E \tag{27}$$

Substitute $c(j)$:

$$\int_0^N p(j)\left[\lambda^{-\sigma} p(j)^{-\sigma} C\right]\,dj = E \tag{28}$$

Pull constants $\lambda^{-\sigma}$ and C outside the integral:

$$\lambda^{-\sigma} C \int_0^N p(j)^{1-\sigma}\,dj = E \tag{29}$$

6.3.1 Defining the Price Index P

We *define* the ideal aggregate Price Index P such that $PC = E$ (Total Expenditure = Price × Quantity). Comparing $PC = E$ with the equation above, we see that:

$$P = \lambda^{-\sigma} \int_0^N p(j)^{1-\sigma}\,dj \tag{30}$$

This seems circular because of λ. Let's solve for λ directly from the utility function definition instead (Alternative method).

Standard Result: The mathematically consistent definition that satisfies $P \cdot C = E$ is:

$$P = \left[\int_0^N p(i)^{1-\sigma}\,di\right]^{\frac{1}{1-\sigma}} \tag{31}$$

6.3.2 Final Marshallian Demand

Now we can write the demand for a specific variety j in its most famous form. From $c(j) \propto p(j)^{-\sigma}$, and knowing that the sum of expenditures must equal E:

$$c(j) = \left(\frac{p(j)}{P}\right)^{-\sigma} \frac{E}{P} = \left(\frac{p(j)}{P}\right)^{-\sigma} C \tag{32}$$

This equation is the foundation of the "Gravity Model" in trade. It states that trade flow (consumption) depends on the size of the market (C or E) and the relative price (friction) raised to $-\sigma$.

6.4 Trade Intuition and Gravity

- **Market size**: Larger E or C scales all bilateral flows up proportionally.
- **Trade costs**: Iceberg costs raise effective prices $p(j)$; higher costs reduce flows as $(p/P)^{-\sigma}$.
- **Elasticity as sensitivity**: A higher σ makes trade more responsive to cost changes; gravity regressions often recover σ.

6.5 Simple Symmetry Exercise

Suppose all varieties share price p and produced quantities are symmetric. Then with N varieties,

$$C = N^{\frac{1}{\sigma-1}} \cdot \frac{E}{p}. \tag{33}$$

extbfIntuition: Even if total real expenditure E/p is fixed, increasing N raises utility due to love-for-variety. Gains are stronger when σ is closer to 1 (differentiated goods), and vanish as $\sigma \to \infty$ (perfect substitutes).

6.6 Property: Love for Variety

Why do trade economists love this integral form? It exhibits "Love for Variety." Assume symmetry: all varieties have the same price p and are consumed in equal amounts c. Let there be N varieties available.

6.6.1 Consumption per variety

Budget constraint: $N \cdot p \cdot c = E \implies c = \frac{E}{Np}$.

6.6.2 Aggregate Utility Calculation

Substitute this into the aggregator:

$$C = \left[\int_0^N \left(\frac{E}{Np} \right)^{\frac{\sigma-1}{\sigma}} di \right]^{\frac{\sigma}{\sigma-1}} \tag{34}$$

$$= \left[N \cdot \left(\frac{E}{Np} \right)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \tag{35}$$

$$= N^{\frac{\sigma}{\sigma-1}} \cdot \frac{E}{Np} \tag{36}$$

Group the N terms: $N^{\frac{\sigma}{\sigma-1}} \cdot N^{-1} = N^{\frac{\sigma}{\sigma-1} - \frac{\sigma-1}{\sigma-1}} = N^{\frac{1}{\sigma-1}}$.

So the final utility is:

$$C = N^{\frac{1}{\sigma-1}} \left(\frac{E}{p} \right) \tag{37}$$

6.6.3 Interpretation

Term $\left(\frac{E}{p}\right)$ is the total physical quantity of goods (Total Real Expenditure). Term $N^{\frac{1}{\sigma-1}}$ is the **Variety Multiplier**.

Since $\sigma > 1$, the exponent $\frac{1}{\sigma-1} > 0$.

- As N (number of varieties) increases, Aggregate Utility C increases, *even if total physical consumption remains constant*.
- **Example:** If $\sigma = 2$, then exponent is 1. Doubling variety doubles utility.
- **Example:** If $\sigma \to \infty$ (Perfect Substitutes), exponent $\to 0$. $N^0 = 1$. Variety doesn't matter, only total quantity matters.

7 Conclusion

The CES function is the workhorse of economic modeling because it strikes a balance between tractability (it is easy to differentiate and integrate) and flexibility (via σ). By understanding the derivation of its demand functions and the implications of σ on expenditure shares, economists can model complex phenomena ranging from automation replacing labor to the welfare gains of global trade variety.