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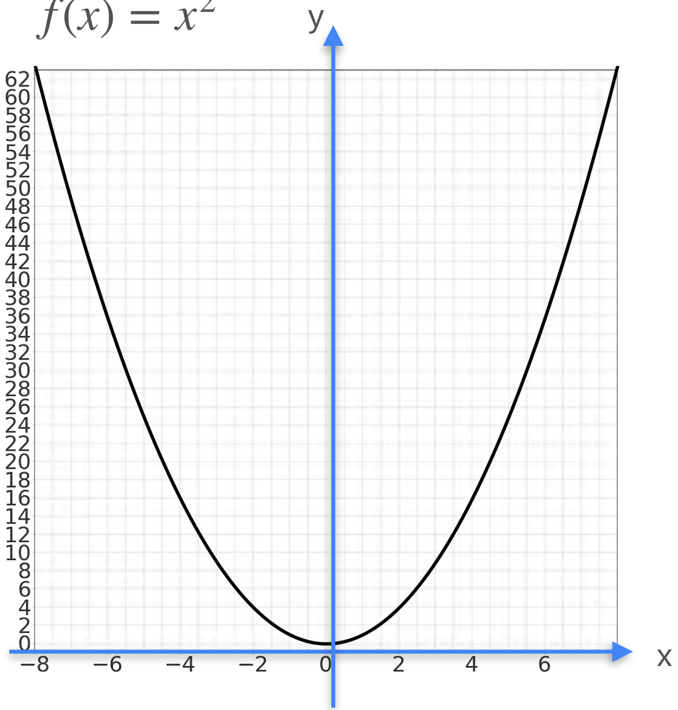
Gradients and Gradient Descent

Tangent planes

Functions of Two Variables

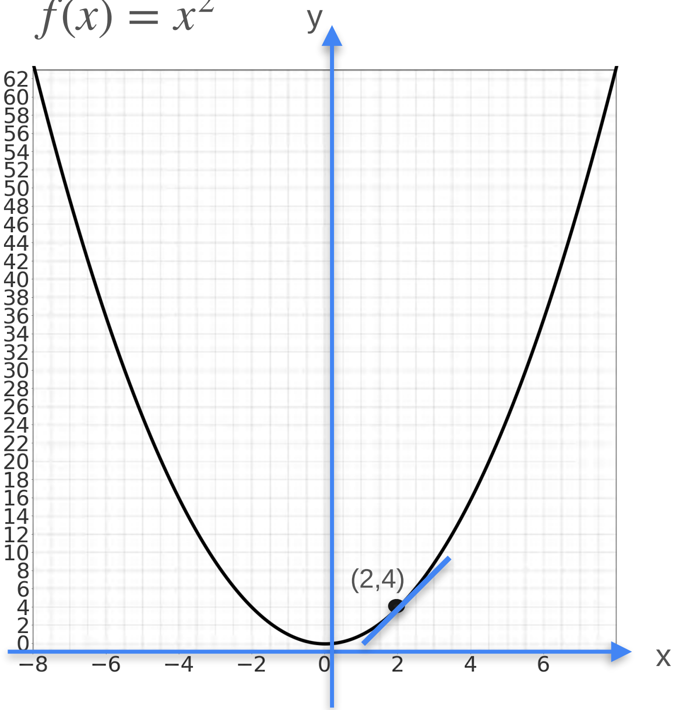
Functions of Two Variables

$f(x) = x^2$



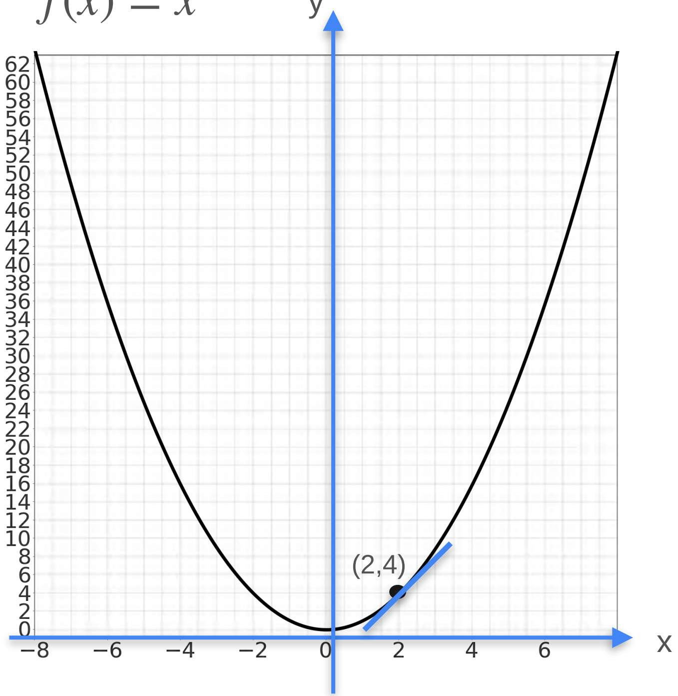
Functions of Two Variables

$f(x) = x^2$

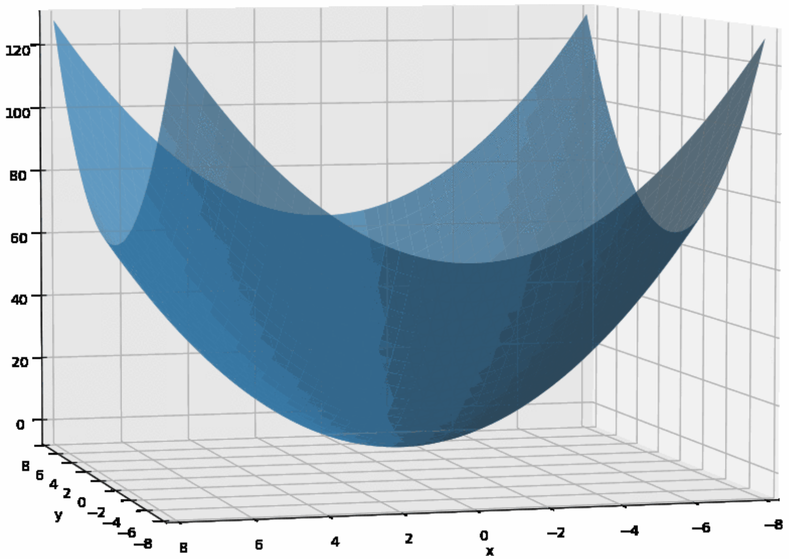


Functions of Two Variables

$f(x) = x^2$

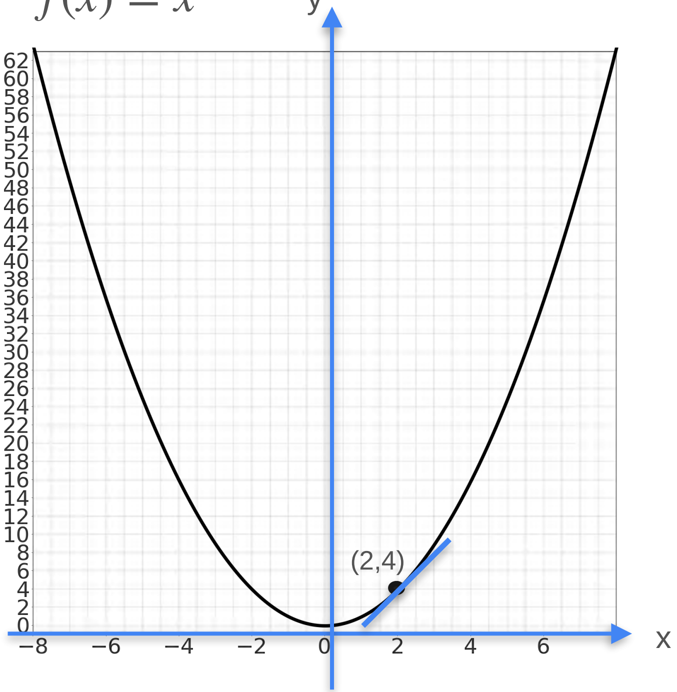


$f(x, y) = x^2 + y^2$

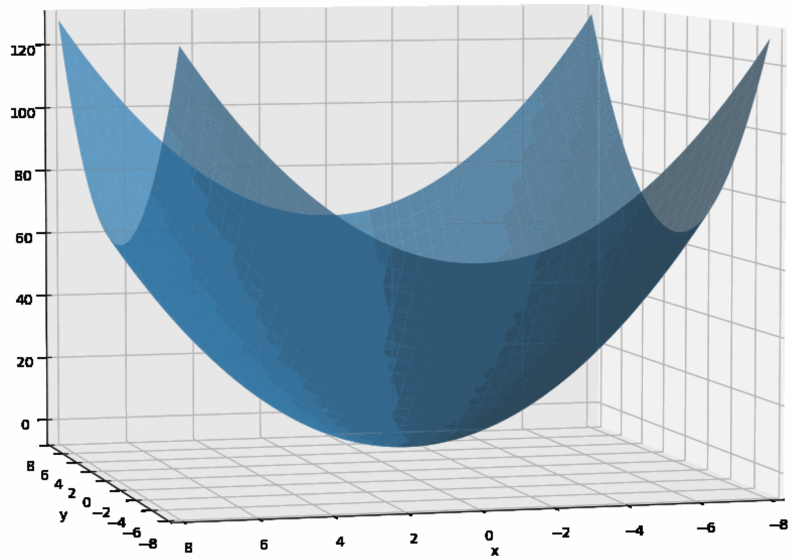


Functions of Two Variables

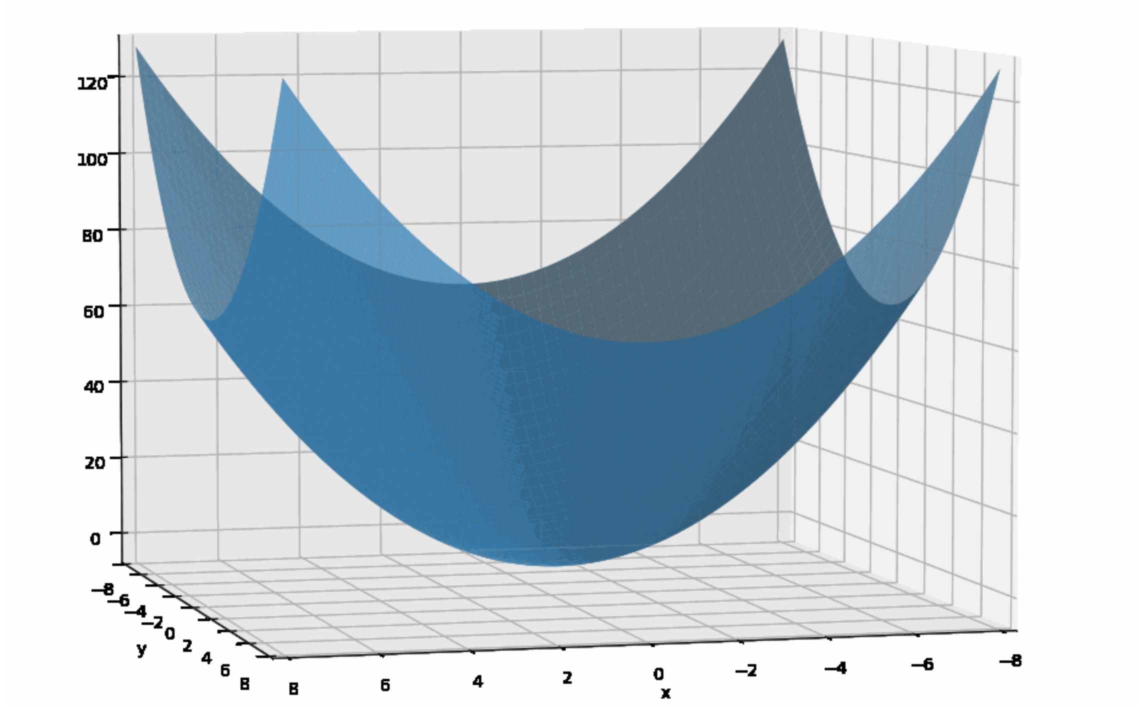
$f(x) = x^2$



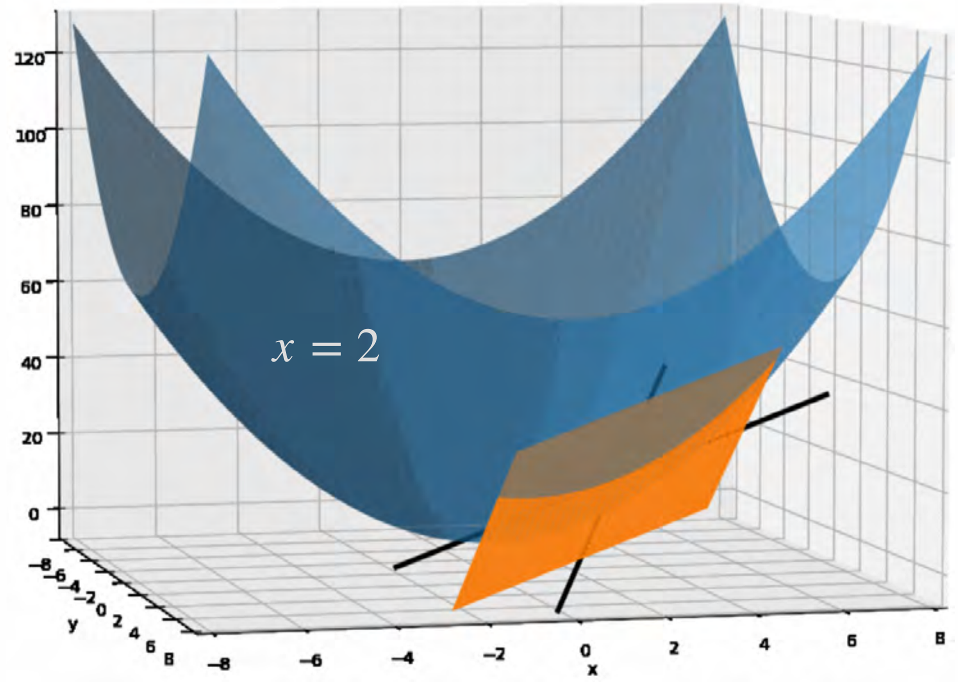
$f(x, y) = x^2 + y^2$



Finding the Tangent Plane



Finding the Tangent Plane



Fix y=4 $f(x,4) = x^2 + 4^2$

$$\frac{d}{dx}\left(f(x,4)\right) = 2x$$

Fix x=2 $f(2,y) = 2^2 + y^2$

$$\frac{d}{dy}\left(f(2,y)\right) = 2y$$

The tangent plane contains both tangent lines.

Video 2: Introduction to Partial Derivatives

Example with the parabola, show tangent plane and slices

Gradients and Gradient Descent

Partial derivatives - Part 1

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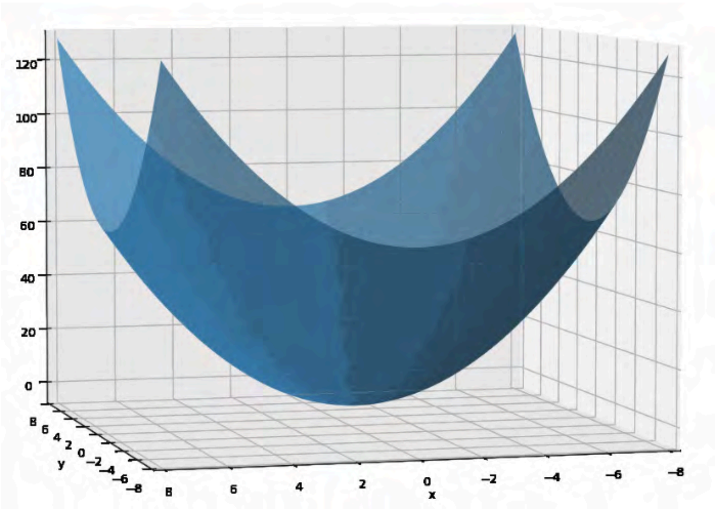
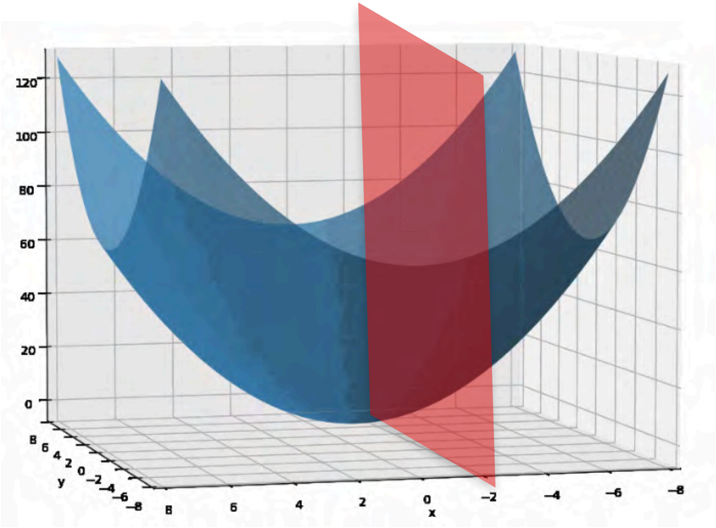
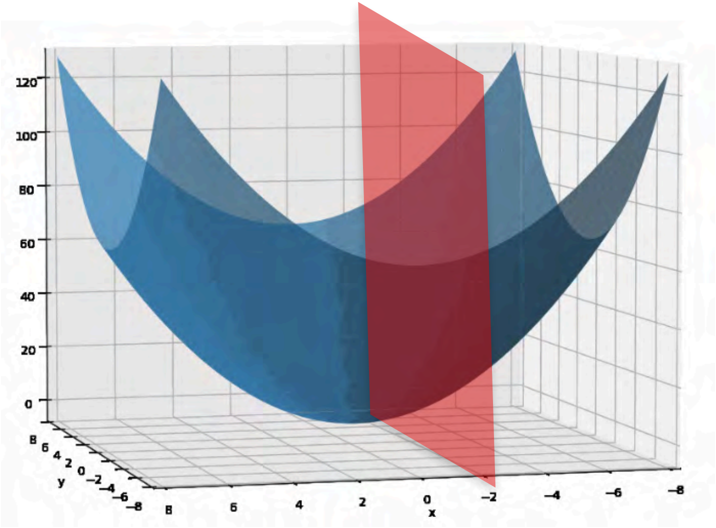
Slicing the Space

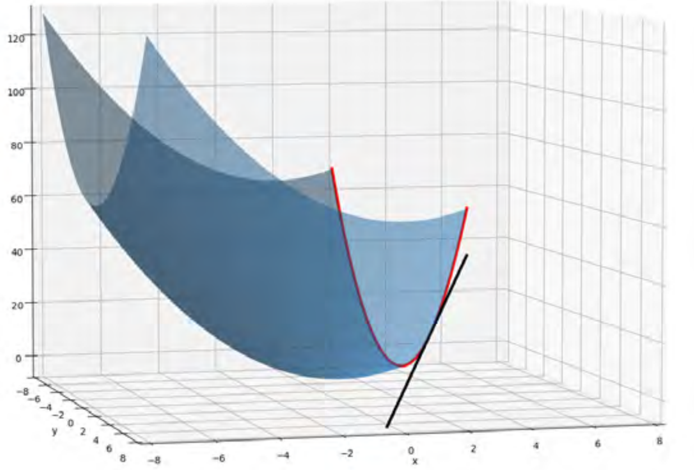


Slicing the Space

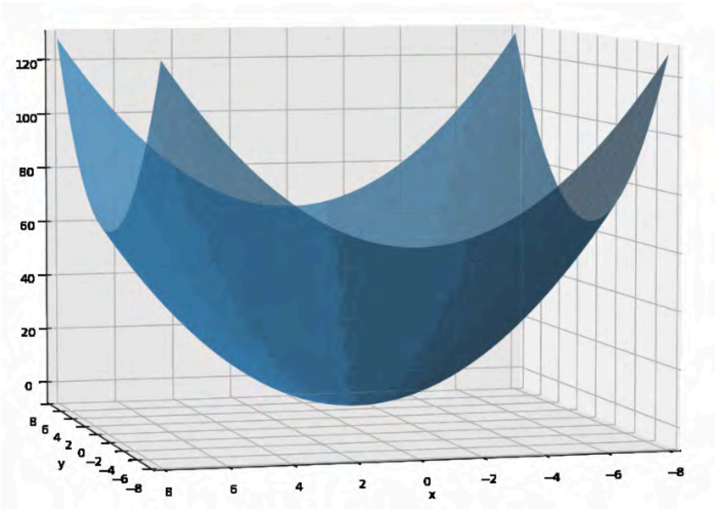


Slicing the Space

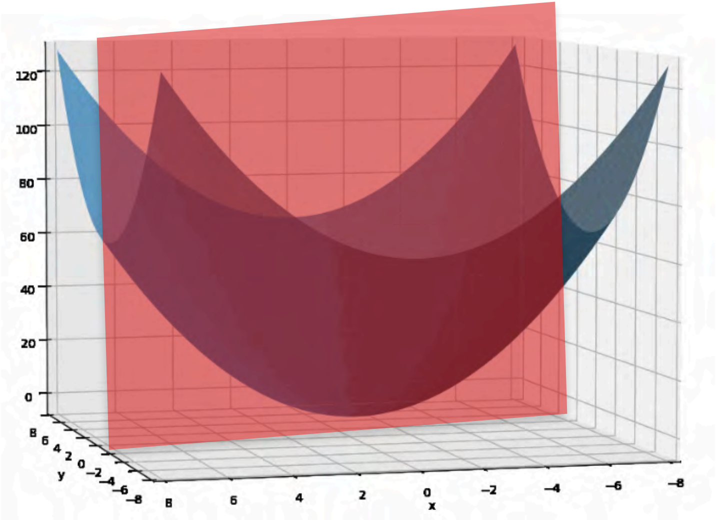




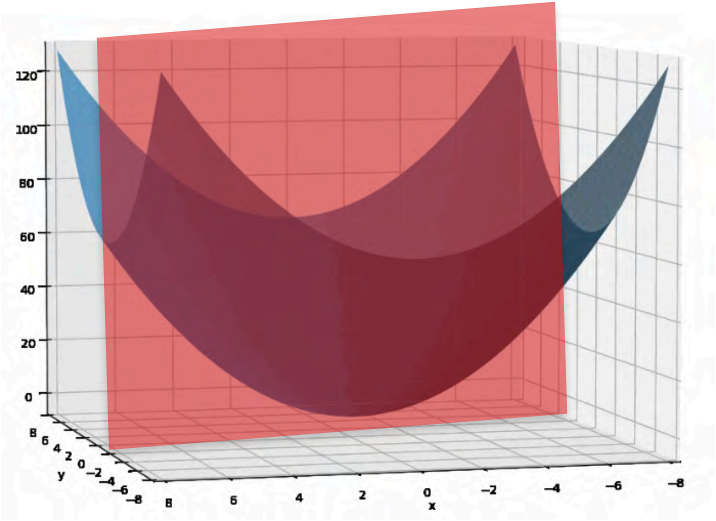
Slicing the Space

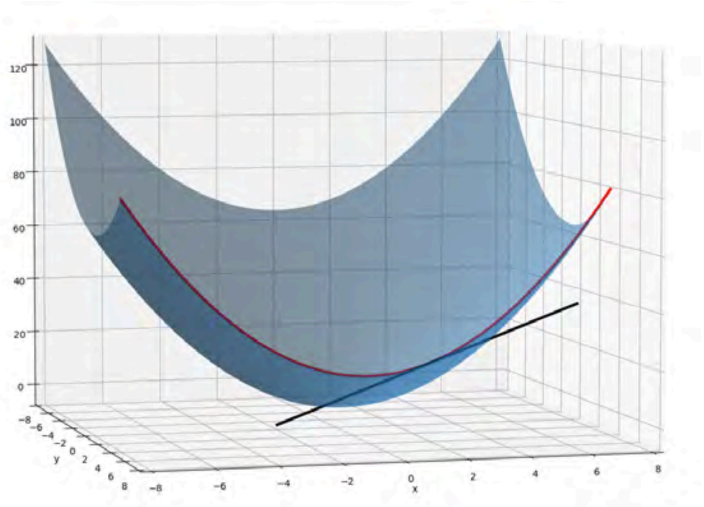


Slicing the Space



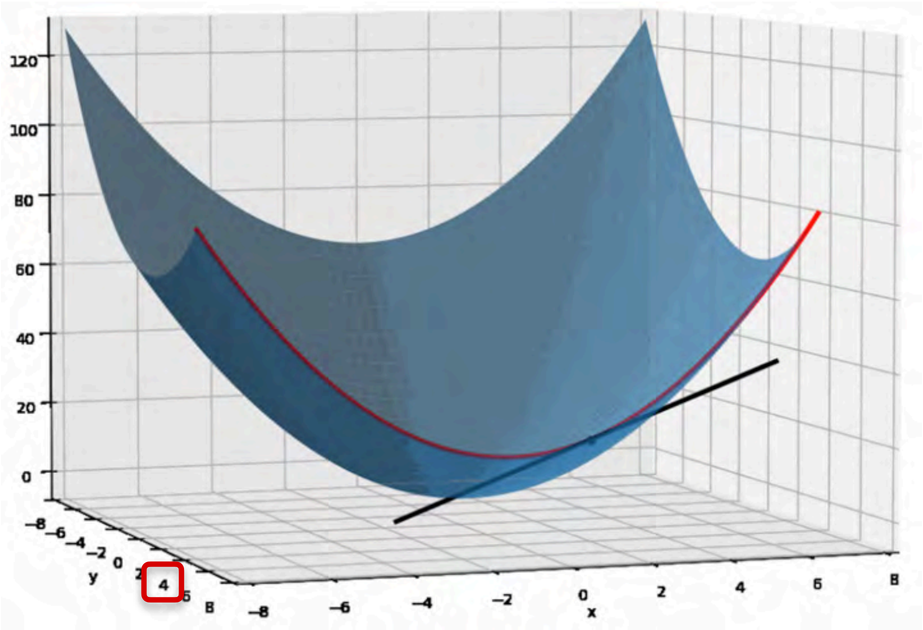
Slicing the Space





Partial Derivatives

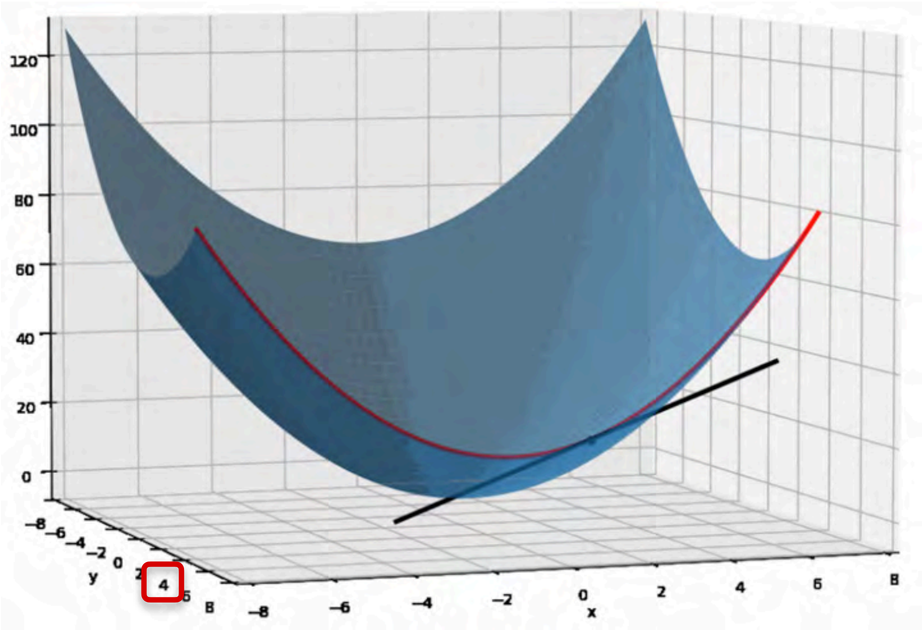
$$f(x, y) = x^2 + y^2$$



Partial Derivatives

$$f(x, y) = x^2 + y^2$$

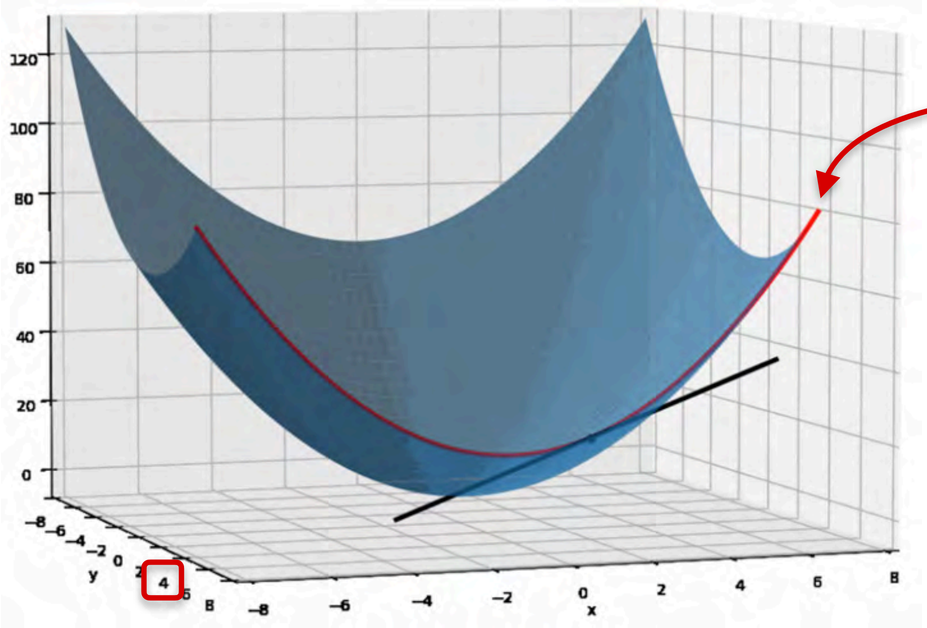
Treat y as a constant



Partial Derivatives

$$f(x, y) = x^2 + y^2$$

Treat y as a constant

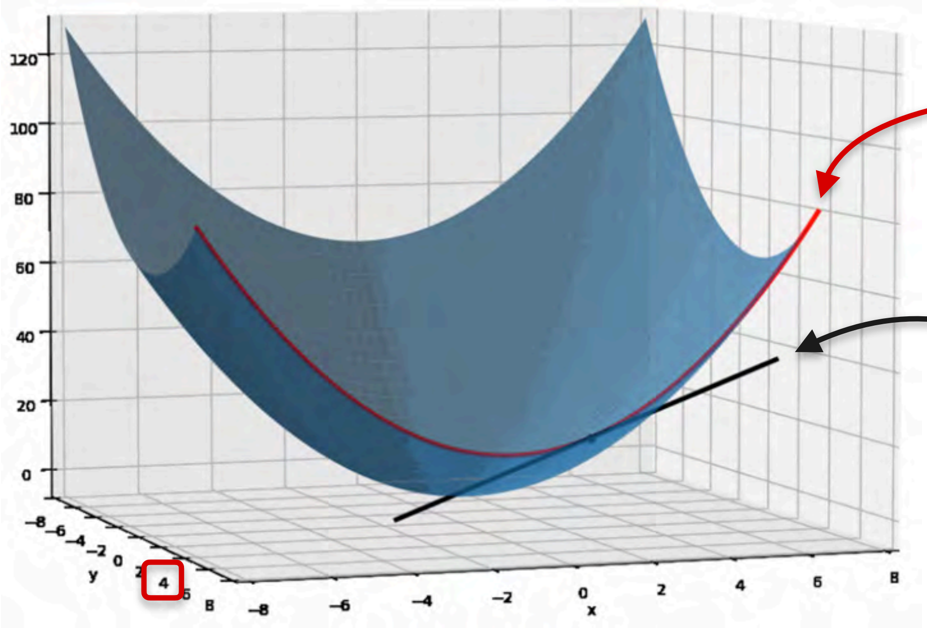


Function of one variable x

Partial Derivatives

$$f(x, y) = x^2 + y^2$$

Treat y as a constant



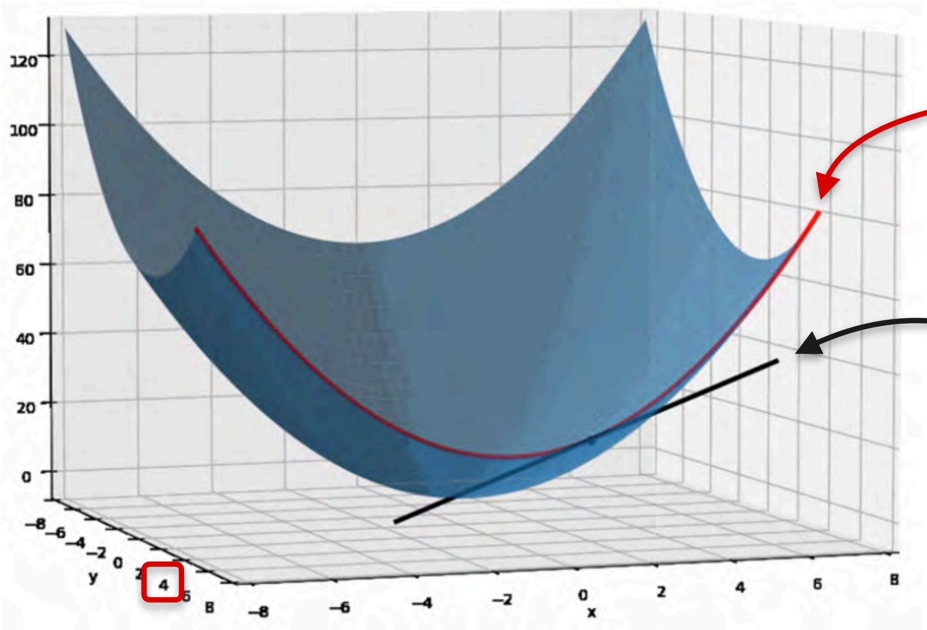
Function of one variable x

Slope?

Partial Derivatives

$$f(x, y) = x^2 + y^2$$

Treat y as a constant



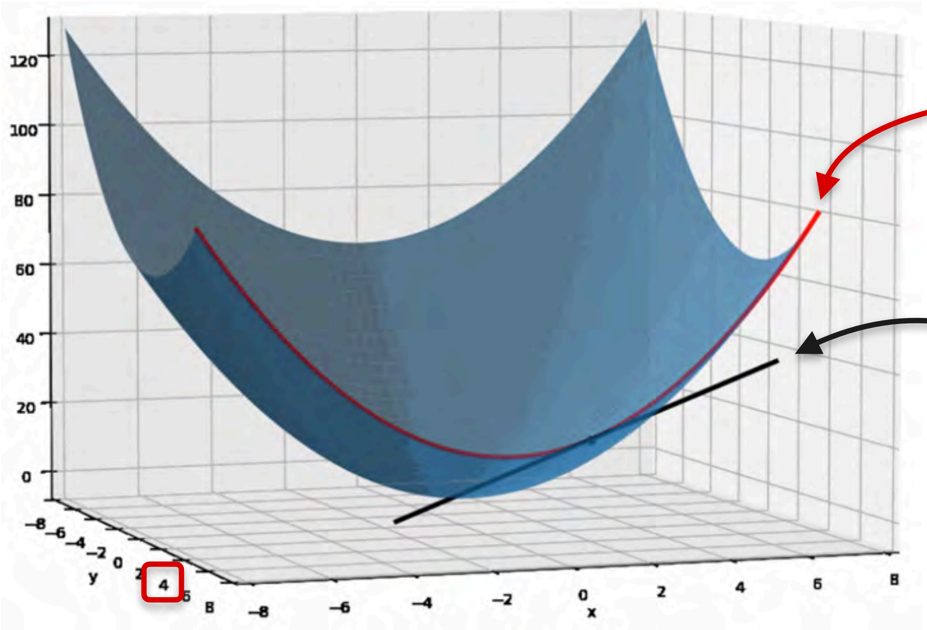
Function of one variable x

Slope?

Partial derivative

Partial Derivatives

$$f(x, y) = x^2 + y^2$$



Treat y as a constant

Function of one variable x

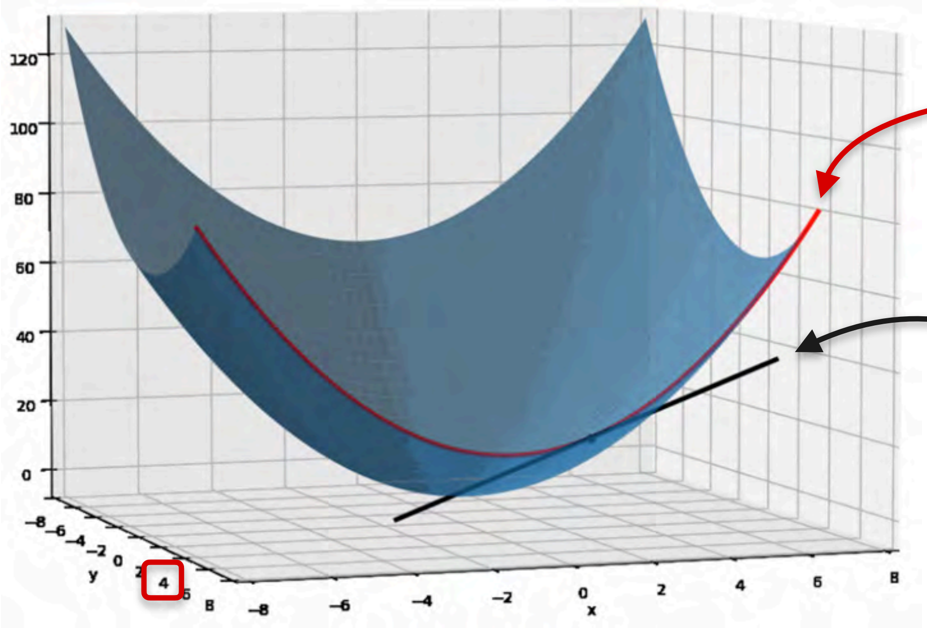
$$f(x, y) = x^2 + y^2$$

Slope?

Partial derivative

Partial Derivatives

$$f(x, y) = x^2 + y^2$$



Treat y as a constant

Function of one variable x

Constant

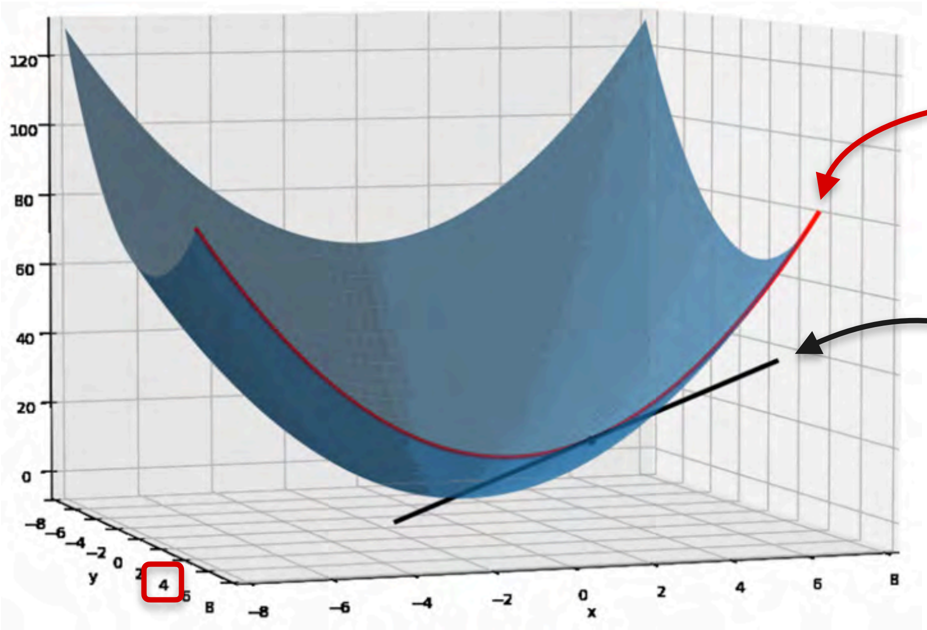
$$f(x, y) = x^2 + \boxed{y^2}$$

Slope?

Partial derivative

Partial Derivatives

$$f(x, y) = x^2 + y^2$$



Treat y as a constant

Function of one variable x

Slope?

Partial derivative

Constant

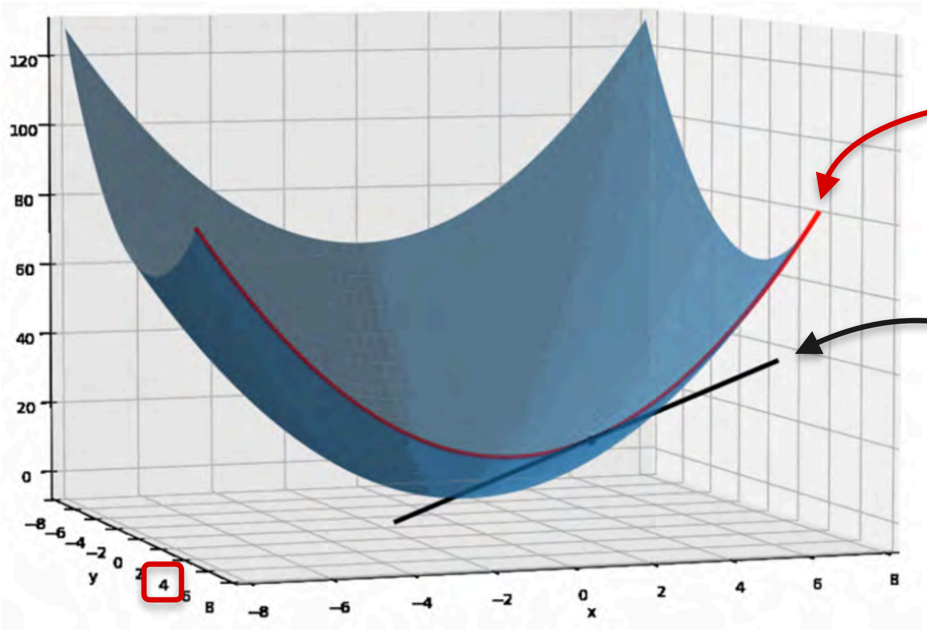
$$f(x, y) = x^2 + \boxed{y^2}$$

$$\frac{\partial f}{\partial x} = 2x + 0$$

Partial Derivatives

$$f(x, y) = x^2 + y^2$$

Treat y as a constant



Function of one variable x

Constant

$$f(x, y) = x^2 + \boxed{y^2}$$

$$\frac{\partial f}{\partial x} = 2x + \boxed{0}$$

Derivative = 0

Slope?

Partial derivative

Partial Derivatives

Partial Derivatives

$$x^2 + y^2$$

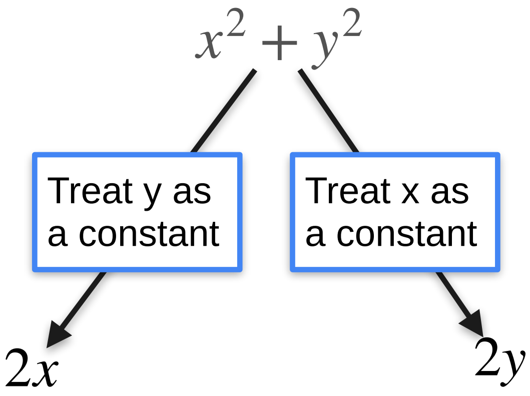
Partial Derivatives

$$x^2 + y^2$$

Treat y as a constant

$$2x$$

Partial Derivatives



Partial Derivatives

$$x^2 + y^2$$

$$f(x, y)$$

Treat y as a constant

$$2x$$

Treat x as a constant

$$2y$$

Partial Derivatives

$$x^2 + y^2$$

Treat y as a constant

Treat x as a constant

$$2x \qquad 2y$$

$$f(x, y)$$

$$f_x = \frac{\partial f}{\partial x}$$

Partial Derivatives

$$x^2 + y^2$$

Treat y as a constant → $2x$

Treat x as a constant → $2y$

$$f(x, y)$$

$$f_x = \frac{\partial f}{\partial x} \qquad f_y = \frac{\partial f}{\partial y}$$

Partial Derivatives

$$x^2 + y^2$$

Treat y as a constant

Treat x as a constant

$2x$ $\qquad$ $2y$

$$f(x, y)$$

$$f_x = \frac{\partial f}{\partial x} \qquad f_y = \frac{\partial f}{\partial y}$$

Partial derivative of f with respect to x

Partial Derivatives

$$x^2 + y^2$$

Treat y as a constant

Treat x as a constant

$$2x \qquad 2y$$

$$f(x, y)$$

$$f_x = \frac{\partial f}{\partial x}$$

Partial derivative of f with respect to x

$$f_y = \frac{\partial f}{\partial y}$$

Partial derivative of f with respect to x

Intro To Partial Derivatives

Intro To Partial Derivatives

$$f(x, y) = x^2 + y^2$$

Intro To Partial Derivatives

$$f(x, y) = x^2 + y^2$$

TASK

Intro To Partial Derivatives

$$f(x, y) = x^2 + y^2$$

TASK

Find partial derivatives of f with respect to x and y

Intro To Partial Derivatives

$$f(x, y) = x^2 + y^2$$

$$\frac{\partial f}{\partial x} =$$

TASK

Find partial derivatives of f with respect to x and y

Intro To Partial Derivatives

$$f(x, y) = x^2 + y^2$$

$$\frac{\partial f}{\partial x} =$$

$$\frac{\partial f}{\partial y} =$$

TASK

Find partial derivatives of f with respect to x and y

Intro To Partial Derivatives

$$f(x, y) = x^2 + y^2$$

$$\frac{\partial f}{\partial x} =$$

$$\frac{\partial f}{\partial y} =$$

?

TASK

Find partial derivatives of f with respect to x and y

Intro To Partial Derivatives

Partial derivative notation

$$f(x, y) = x^2 + y^2$$

TASK

Find partial derivatives of f with respect to x and y

$$\frac{\partial f}{\partial x} =$$

?

$$\frac{\partial f}{\partial y} =$$

Intro To Partial Derivatives

Partial derivative notation

$$f(x, y) = x^2 + y^2$$

$$\frac{\partial f}{\partial x} =$$

?

$$\frac{\partial f}{\partial y} =$$

TASK

Find partial derivatives of f with respect to x and y

Intro To Partial Derivatives

$$f(x, y) = x^2 + \boxed{y^2}$$

TASK

$$\frac{\partial f}{\partial x} =$$

?

$$\frac{\partial f}{\partial y} =$$

Intro To Partial Derivatives

$$f(x, y) = x^2 + \boxed{y^2}$$

TASK

Find partial derivative of f with respect to x

$$\frac{\partial f}{\partial x} =$$

?

$$\frac{\partial f}{\partial y} =$$

Intro To Partial Derivatives

$$f(x, y) = x^2 + \boxed{y^2}$$

TASK

Find partial derivative of f with respect to x

$$\frac{\partial f}{\partial x} =$$

?

Step 1:

$$\frac{\partial f}{\partial y} =$$

Intro To Partial Derivatives

$$f(x, y) = x^2 + \boxed{y^2}$$

$$\frac{\partial f}{\partial x} =$$

?

$$\frac{\partial f}{\partial y} =$$

TASK

Find partial derivative of f with respect to x

Step 1: Treat all other variables as a constant. In our case y.

Intro To Partial Derivatives

$$f(x, y) = x^2 + \boxed{y^2}$$

$$\frac{\partial f}{\partial x} =$$

?

$$\frac{\partial f}{\partial y} =$$

TASK

Find partial derivative of f with respect to x

Step 1: Treat all other variables as a constant. In our case y.

Step 2:

Intro To Partial Derivatives

$$f(x, y) = x^2 + \boxed{y^2}$$

TASK

Find partial derivative of f with respect to x

$$\frac{\partial f}{\partial x} =$$

$$\frac{\partial f}{\partial y} =$$

?

Step 1: Treat all other variables as a constant. In our case y.

Step 2: Differentiate the function using the normal rules of differentiation.

Intro To Partial Derivatives

$$f(x, y) = x^2 + y^2$$

TASK

Find partial derivative of f with respect to x

Step 1: Treat all other variables as a constant. In our case y.

Step 2: Differentiate the function using the normal rules of differentiation.

Intro To Partial Derivatives

$$f(x, y) = x^2 + y^2$$

TASK

Find partial derivative of f with respect to x

Step 1: Treat all other variables as a constant. In our case y.

Step 2: Differentiate the function using the normal rules of differentiation.

Intro To Partial Derivatives

$$f(x, y) = x^2 + \boxed{y^2}$$

TASK

Find partial derivative of f with respect to x

Step 1: Treat all other variables as a constant. In our case y.

Step 2: Differentiate the function using the normal rules of differentiation.

Intro To Partial Derivatives

$$f(x, y) = x^2 + 1$$

$$f(x, y) = x^2 + \boxed{y^2}$$

TASK

Find partial derivative of f with respect to x

Step 1: Treat all other variables as a constant. In our case y.

Step 2: Differentiate the function using the normal rules of differentiation.

Intro To Partial Derivatives

$$f(x, y) = x^2 + 1$$

$$f(x, y) = x^2 + \boxed{y^2}$$

$$\frac{\partial f}{\partial x} = 2x$$

TASK

Find partial derivative of f with respect to x

Step 1: Treat all other variables as a constant. In our case y.

Step 2: Differentiate the function using the normal rules of differentiation.

Intro To Partial Derivatives

$$f(x, y) = x^2 + y^2$$

$$\frac{\partial f}{\partial x} = 2x$$

$$\frac{\partial f}{\partial y} =$$

TASK

Find partial derivative of f with respect to x

Step 1: Treat all other variables as a constant. In our case x.

Step 2: Differentiate the function using the normal rules of differentiation.

Intro To Partial Derivatives

$$f(x, y) = x^2 + y^2$$

TASK

Find partial derivative of f with respect to x

Step 1: Treat all other variables as a constant. In our case x.

Step 2: Differentiate the function using the normal rules of differentiation.

$$\frac{\partial f}{\partial x} = 2x$$

$$\frac{\partial f}{\partial y} =$$

Intro To Partial Derivatives

$$f(x, y) = \boxed{x^2} + y^2$$

$$\frac{\partial f}{\partial x} = 2x$$

$$\frac{\partial f}{\partial y} =$$

TASK

Find partial derivative of f with respect to x

Step 1: Treat all other variables as a constant. In our case x.

Step 2: Differentiate the function using the normal rules of differentiation.

Intro To Partial Derivatives

$$f(x, y) = 1 + y^2$$

$$f(x, y) = \boxed{x^2} + y^2$$

$$\frac{\partial f}{\partial x} = 2x$$

$$\frac{\partial f}{\partial y} =$$

TASK

Find partial derivative of f with respect to x

Step 1: Treat all other variables as a constant. In our case x.

Step 2: Differentiate the function using the normal rules of differentiation.

Intro To Partial Derivatives

$$f(x, y) = 1 + y^2$$

$$f(x, y) = \boxed{x^2} + y^2$$

$$\frac{\partial f}{\partial x} = 2x$$

$$\frac{\partial f}{\partial y} = 2y$$

TASK

Find partial derivative of f with respect to x

Step 1: Treat all other variables as a constant. In our case x.

Step 2: Differentiate the function using the normal rules of differentiation.

Gradients and Gradient Descent

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Partial derivatives -Part 2

Partial Derivatives (More Examples)

Partial Derivatives (More Examples)

$$f(x, y) = 3x^2y^3$$

Partial Derivatives (More Examples)

$$f(x, y) = 3x^2y^3$$

TASK

What is the partial derivate of f with respect to x?

Partial Derivatives (More Examples)

$$f(x, y) = 3x^2y^3$$

$$\frac{\partial f}{\partial x} = \quad ?$$

TASK

What is the partial derivate of f with respect to x?

Partial Derivatives (More Examples)

$$f(x, y) = 3x^2y^3$$

$$\frac{\partial f}{\partial x} =$$

TASK

Find partial derivate of f with respect to x

Partial Derivatives (More Examples)

$$f(x, y) = 3x^2y^3$$

$$\frac{\partial f}{\partial x} =$$

TASK

Find partial derivate of f with respect to x

Step 1: Treat all other variables as a constant. In our case y.

Partial Derivatives (More Examples)

$$f(x, y) = 3x^2$$

$$\frac{\partial f}{\partial x} =$$

TASK

Find partial derivate of f with respect to x

Step 1: Treat all other variables as a constant. In our case y.

Partial Derivatives (More Examples)

$$f(x, y) = 3x^2$$

$$\frac{\partial f}{\partial x} =$$

TASK

Find partial derivate of f with respect to x

Step 1: Treat all other variables as a constant. In our case y.

Step 2: Differentiate the function using the normal rules of differentiation.

Partial Derivatives (More Examples)

$$f(x, y) = 3x^2$$

$$\frac{\partial f}{\partial x} =$$

TASK

Find partial derivate of f with respect to x

Step 1: Treat all other variables as a constant. In our case y.

Step 2: Differentiate the function using the normal rules of differentiation.

Partial Derivatives (More Examples)

$$f(x, y) = 3x^2$$

Constant coefficient

$$\frac{\partial f}{\partial x} =$$

TASK

Find partial derivate of f with respect to x

Step 1: Treat all other variables as a constant. In our case y.

Step 2: Differentiate the function using the normal rules of differentiation.

Partial Derivatives (More Examples)

$$f(x, y) = 3x^2$$

Constant coefficient

$$\frac{\partial f}{\partial x} = 3$$

TASK

Find partial derivate of f with respect to x

Step 1: Treat all other variables as a constant. In our case y.

Step 2: Differentiate the function using the normal rules of differentiation.

Partial Derivatives (More Examples)

$$f(x, y) = 3x^2$$

$$\frac{\partial f}{\partial x} = 3$$

TASK

Find partial derivate of f with respect to x

Step 1: Treat all other variables as a constant. In our case y.

Step 2: Differentiate the function using the normal rules of differentiation.

Partial Derivatives (More Examples)

$$f(x, y) = 3x^2$$

Differentiate with respect to x

$$\frac{\partial f}{\partial x} = 3$$

TASK

Find partial derivate of f with respect to x

Step 1: Treat all other variables as a constant. In our case y.

Step 2: Differentiate the function using the normal rules of differentiation.

Partial Derivatives (More Examples)

$$f(x, y) = 3x^2$$

Differentiate with respect to x

$$\frac{\partial f}{\partial x} = 3(2x)$$

TASK

Find partial derivate of f with respect to x

Step 1: Treat all other variables as a constant. In our case y.

Step 2: Differentiate the function using the normal rules of differentiation.

Partial Derivatives (More Examples)

$$f(x, y) = 3x^2$$

$$\frac{\partial f}{\partial x} = 3(2x)$$

TASK

Find partial derivate of f with respect to x

Step 1: Treat all other variables as a constant. In our case y.

Step 2: Differentiate the function using the normal rules of differentiation.

Partial Derivatives (More Examples)

$$f(x, y) = 3x^2$$

treat as constant coefficient

$$\frac{\partial f}{\partial x} = 3(2x)$$

TASK

Find partial derivate of f with respect to x

Step 1: Treat all other variables as a constant. In our case y.

Step 2: Differentiate the function using the normal rules of differentiation.

Partial Derivatives (More Examples)

$$f(x, y) = 3x^2$$

treat as constant coefficient

$$\frac{\partial f}{\partial x} = 3(2x)$$

TASK

Find partial derivate of f with respect to x

Step 1: Treat all other variables as a constant. In our case y.

Step 2: Differentiate the function using the normal rules of differentiation.

Partial Derivatives (More Examples)

$$f(x, y) = 3x^2y^3$$

$$\frac{\partial f}{\partial x} = 3(2x)y^3$$

TASK

Find partial derivate of f with respect to x

Step 1: Treat all other variables as a constant. In our case y.

Step 2: Differentiate the function using the normal rules of differentiation.

Partial Derivatives (More Examples)

$$f(x, y) = 3x^2y^3$$

$$\frac{\partial f}{\partial x} = 3(2x)y^3$$

$$= 6xy^3$$

TASK

Find partial derivate of f with respect to x

Step 1: Treat all other variables as a constant. In our case y.

Step 2: Differentiate the function using the normal rules of differentiation.

Partial Derivatives (More Examples)

$$f(x, y) = 3x^2y^3$$

TASK

What is the partial derivate of f with respect to y?

Partial Derivatives (More Examples)

$$f(x, y) = 3x^2y^3$$

$$\frac{\partial f}{\partial y} =$$

TASK

What is the partial derivate of f with respect to y?

Partial Derivatives (More Examples)

$$f(x, y) = 3x^2y^3$$

$$\frac{\partial f}{\partial y} = \ ?$$

TASK

What is the partial derivate of f with respect to y?

Partial Derivatives (More Examples)

$$f(x, y) = 3x^2y^3$$

$$\frac{\partial f}{\partial y} =$$

TASK

What is the partial derivate of f with respect to y?

Step 1: Treat all other variables as a constant. In our case x.

Step 2: Differentiate the function using the normal rules of differentiation.

Partial Derivatives (More Examples)

$$f(x, y) = 3\,\text{[illegible]}\,y^3$$

$$\frac{\partial f}{\partial y} =$$

TASK

What is the partial derivate of f with respect to y?

Step 1: Treat all other variables as a constant. In our case x.

Step 2: Differentiate the function using the normal rules of differentiation.

Partial Derivatives (More Examples)

$$f(x, y) = 3 \quad y^3$$

$$\frac{\partial f}{\partial y} = 3$$

TASK

What is the partial derivate of f with respect to y?

Step 1: Treat all other variables as a constant. In our case x.

Step 2: Differentiate the function using the normal rules of differentiation.

Partial Derivatives (More Examples)

$$f(x, y) = 3\,\square\,y^3$$

$$\frac{\partial f}{\partial y} = 3\,\square$$

TASK

What is the partial derivate of f with respect to y?

Step 1: Treat all other variables as a constant. In our case x.

Step 2: Differentiate the function using the normal rules of differentiation.

Partial Derivatives (More Examples)

$$f(x, y) = 3\,\Box\,y^3$$

$$\frac{\partial f}{\partial y} = 3\,\Box\,(3y^2)$$

TASK

What is the partial derivate of f with respect to y?

Step 1: Treat all other variables as a constant. In our case x.

Step 2: Differentiate the function using the normal rules of differentiation.

Partial Derivatives (More Examples)

$$f(x, y) = 3x^2y^3$$

$$\frac{\partial f}{\partial y} = 3\,(x^2)\,(3y^2)$$

$$= 9x^2y^2$$

TASK

What is the partial derivate of f with respect to y?

Step 1: Treat all other variables as a constant. In our case x.

Step 2: Differentiate the function using the normal rules of differentiation.

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Gradients and Gradient Descent

DeepLearning.AI

Gradients

Gradient

$$f(x, y) = x^2 + y^2$$

Gradient

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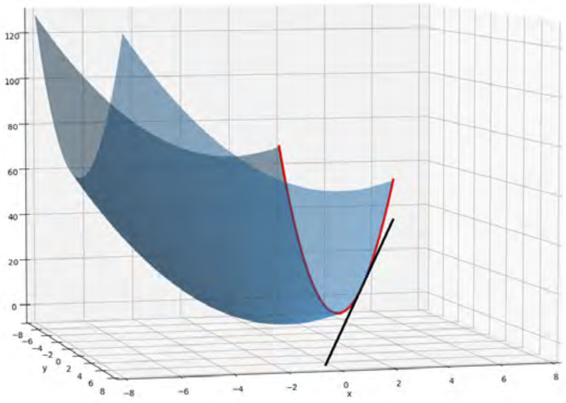
Treat y as a constant



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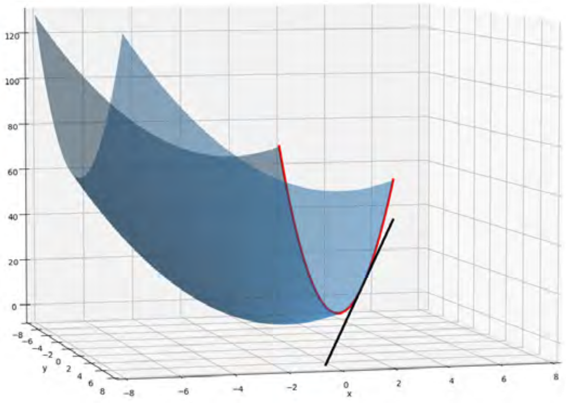
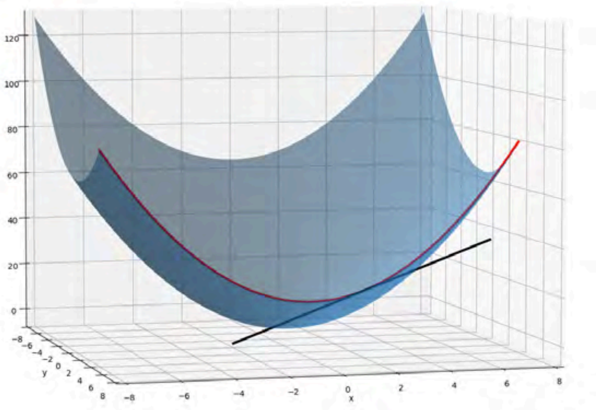


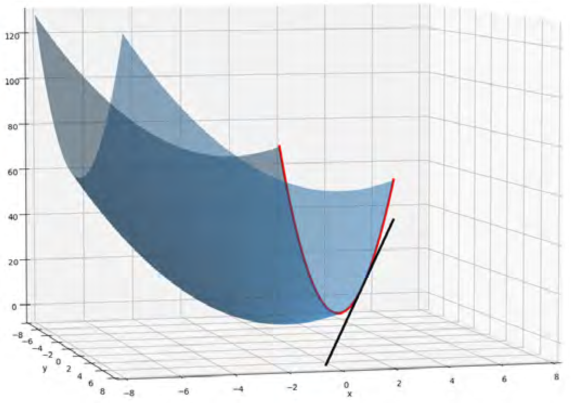
Treat x as a constant



Gradient

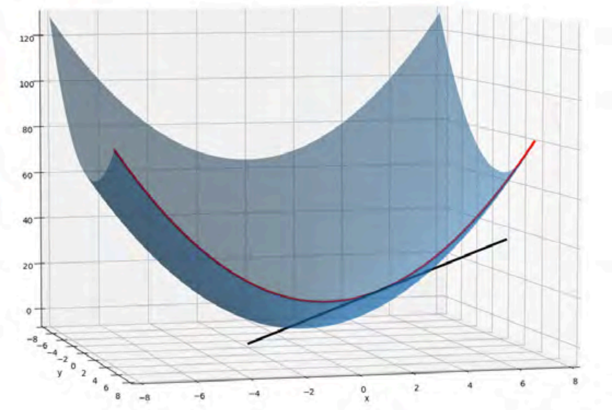
$$f(x, y) = x^2 + y^2$$

Treat y as a constant



$$\frac{\partial f}{\partial x} = 2x$$

Treat x as a constant



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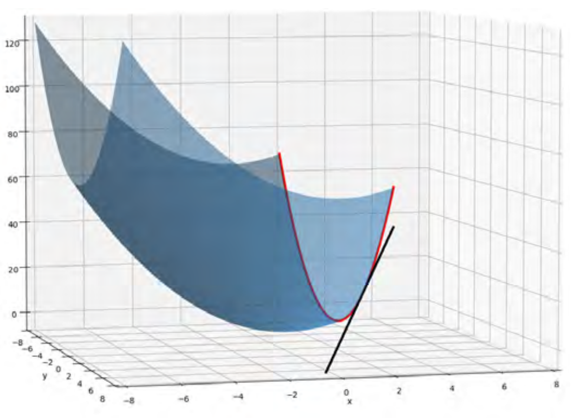

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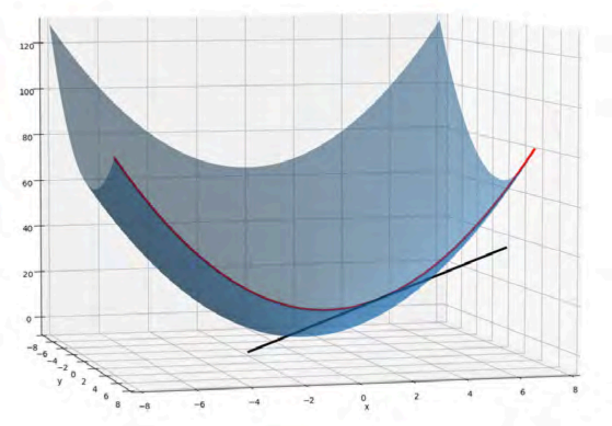

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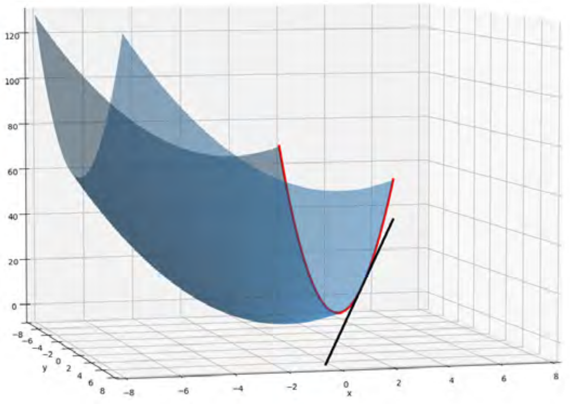

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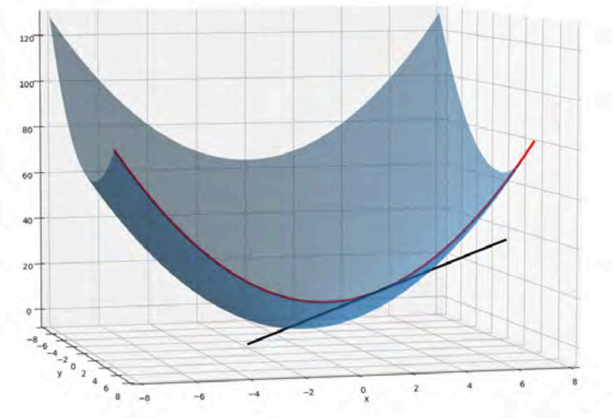

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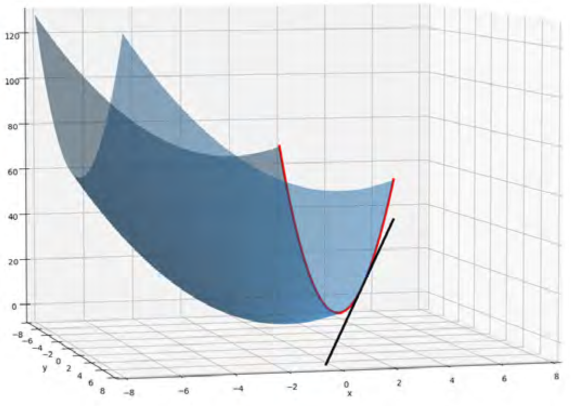
Gradient

$$f(x, y) = x^2 + y^2$$

Gradient

$$\begin{bmatrix} 2x \\ 2y \end{bmatrix}$$

Treat y as a constant



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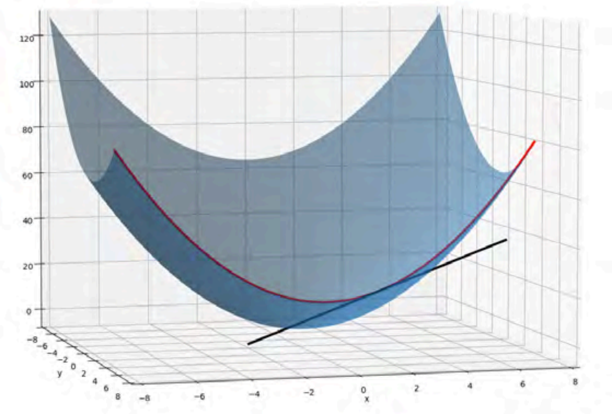

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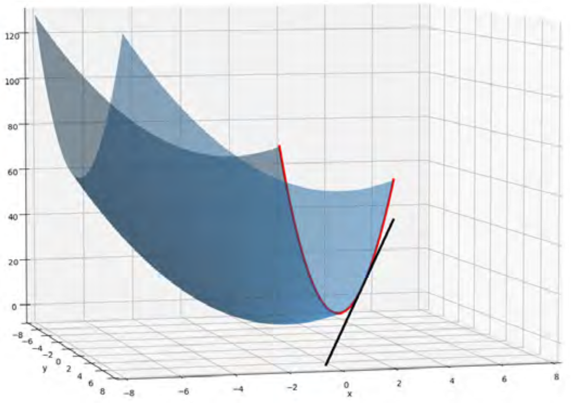

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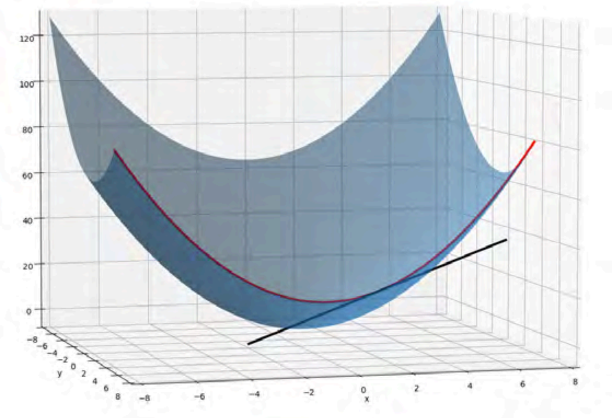
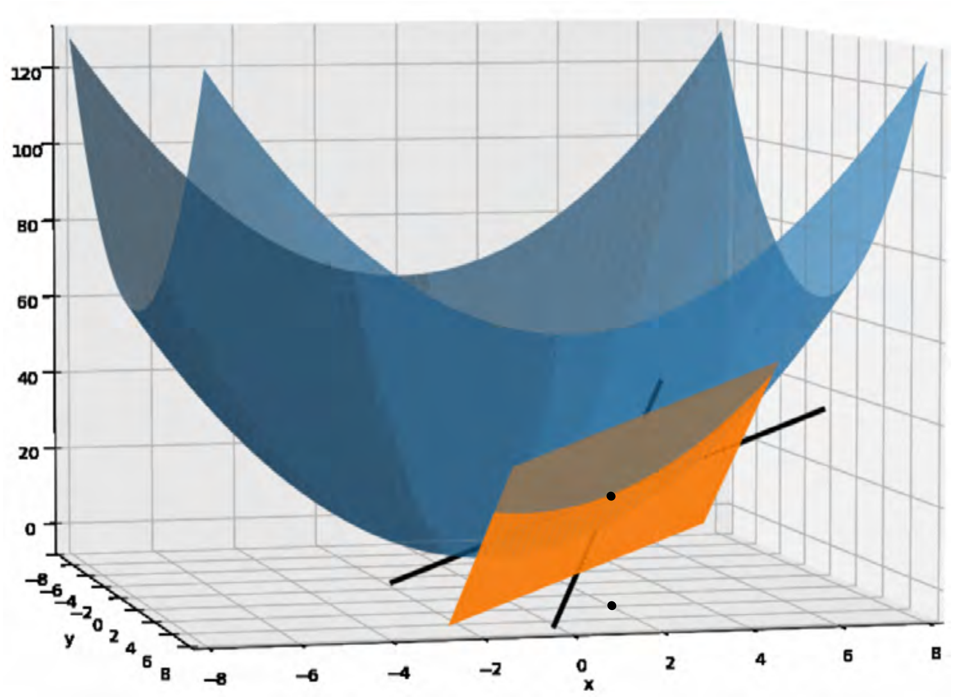


$$\frac{\partial f}{\partial y} = 2y$$

$$\nabla f = \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{bmatrix}$$

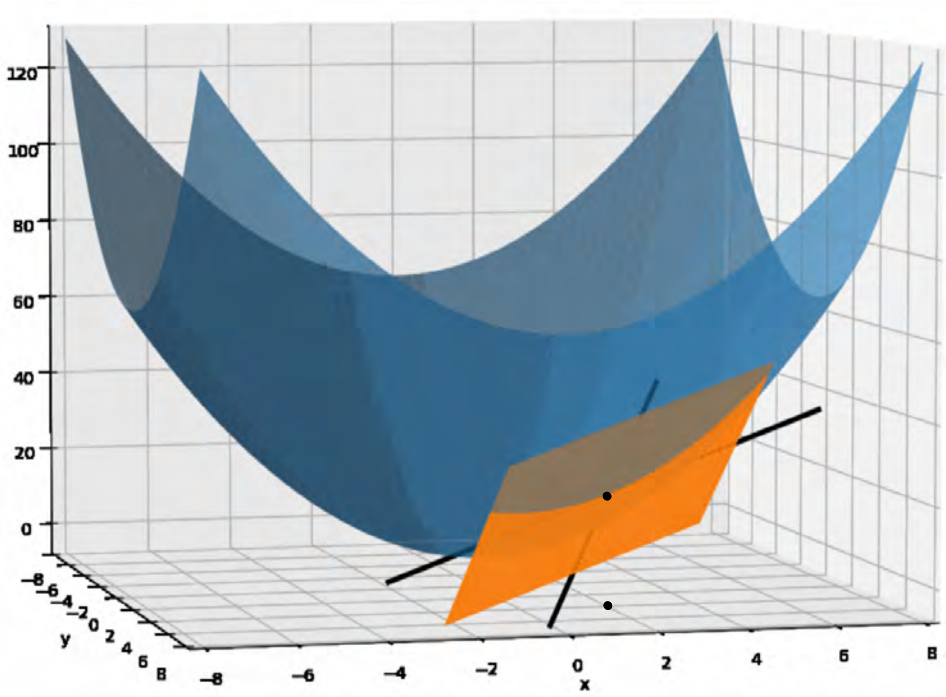
Gradient



$$f(x, y) = x^2 + y^2$$

The gradient of $f(x, y)$ is: $\nabla f = \begin{bmatrix} 2x \\ 2y \end{bmatrix}$

Gradient



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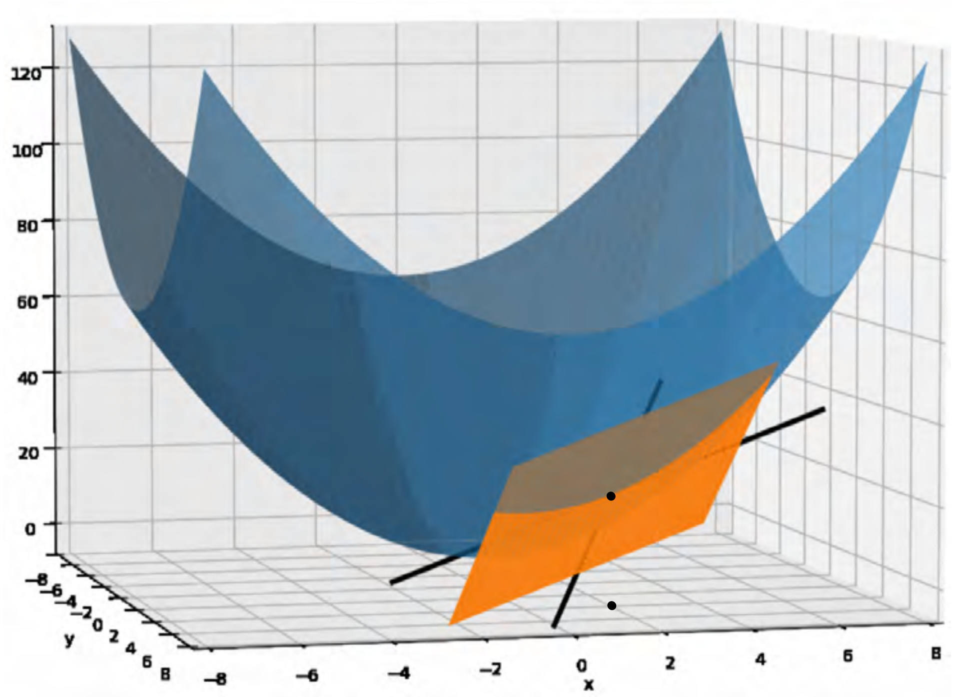
TASK

Find the gradient of $f(x, y)$ at $(2,3)$

Gradient



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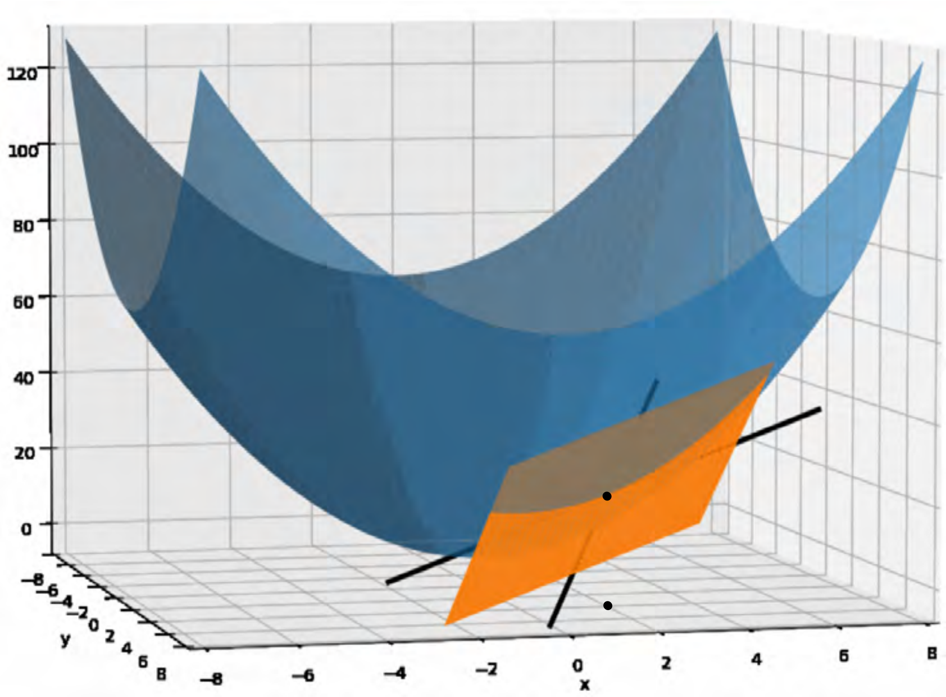
TASK

Find the gradient of $f(x, y)$ at $(2,3)$

The gradient of $f(x, y)$ is given as:

Gradient



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The gradient of $f(x, y)$ is: $\nabla f = \begin{bmatrix} 2x \\ 2y \end{bmatrix}$

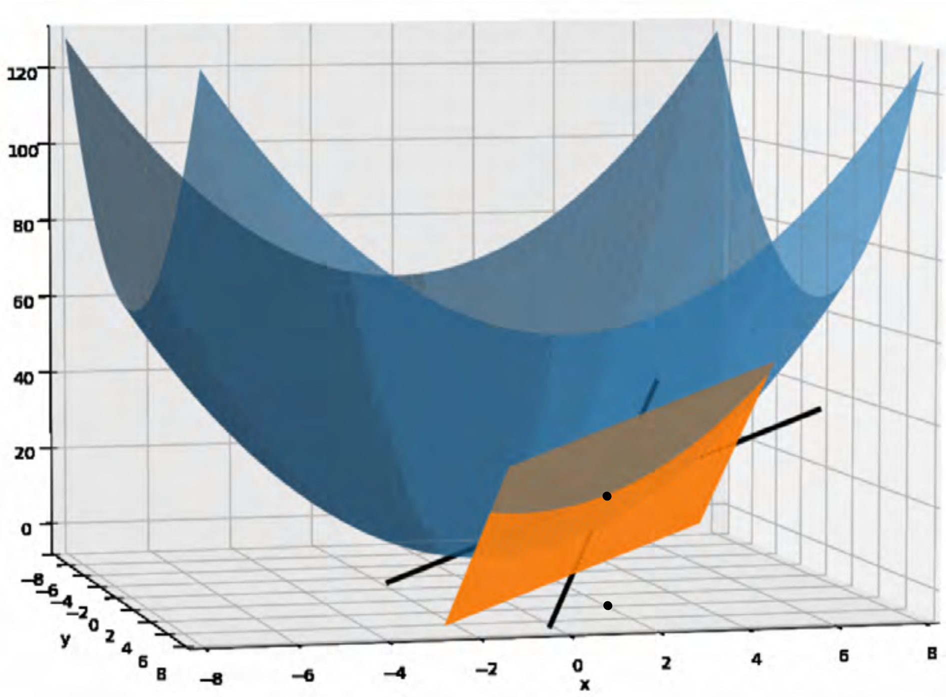
TASK

Find the gradient of $f(x, y)$ at $(2,3)$

The gradient of $f(x, y)$ is given as:

$$\nabla f = \begin{bmatrix} 2 \cdot 2 \\ 2 \cdot 3 \end{bmatrix}$$

Gradient



$$f(x, y) = x^2 + y^2$$

The gradient of $f(x, y)$ is: $\nabla f = \begin{bmatrix} 2x \\ 2y \end{bmatrix}$

TASK

Find the gradient of $f(x, y)$ at $(2,3)$

The gradient of $f(x, y)$ is given as:

$$\nabla f = \begin{bmatrix} 2 \cdot 2 \\ 2 \cdot 3 \end{bmatrix} = \begin{bmatrix} 4 \\ 6 \end{bmatrix}$$

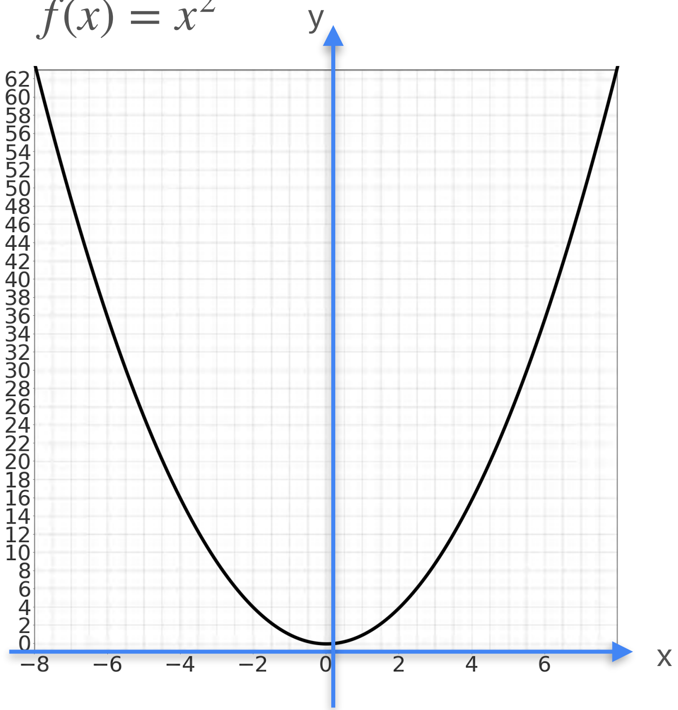
Gradients and Gradient Descent

Gradients and maxima/ minima

Functions of Two Variables

Functions of Two Variables

$f(x) = x^2$



Functions of Two Variables

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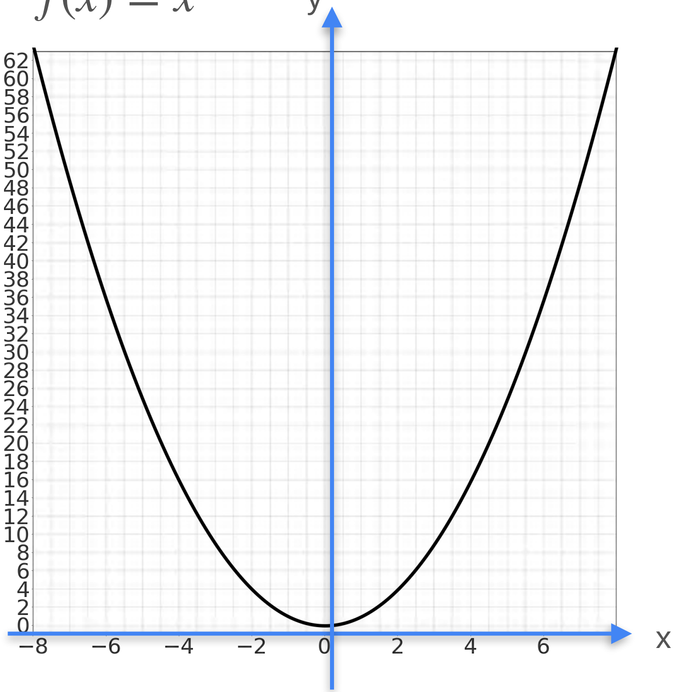

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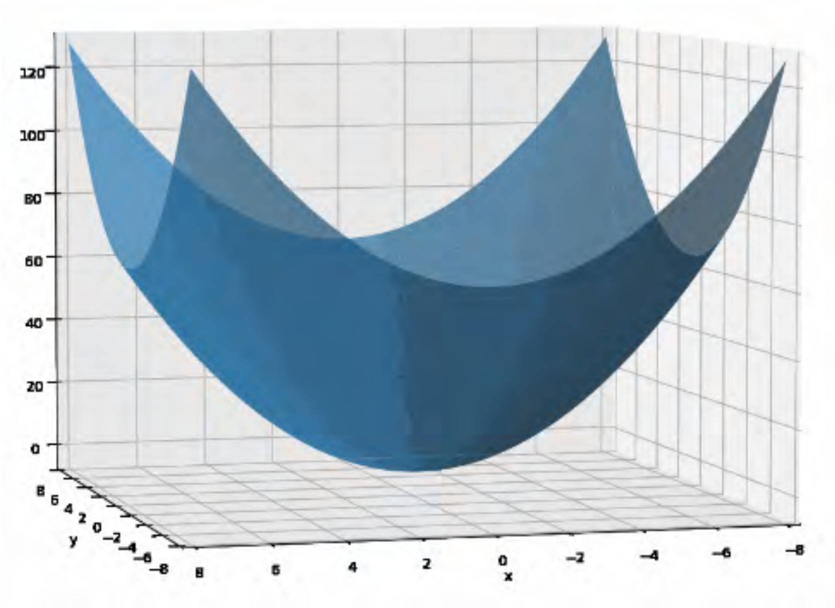

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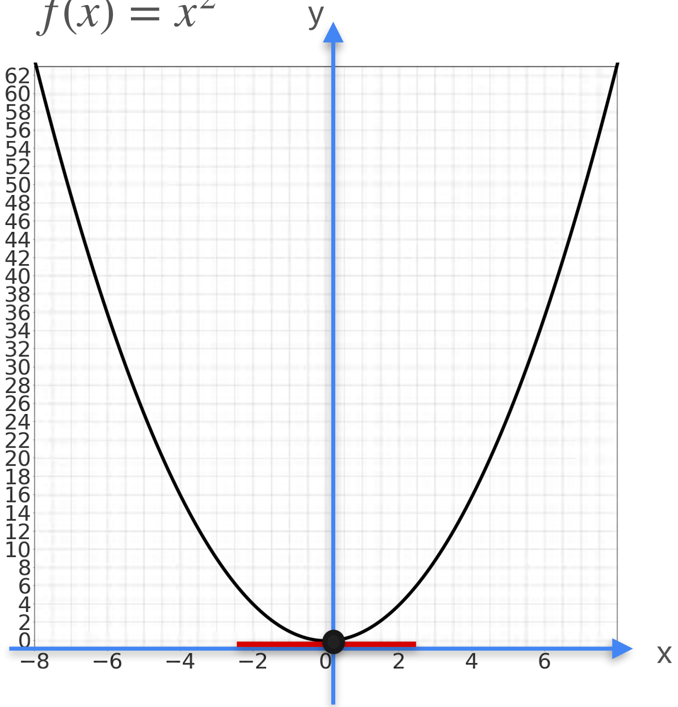

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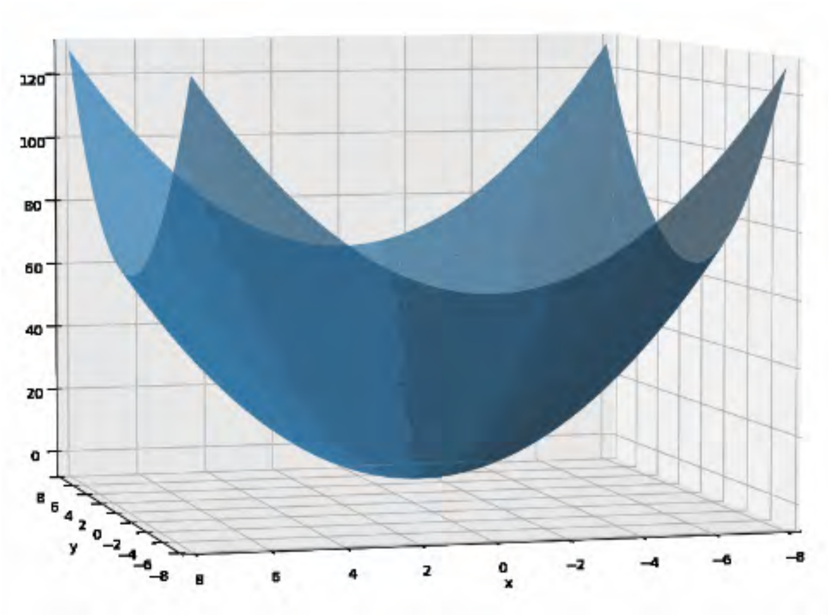

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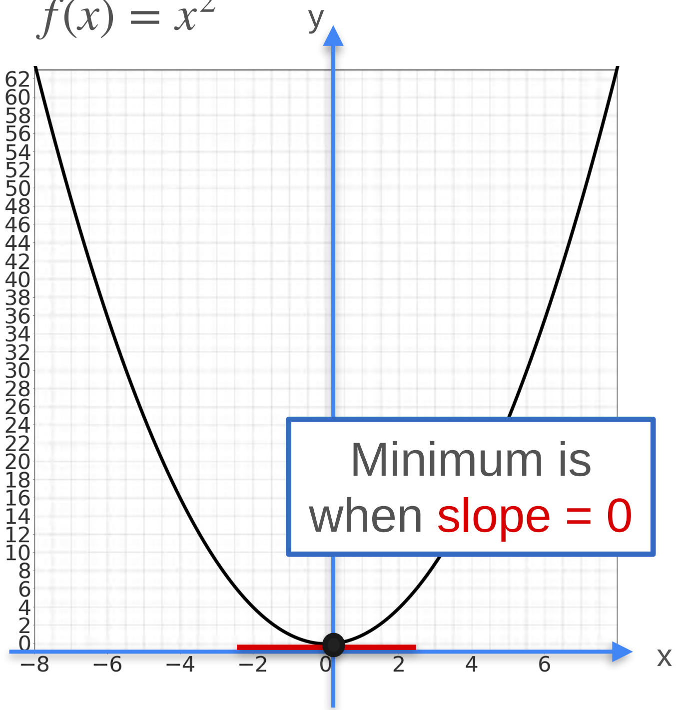

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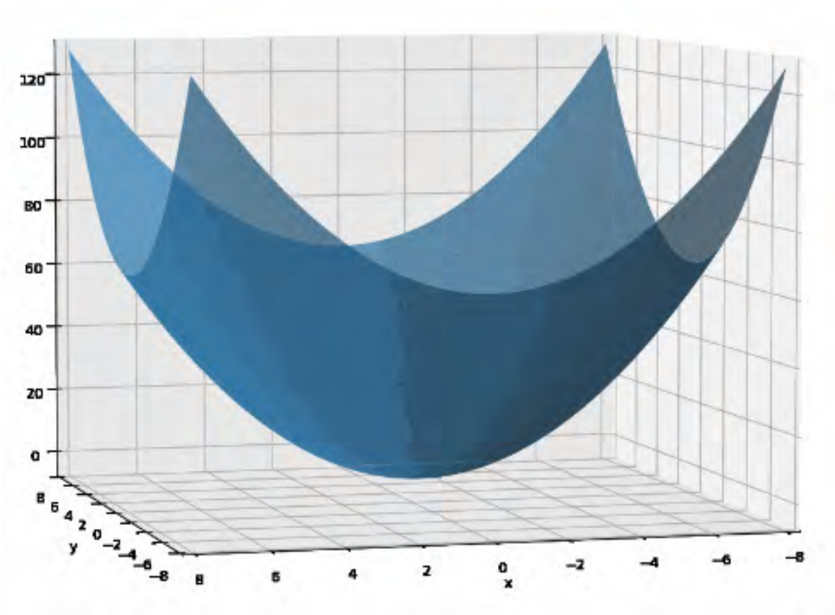

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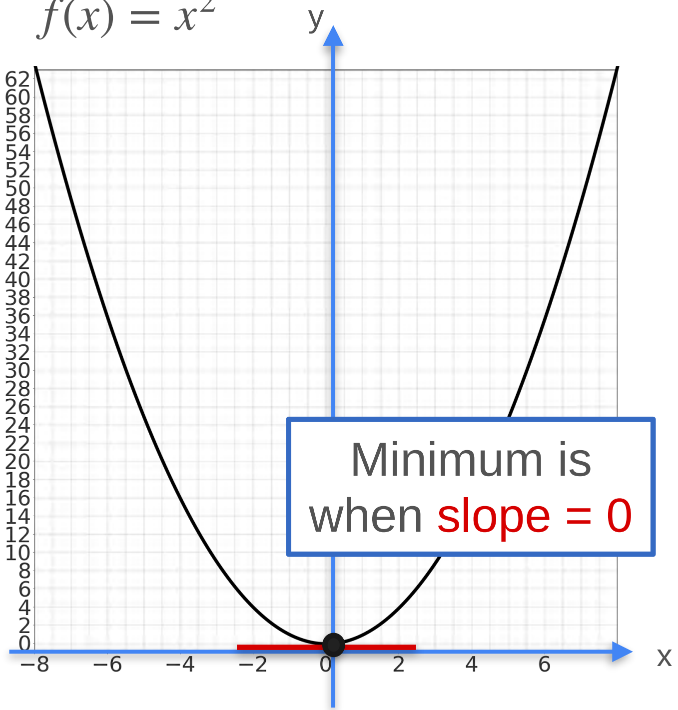

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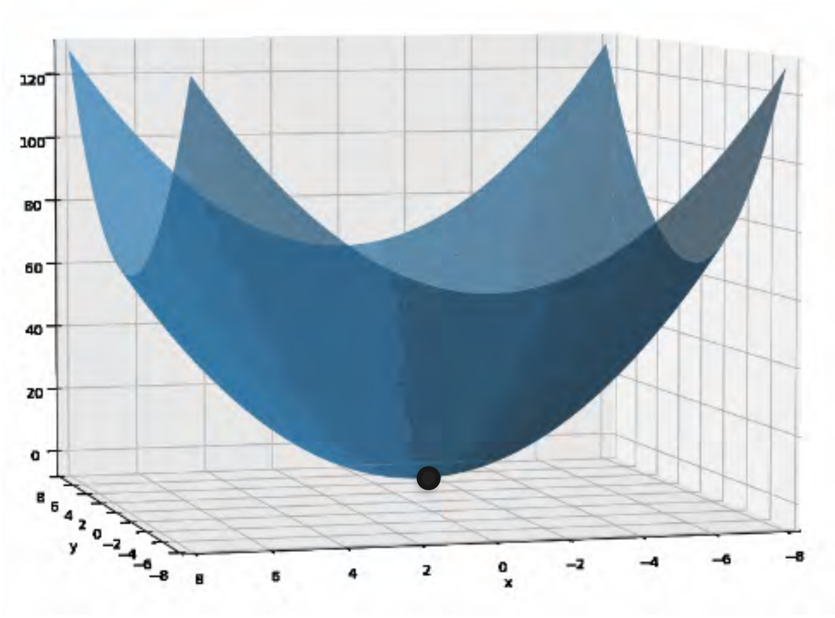

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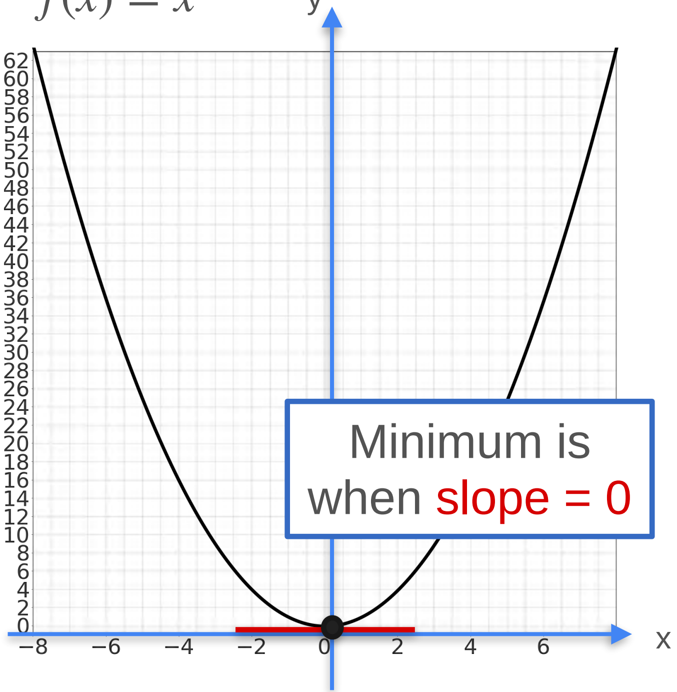

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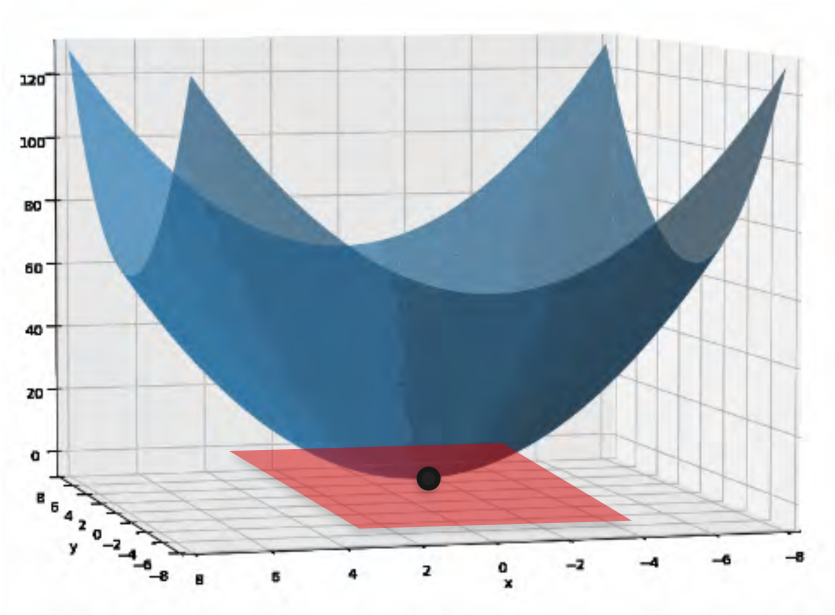

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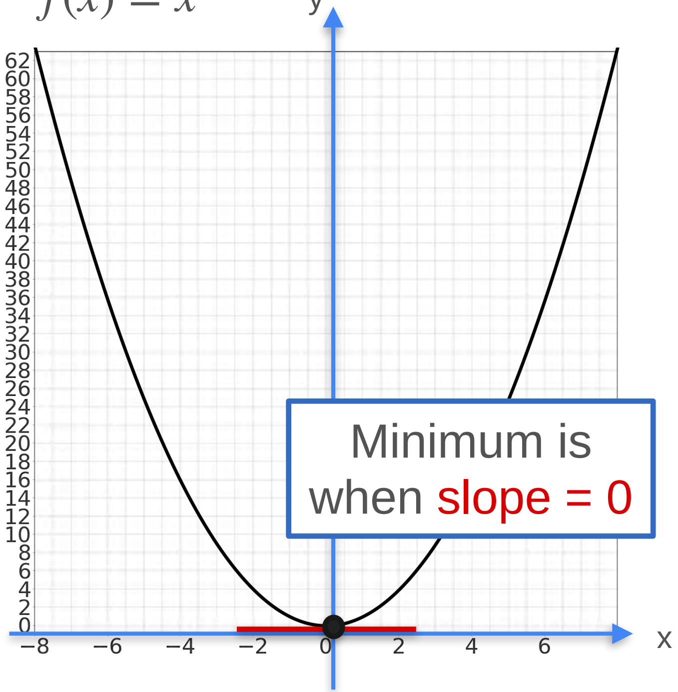

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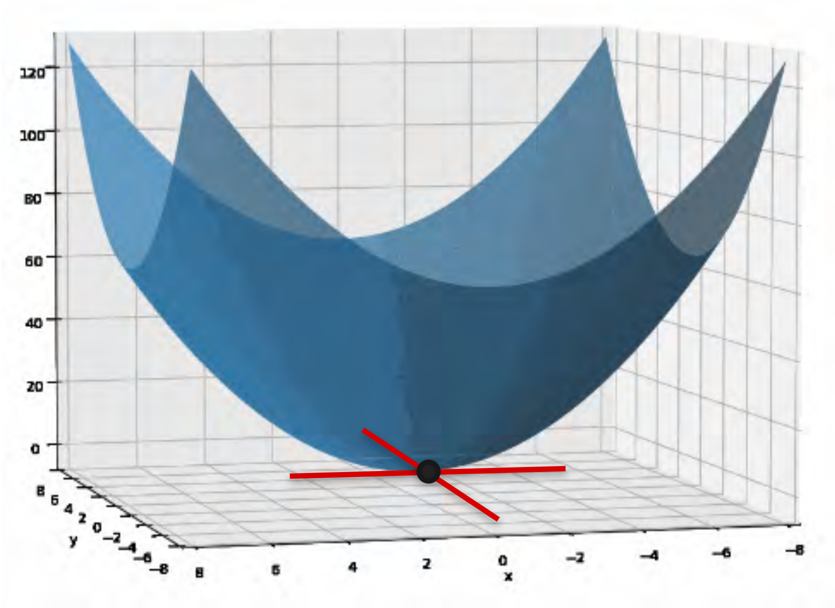

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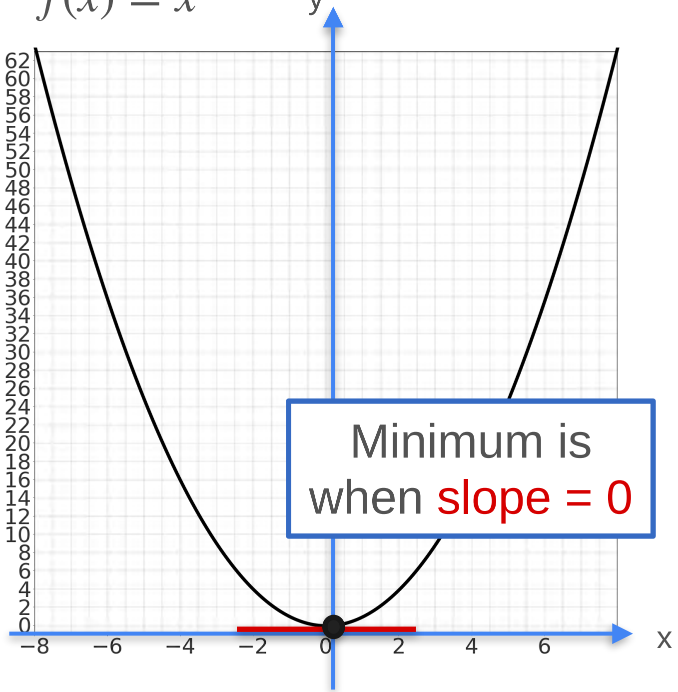

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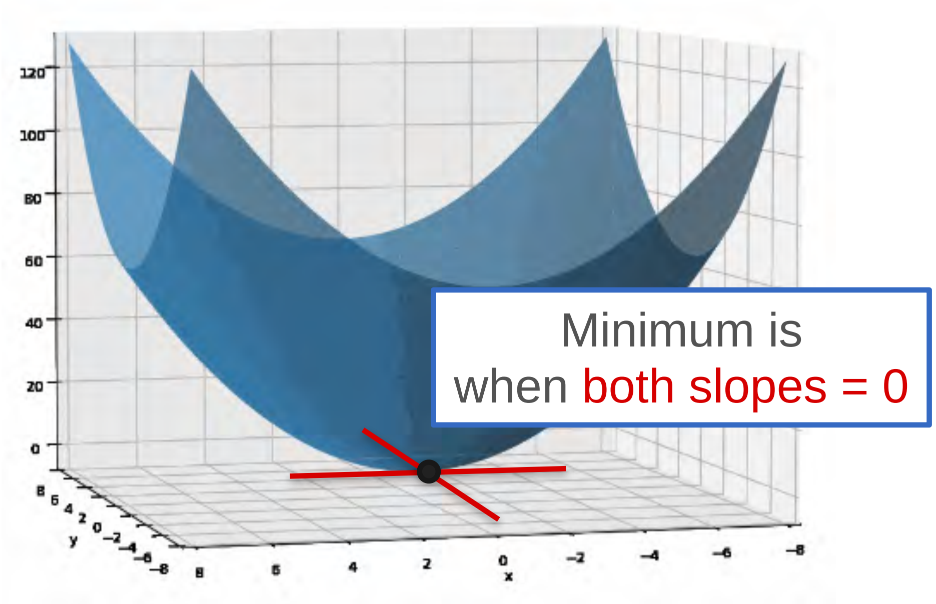

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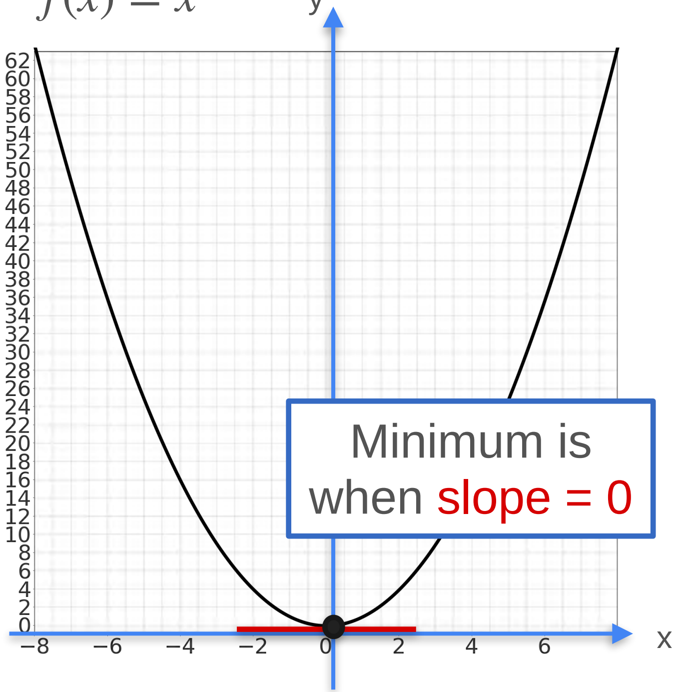

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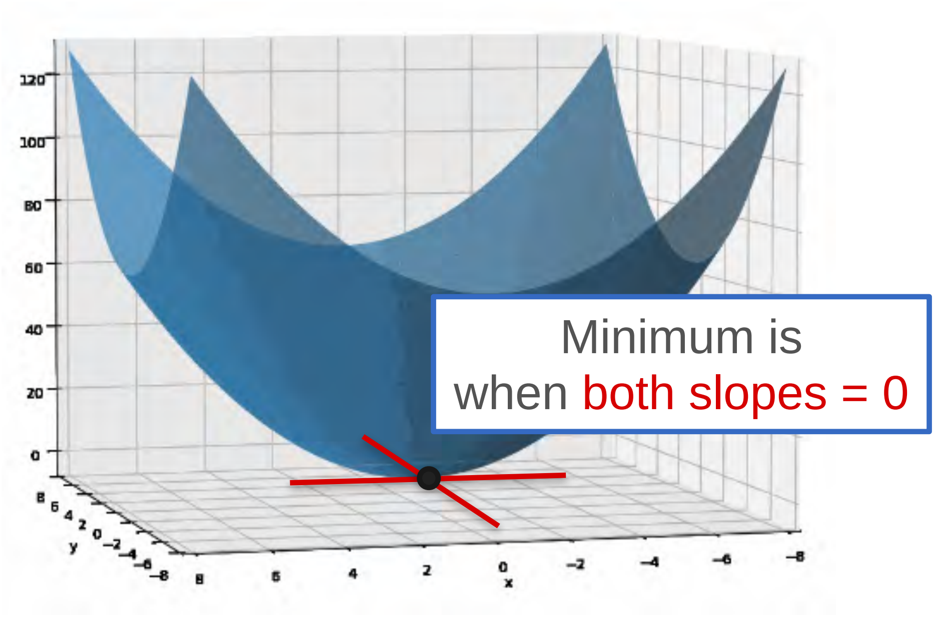

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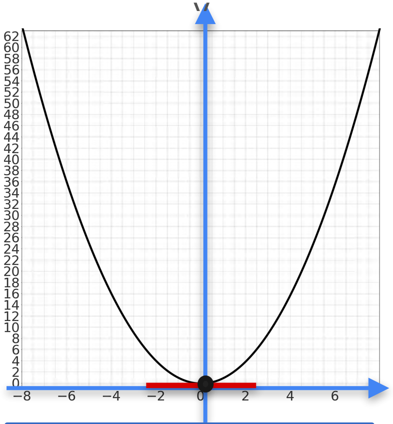
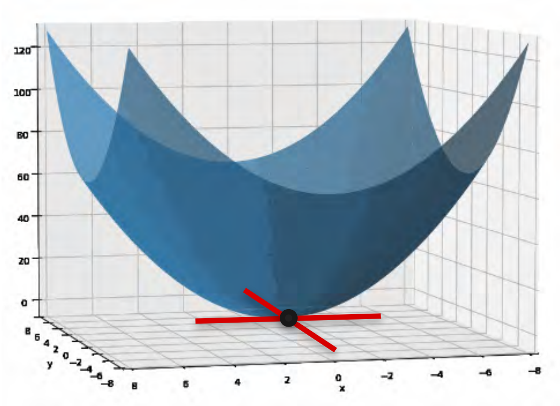


Minimum is when slope = 0

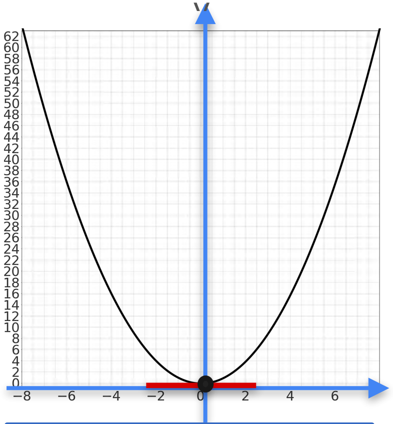
$f(x, y) = x^2 + y^2$



Minimum is when both slopes = 0

Functions of Two Variables

$f(x) = x^2$



$f'(x) = 0$

Minimum is when slope = 0

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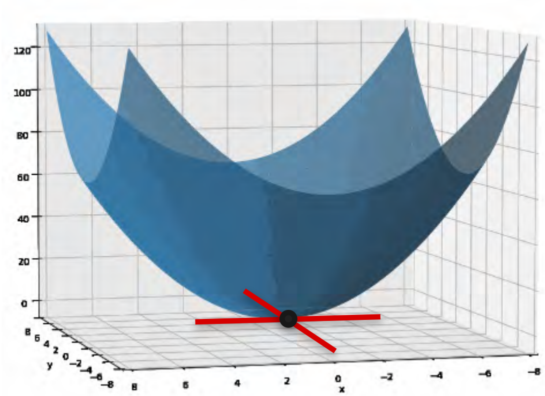

Minimum is when both slopes = 0

Functions of Two Variables

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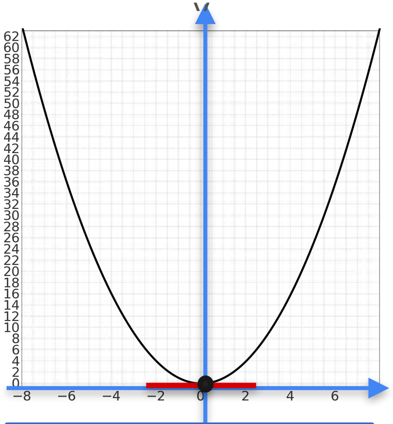

Minimum is when slope = 0

$$f'(x) = 0$$

$$2x = 0$$

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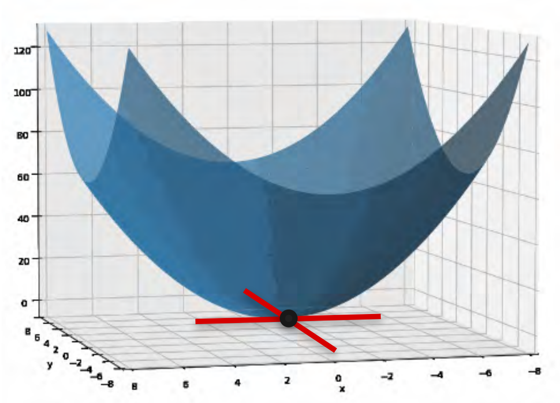

Minimum is when both slopes = 0

Functions of Two Variables

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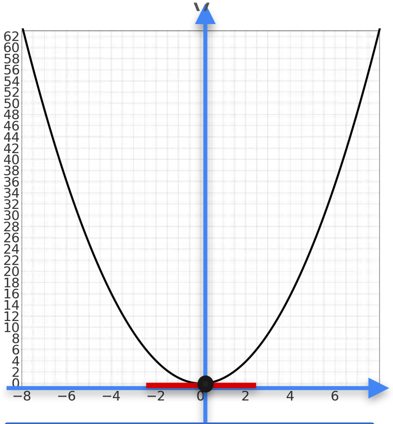

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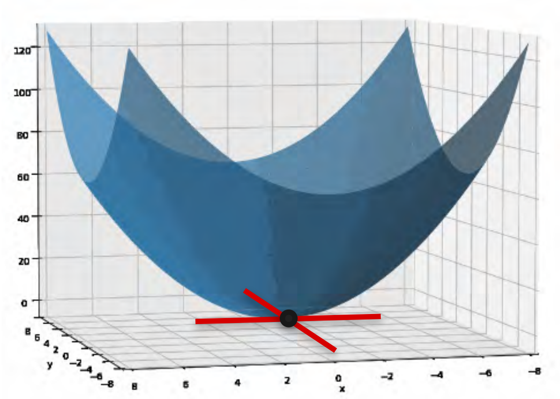

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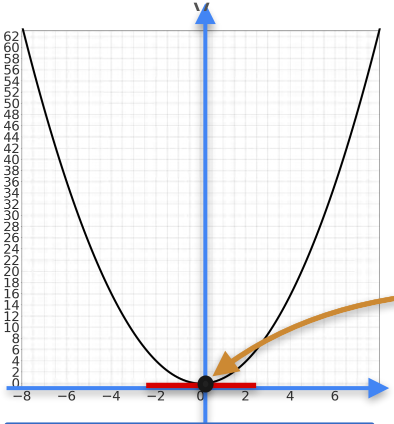

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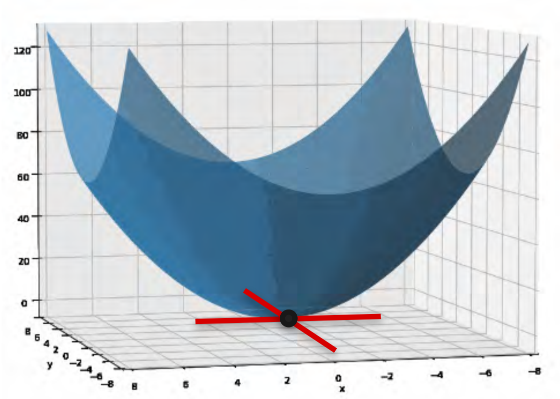

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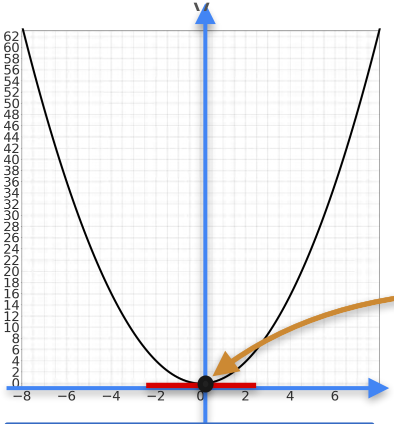

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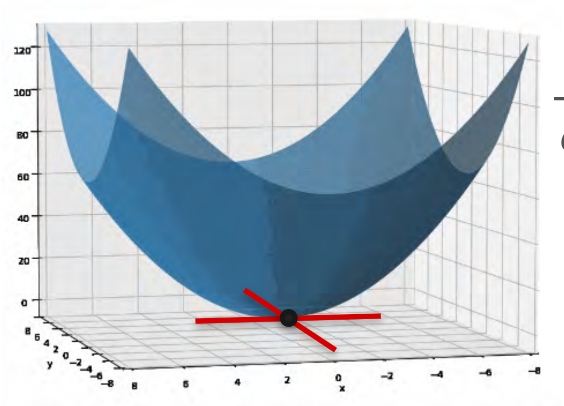
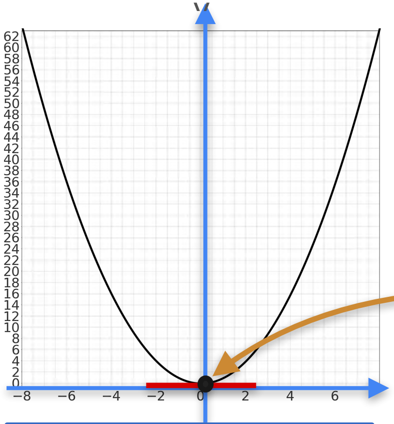


$$\frac{\partial f}{\partial x} = 0 \text{ and } \frac{\partial f}{\partial y} = 0$$

Minimum is when both slopes = 0

Functions of Two Variables

$f(x) = x^2$



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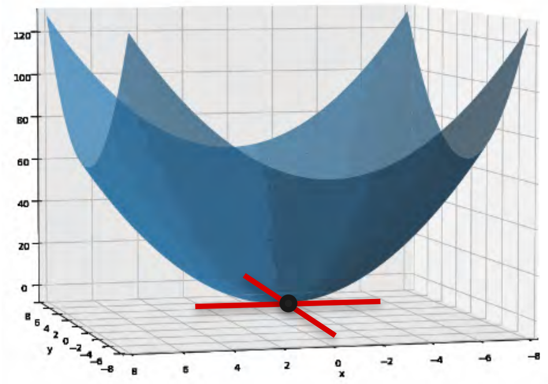

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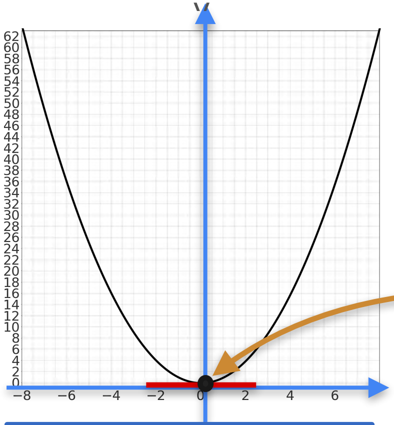

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$$x = 0$$

Minimum is
when slope = 0

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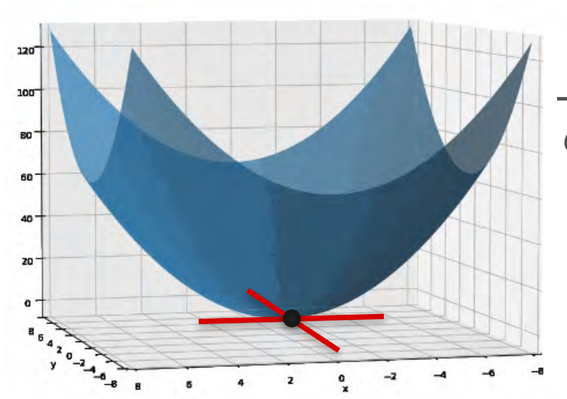


$$\frac{\partial f}{\partial x} = 0 \text{ and } \frac{\partial f}{\partial y} = 0$$

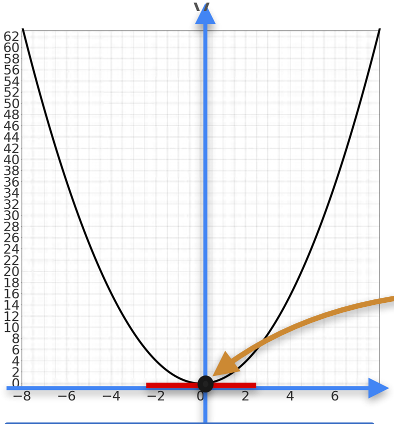
$$2x = 0 \text{ and } 2y = 0$$

$$(x, y) = (0,0)$$

Minimum is
when both slopes = 0

Functions of Two Variables

$f(x) = x^2$



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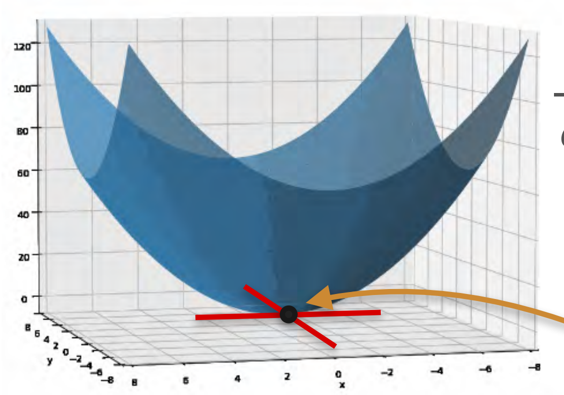
Minimum is when slope = 0

$f(x, y) = x^2 + y^2$



$$\frac{\partial f}{\partial x} = 0 \text{ and } \frac{\partial f}{\partial y} = 0$$

$$2x = 0 \text{ and } 2y = 0$$

$$(x, y) = (0,0)$$

Minimum is when both slopes = 0

Gradients and Gradient Descent

Optimization with gradients: An example

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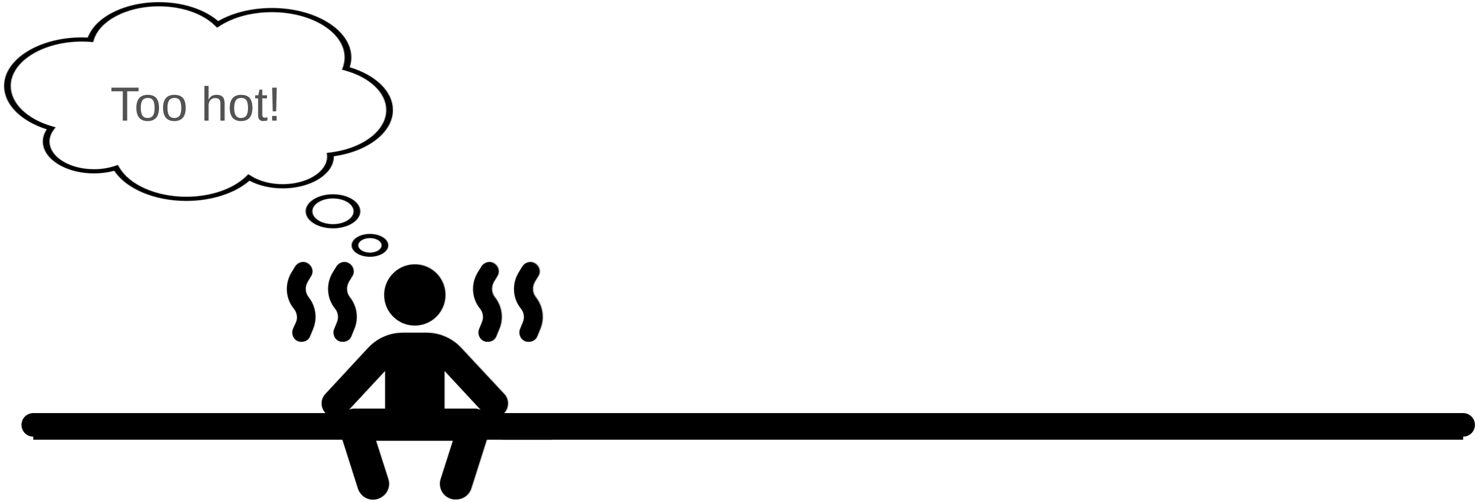
Motivation for Optimization in Two Variables



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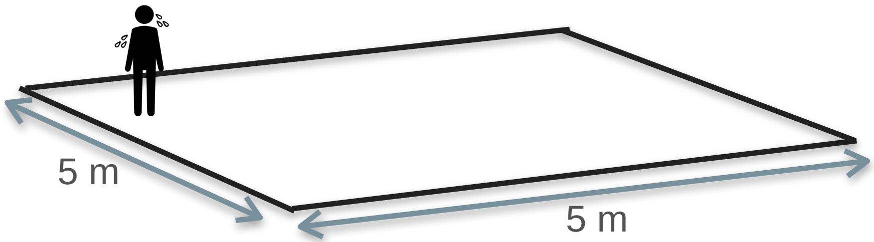
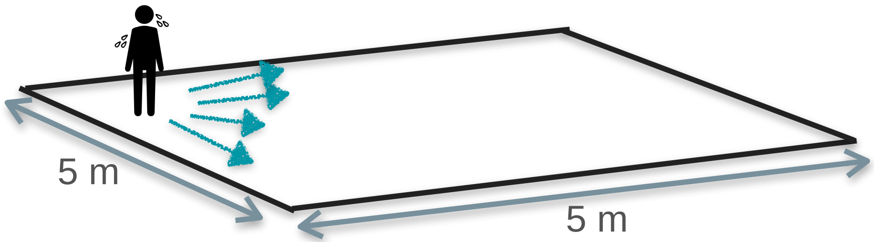
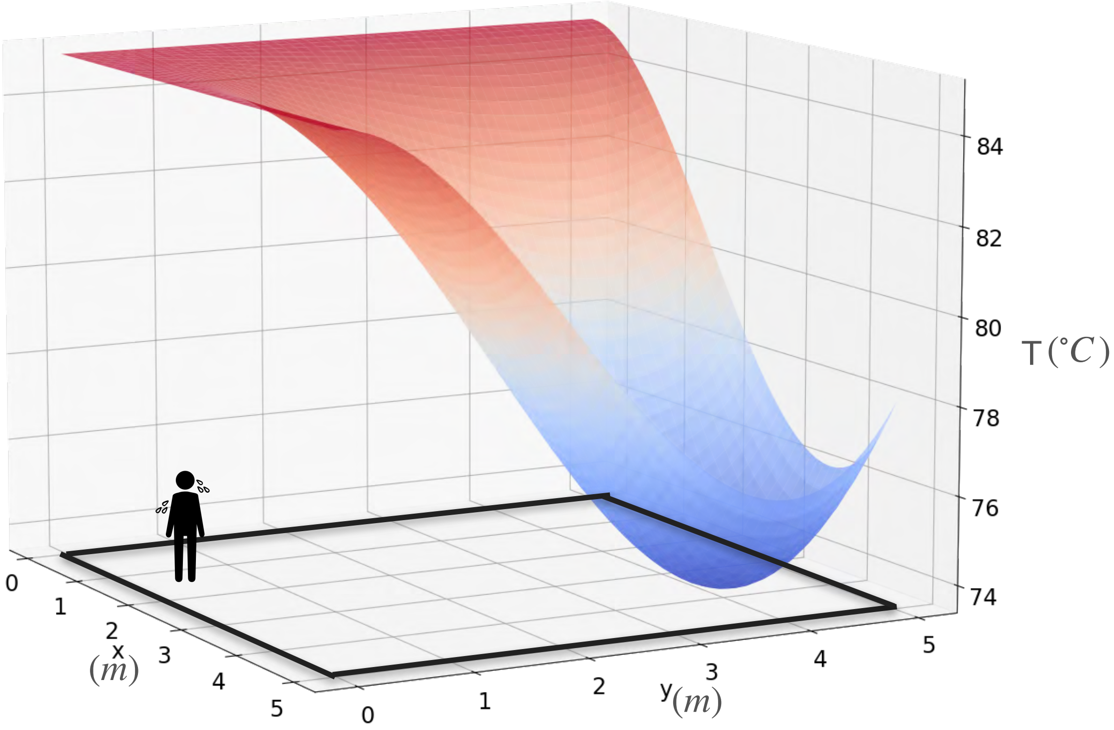


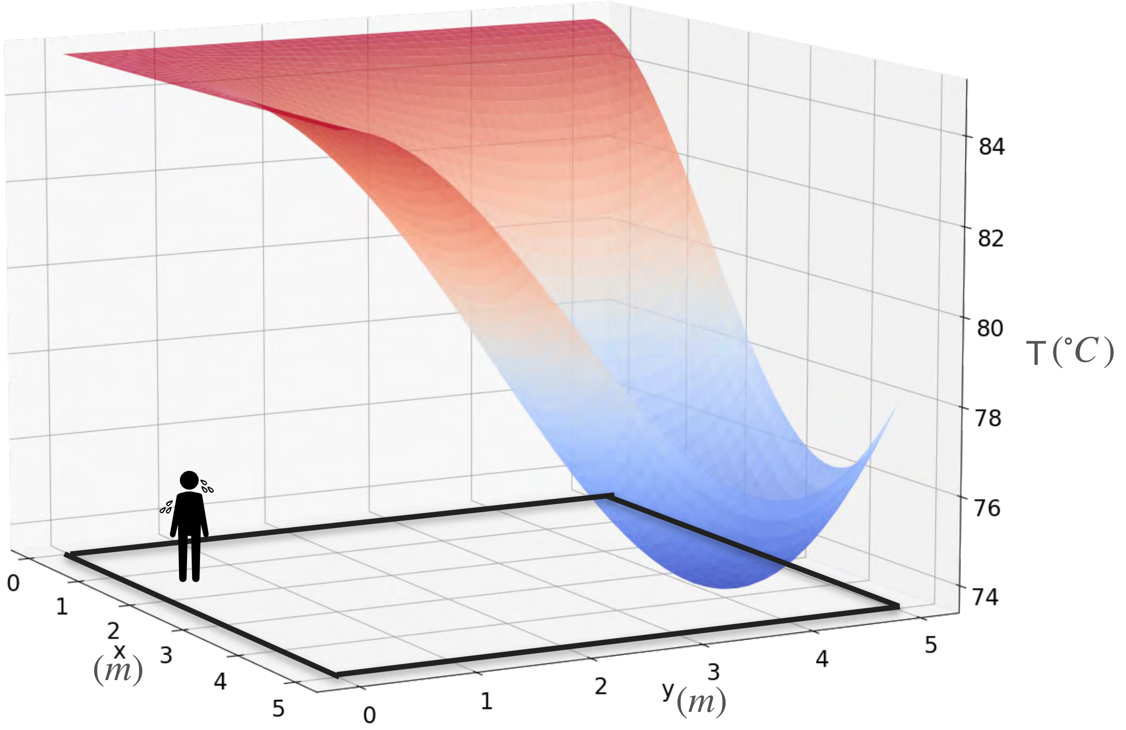
Motivation for Optimization in Two Variables



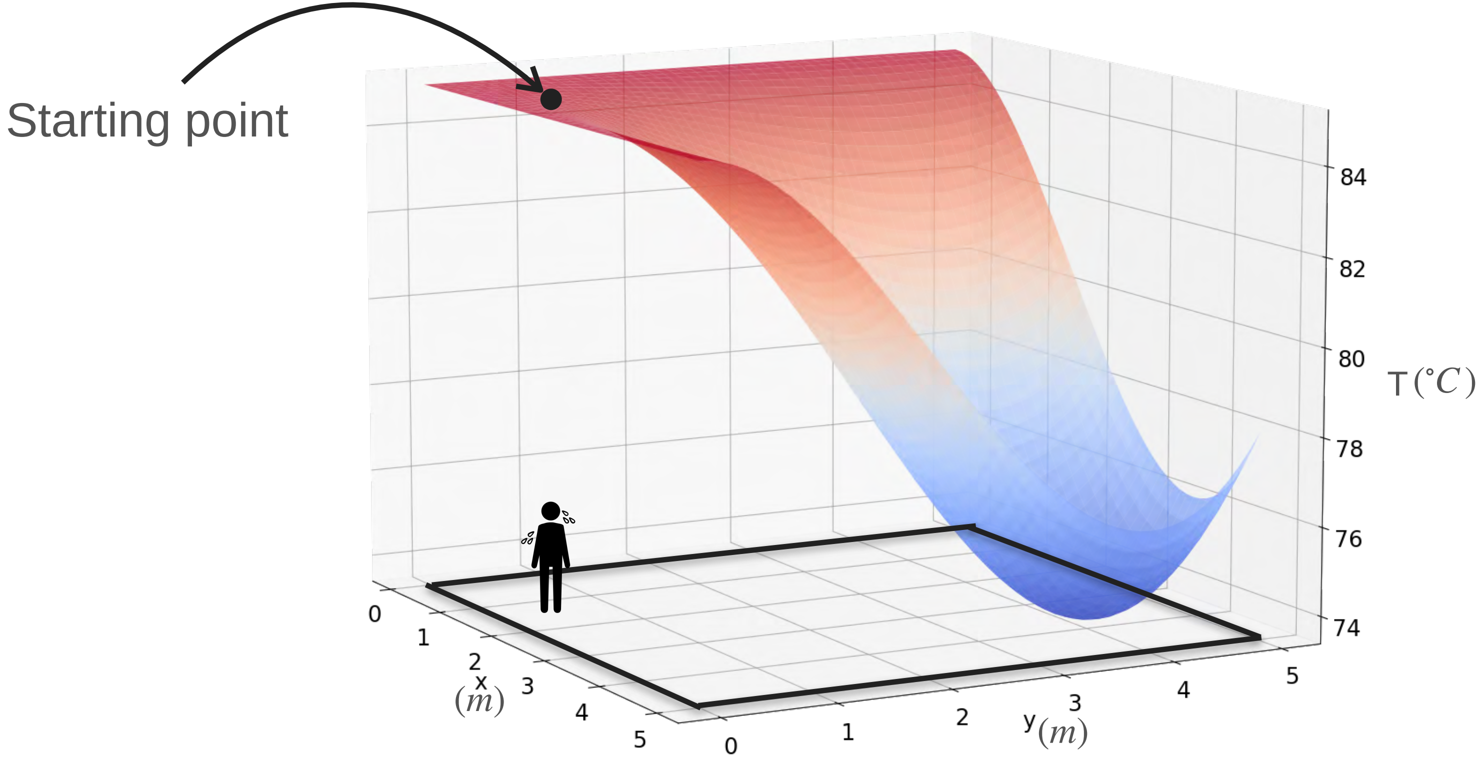
Motivation for Optimization in Two Variables



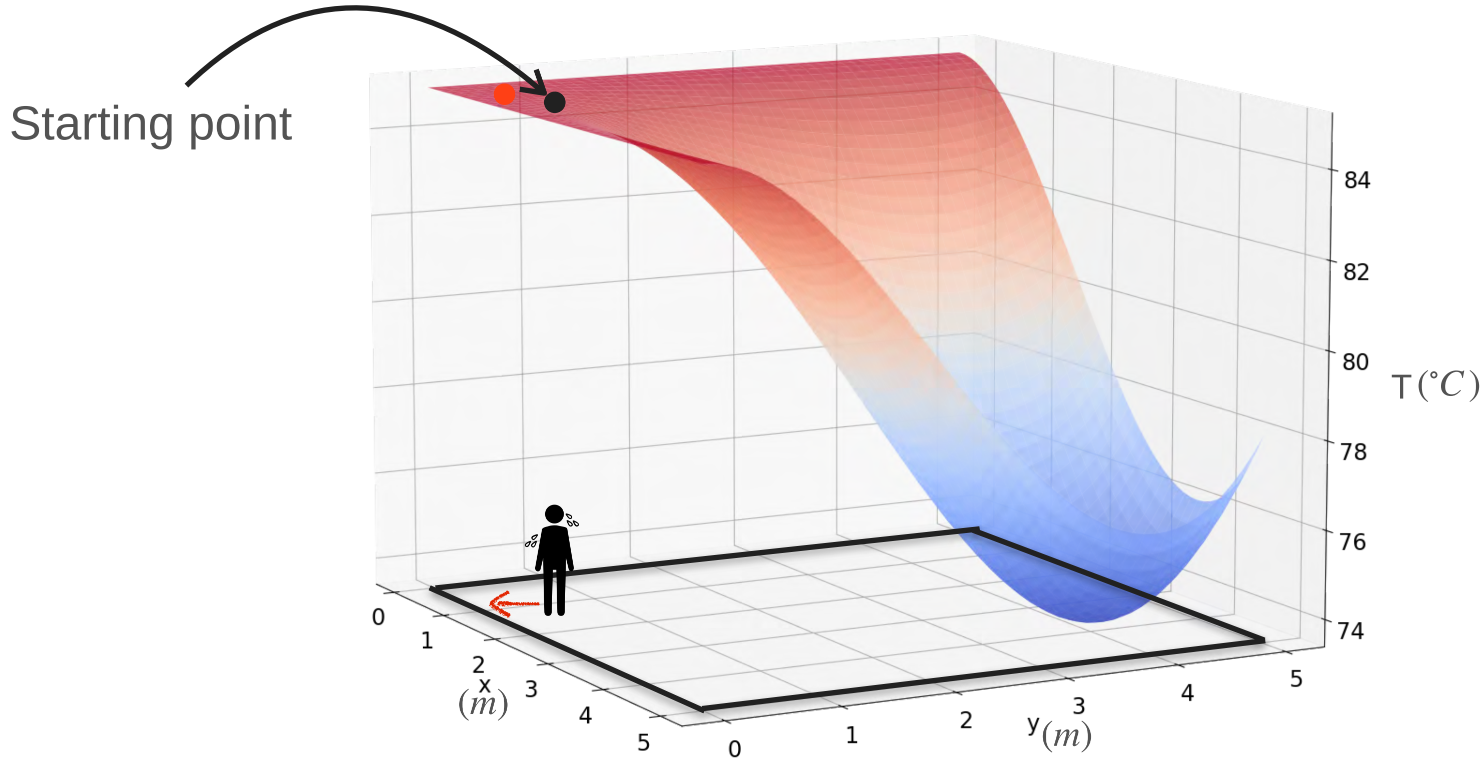
Motivation for Optimization in Two Variables



Motivation for Optimization in Two Variables

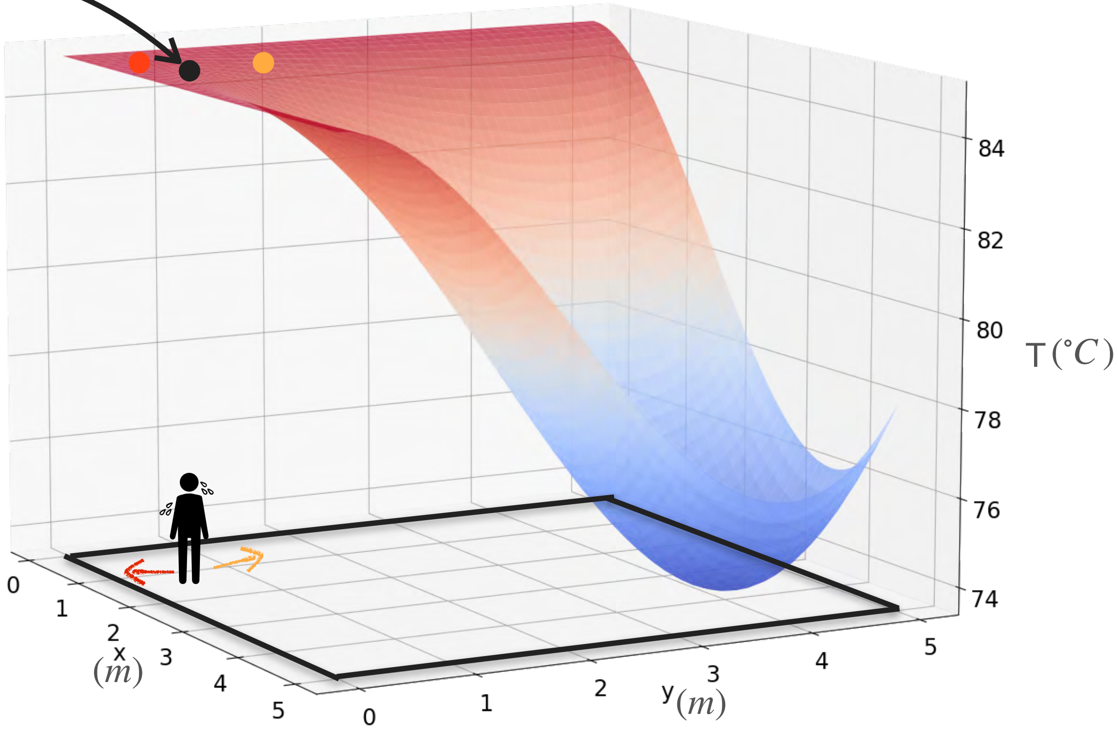


Motivation for Optimization in Two Variables

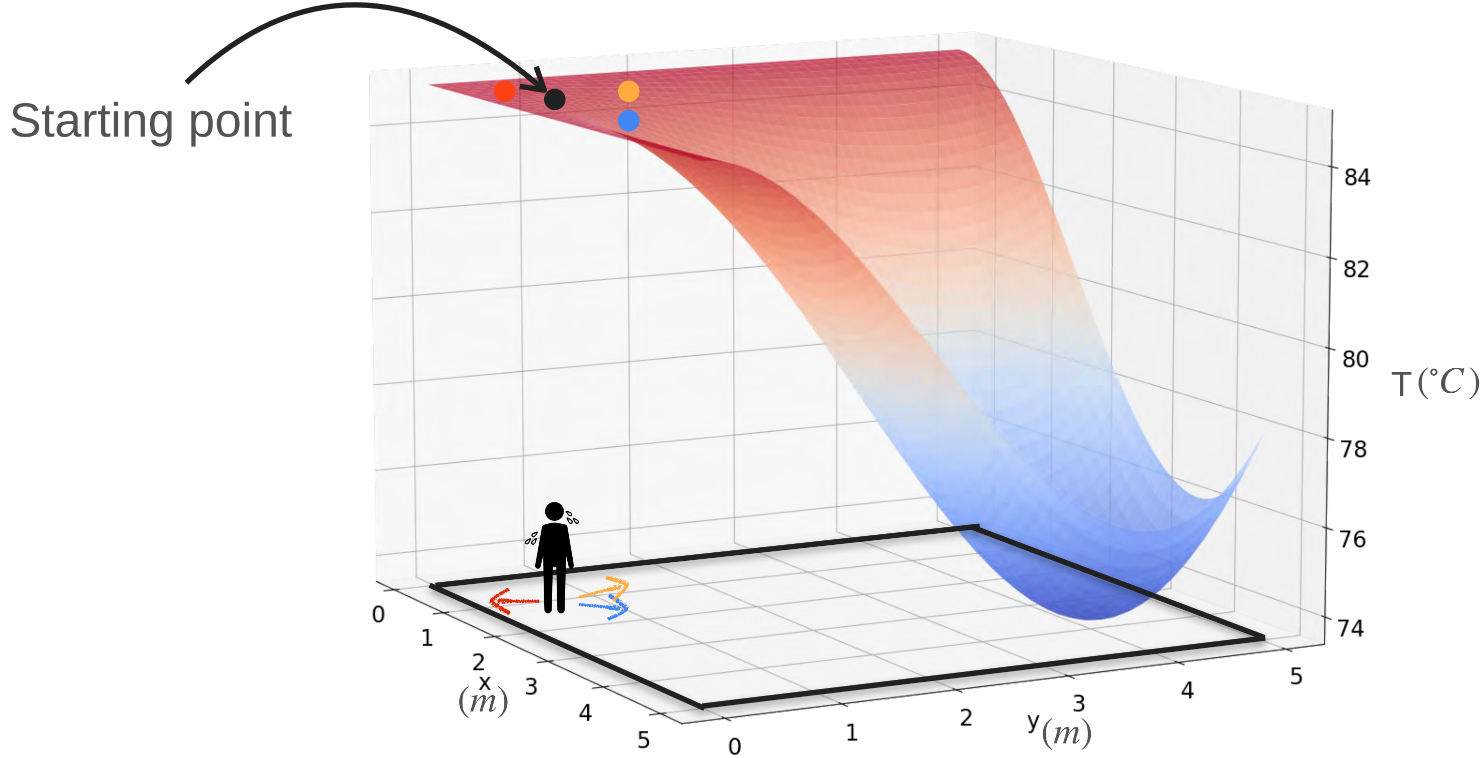


Motivation for Optimization in Two Variables

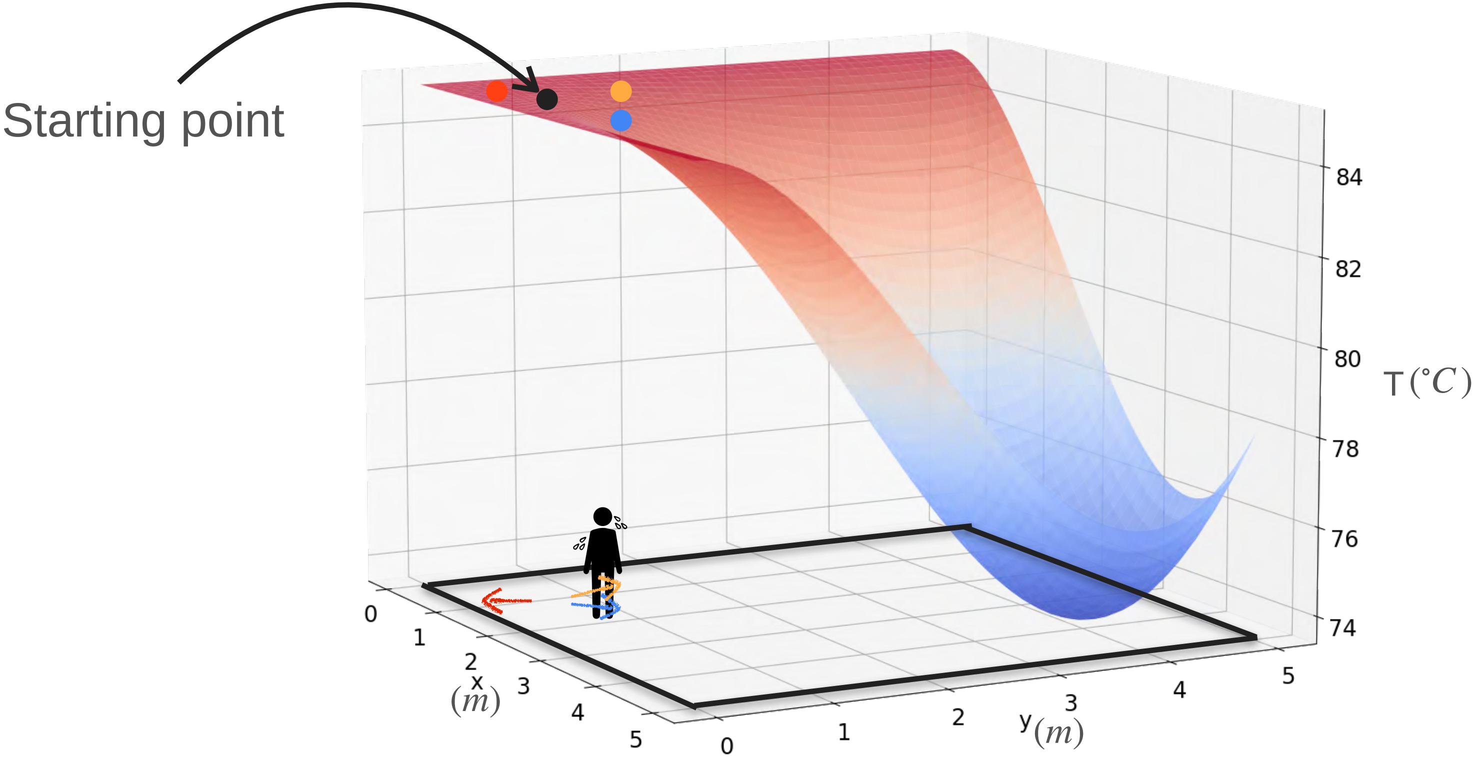
Starting point



Motivation for Optimization in Two Variables



Motivation for Optimization in Two Variables



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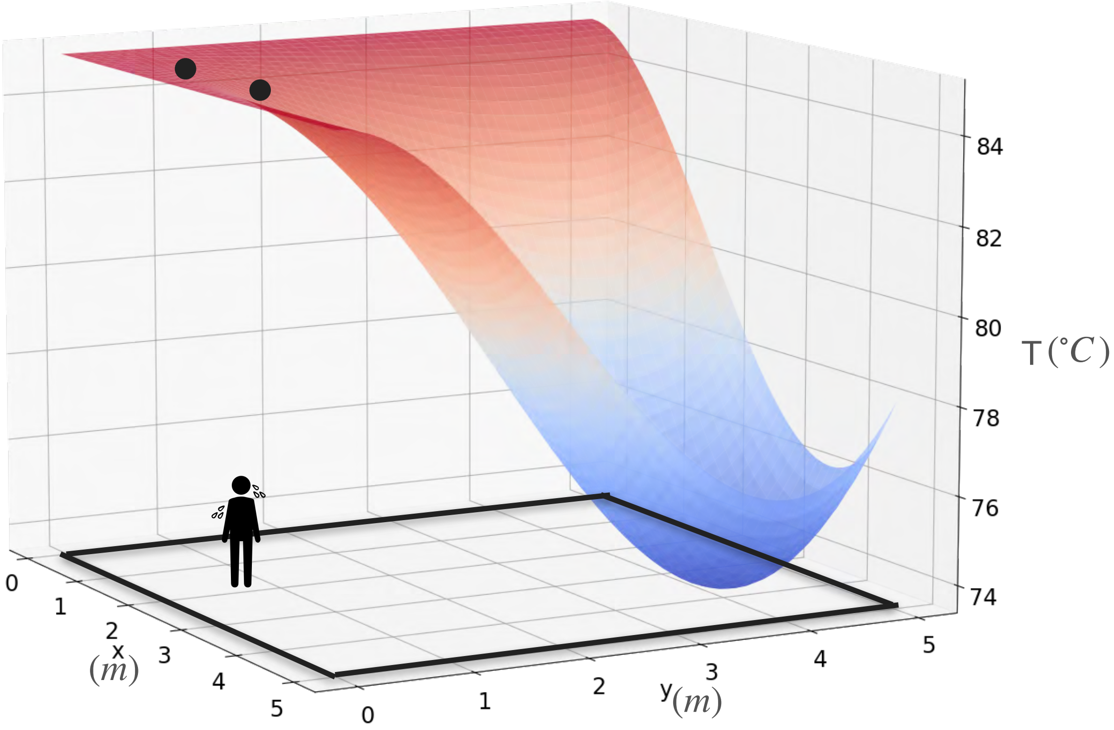
Starting point



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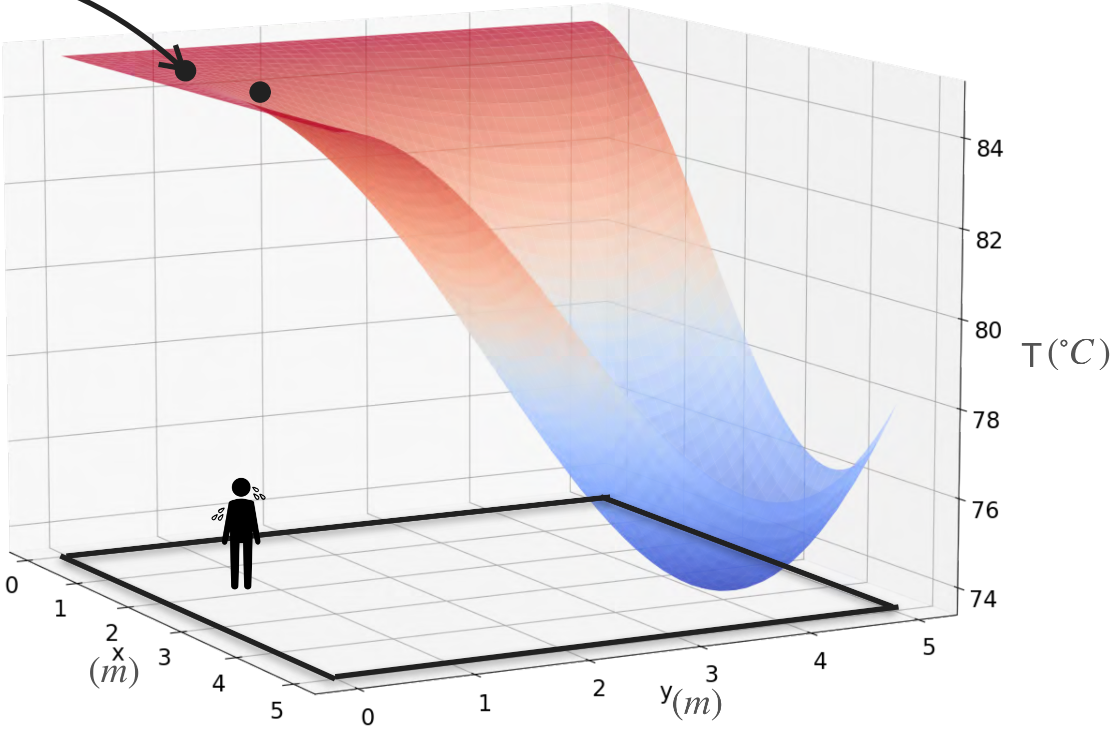
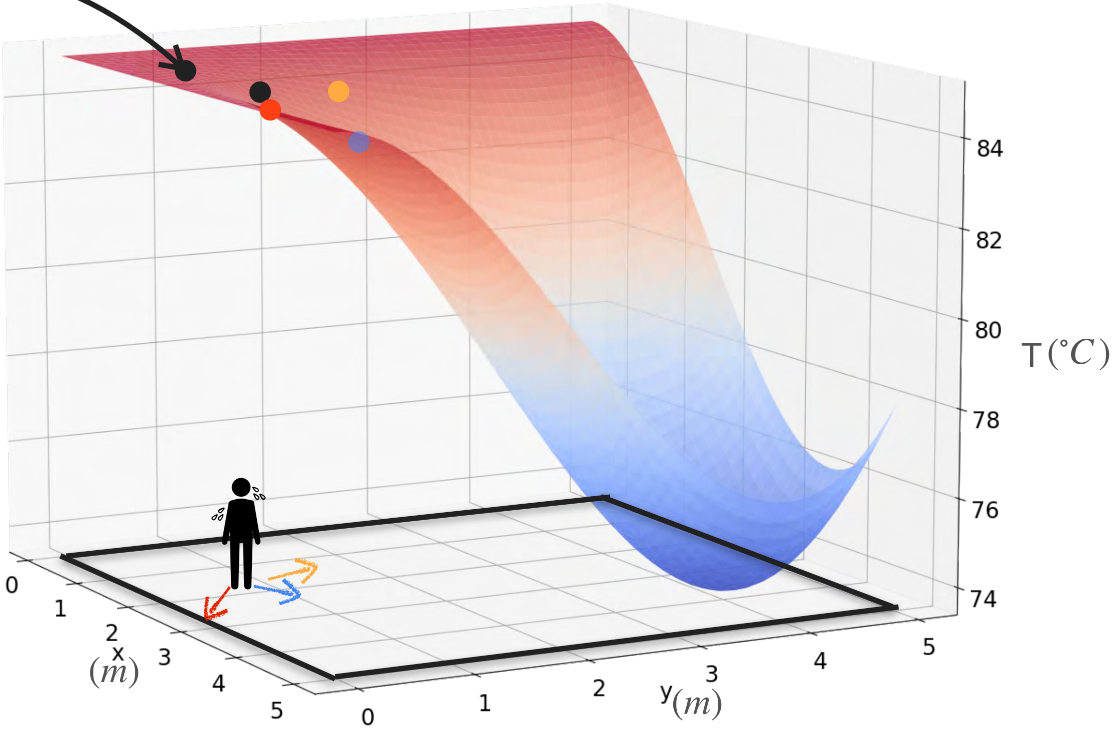


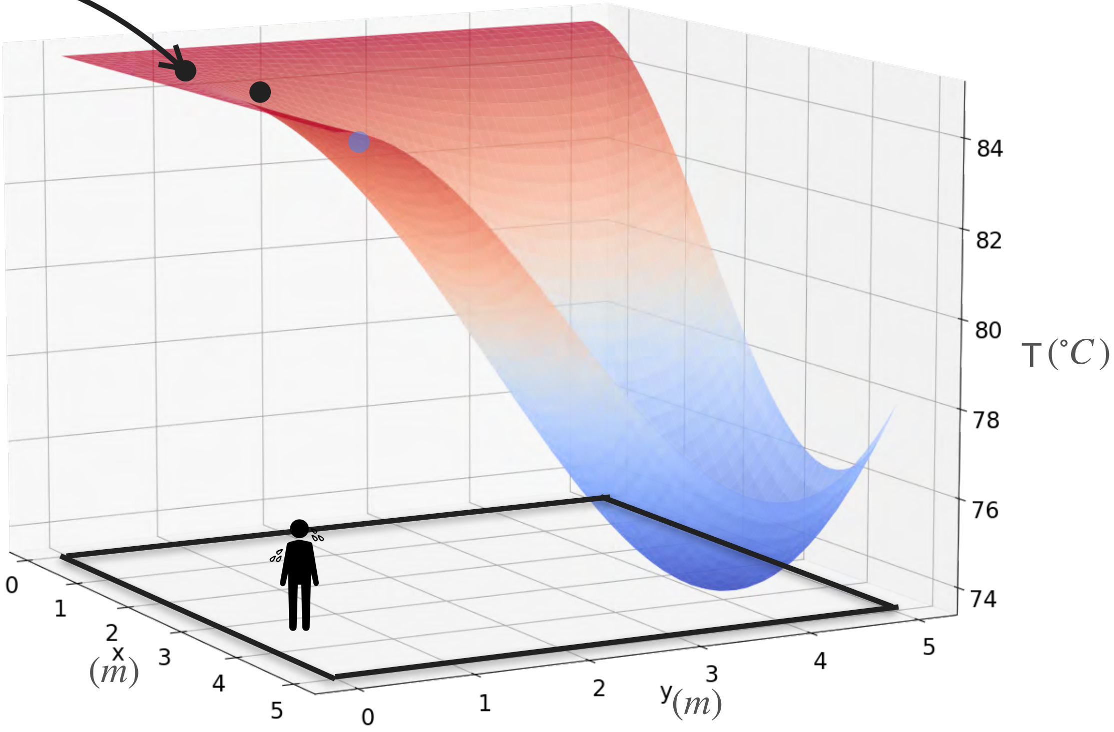
Motivation for Optimization in Two Variables

Starting point

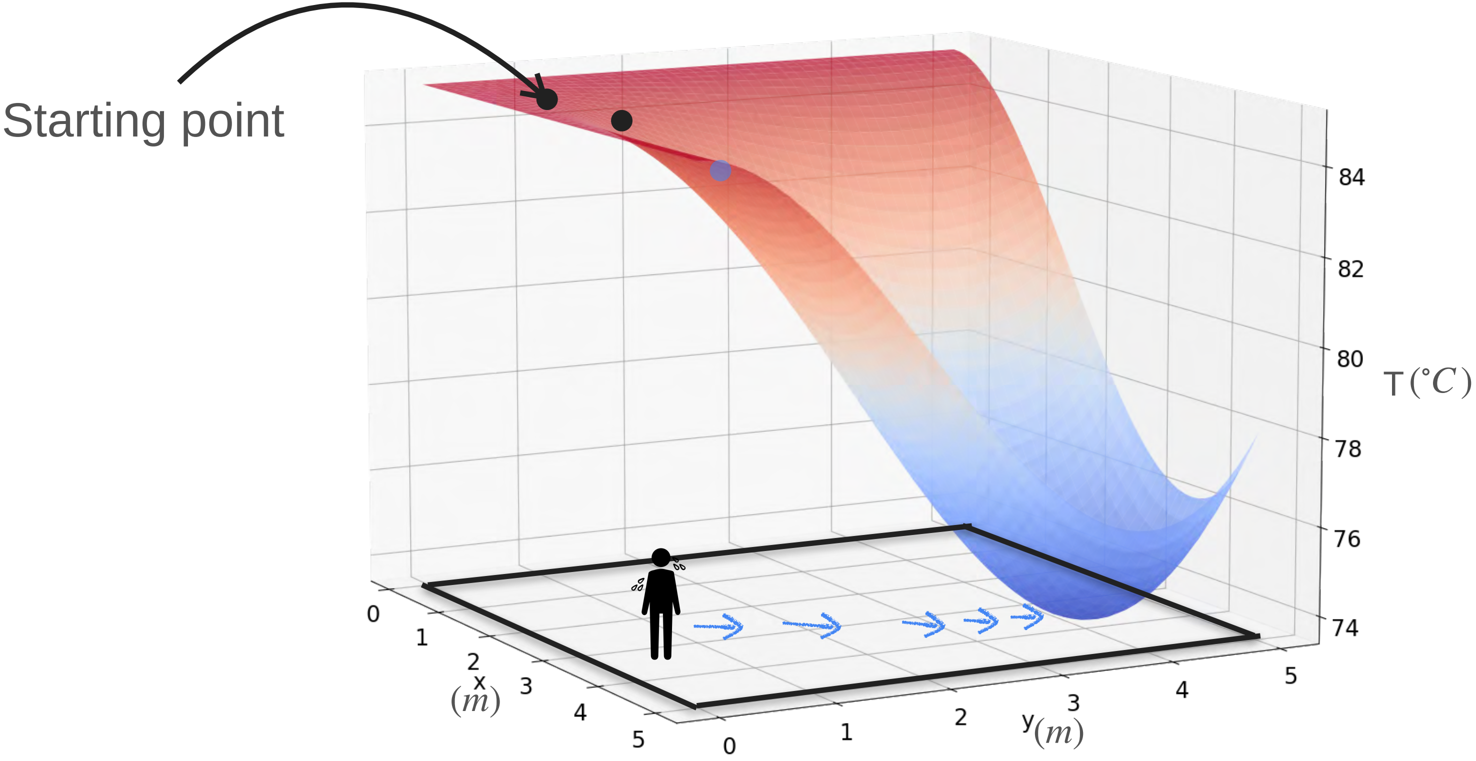


Motivation for Optimization in Two Variables

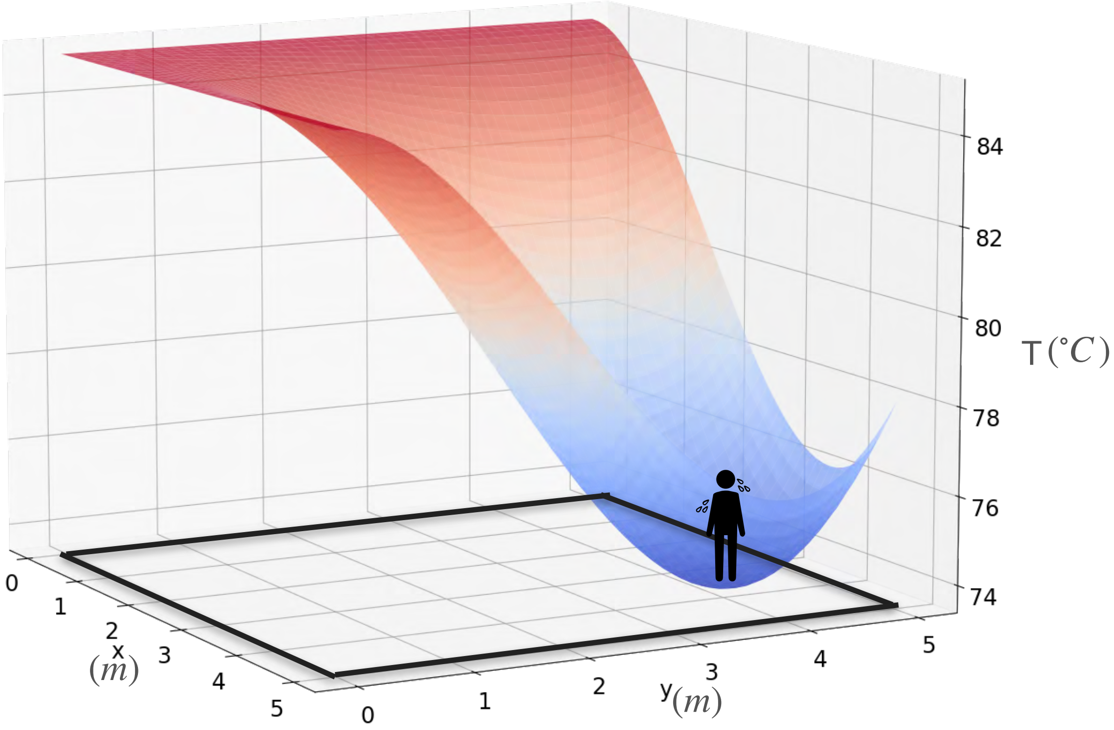
Starting point



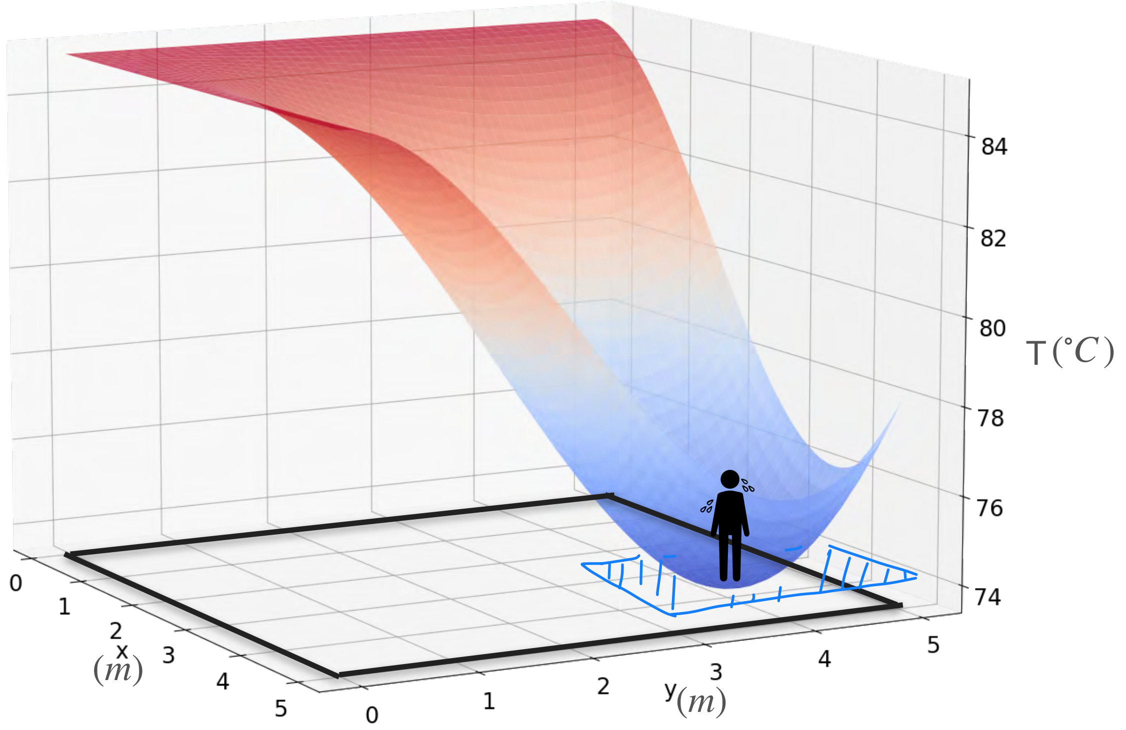
Motivation for Optimization in Two Variables



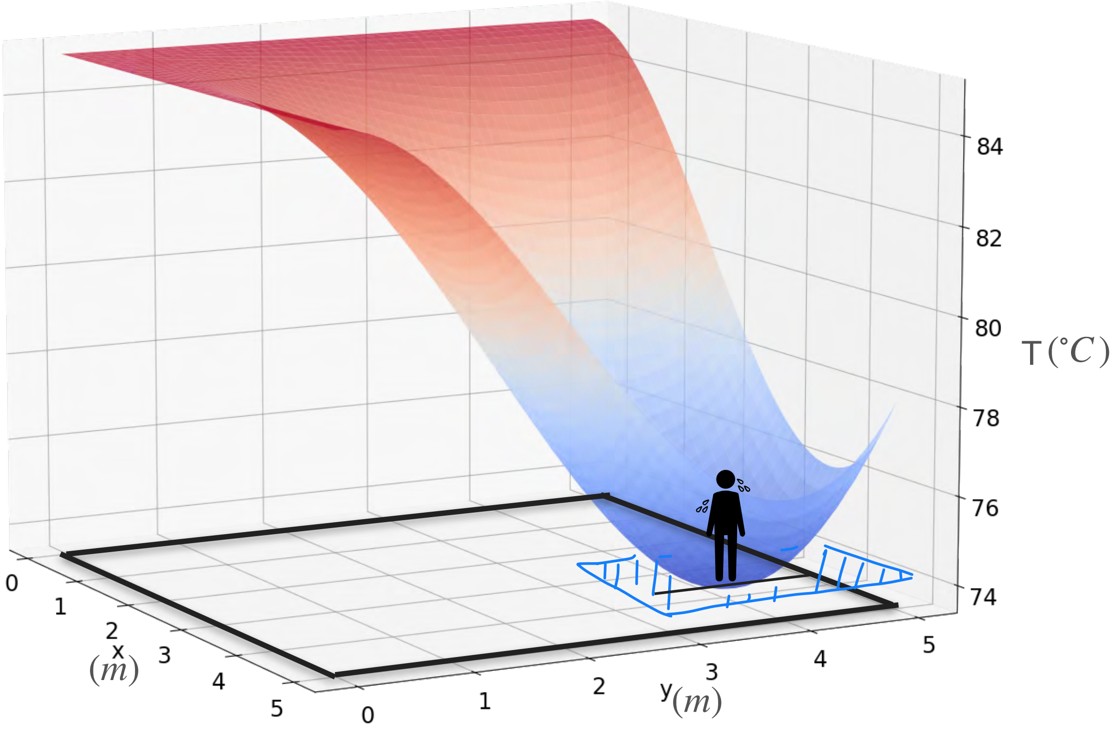
Motivation for Optimization in Two Variables



Motivation for Optimization in Two Variables

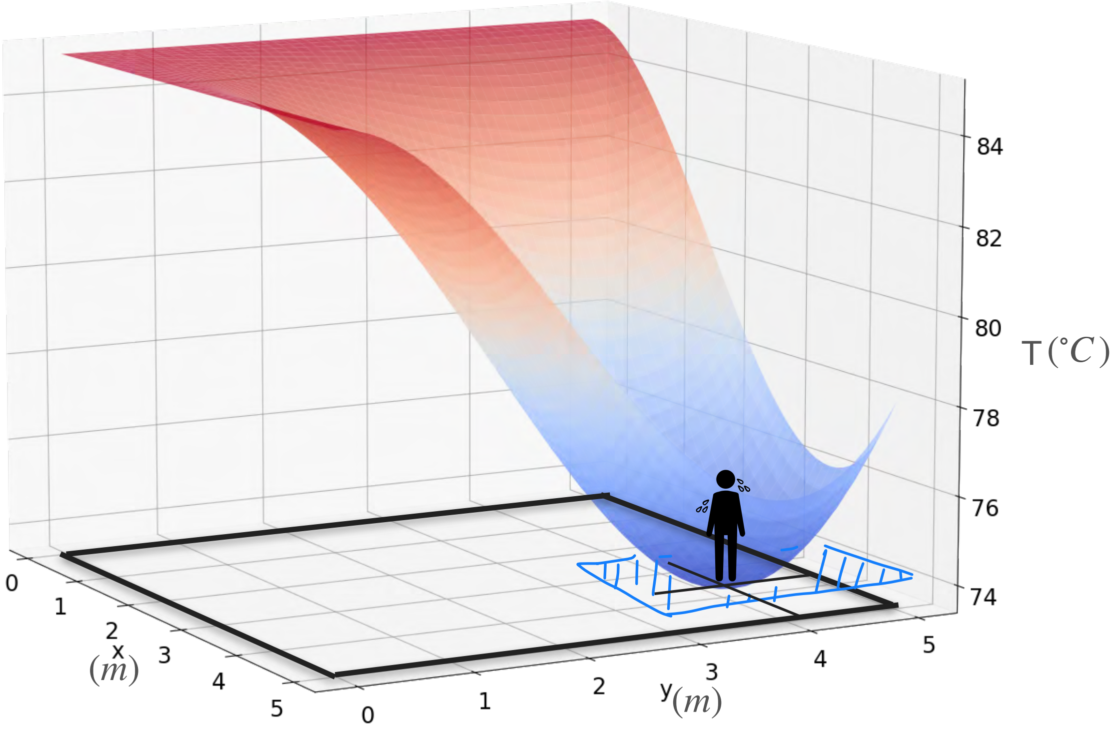


Motivation for Optimization in Two Variables



$$\frac{\partial T}{\partial x} = 0$$

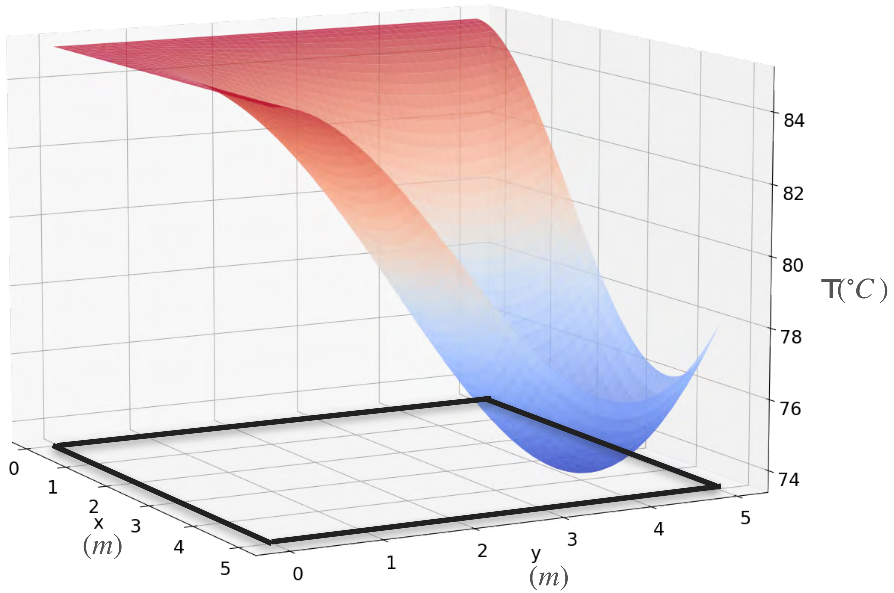
Motivation for Optimization in Two Variables



$$\frac{\partial T}{\partial x} = 0$$

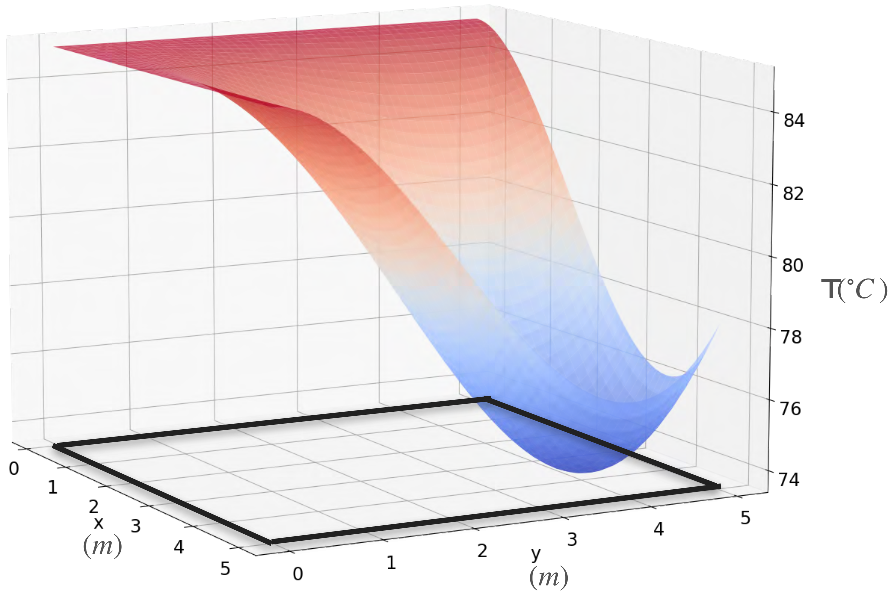
$$\frac{\partial T}{\partial y} = 0$$

Exercise



Exercise

$$T = f(x, y) = 85 - \frac{1}{90}x^2(x-6)y^2(y-6)$$



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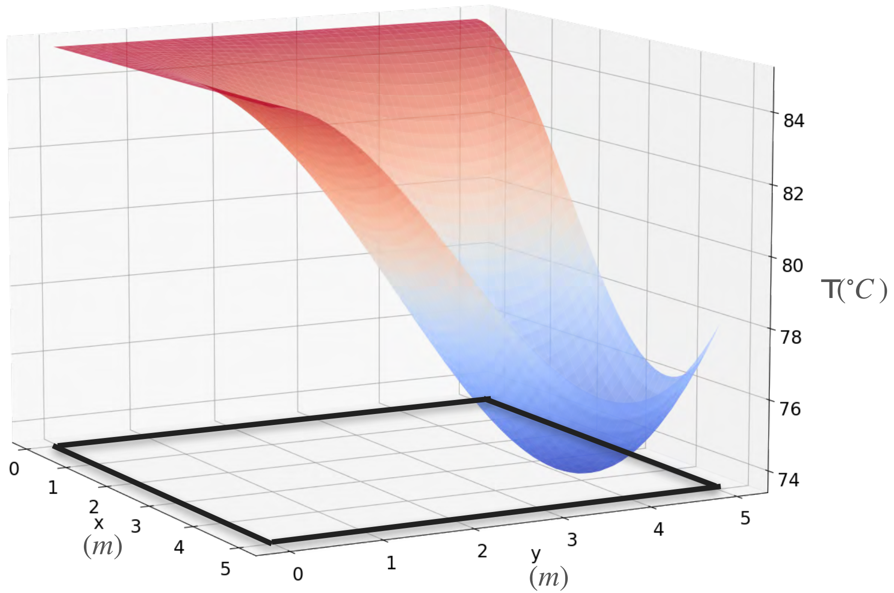

Try and calculate $\frac{\partial f}{\partial x}$

Exercise

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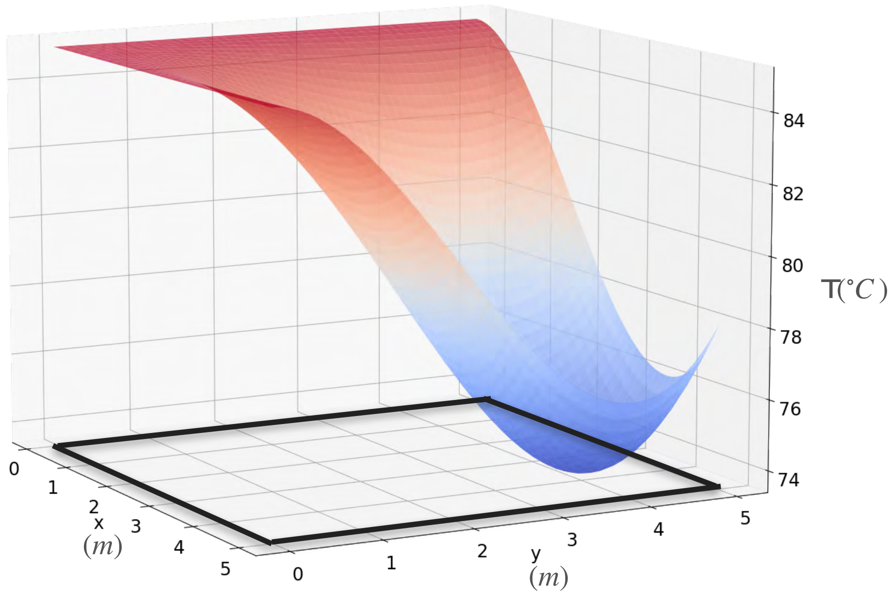
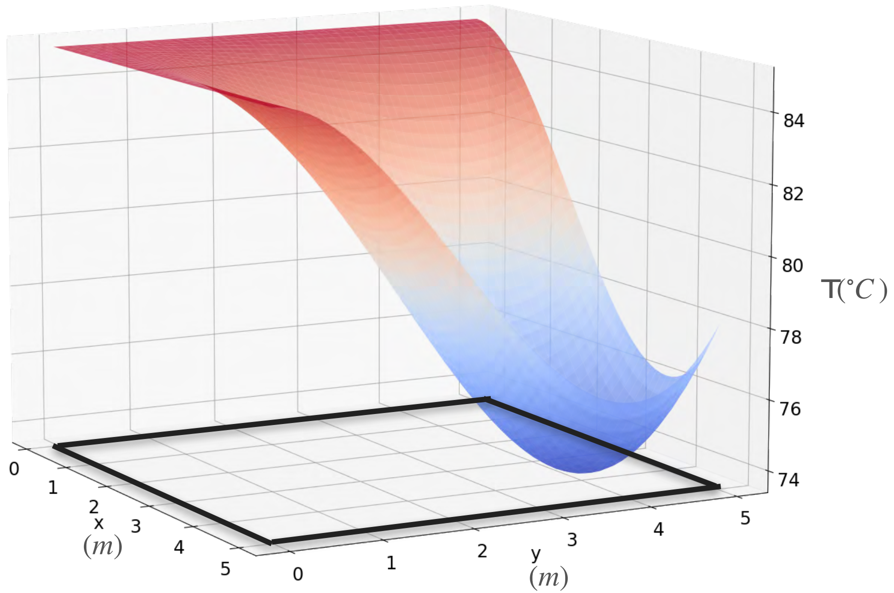


Try and calculate $\frac{\partial f}{\partial x}$

and $\frac{\partial f}{\partial y}$

Motivation for Optimization in Two Variables

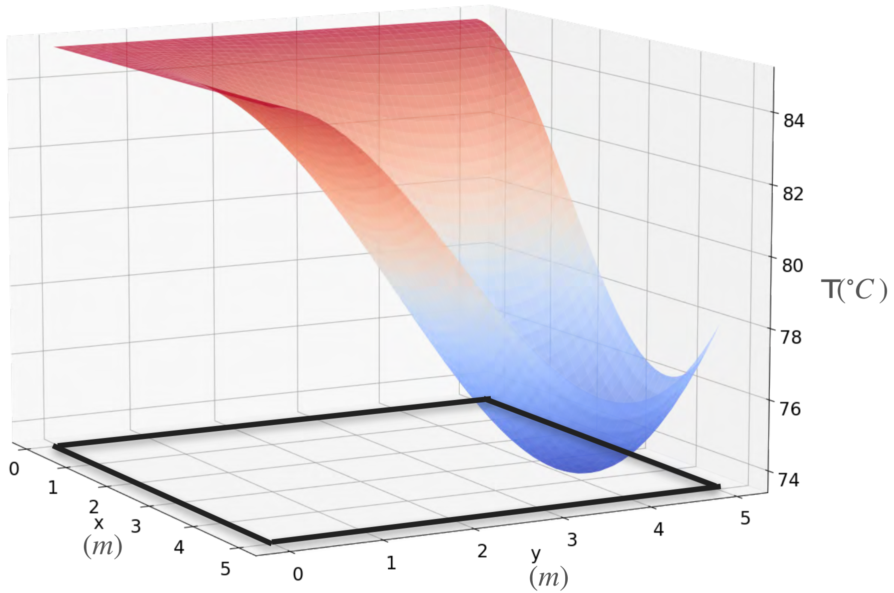
$$T = f(x, y) = 85 - \frac{1}{90}x^2(x-6)y^2(y-6)$$



Motivation for Optimization in Two Variables

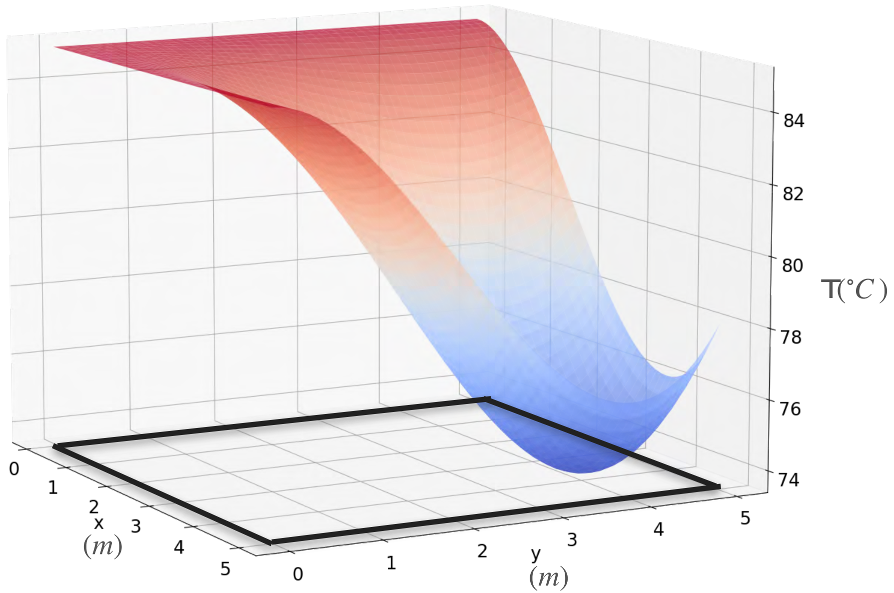
$$T = f(x, y) = 85 - \frac{1}{90}x^2(x-6)y^2(y-6)$$

$$\frac{\partial f}{\partial x} = -\frac{1}{90}x(3x-12)y^2(y-6)$$



Motivation for Optimization in Two Variables

$$T = f(x, y) = 85 - \frac{1}{90}x^2(x-6)y^2(y-6)$$

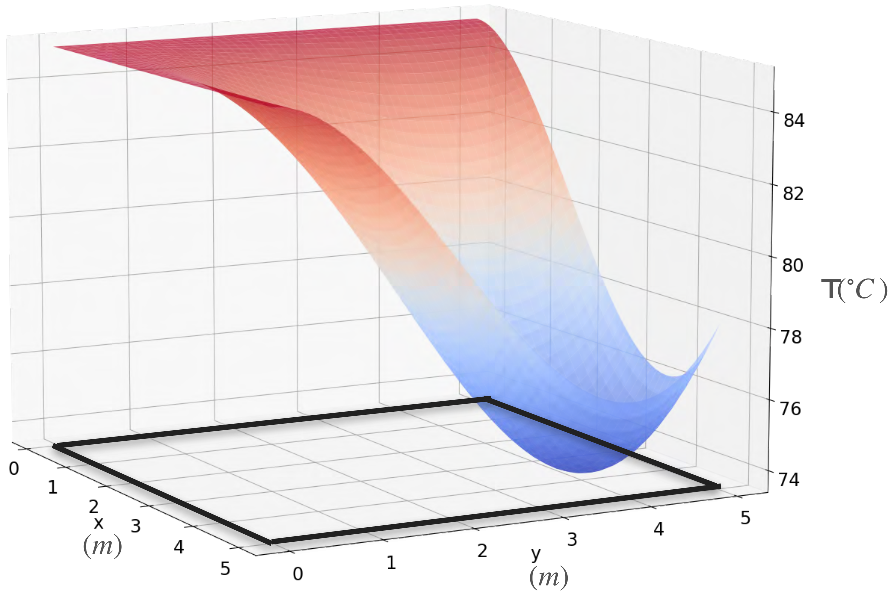


$$\frac{\partial f}{\partial x} = -\frac{1}{90}x(3x-12)y^2(y-6)$$

$$\frac{\partial f}{\partial y} = -\frac{1}{90}x^2(x-6)y(3y-12)$$

Motivation for Optimization in Two Variables

$$T = f(x, y) = 85 - \frac{1}{90}x^2(x-6)y^2(y-6)$$



$$\frac{\partial f}{\partial x} = -\frac{1}{90}x(3x-12)y^2(y-6) = 0$$

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Motivation for Optimization in Two Variables

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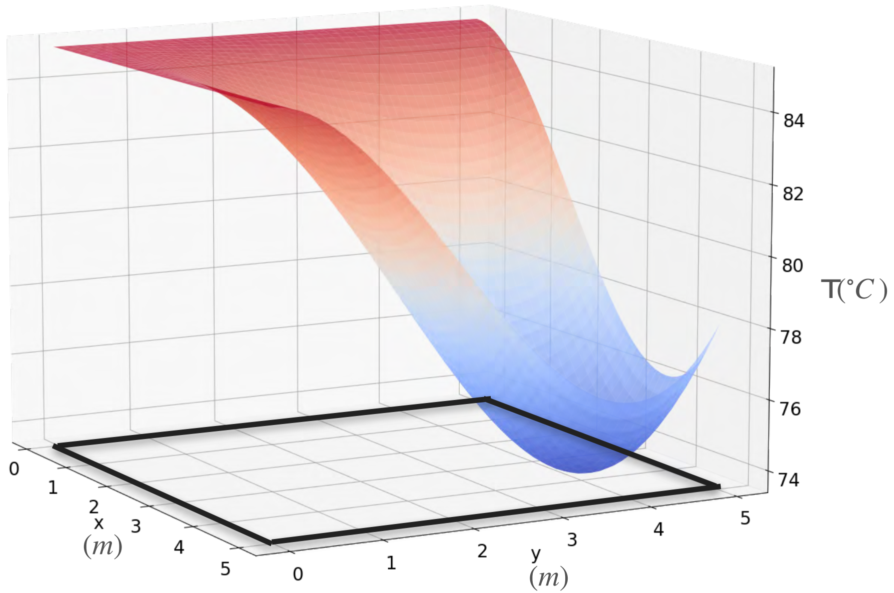


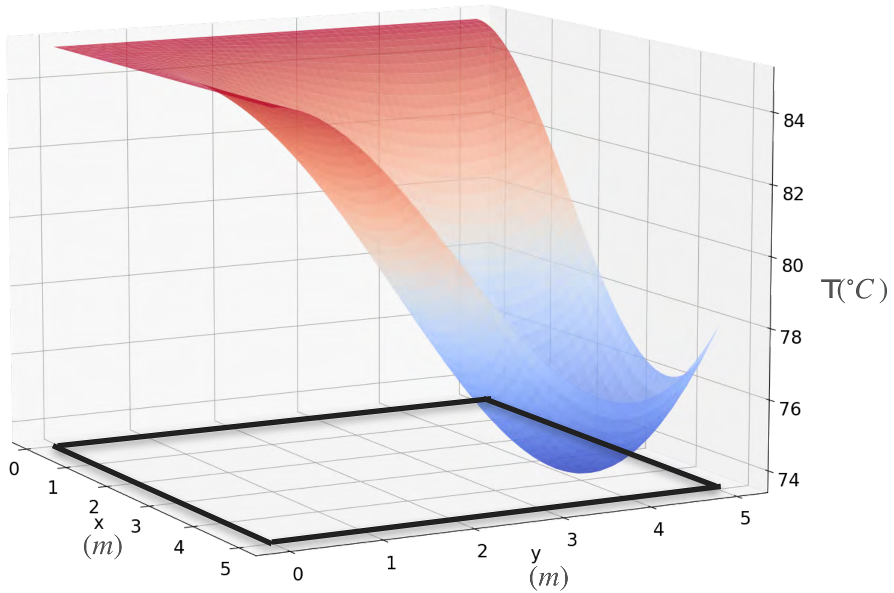
$$\frac{\partial f}{\partial x} = -\frac{1}{90}\boxed{x}(3x-12)y^2(y-6) = 0$$

$$x = 0$$

$$\frac{\partial f}{\partial y} = -\frac{1}{90}x^2(x-6)y(3y-12) = 0$$

Motivation for Optimization in Two Variables

$$T = f(x, y) = 85 - \frac{1}{90}x^2(x-6)y^2(y-6)$$

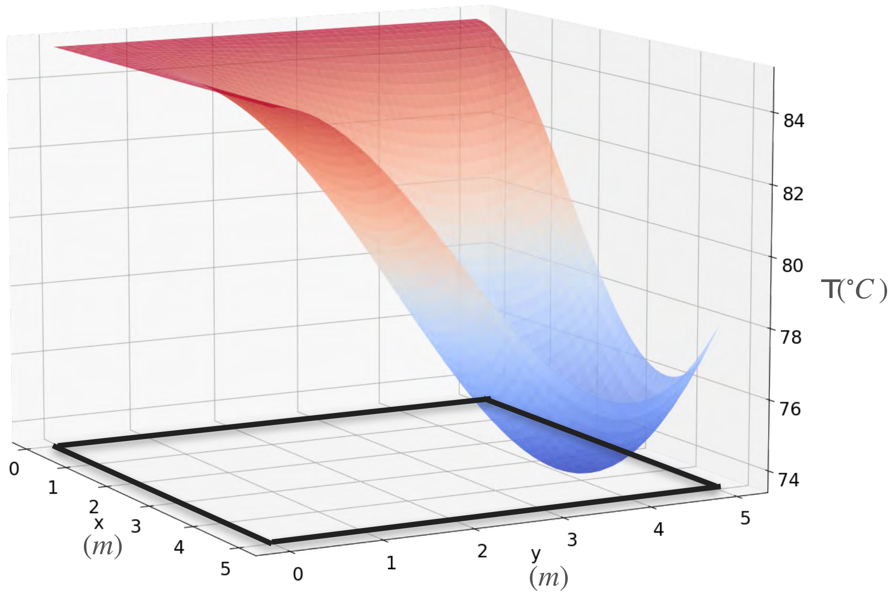


$$\frac{\partial f}{\partial x} = -\frac{1}{90}\underbrace{\boxed{x}}_{x=0}\underbrace{\boxed{(3x-12)}}_{x=4}y^2(y-6) = 0$$

$$\frac{\partial f}{\partial y} = -\frac{1}{90}x^2(x-6)y(3y-12) = 0$$

Motivation for Optimization in Two Variables

$$T = f(x, y) = 85 - \frac{1}{90}x^2(x-6)y^2(y-6)$$



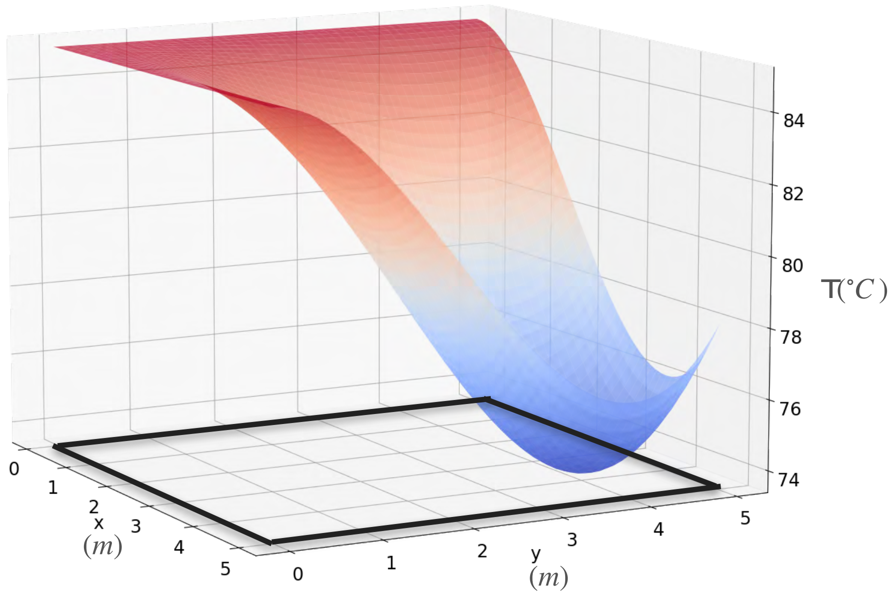
$$\frac{\partial f}{\partial x} = -\frac{1}{90}\boxed{x}\boxed{(3x-12)}\boxed{y^2}(y-6) = 0$$

$x = 0$ $\quad x = 4$ $\quad y = 0$

$$\frac{\partial f}{\partial y} = -\frac{1}{90}x^2(x-6)y(3y-12) = 0$$

Motivation for Optimization in Two Variables

$$T = f(x, y) = 85 - \frac{1}{90}x^2(x-6)y^2(y-6)$$



$$\frac{\partial f}{\partial x} = -\frac{1}{90}\underbrace{\boxed{x}}_{x=0}\underbrace{\boxed{(3x-12)}}_{x=4}\underbrace{\boxed{y^2}}_{y=0}\underbrace{\boxed{(y-6)}}_{y=6} = 0$$

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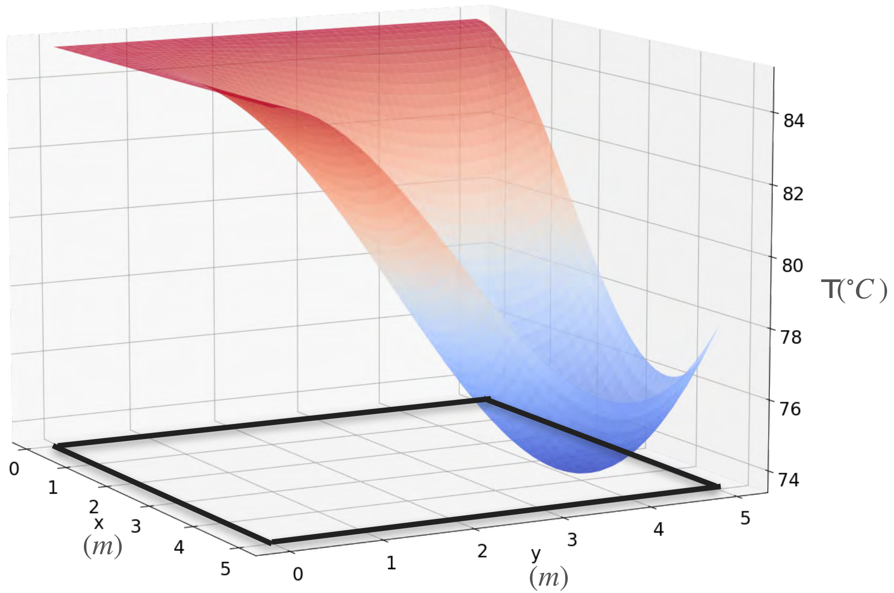

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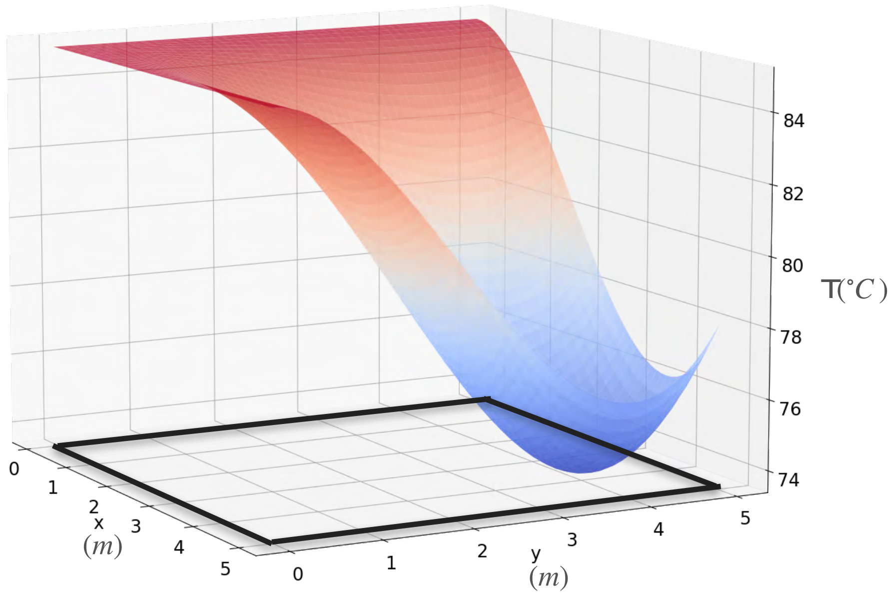

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Motivation for Optimization in Two Variables

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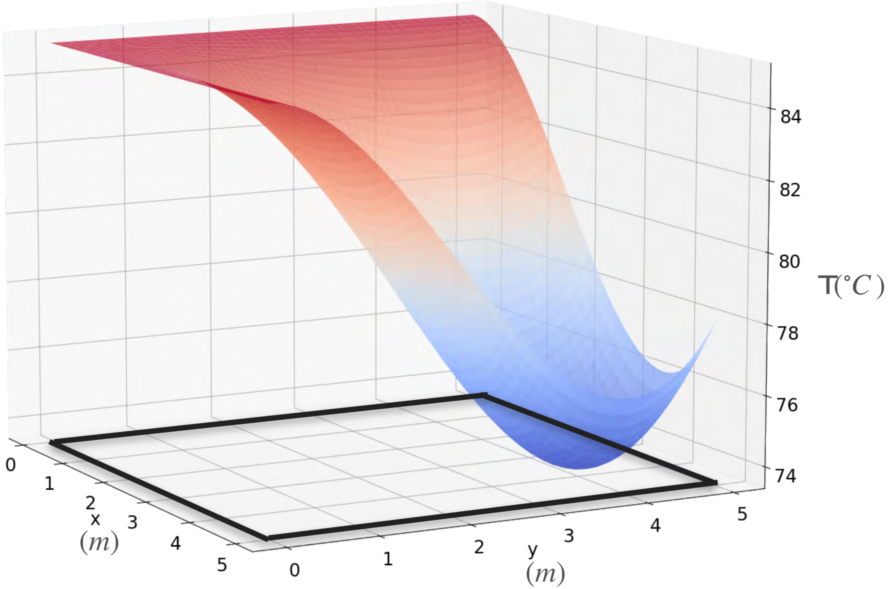

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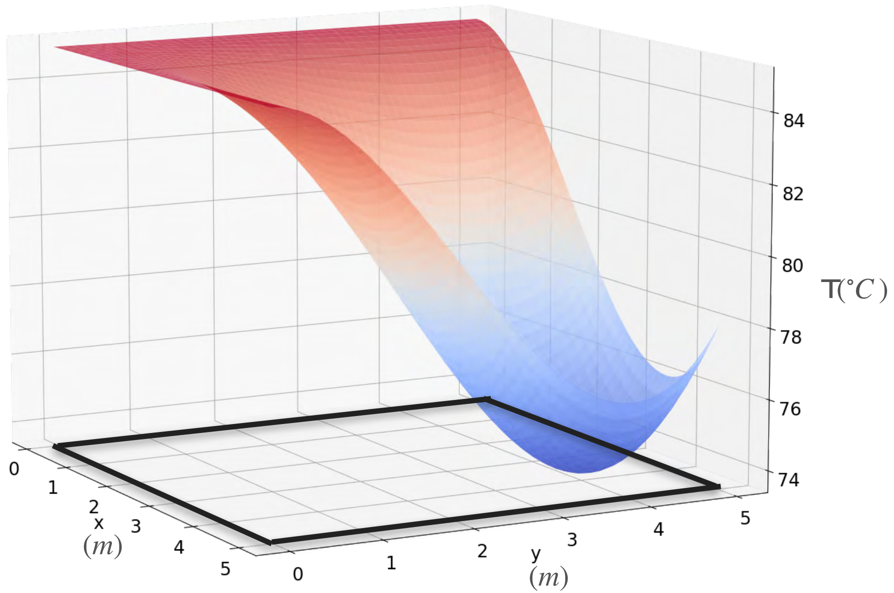

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Motivation for Optimization in Two Variables

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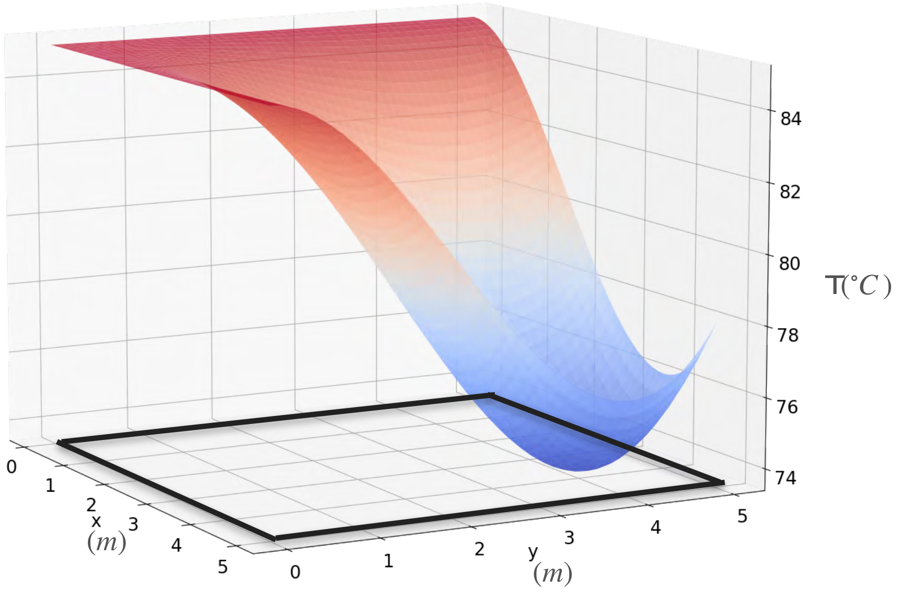
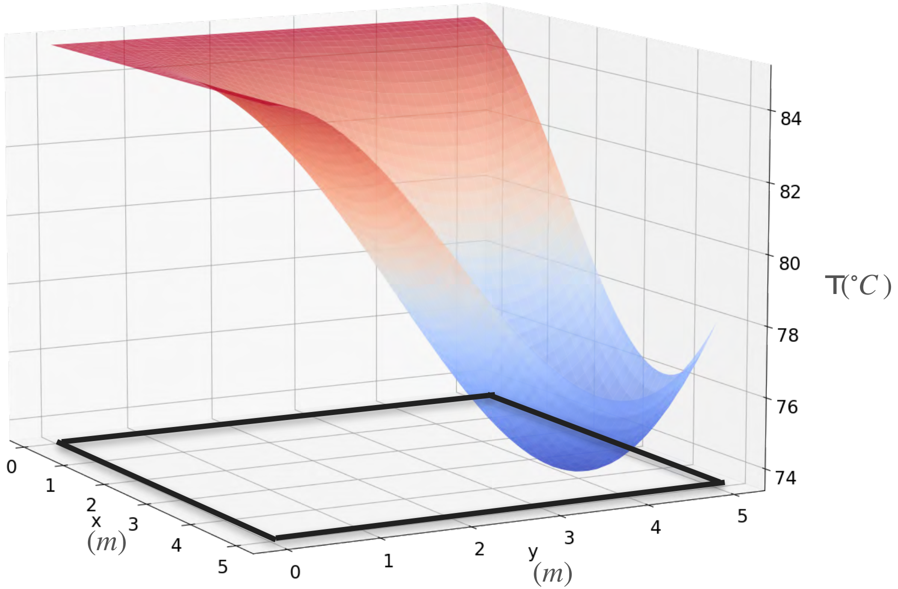


Motivation for Optimization in Two Variables

$$T = f(x, y) = 85 - \frac{1}{90}x^2(x-6)y^2(y-6)$$

Candidate points for the minima



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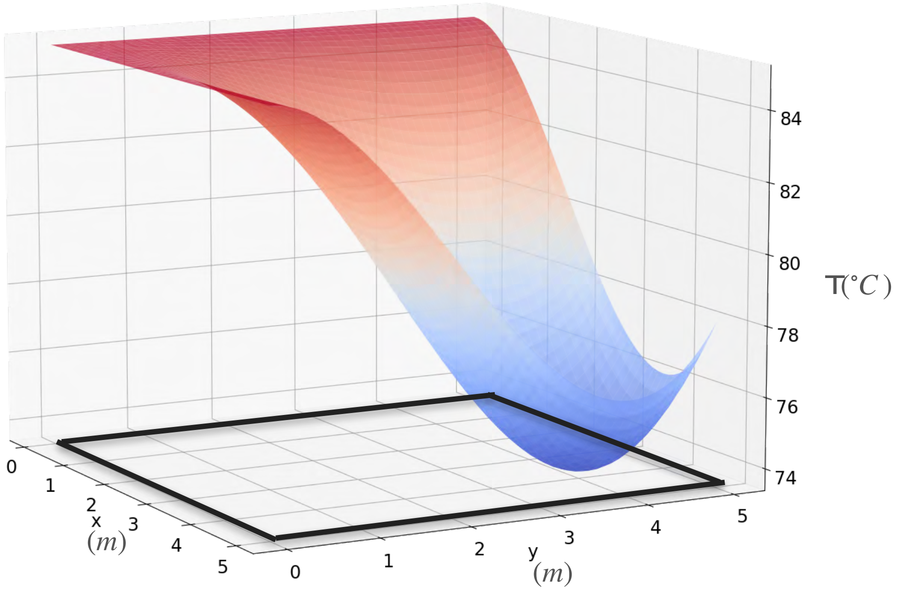


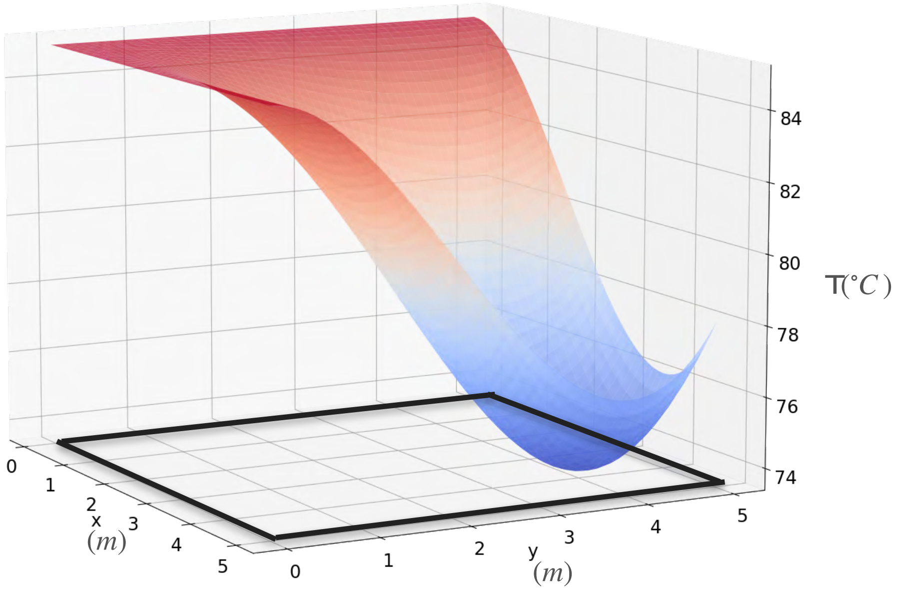
Candidate points for the minima

$$x = 0$$
$$y = 0$$

$$x = 0,\ y = 0$$
$$x = 0,\ y = 4$$
$$x = 0,\ y = 6$$
$$x = 4,\ y = 0$$
$$x = 4,\ y = 4$$
$$x = 6,\ y = 0$$
$$x = 6,\ y = 6$$

Motivation for Optimization in Two Variables

$$T = f(x, y) = 85 - \frac{1}{90}x^2(x-6)y^2(y-6)$$



Candidate points for the minima

$x = 0$

$y = 0$

$x = 0,\ y = 0$

$x = 0,\ y = 4$

$\boxed{x = 0,\ y = 6}$ Outside

$x = 4,\ y = 0$

$x = 4,\ y = 4$

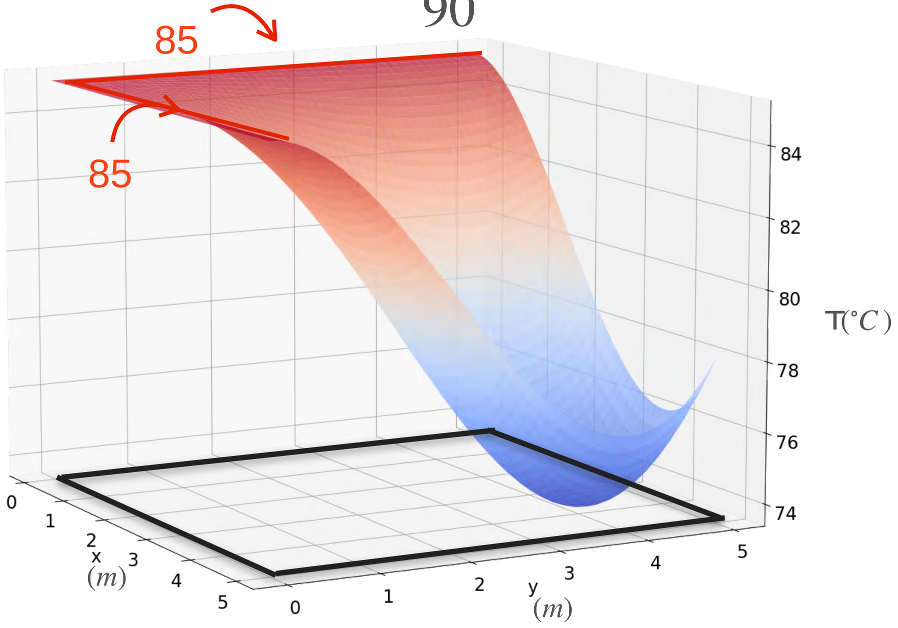
$x = 6,\ y = 0$

$x = 6,\ y = 6$

Outside

Motivation for Optimization in Two Variables

$$T = f(x, y) = 85 - \frac{1}{90}x^2(x-6)y^2(y-6)$$



Candidate points for the minima

$\boxed{\begin{matrix} x = 0 \\ y = 0 \end{matrix}}$ Maxima

$x = 0,\ y = 0$

$x = 0,\ y = 4$

$\boxed{x = 0,\ y = 6}$ Outside

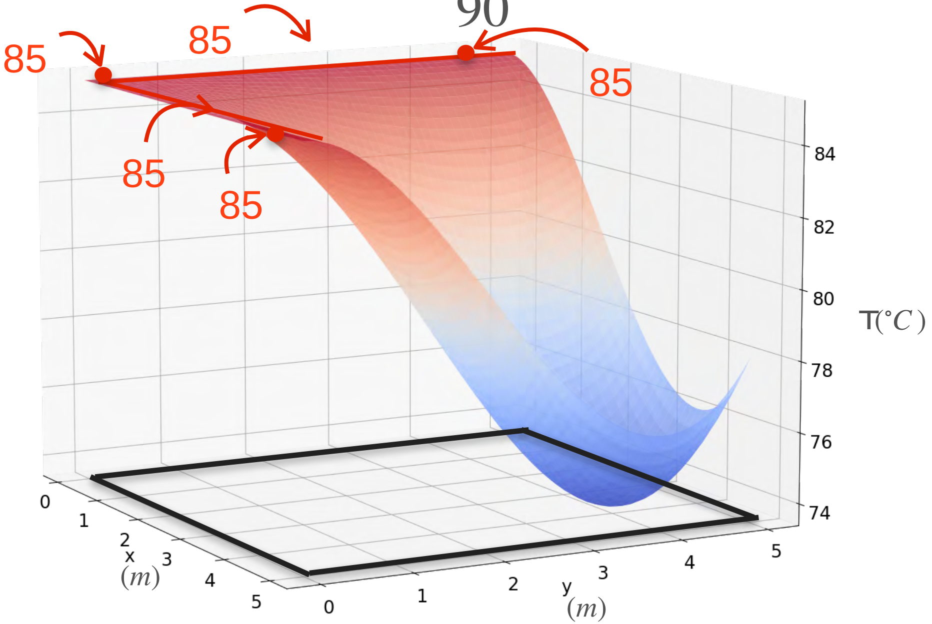
$x = 4,\ y = 0$

$x = 4,\ y = 4$

$\boxed{\begin{matrix} x = 6,\ y = 0 \\ x = 6,\ y = 6 \end{matrix}}$ Outside

Motivation for Optimization in Two Variables

$$T = f(x, y) = 85 - \frac{1}{90}x^2(x-6)y^2(y-6)$$



Candidate points for the minima

$x = 0$, $y = 0$	Maxima
$x = 0,\ y = 0$; $x = 0,\ y = 4$	Maxima
$x = 0,\ y = 6$	Outside
$x = 4,\ y = 0$	Maxima
$x = 4,\ y = 4$	
$x = 6,\ y = 0$; $x = 6,\ y = 6$	Outside

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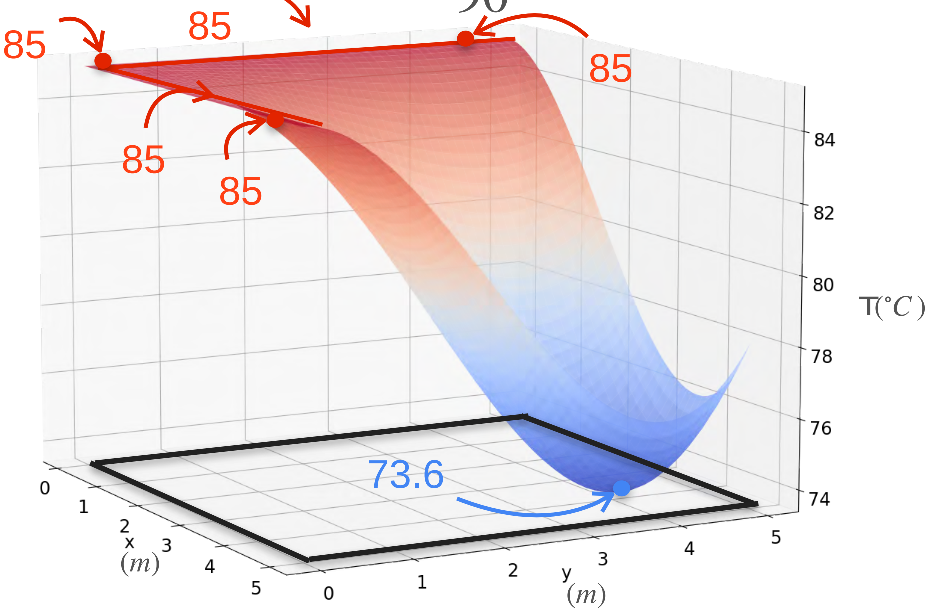
Motivation for Optimization in Two Variables

$$T = f(x,y) = 85 - \frac{1}{90}x^2(x-6)y^2(y-6)$$

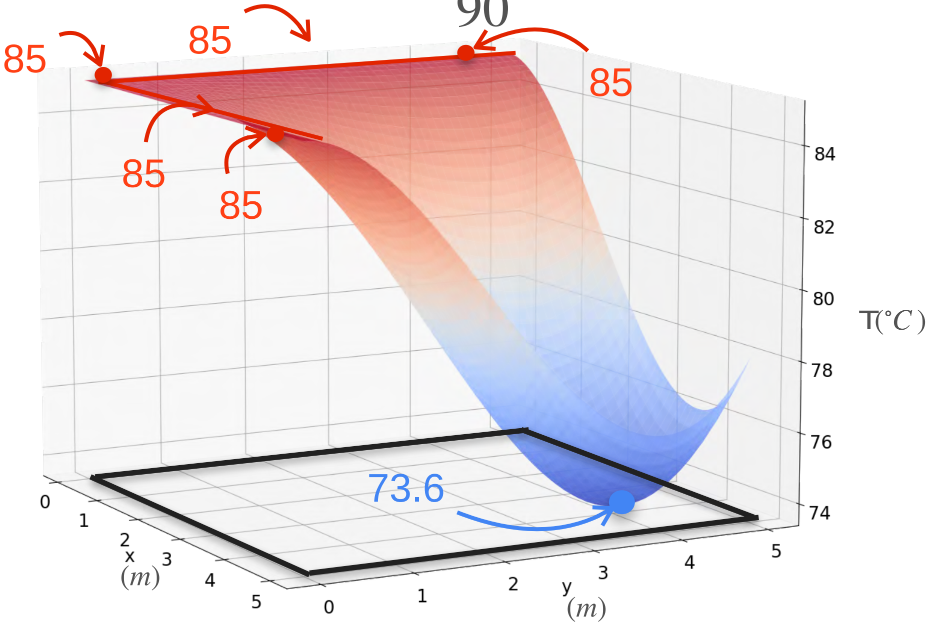


Candidate points for the minima

$x = 0$ $y = 0$	Maxima
$x = 0,\ y = 0$ $x = 0,\ y = 4$	Maxima
$x = 0,\ y = 6$	Outside
$x = 4,\ y = 0$	Maxima
$x = 4,\ y = 4$	
$x = 6,\ y = 0$ $x = 6,\ y = 6$	Outside

Motivation for Optimization in Two Variables

$$T = f(x, y) = 85 - \frac{1}{90}x^2(x-6)y^2(y-6)$$



Candidate points for the minima

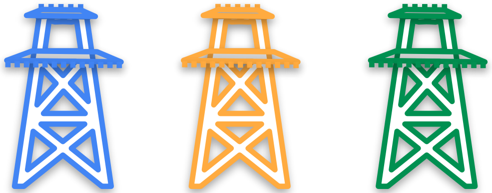
- $x = 0$, $y = 0$ — Maxima
- $x = 0, \ y = 0$; $x = 0, \ y = 4$ — Maxima
- $x = 0, \ y = 6$ — Outside
- $x = 4, \ y = 0$ — Maxima
- $x = 4, \ y = 4$ — Minimum
- $x = 6, \ y = 0$; $x = 6, \ y = 6$ — Outside

Gradients and Gradient Descent

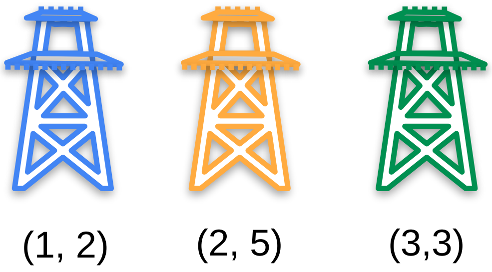
Optimization using gradients - Analytical method

Linear Regression: Analytical Approach

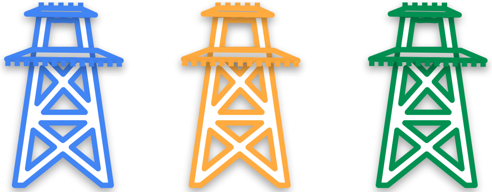
Linear Regression: Analytical Approach



Linear Regression: Analytical Approach

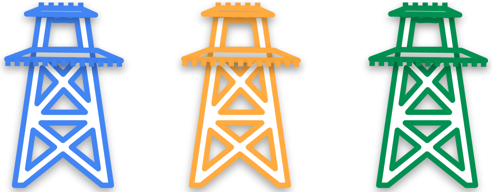


Linear Regression: Analytical Approach

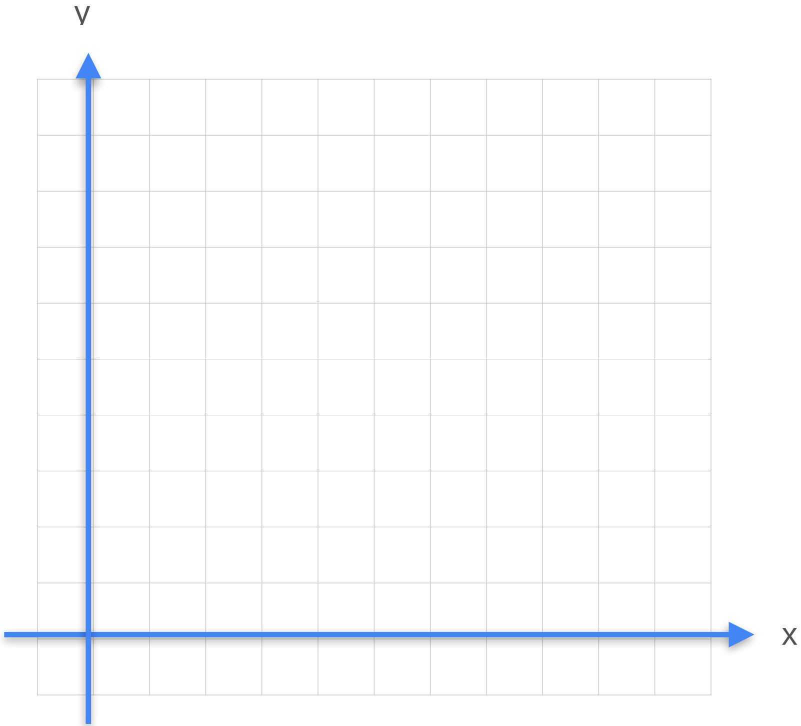


Location on the x,y plane (1, 2) (2, 5) (3,3)

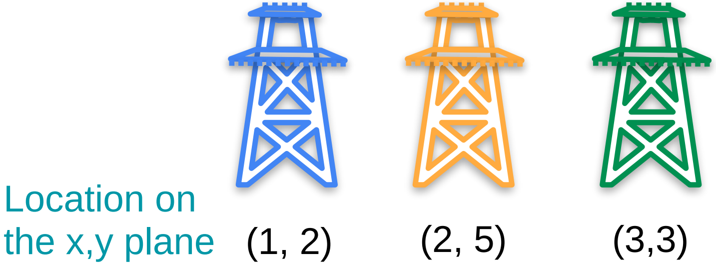
Linear Regression: Analytical Approach

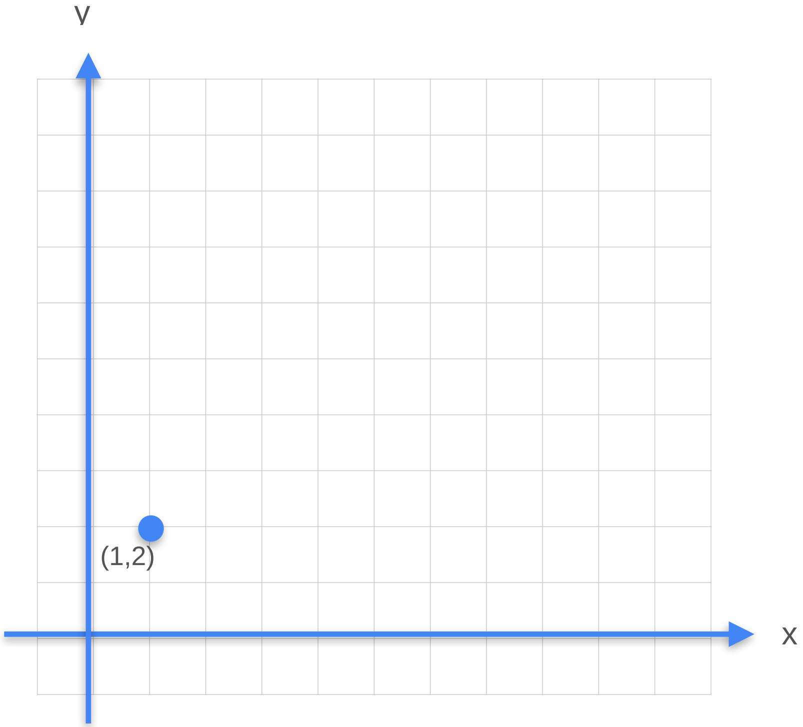


Location on the x,y plane (1, 2) (2, 5) (3,3)

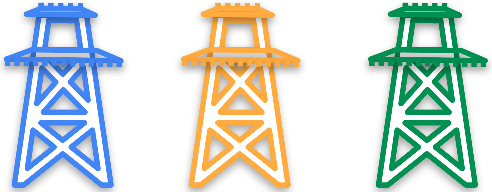


Linear Regression: Analytical Approach

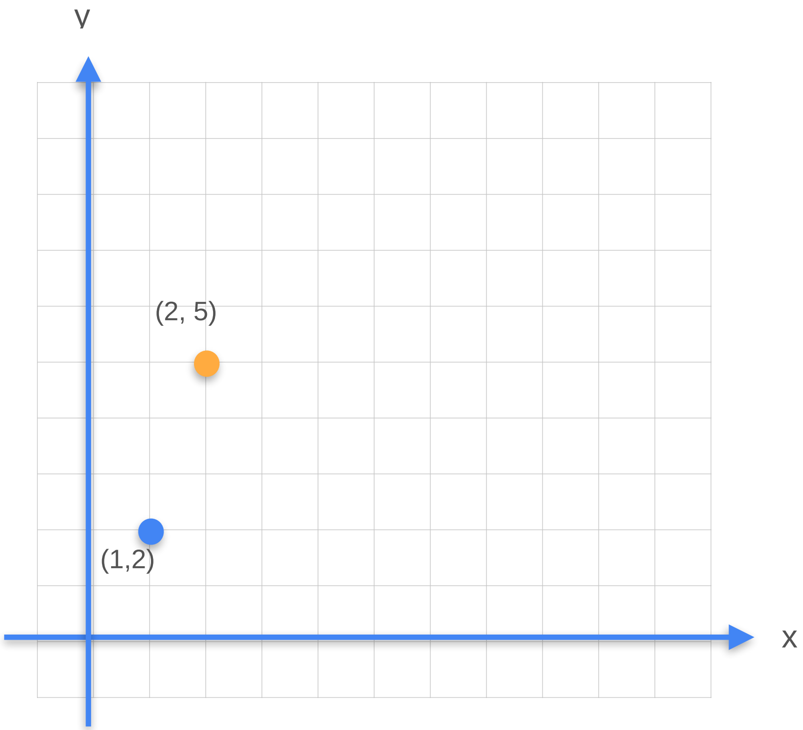




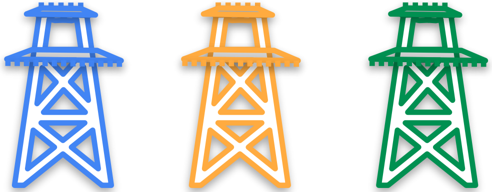
Linear Regression: Analytical Approach



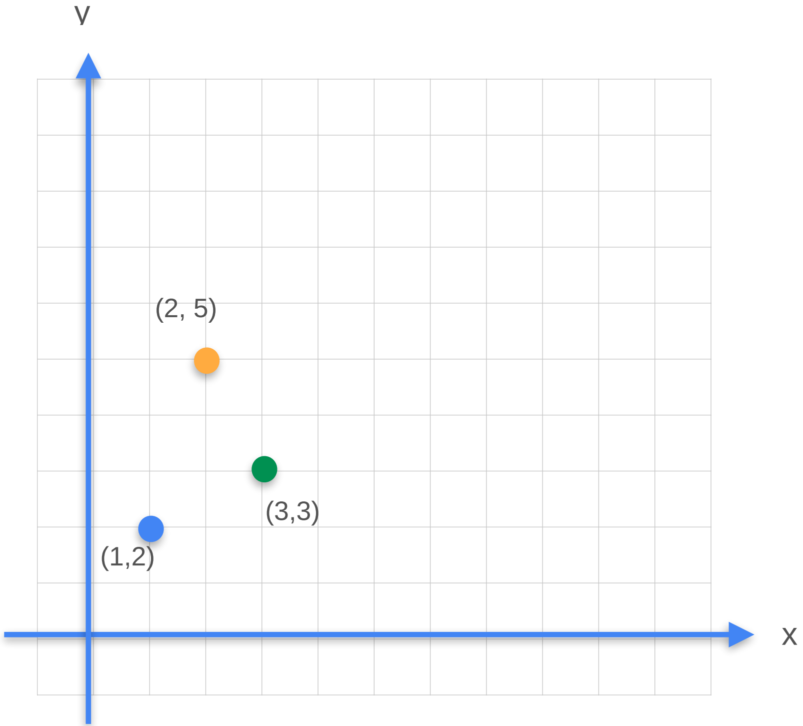
Location on the x,y plane (1, 2) (2, 5) (3,3)



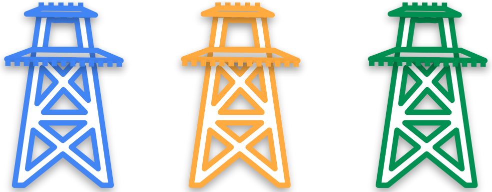
Linear Regression: Analytical Approach



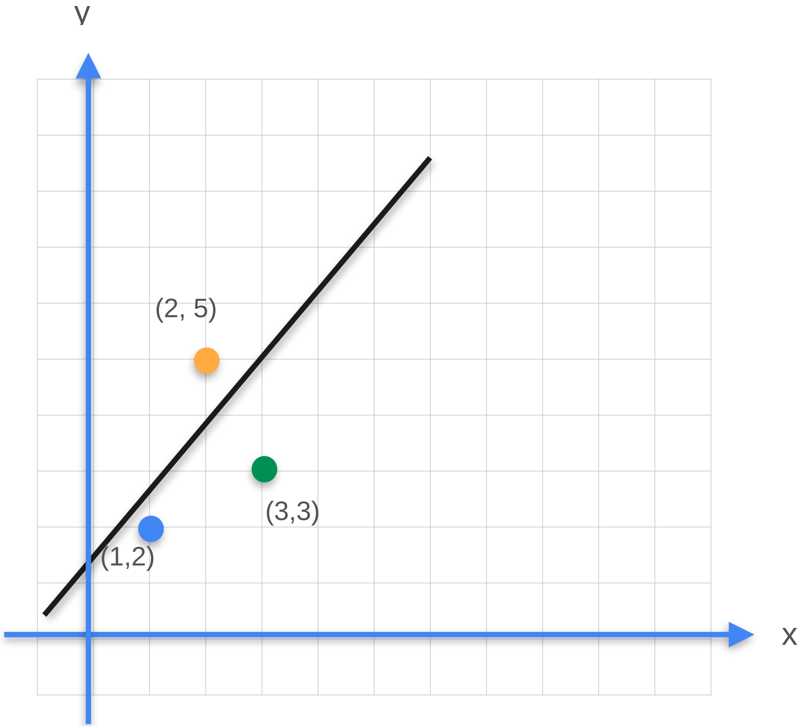
Location on the x,y plane (1, 2) (2, 5) (3,3)



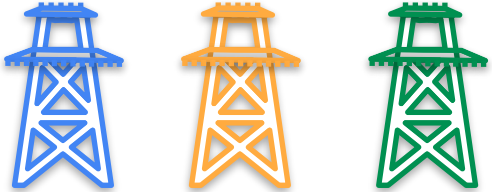
Linear Regression: Analytical Approach



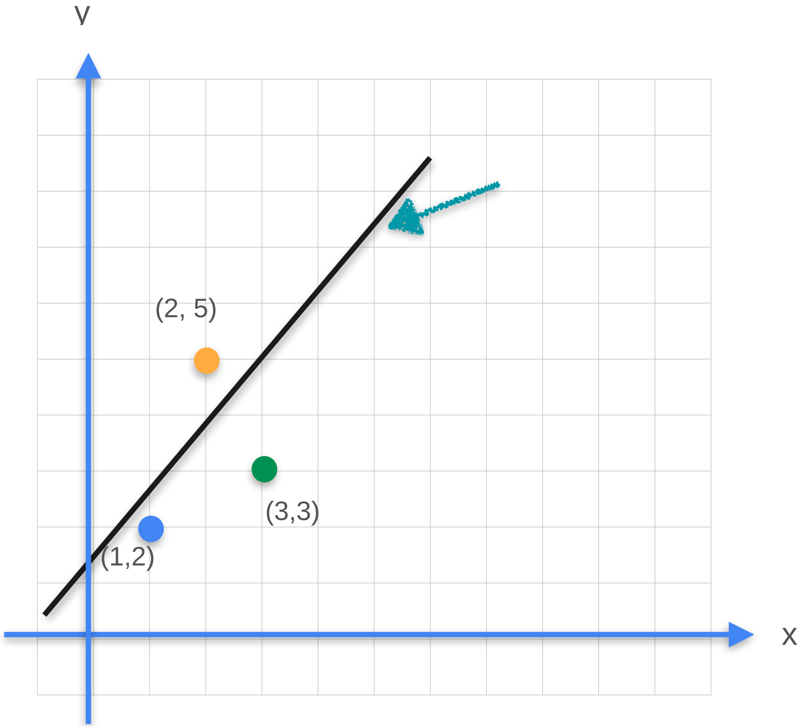
Location on the x,y plane (1, 2) (2, 5) (3,3)



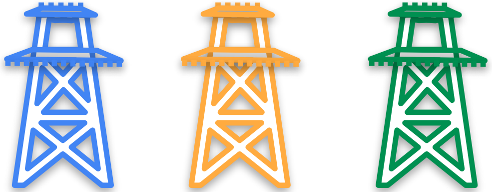
Linear Regression: Analytical Approach



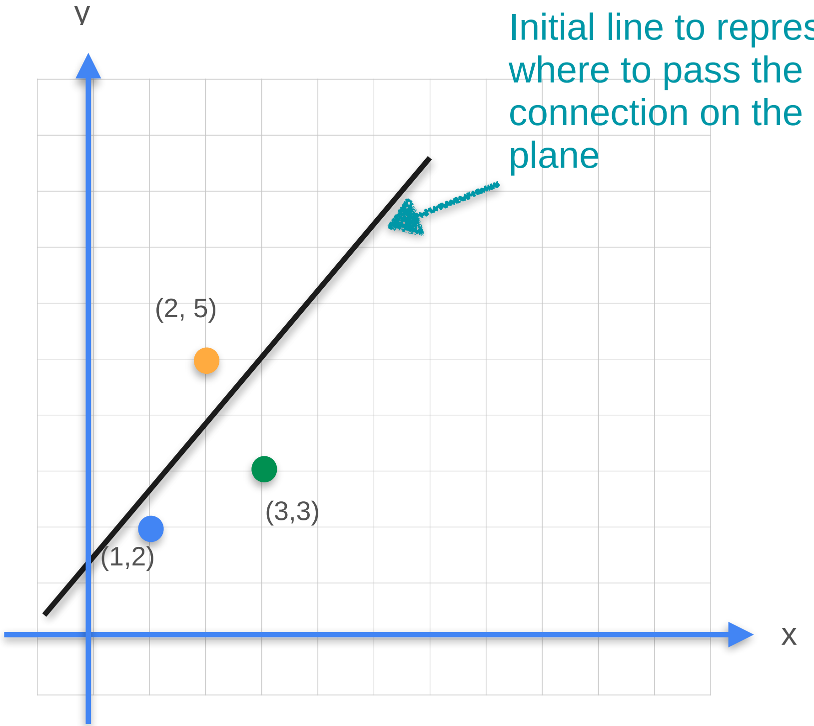
Location on the x,y plane (1, 2) (2, 5) (3,3)



Linear Regression: Analytical Approach

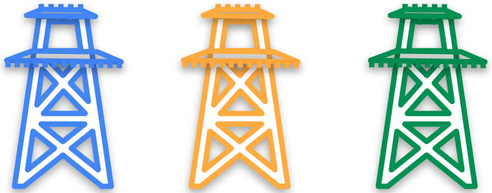


Location on the x,y plane (1, 2) (2, 5) (3,3)

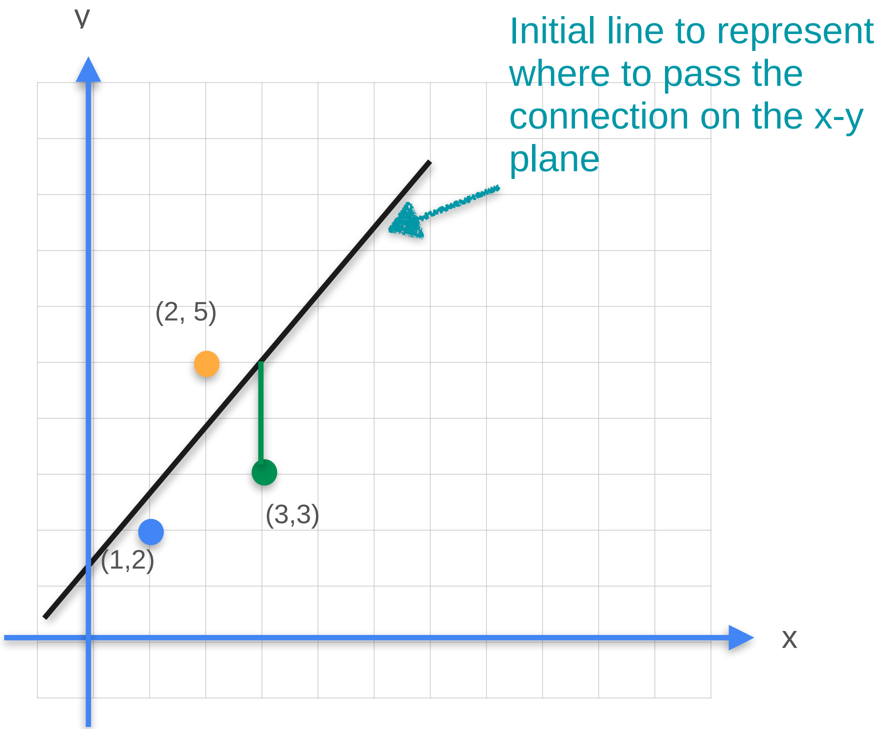


Initial line to represent where to pass the connection on the x-y plane

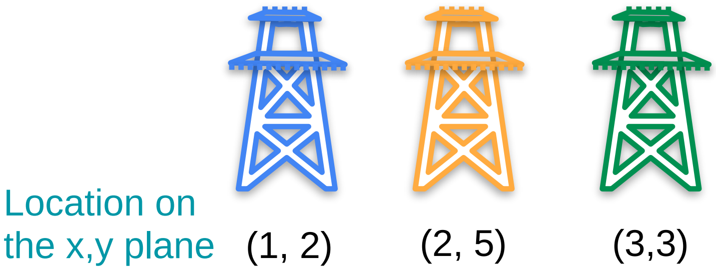
Linear Regression: Analytical Approach

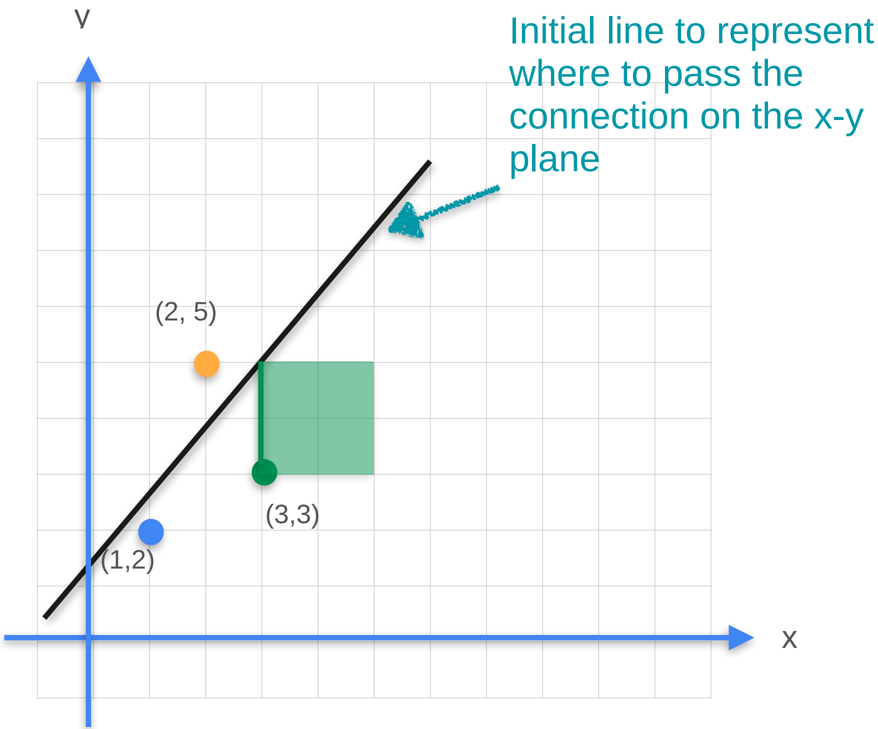


Location on the x,y plane (1, 2) (2, 5) (3,3)

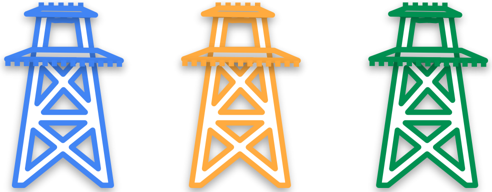


Linear Regression: Analytical Approach





Linear Regression: Analytical Approach



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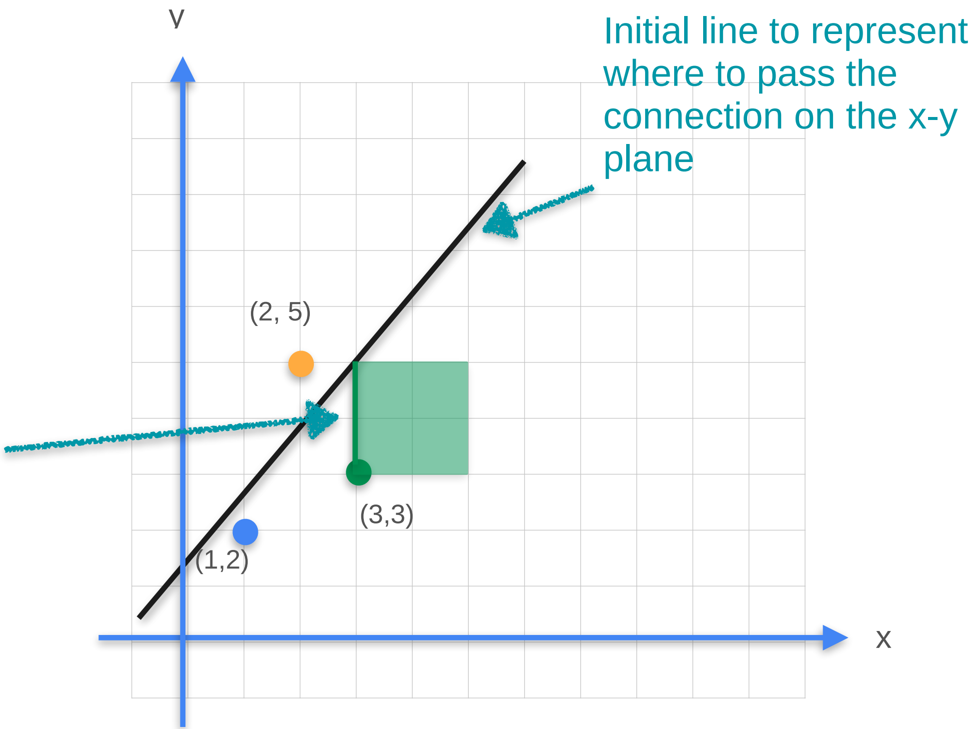
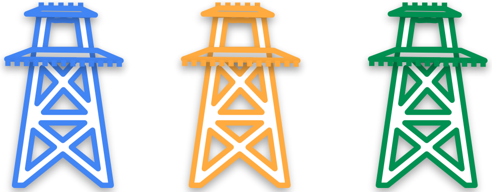
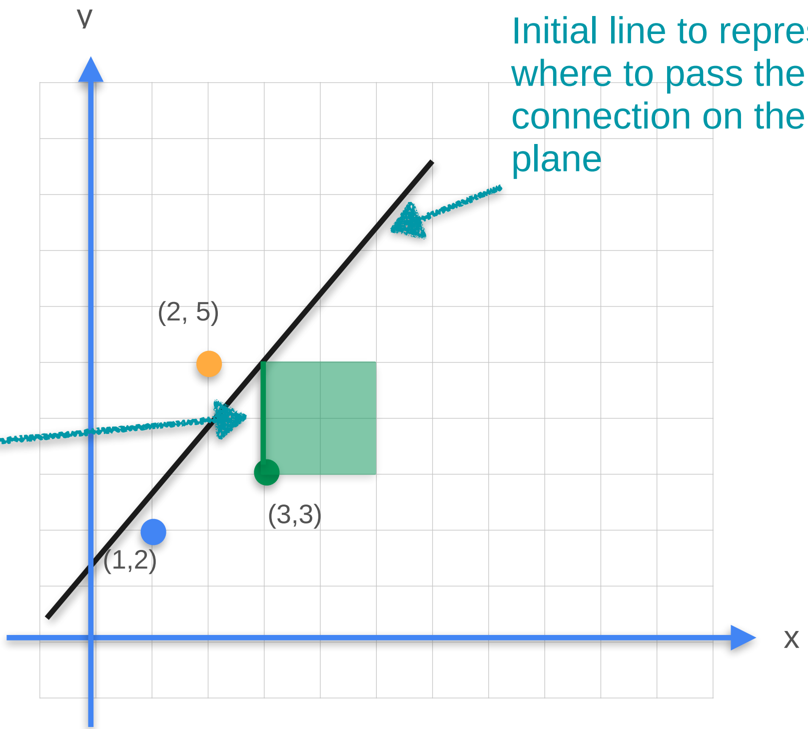


Linear Regression: Analytical Approach



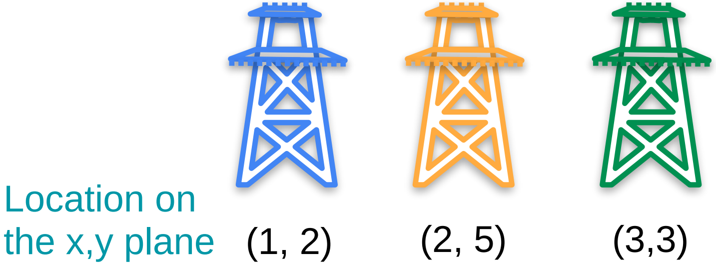
Location on the x,y plane (1, 2) (2, 5) (3,3)

The cost of connecting connection to the powerline

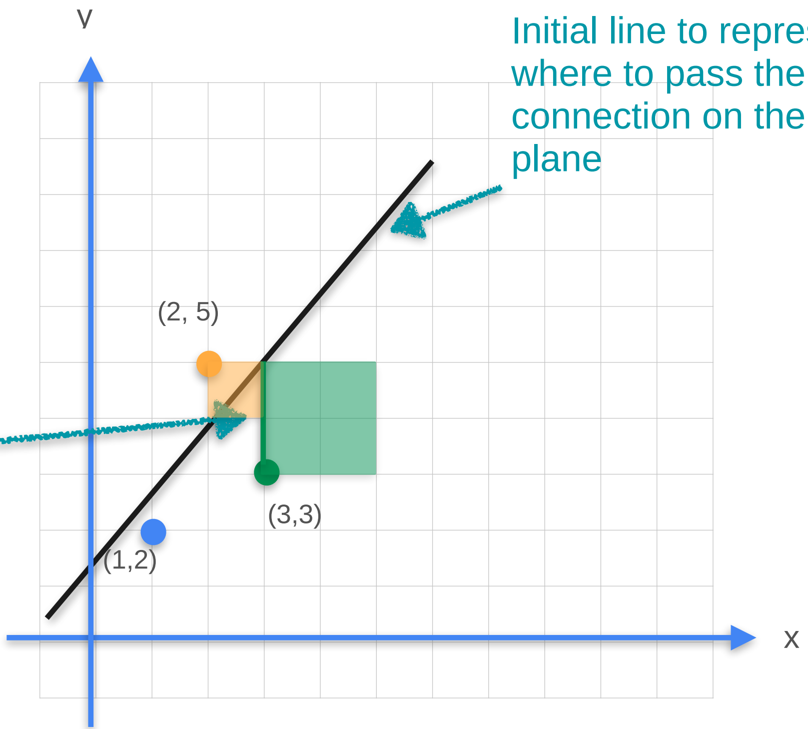


Initial line to represent where to pass the connection on the x-y plane

Linear Regression: Analytical Approach

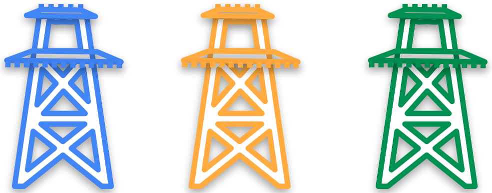


The cost of connecting connection to the powerline



Initial line to represent where to pass the connection on the x-y plane

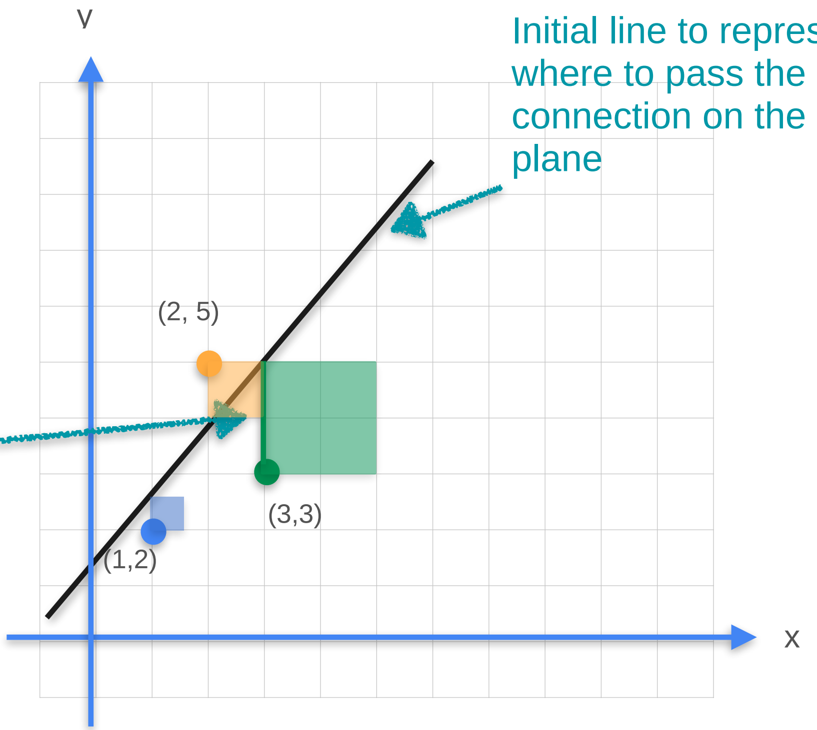
Linear Regression: Analytical Approach



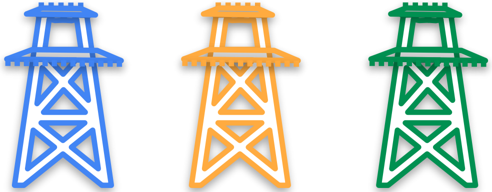
Location on the x,y plane (1, 2) (2, 5) (3,3)

The cost of connecting connection to the powerline

Initial line to represent where to pass the connection on the x-y plane

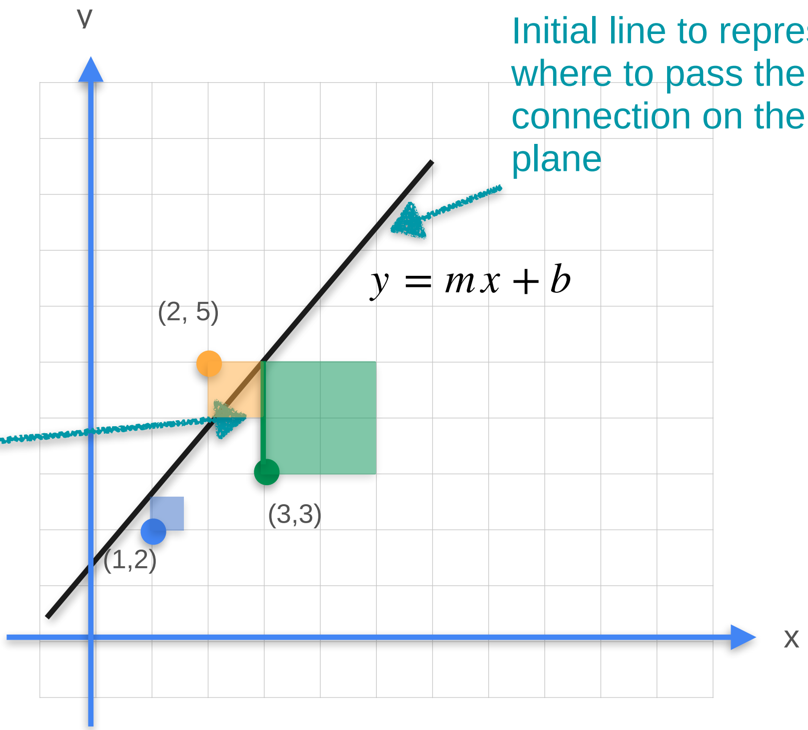


Linear Regression: Analytical Approach



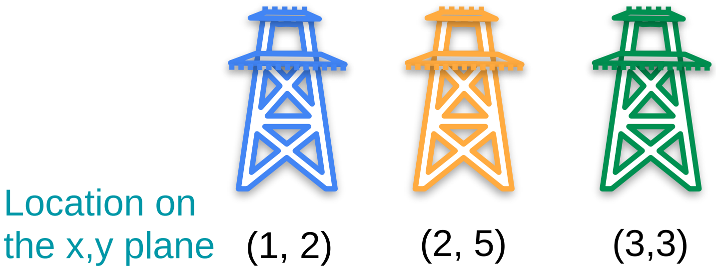
Location on the x,y plane (1, 2) (2, 5) (3,3)

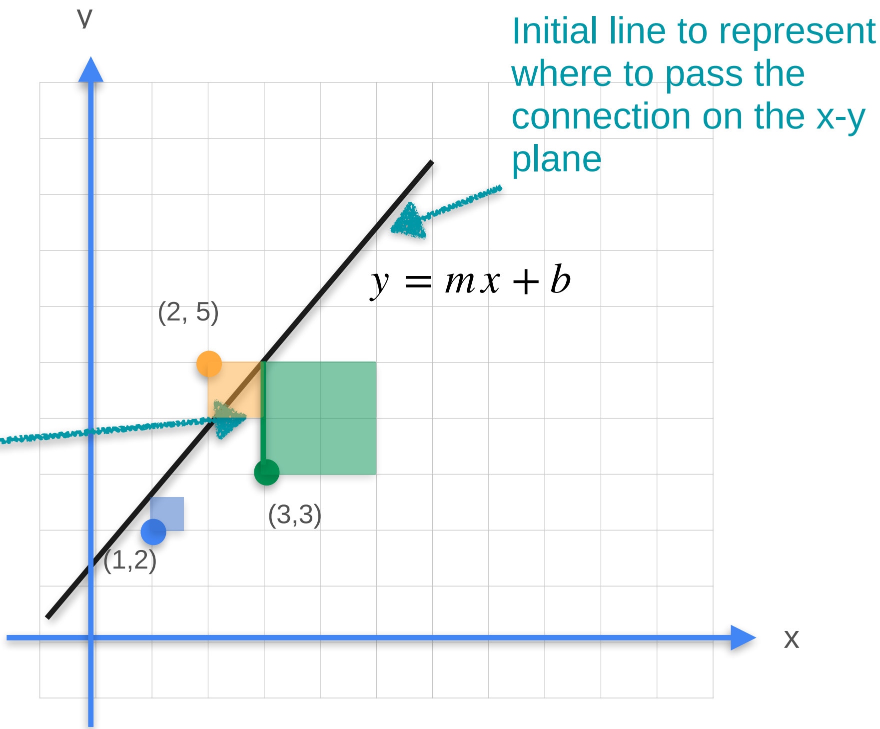
The cost of connecting connection to the powerline



Initial line to represent where to pass the connection on the x-y plane

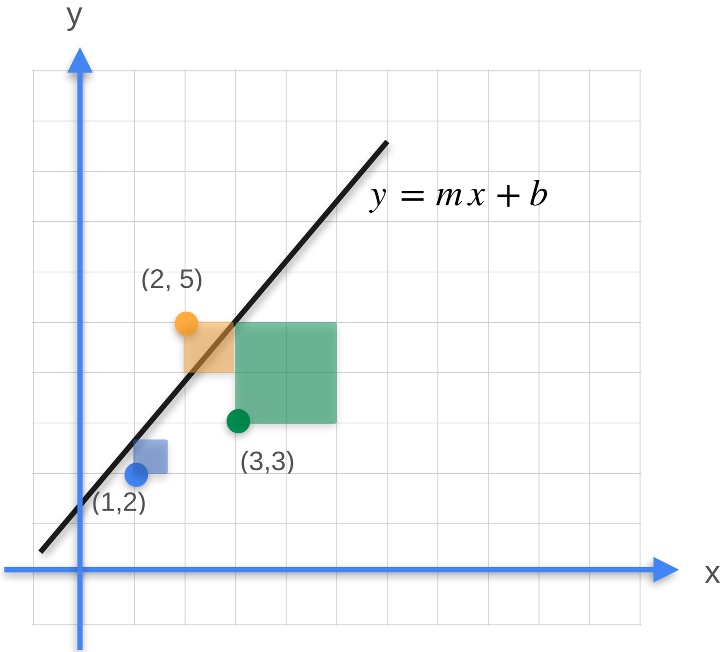
Linear Regression: Analytical Approach



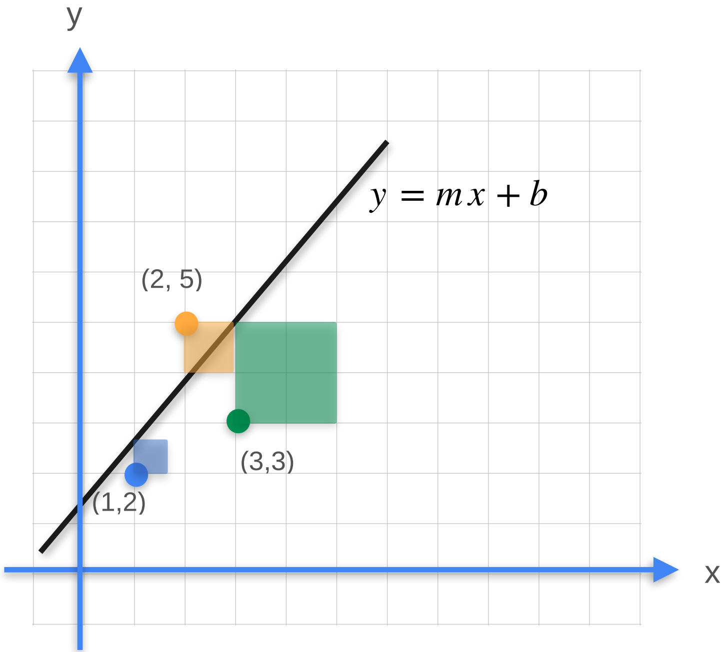


Goal: Find m, b such that you minimize sum of squares cost

Linear Regression: Analytical Approach

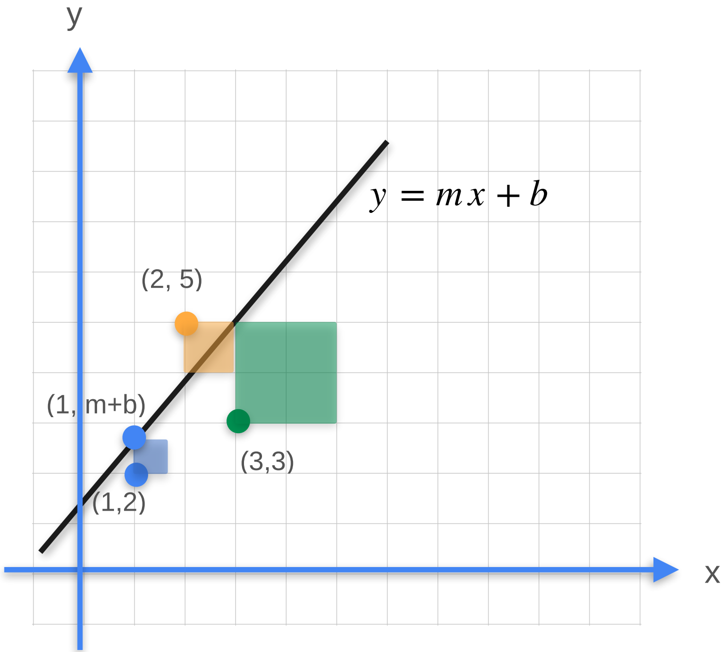


Linear Regression: Analytical Approach



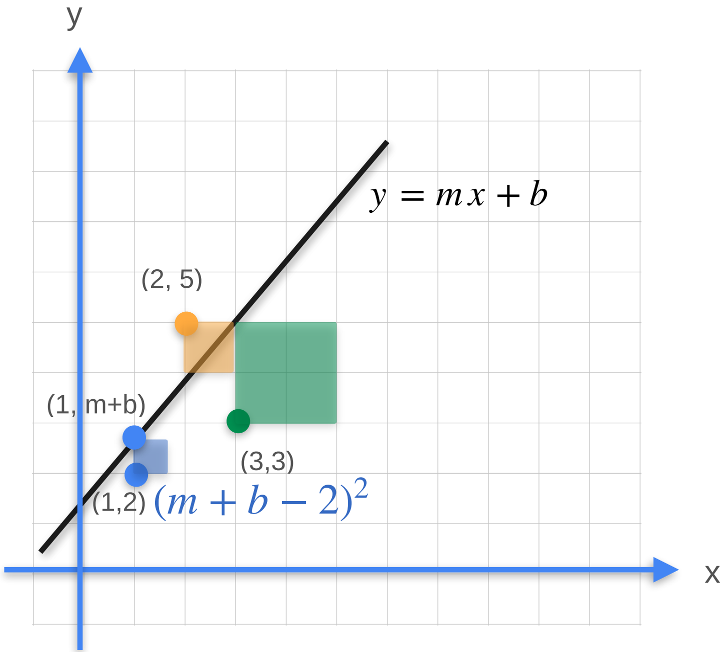
Goal: Minimize sum of squares cost

Linear Regression: Analytical Approach



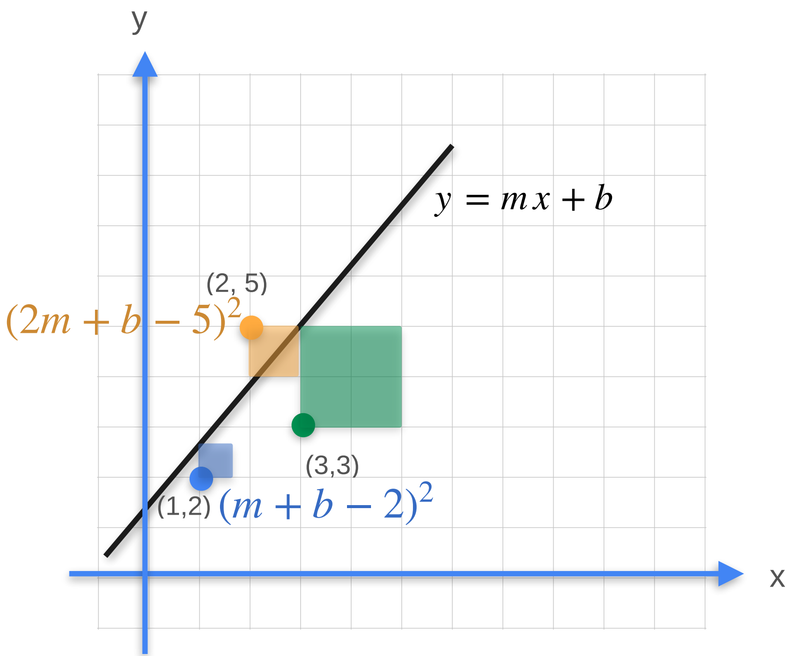
Goal: Minimize sum of squares cost

Linear Regression: Analytical Approach



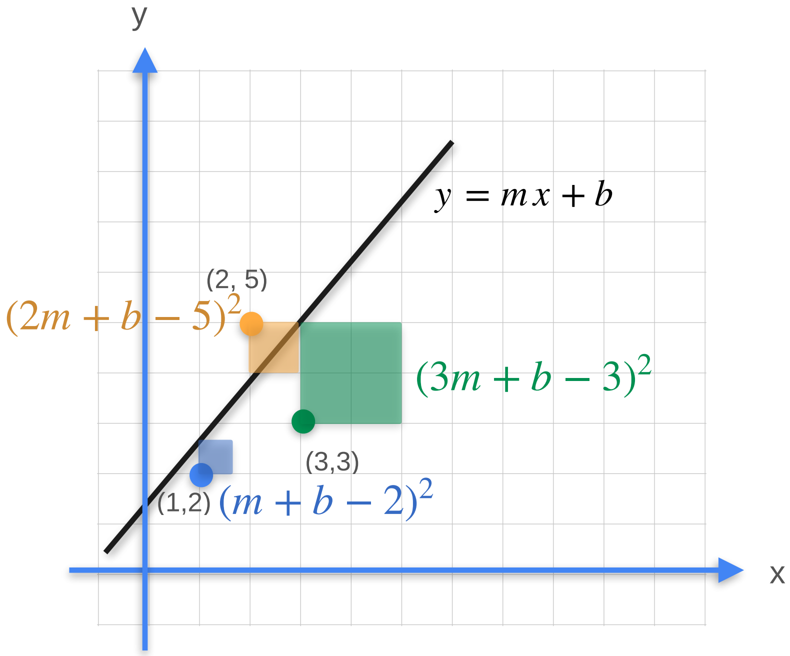
Goal: Minimize sum of squares cost

Linear Regression: Analytical Approach



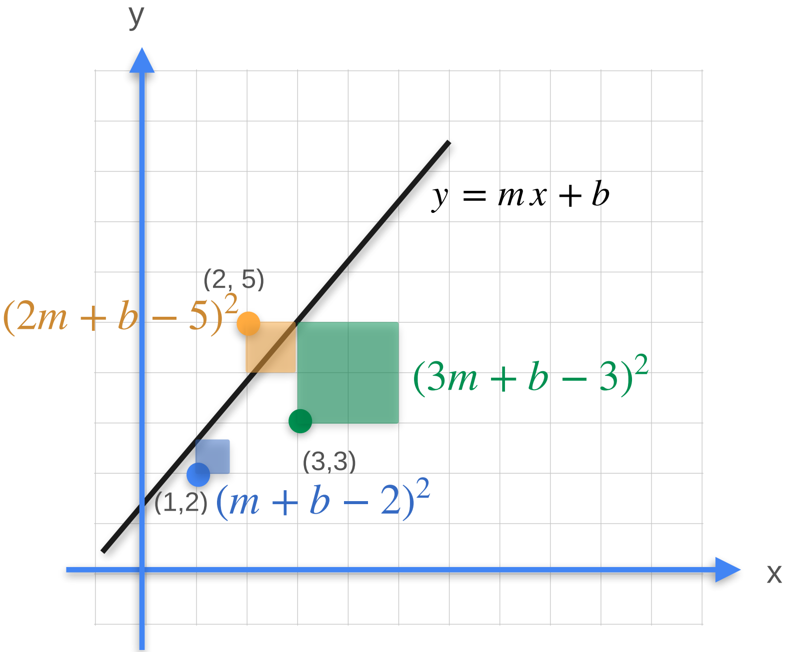
Goal: Minimize sum of squares cost

Linear Regression: Analytical Approach



Goal: Minimize sum of squares cost

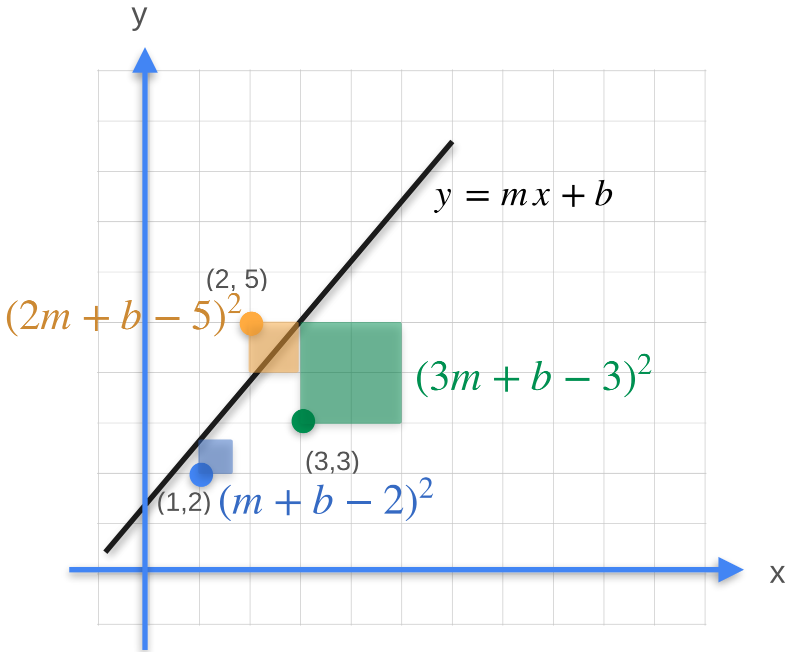
Linear Regression: Analytical Approach



Goal: Minimize sum of squares cost

$$(m + b - 2)^2 + (2m + b - 5)^2 + (3m + b - 3)^2$$

Linear Regression: Analytical Approach

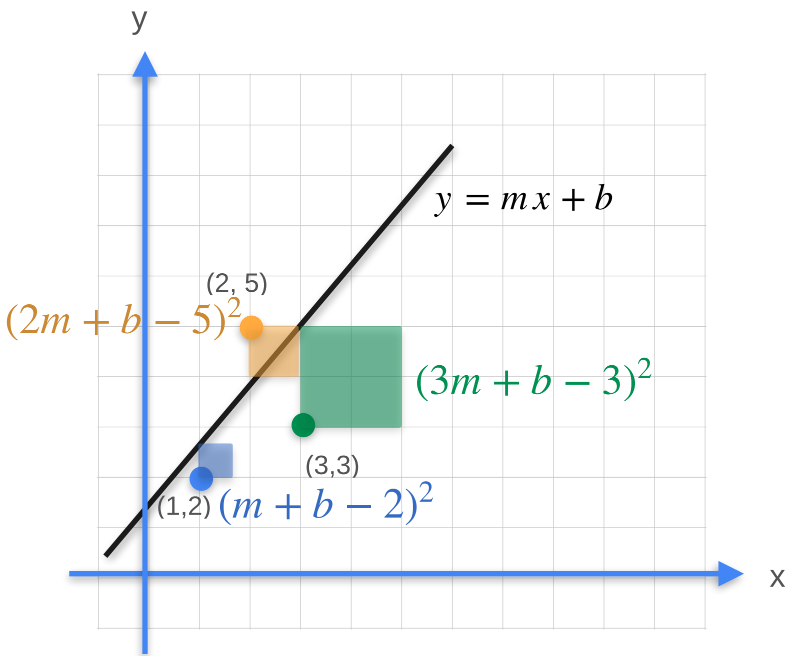


Goal: Minimize sum of squares cost

$$(m + b - 2)^2 + (2m + b - 5)^2 + (3m + b - 3)^2$$

$$m^2 + b^2 + 4 + 2mb - 4m - 4b$$

Linear Regression: Analytical Approach



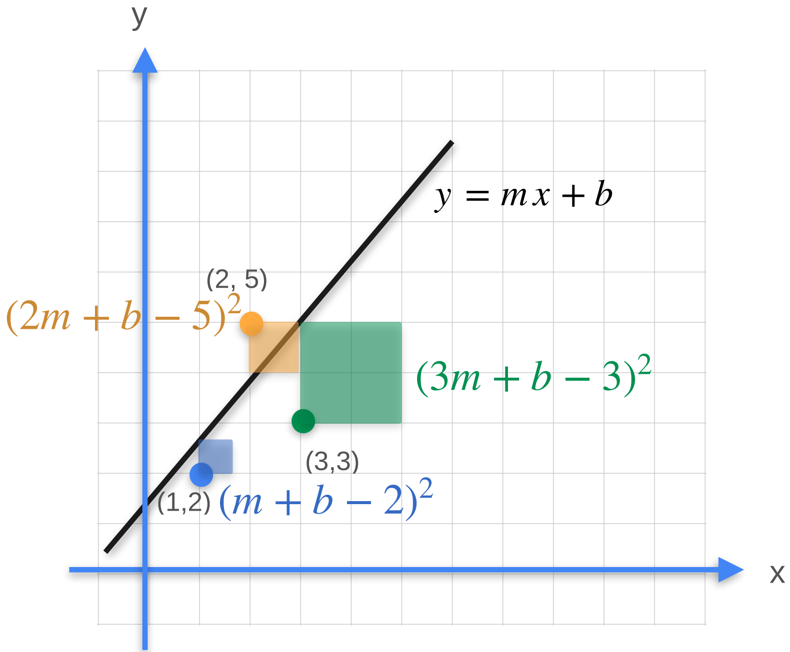
Goal: Minimize sum of squares cost

$$(m + b - 2)^2 + (2m + b - 5)^2 + (3m + b - 3)^2$$

$$m^2 + b^2 + 4 + 2mb - 4m - 4b$$

$$+4m^2 + b^2 + 25 + 4mb - 20m - 10b$$

Linear Regression: Analytical Approach



Goal: Minimize sum of squares cost

$$(m + b - 2)^2 + (2m + b - 5)^2 + (3m + b - 3)^2$$

$$m^2 + b^2 + 4 + 2mb - 4m - 4b$$

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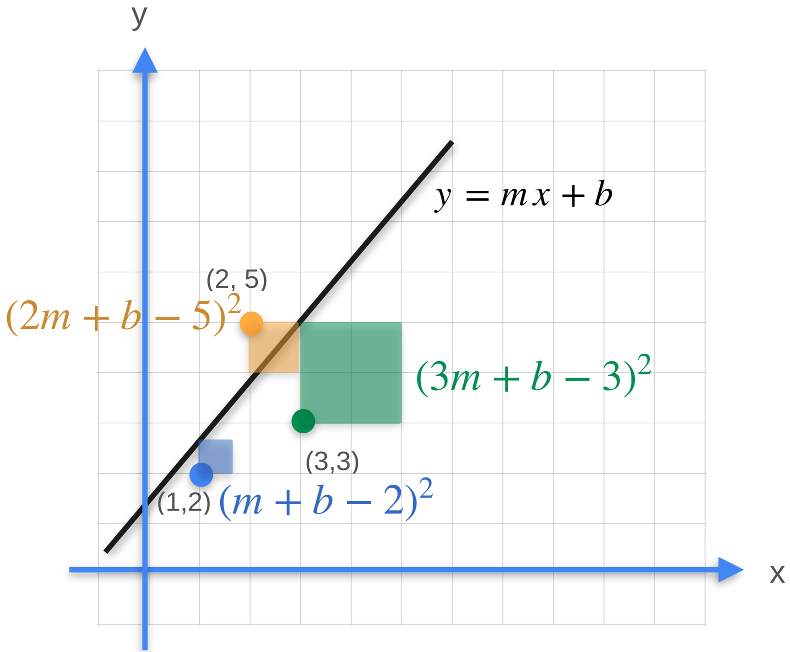
$$+9m^2 + b^2 + 9 + 6mb - 18m - 6b$$

Linear Regression: Analytical Approach



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$$(m + b - 2)^2 + (2m + b - 5)^2 + (3m + b - 3)^2$$

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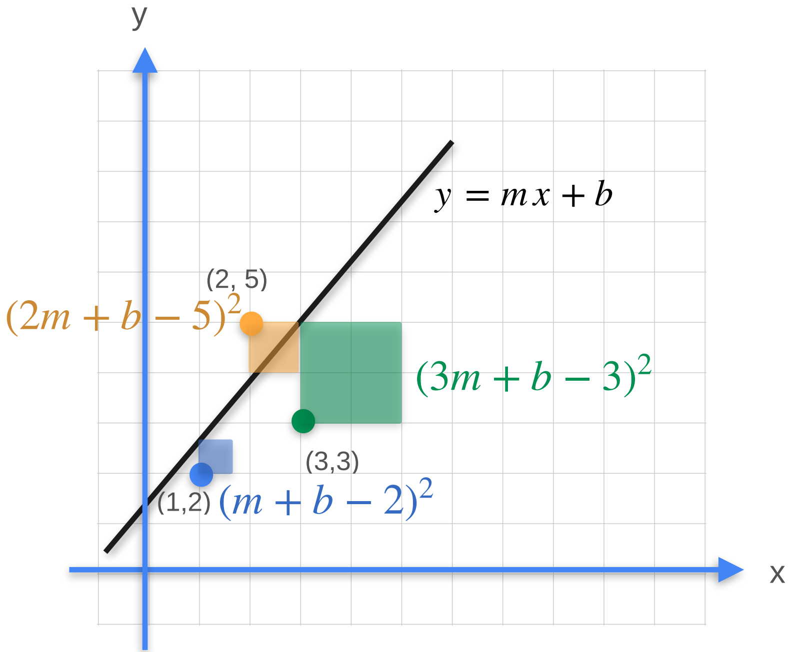
Linear Regression: Analytical Approach



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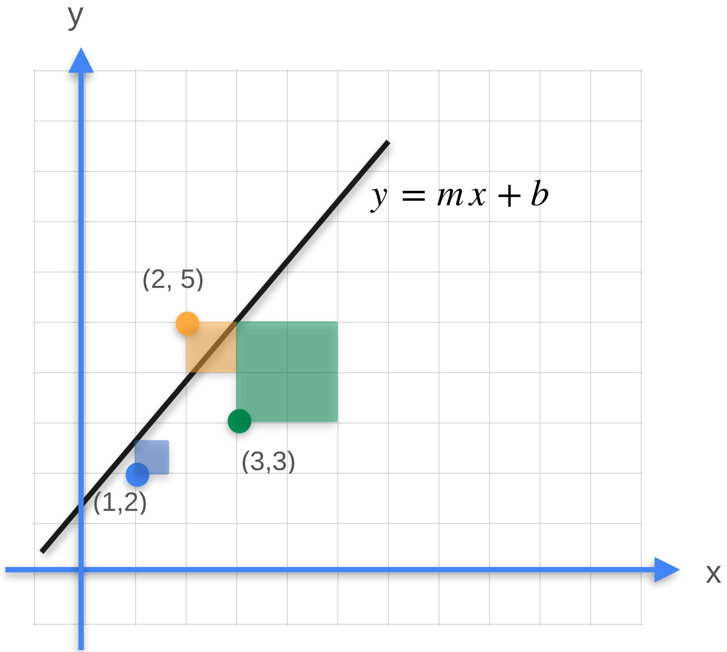
$$E(m, b) = 14m^2 + 3b^2 + 38 + 12mb - 42m - 20b$$

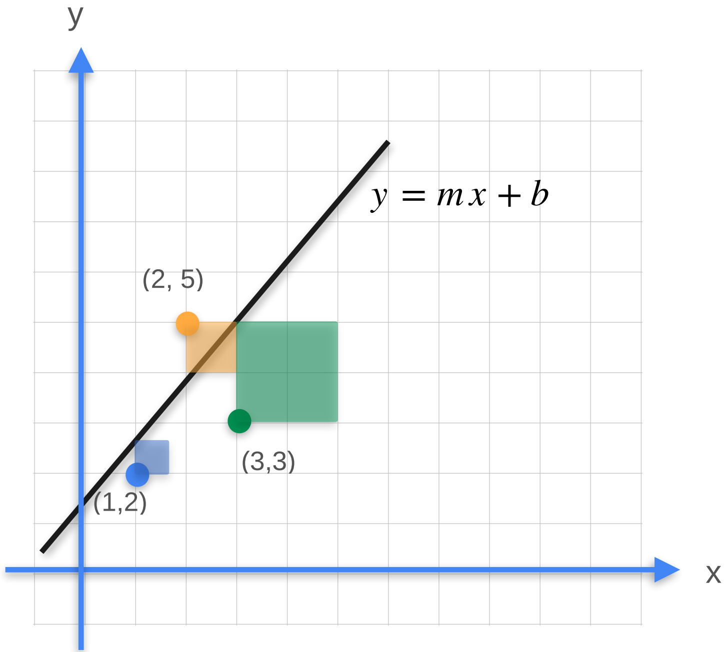
Linear Regression: Analytical Approach



Goal: Minimize sum of squares cost

$$E(m, b) = 14m^2 + 3b^2 + 38 + 12mb - 42m - 20b$$

Linear Regression: Analytical Approach

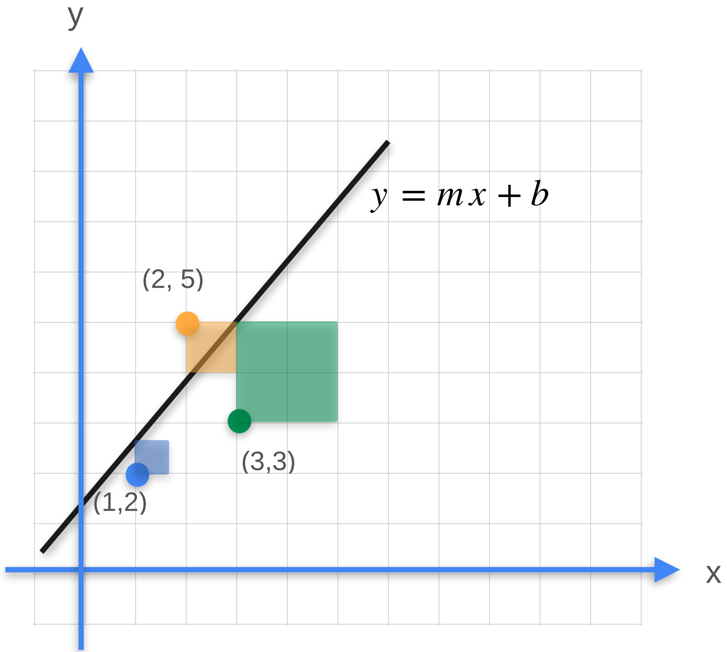


Goal: Minimize sum of squares cost

$$E(m, b) = 14m^2 + 3b^2 + 38 + 12mb - 42m - 20b$$

$$\frac{\partial E}{\partial m} = 0$$

Linear Regression: Analytical Approach



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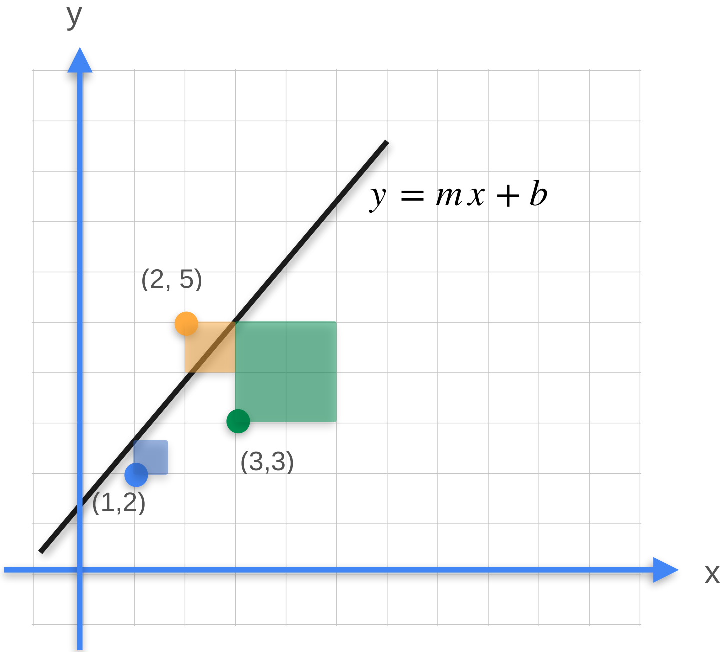
Linear Regression: Analytical Approach



Goal: Minimize sum of squares cost

$$E(m,b) = 14m^2 + 3b^2 + 38 + 12mb - 42m - 20b$$

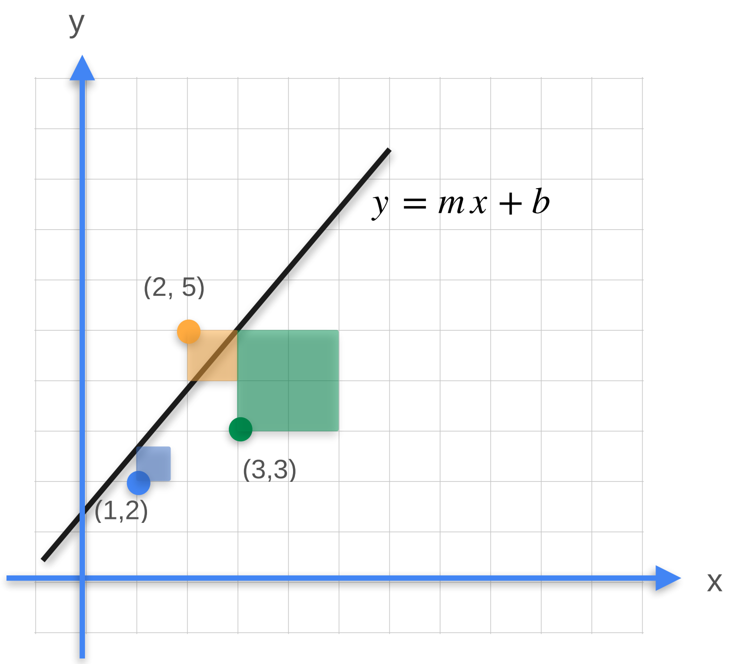
$$\frac{\partial E}{\partial m} = 0$$

$$\frac{\partial E}{\partial b} = 0$$

Quiz:

Find the partial derivative of E with respect to m

Linear Regression: Analytical Approach



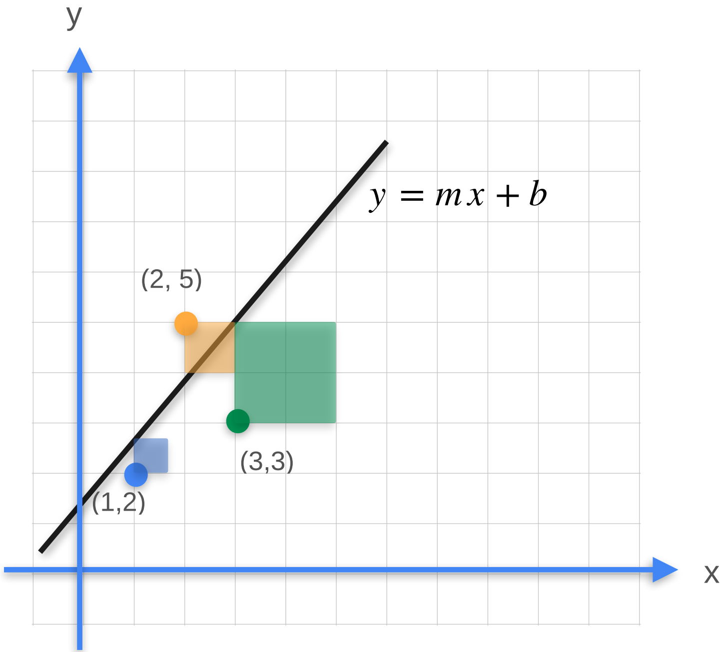
Goal: Minimize sum of squares cost

$$E(m, b) = 14m^2 + 3b^2 + 38 + 12mb - 42m - 20b$$

$$\frac{\partial E}{\partial m} = 28m + 12b - 42$$

$$\frac{\partial E}{\partial b} =$$

Linear Regression: Analytical Approach



Goal: Minimize sum of squares cost

$$E(m, b) = 14m^2 + 3b^2 + 38 + 12mb - 42m - 20b$$

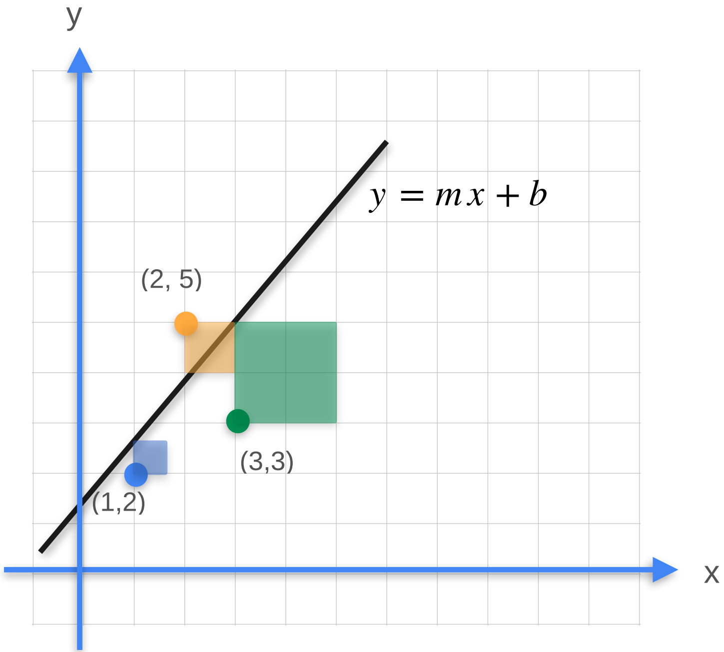
$$\frac{\partial E}{\partial m} = 28m + 12b - 42$$

$$\frac{\partial E}{\partial b} =$$

Quiz:

Find the partial derivative of E with respect to b

Linear Regression: Analytical Approach



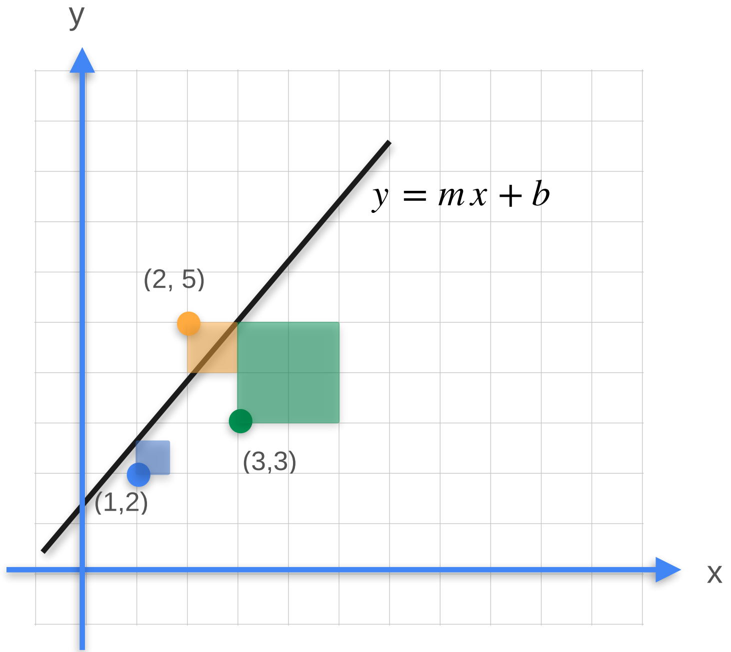
Goal: Minimize sum of squares cost

$$E(m, b) = 14m^2 + 3b^2 + 38 + 12mb - 42m - 20b$$

$$\frac{\partial E}{\partial m} = 28m + 12b - 42$$

$$\frac{\partial E}{\partial b} = 6b + 12m - 20$$

Linear Regression: Analytical Approach



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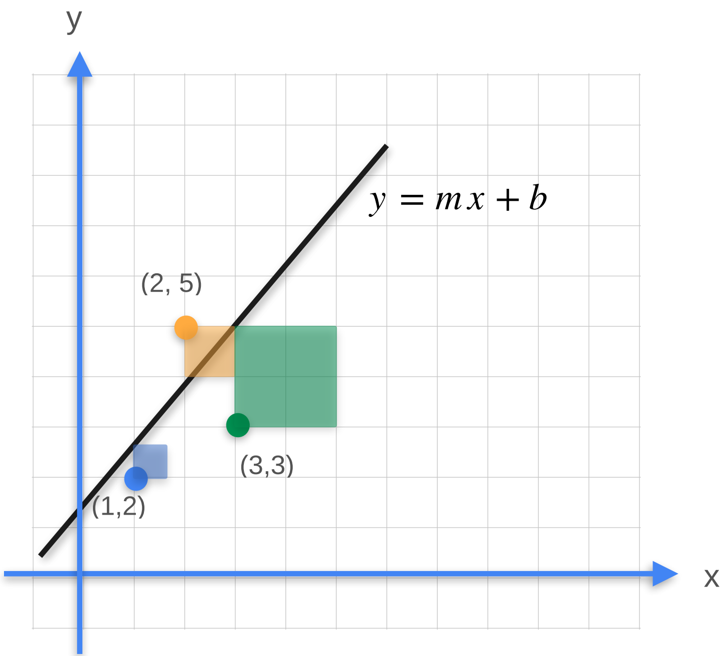
Linear Regression: Analytical Approach



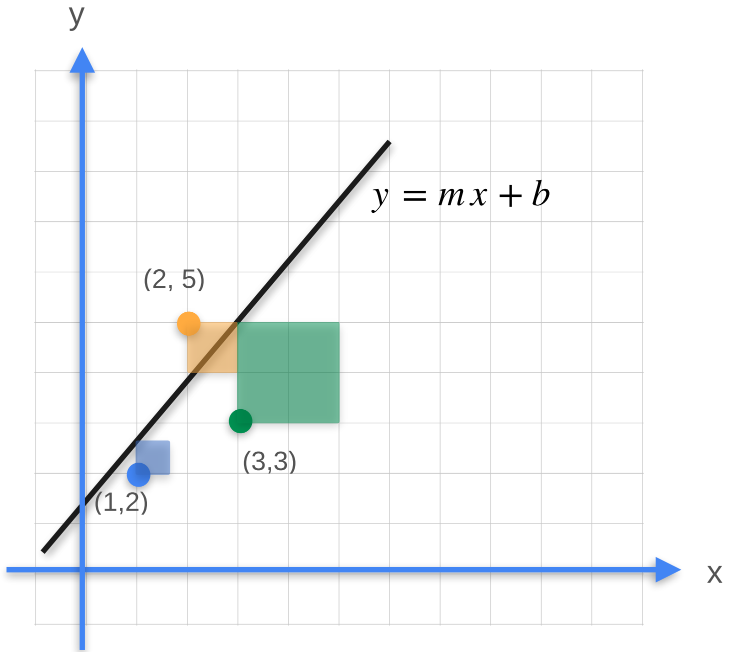
Goal: Minimize sum of squares cost

$$E(m, b) = 14m^2 + 3b^2 + 38 + 12mb - 42m - 20b$$

$$\frac{\partial E}{\partial m} = 28m + 12b - 42 = 0$$

$$\frac{\partial E}{\partial b} = 6b + 12m - 20$$

Linear Regression: Analytical Approach



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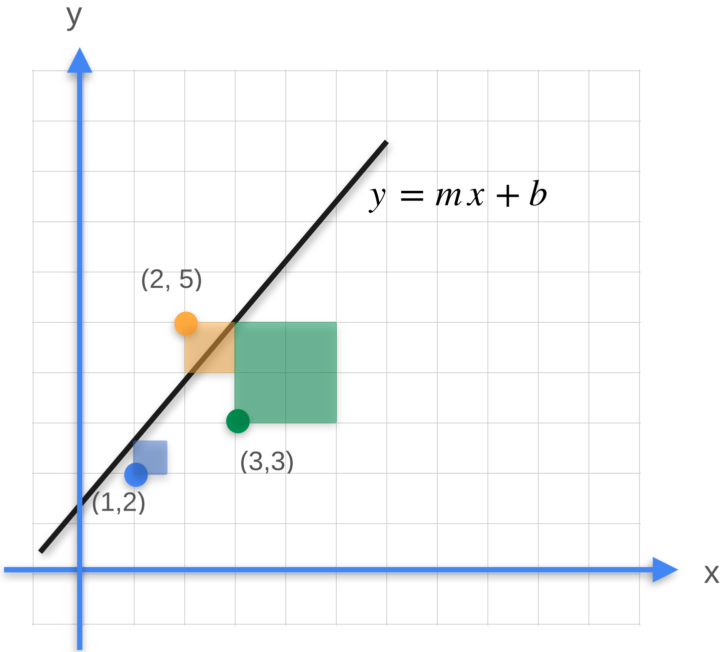
Linear Regression: Analytical Approach



Goal: Minimize sum of squares cost

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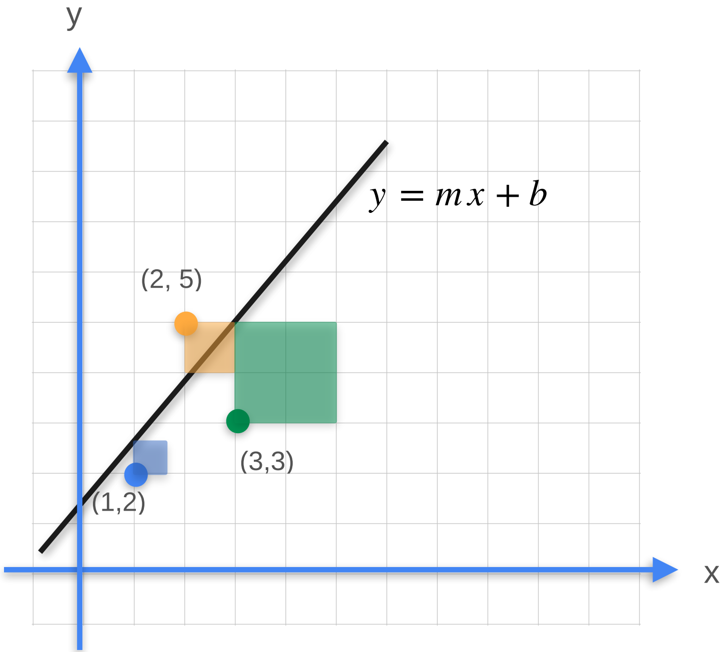
$$\frac{\partial E}{\partial m} = 28m + 12b - 42 = 0$$

$$\frac{\partial E}{\partial b} = 6b + 12m - 20 = 0$$

$$m =$$

Linear Regression: Analytical Approach



Goal: Minimize sum of squares cost

$$E(m, b) = 14m^2 + 3b^2 + 38 + 12mb - 42m - 20b$$

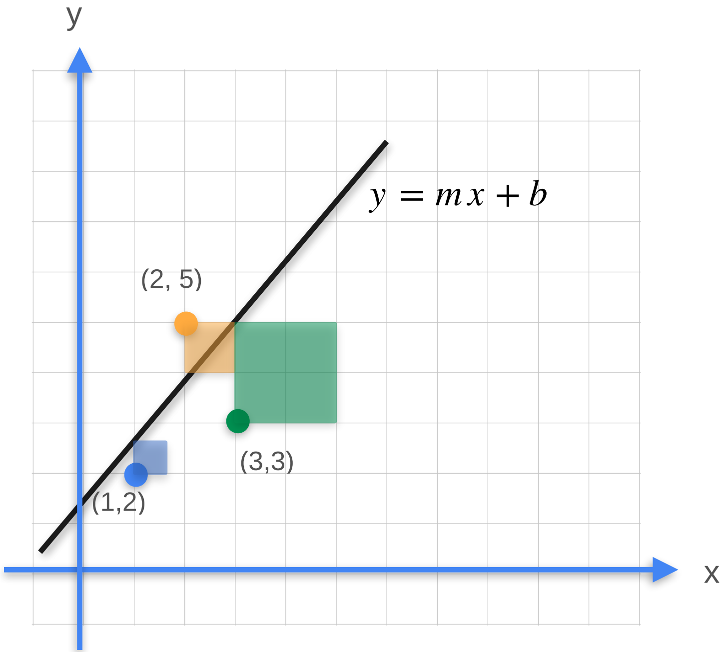
$$\frac{\partial E}{\partial m} = \boxed{28m + 12b - 42} = 0$$

$$\frac{\partial E}{\partial b} = \boxed{6b + 12m - 20} = 0$$

$m =$

$b =$

Linear Regression: Analytical Approach



Goal: Minimize sum of squares cost

$$E(m,b) = 14m^2 + 3b^2 + 38 + 12mb - 42m - 20b$$

$$\frac{\partial E}{\partial m} = \boxed{28m + 12b - 42} = 0$$

$$\frac{\partial E}{\partial b} = \boxed{6b + 12m - 20} = 0$$

$m =$

$b =$ **?**

Linear Regression: Analytical Approach

Goal: Minimize sum of squares cost

$$E(m,b) = 14m^2 + 3b^2 + 38 + 12mb - 42m - 20b$$

$$\frac{\partial E}{\partial m} = \boxed{28m + 12b - 42} = 0$$

$$\frac{\partial E}{\partial b} = \boxed{6b + 12m - 20} = 0$$

$$m = \quad ?$$

$$b = $$

Linear Regression: Analytical Approach

Goal: Minimize sum of squares cost

$$E(m,b) = 14m^2 + 3b^2 + 38 + 12mb - 42m - 20b$$

$$\frac{\partial E}{\partial m} = \boxed{28m + 12b - 42} = 0$$

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$$\frac{\partial E}{\partial b} = \boxed{6b + 12m - 20} = 0$$

$m =$

$b =$

?

$$12b + 24m - 40 = 0$$

Linear Regression: Analytical Approach

Goal: Minimize sum of squares cost

$$E(m,b) = 14m^2 + 3b^2 + 38 + 12mb - 42m - 20b$$

$$\frac{\partial E}{\partial m} = 28m + 12b - 42 = 0$$

$$\frac{\partial E}{\partial b} = 6b + 12m - 20 = 0$$

$m =$

$b =$

?

$$12b + 24m - 40 = 0$$

$$4m - 2 = 0$$

Linear Regression: Analytical Approach

Goal: Minimize sum of squares cost

$$E(m,b) = 14m^2 + 3b^2 + 38 + 12mb - 42m - 20b$$

$$\frac{\partial E}{\partial m} = \boxed{28m + 12b - 42} = 0$$

$$\frac{\partial E}{\partial b} = \boxed{6b + 12m - 20} = 0$$

$m =$

$b =$ **?**

$$12b + 24m - 40 = 0$$

$$4m - 2 = 0$$

$$m = \frac{2}{4} = 0.5$$

Linear Regression: Analytical Approach

Goal: Minimize sum of squares cost

$$E(m,b) = 14m^2 + 3b^2 + 38 + 12mb - 42m - 20b$$

$$\frac{\partial E}{\partial m} = 28m + 12b - 42 = 0$$

$$\frac{\partial E}{\partial b} = 6b + 12m - 20 = 0$$

$m =$

$b =$?

$$12b + 24m - 40 = 0$$

$$4m - 2 = 0$$

$$m = \frac{2}{4} = 0.5$$

$$6b + 12(0.5) - 20 = 0$$

Linear Regression: Analytical Approach

Goal: Minimize sum of squares cost

$$E(m,b) = 14m^2 + 3b^2 + 38 + 12mb - 42m - 20b$$

$$\frac{\partial E}{\partial m} = \boxed{28m + 12b - 42} = 0$$

$$\frac{\partial E}{\partial b} = \boxed{6b + 12m - 20} = 0$$

$m =$

$b =$?

$$12b + 24m - 40 = 0$$

$$4m - 2 = 0$$

$$m = \frac{2}{4} = 0.5$$

$$6b + 12(0.5) - 20 = 0$$

$$6b + 6 - 20 = 0$$

Linear Regression: Analytical Approach

Goal: Minimize sum of squares cost

$$E(m,b) = 14m^2 + 3b^2 + 38 + 12mb - 42m - 20b$$

$$\frac{\partial E}{\partial m} = \boxed{28m + 12b - 42} = 0$$

$$\frac{\partial E}{\partial b} = \boxed{6b + 12m - 20} = 0$$

$m =$

$b =$?

$$12b + 24m - 40 = 0$$

$$4m - 2 = 0$$

$$m = \frac{2}{4} = 0.5$$

$$6b + 12(0.5) - 20 = 0$$

$$6b + 6 - 20 = 0$$

$$6b - 14 = 0$$

Linear Regression: Analytical Approach

Goal: Minimize sum of squares cost

$$E(m,b) = 14m^2 + 3b^2 + 38 + 12mb - 42m - 20b$$

$$\frac{\partial E}{\partial m} = \boxed{28m + 12b - 42} = 0$$

$$\frac{\partial E}{\partial b} = \boxed{6b + 12m - 20} = 0$$

$m =$

$b =$ **?**

$$12b + 24m - 40 = 0$$

$$4m - 2 = 0$$

$$m = \frac{2}{4} = 0.5$$

$$6b + 12(0.5) - 20 = 0$$

$$6b + 6 - 20 = 0$$

$$6b - 14 = 0$$

$$b = \frac{14}{6} = \frac{7}{3}$$

Linear Regression: Analytical Approach

Goal: Minimize sum of squares cost

$$E(m, b) = 14m^2 + 3b^2 + 38 + 12mb - 42m - 20b$$

$$\frac{\partial E}{\partial m} = 28m + 12b - 42 = 0$$

$$\frac{\partial E}{\partial b} = 6b + 12m - 20 = 0$$

$m =$

$b =$

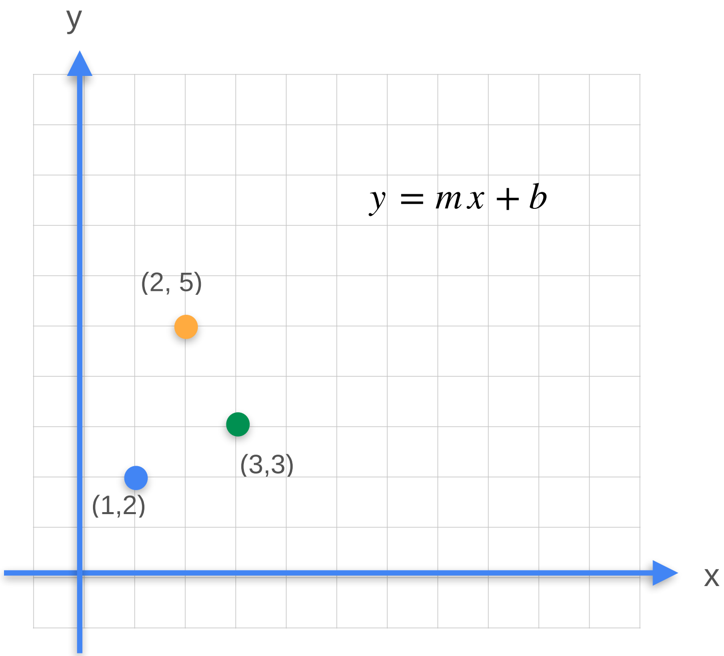
?

$$m = \frac{1}{2}$$

$$b = \frac{7}{3}$$

$$E(m = \frac{1}{2}, b = \frac{7}{3}) \approx 4.167$$

Linear Regression: Optimal Solution

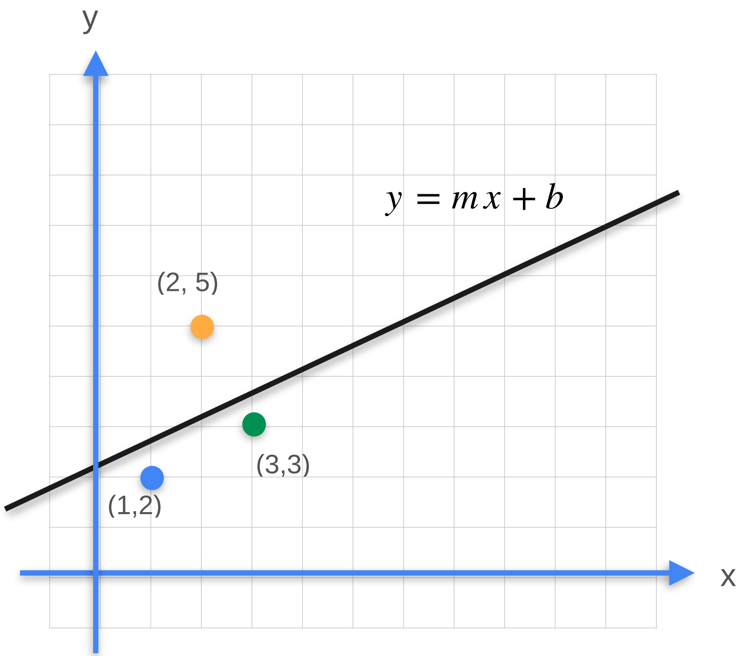


$$m = \frac{1}{2}$$

$$b = \frac{7}{3}$$

$$E(m = \frac{1}{2}, b = \frac{7}{3}) \approx 4.167$$

Linear Regression: Optimal Solution



$$m = \frac{1}{2}$$

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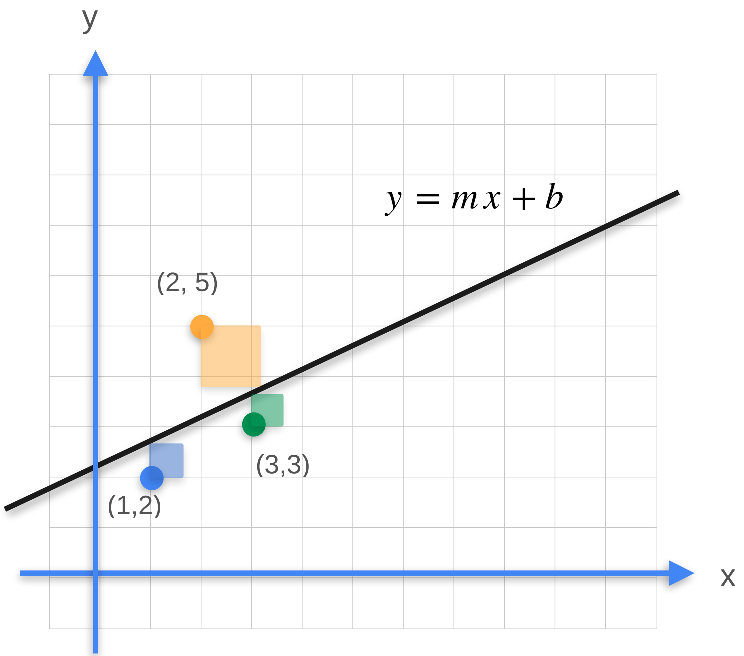
$$E(m = \frac{1}{2}, b = \frac{7}{3}) \approx 4.167$$

Linear Regression: Optimal Solution



$$m = \frac{1}{2}$$

$$b = \frac{7}{3}$$

$$E(m = \frac{1}{2}, b = \frac{7}{3}) \approx 4.167$$

Linear Regression: Gradient Descent

Goal: Minimize sum of squares cost

$$E(m,b) = 14m^2 + 3b^2 + 38 + 12mb - 42m - 20b$$

$$\frac{\partial E}{\partial m} = \boxed{28m + 12b - 42} = 0$$

$$\frac{\partial E}{\partial b} = \boxed{6b + 12m - 20} = 0$$

$$m = \quad ?$$
$$b = $$

Gradient Descent to the rescue

Gradients and Gradient Descent

Optimization using Gradient Descent in one variable - Part 1

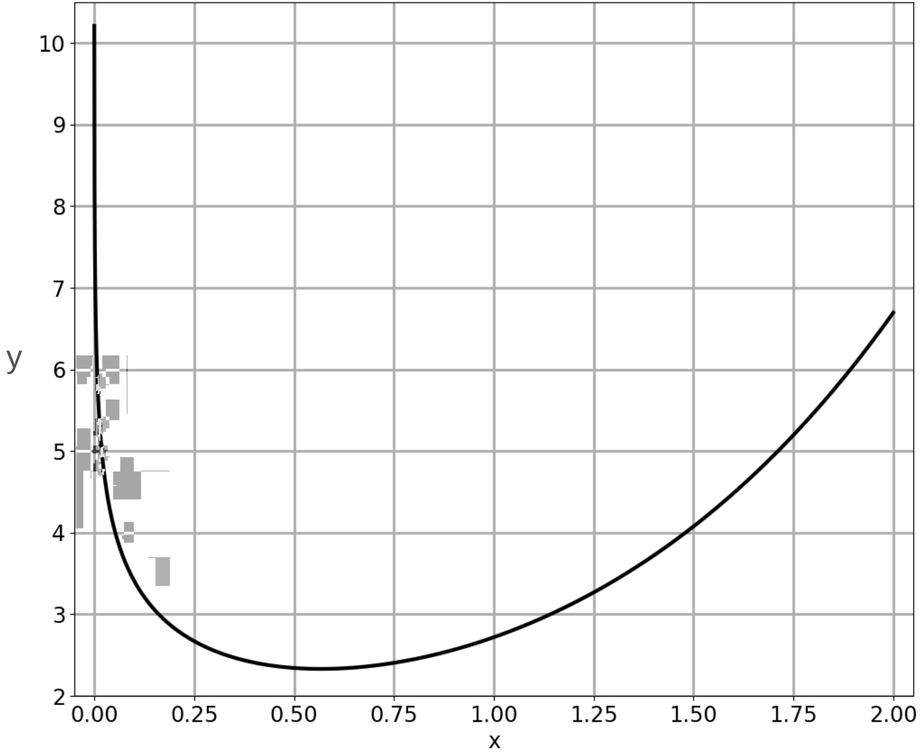
Hard To Optimize Functions

Hard To Optimize Functions

$$f(x) = e^x - \log(x)$$

Hard To Optimize Functions

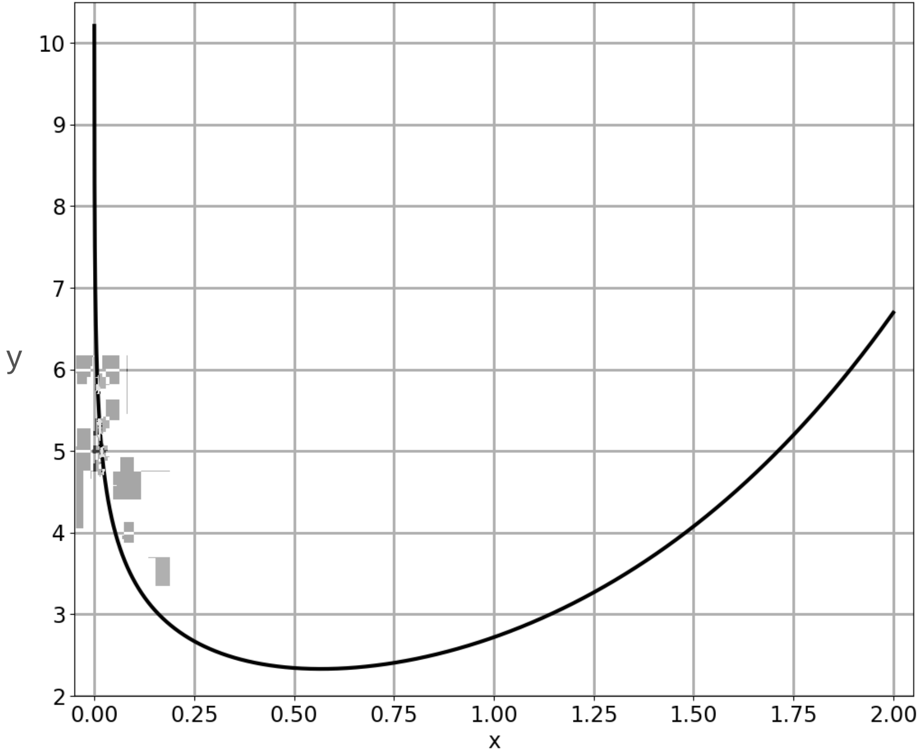
$$f(x) = e^x - \log(x)$$



Hard To Optimize Functions

$$f(x) = e^x - \log(x)$$

Minimum?



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$$f(x) = e^x - \log(x)$$

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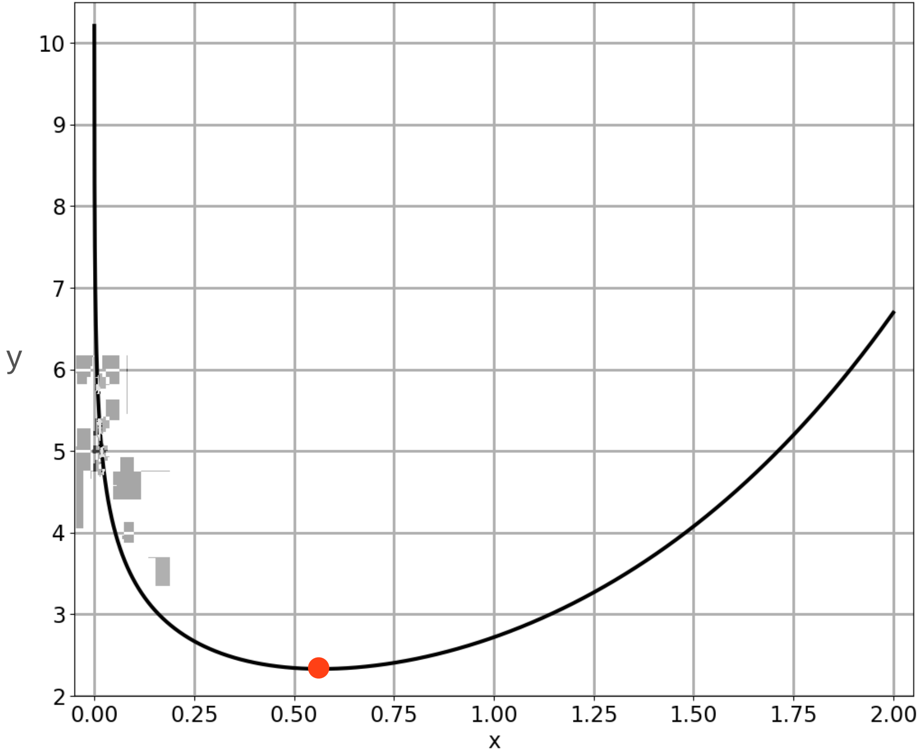
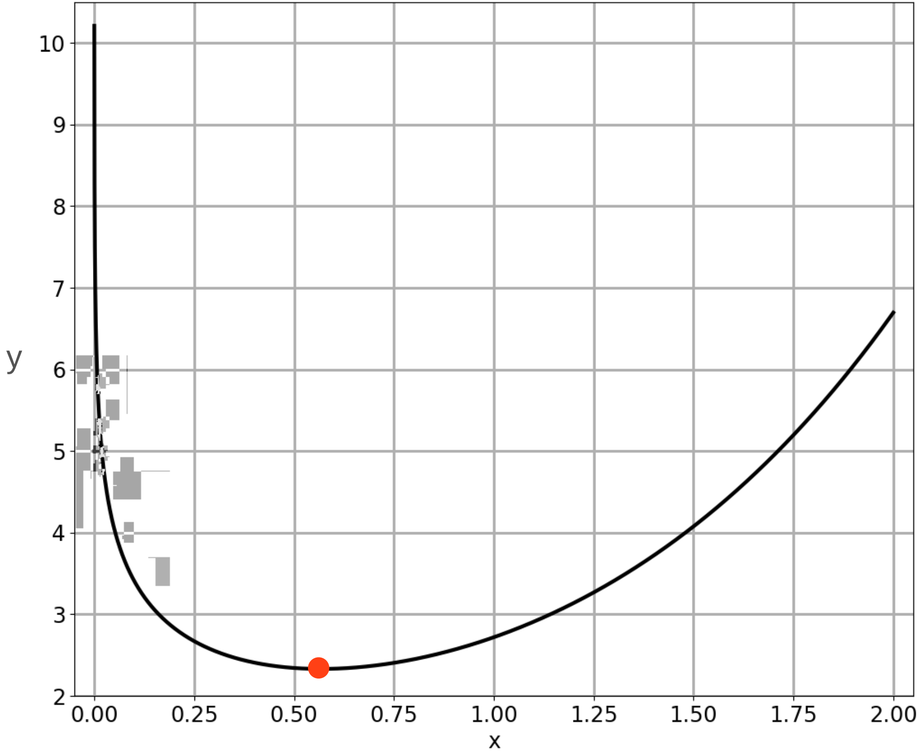


Hard To Optimize Functions

$$f(x) = e^x - \log(x)$$

Minimum?

$$f'(x) = 0$$

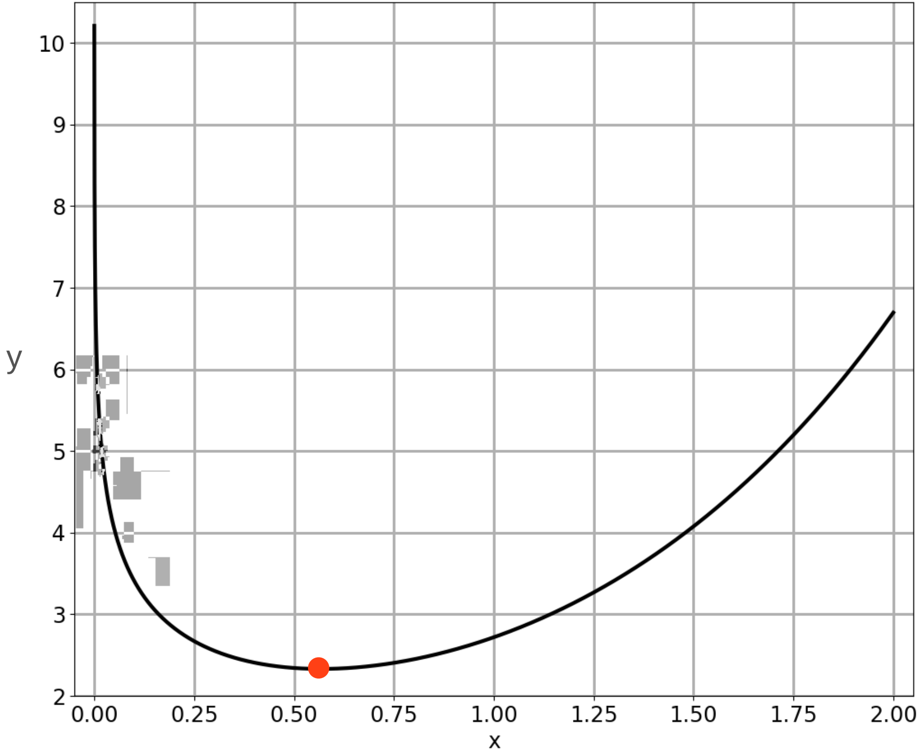


Hard To Optimize Functions

$$f(x) = e^x - \log(x)$$

Minimum?

$$f'(x) = e^x - \frac{1}{x} = 0$$

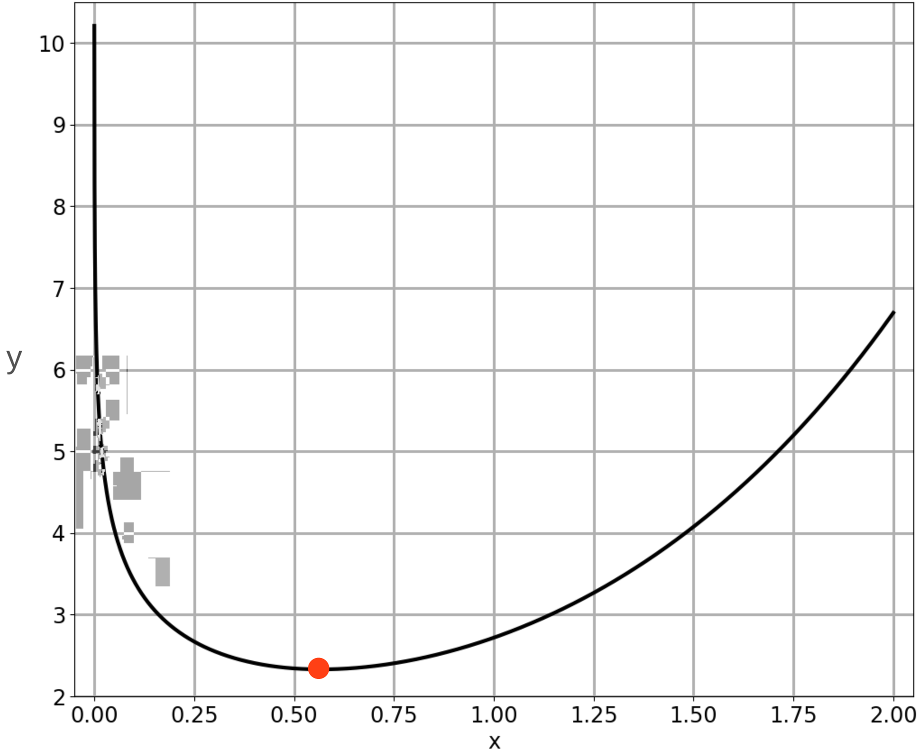


Hard To Optimize Functions

$$f(x) = \boxed{e^x} - \log(x)$$

Minimum?

$$f'(x) = \boxed{e^x} - \frac{1}{x} = 0$$

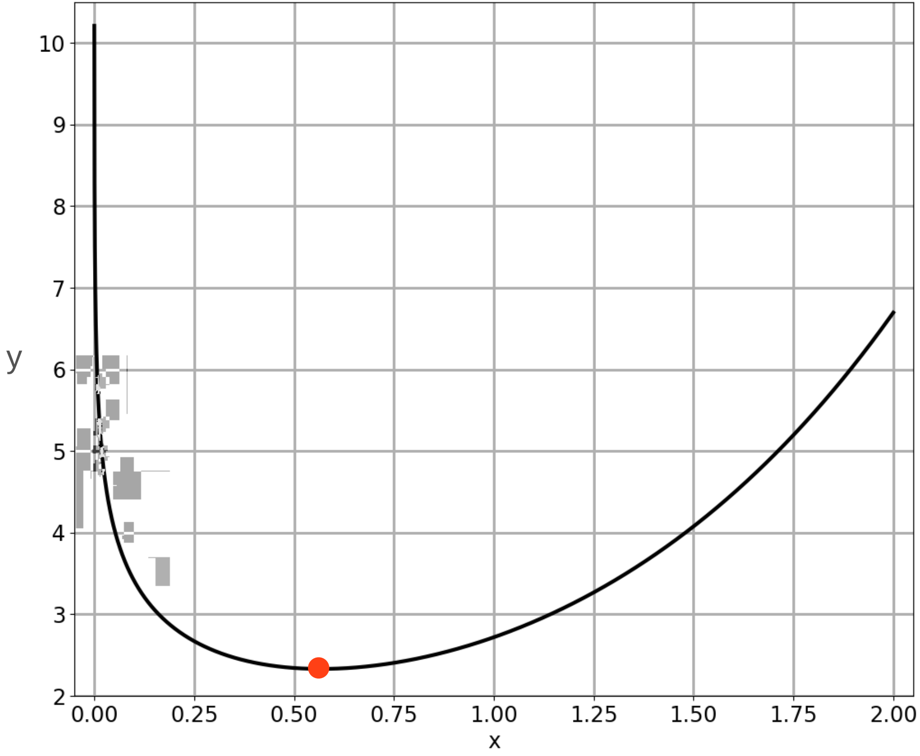


Hard To Optimize Functions

$$f(x) = \boxed{e^x} - \boxed{\log(x)}$$

Minimum?

$$f'(x) = \boxed{e^x} - \boxed{\frac{1}{x}} = 0$$

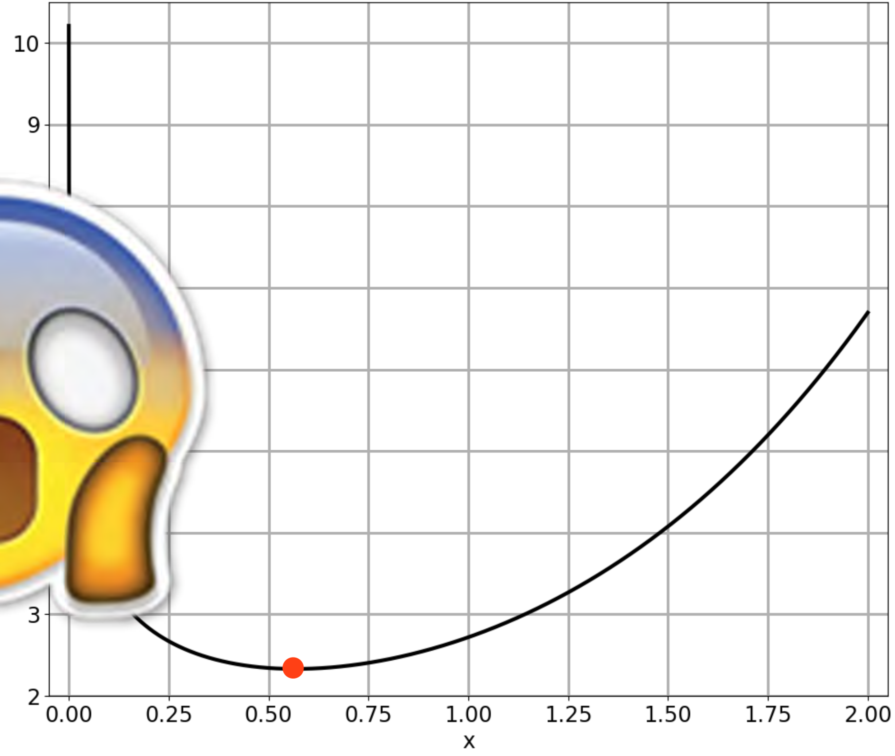


Hard To Optimize Functions

$$f(x) = \boxed{e^x} - \boxed{\log(x)}$$

Minimum?

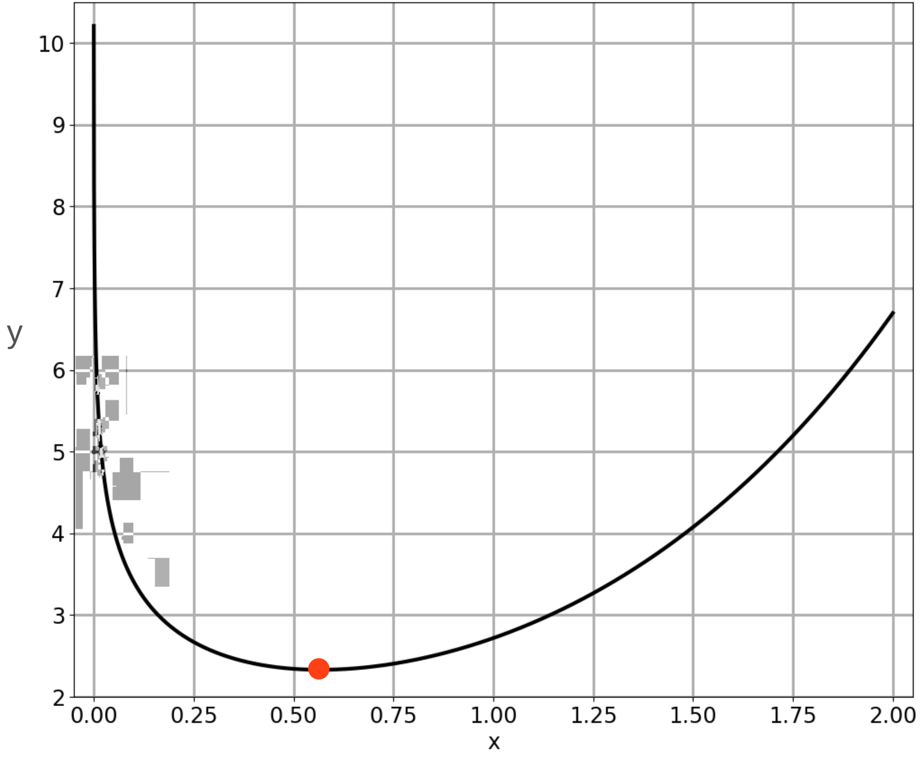
$$f'(x) = \boxed{e^x} - \boxed{\frac{1}{x}} = 0$$



Hard To Optimize Functions

$$f(x) = e^x - \log(x)$$

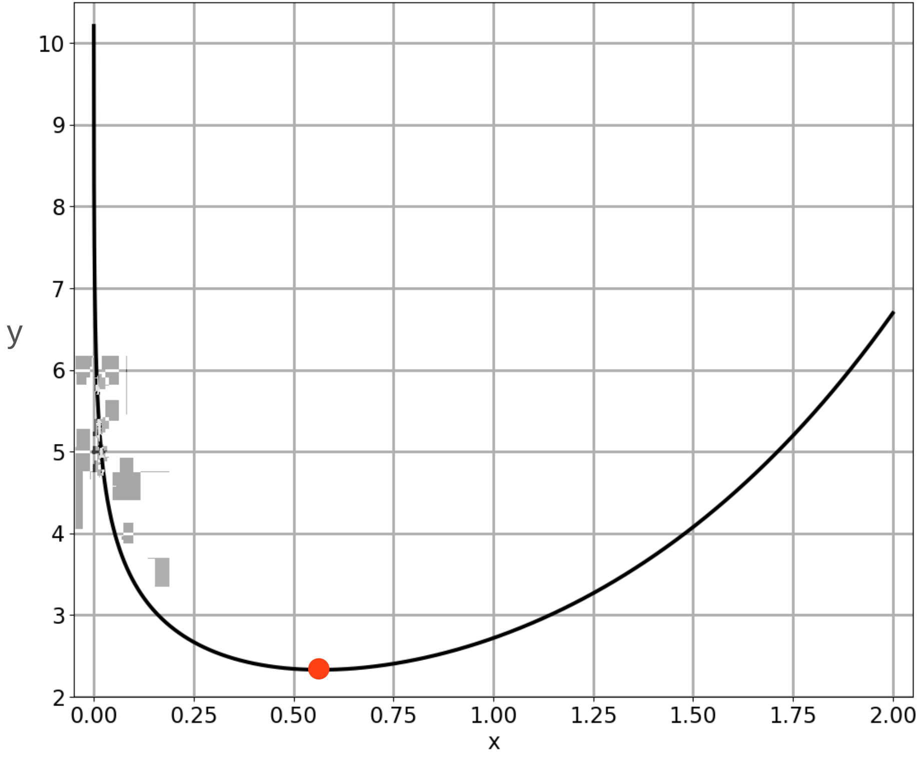
$$f'(x) = e^x - \frac{1}{x}$$



Hard To Optimize Functions

$$f(x) = e^x - \log(x)$$

$$f'(x) = e^x - \frac{1}{x} = 0$$

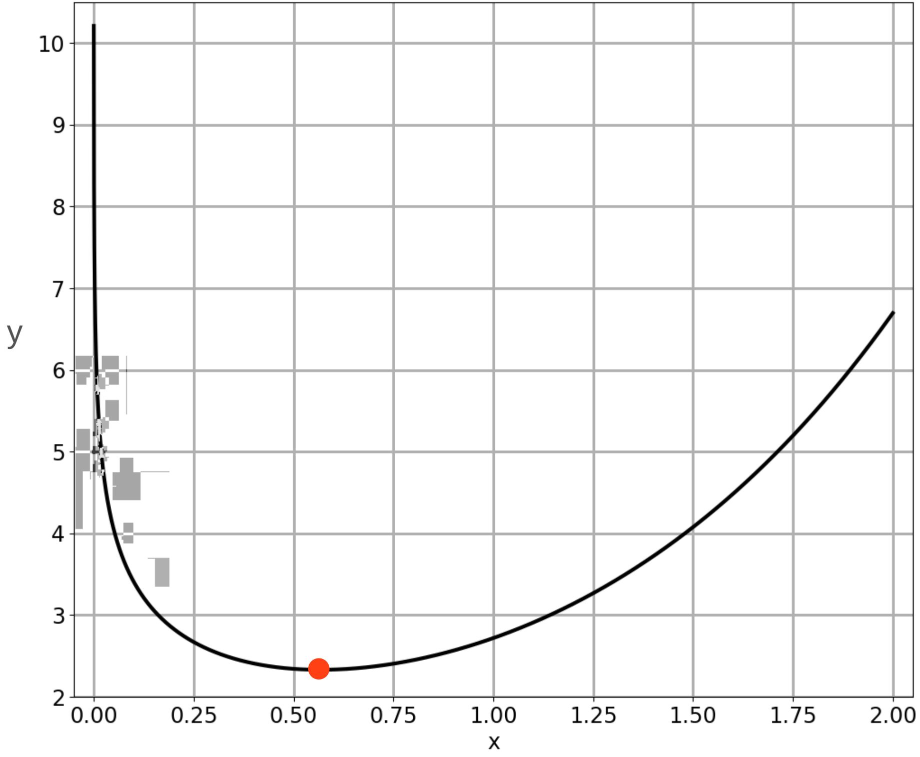


Hard To Optimize Functions

$$f(x) = e^x - \log(x)$$

$$f'(x) = e^x - \frac{1}{x} = 0$$

$$e^x = \frac{1}{x}$$



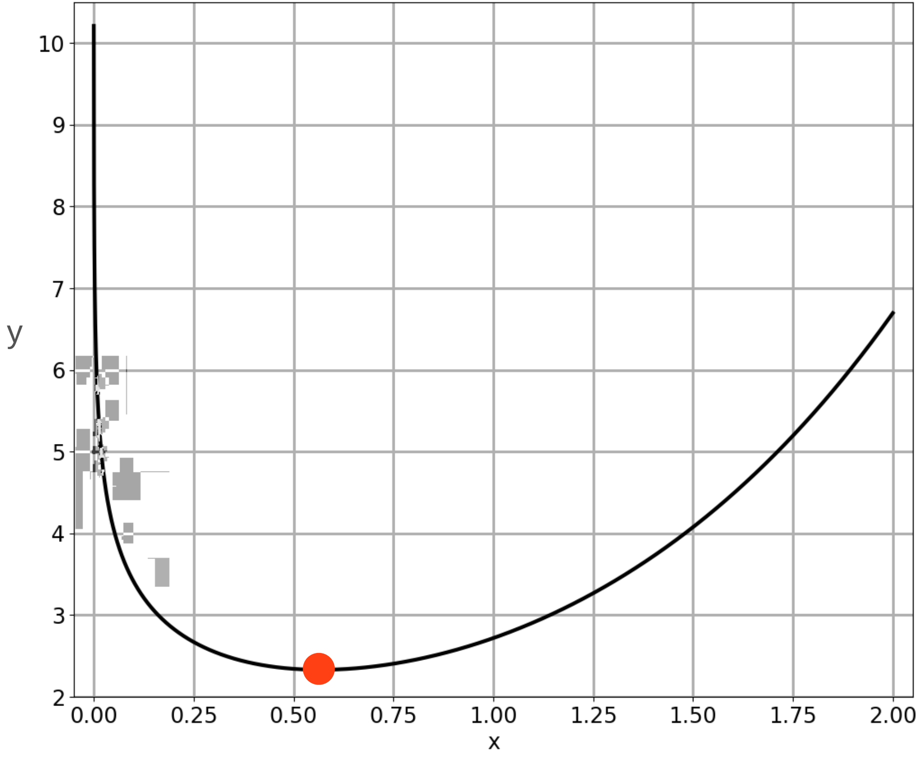
Hard To Optimize Functions

$$f(x) = e^x - \log(x)$$

$$f'(x) = e^x - \frac{1}{x} = 0$$

$$e^x = \frac{1}{x}$$

Solution: $x = 0.5671...$



Hard To Optimize Functions

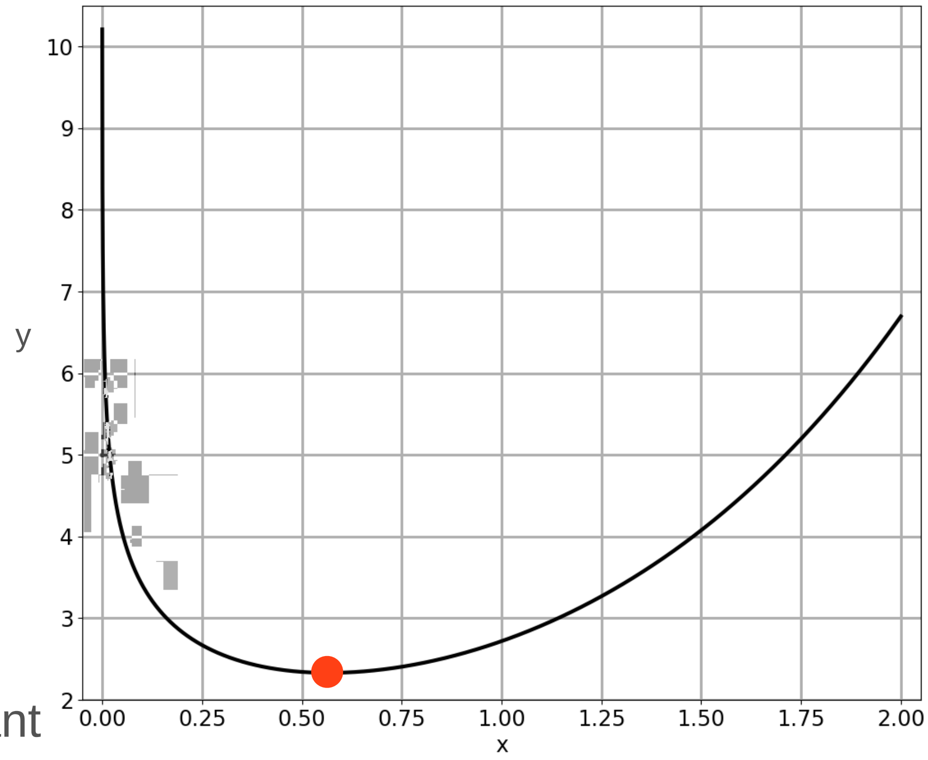
$$f(x) = e^x - \log(x)$$

$$f'(x) = e^x - \frac{1}{x} = 0$$

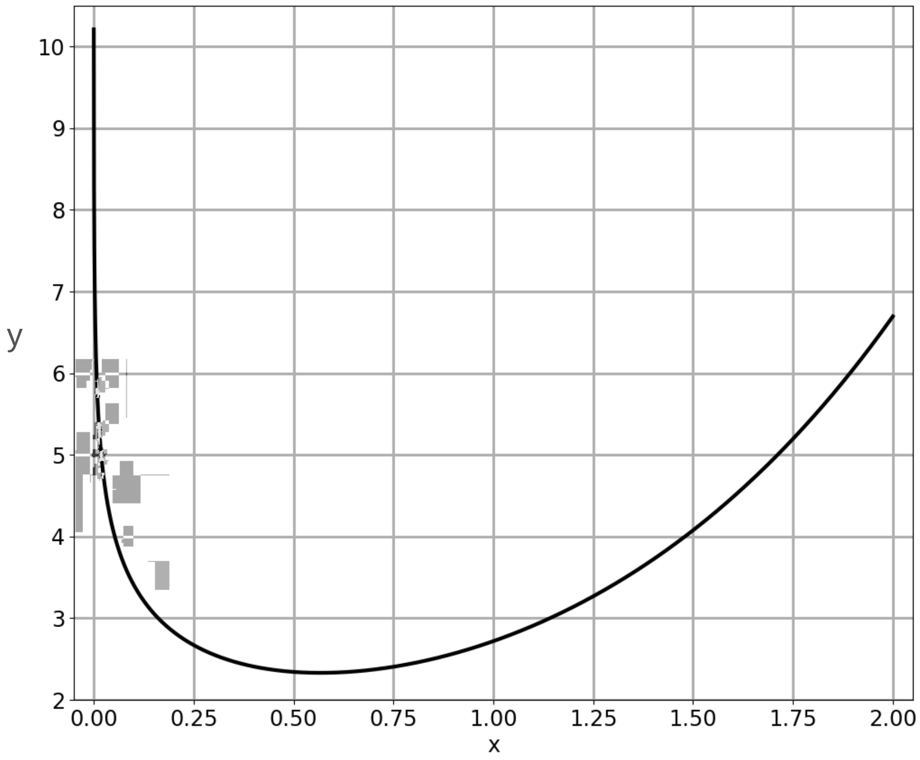
$$\Rightarrow \quad e^x = \frac{1}{x}$$

Solution: $x = 0.5671...$

Also known as the Omega constant

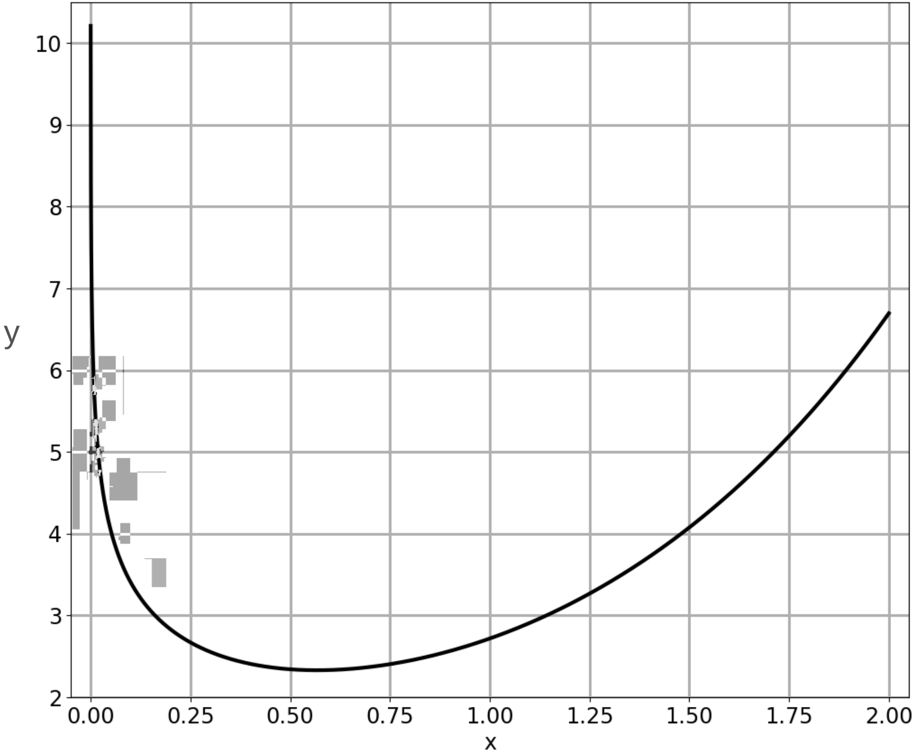


Method 1: Try Both Directions



Method 1: Try Both Directions

Is there any other way?



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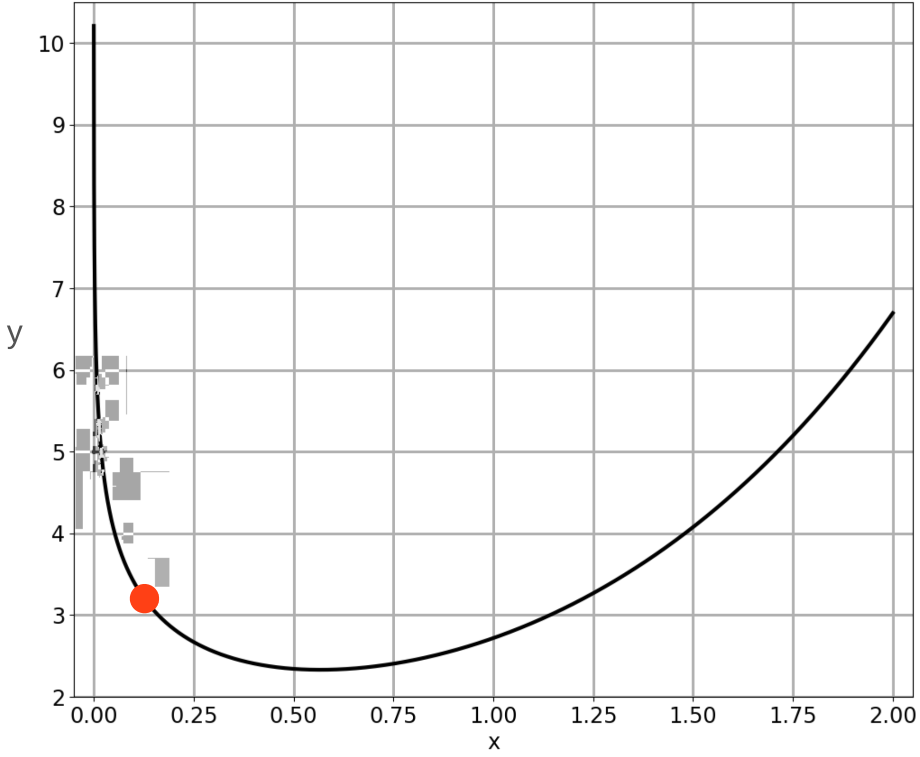

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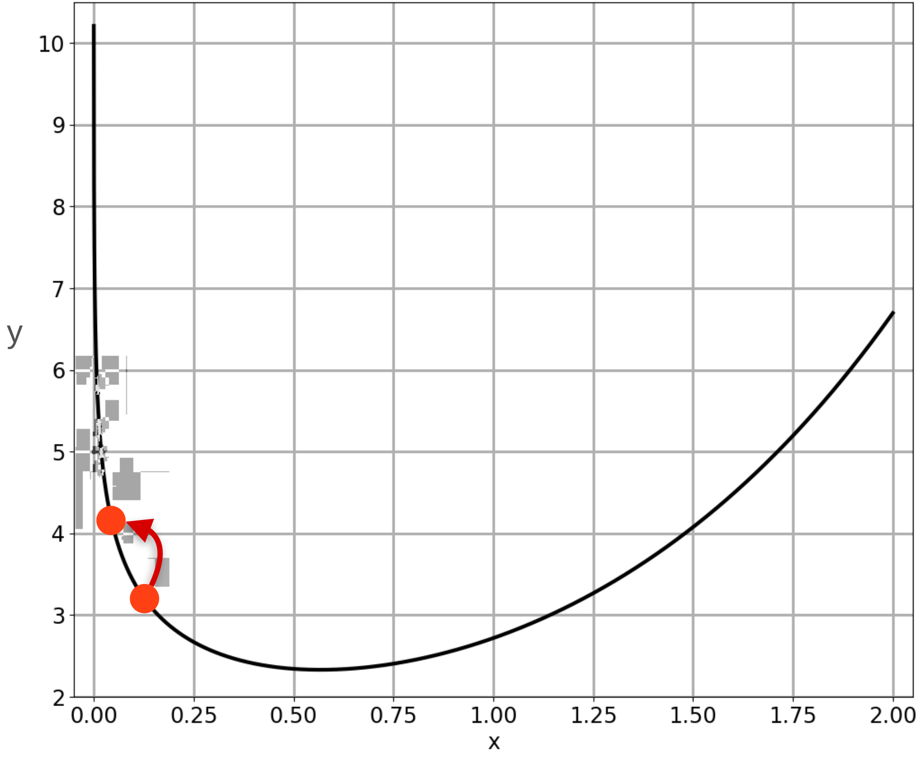

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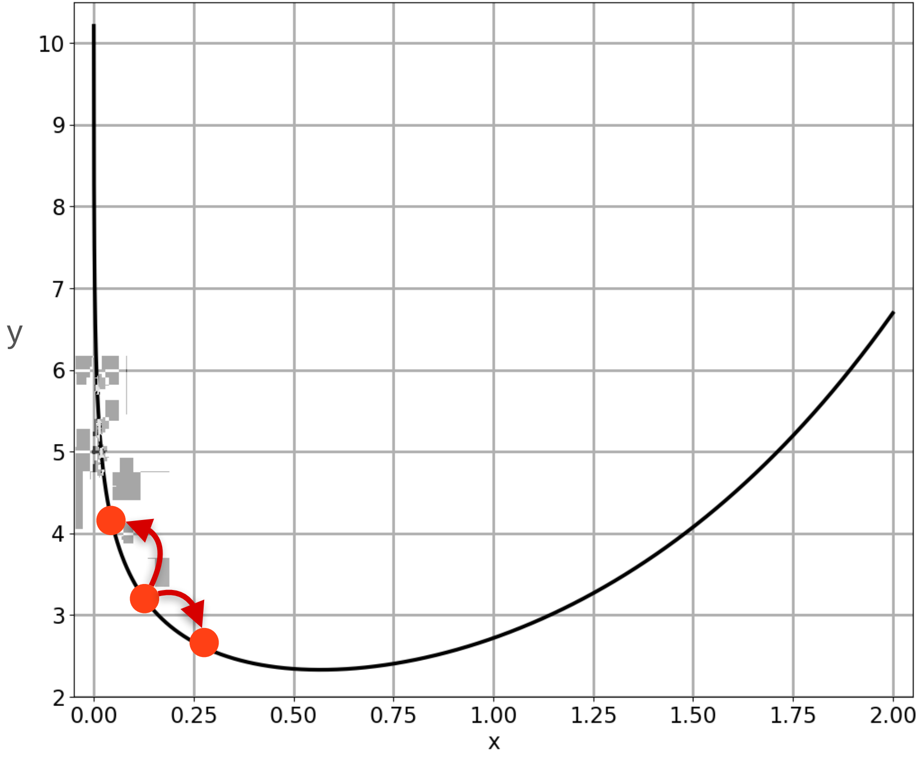

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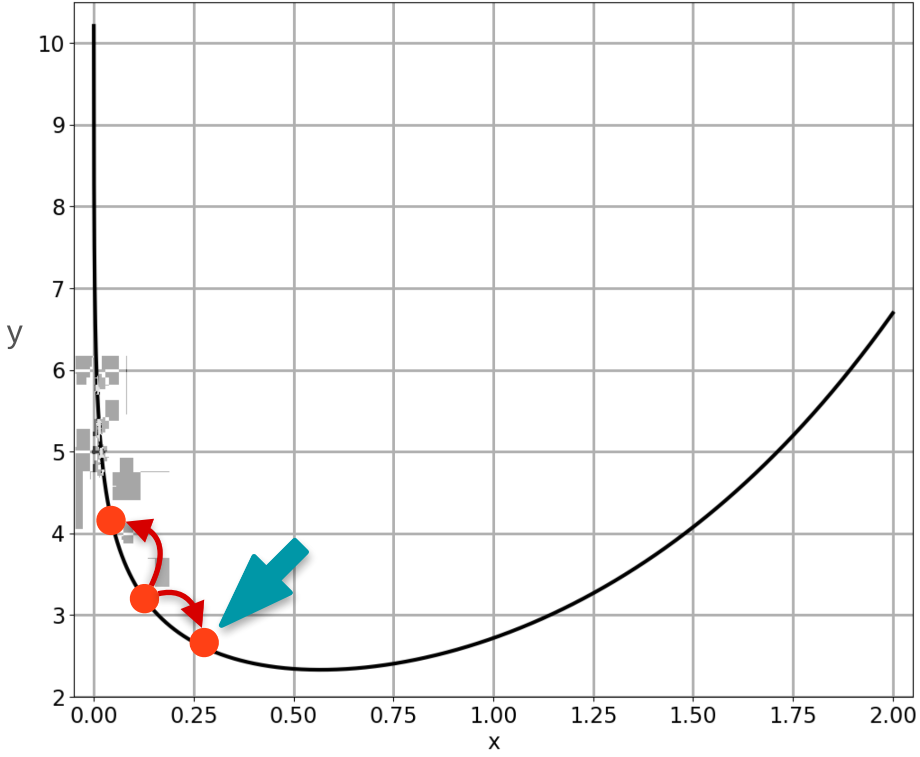

Method 1: Try Both Directions

Is there any
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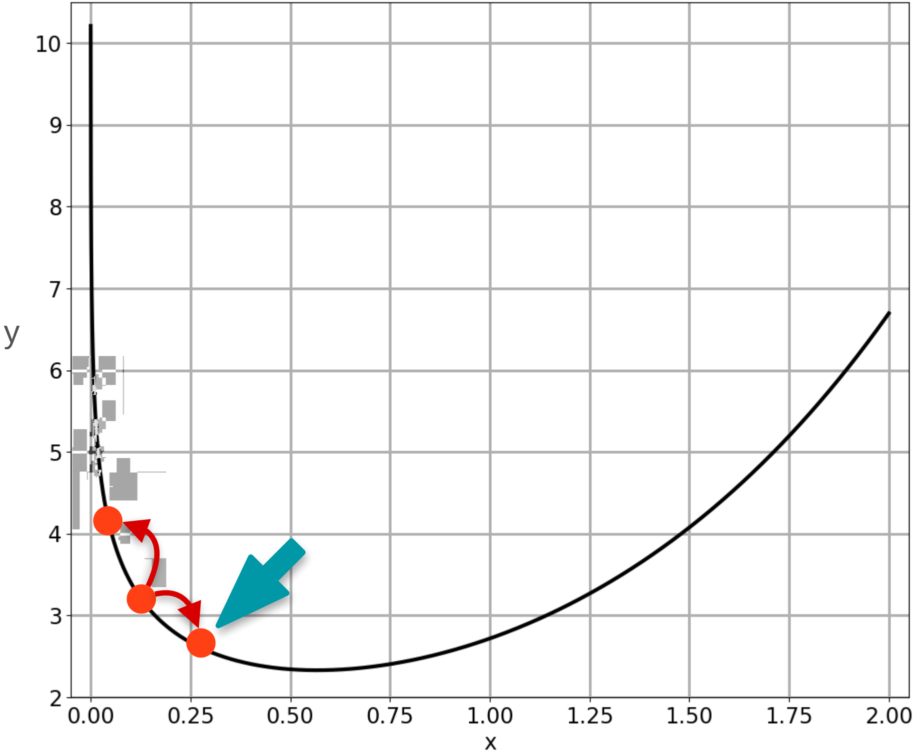
Repeat!



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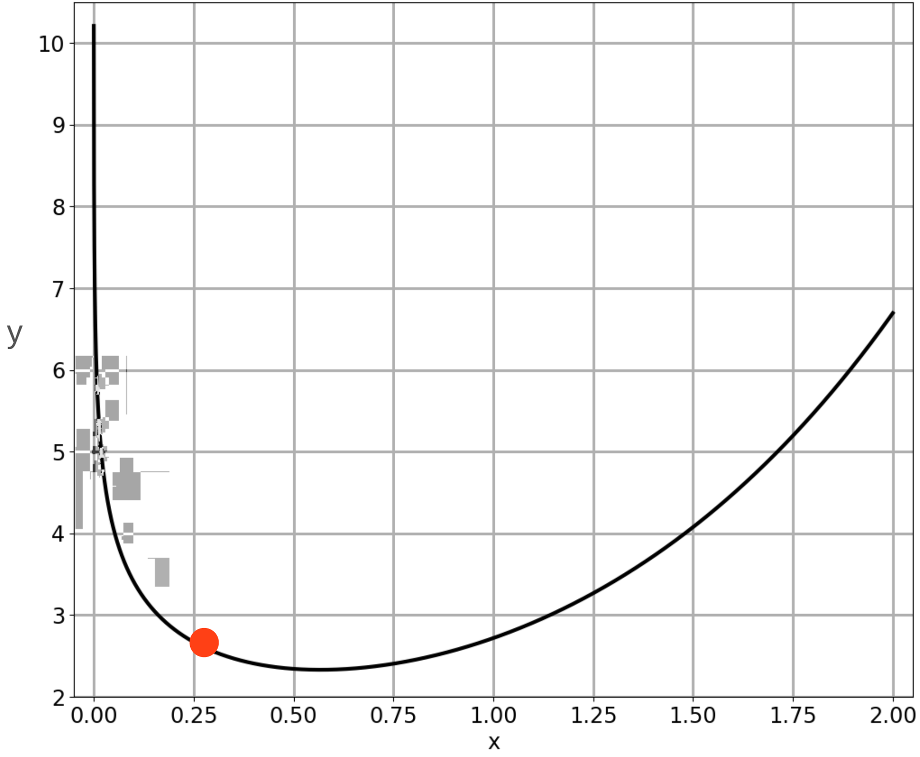

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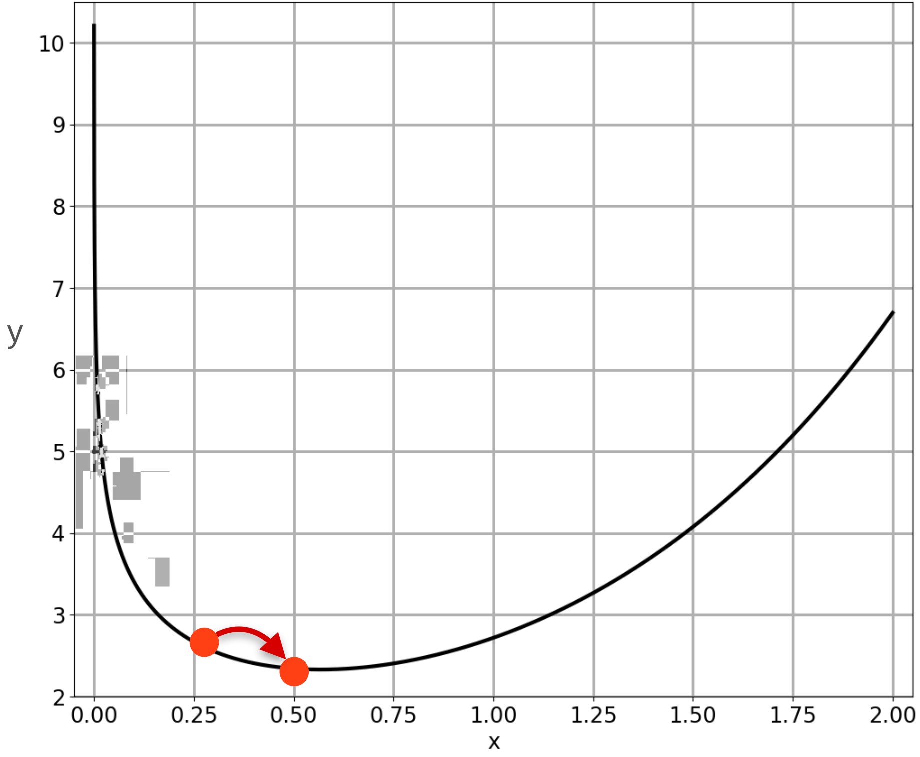

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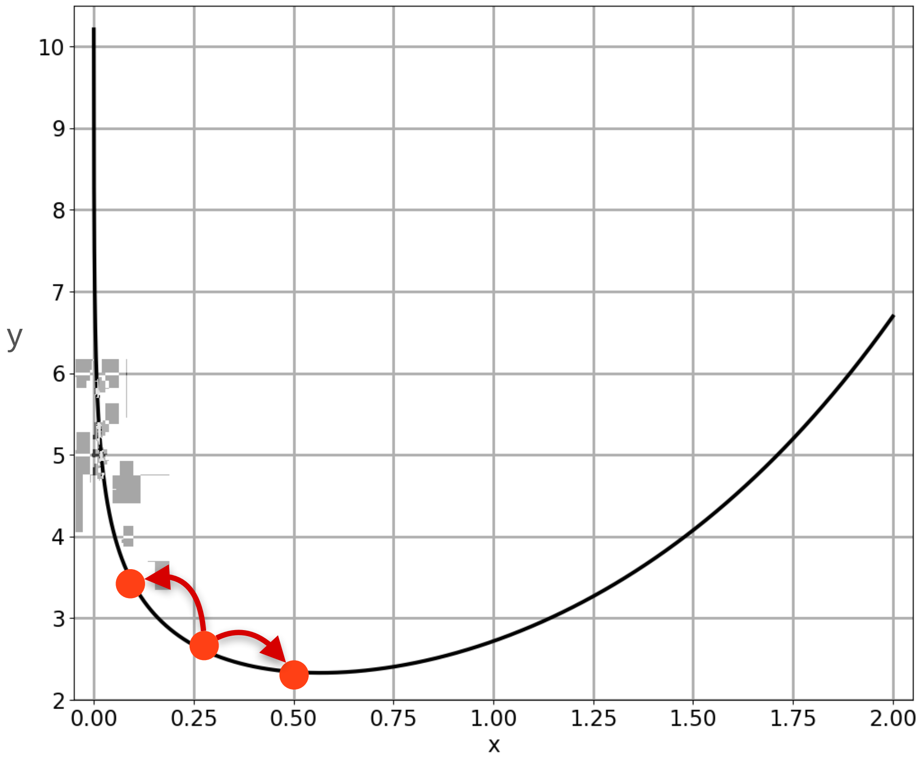

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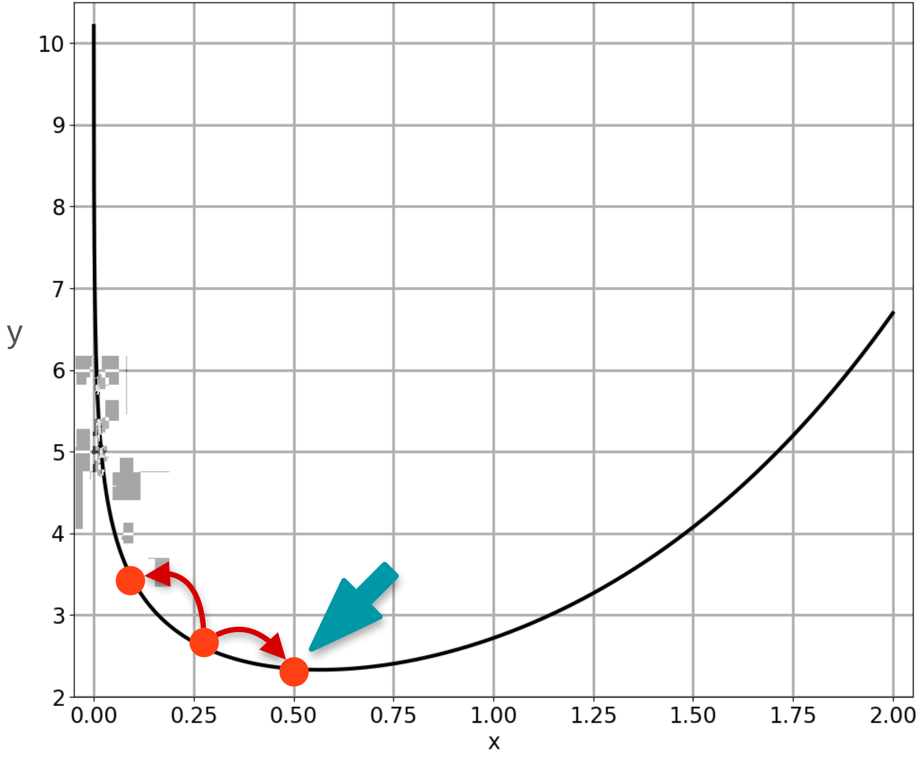

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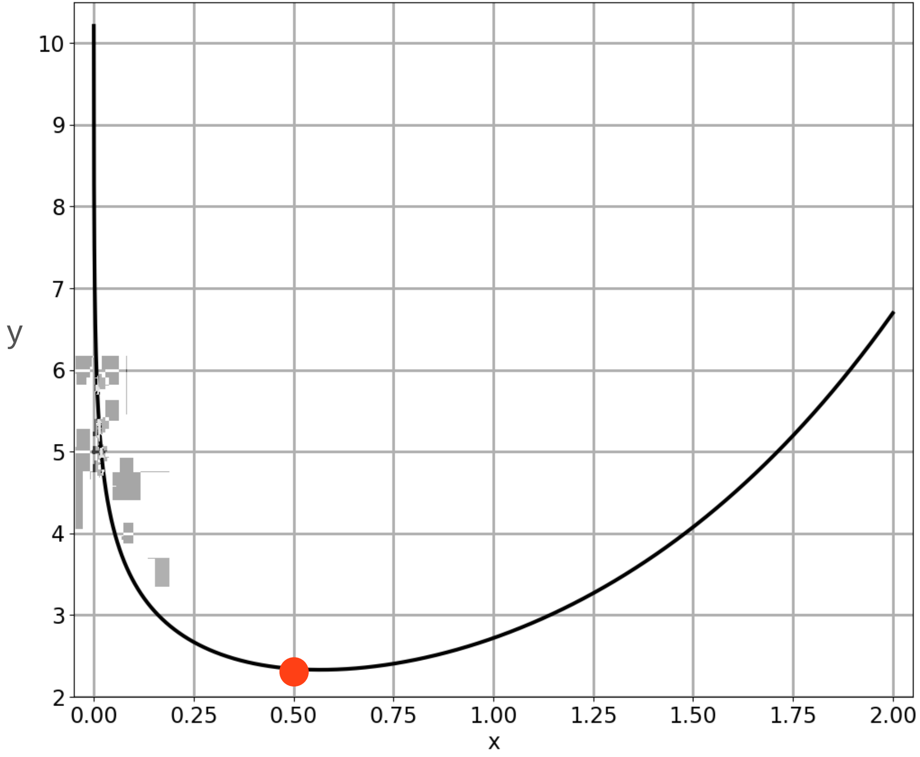

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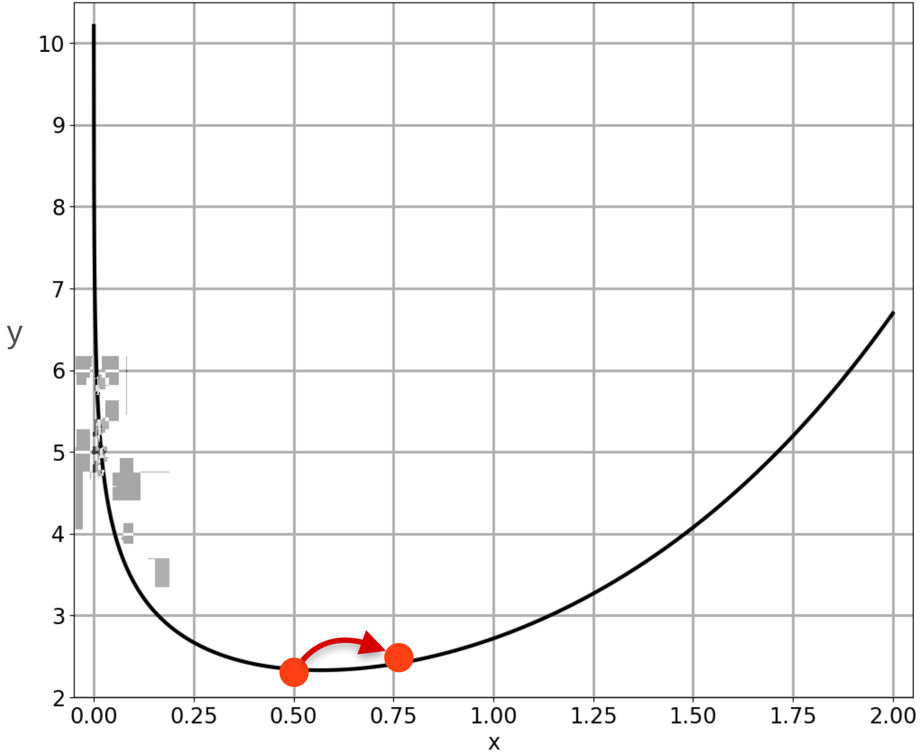

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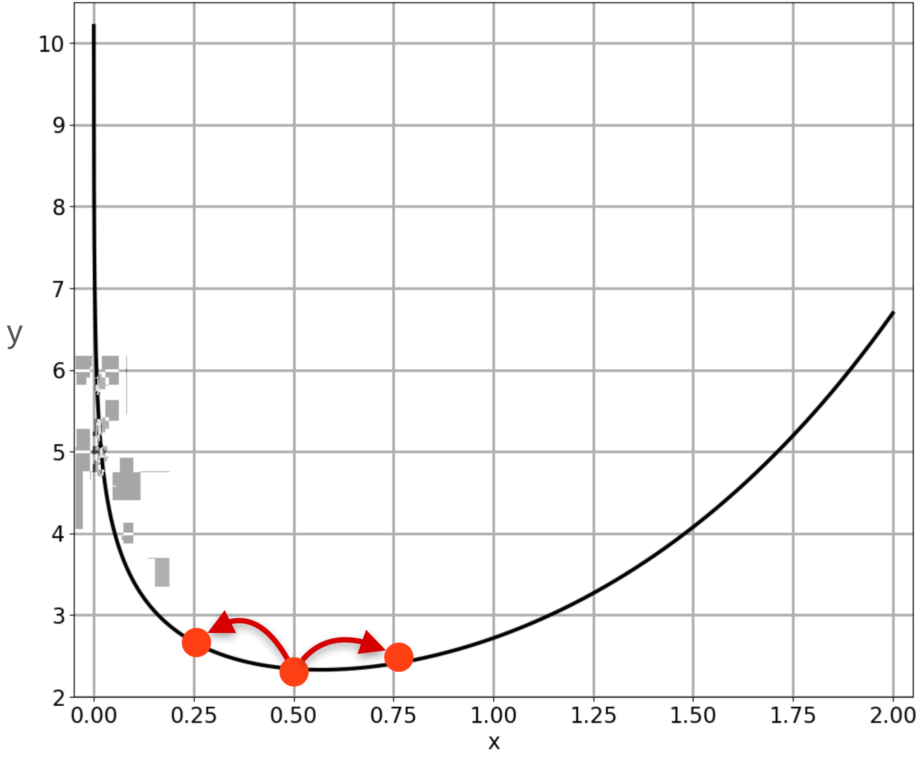

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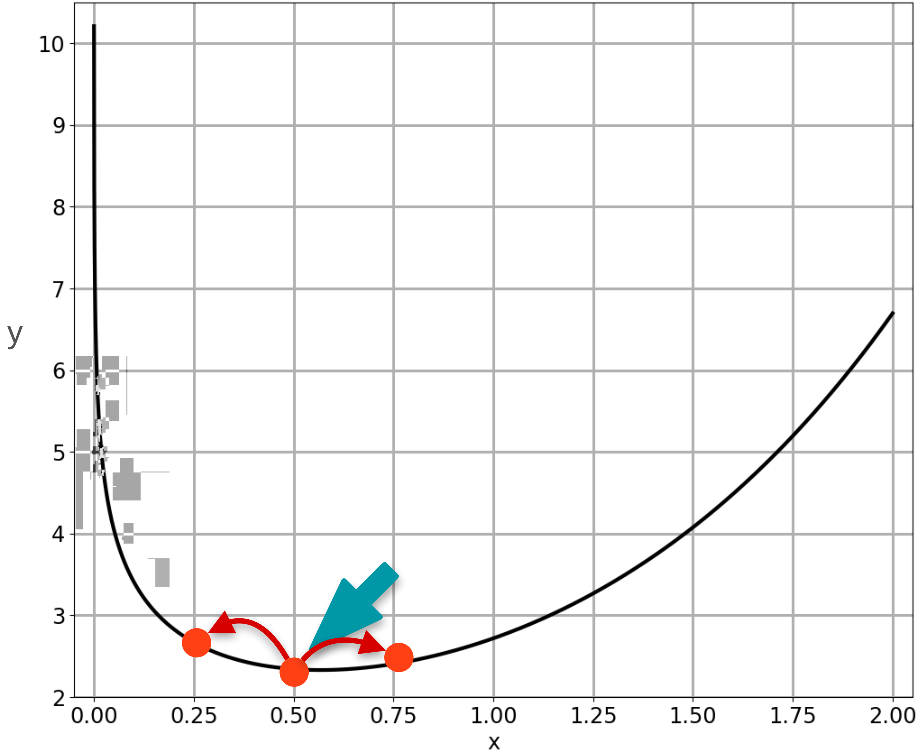

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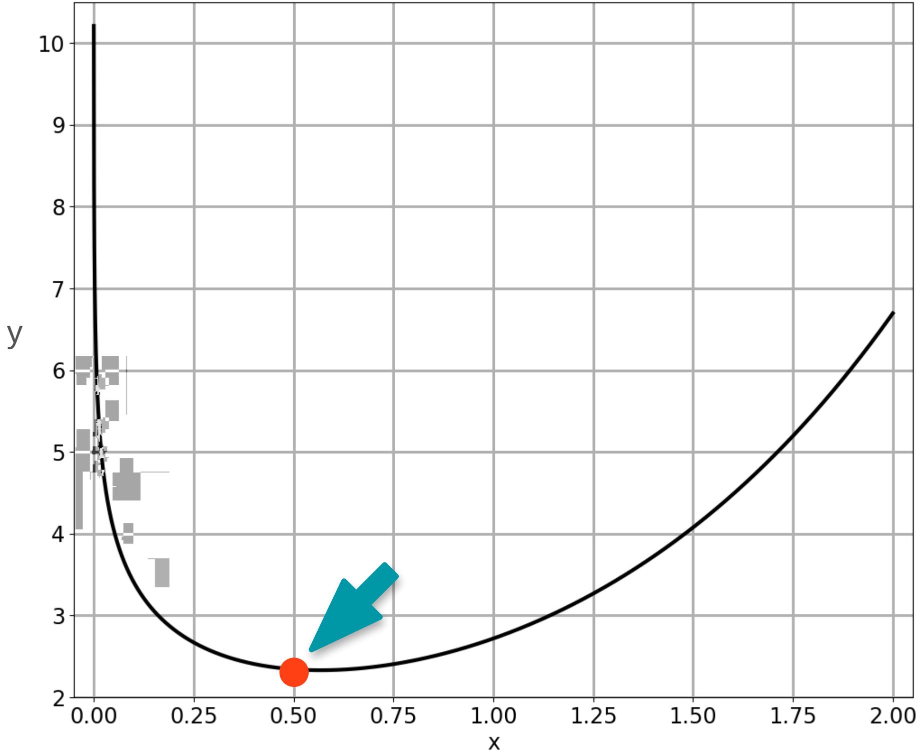
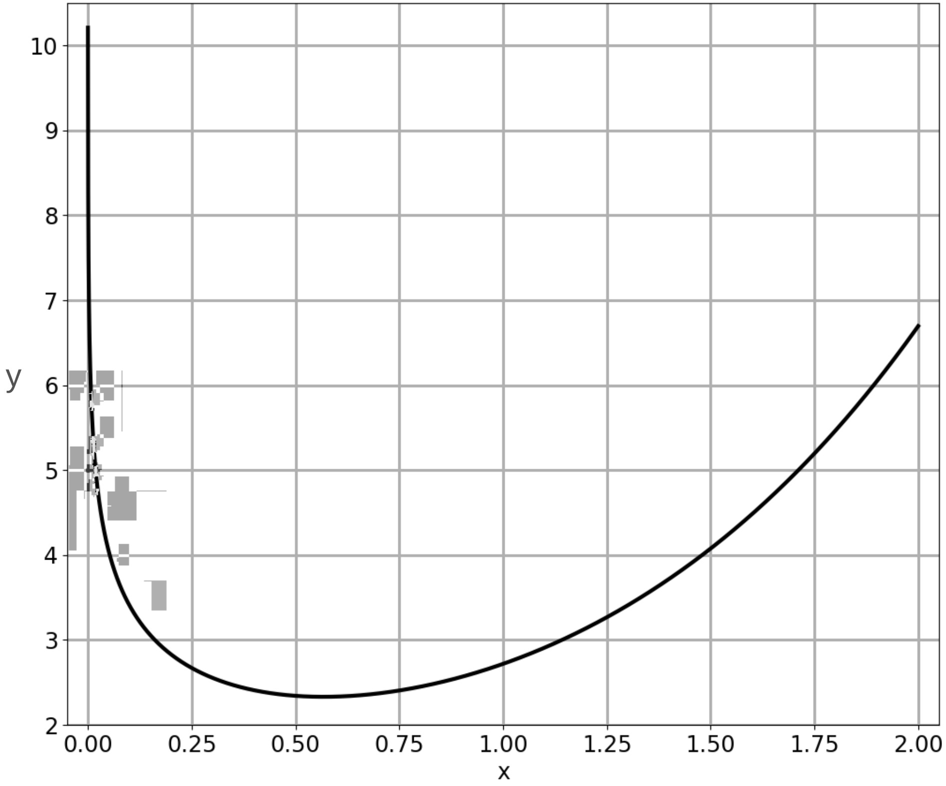


DeepLearning.AI

Gradients and Gradient Descent

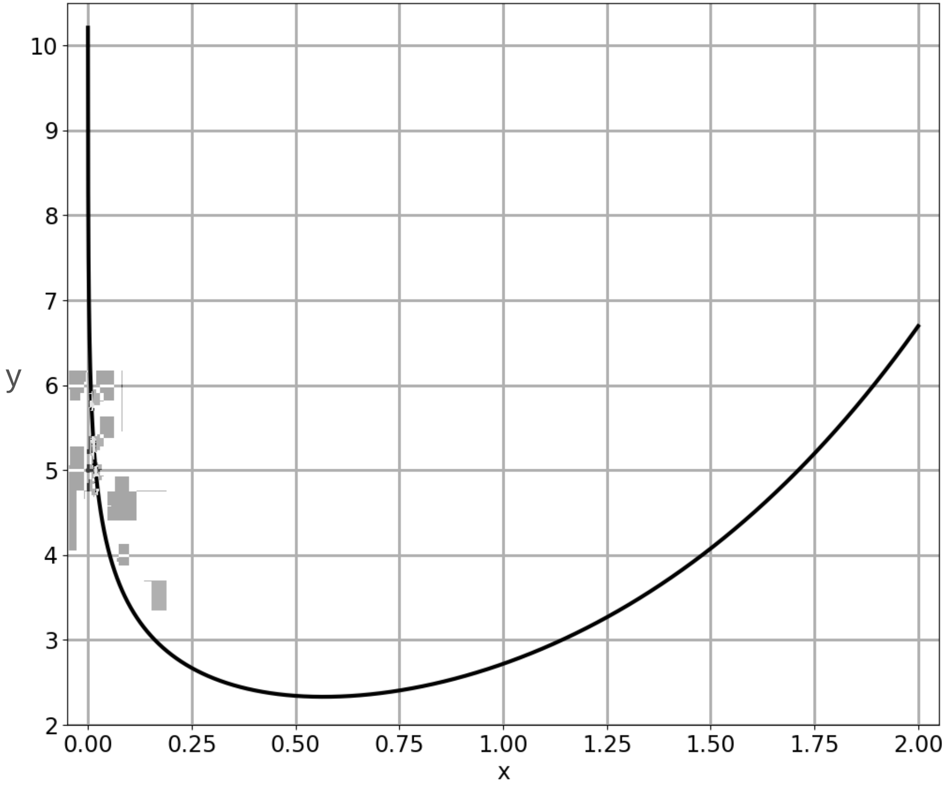
Optimization using Gradient Descent in one variable - Part 2

Method 2: Be Clever



Method 2: Be Clever

Try something smarter…



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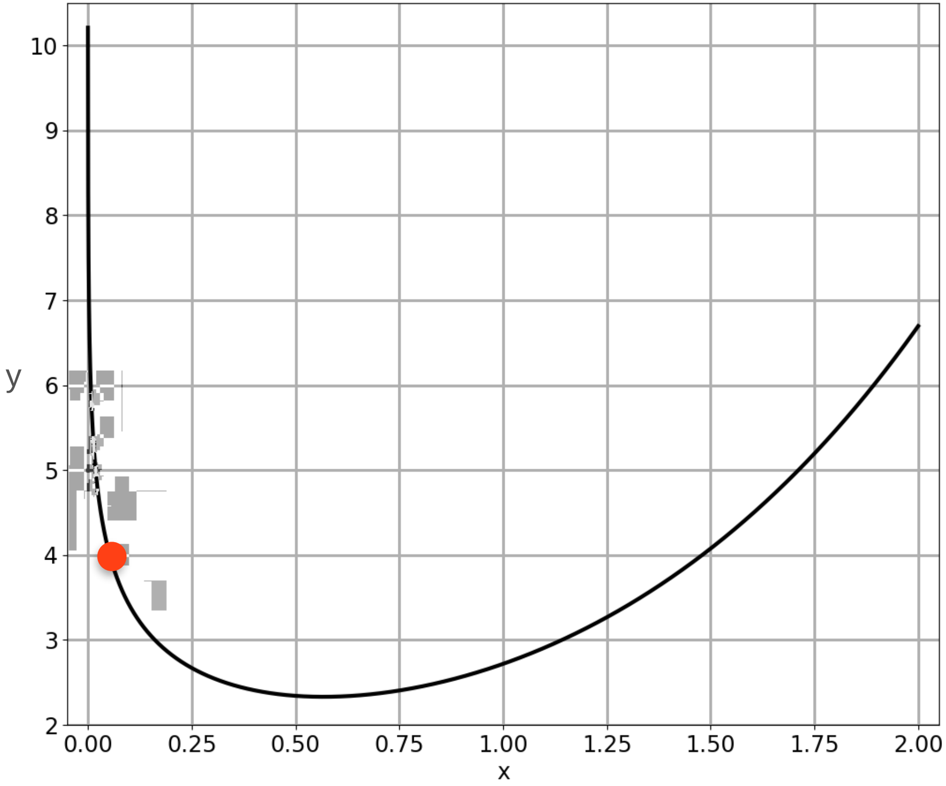

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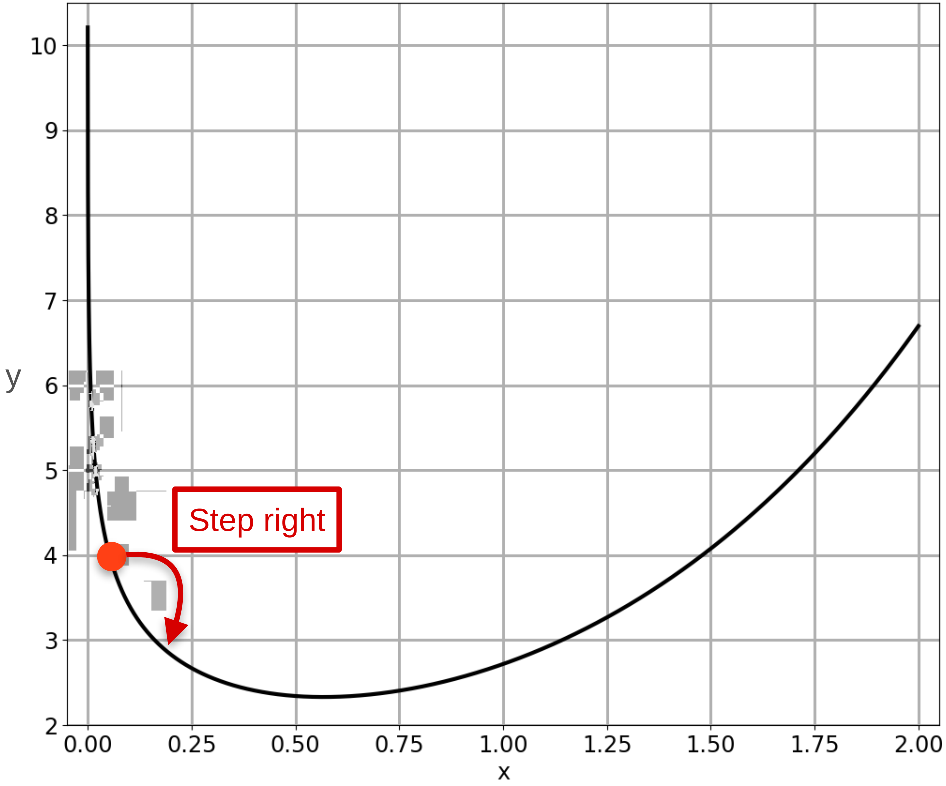

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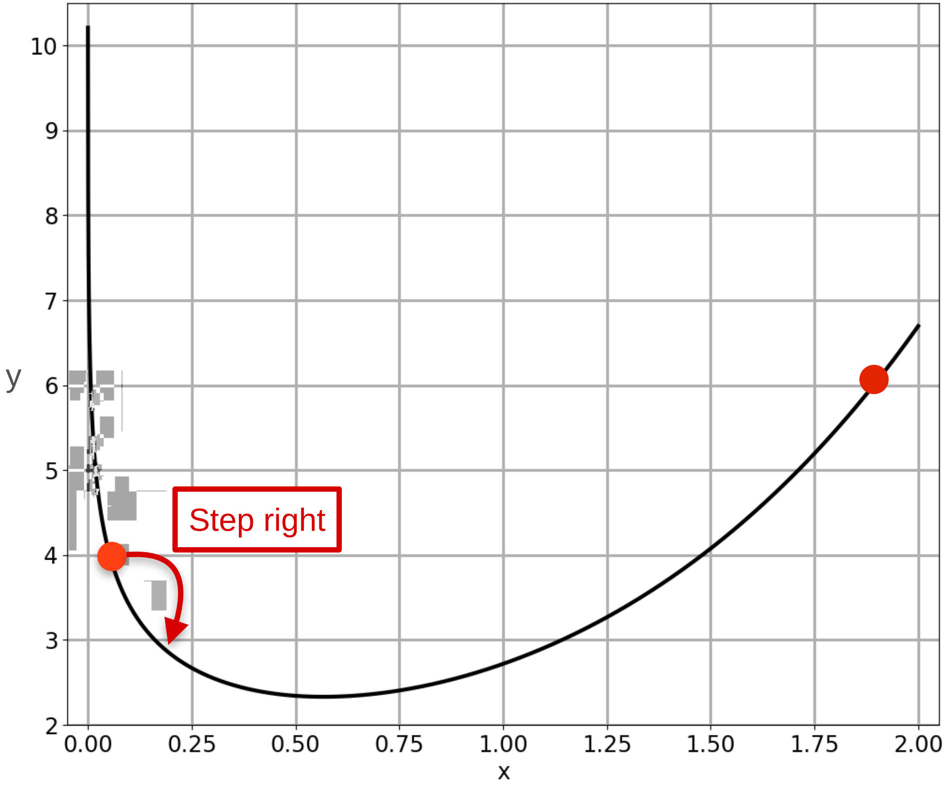

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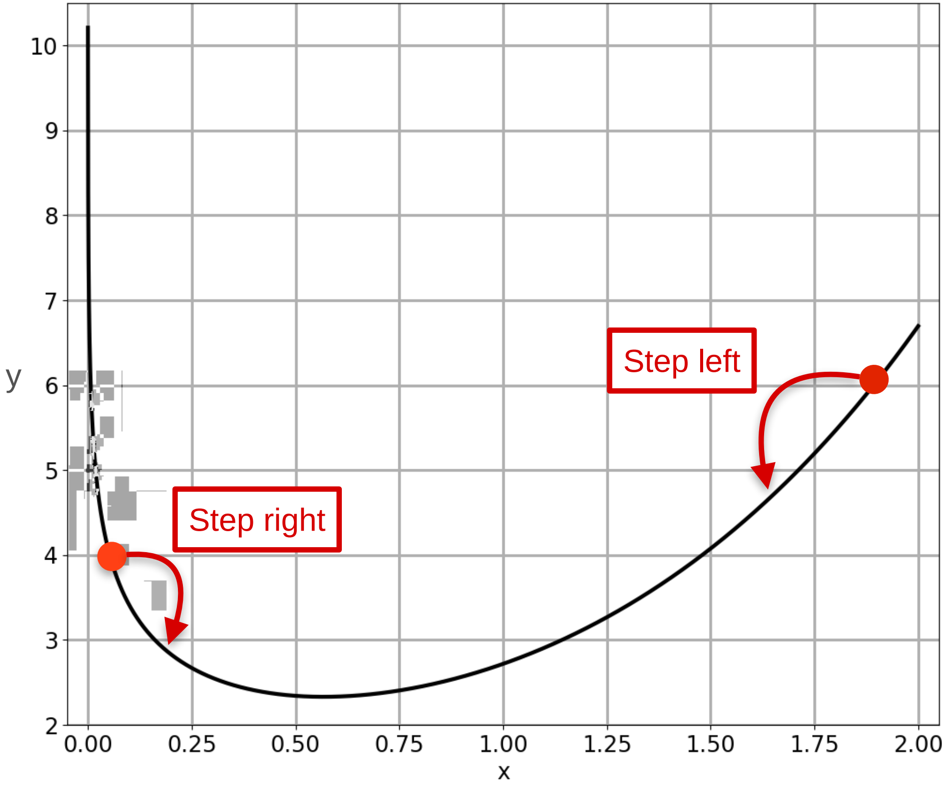

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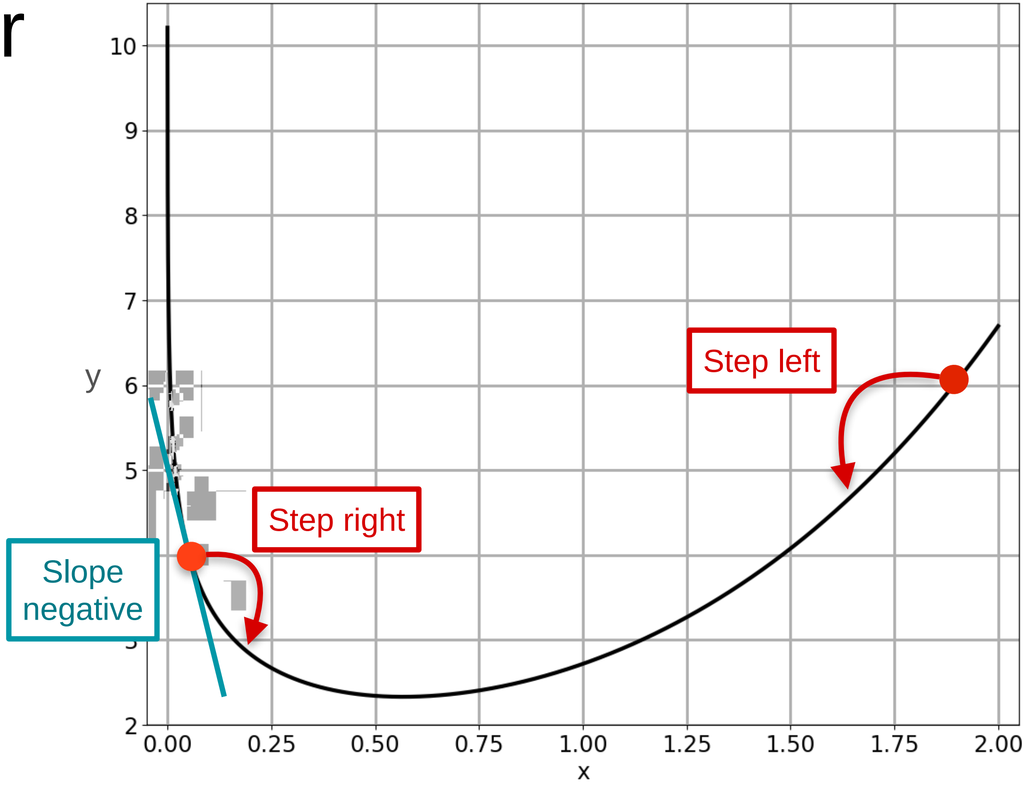

Method 2: Be Clever

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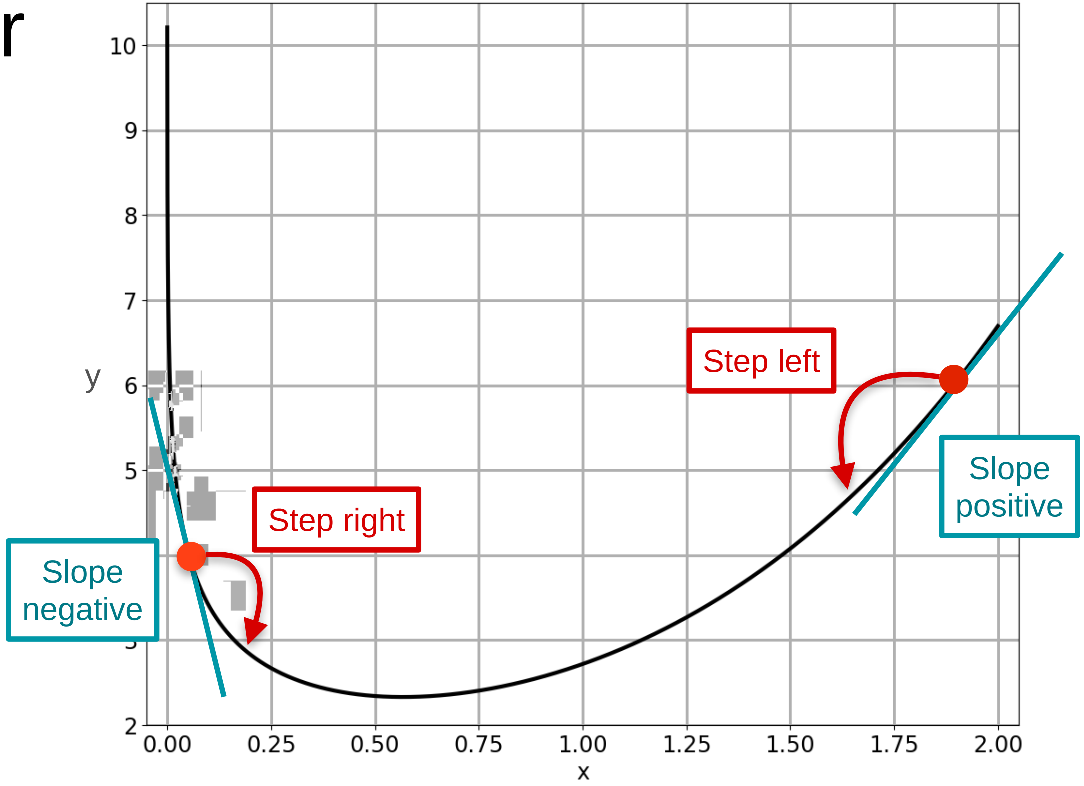
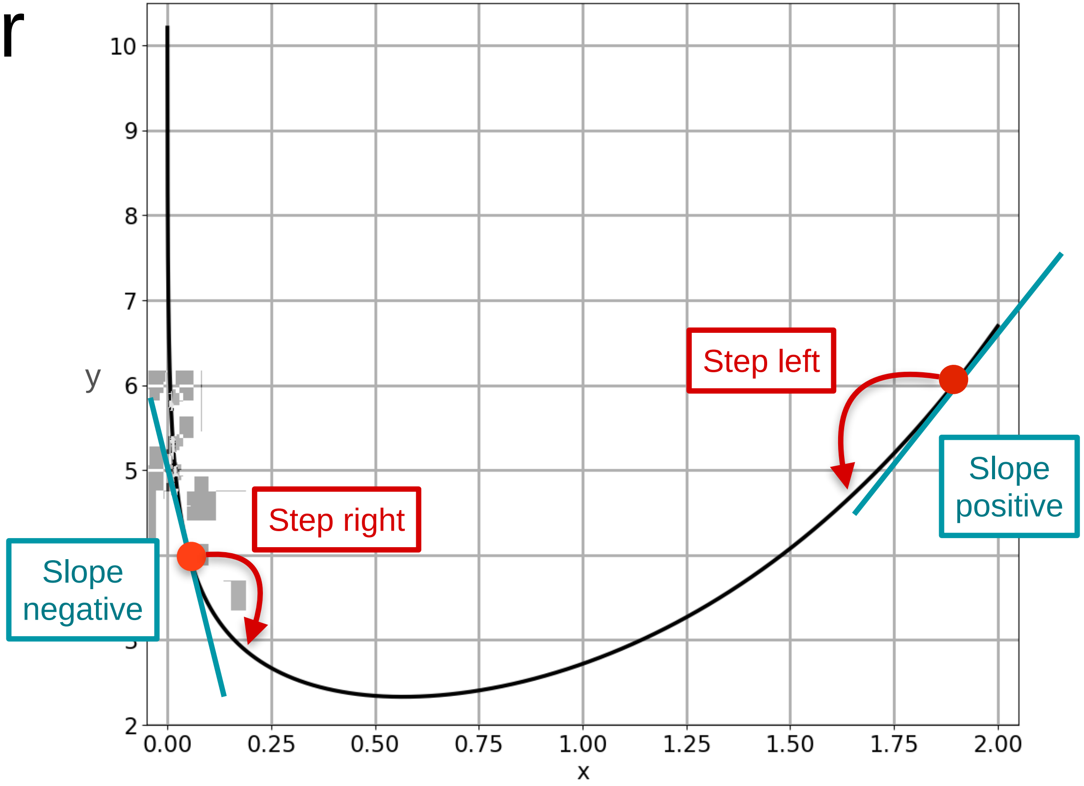


Method 2: Be Clever

Try something smarter…

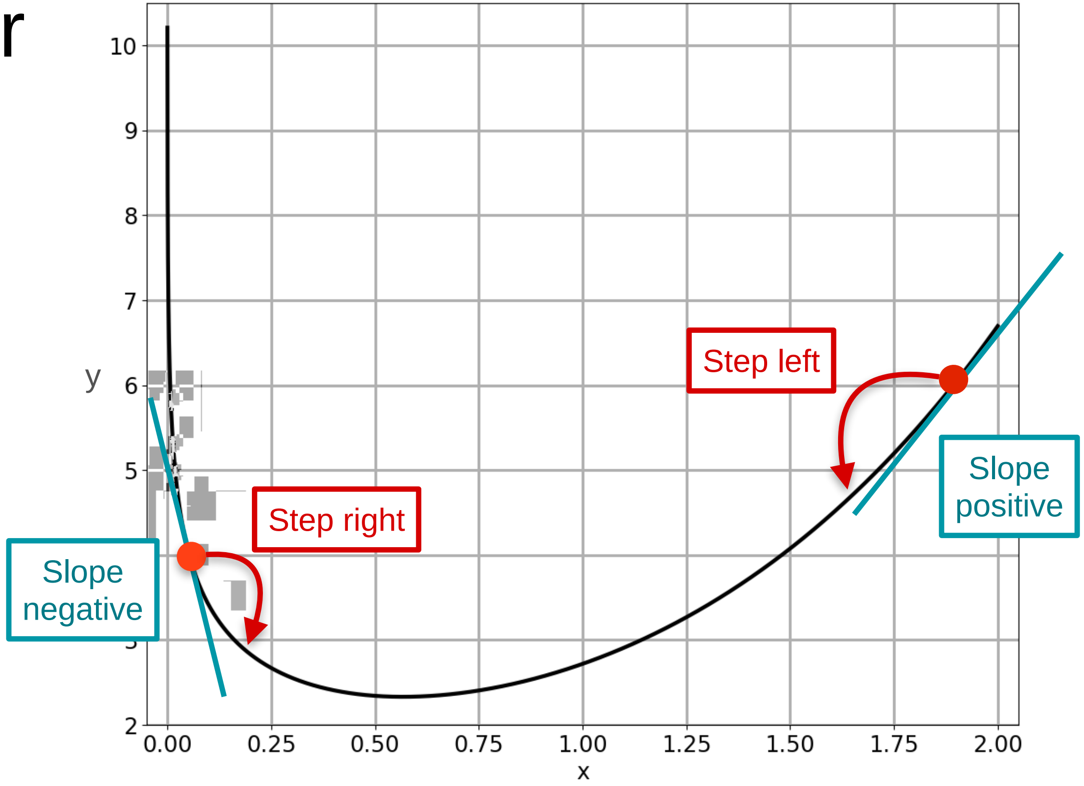
new point



Method 2: Be Clever

Try something smarter…

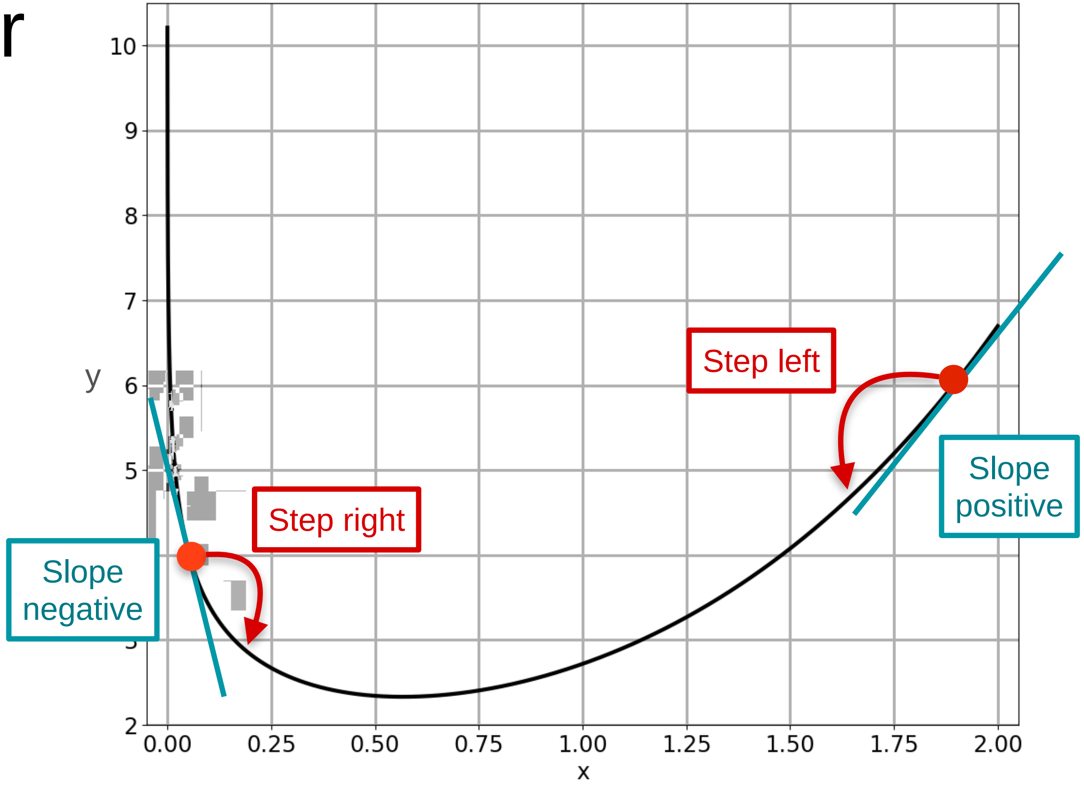
new point = old point



Method 2: Be Clever

Try something smarter…

new point = old point - slope

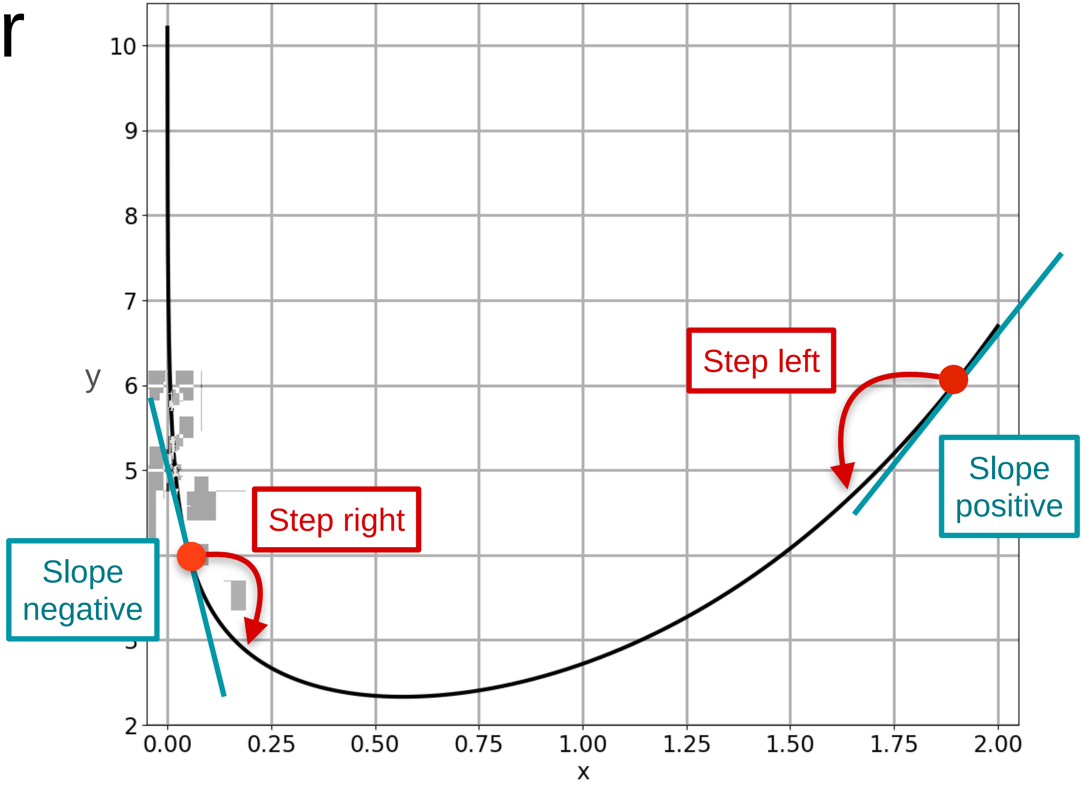


Method 2: Be Clever

Try something smarter…

new point = old point - slope

x_1

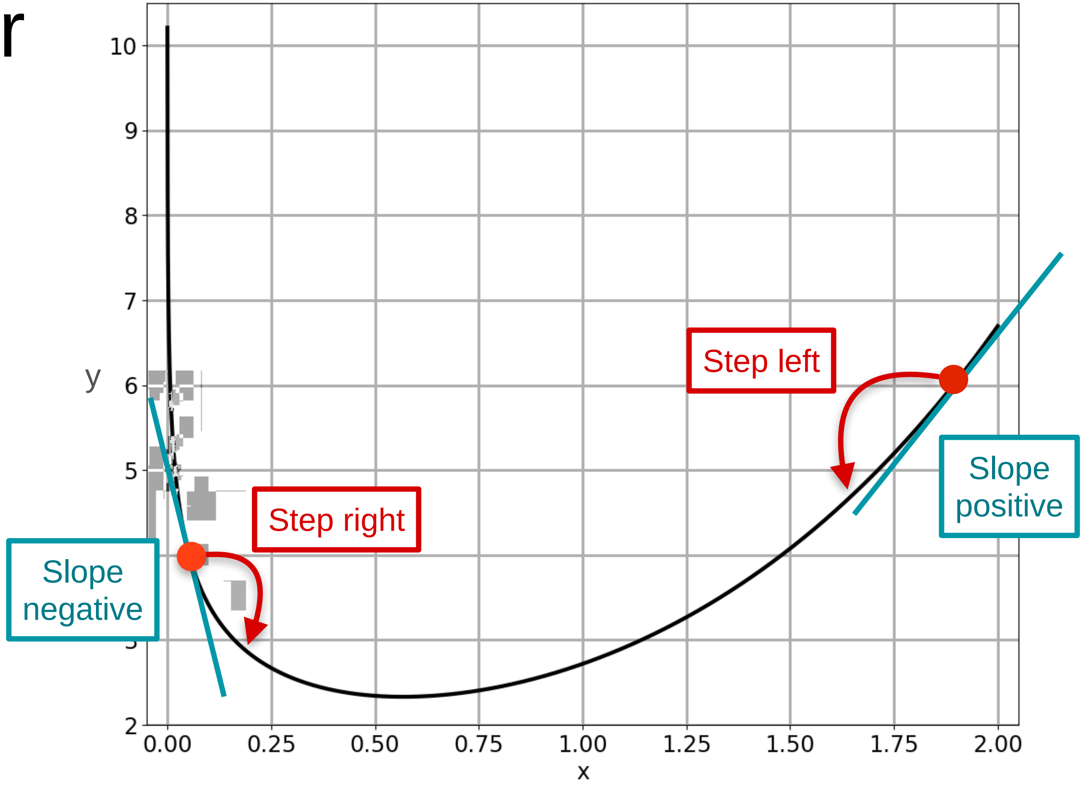


Method 2: Be Clever

Try something smarter…

new point = old point - slope

$$x_1 = x_0$$

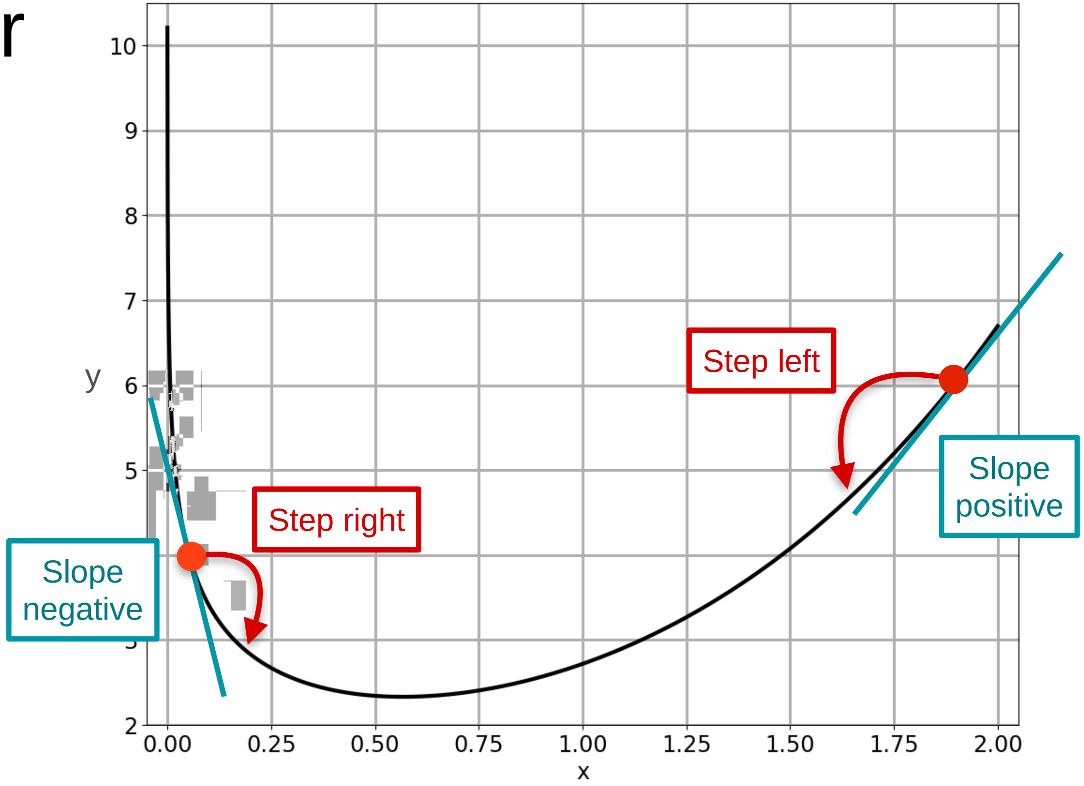


Method 2: Be Clever

Try something smarter…

new point = old point - slope

$$x_1 = x_0 - f'(x_0)$$

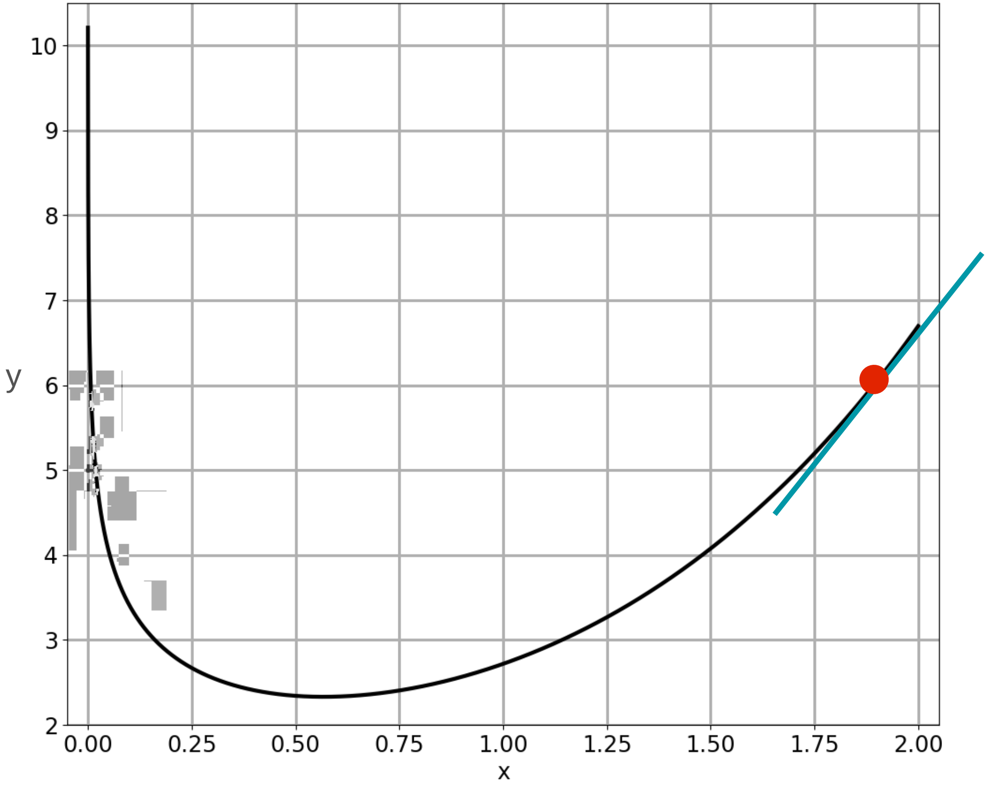


Method 2: Be Clever

Try something smarter…

new point = old point - slope

$$x_1 = x_0 - f'(x_0)$$

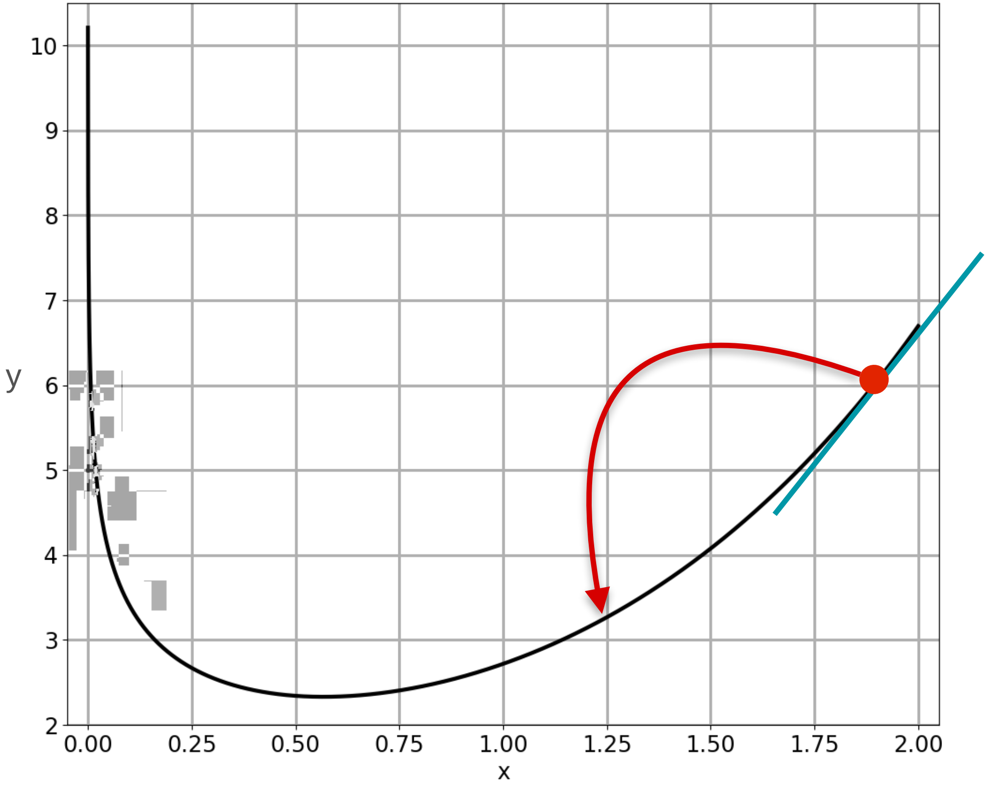


Method 2: Be Clever

Try something smarter…

new point = old point - slope

$$x_1 = x_0 - f'(x_0)$$

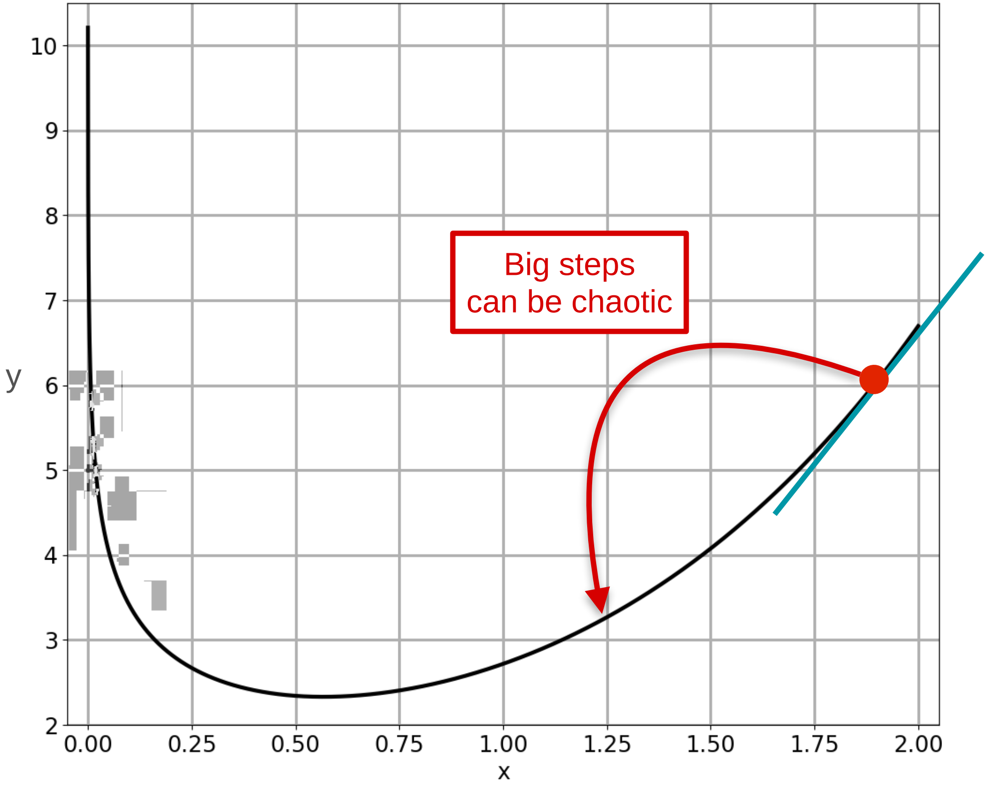


Method 2: Be Clever

Try something smarter…

new point = old point - slope

$$x_1 = x_0 - f'(x_0)$$

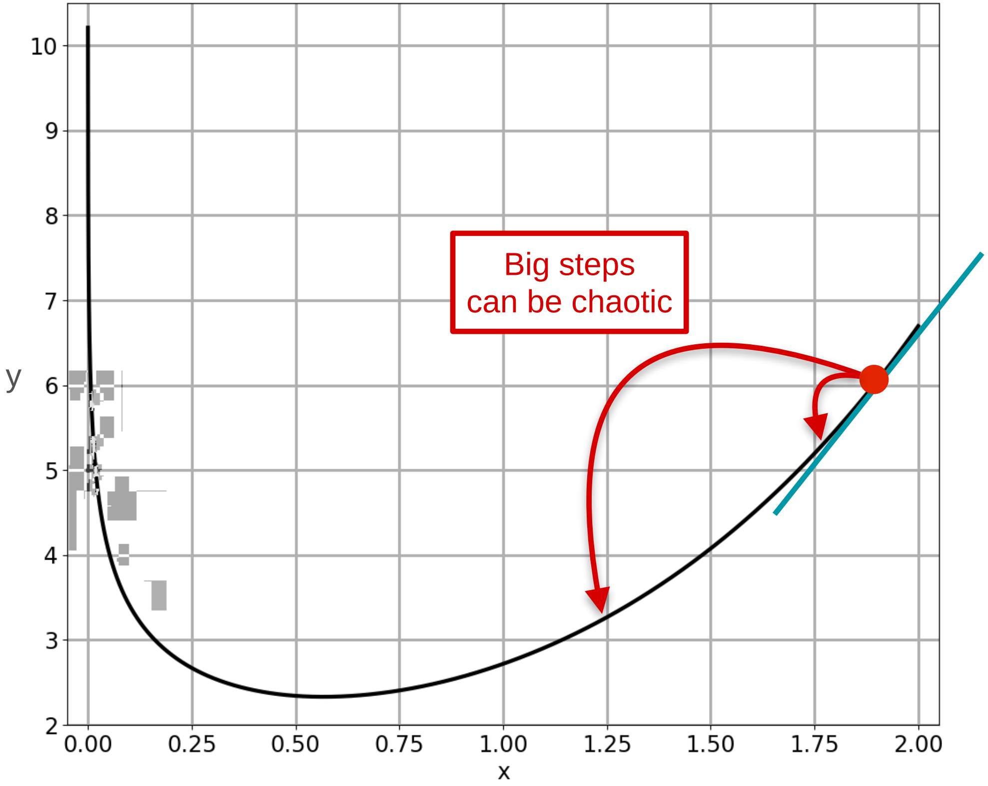


Method 2: Be Clever

Try something smarter…

new point = old point - slope

$$x_1 = x_0 - f'(x_0)$$



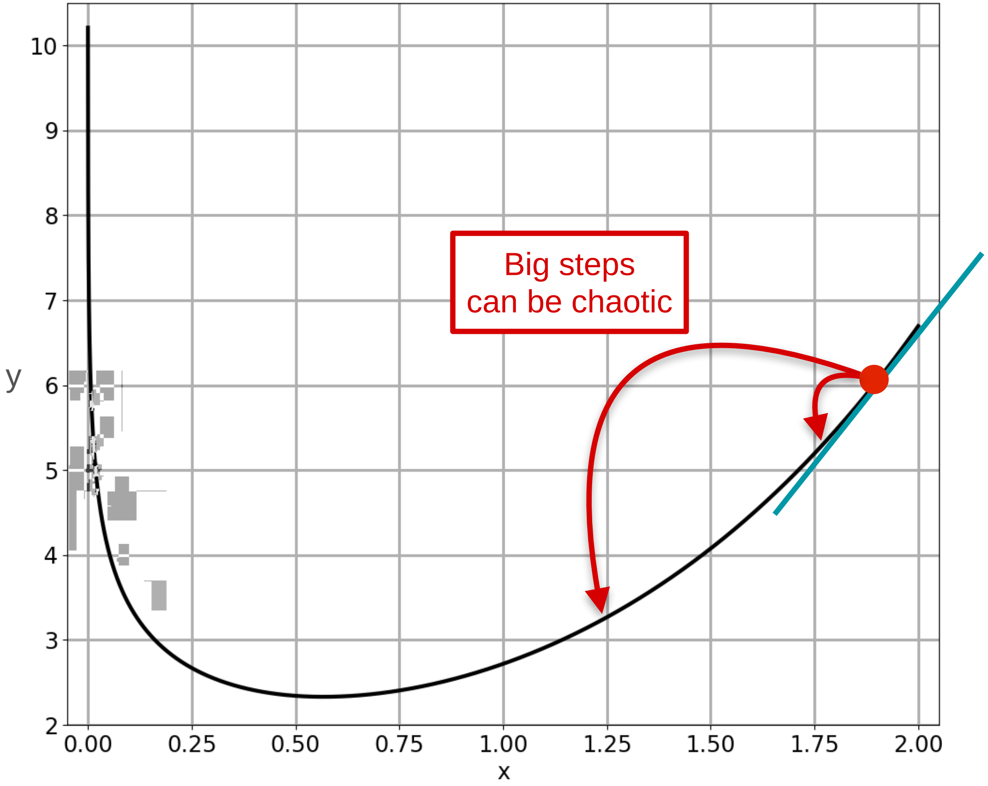
Method 2: Be Clever

Try something smarter…

new point = old point - slope

$$x_1 = x_0 - f'(x_0)$$

$$x_1 = x_0 - 0.01\, f'(x_0)$$



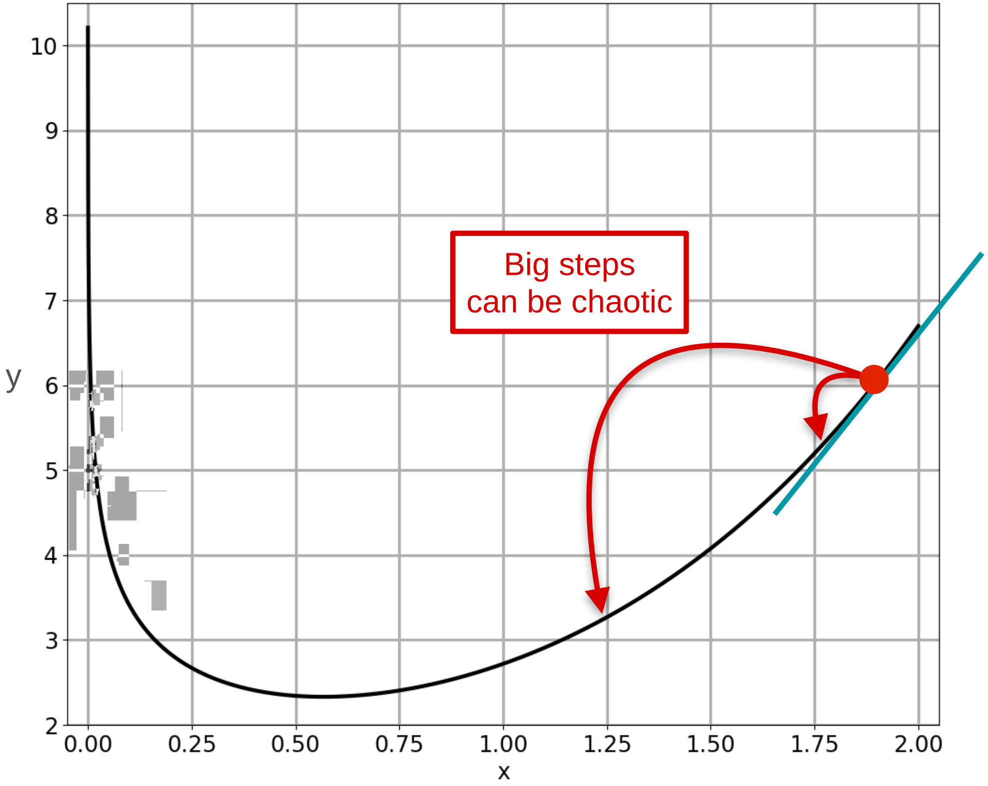
Method 2: Be Clever

Try something smarter…

new point = old point - slope

$$x_1 = x_0 - f'(x_0)$$

$$x_1 = x_0 - \alpha \, f'(x_0)$$



Method 2: Be Clever

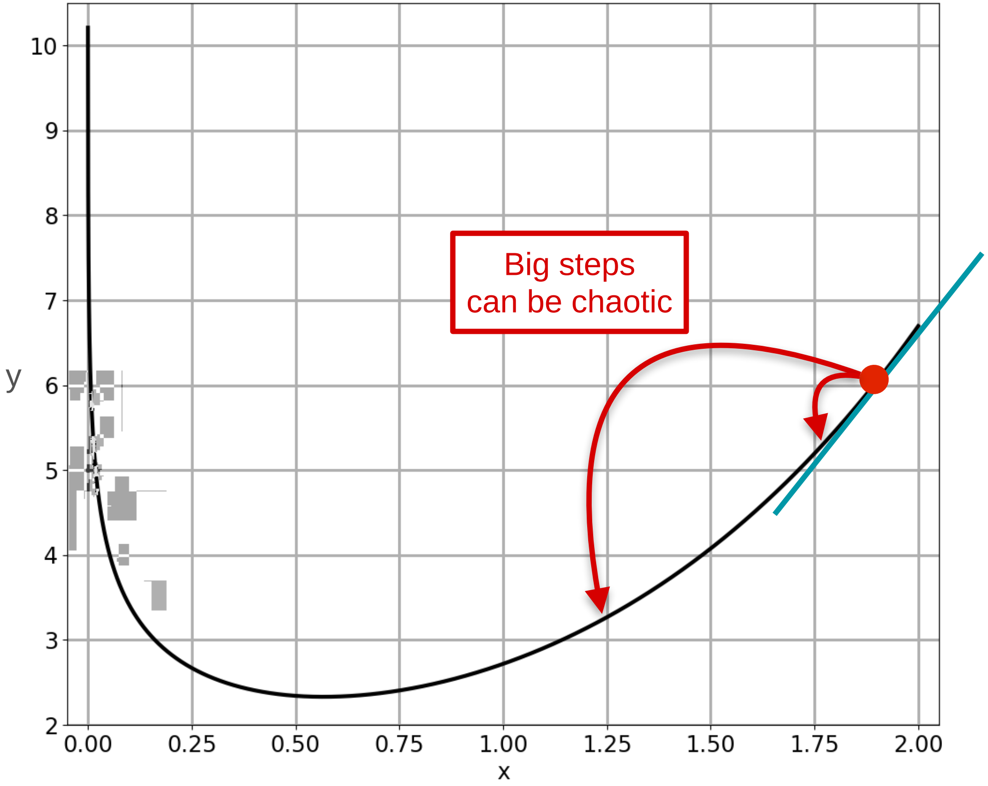
Try something smarter…

new point = old point - slope

$$x_1 = x_0 - f'(x_0)$$

$$x_1 = x_0 - \alpha \; f'(x_0)$$

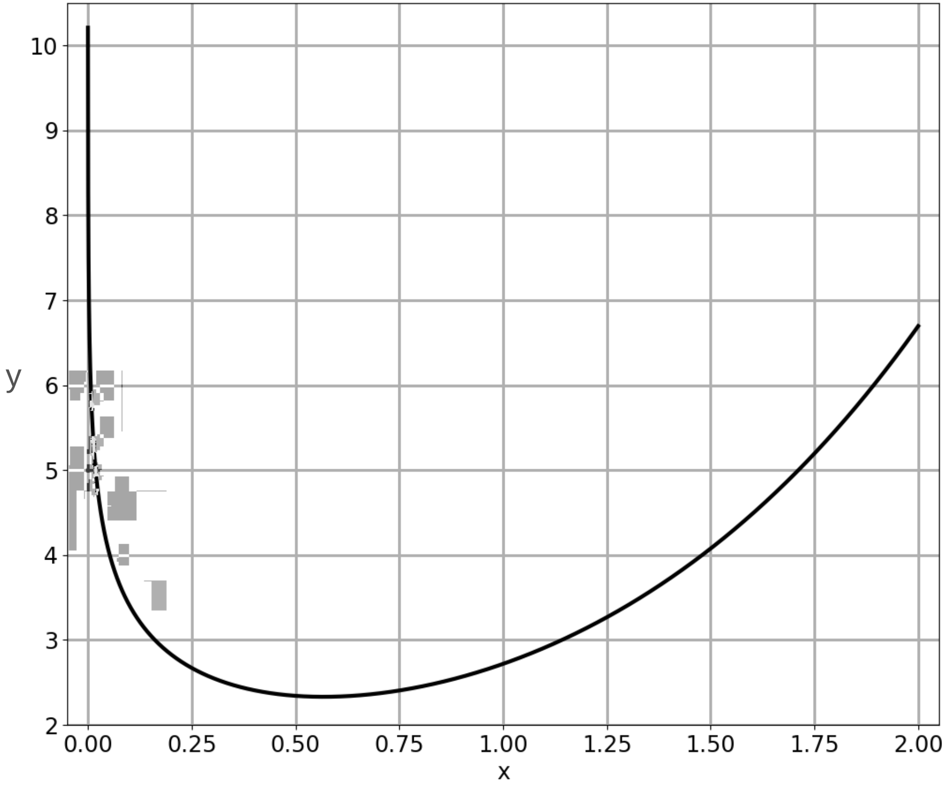
Learning rate



Method 2: Be Clever

Try something smarter…

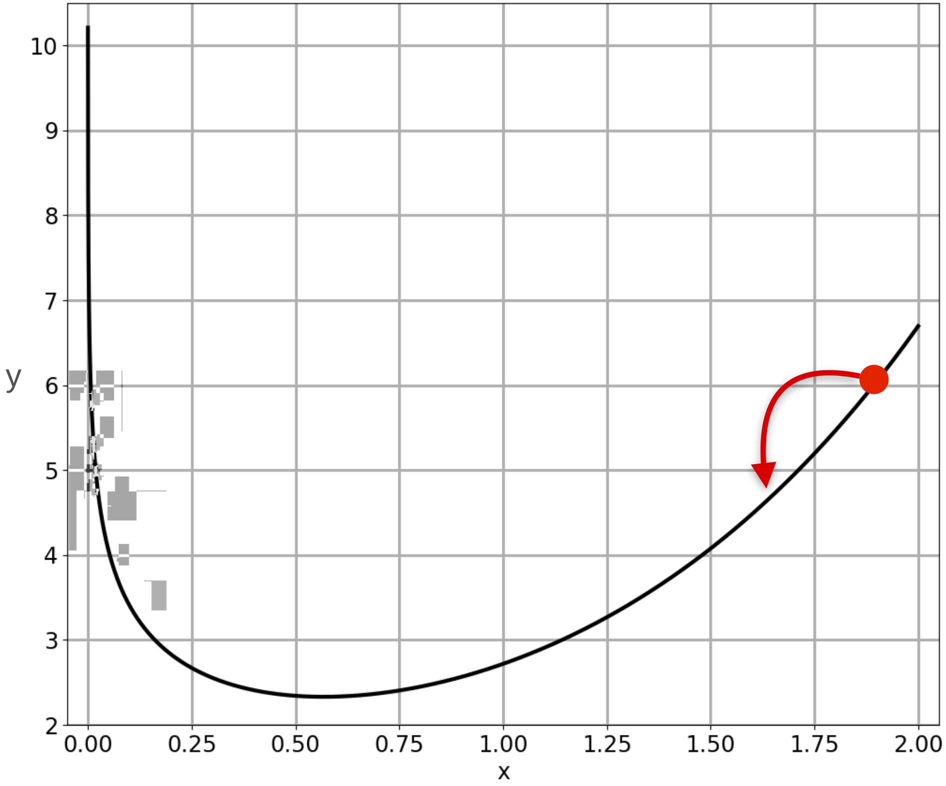
$$x_1 = x_0 - \alpha \, f'(x_0)$$



Method 2: Be Clever

Try something smarter…

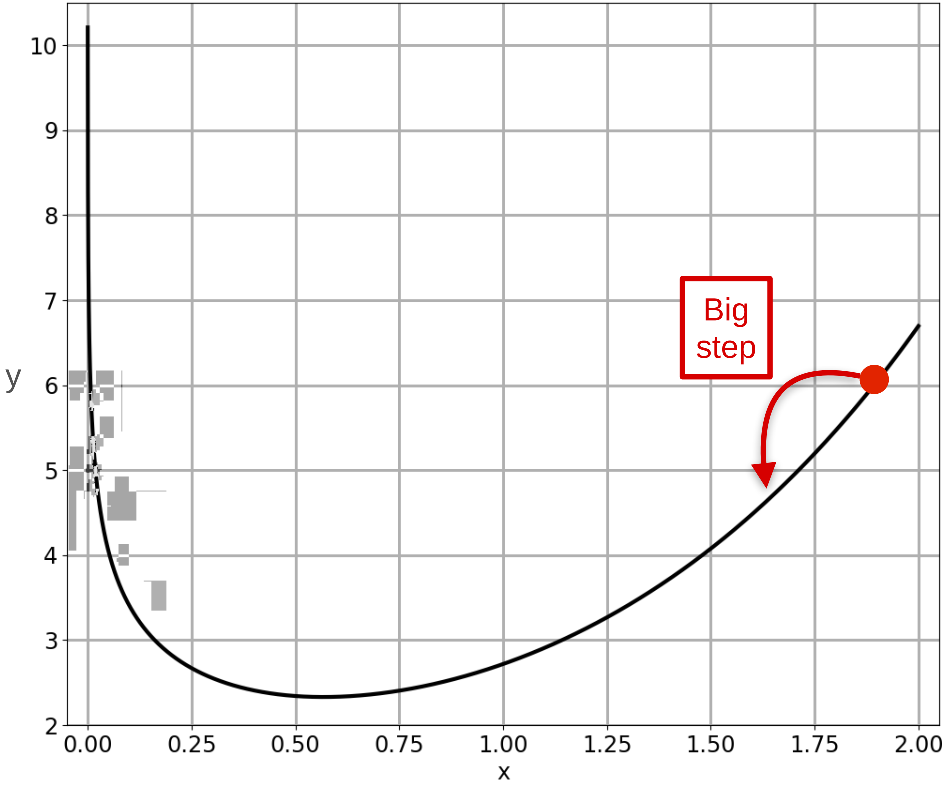
$$x_1 = x_0 - \alpha \, f'(x_0)$$



Method 2: Be Clever

Try something smarter…

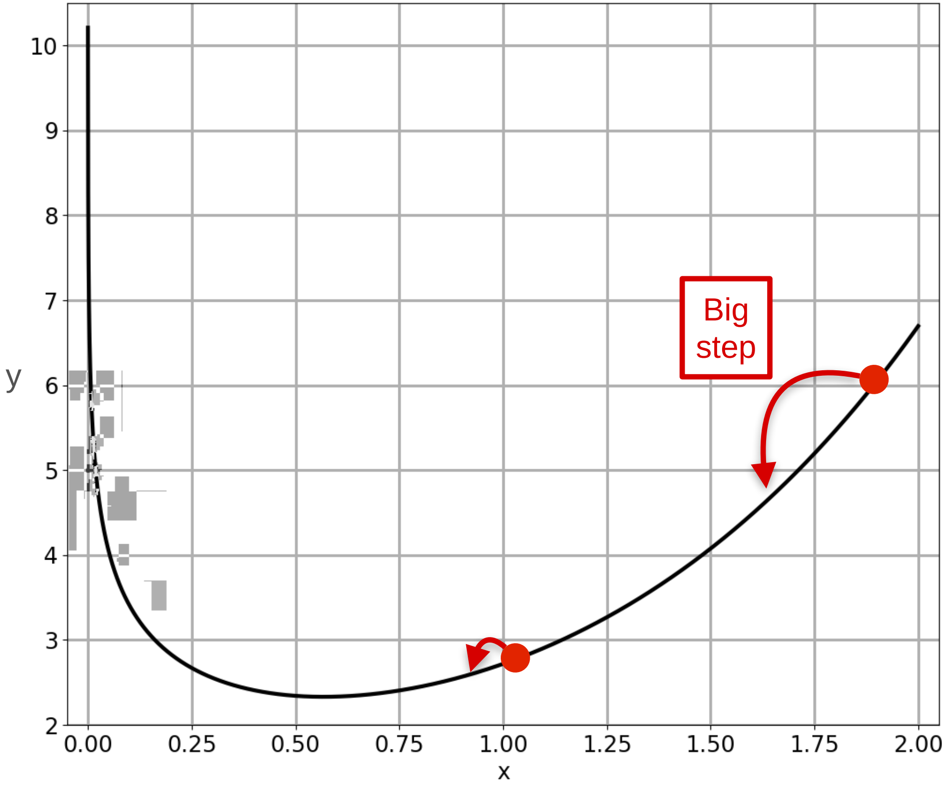
$$x_1 = x_0 - \alpha f'(x_0)$$



Method 2: Be Clever

Try something smarter…

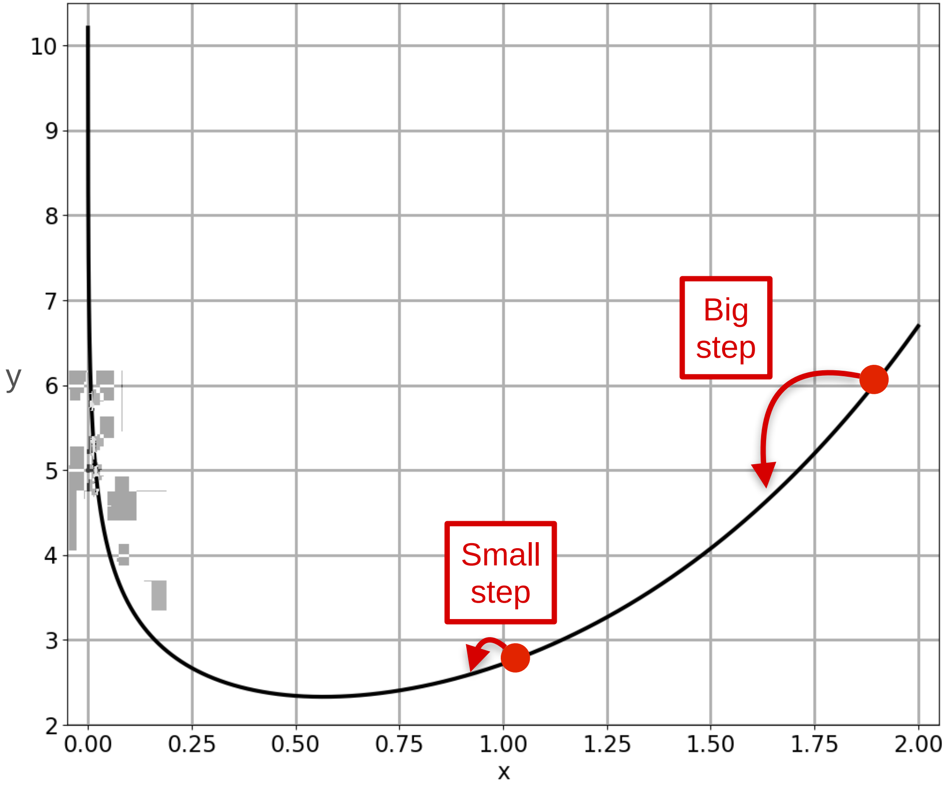
$$x_1 = x_0 - \alpha \, f'(x_0)$$



Method 2: Be Clever

Try something smarter…

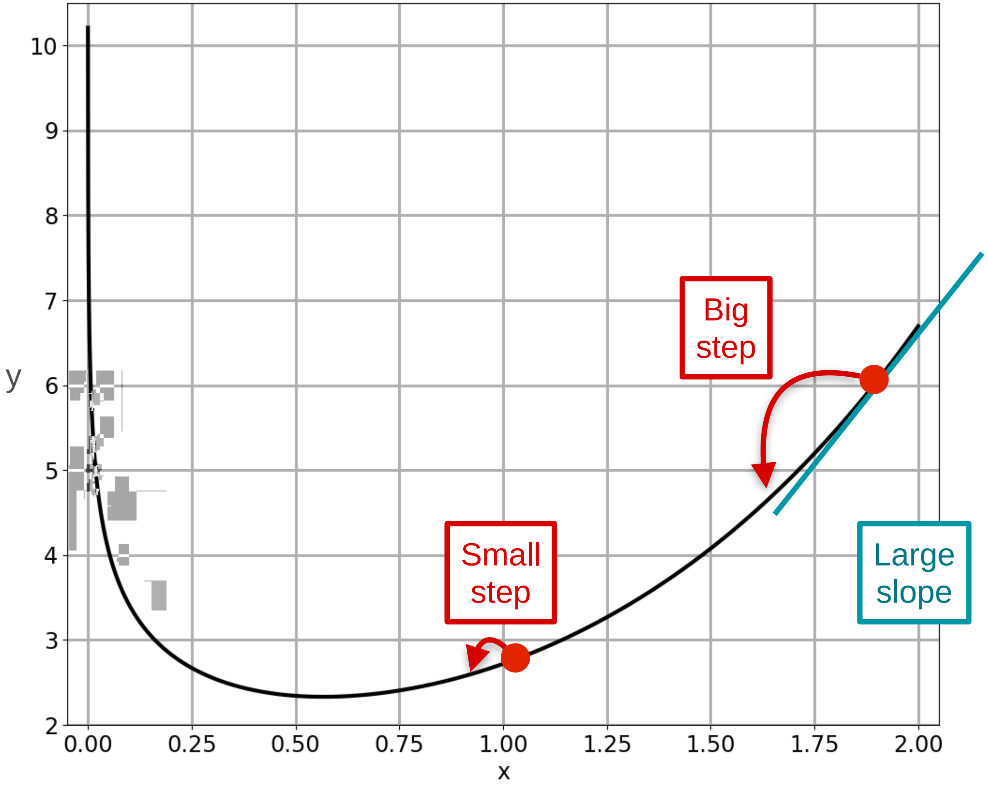
$$x_1 = x_0 - \alpha \, f'(x_0)$$



Method 2: Be Clever

Try something smarter…

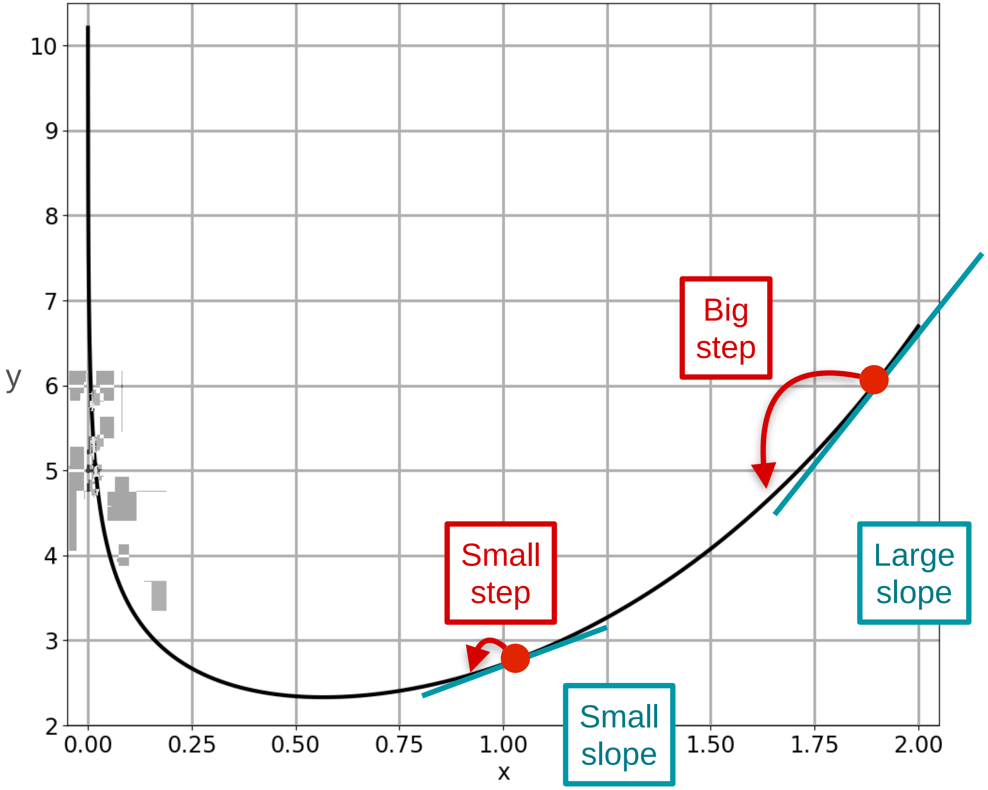
$$x_1 = x_0 - \alpha f'(x_0)$$



Method 2: Be Clever

Try something smarter…

$$x_1 = x_0 - \alpha \ f'(x_0)$$

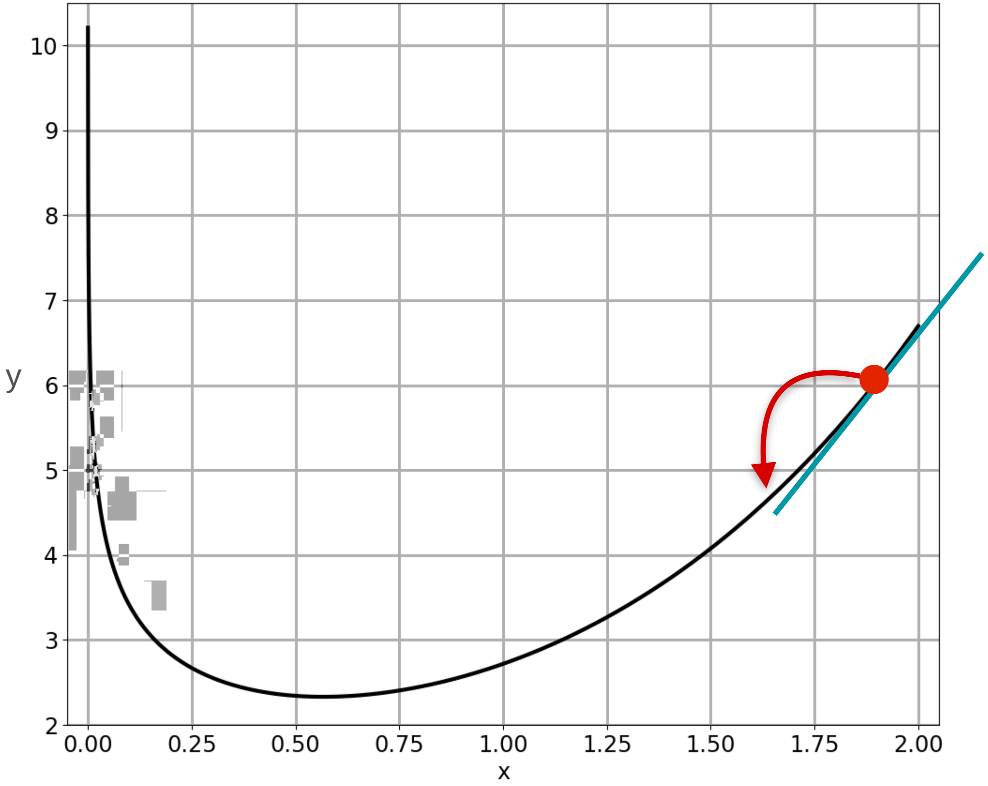


Method 2: Be Clever

Try something smarter…

$$x_1 = x_0 - \alpha f'(x_0)$$

Gradient descent

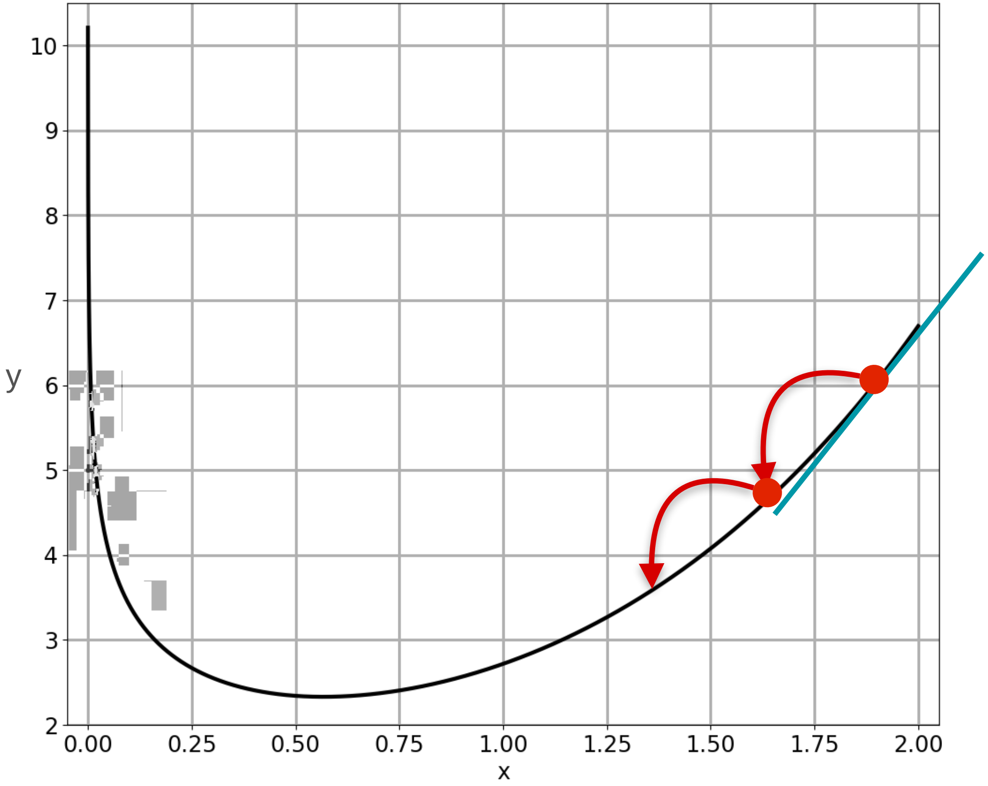


Method 2: Be Clever

Try something smarter…

$$x_2 = x_1 - \alpha \; f'(x_1)$$

Gradient descent

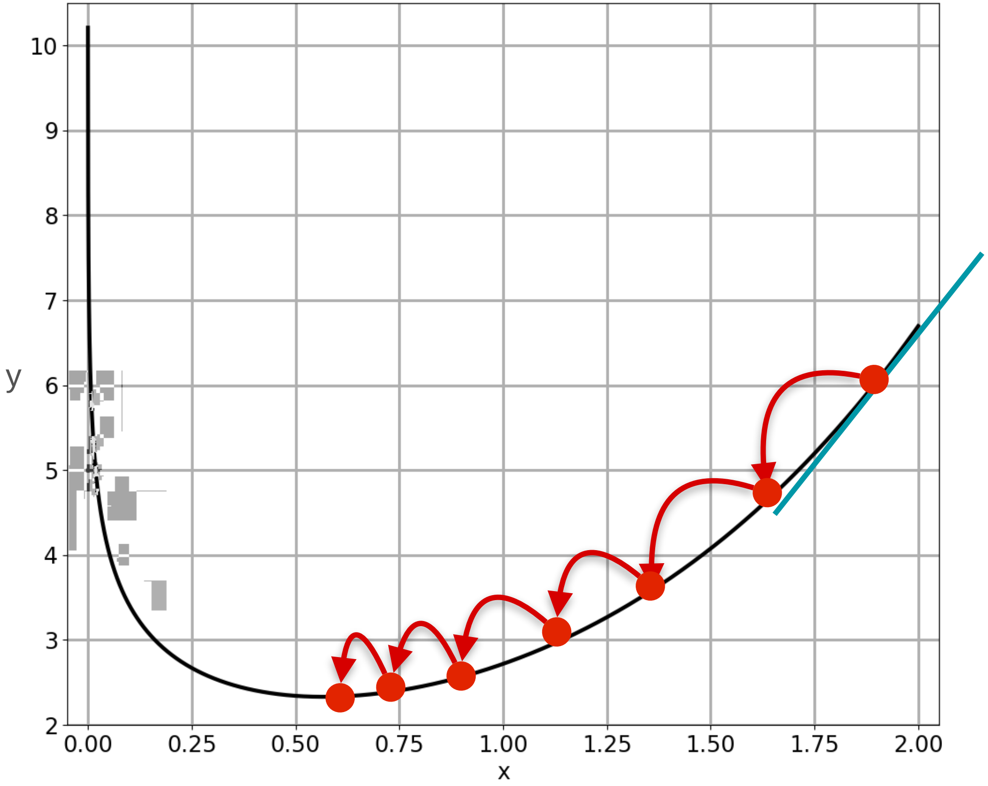


Method 2: Be Clever

Try something smarter…

$$x_{20} = x_{19} - \alpha \; f'(x_{19})$$

Gradient descent



Gradient Descent

Function: $f(x)$ Goal: find minimum of $f(x)$

Step 1:

Define a learning rate α

Choose a starting point x_0

Step 2:

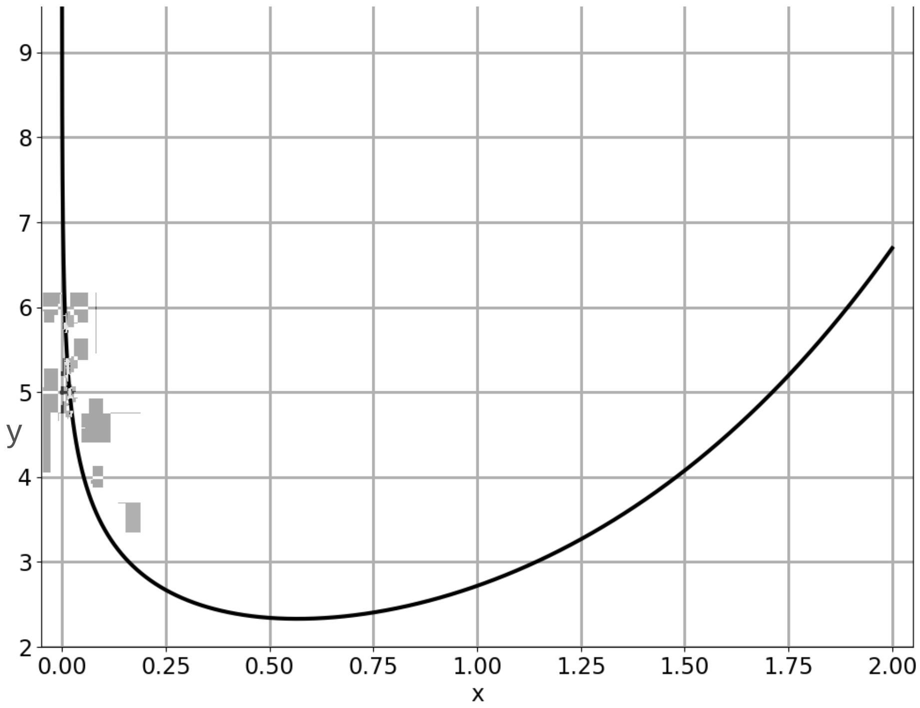
Update: $x_k = x_{k-1} - af'(x_{k-1})$

Step 3:

Repeat Step 2 until you are close enough to the true minimum $x*$

Gradient Descent

$$f(x) = e^x - \log(x) \qquad f'(x) = e^x - \frac{1}{x}$$



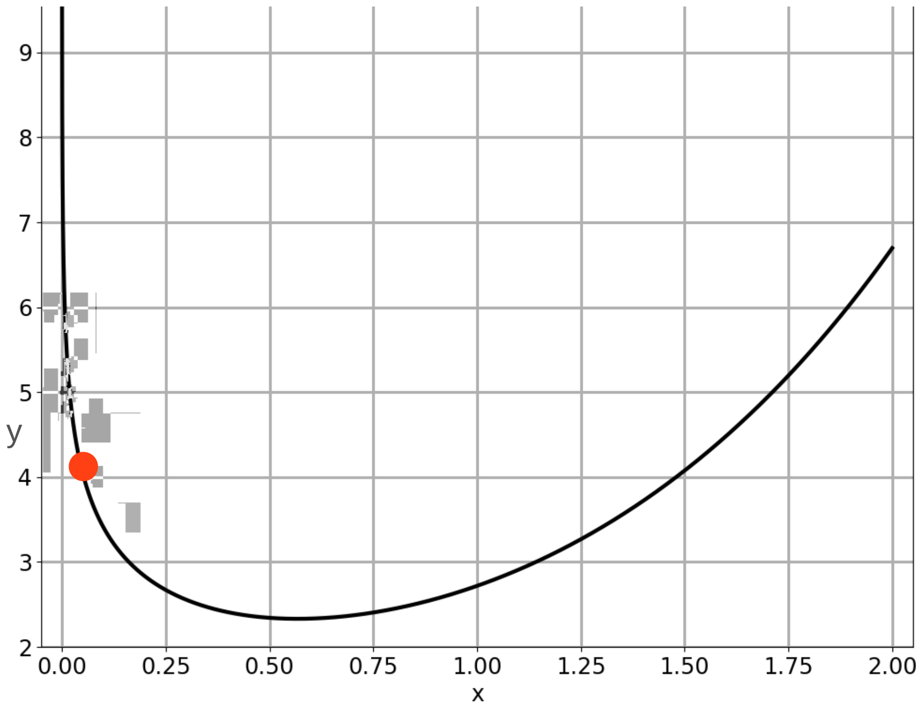
Gradient Descent

$f(x) = e^x - \log(x)$

$f'(x) = e^x - \dfrac{1}{x}$

Start: $x = 0.05$

Rate: $\alpha = 0.005$



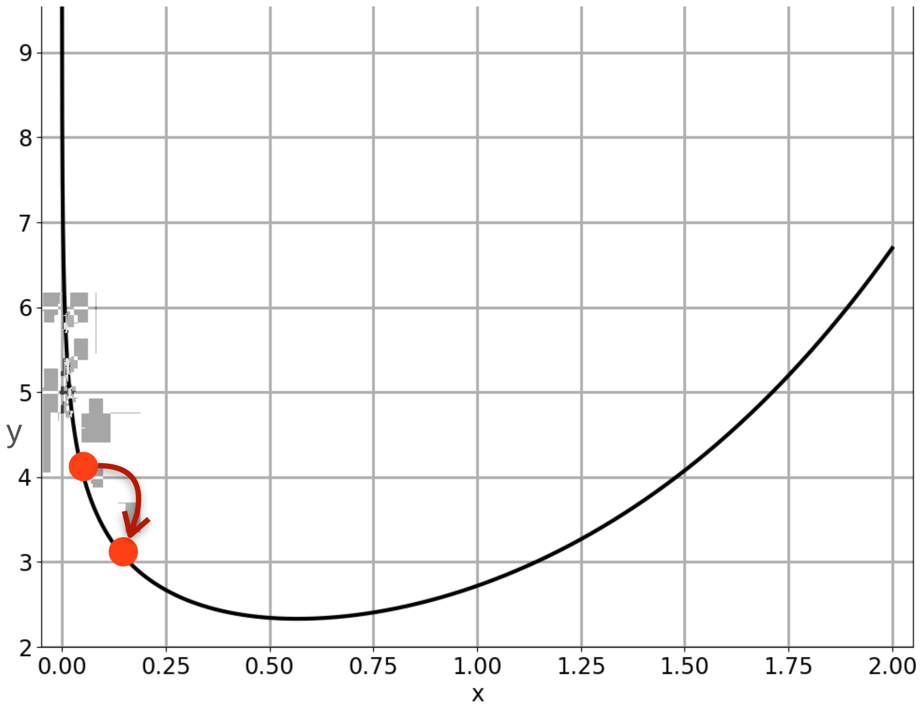
Gradient Descent

$f(x) = e^x - \log(x)$ $\qquad$ $f'(x) = e^x - \dfrac{1}{x}$

Start: $x = 0.05$ $\qquad$ Rate: $\alpha = 0.005$

Find:
$f'(0.05) = -18.9$

Move by $-0.005 f'(0.05)$ $\qquad$ $x \mapsto 0.1447$



Gradient Descent

$$f(x) = e^x - \log(x) \qquad f'(x) = e^x - \frac{1}{x}$$

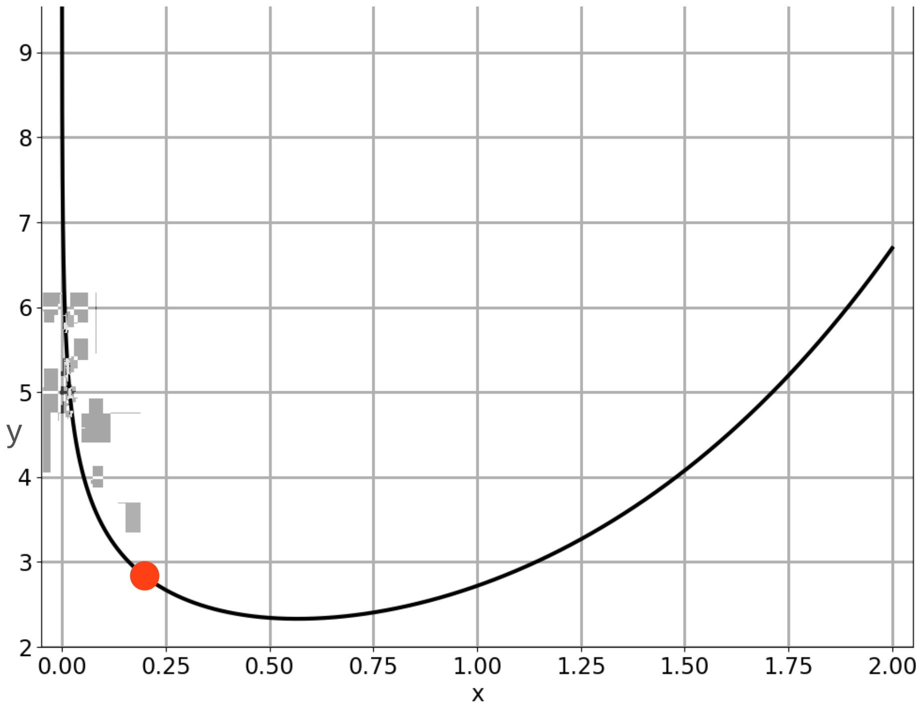
Start: $x = 0.05$ Rate: $\alpha = 0.005$

Find:
$f'(0.05) = -18.9$

Move by $-0.005 f'(0.05)$ $x \mapsto 0.1447$

Find:
$f'(0.1447) = -5.7552$

Move by $-0.005 f'(0.05)$ $x \mapsto 0.1735$



Gradient Descent

$f(x) = e^x - \log(x)$ $\qquad$ $f'(x) = e^x - \dfrac{1}{x}$

Start: $x = 0.05$ $\qquad$ Rate: $\alpha = 0.005$

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Find:
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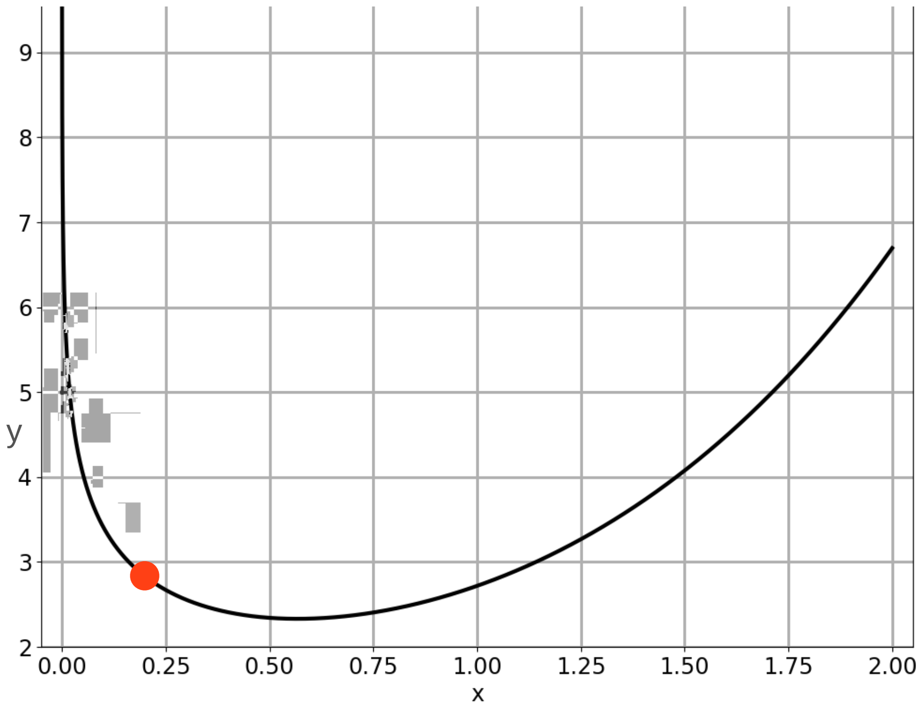
Move by $-0.005 f'(0.05)$ $\qquad$ $x \mapsto 0.1735$



Repeat!

Gradient Descent

$f(x) = e^x - \log(x)$ $f'(x) = e^x - \frac{1}{x}$

Start: $x = 0.05$ Rate: $\alpha = 0.005$

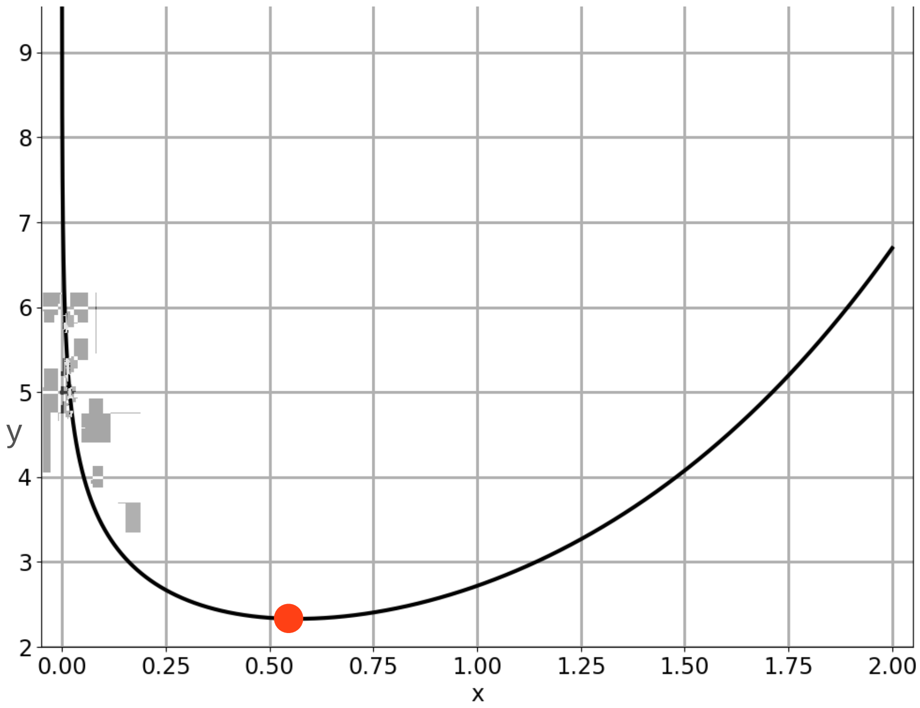
Find:
$f'(0.05) = -18.9$

Move by $-0.005 f'(0.05)$ $x \mapsto 0.1447$

Find:
$f'(0.1447) = -5.7552$

Move by $-0.005 f'(0.05)$ $x \mapsto 0.1735$

Repeat!



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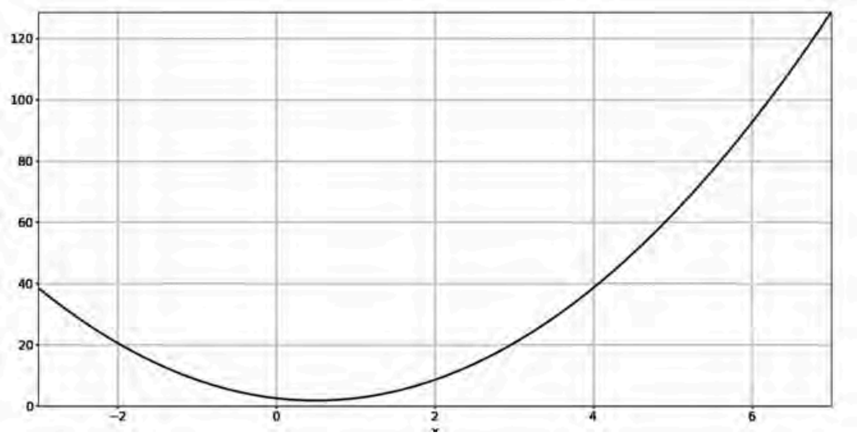
Gradients and Gradient Descent

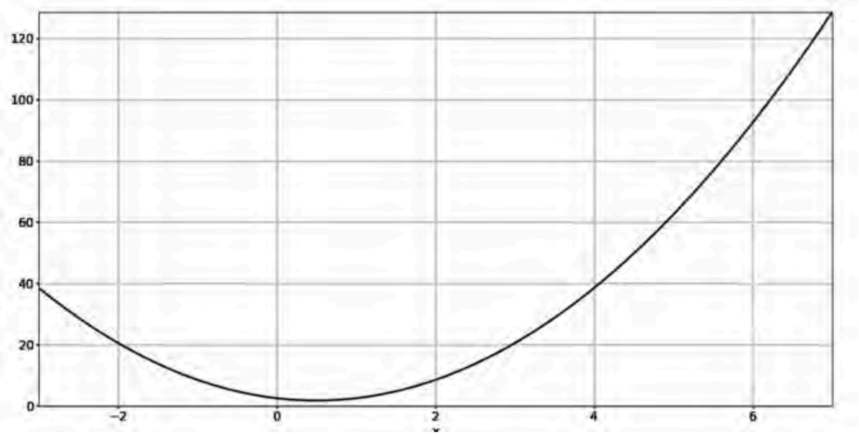
Optimization using Gradient Descent in one variable - Part 3

What Is a Good Learning Rate?

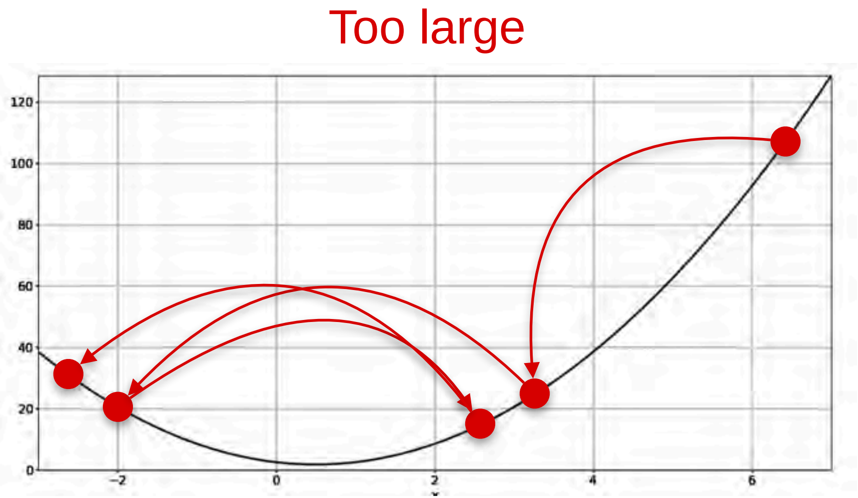
Too large

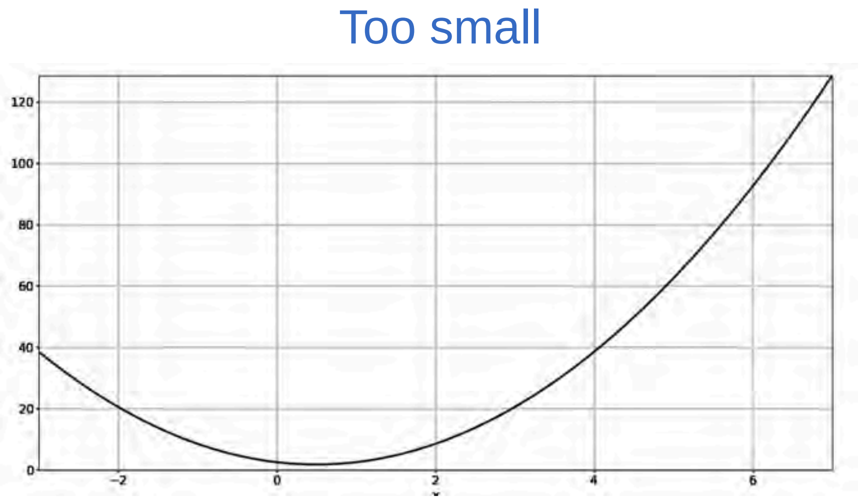


Too small



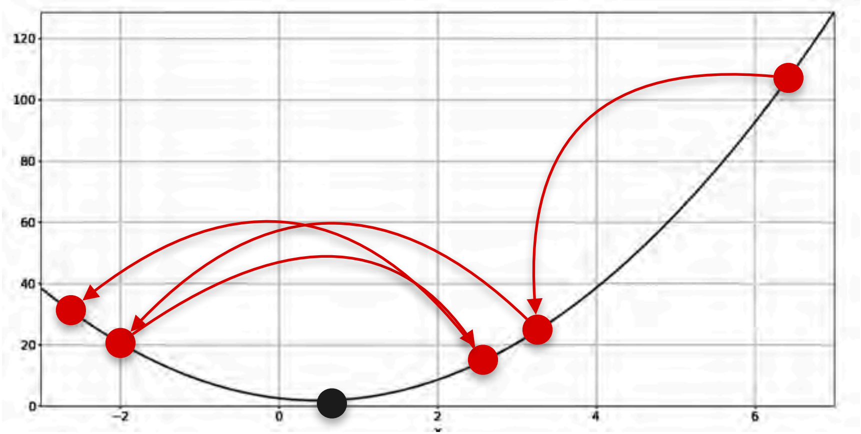
What Is a Good Learning Rate?



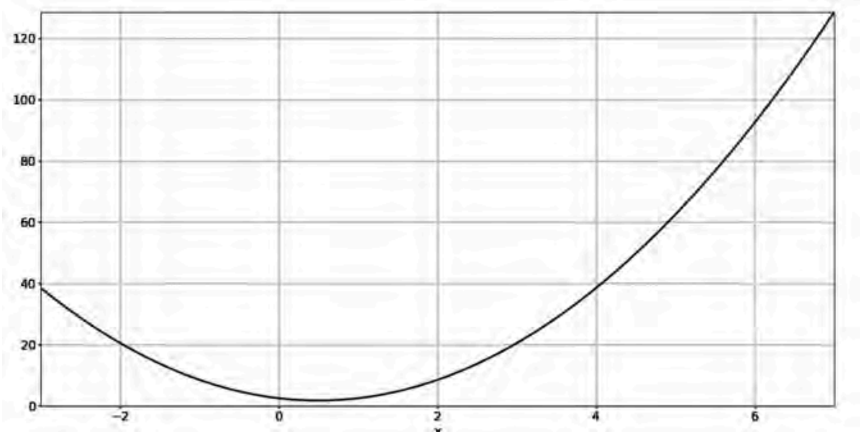


What Is a Good Learning Rate?

Too large

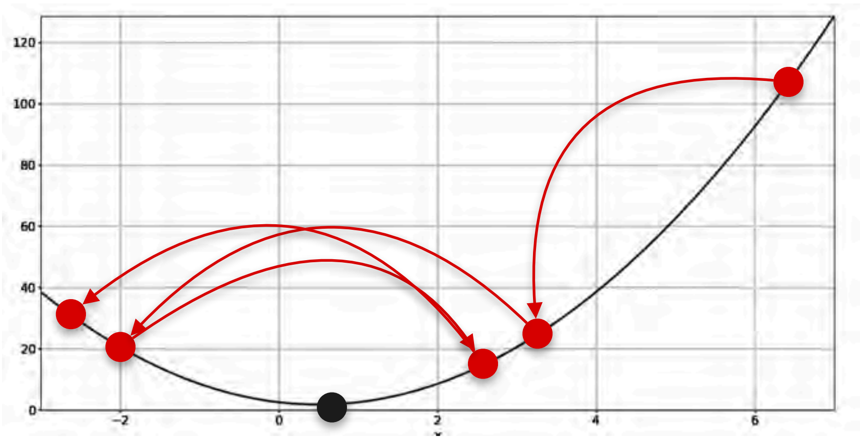


Too small

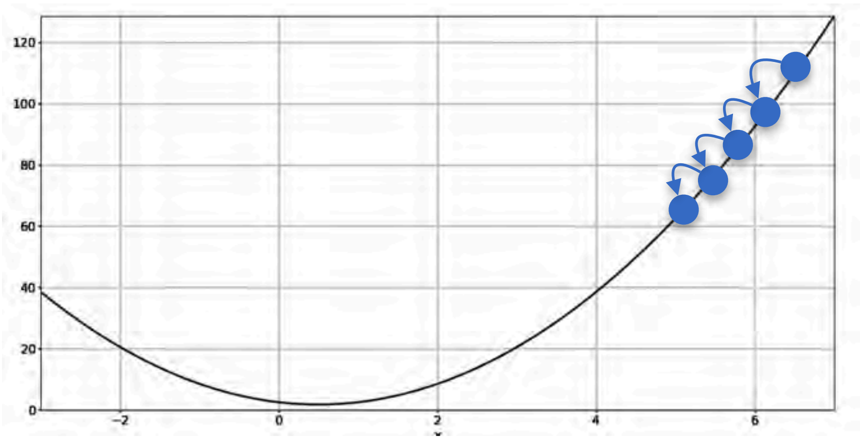


What Is a Good Learning Rate?

Too large

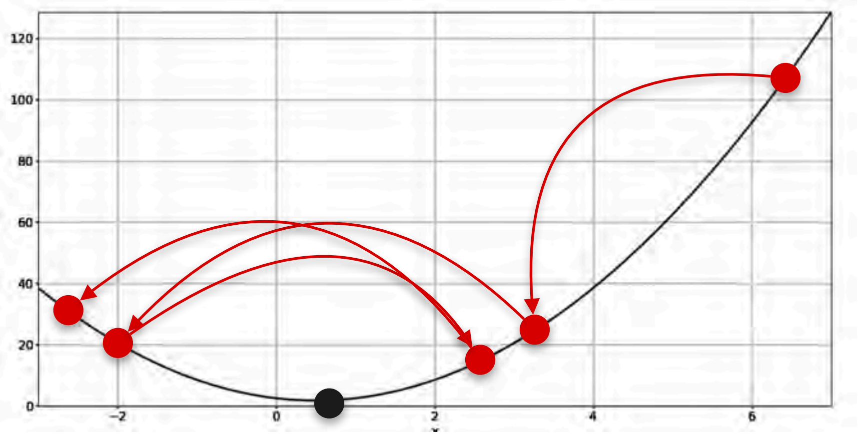


Too small

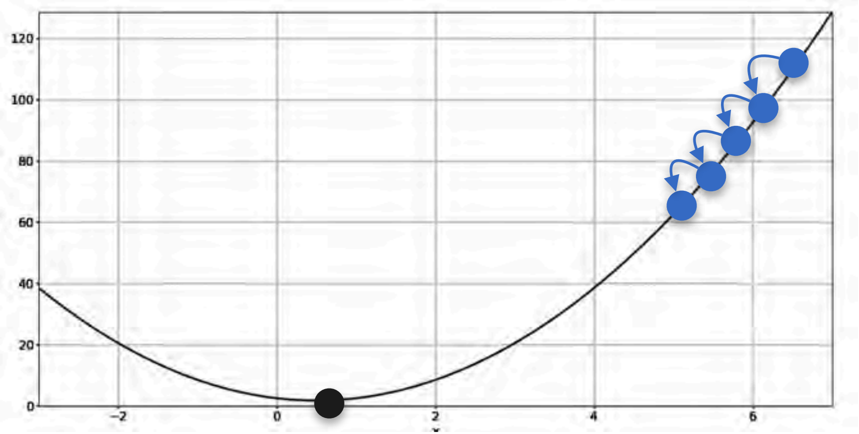


What Is a Good Learning Rate?

Too large

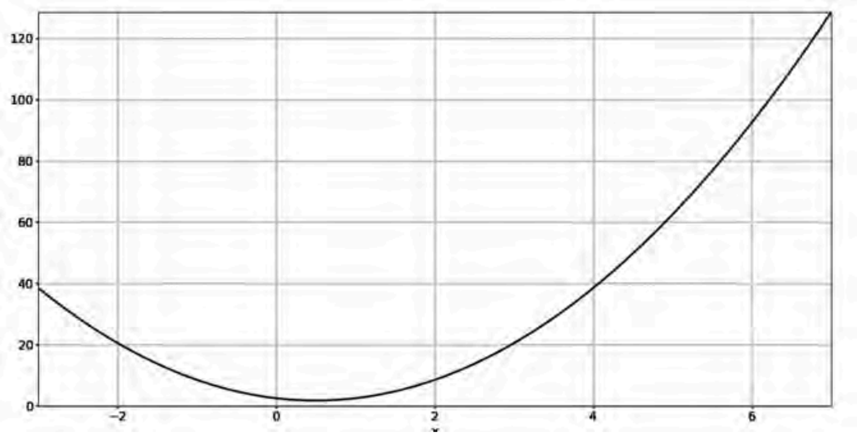


Too small



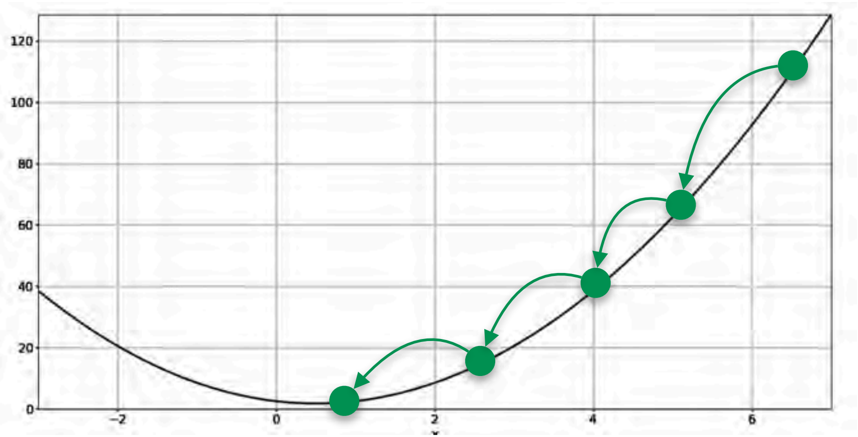
What Is a Good Learning Rate?

Just right



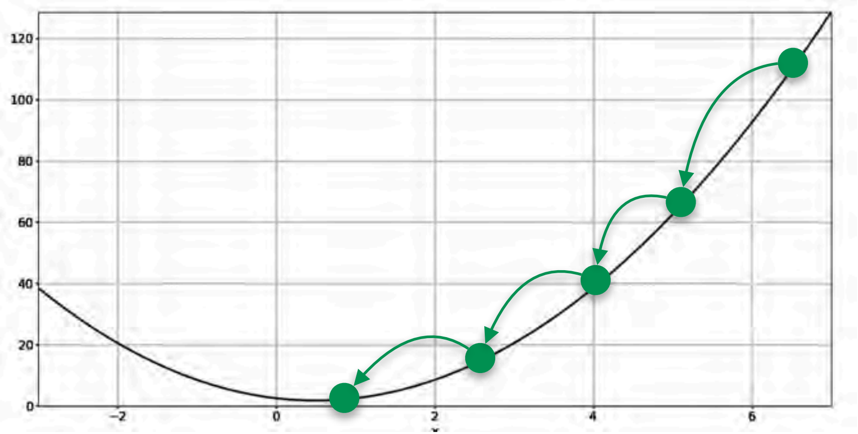
What Is a Good Learning Rate?

Just right



What Is a Good Learning Rate?

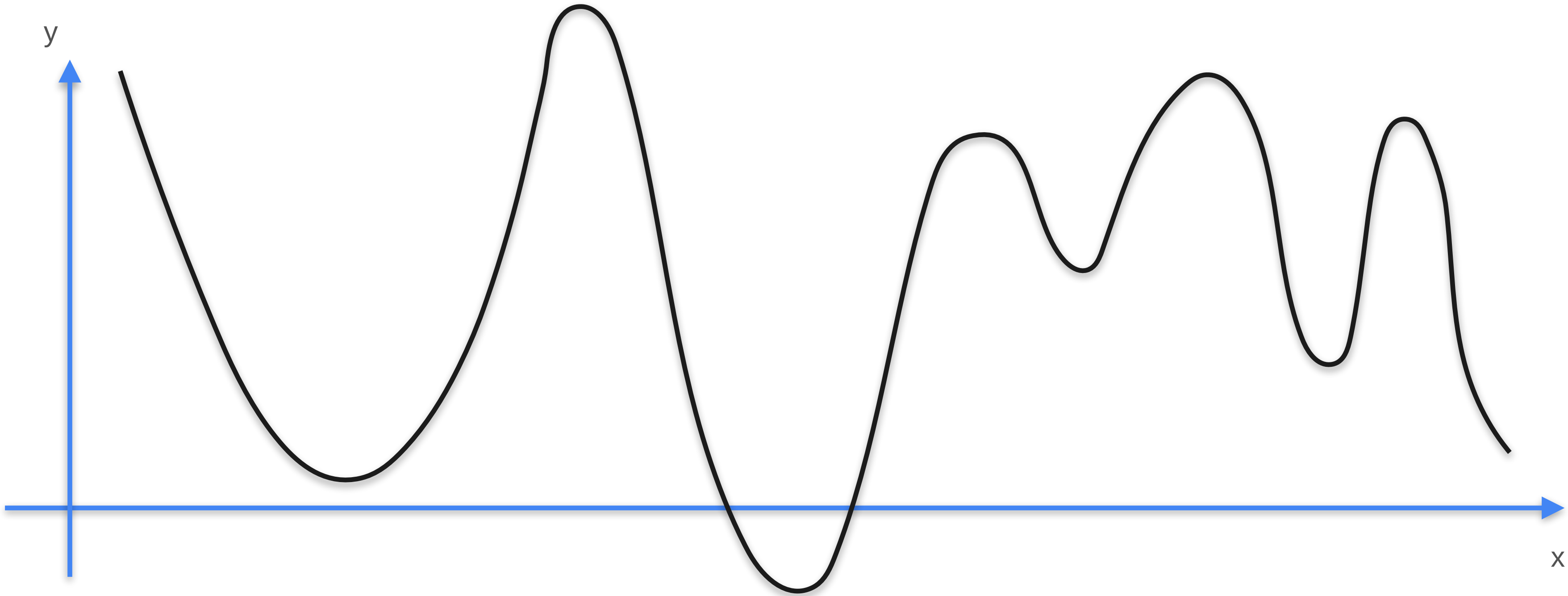
Just right



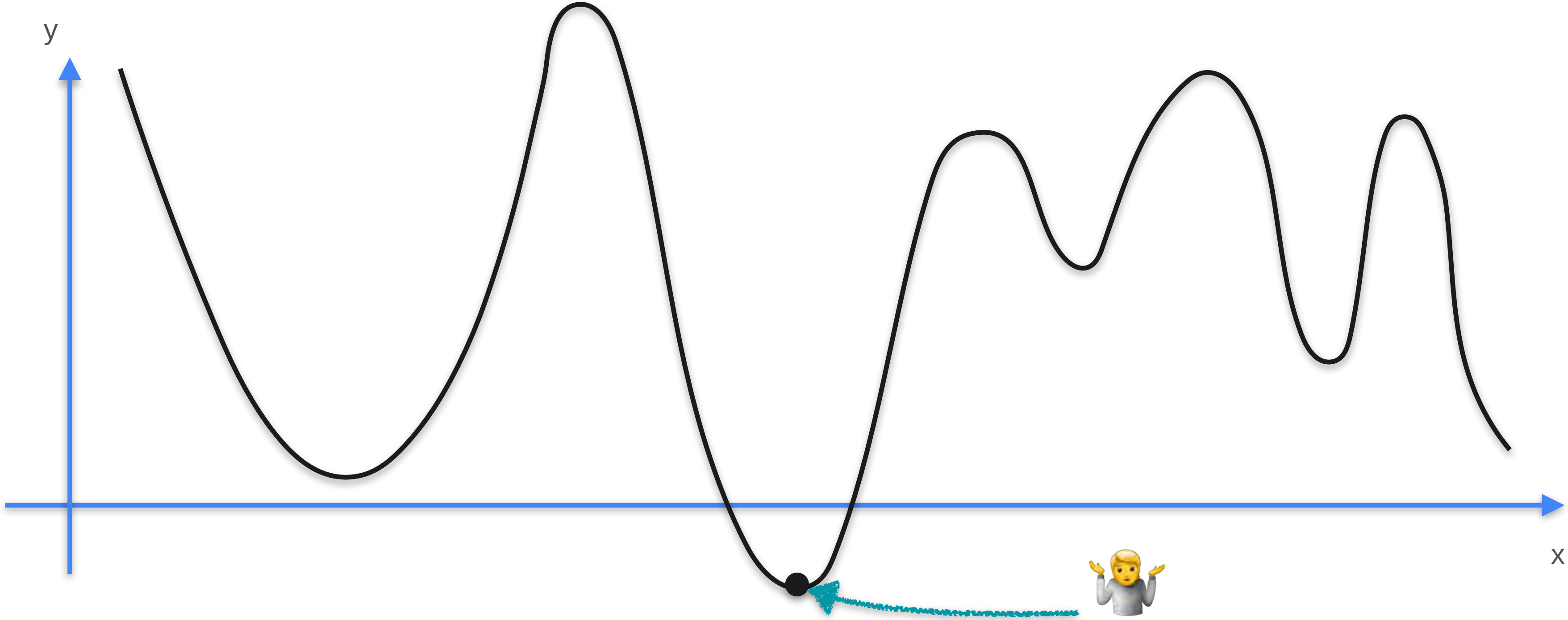
Unfortunately, there is no rule to give the best learning rate α

Drawbacks of Gradient Descent

Drawbacks of Gradient Descent

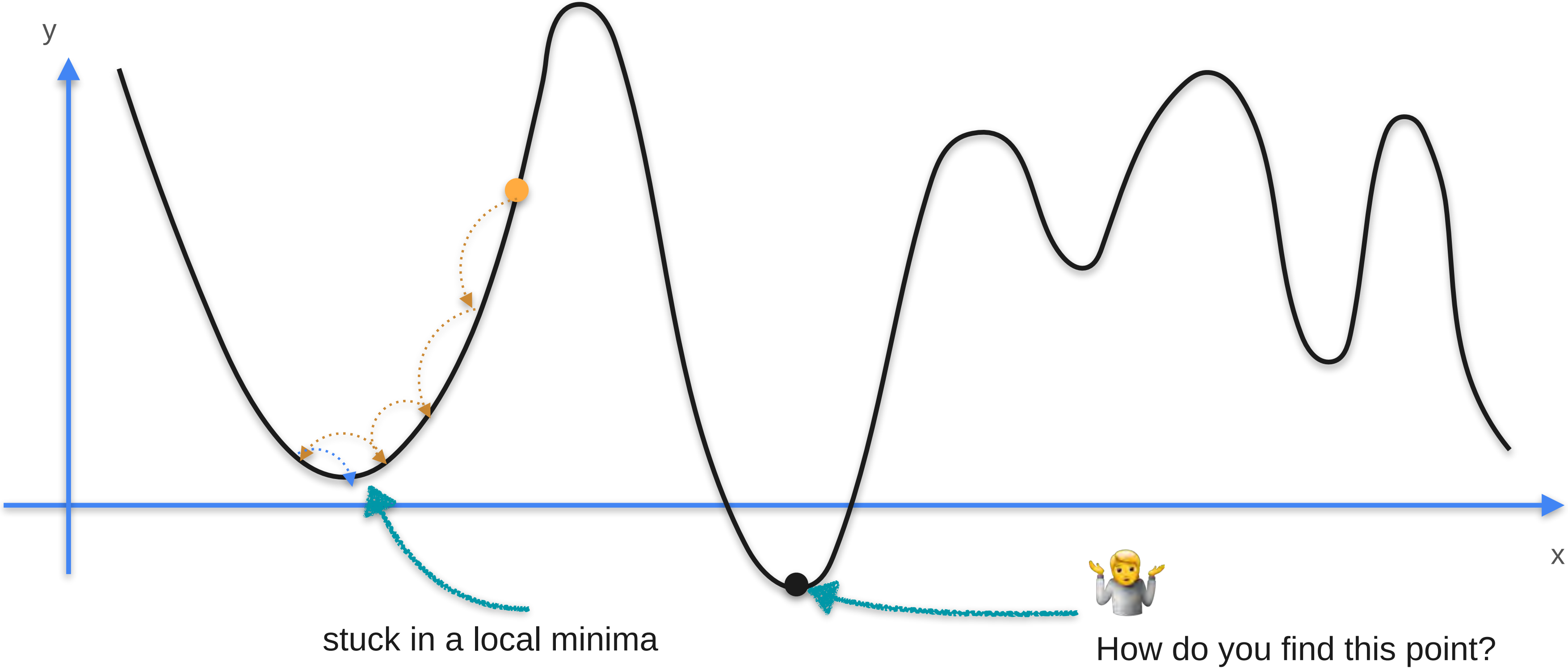


Drawbacks of Gradient Descent

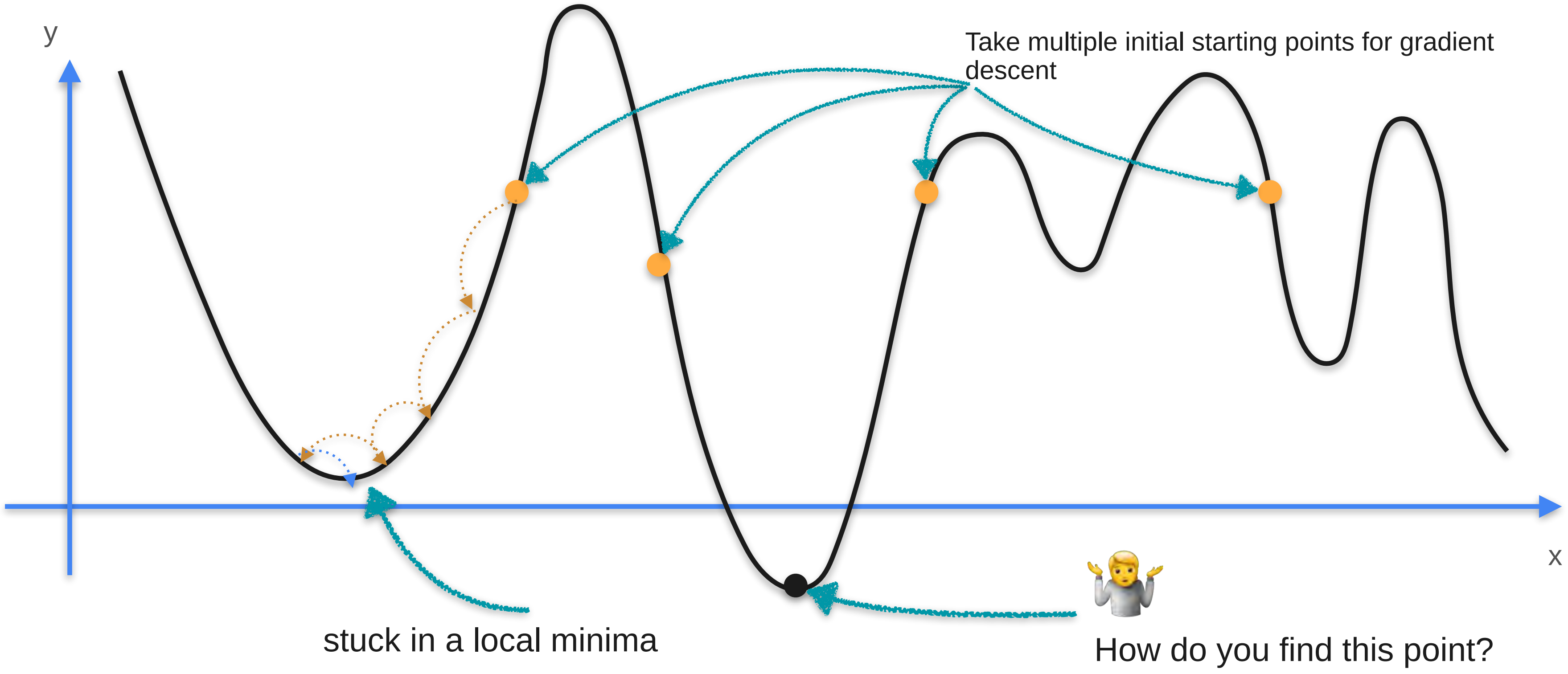


How do you find this point?

Drawbacks of Gradient Descent



Drawbacks of Gradient Descent



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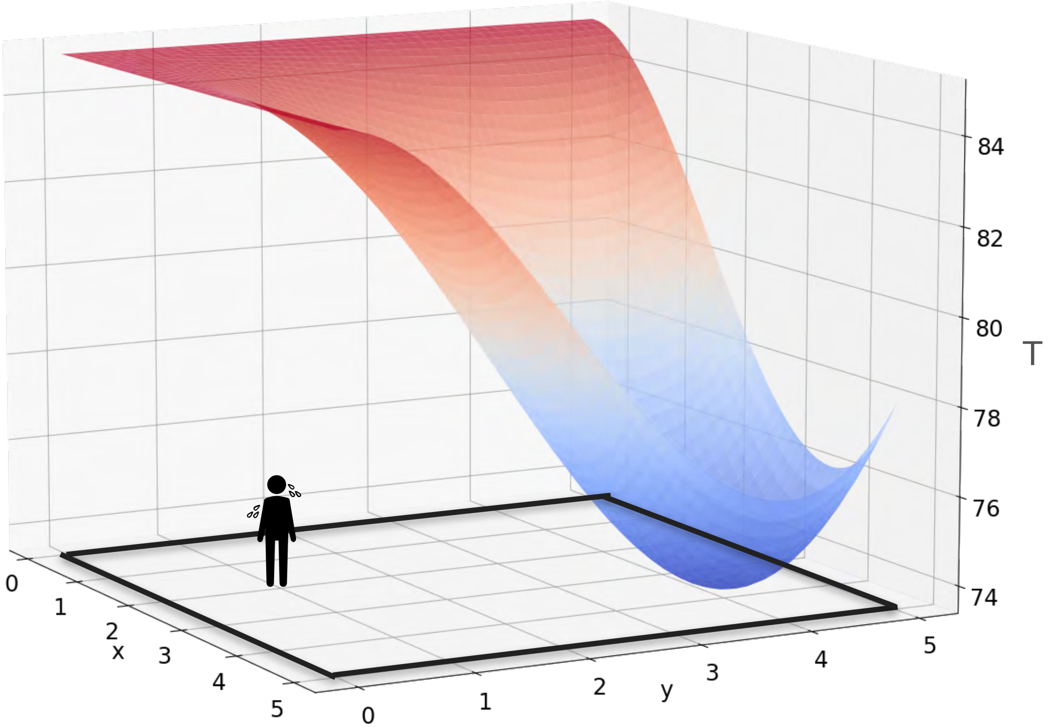
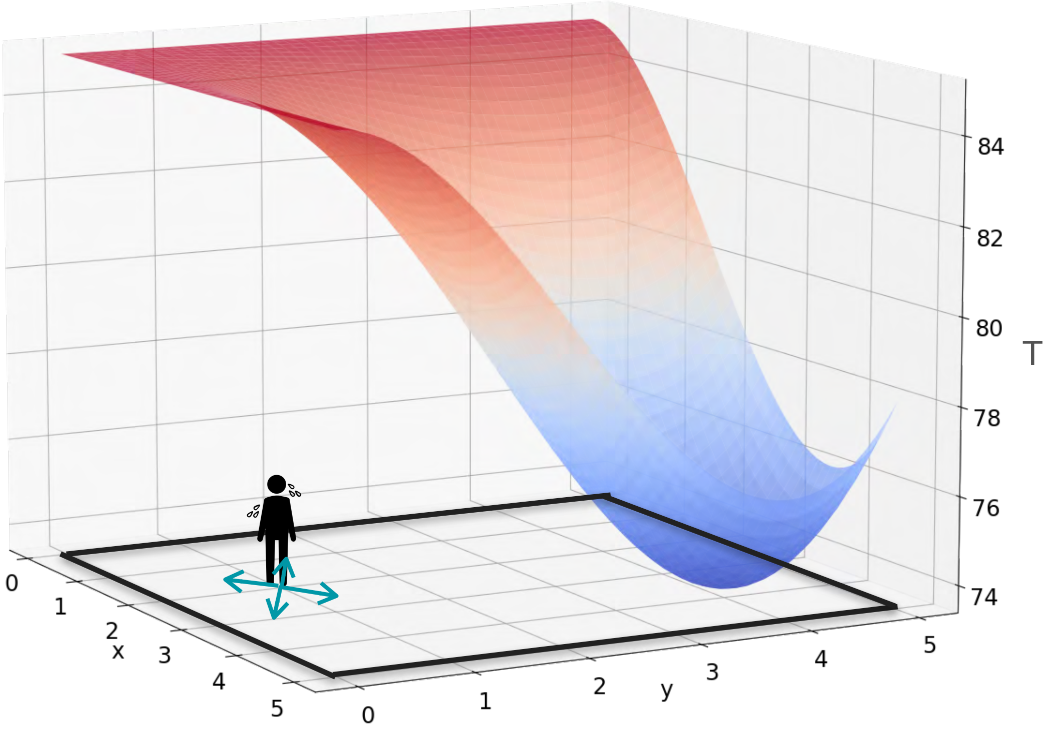
Gradients and Gradient Descent

Optimization using Gradient Descent in two variables - Part 1

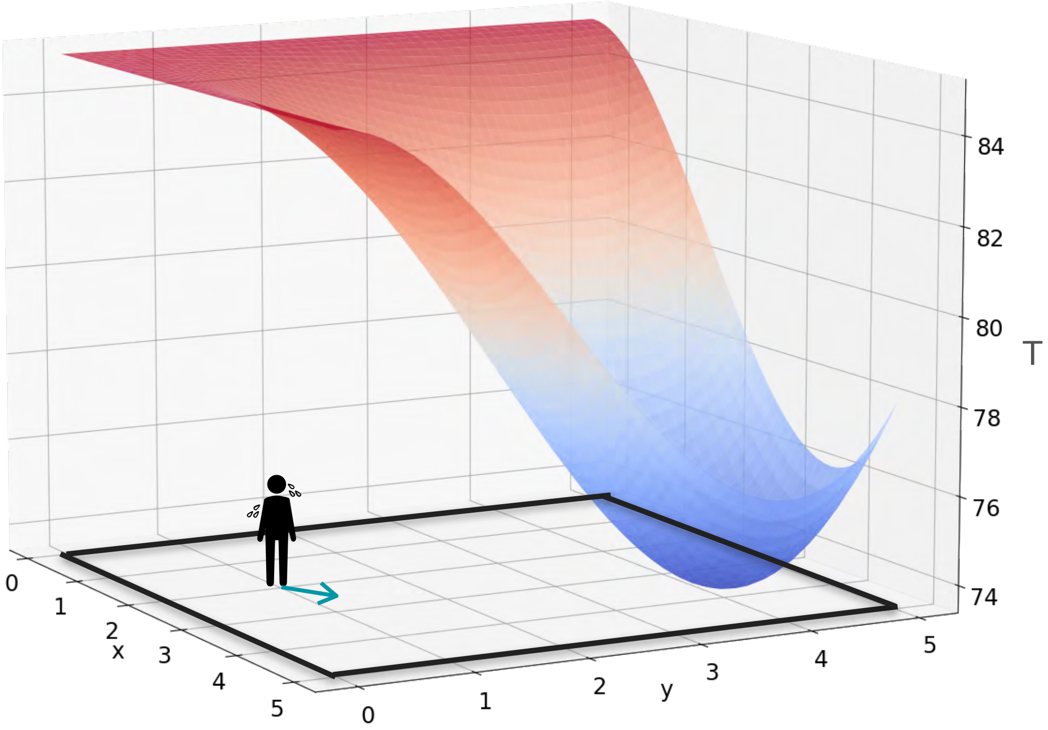
Gradient Descent With Heat Example



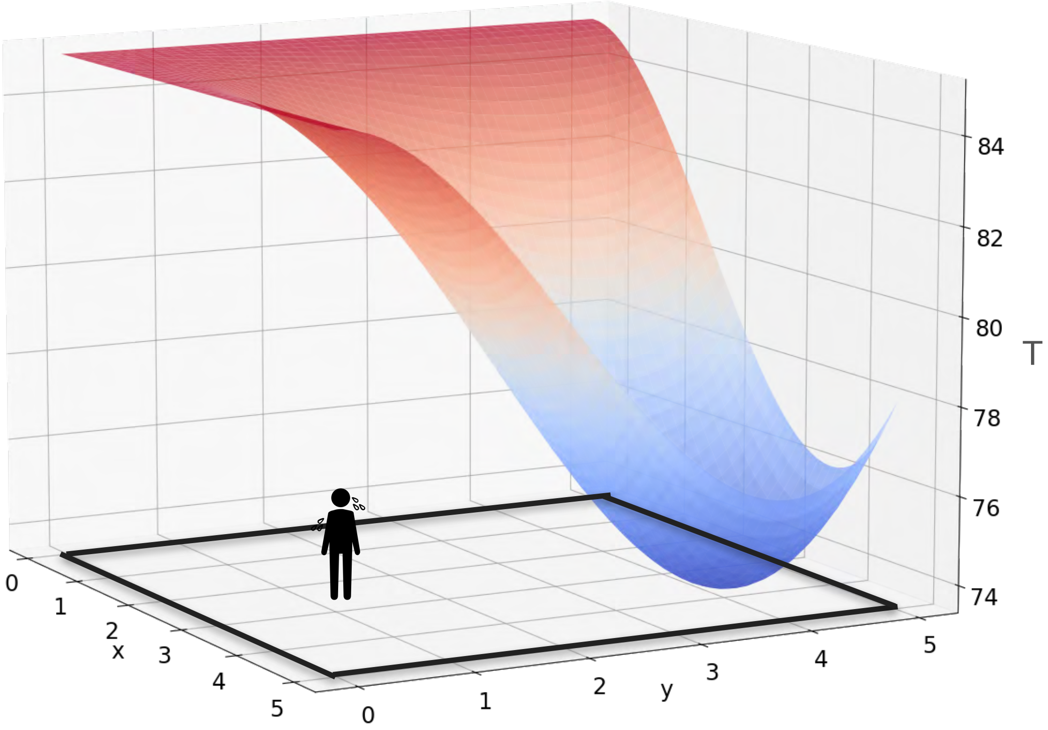
Gradient Descent With Heat Example



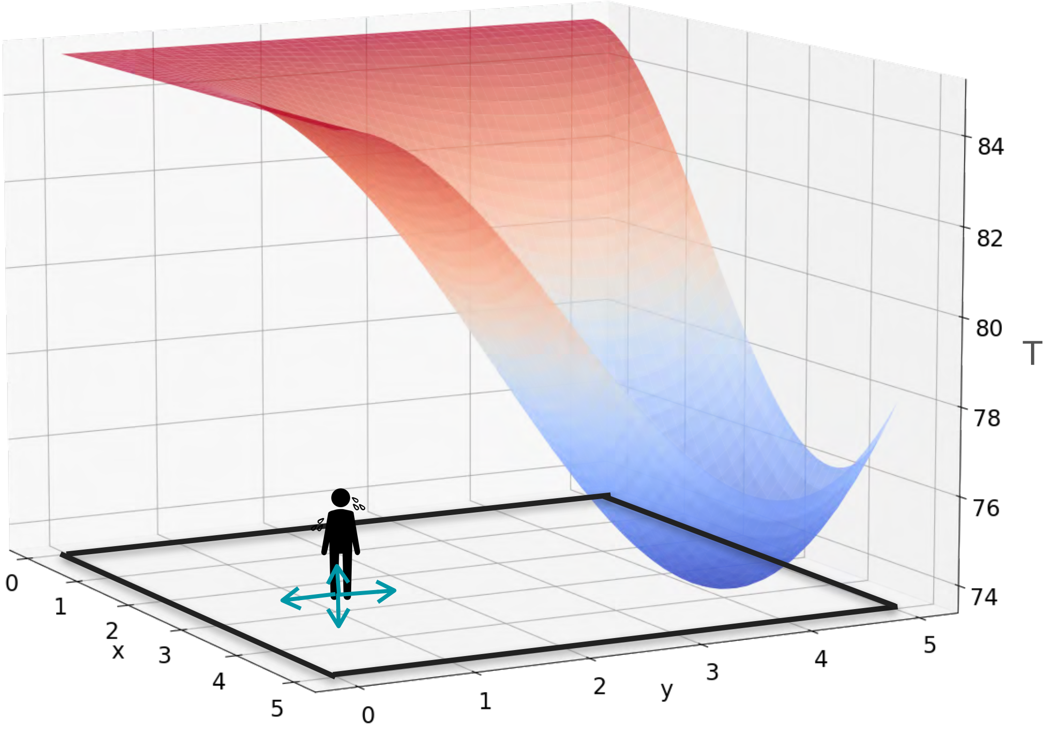
Gradient Descent With Heat Example



Gradient Descent With Heat Example

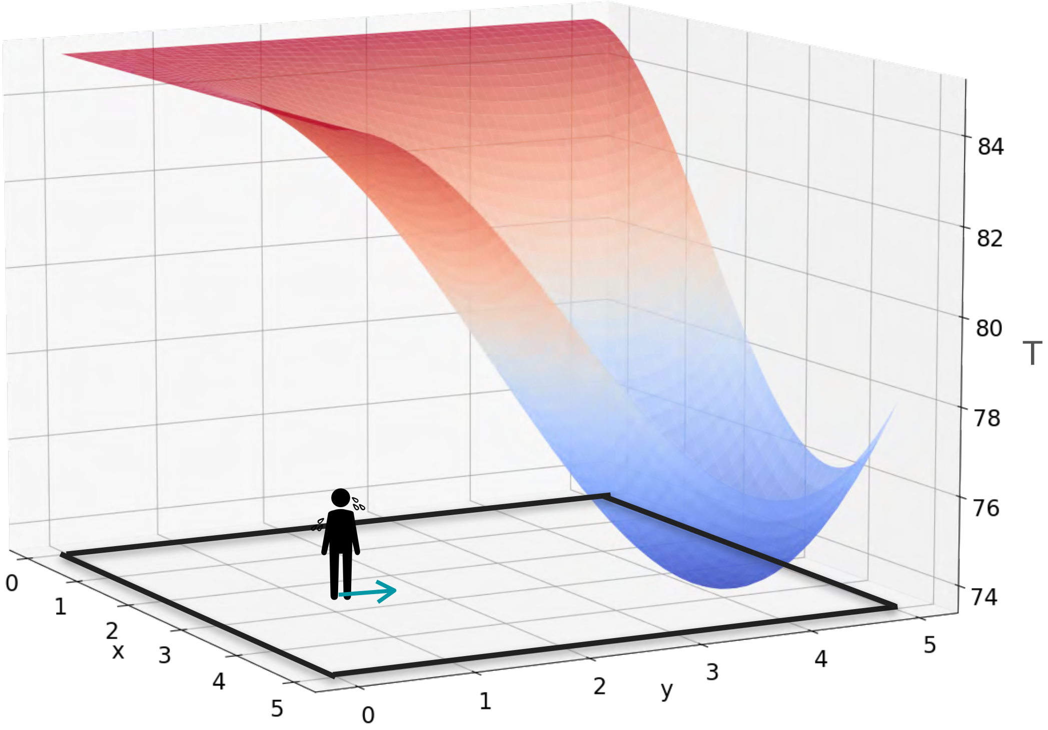


Gradient Descent With Heat Example

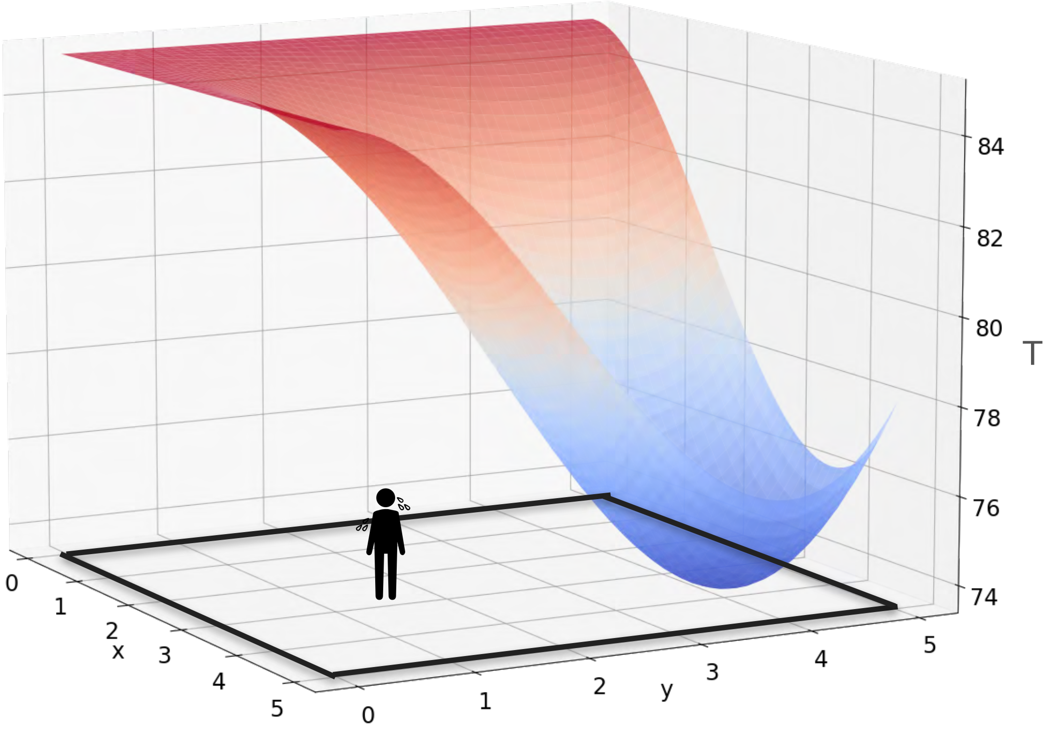


Gradient Descent With Heat Example

Repeat!

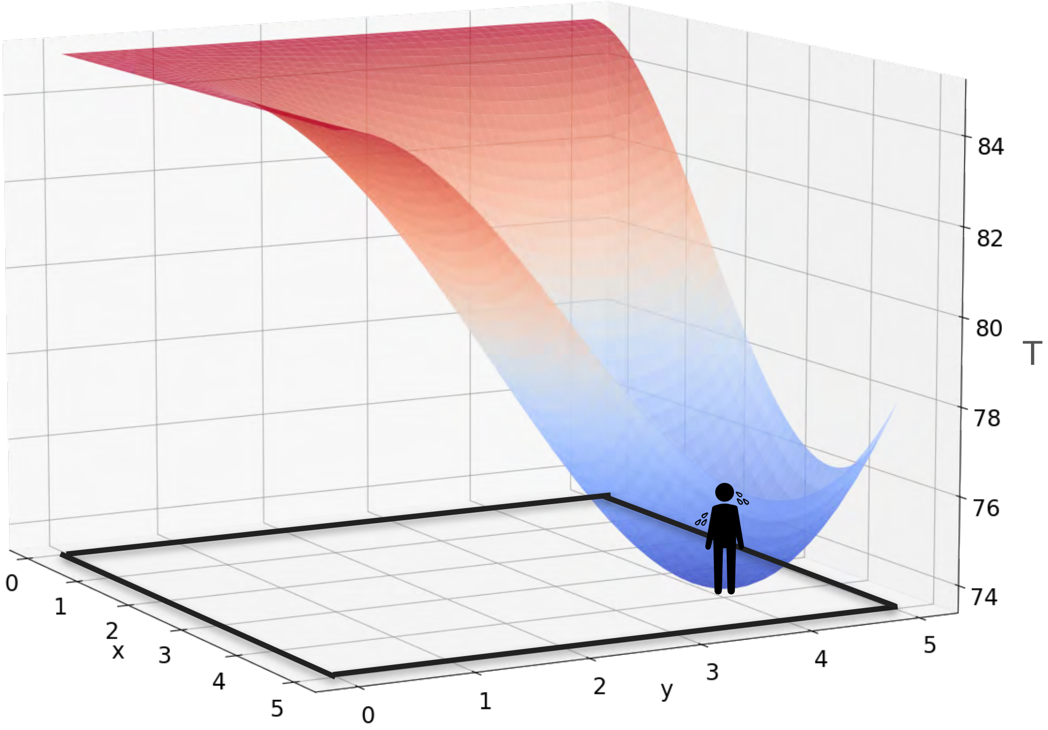


Gradient Descent With Heat Example



Gradient Descent With Heat Example

Repeat!

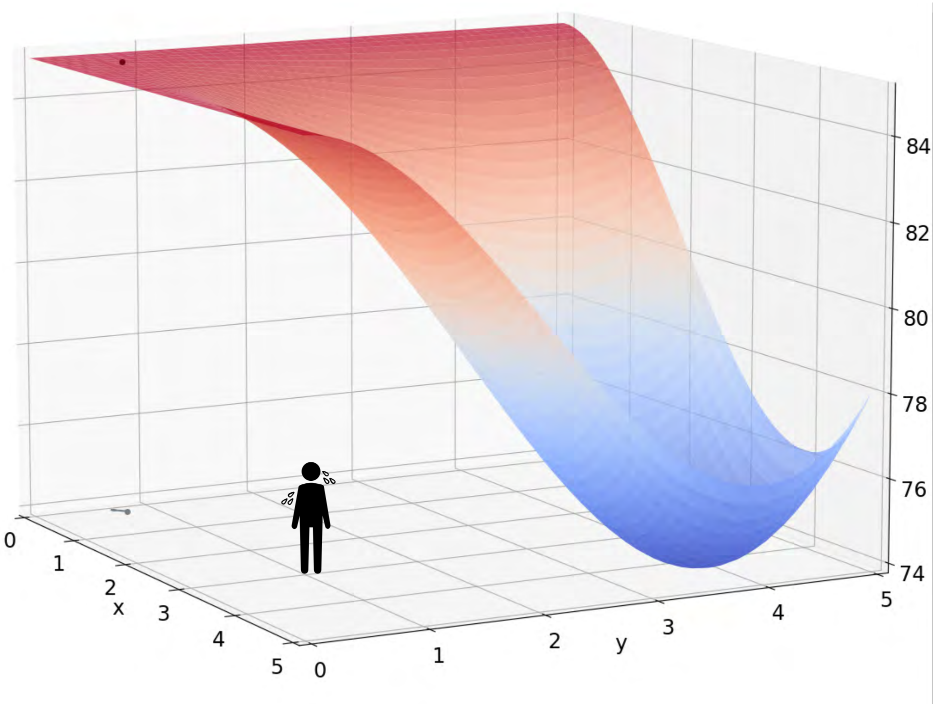


DeepLearning.AI

Gradients and Gradient Descent

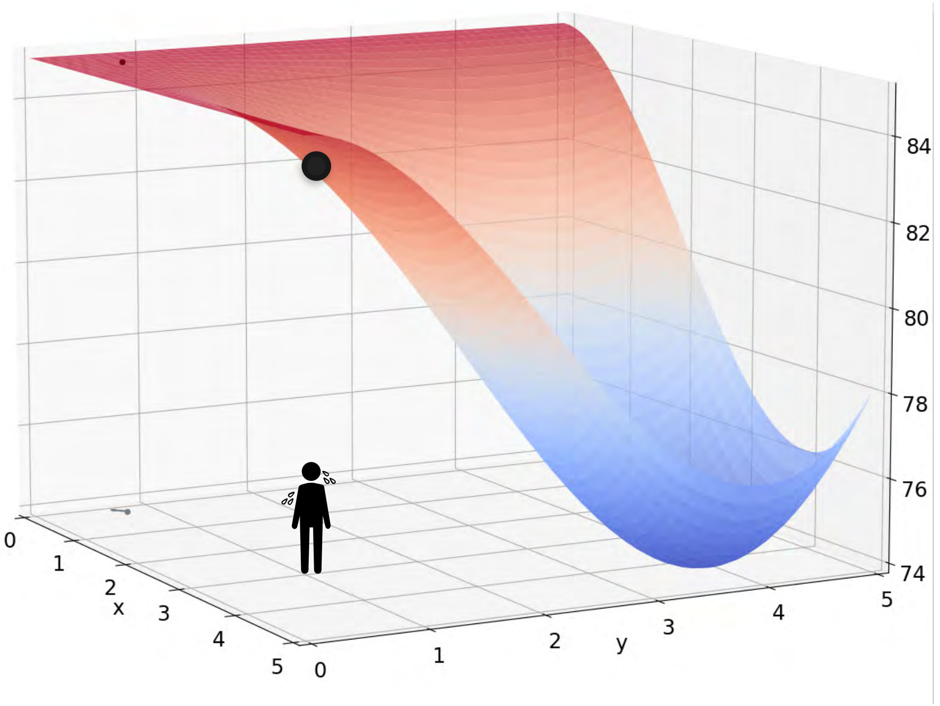
Optimization using Gradient Descent in two variables - Part 2

Idea for Gradient Descent



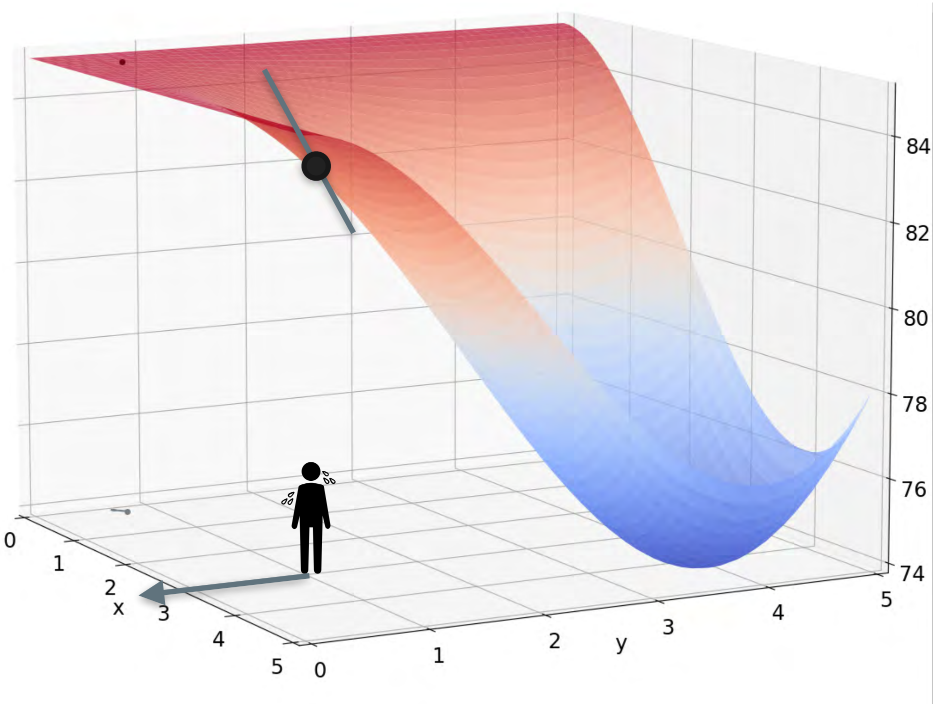
Idea for Gradient Descent

Initial position: (x_0, y_0)



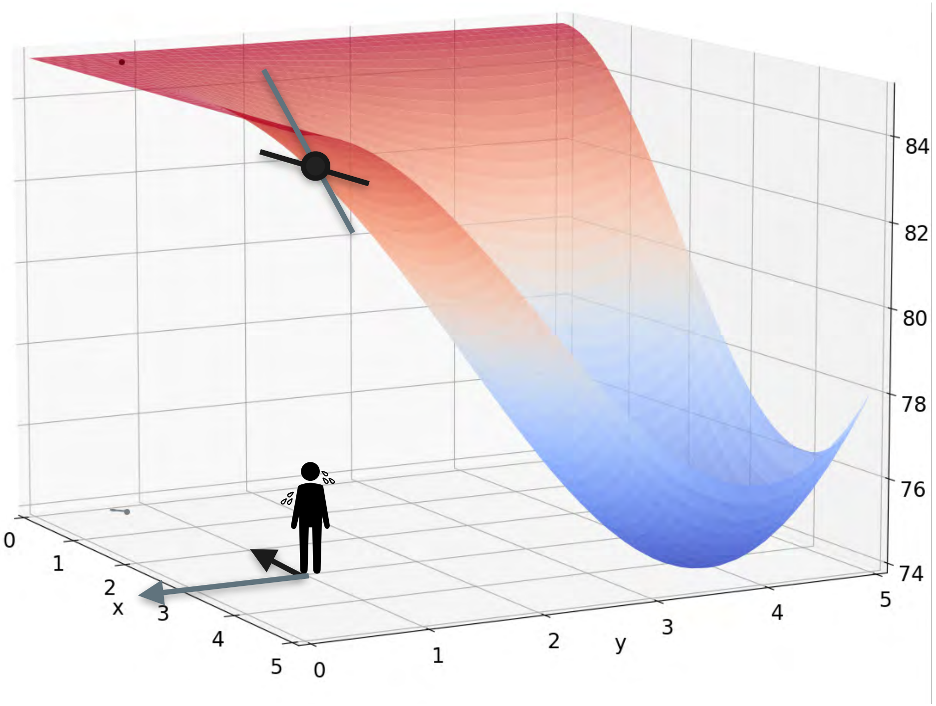
Idea for Gradient Descent

Initial position: (x_0, y_0)



Idea for Gradient Descent

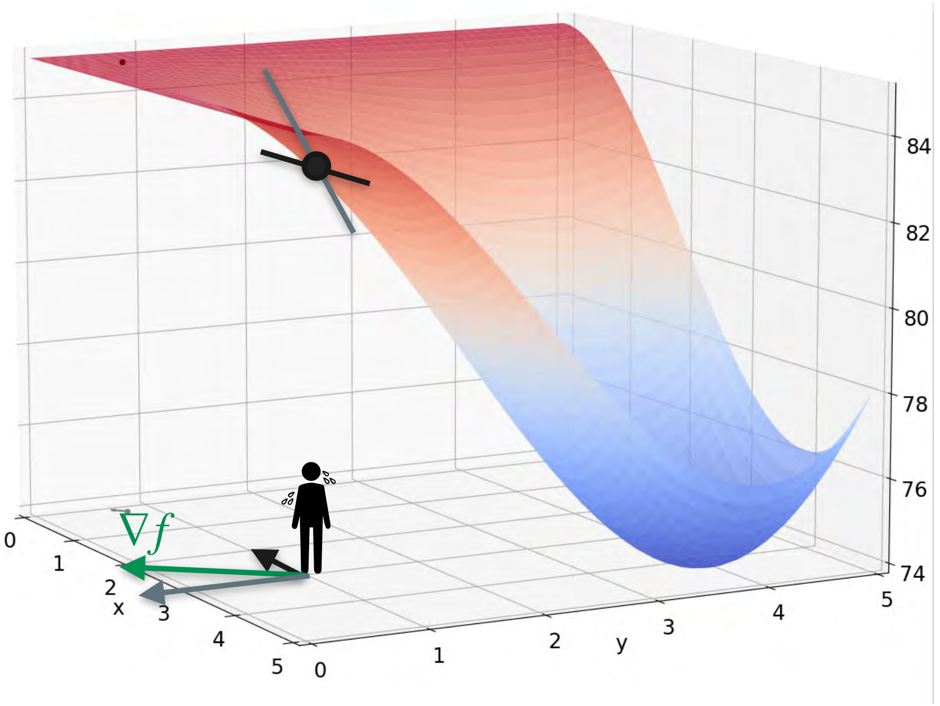
Initial position: (x_0, y_0)



Idea for Gradient Descent

Initial position: (x_0, y_0)

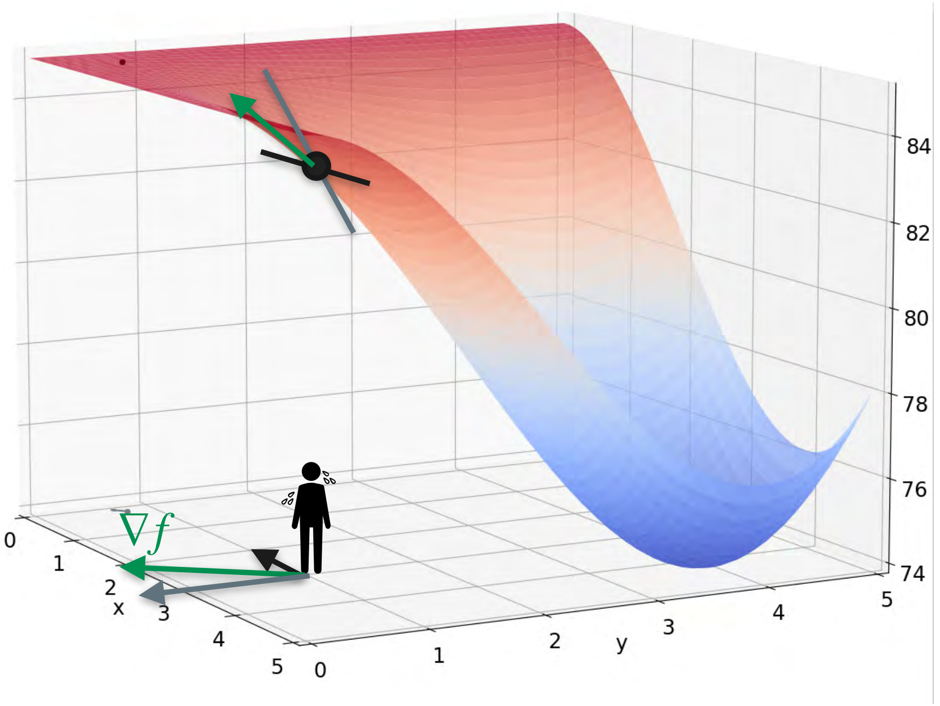
Direction of greatest ascent: ∇f



Idea for Gradient Descent

Initial position: (x_0, y_0)

Direction of greatest ascent: ∇f

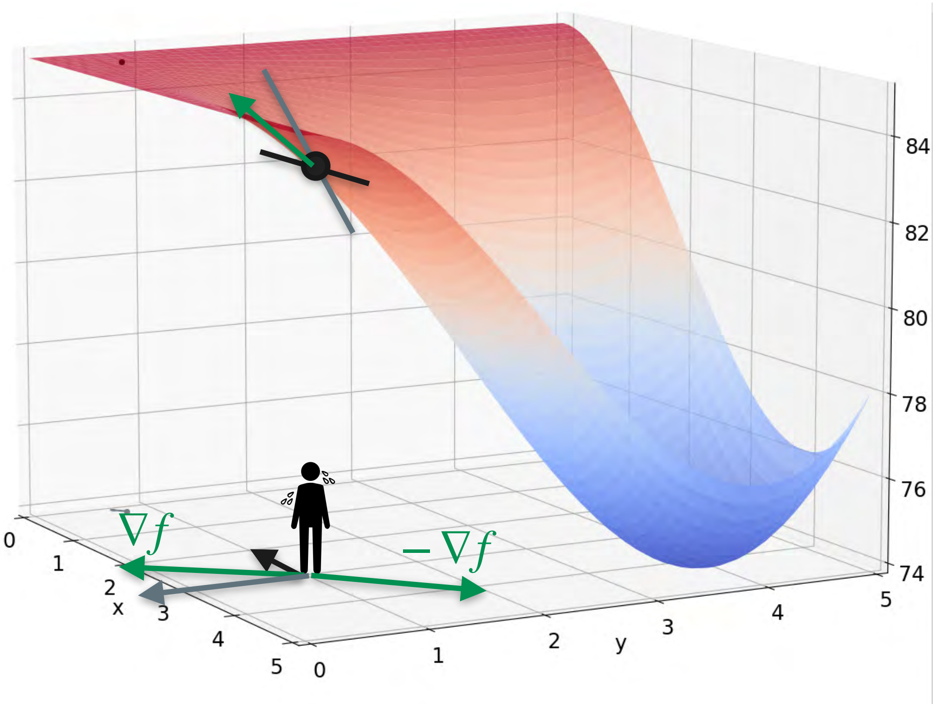


Idea for Gradient Descent

Initial position: (x_0, y_0)

Direction of greatest ascent: ∇f

Direction of greatest descent: $-\nabla f$

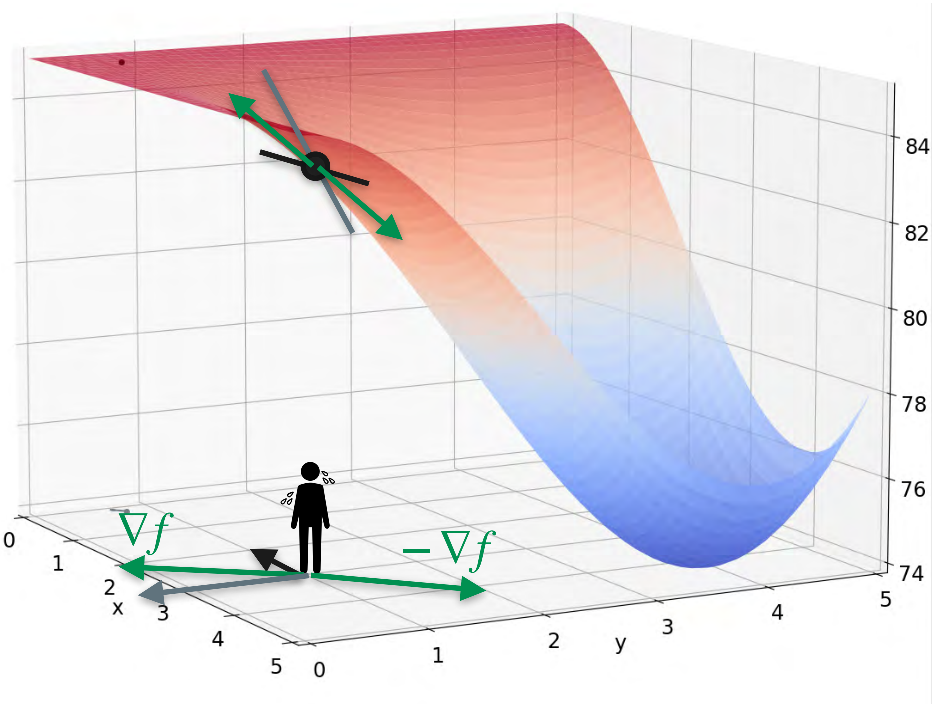


Idea for Gradient Descent

Initial position: (x_0, y_0)

Direction of greatest ascent: ∇f

Direction of greatest descent: $-\nabla f$

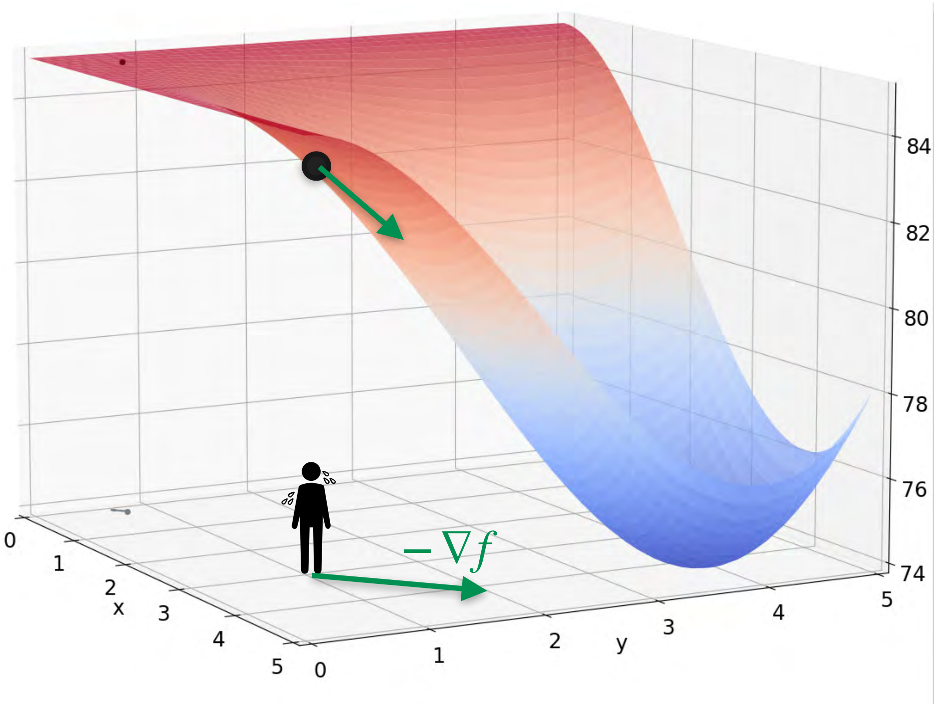


Idea for Gradient Descent

Initial position: (x_0, y_0)

Direction of greatest ascent: ∇f

Direction of greatest descent: $-\nabla f$



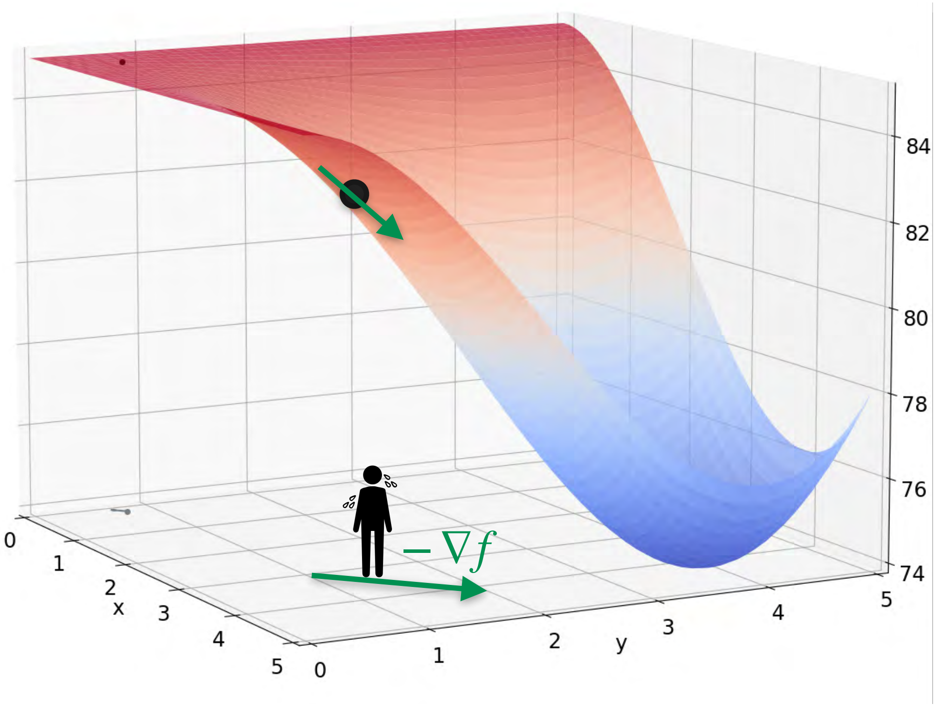
Idea for Gradient Descent

Initial position: (x_0, y_0)

Direction of greatest ascent: ∇f

Direction of greatest descent: $-\nabla f$

Updated position: $(x_0, y_0) - \alpha \nabla f$



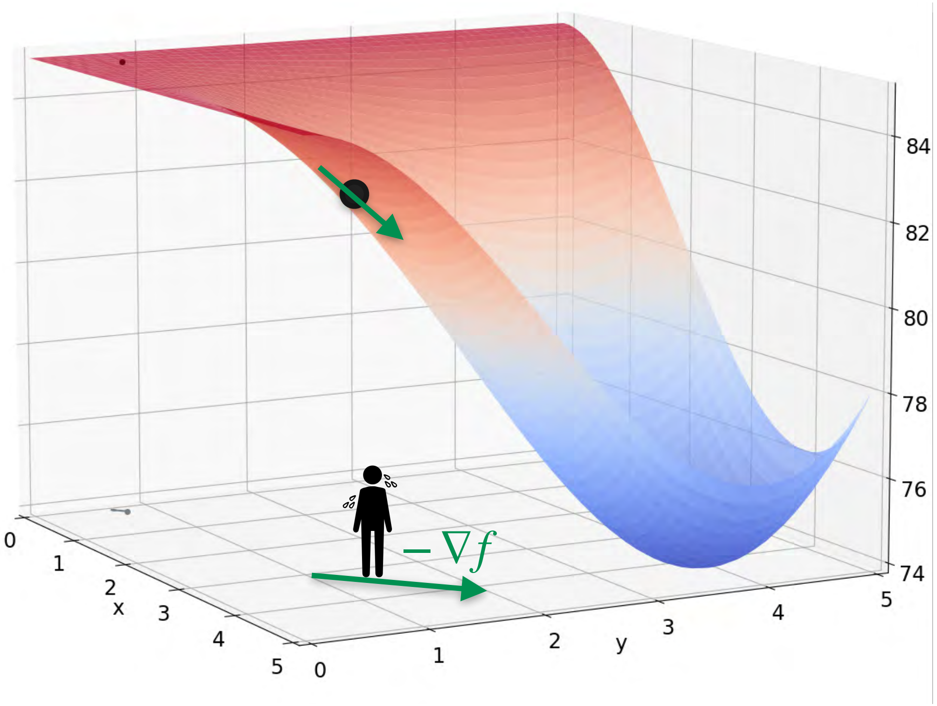
Idea for Gradient Descent

Initial position: (x_0, y_0)

Direction of greatest ascent: ∇f

Direction of greatest descent: $-\nabla f$

Updated position: $\underbrace{(x_0, y_0) - \alpha \nabla f}_{(x_1, y_1)}$



Idea for Gradient Descent

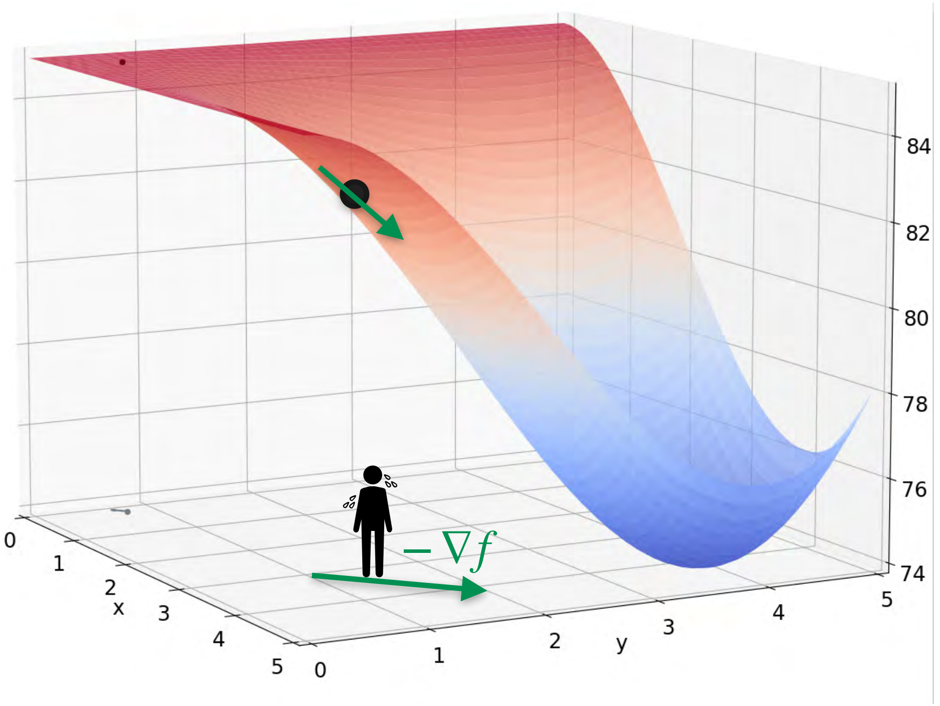
Initial position: (x_0, y_0)

Direction of greatest ascent: ∇f

Direction of greatest descent: $-\nabla f$

Updated position: $\underbrace{(x_0, y_0) - \alpha \nabla f}_{(x_1, y_1)}$

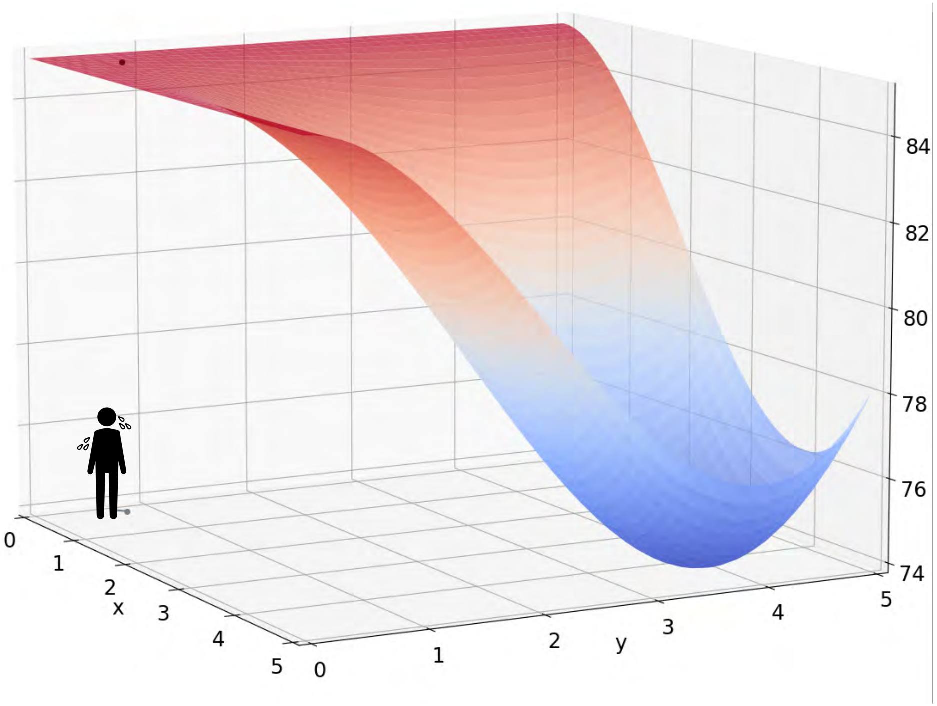
Better point!



Method 2: Gradient Descent

$$T = f(x, y) = 85 - \frac{1}{90}x^2(x-6)y^2(y-6)$$

Start: $x = 0.5,\ y = 0.6$



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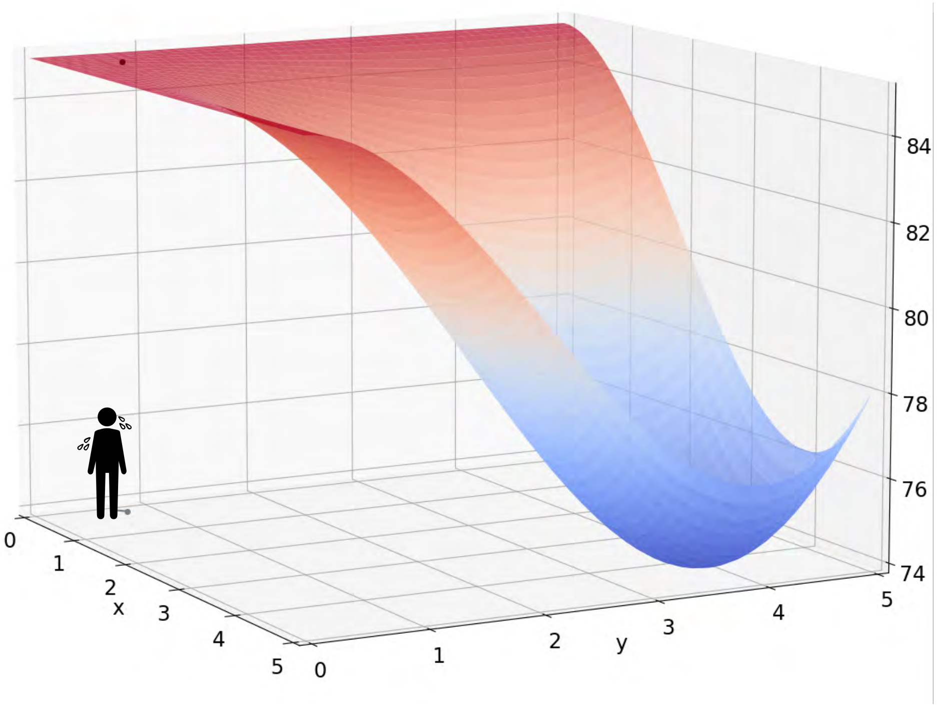

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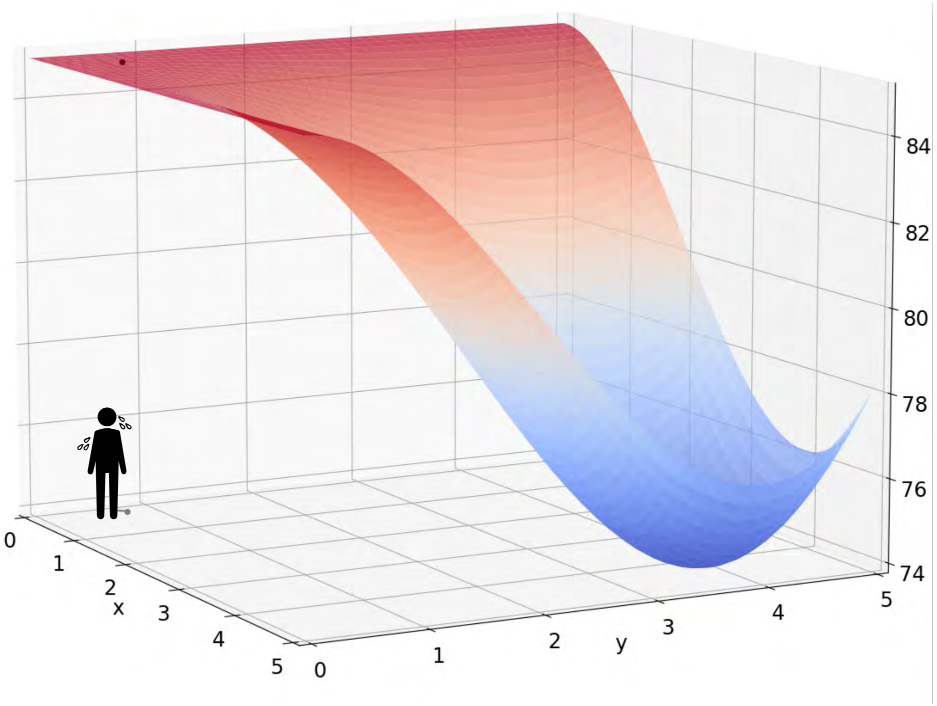
$$\nabla f = \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{bmatrix}$$

$$\frac{\partial f}{\partial x} = -\frac{1}{90}x(3x-12)y^2(y-6)$$

$$\frac{\partial f}{\partial y} = -\frac{1}{90}x^2(x-6)y(3y-12)$$



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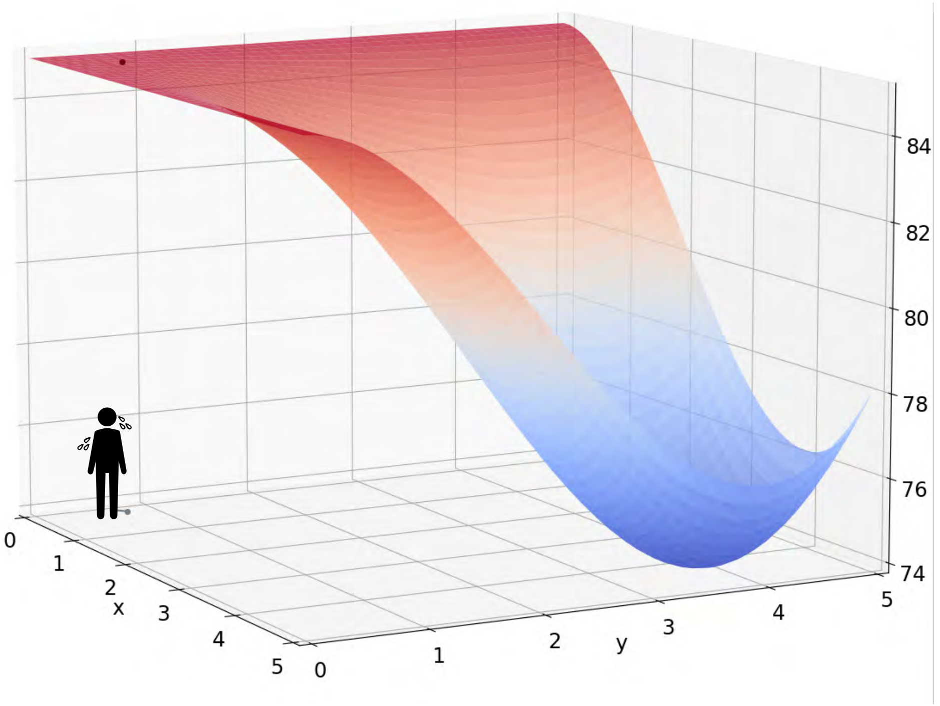

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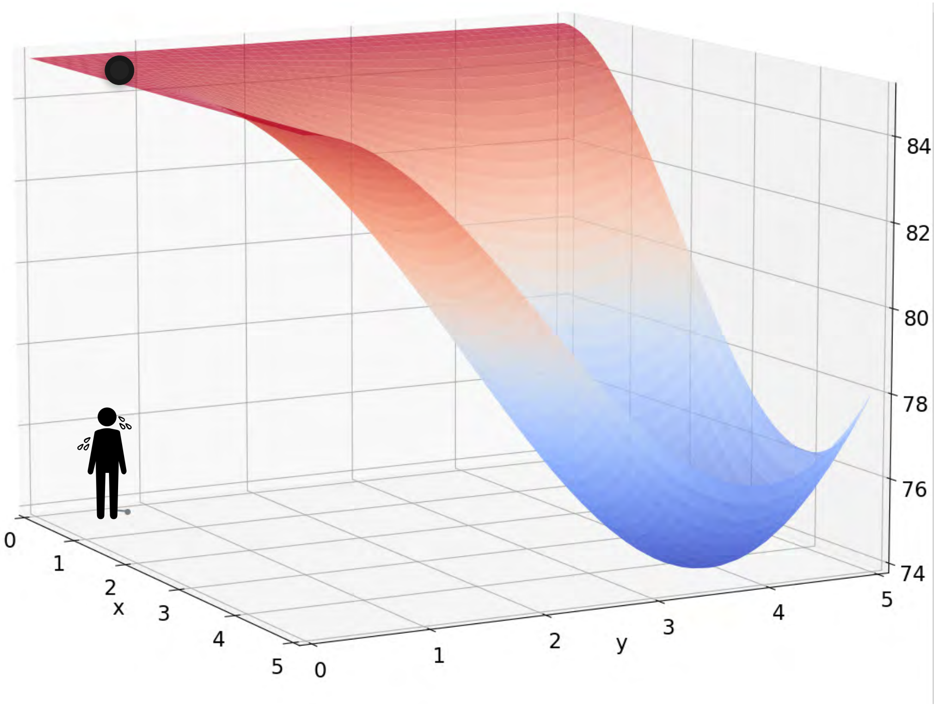
$$\nabla f(0.5, 0.6) = \begin{bmatrix} 0.1134 \\ -0.0935 \end{bmatrix}$$



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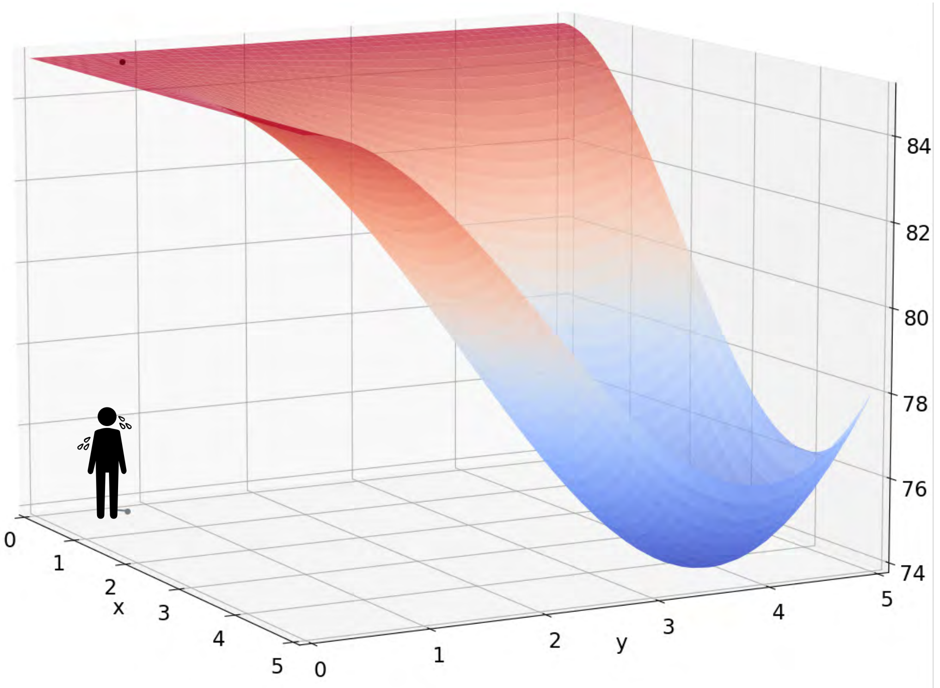


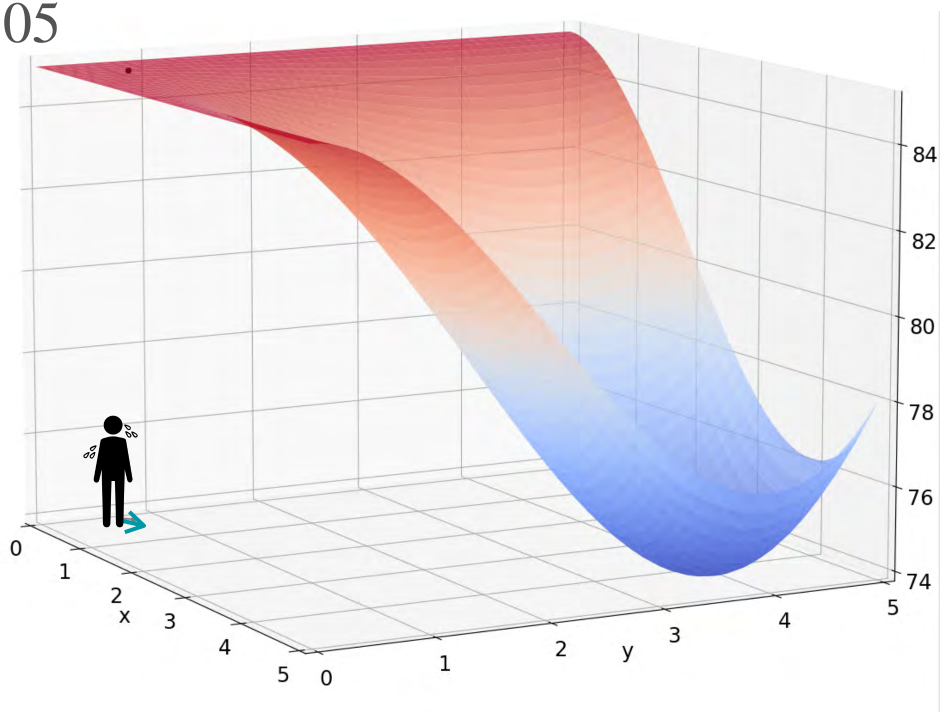
Method 2: Gradient Descent

Start: $x = 0.5,\ y = 0.6$ Rate: $\alpha = 0.05$

$$\nabla f(0.5, 0.6) = \begin{bmatrix} 0.1134 \\ -0.0935 \end{bmatrix}$$

Move by

$-0.05\ \nabla f(0.5, 0.6)$



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Move by

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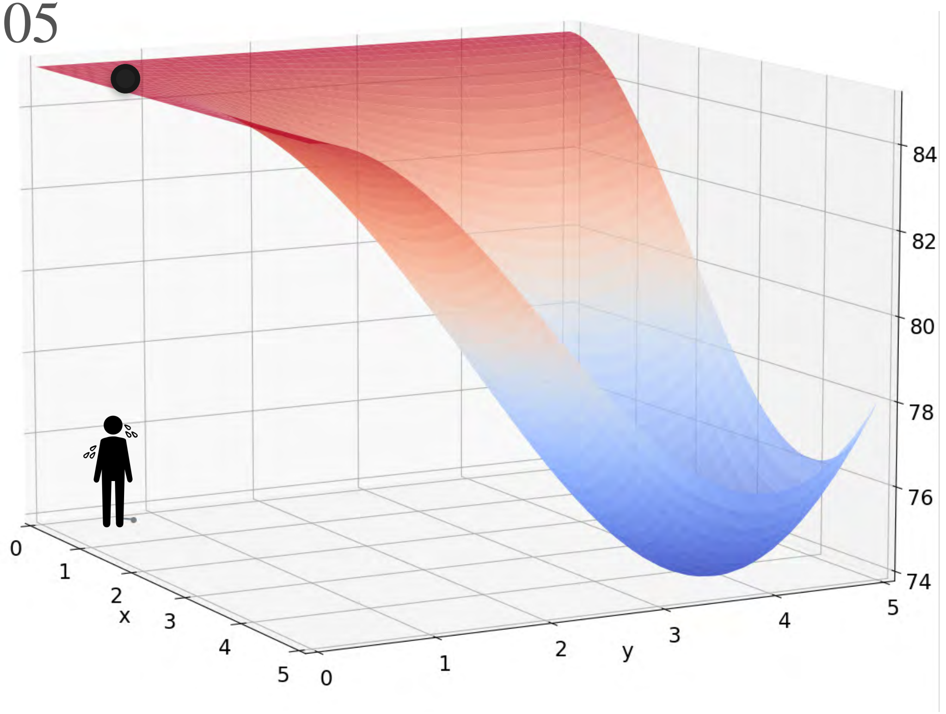
$x \mapsto 0.5057$
$y \mapsto 0.6047$



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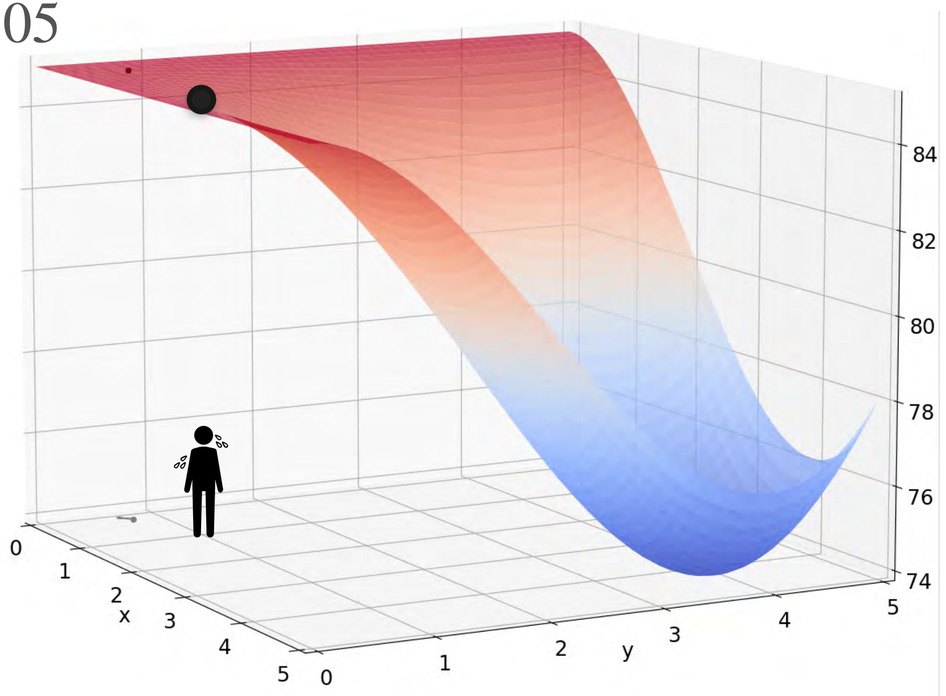

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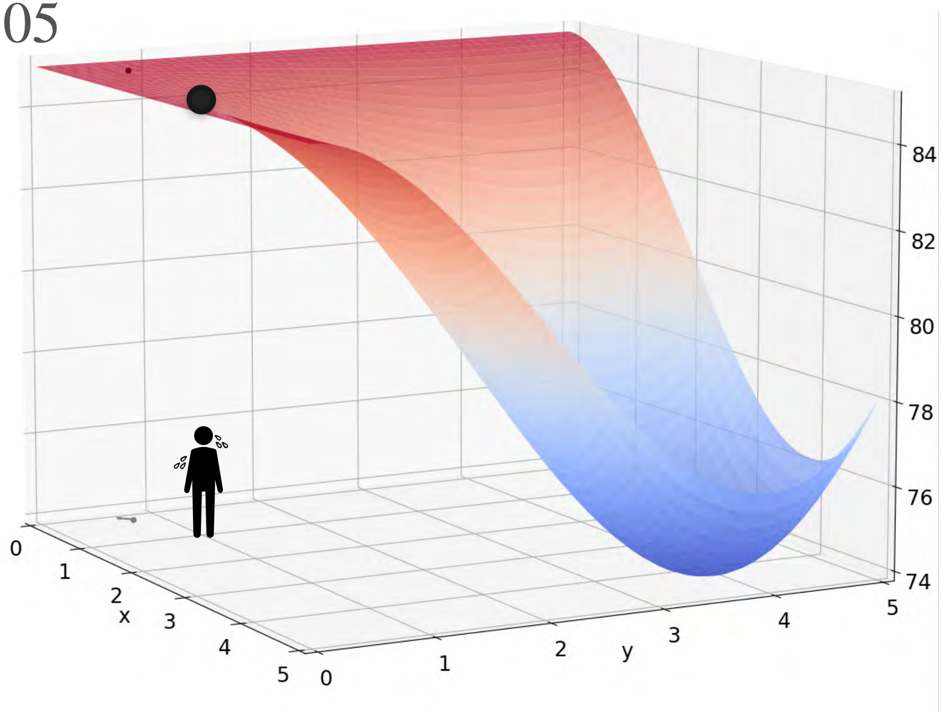


Method 2

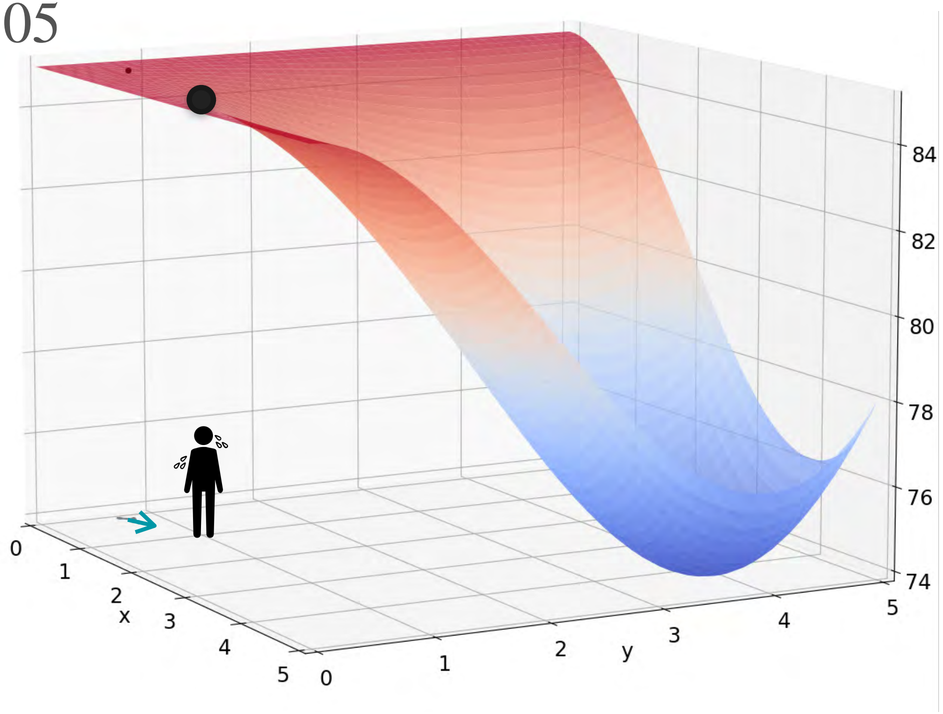
Start: $x = 0.5,\ y = 0.6$ Rate: $\alpha = 0.05$

Find:

$$\nabla f(0.5057, 0.6047) = \begin{bmatrix} -0.1162 \\ -0.0961 \end{bmatrix}$$

Move by

$$-0.05\, \nabla f(0.5057, 0.6047)$$



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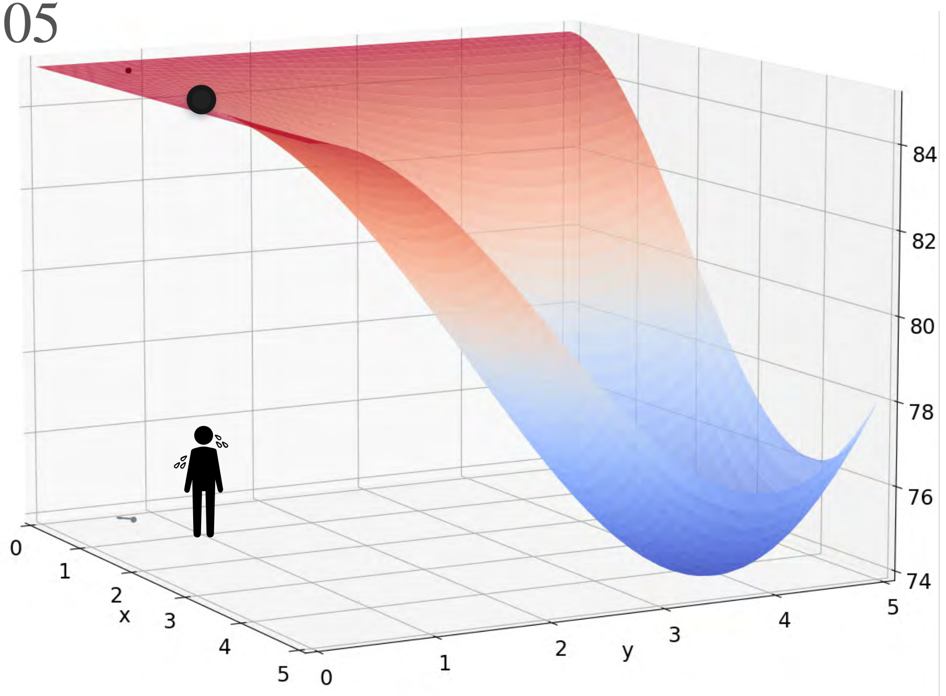
Move by

$$-0.05\ \nabla f(0.5057, 0.6047)$$

$$x \mapsto 0.5115$$
$$y \mapsto 0.6095$$

Repeat!



Gradient Descent

Start: $x = 0.5,\ y = 0.6$ Rate: $\alpha = 0.05$

Find:

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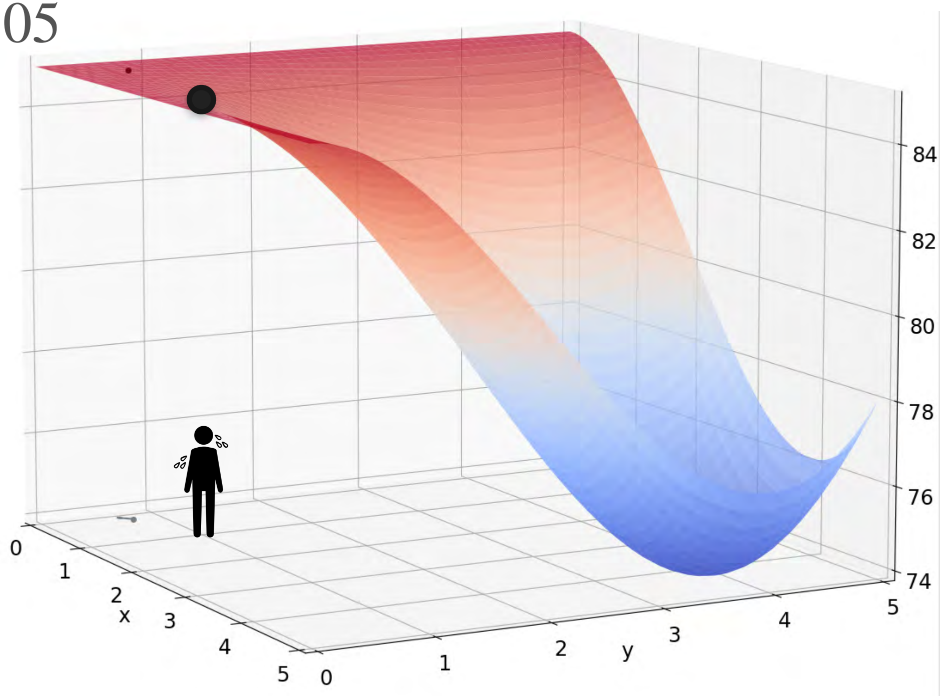

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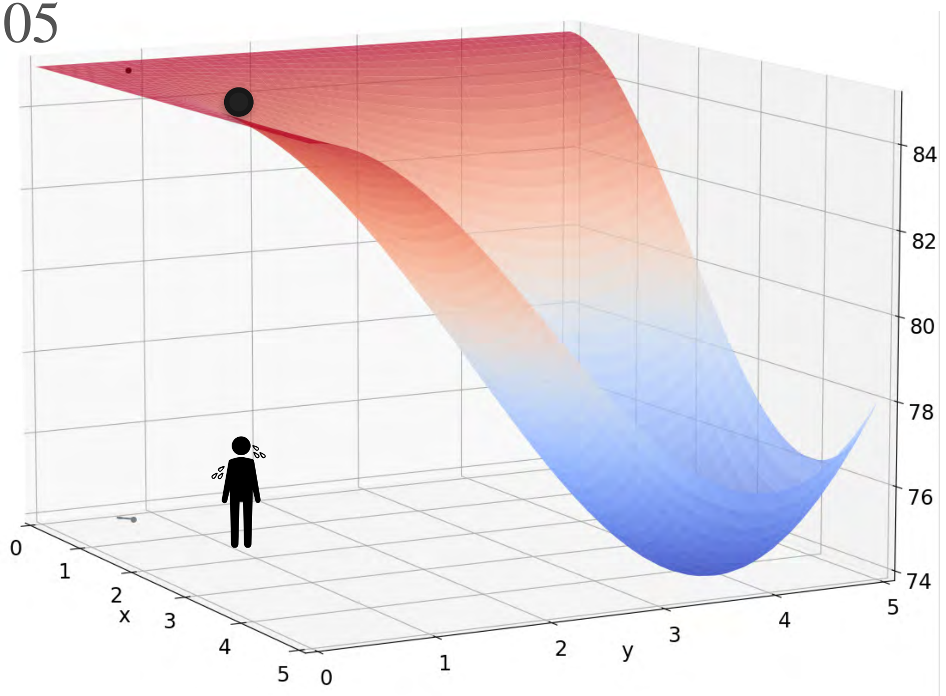

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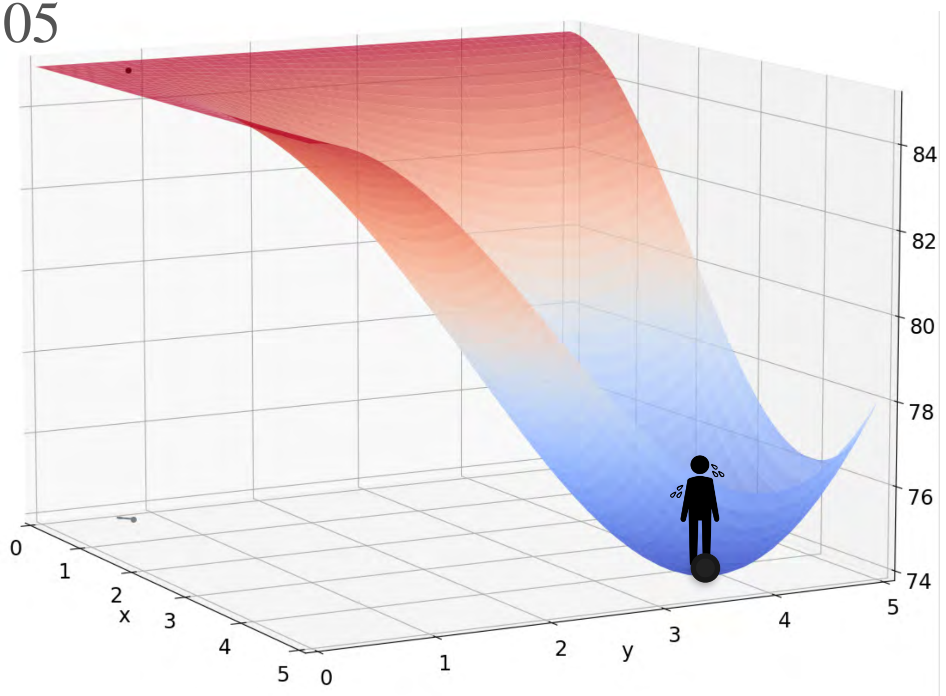


Gradient Descent

Gradient Descent

Function: $f(x, y)$

Gradient Descent

Function: $f(x, y)$

Goal: find minimum of $f(x, y)$

Gradient Descent

Function: $f(x, y)$

Goal: find minimum of $f(x, y)$

Step 1:

Define a learning rate α

Choose a starting point (x_0, y_0)

Gradient Descent

Function: $f(x, y)$

Goal: find minimum of $f(x, y)$

Step 1:

Define a learning rate α

Choose a starting point (x_0, y_0)

Step 2:

Update: $\begin{bmatrix} x_k \\ y_k \end{bmatrix} = \begin{bmatrix} x_{k-1} \\ y_{k-1} \end{bmatrix} - \alpha \nabla f(x_{k-1}, y_{k-1})$

Gradient Descent

Function: $f(x, y)$ Goal: find minimum of $f(x, y)$

Step 1:

Define a learning rate α

Choose a starting point (x_0, y_0)

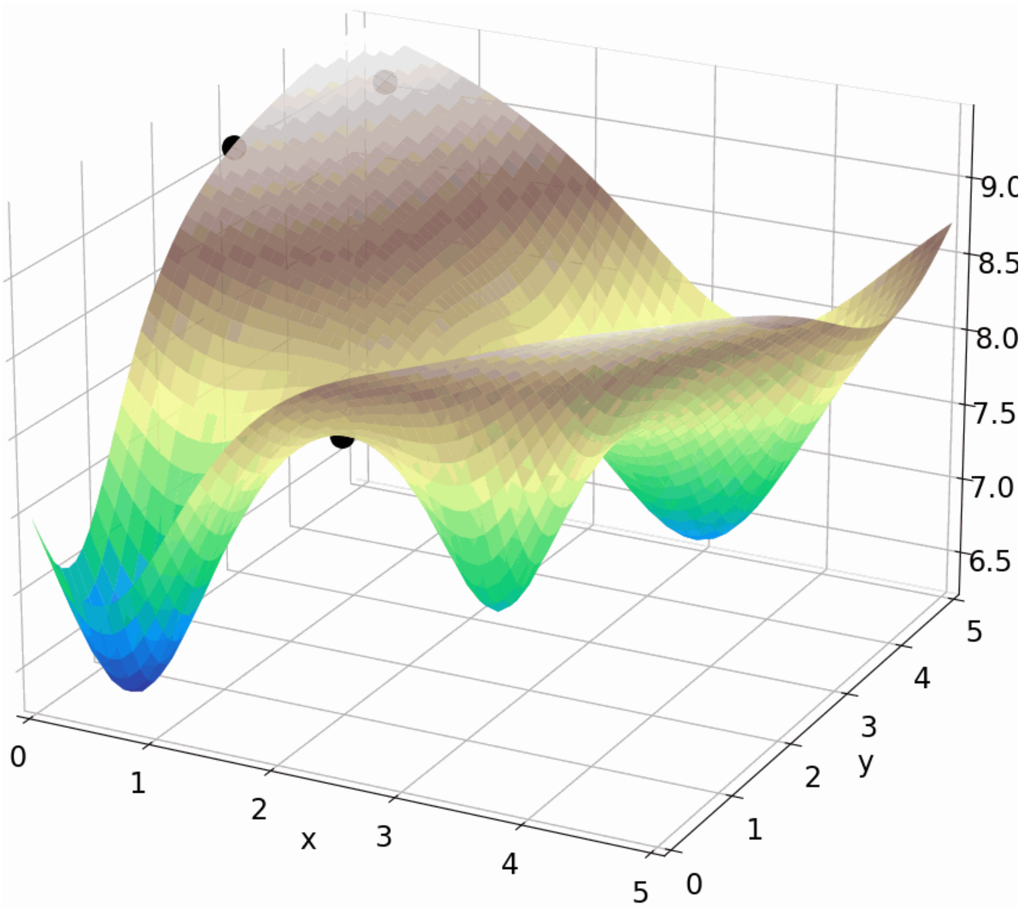
Step 2:

Update: $\begin{bmatrix} x_k \\ y_k \end{bmatrix} = \begin{bmatrix} x_{k-1} \\ y_{k-1} \end{bmatrix} - \alpha \nabla f(x_{k-1}, y_{k-1})$

Step 3:

Repeat Step 2 until you are close enough to the true minimum (x^*, y^*)

Gradient Descent With Local Minima



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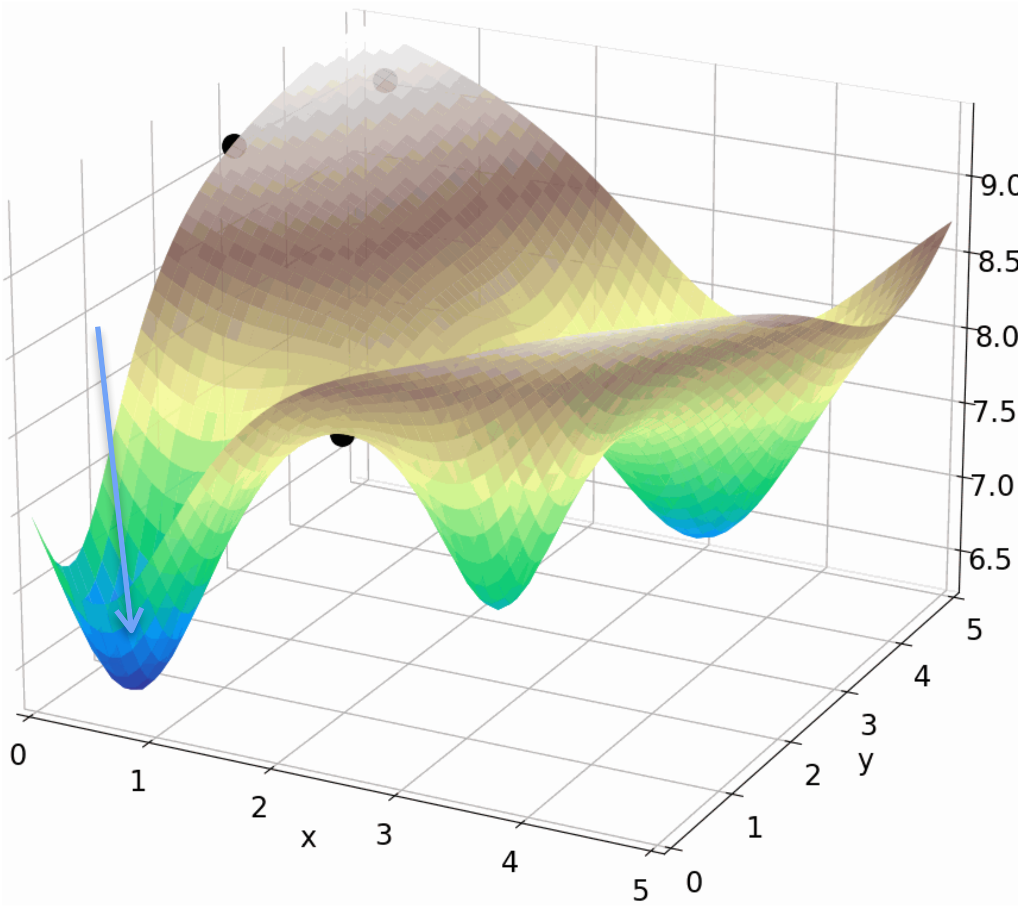
Gradient Descent With Local Minima



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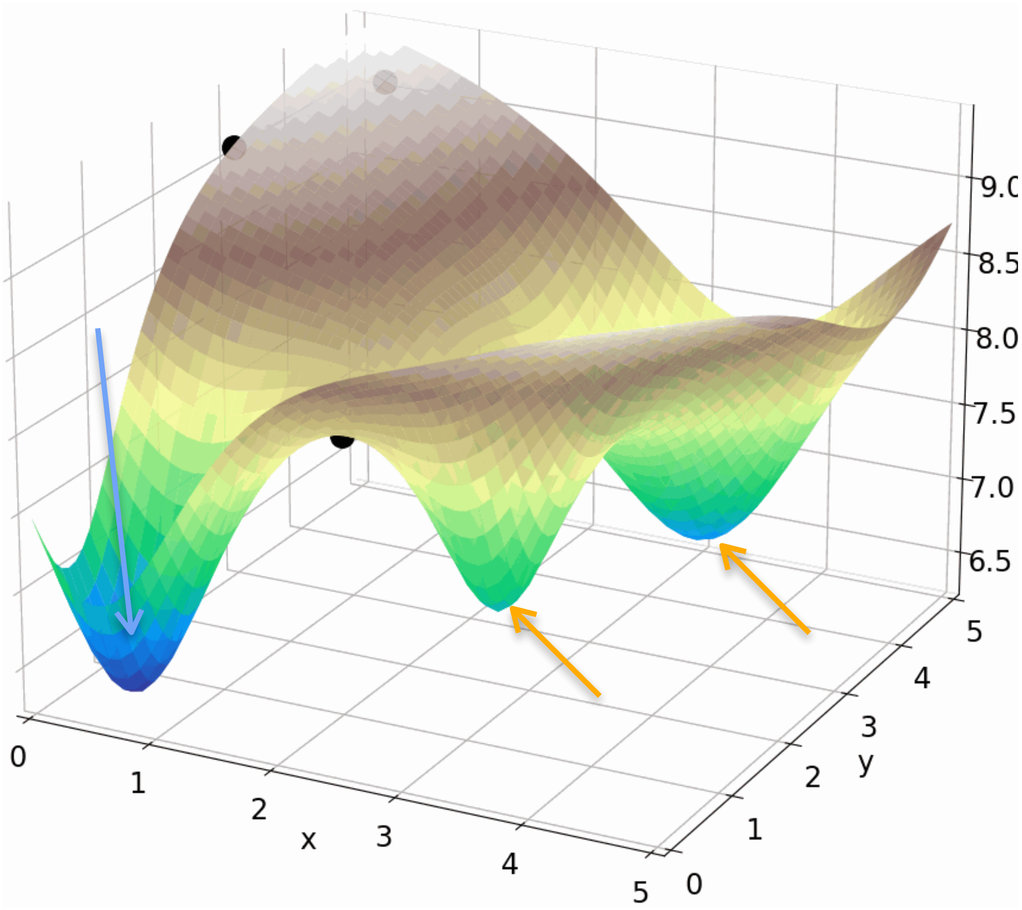
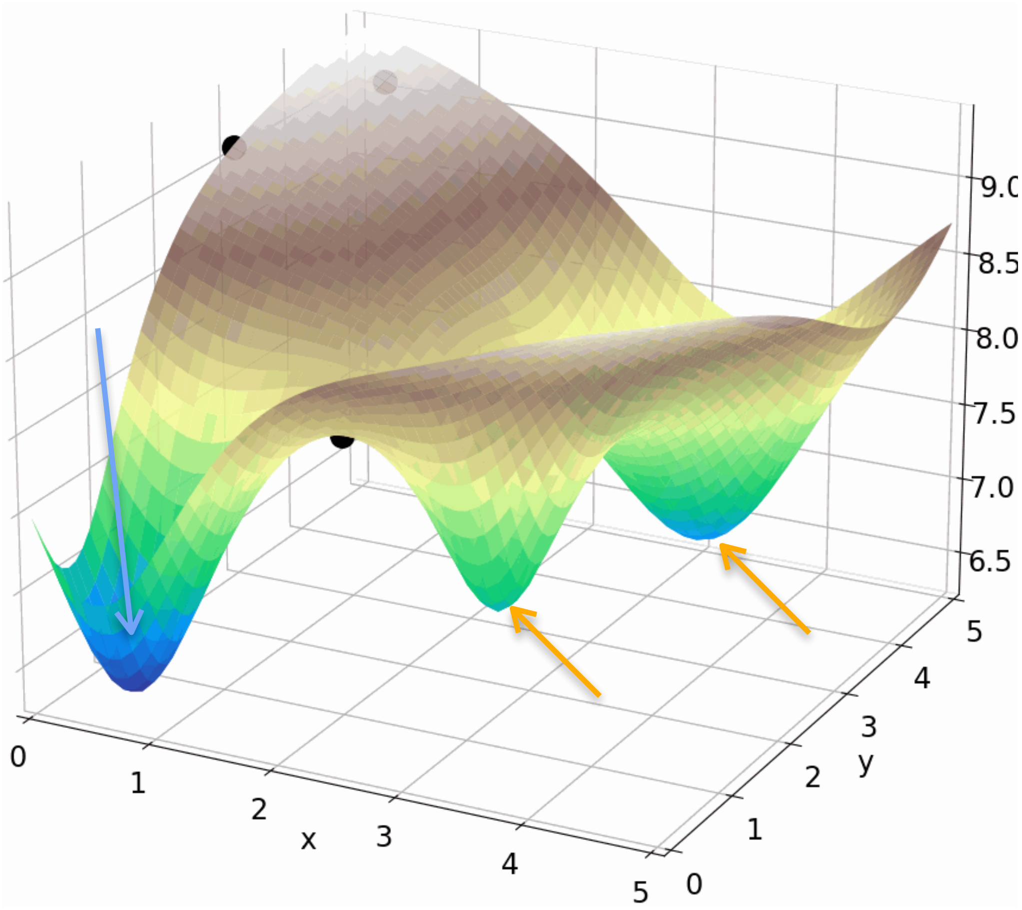


Gradient Descent With Local Minima



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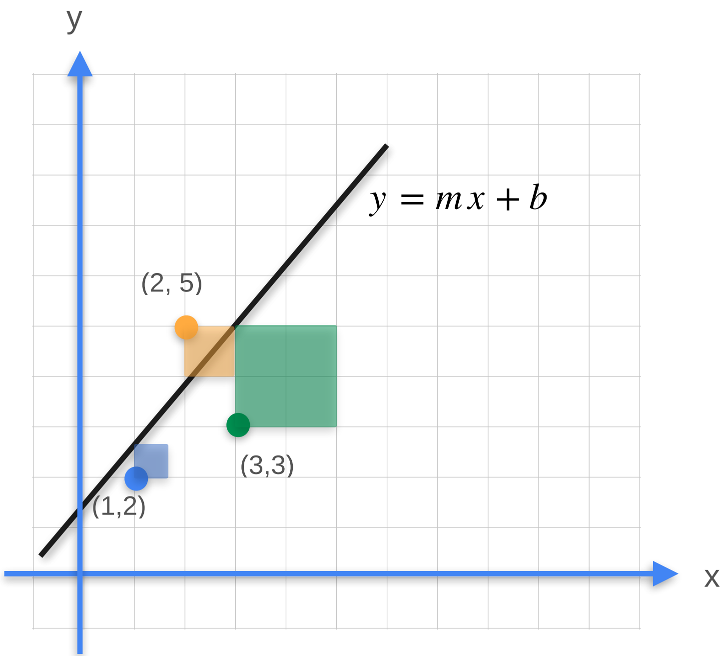
Gradients and Gradient Descent

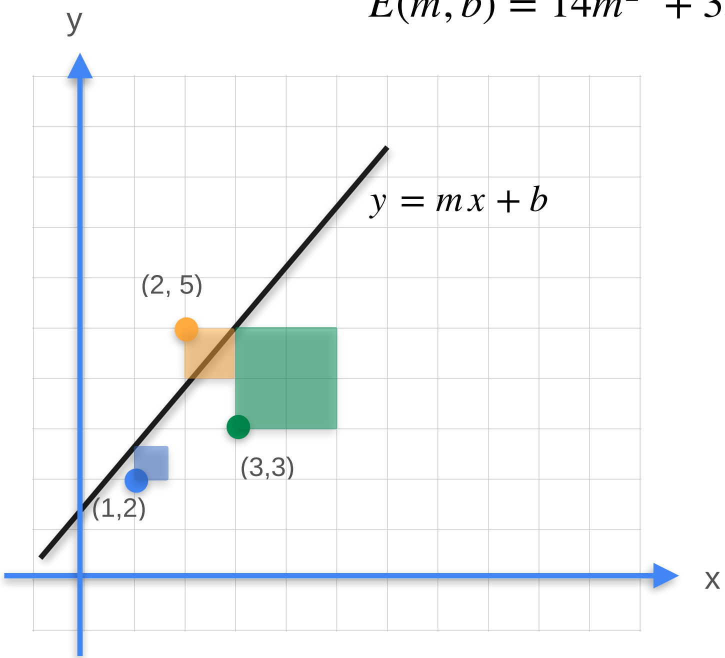
Optimization using Gradient Descent - Least squares

Gradient Descent With Power Lines Example



Gradient Descent With Power Lines Example

$$E(m,b) = 14m^2 + 3b^2 + 38 + 12mb - 42m - 20b$$



Gradient Descent With Power Lines Example

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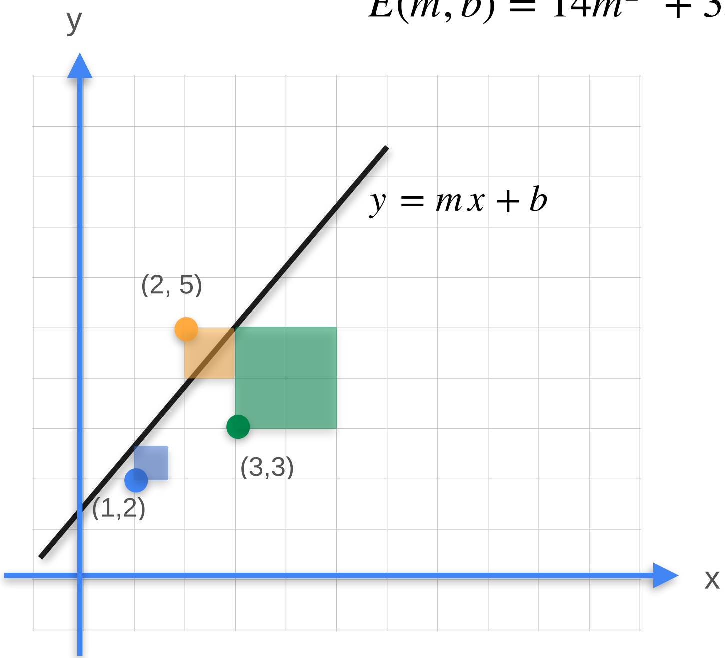
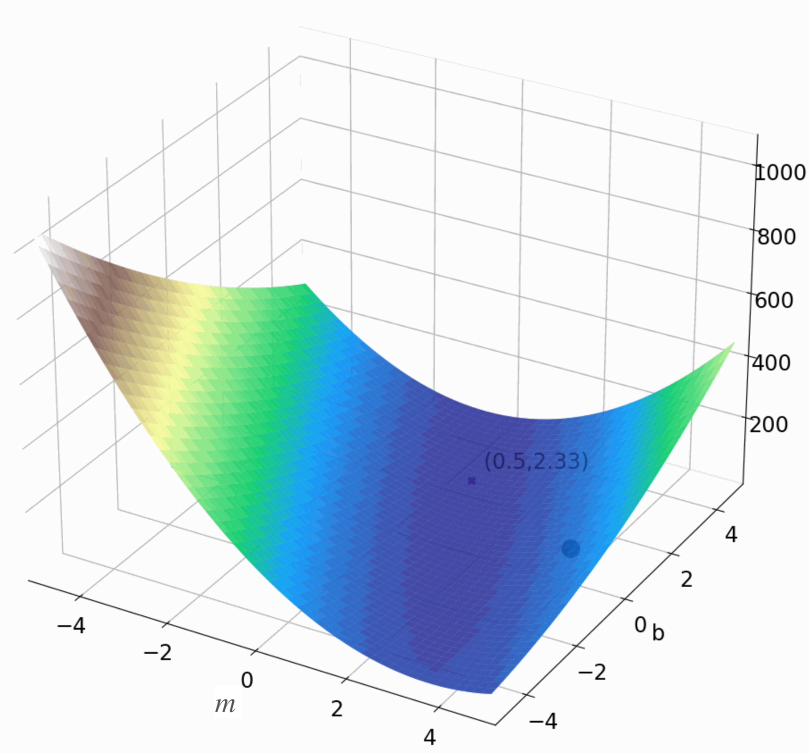


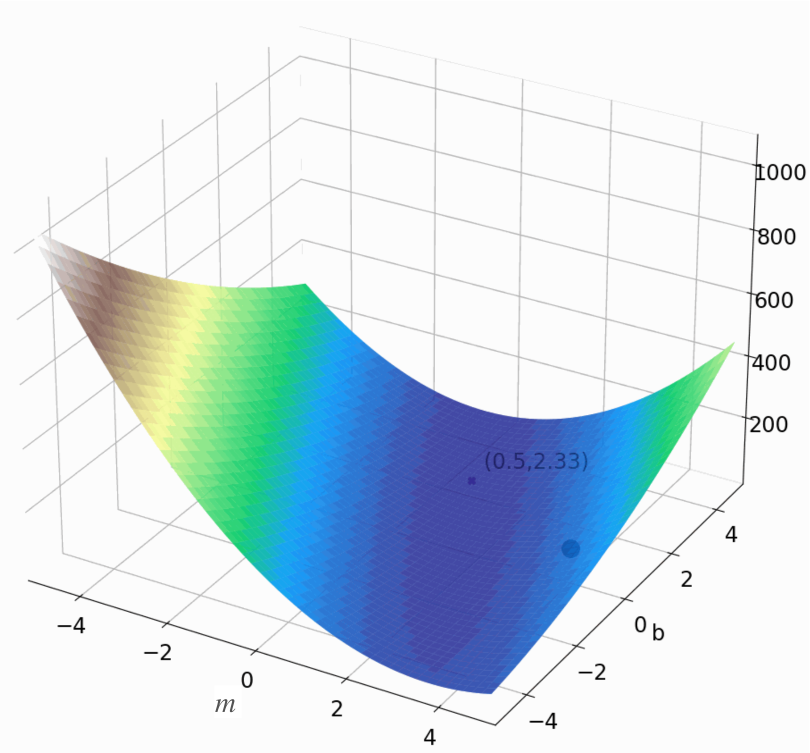
$$E(m = \frac{1}{2}, b = \frac{7}{3}) \approx 4.167$$

Linear Regression: Gradient Descent



Linear Regression: Gradient Descent

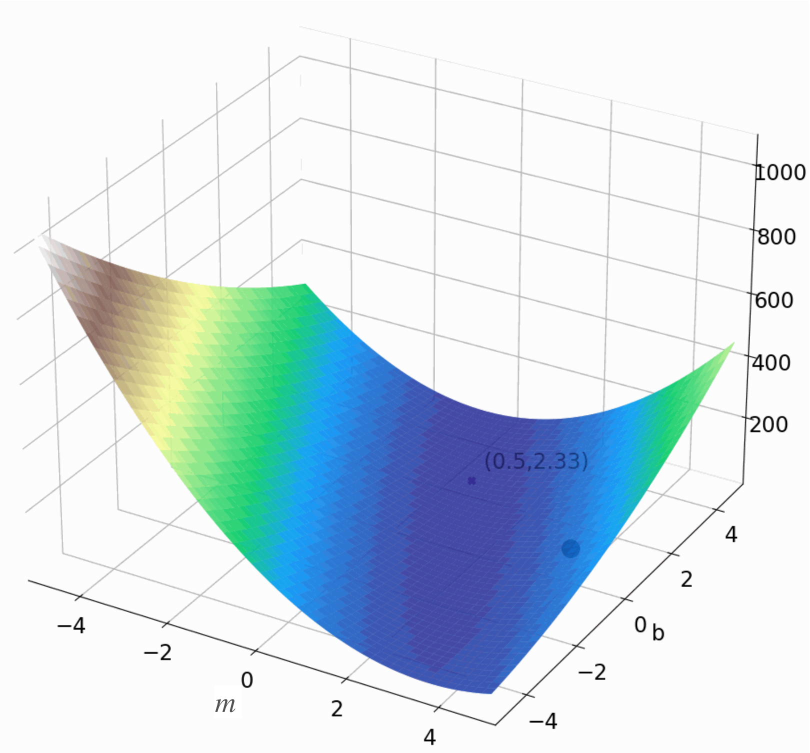
Goal: Minimize sum of squares cost



Linear Regression: Gradient Descent

Goal: Minimize sum of squares cost

$$\nabla E = \left[28m + 12b - 42,\ 6b + 12m - 20\right]$$



Linear Regression: Gradient Descent

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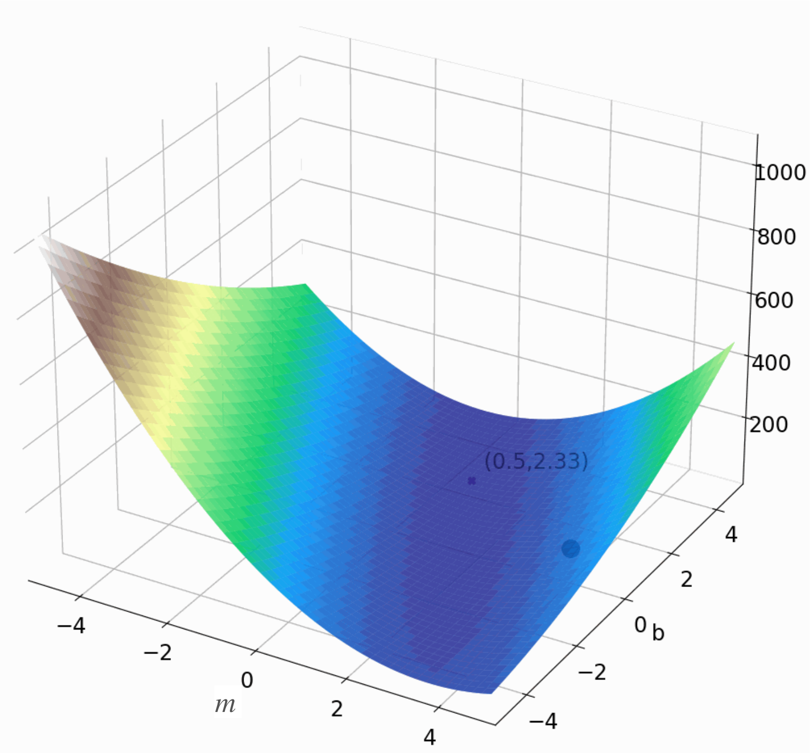
$$m =$$

$$b =$$



Linear Regression: Gradient Descent

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$$m = \quad ?$$
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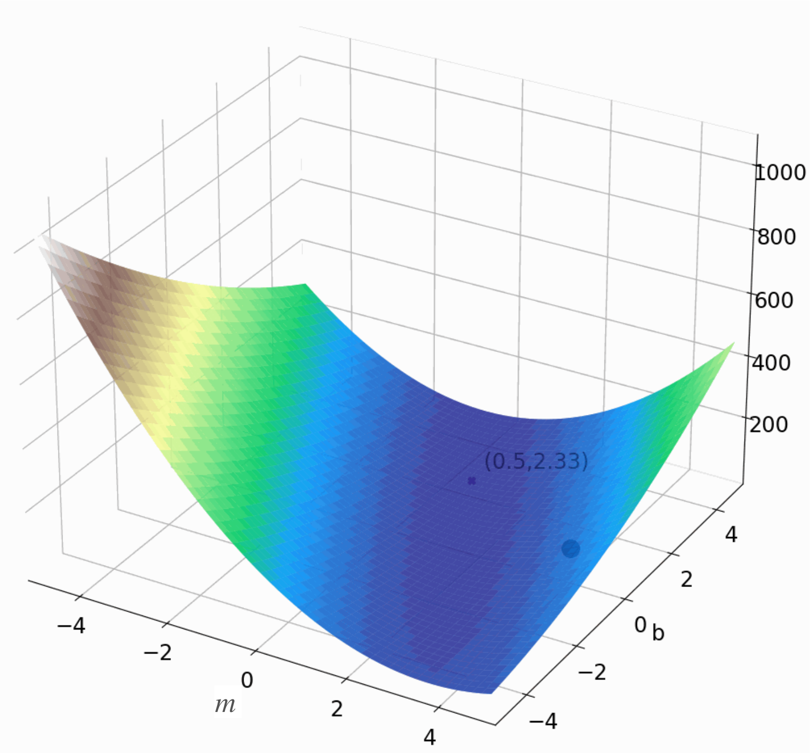

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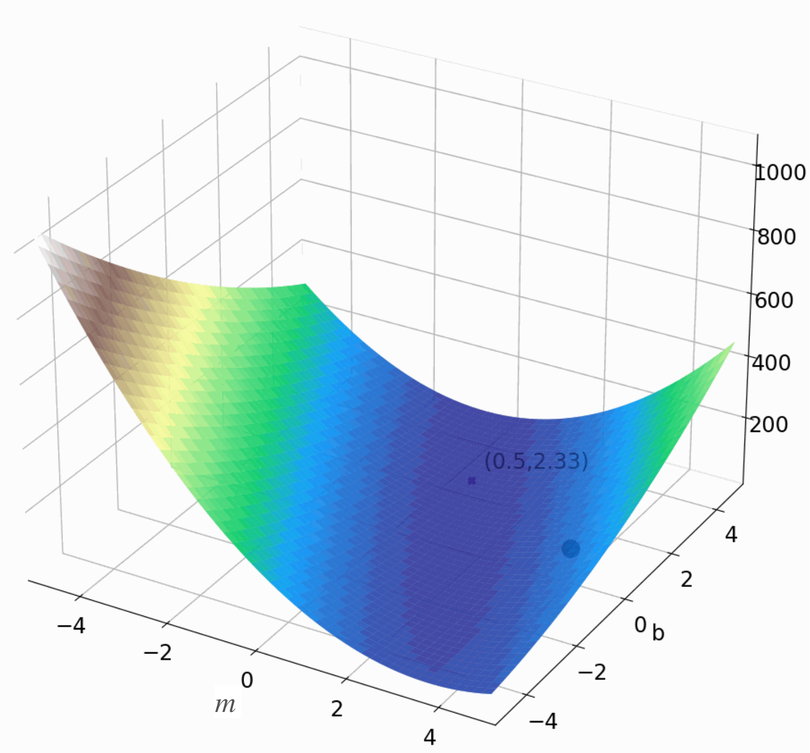

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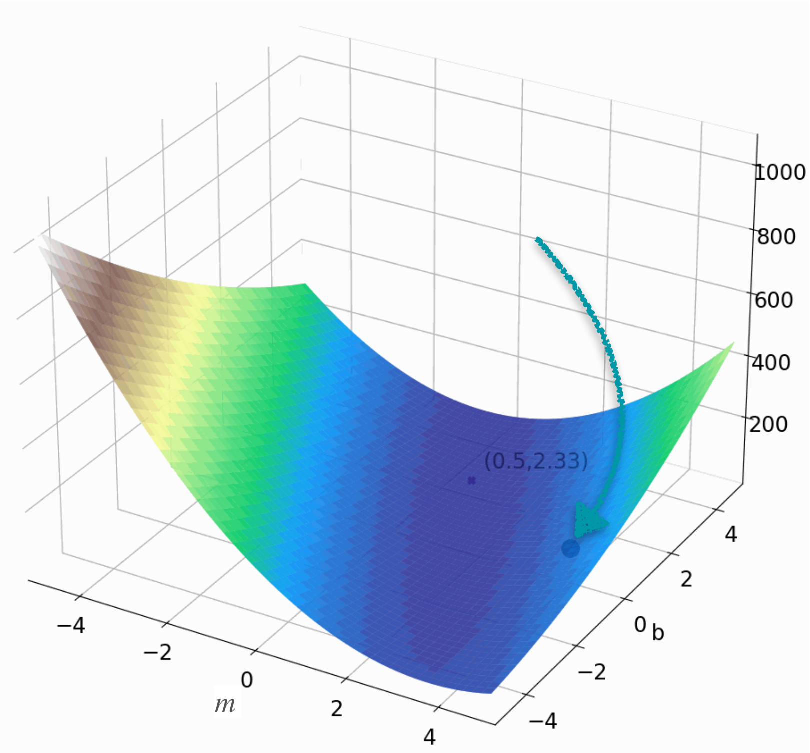
?



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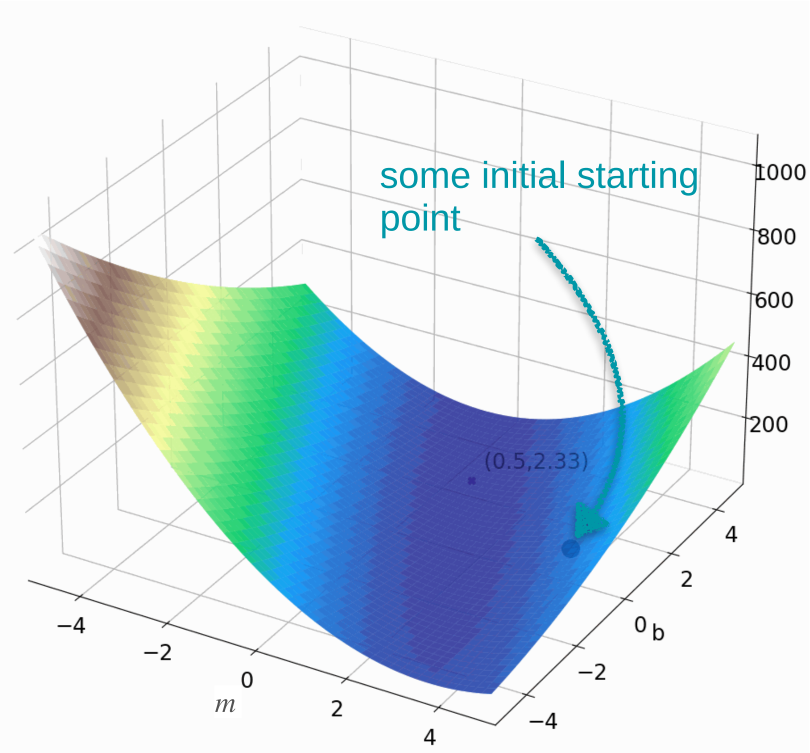

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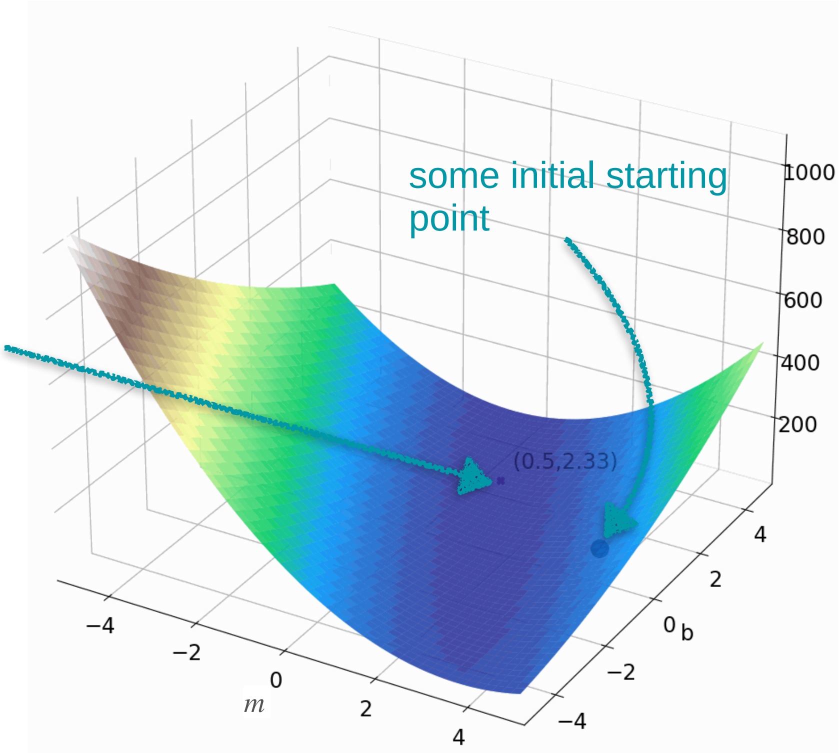


Linear Regression: Gradient Descent

Goal: Minimize sum of squares cost

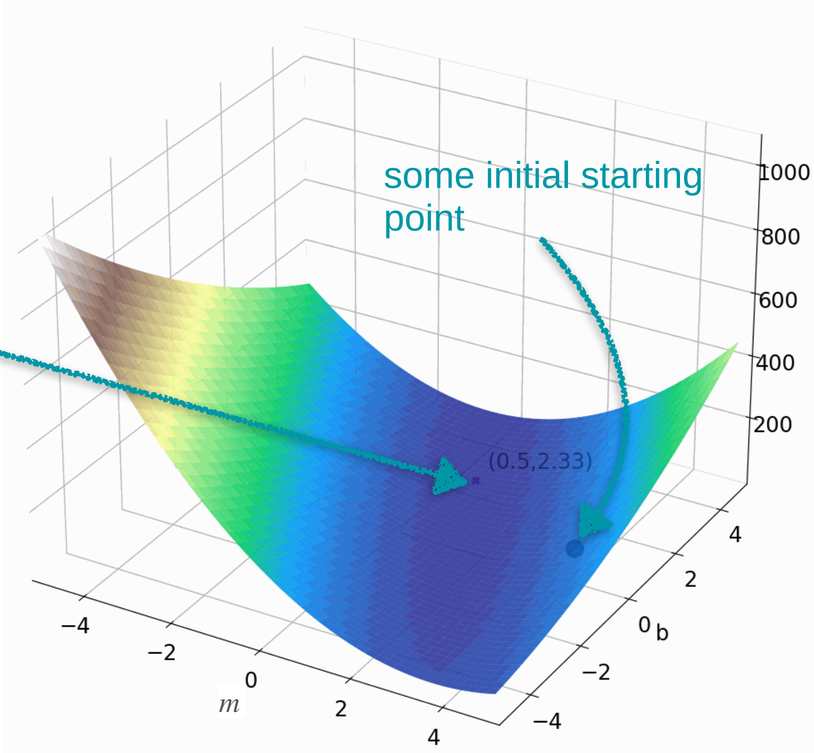
$$\nabla E = \left[28m + 12b - 42,\ 6b + 12m - 20\right]$$

$m =$

$b =$?

The points m,b such that the cost is minimum

some initial starting point



Linear Regression: Gradient Descent

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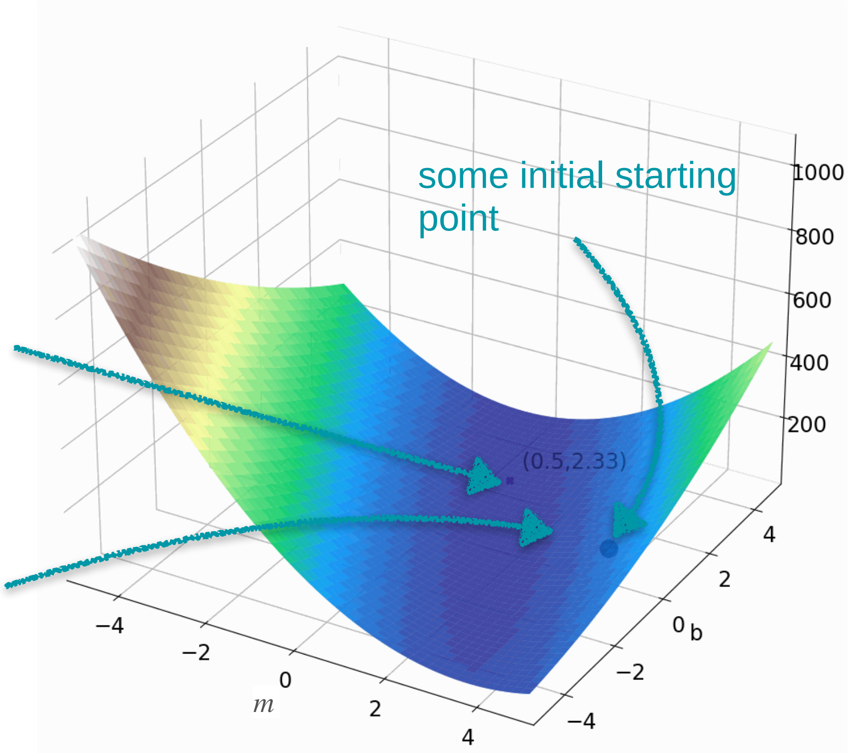
$m =$

$b =$ **?**

The points m,b such that the cost is minimum



Linear Regression: Gradient Descent

Goal: Minimize sum of squares cost

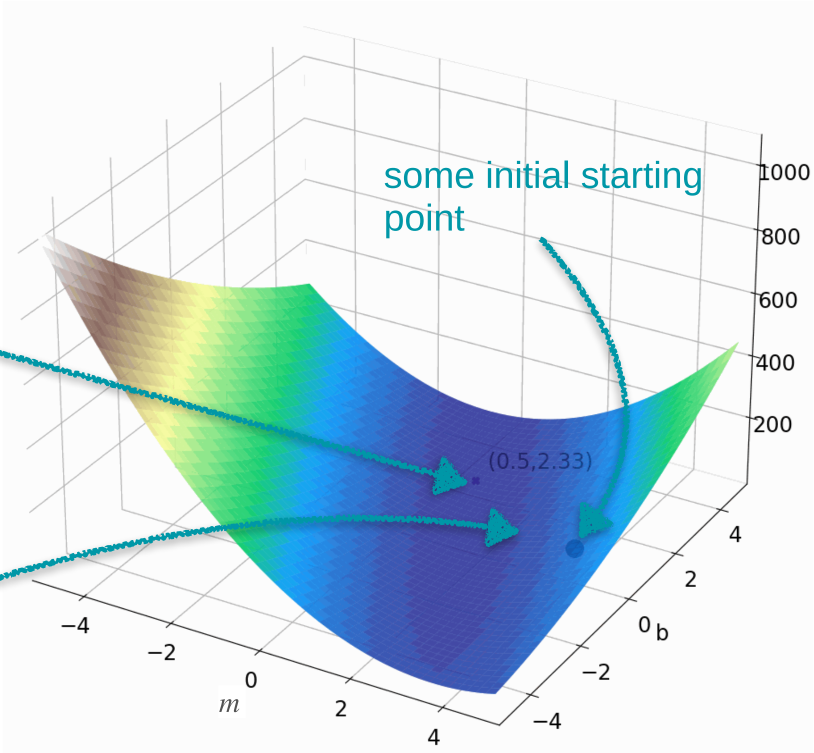
$$\nabla E = \left[28m + 12b - 42,\ 6b + 12m - 20\right]$$

$m =$

$b =$?

The points m,b such that the cost is minimum

descend until you find the minimum



Linear Regression: Gradient Descent

Goal: Minimize sum of squares cost

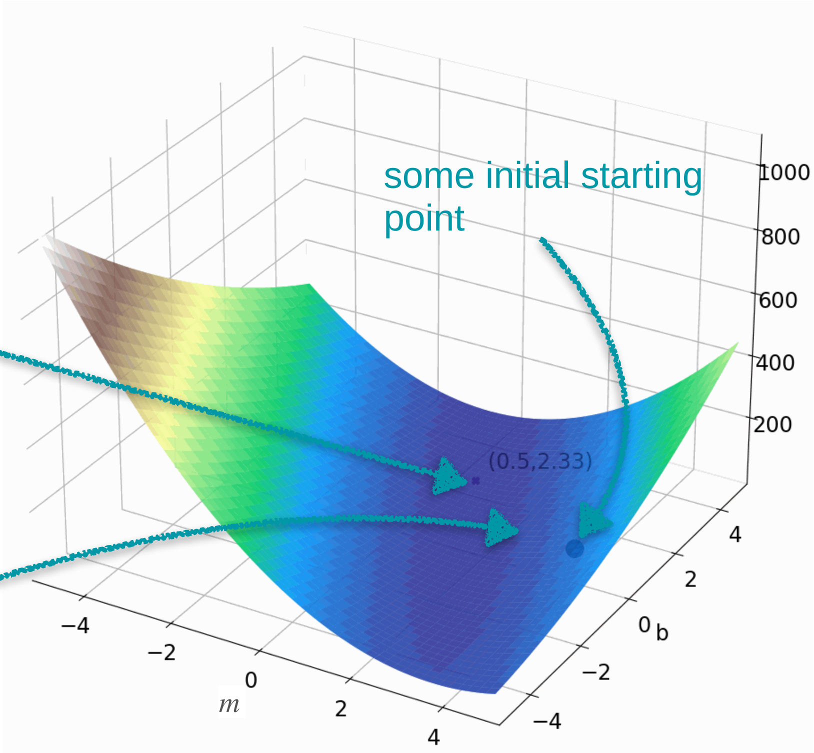
$$\nabla E = \left[28m + 12b - 42,\ 6b + 12m - 20\right]$$

$$m = \quad ?$$
$$b =$$

Steps:

The points m,b such that the cost is minimum

descend until you find the minimum



Linear Regression: Gradient Descent

Goal: Minimize sum of squares cost

$$\nabla E = \left[28m + 12b - 42,\ 6b + 12m - 20\right]$$

$m =$

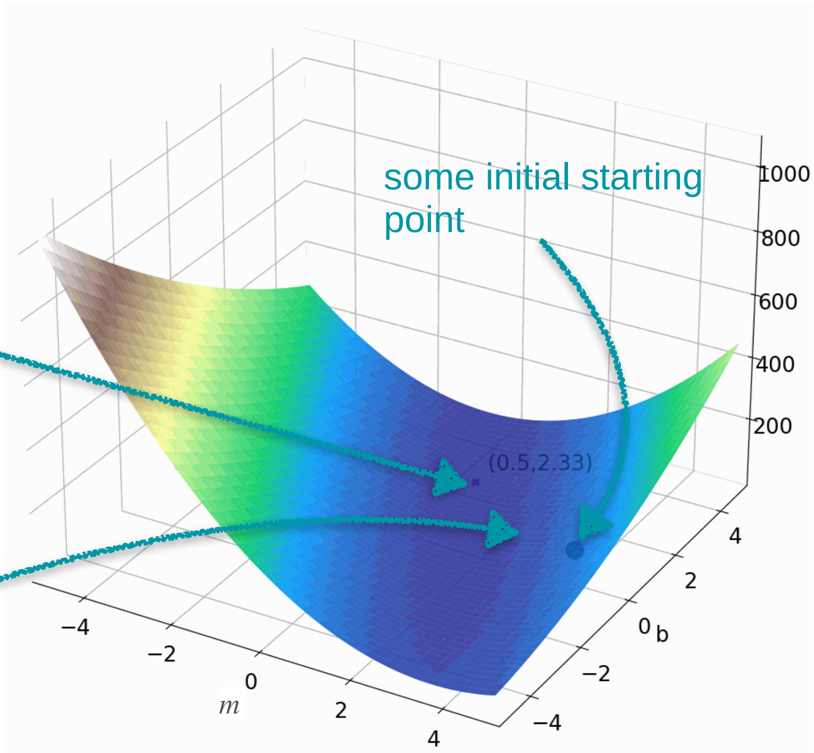
$b =$ **?**

The points m,b such that the cost is minimum

Steps:

Start with (m_0, b_0)

descend until you find the minimum



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Linear Regression: Gradient Descent

Goal: Minimize sum of squares cost

$$\nabla E = \left[28m + 12b - 42,\ 6b + 12m - 20\right]$$

$m =$

$b =$?

The points m,b such that the cost is minimum

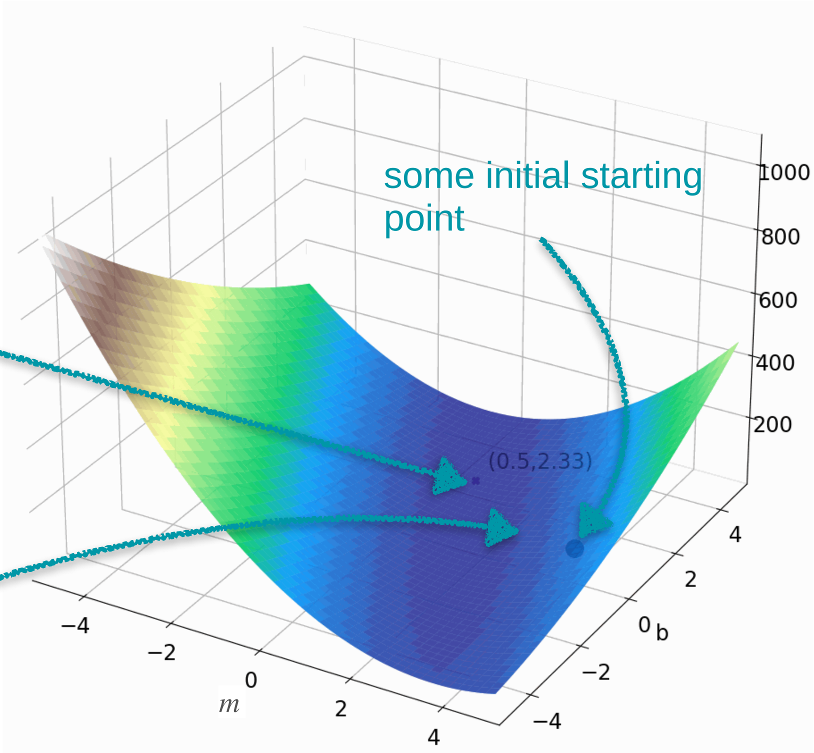
Steps:

Start with (m_0, b_0)

descend until you find the minimum

Iterate

$$(m_{k+1}, b_{k+1}) = (m_k, b_k) - \alpha \nabla E(m_k, b_k)$$



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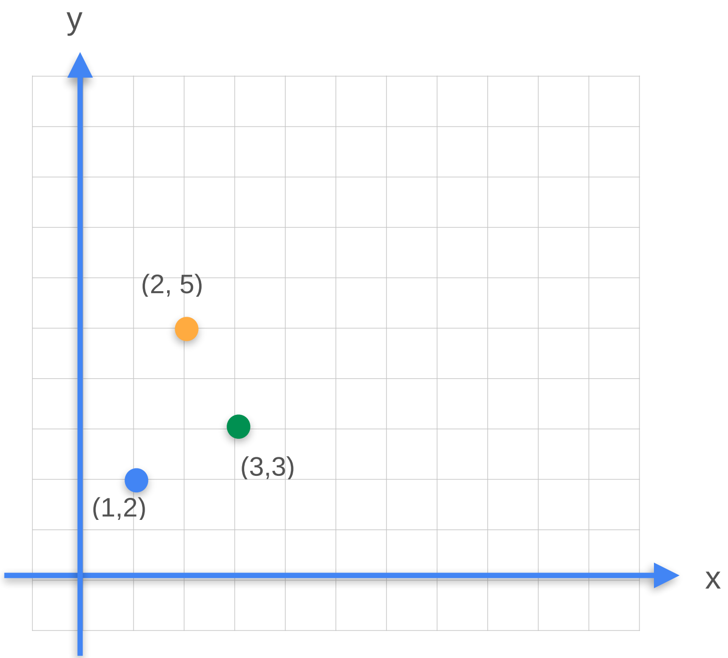
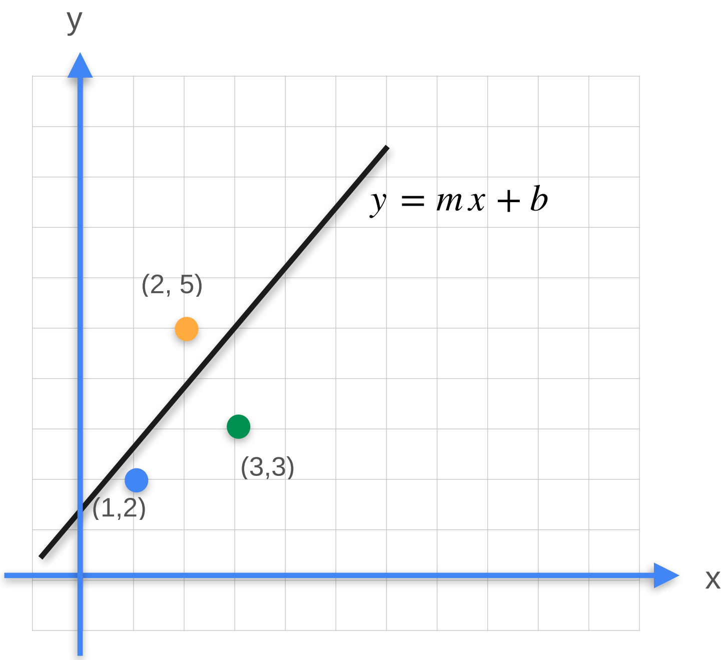
Gradients and Gradient Descent

Optimization using Gradient Descent - Least squares with multiple observations

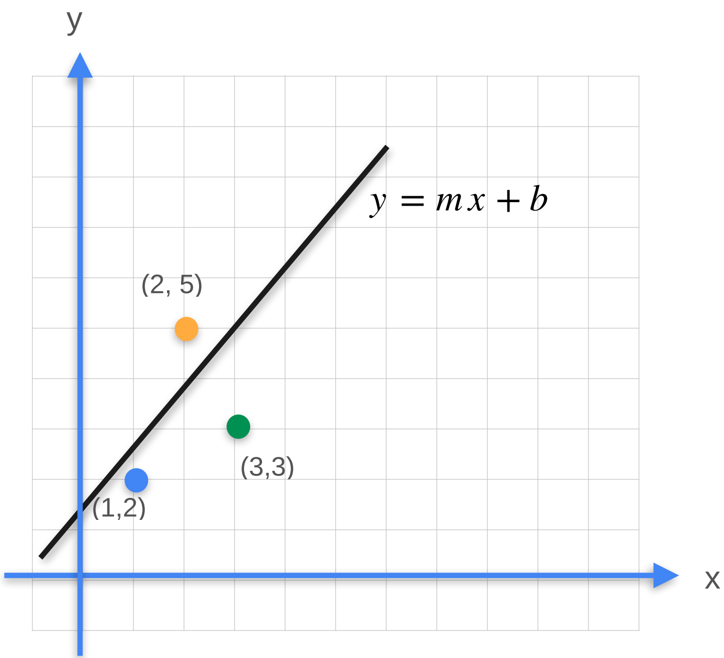
Gradient Descent

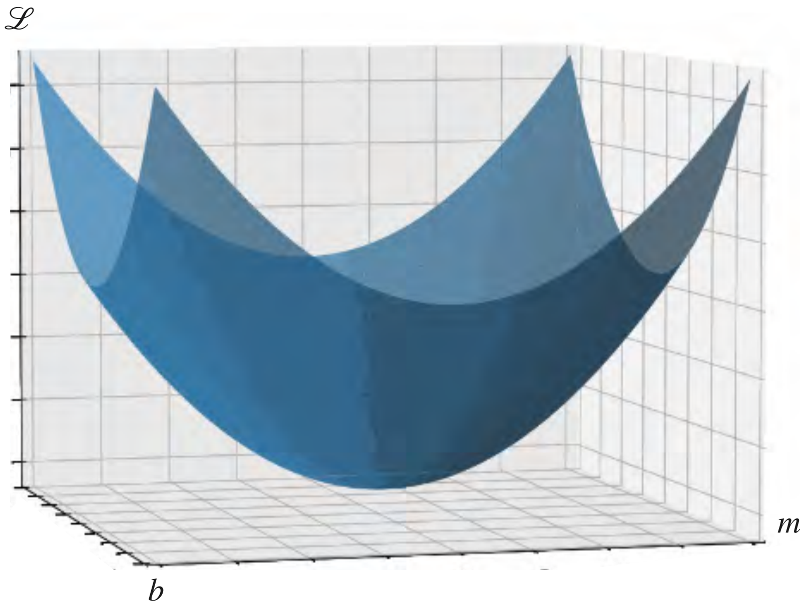


Gradient Descent

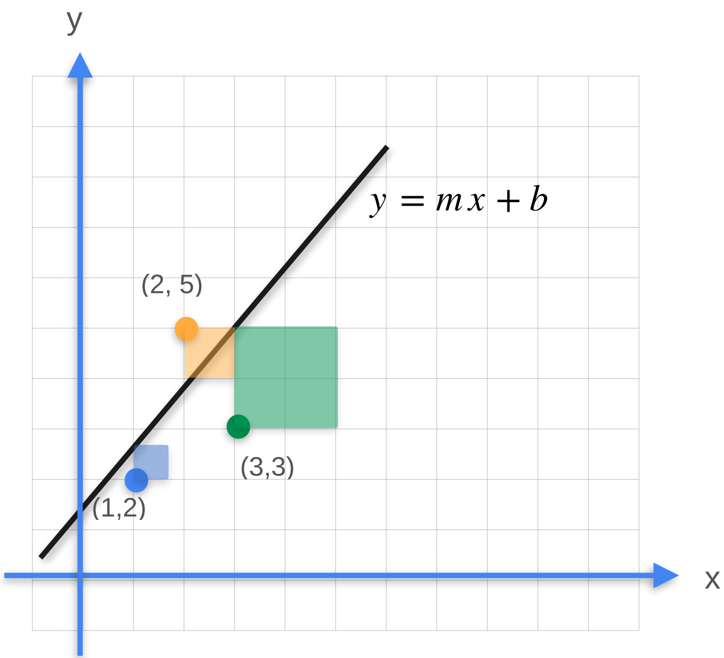


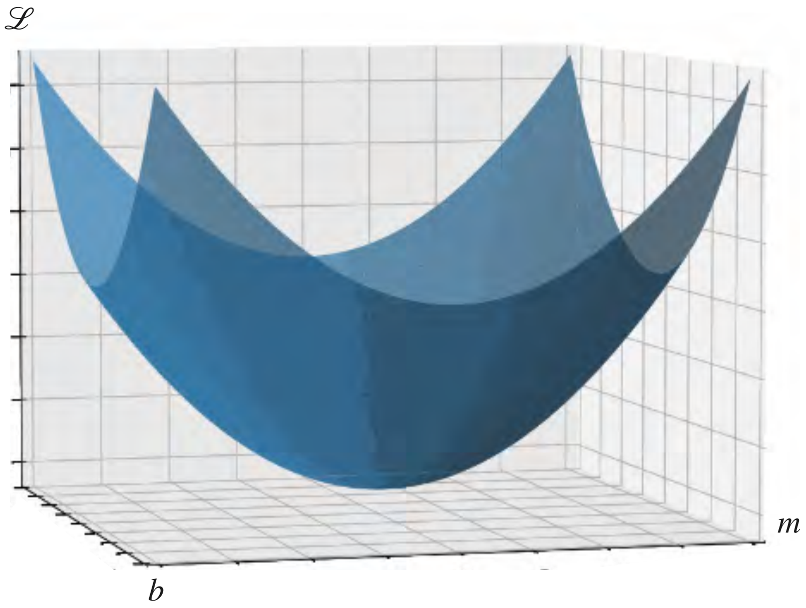
Gradient Descent



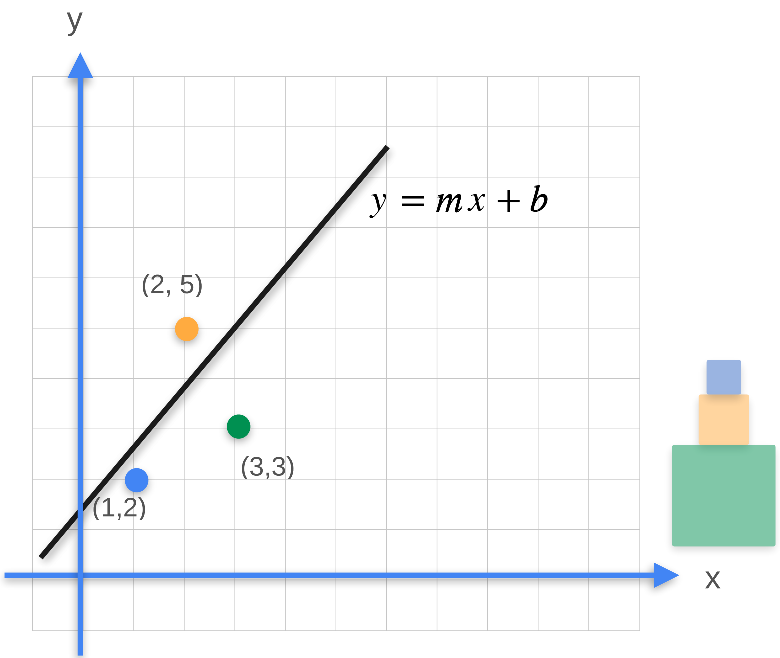


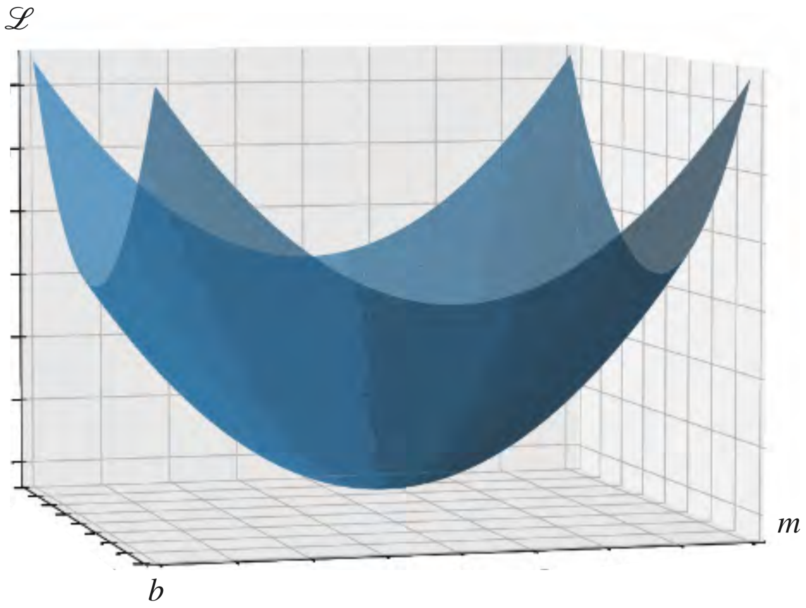
Gradient Descent



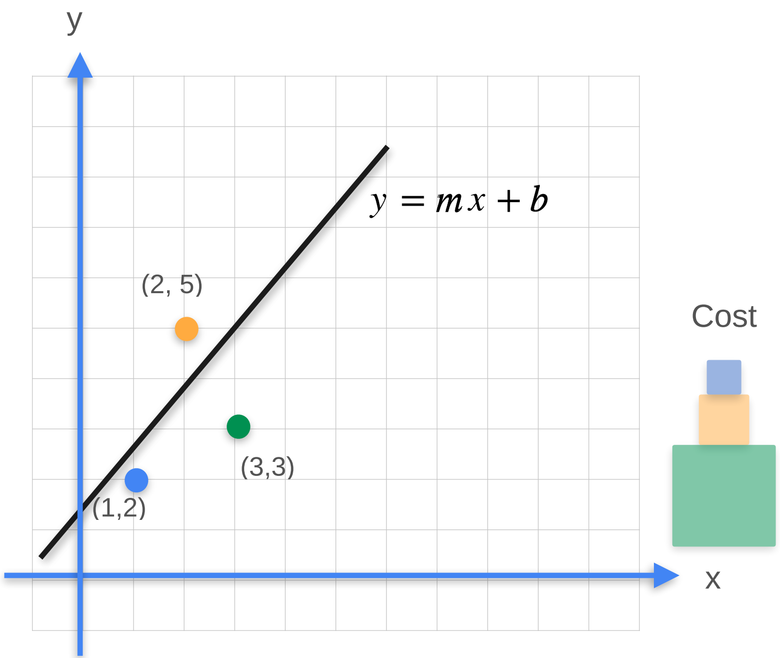


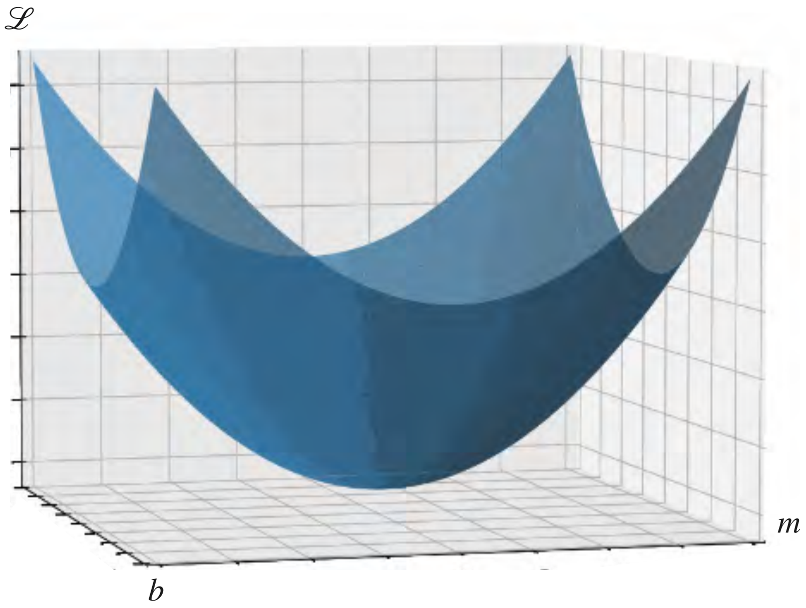
Gradient Descent



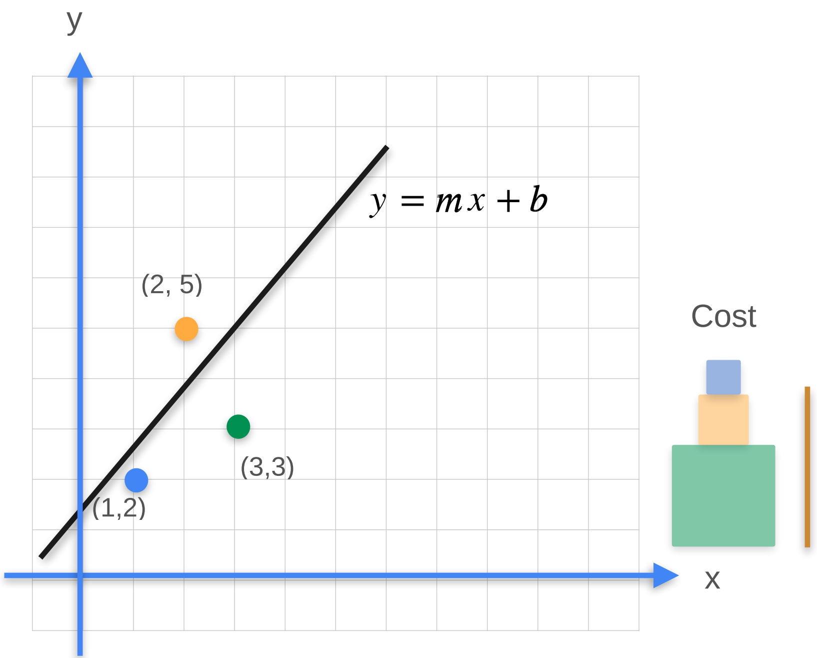


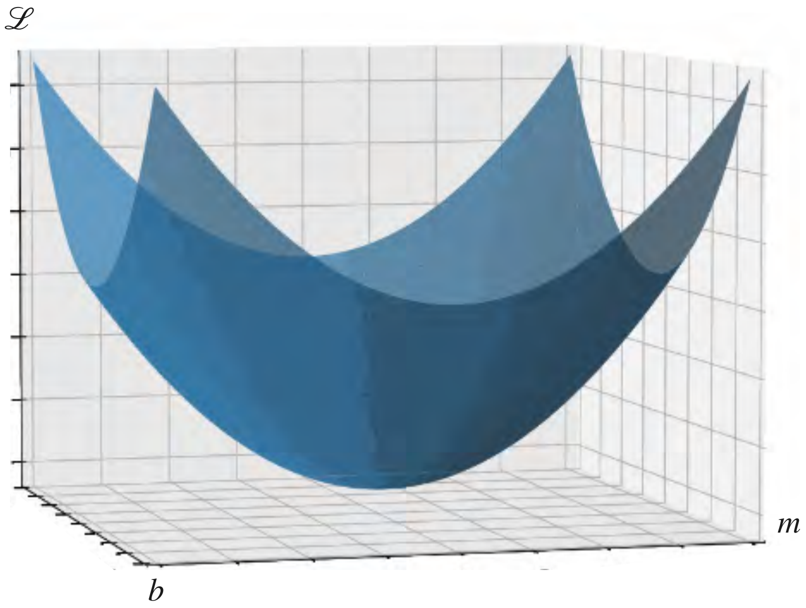
Gradient Descent



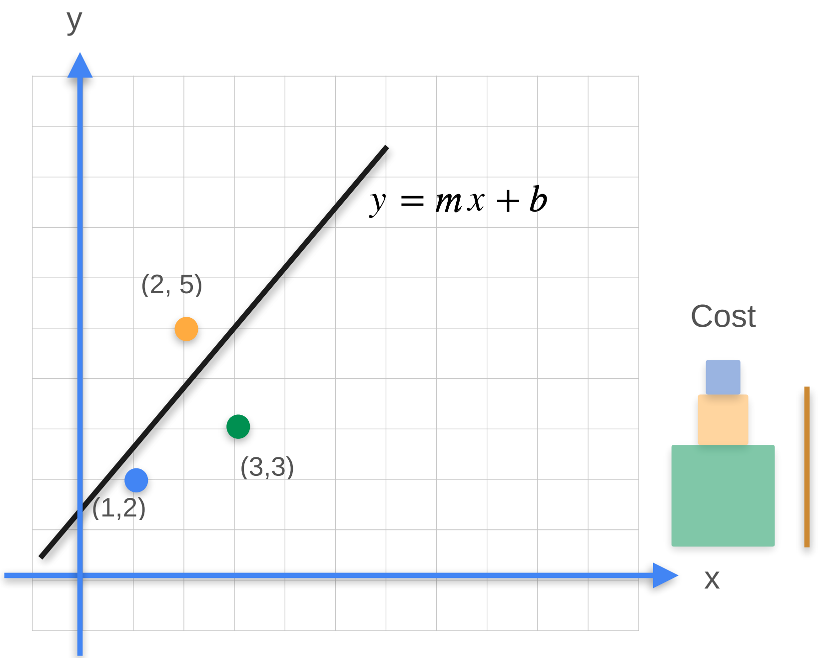


Gradient Descent

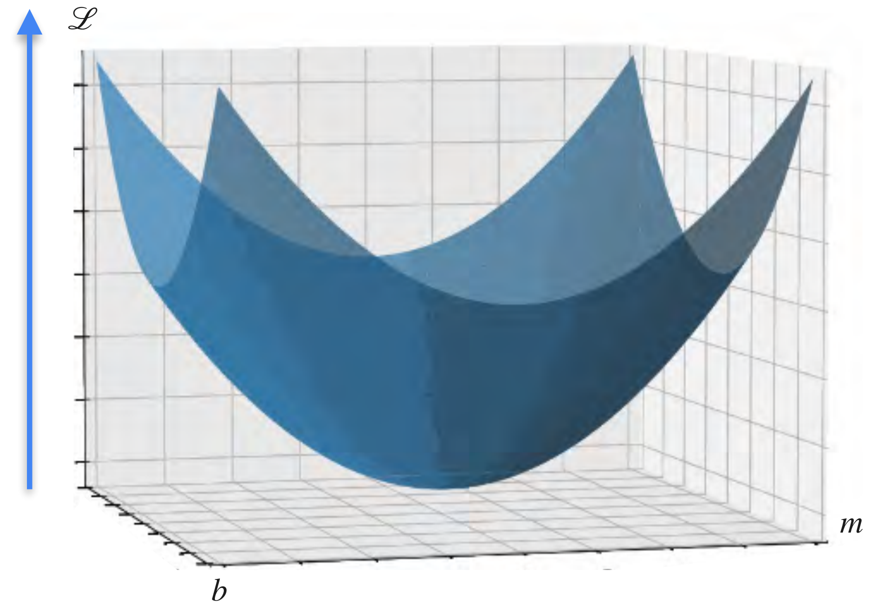




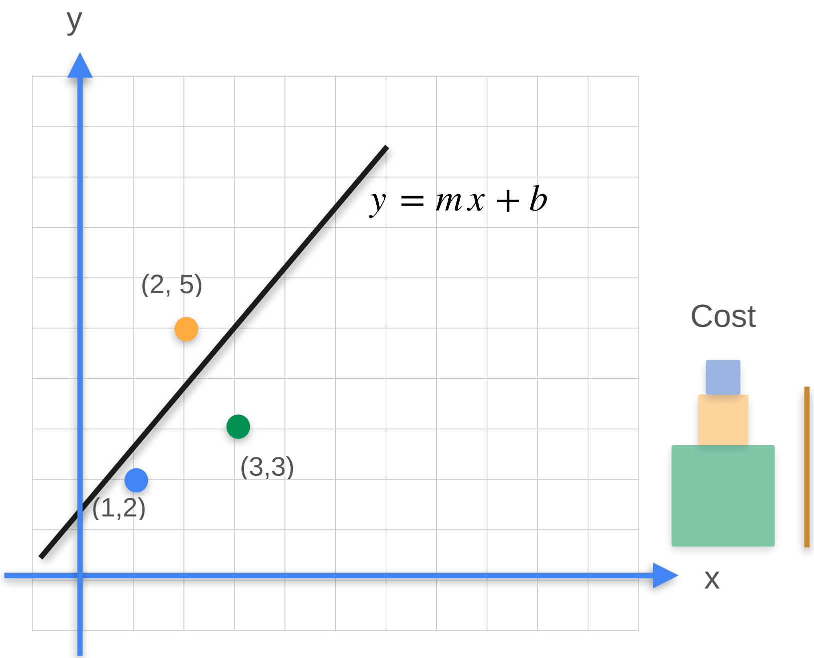
Gradient Descent



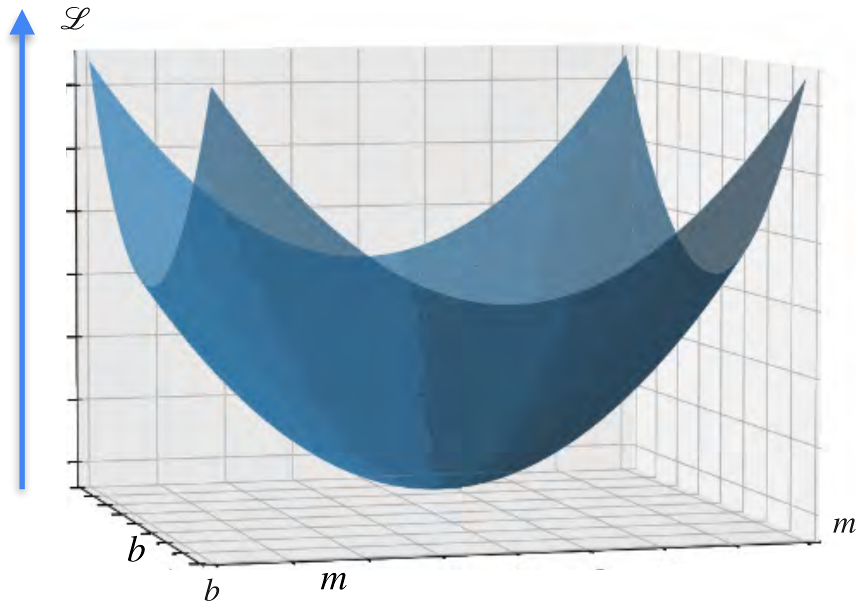
Square loss



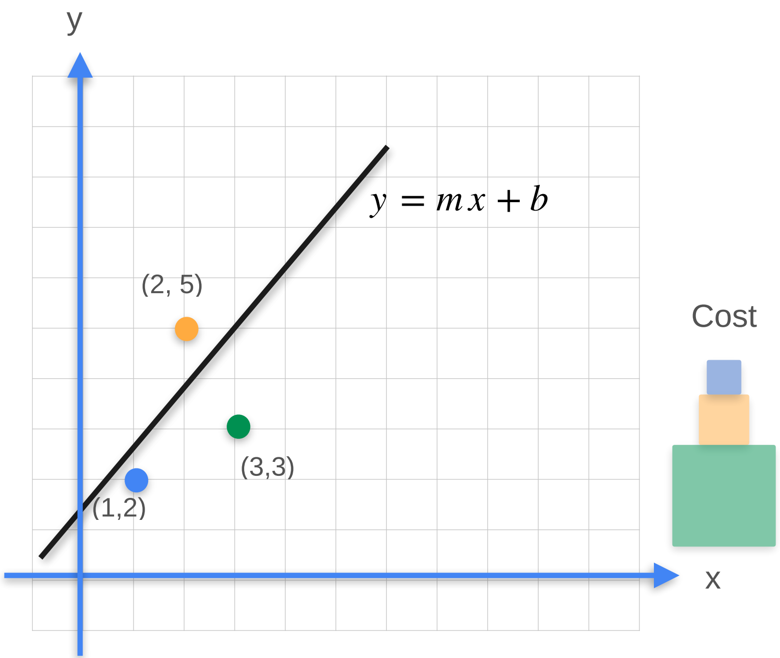
Gradient Descent

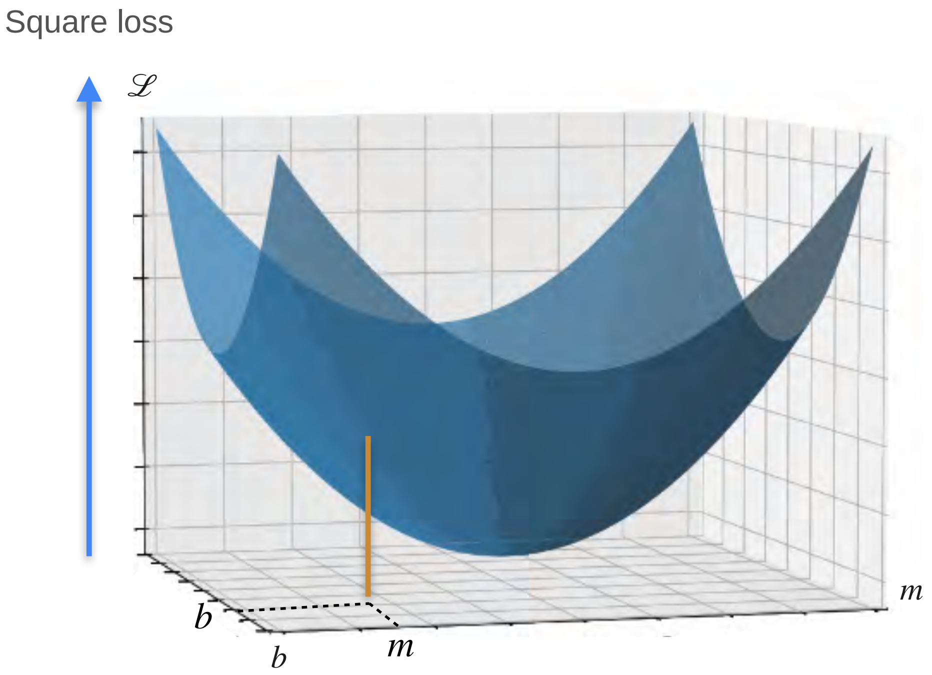


Square loss

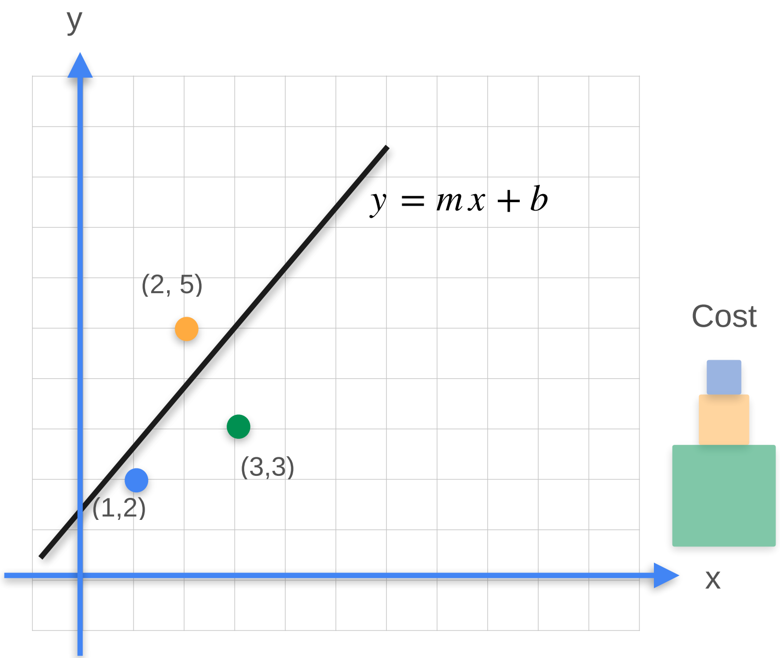


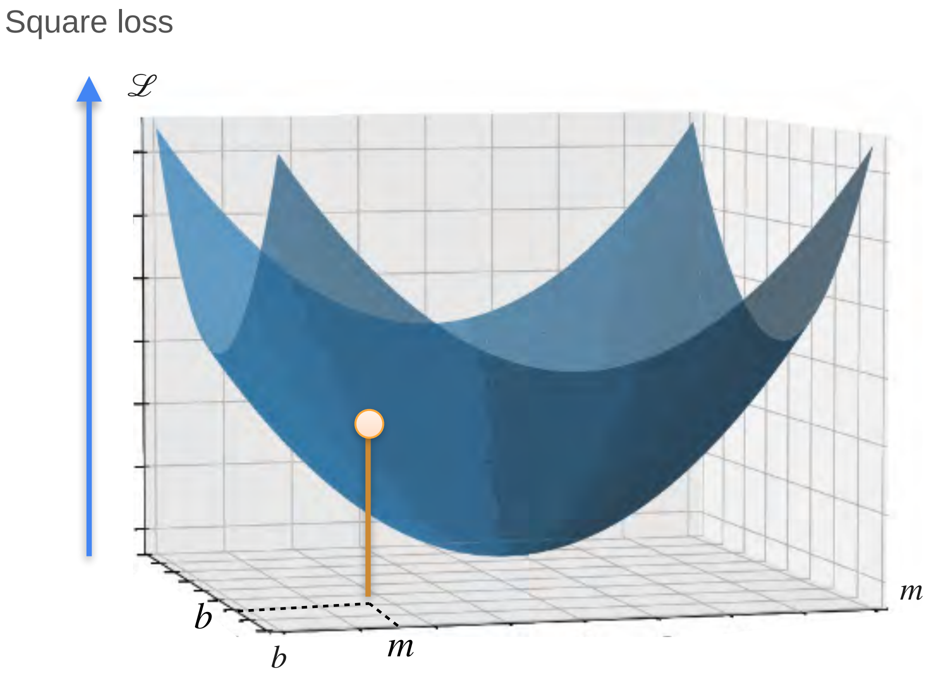
Gradient Descent



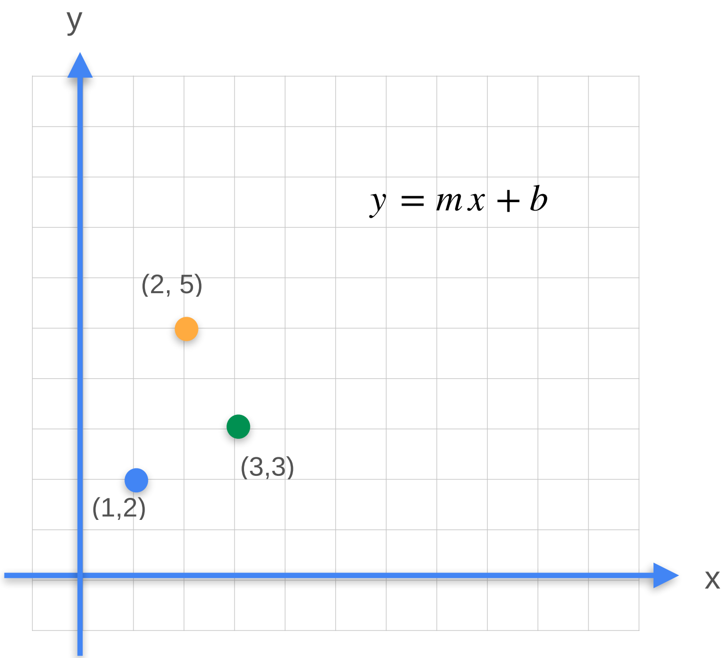


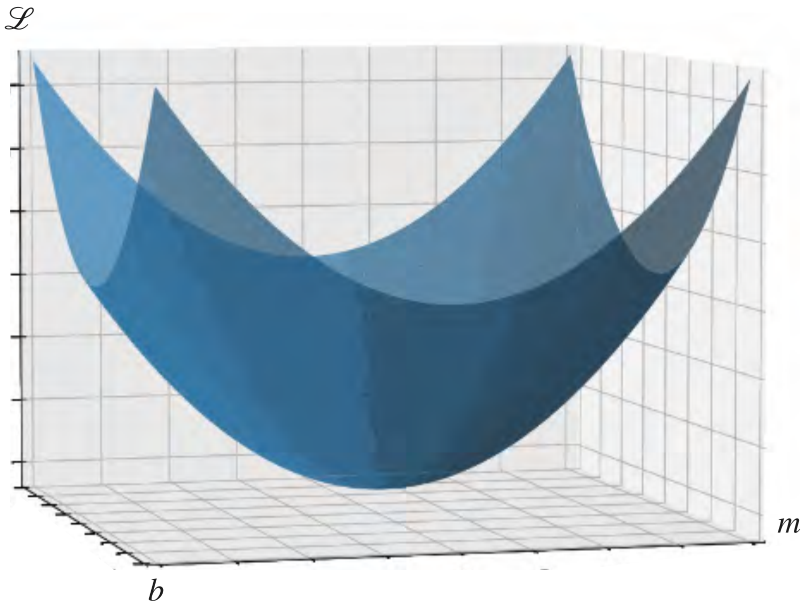
Gradient Descent



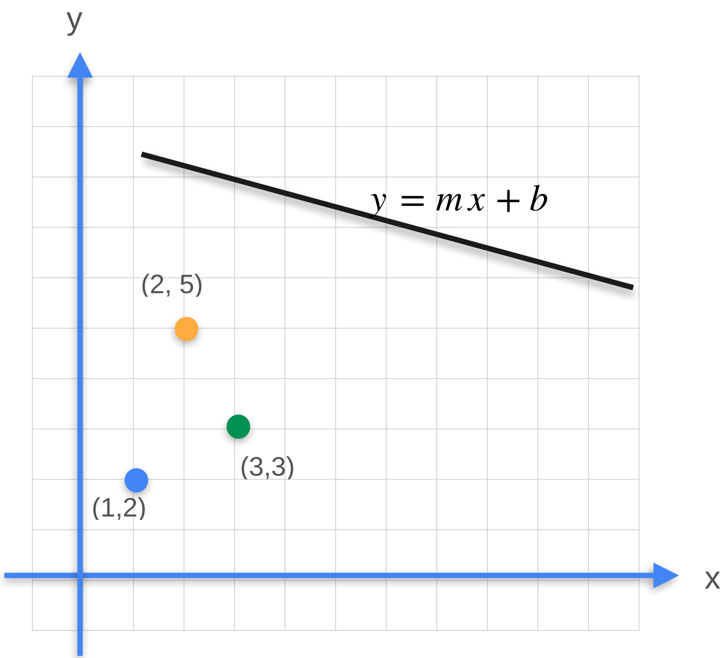


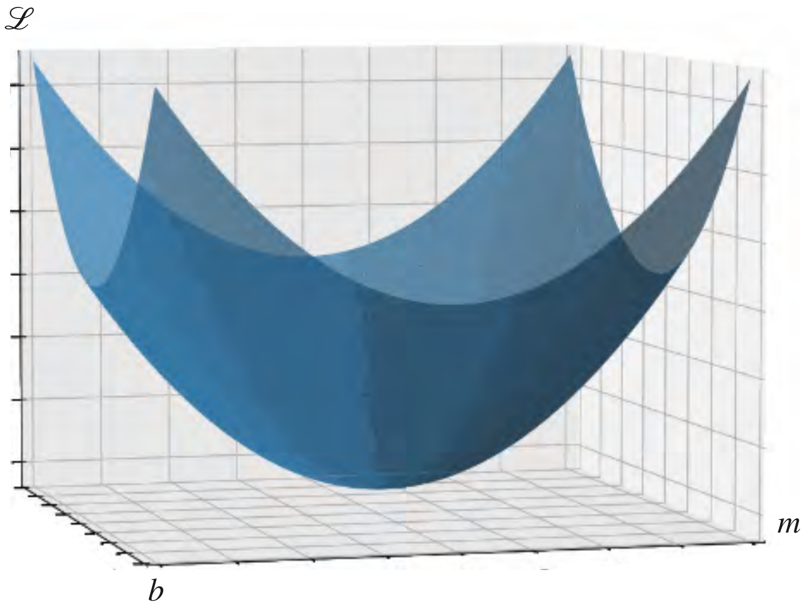
Gradient Descent



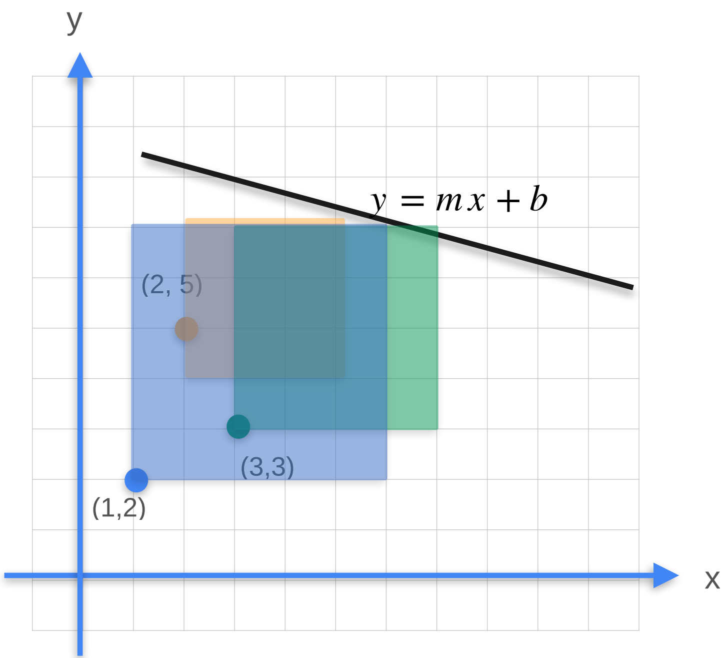


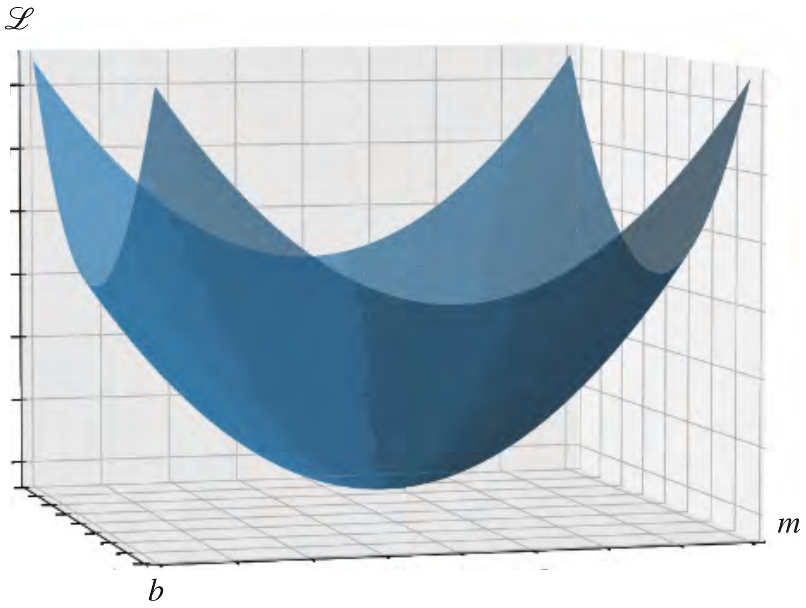
Gradient Descent



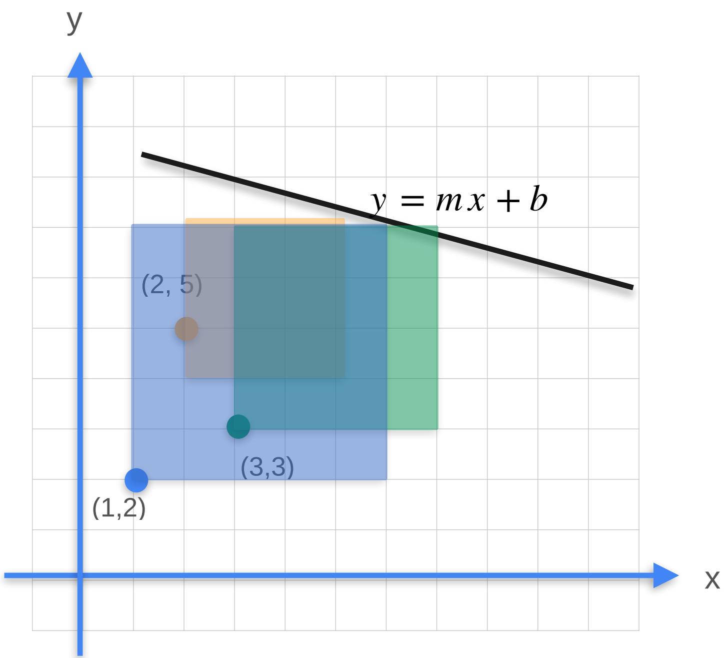


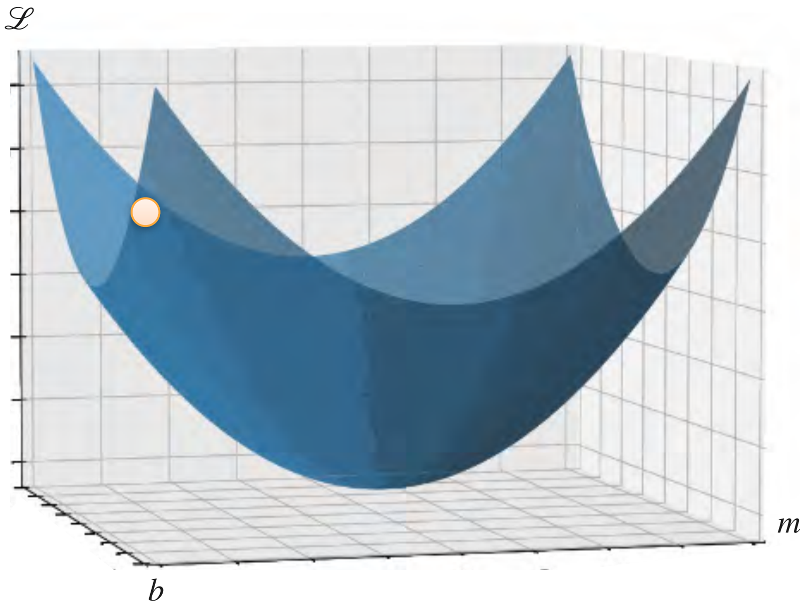
Gradient Descent



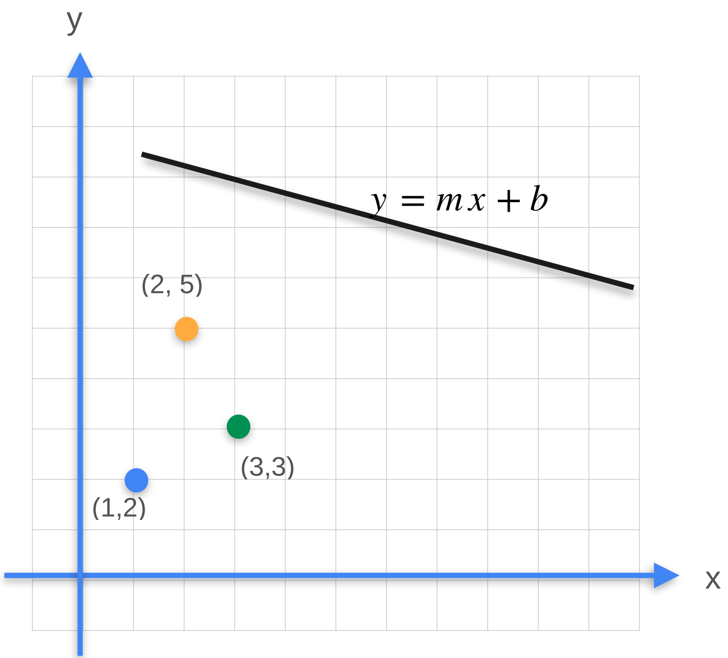


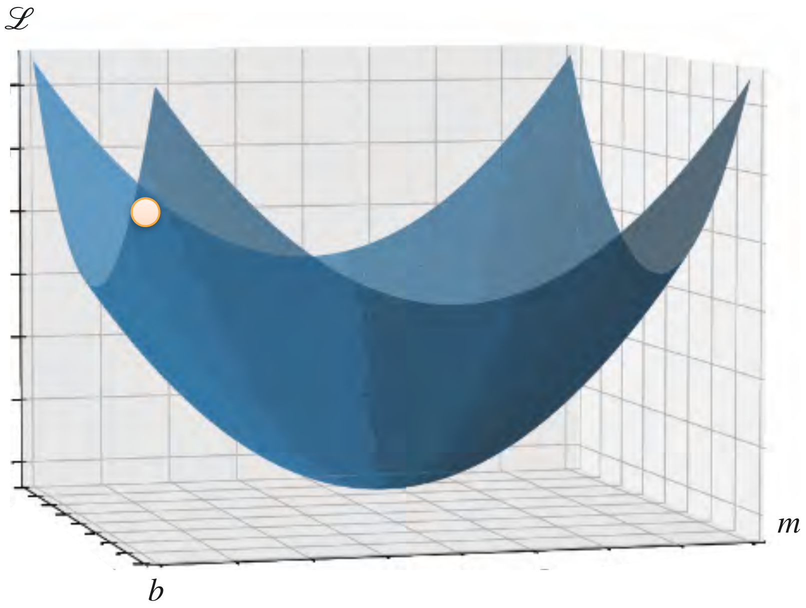
Gradient Descent



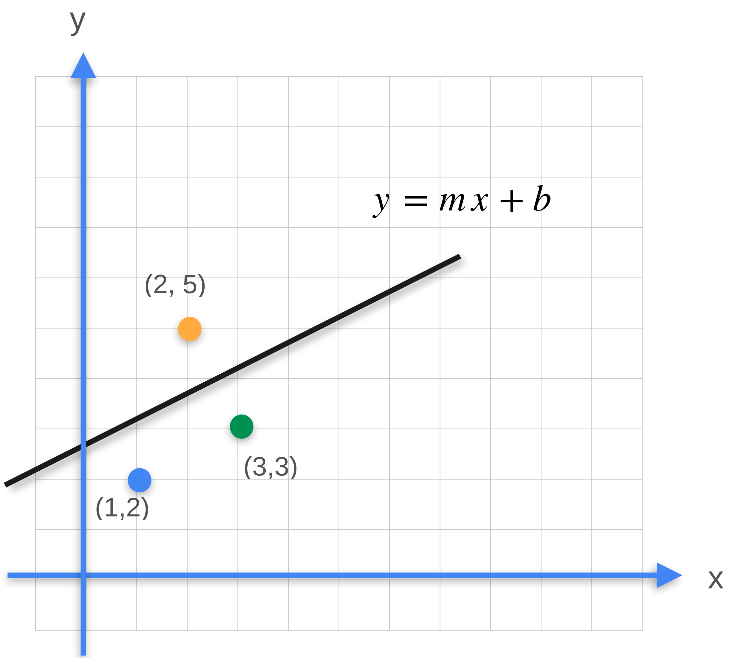


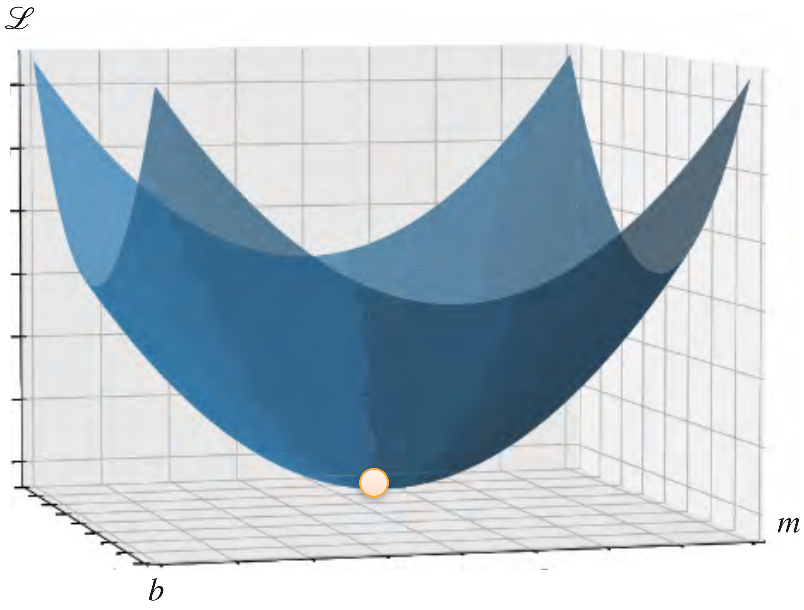
Gradient Descent





Gradient Descent





Another Example

Another Example

Another Example

TV advertisement
budget

Another Example

TV advertisement
budget

Another Example

TV advertisement
budget

Number of sales

Another Example

Another Example

TV budget	Sales

Another Example

TV budget	Sales
230.1	22.1

Another Example

TV budget	Sales
230.1	22.1
44.5	10.4

Another Example

TV budget	Sales
230.1	22.1
44.5	10.4
17.2	9.3

Another Example

TV budget	Sales
230.1	22.1
44.5	10.4
17.2	9.3

Goal: Predict sales in terms of TV budget

Another Example

TV budget	Sales
230.1	22.1
44.5	10.4
17.2	9.3

Goal: Predict sales in terms of TV budget

Tool: Linear regression

Another Example

TV budget	Sales
230.1	22.1
44.5	10.4
17.2	9.3

Goal: Predict sales in terms of TV budget

Tool: Linear regression

$$y = mx + b$$

Another Example

TV budget	Sales
230.1	22.1
44.5	10.4
17.2	9.3

Goal: Predict sales in terms of TV budget

Tool: Linear regression

$$y = mx + b$$

Another Example

TV budget	Sales
230.1	22.1
44.5	10.4
17.2	9.3

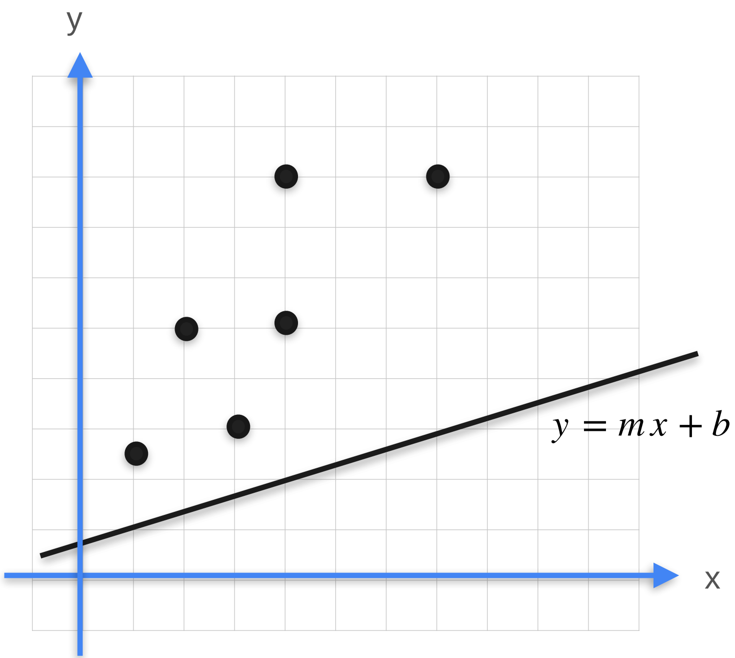
Multiple observations

Goal: Predict sales in terms of TV budget

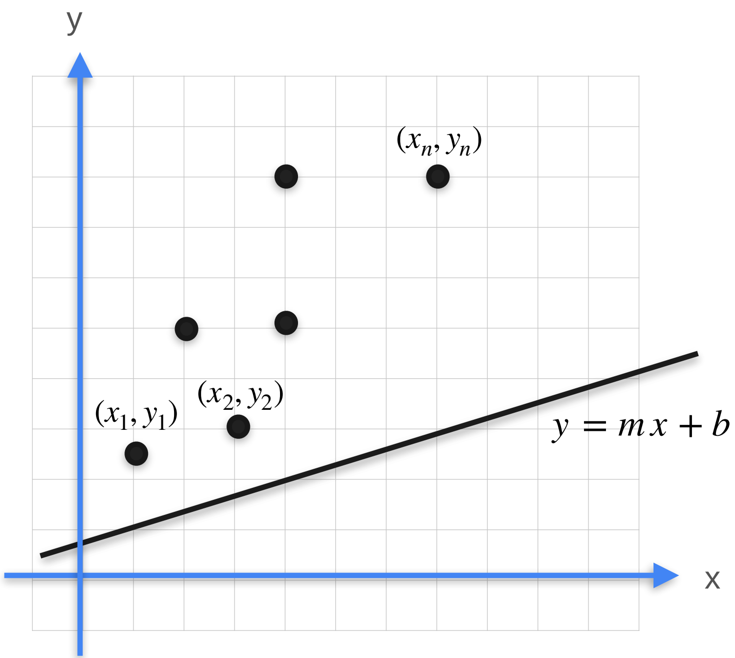
Tool: Linear regression

$$y = mx + b$$

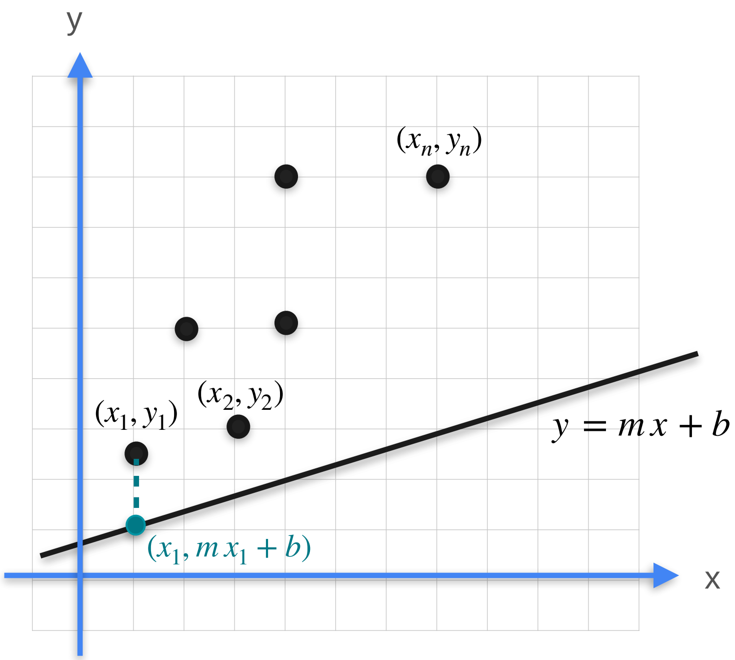
Gradient Descent



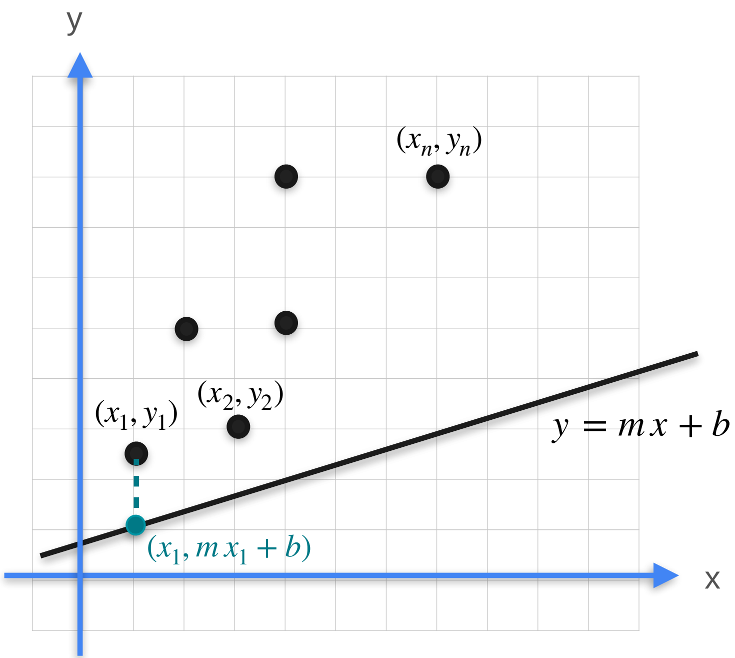
Gradient Descent



Gradient Descent



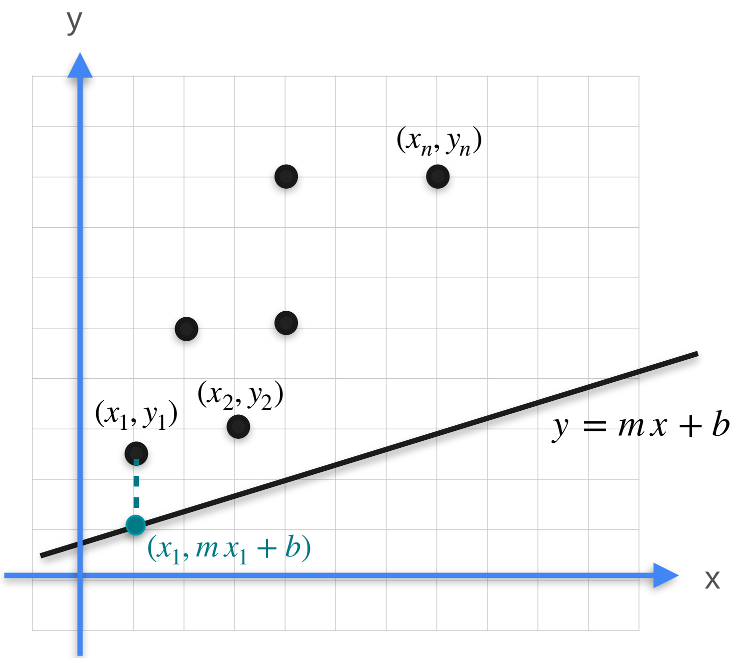
Gradient Descent



Loss

$$mx_1 + b - y_1$$

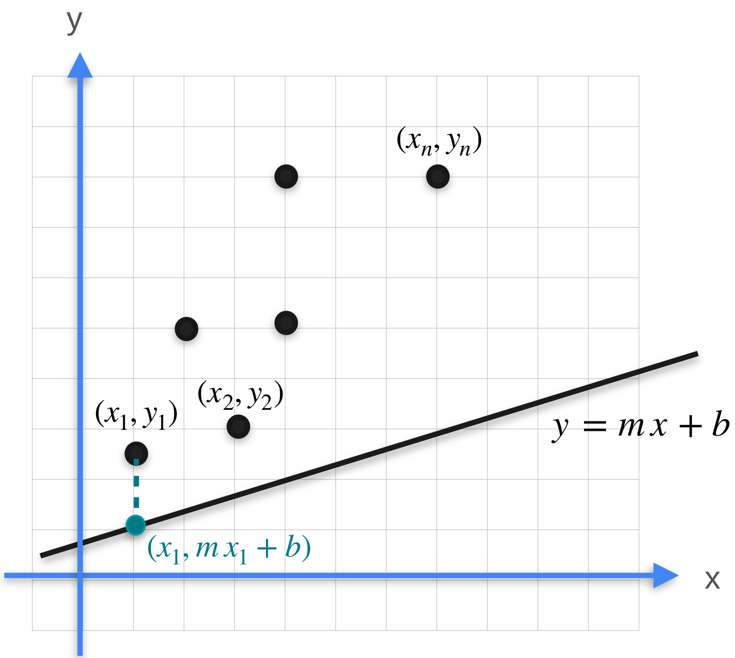
Gradient Descent



Loss

$$(mx_1 + b - y_1)^2$$

Gradient Descent



Cost

Loss

$$\underbrace{\mathscr{L}(m,b)}_{\text{Cost}} = \frac{1}{2m}\left[\underbrace{(mx_1 + b - y_1)^2}_{\text{Loss}} + \cdots + (mx_n - y_n)^2\right]$$

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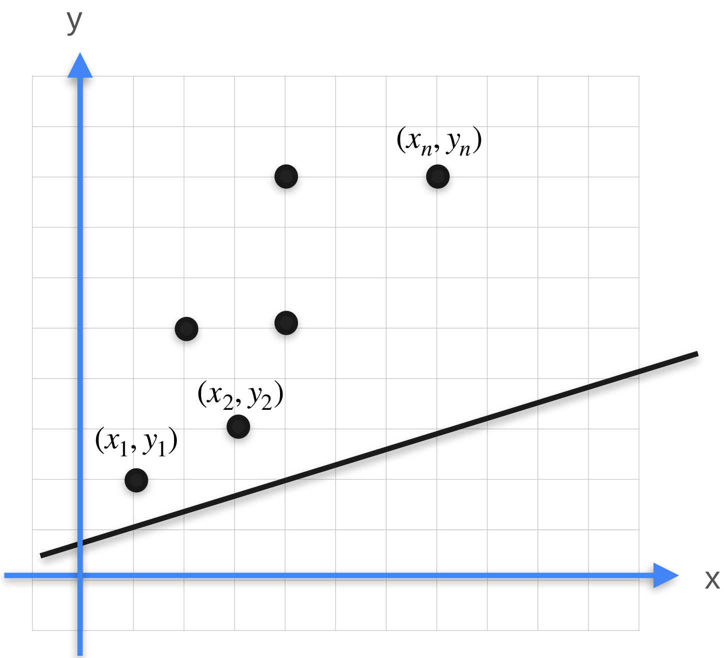
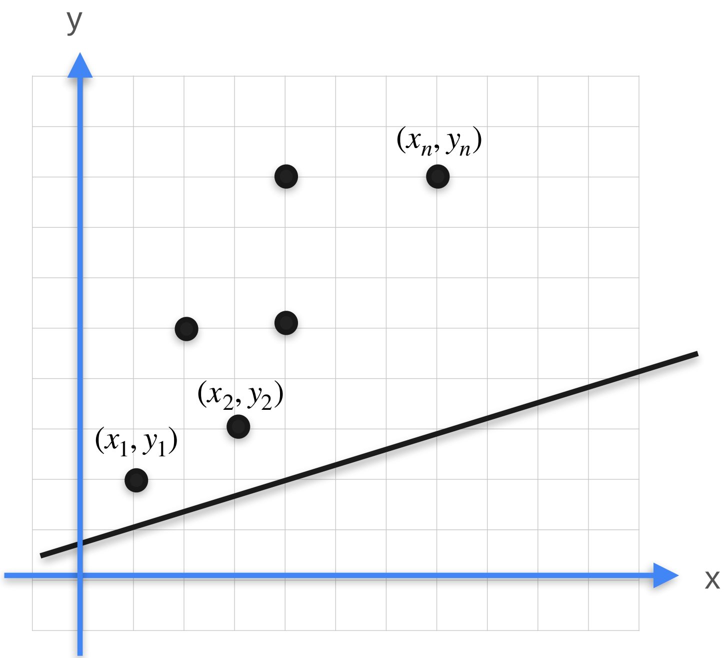
Gradient Descent



$$\mathscr{L}(m,b) = \frac{1}{2m}\left[(mx_1 + b - y_1)^2 + \cdots + (mx_n - y_n)^2\right]$$

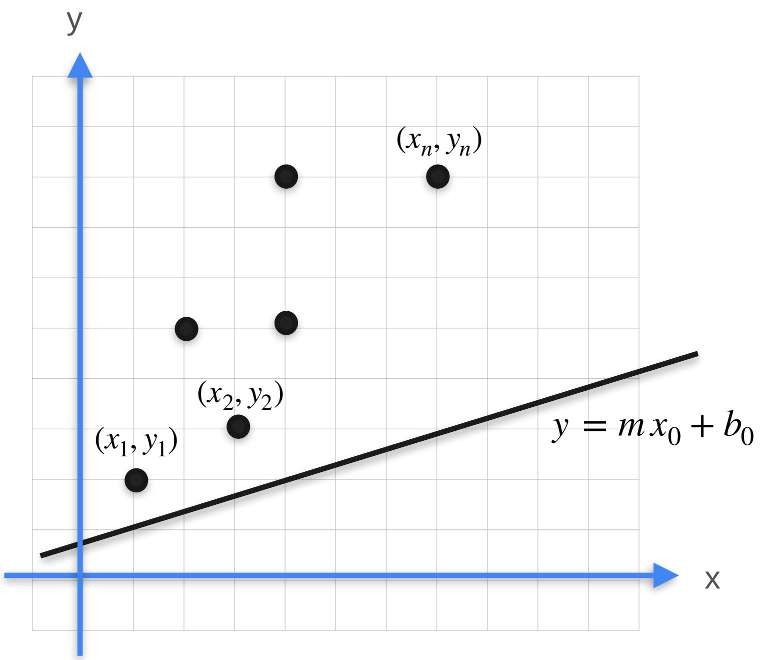
Gradient Descent



$$\mathscr{L}(m, b) = \frac{1}{2m}\left[(mx_1 + b - y_1)^2 + \cdots + (mx_n - y_n)^2\right]$$

$$\begin{bmatrix} m_0 \\ b_0 \end{bmatrix}$$

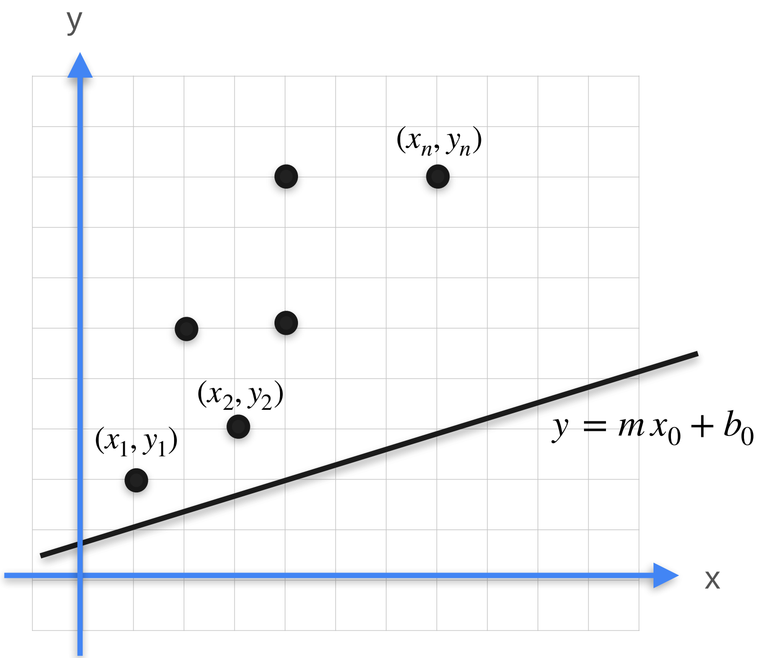
Gradient Descent



$$\mathscr{L}(m, b) = \frac{1}{2m}\left[(mx_1 + b - y_1)^2 + \cdots + (mx_n - y_n)^2\right]$$

$$\begin{bmatrix} m_0 \\ b_0 \end{bmatrix}$$

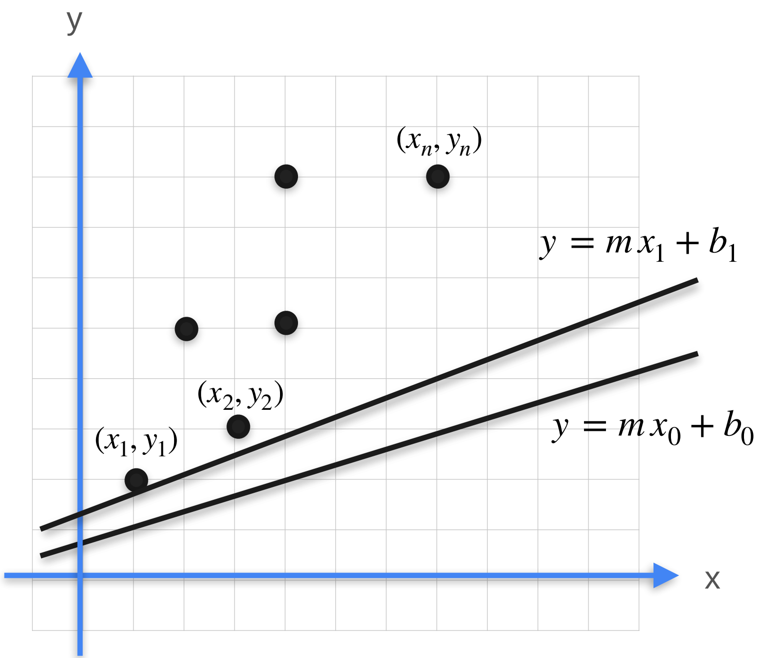
Gradient Descent



$$\mathscr{L}(m,b) = \frac{1}{2m}\left[(mx_1 + b - y_1)^2 + \cdots + (mx_n - y_n)^2\right]$$

$$\begin{bmatrix} m_0 \\ b_0 \end{bmatrix} \rightarrow \begin{bmatrix} m_1 \\ b_1 \end{bmatrix} = \begin{bmatrix} m_0 \\ b_0 \end{bmatrix} - \alpha \nabla \mathscr{L}_1(m_0, b_0)$$

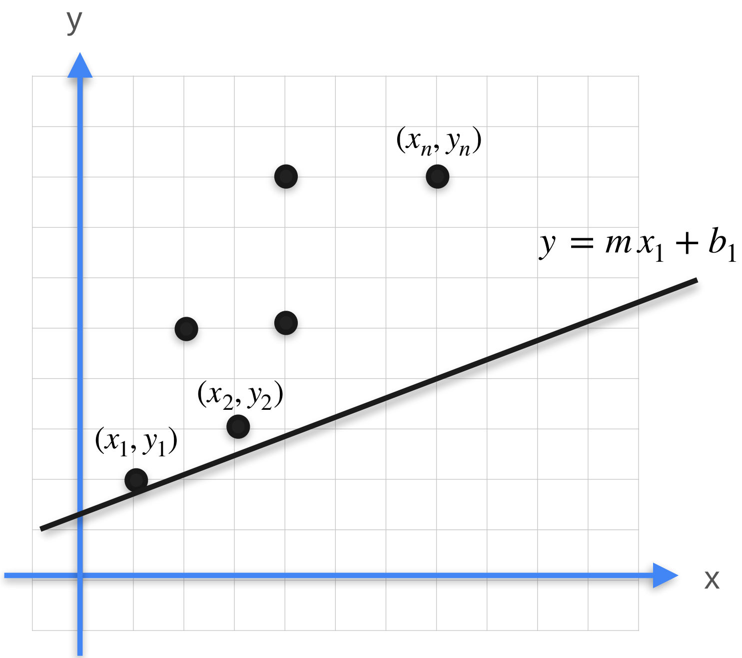
Gradient Descent



$$\mathscr{L}(m, b) = \frac{1}{2m}\left[(mx_1 + b - y_1)^2 + \cdots + (mx_n - y_n)^2\right]$$

$$\begin{bmatrix} m_0 \\ b_0 \end{bmatrix} \rightarrow \begin{bmatrix} m_1 \\ b_1 \end{bmatrix} = \begin{bmatrix} m_0 \\ b_0 \end{bmatrix} - \alpha \nabla \mathscr{L}_1(m_0, b_0)$$

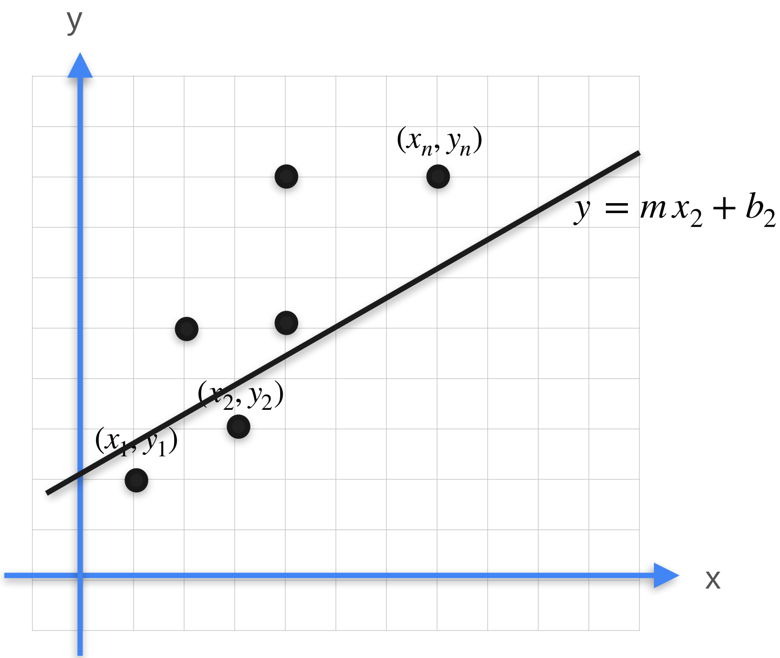
Gradient Descent



$$\mathcal{L}(m, b) = \frac{1}{2m}\left[(mx_1 + b - y_1)^2 + \cdots + (mx_n - y_n)^2\right]$$

$$\begin{bmatrix} m_0 \\ b_0 \end{bmatrix} \rightarrow \begin{bmatrix} m_1 \\ b_1 \end{bmatrix} = \begin{bmatrix} m_0 \\ b_0 \end{bmatrix} - \alpha \nabla \mathcal{L}_1(m_0, b_0)$$

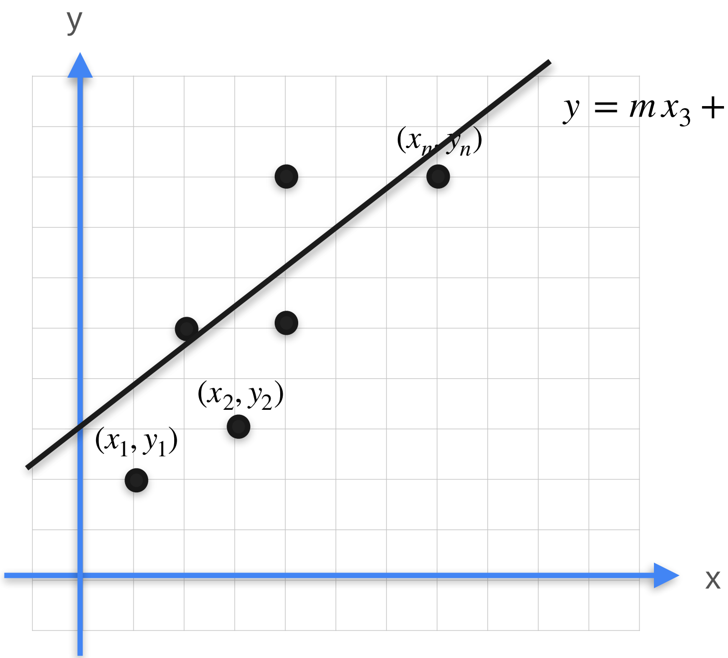
Gradient Descent



$$\mathcal{L}(m, b) = \frac{1}{2m}\left[(mx_1 + b - y_1)^2 + \cdots + (mx_n - y_n)^2\right]$$

$$\begin{bmatrix} m_1 \\ b_1 \end{bmatrix} \rightarrow \begin{bmatrix} m_2 \\ b_2 \end{bmatrix} = \begin{bmatrix} m_1 \\ b_1 \end{bmatrix} - \alpha \nabla \mathcal{L}_1(m_1, b_1)$$

Gradient Descent

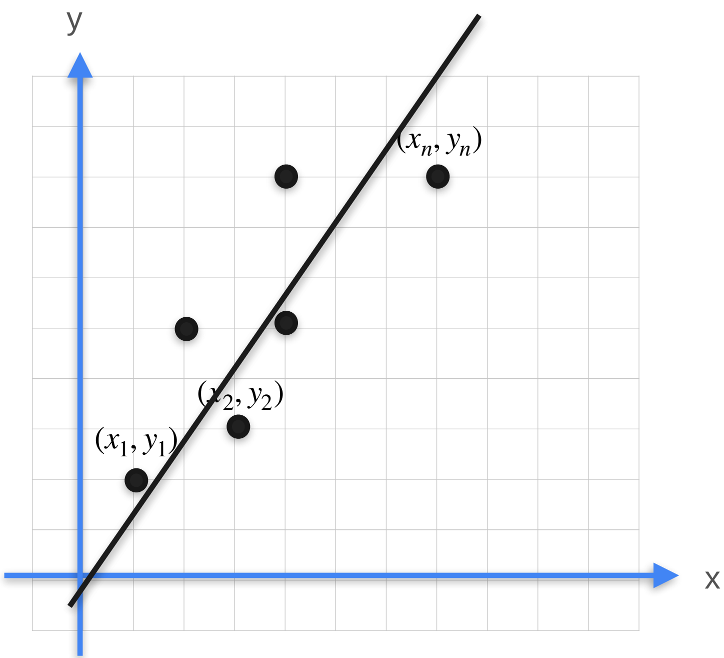


$$y = mx_3 + b_3$$

$$\mathscr{L}(m, b) = \frac{1}{2m}\left[(mx_1 + b - y_1)^2 + \cdots + (mx_n - y_n)^2\right]$$

$$\begin{bmatrix} m_2 \\ b_2 \end{bmatrix} \rightarrow \begin{bmatrix} m_3 \\ b_3 \end{bmatrix} = \begin{bmatrix} m_2 \\ b_2 \end{bmatrix} - \alpha \nabla \mathscr{L}_1(m_2, b_2)$$

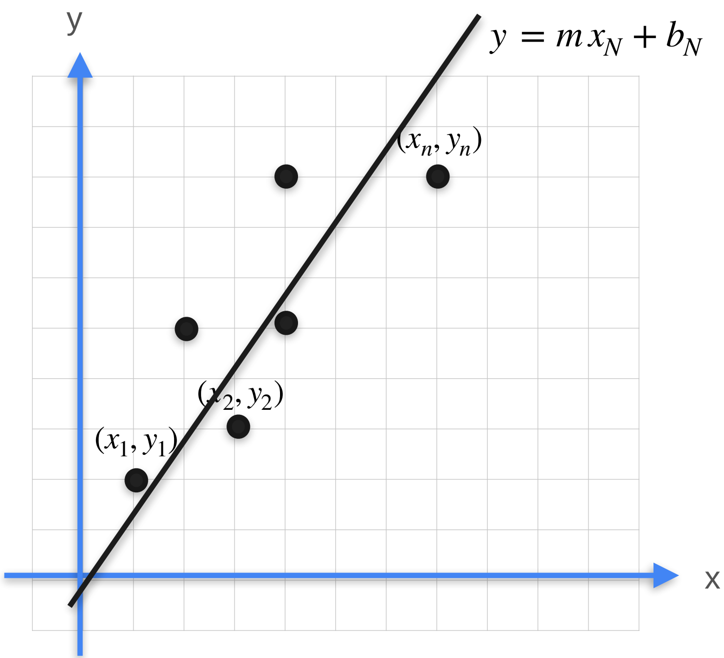
Gradient Descent



$$\mathscr{L}(m,b) = \frac{1}{2m}\left[(mx_1 + b - y_1)^2 + \cdots + (mx_n - y_n)^2\right]$$

$$\begin{bmatrix} m_N \\ b_N \end{bmatrix} \rightarrow \begin{bmatrix} m_N \\ b_N \end{bmatrix} = \begin{bmatrix} m_{N-1} \\ b_{N-1} \end{bmatrix} - \alpha \nabla \mathscr{L}_1(m_{N-1}, b_{N-1})$$

Gradient Descent



$$\mathscr{L}(m, b) = \frac{1}{2m}\left[(mx_1 + b - y_1)^2 + \cdots + (mx_n - y_n)^2\right]$$

$$\begin{bmatrix} m_N \\ b_N \end{bmatrix} \rightarrow \begin{bmatrix} m_N \\ b_N \end{bmatrix} = \begin{bmatrix} m_{N-1} \\ b_{N-1} \end{bmatrix} - \alpha \nabla \mathscr{L}_1(m_{N-1}, b_{N-1})$$

Gradients and Gradient Descent

Conclusion

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