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DeepLearning.AI

## Math for Machine Learning

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# Probability and Statistics - Week 4

# W4 Lesson 1



DeepLearning.AI

## Confidence Interval

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**Confidence Interval  
(Known Standard Deviation)**

# Confidence Interval - Intuition

# Confidence Interval - Intuition

Statistopia

10,000 people

# Confidence Interval - Intuition

Statistopia

10,000 people

Estimate  $\mu$

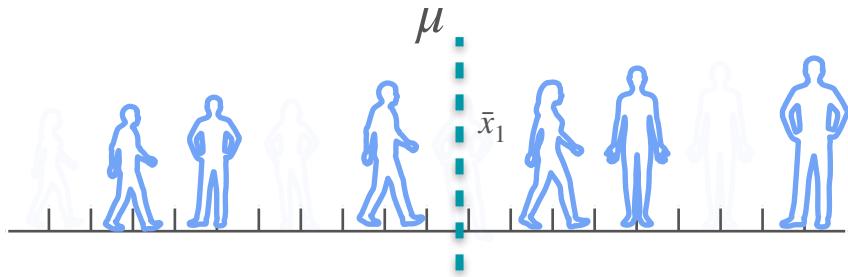
# Confidence Interval - Intuition

Statistopia

**10,000 people**

Estimate  $\mu$   
(mean height of the population)

# Confidence Interval - Intuition

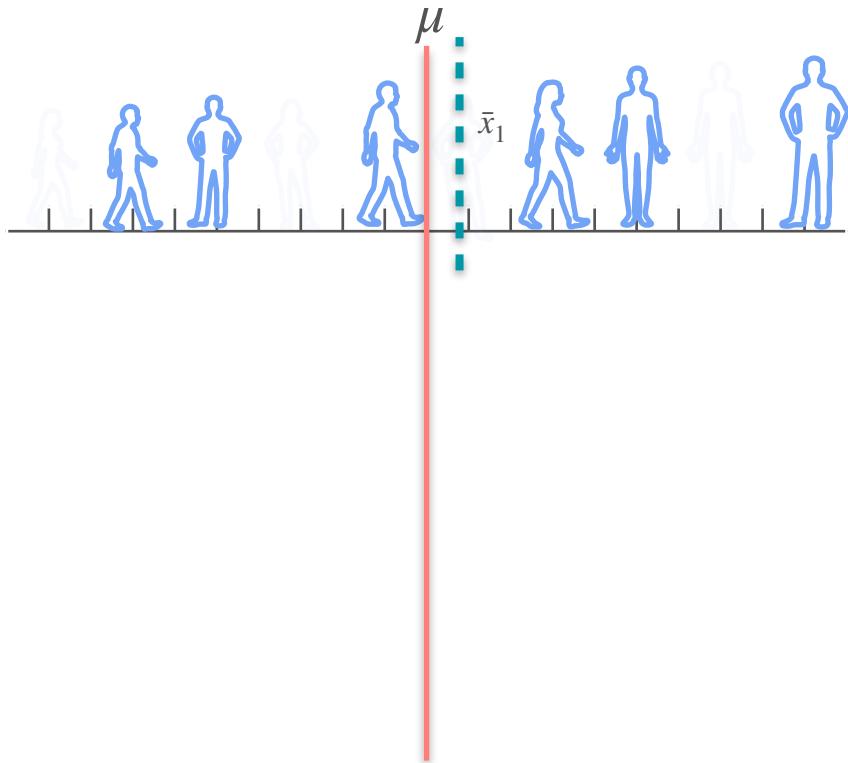


Statistopia

10,000 people

Estimate  $\mu$   
(mean height of the population)

# Confidence Interval - Intuition

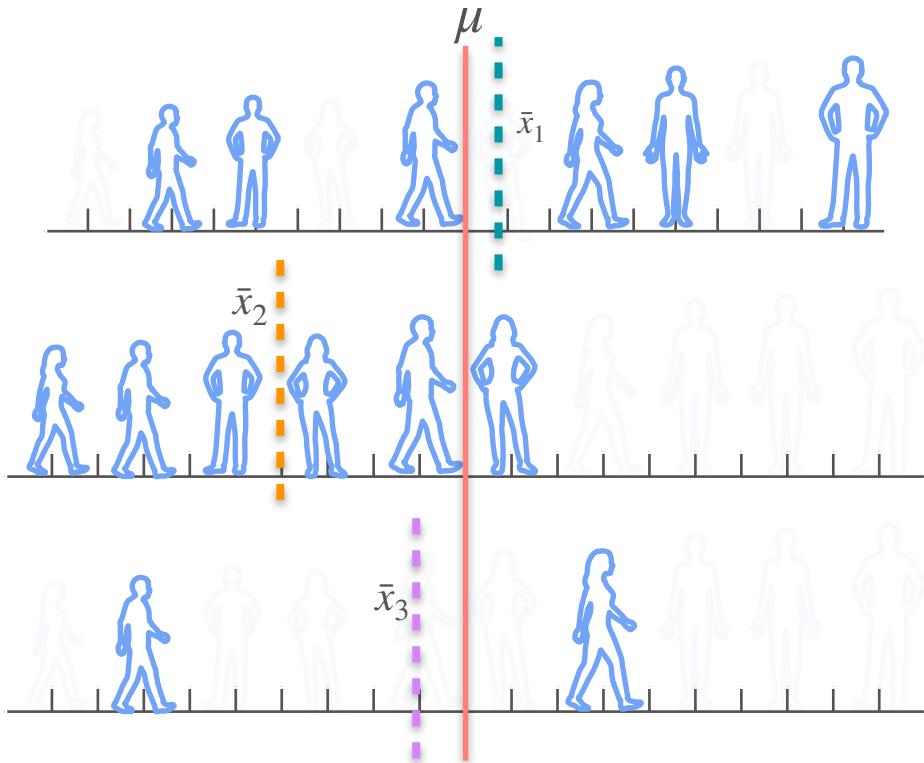


Statistopia

**10,000 people**

Estimate  $\mu$   
(mean height of the population)

# Confidence Interval - Intuition

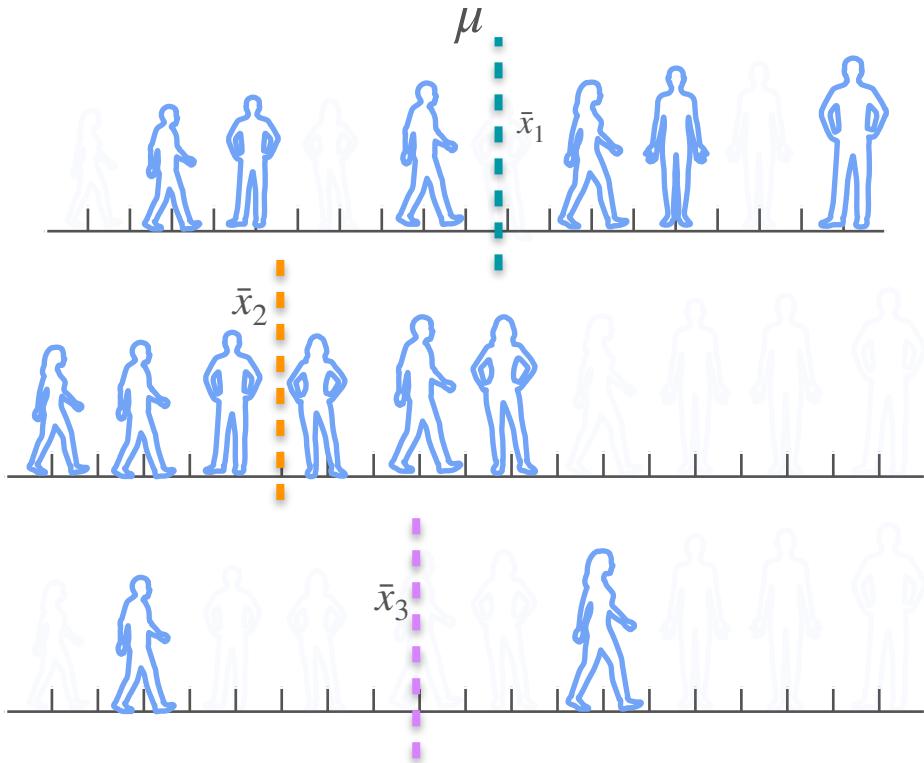


Statistopia

**10,000 people**

Estimate  $\mu$   
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# Confidence Interval - Intuition

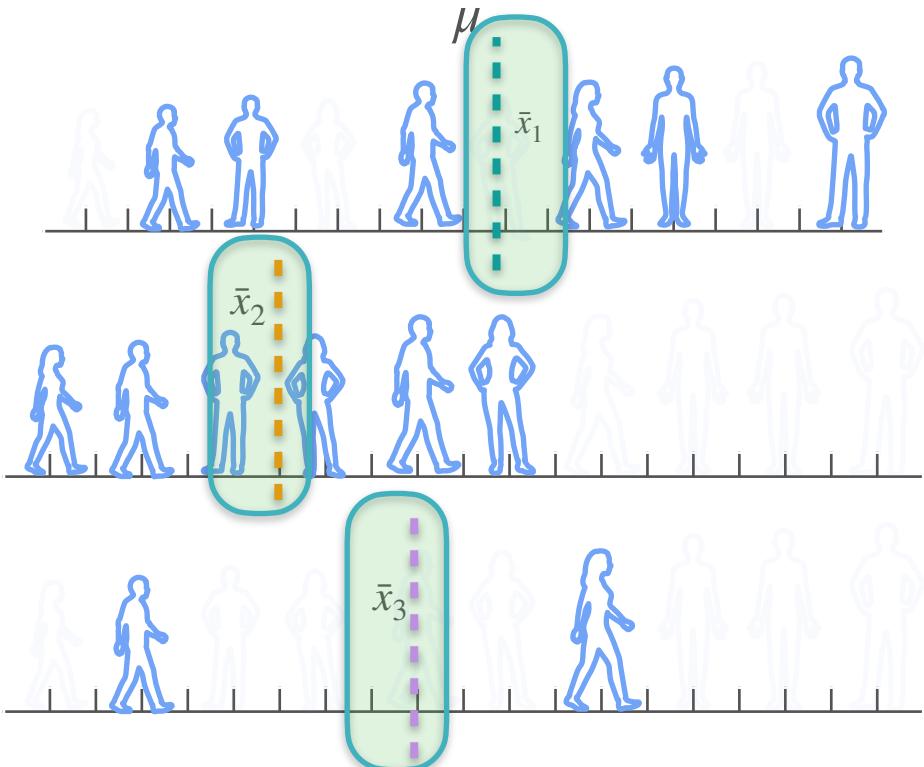


Statistopia

**10,000 people**

Estimate  $\mu$   
(mean height of the population)

# Confidence Interval - Intuition



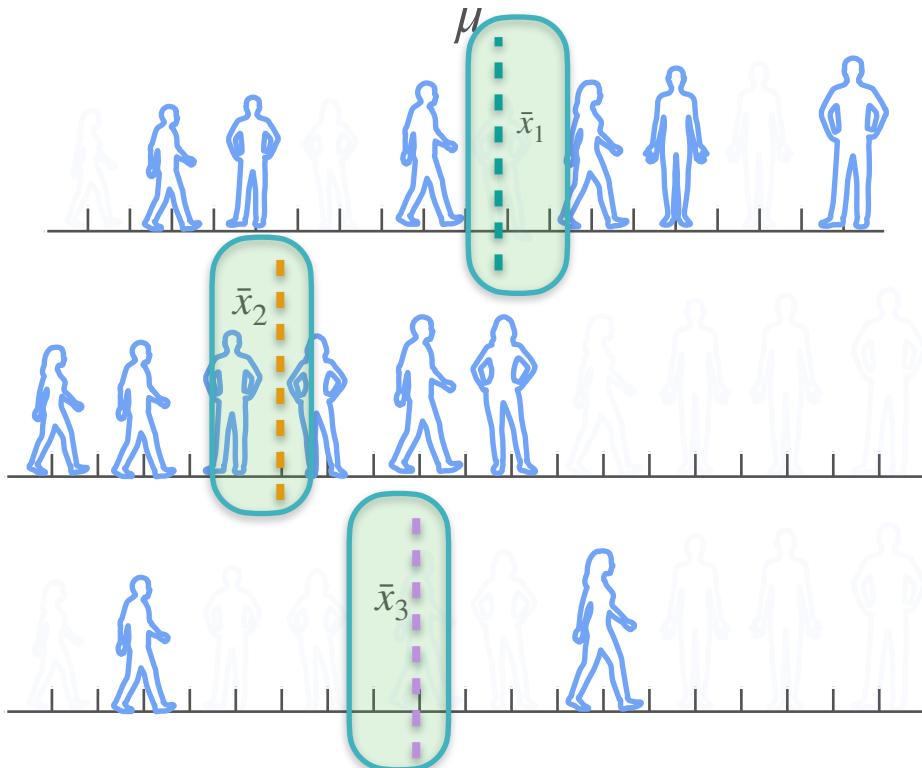
Statistopia

10,000 people

Estimate  $\mu$   
(mean height of the population)

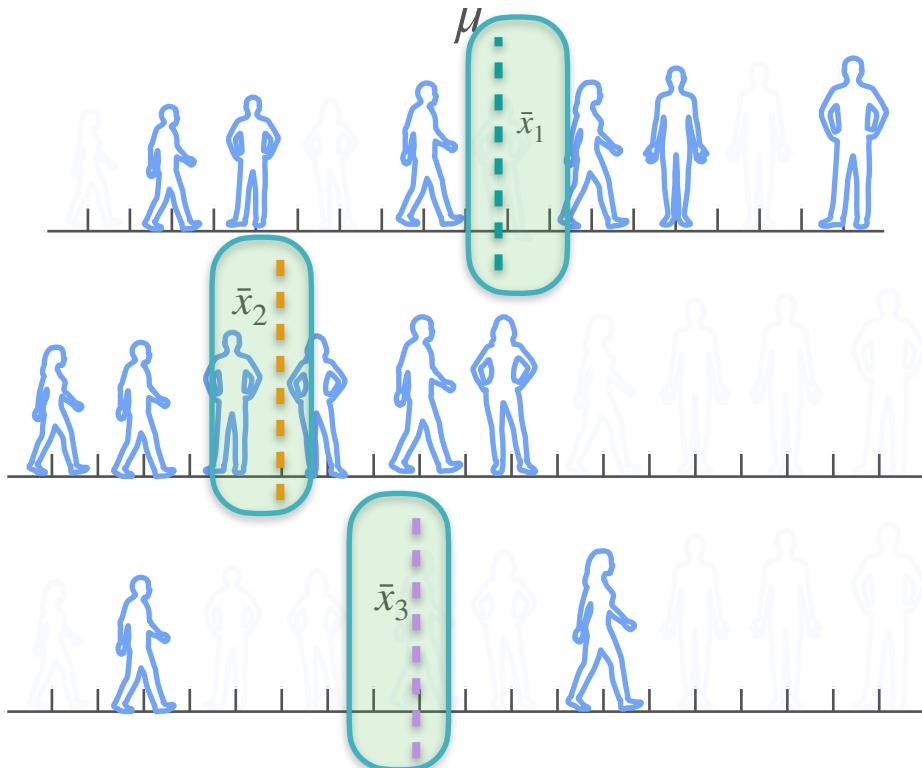
Can you use these sample means with  
some degree of certainty?

# Confidence Interval - Intuition



Can you use these sample means with some degree of certainty?

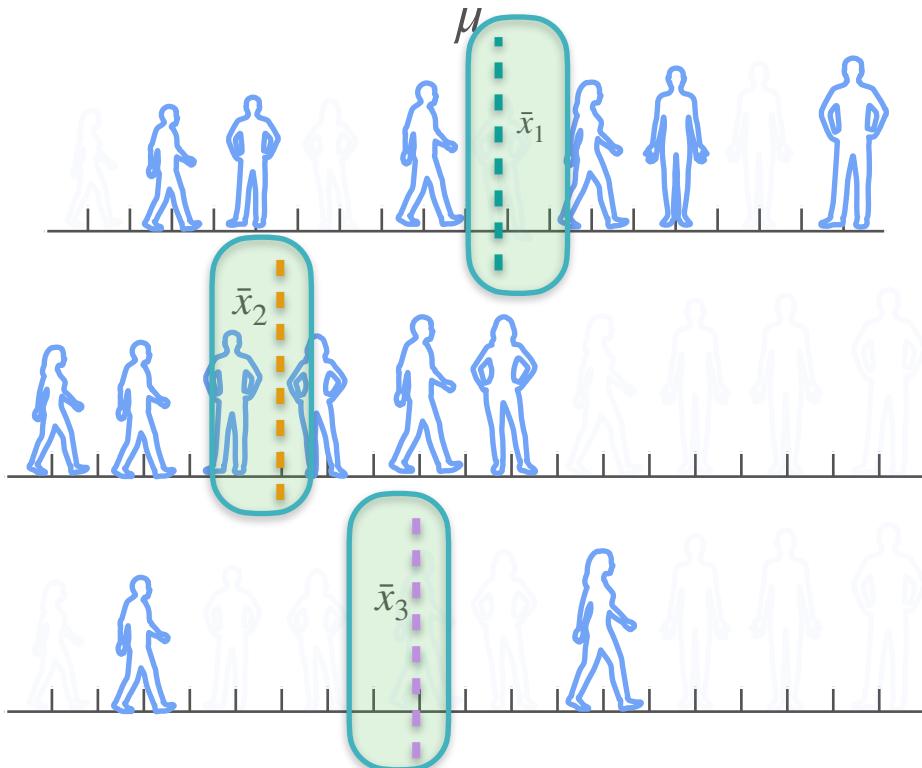
# Confidence Interval - Intuition



Can you use these sample means with some degree of certainty?

**Confidence  
Interval**

# Confidence Interval - Intuition



Can you use these sample means with some degree of certainty?

## Confidence Interval

$$\text{lower limit} < \bar{x} < \text{upper limit}$$

# Confidence Interval - Intuition

# Confidence Interval - Intuition

$$n = 1$$



# Confidence Interval - Intuition

$$n = 1$$



$$\bar{x}$$

# Confidence Interval - Intuition

$n = 1$



$\bar{x}$

**Central Limit Theorem**

# Confidence Interval - Intuition

$$n = 1$$

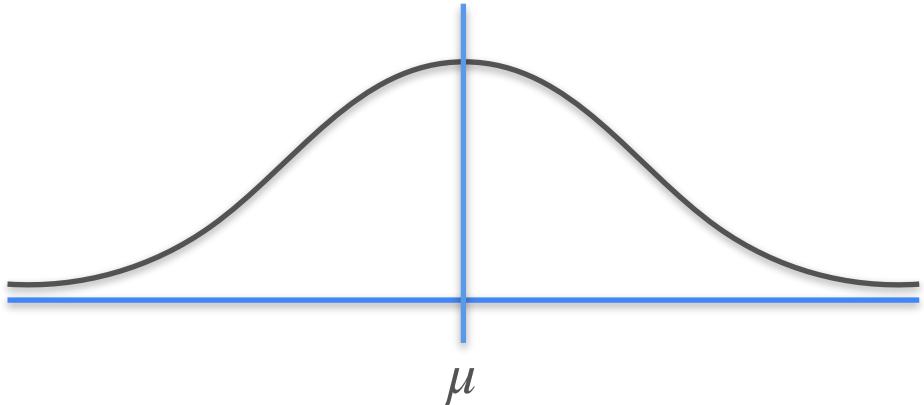


$$\bar{x}$$

**Central Limit Theorem**

population standard deviation ( $\sigma$ )

$$\bar{X} \sim \mathcal{N}(\mu, \sigma^2)$$



# Confidence Interval - Intuition

$$n = 1$$

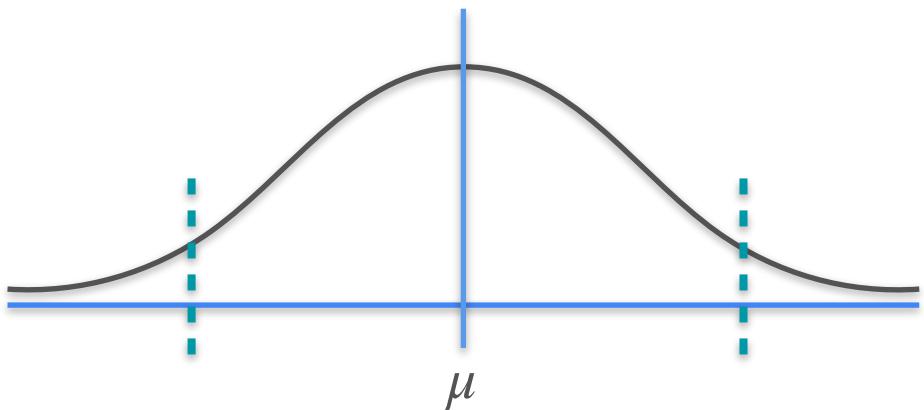


$$\bar{x}$$

**Central Limit Theorem**

population standard deviation ( $\sigma$ )

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# Confidence Interval - Intuition

$$n = 1$$

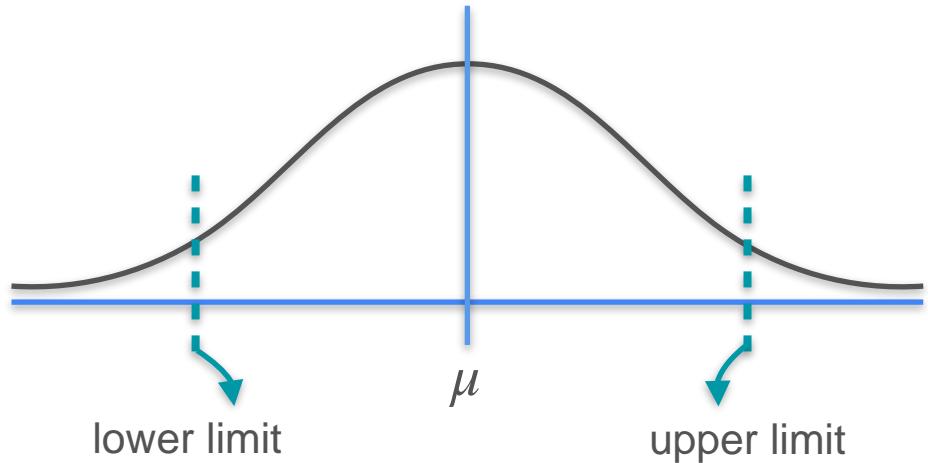


$$\bar{x}$$

**Central Limit Theorem**

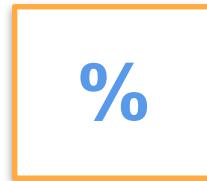
population standard deviation ( $\sigma$ )

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# Confidence Interval - Intuition

$$n = 1$$

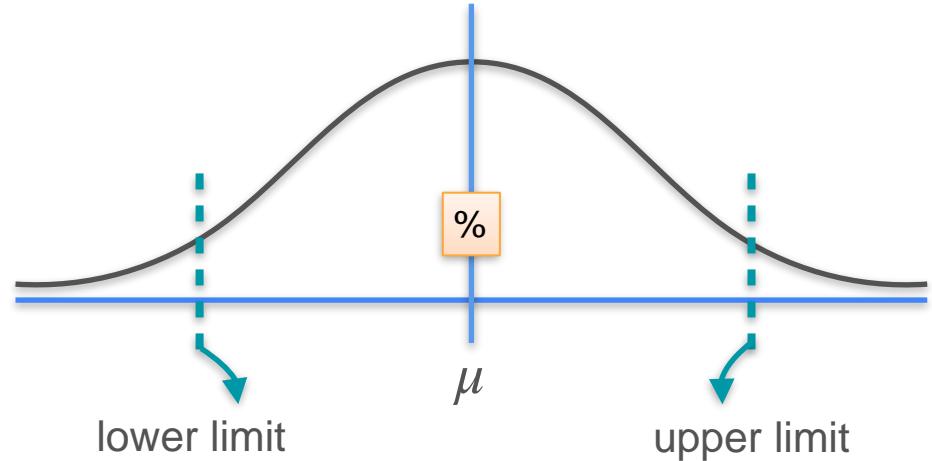


$$\bar{x}$$

Central Limit Theorem

population standard deviation ( $\sigma$ )

$$\bar{X} \sim \mathcal{N}(\mu, \sigma^2)$$



# Confidence Interval - Intuition

$$n = 1$$



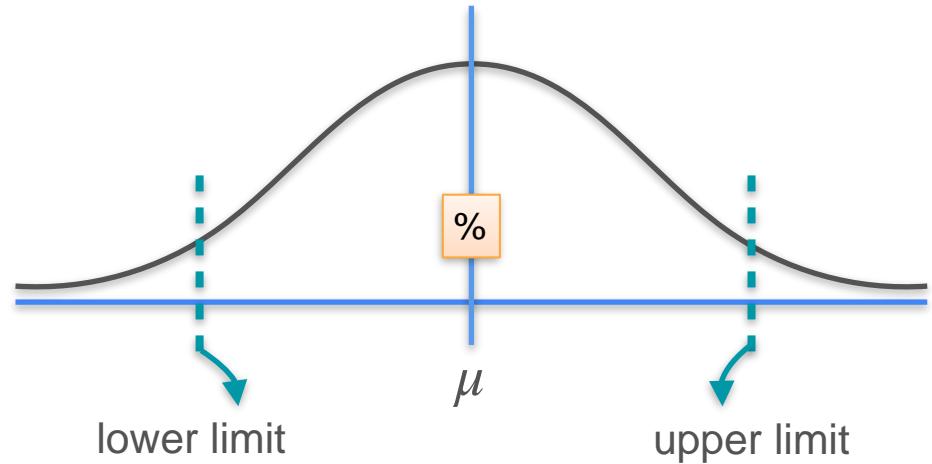
$$\bar{x}$$

$\alpha$   
significance level

**Central Limit Theorem**

population standard deviation ( $\sigma$ )

$$\bar{X} \sim \mathcal{N}(\mu, \sigma^2)$$



# Confidence Interval - Intuition

$n = 1$



$\bar{x}$

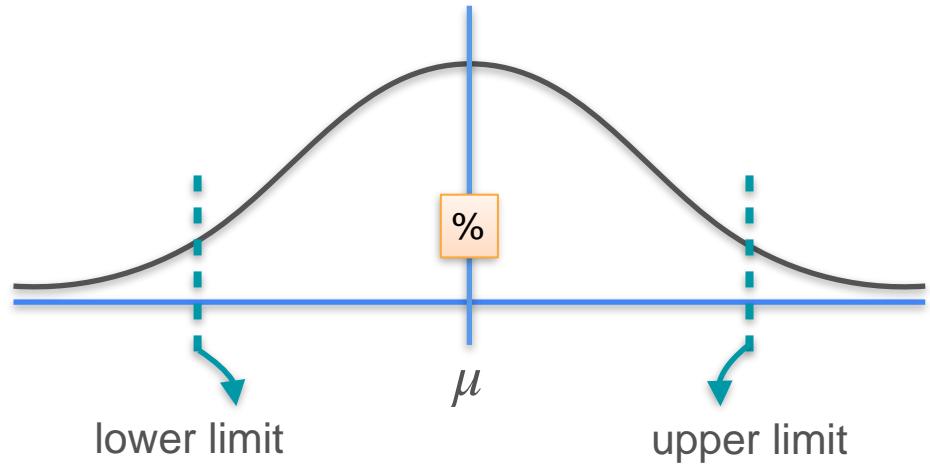
$1 - \alpha$

$\alpha$   
significance level

**Central Limit Theorem**

population standard deviation ( $\sigma$ )

$$\bar{X} \sim \mathcal{N}(\mu, \sigma^2)$$

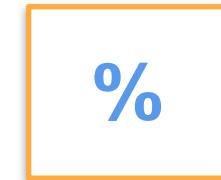


# Confidence Interval - Intuition

$n = 1$



Confidence level



$\bar{x}$

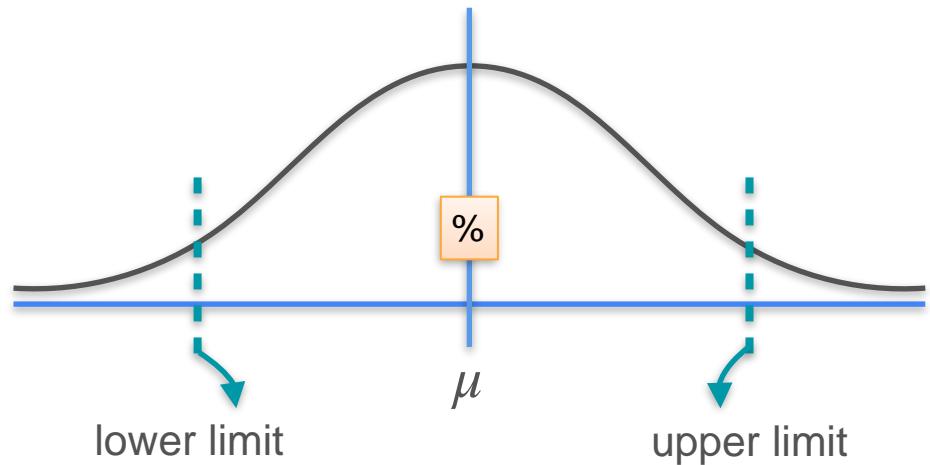
$1 - \alpha$

$\alpha$   
significance level

**Central Limit Theorem**

population standard deviation ( $\sigma$ )

$$\bar{X} \sim \mathcal{N}(\mu, \sigma^2)$$



# Confidence Interval - Intuition

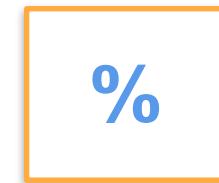
$n = 1$



$\bar{x}$

$\alpha$   
significance level

Confidence level

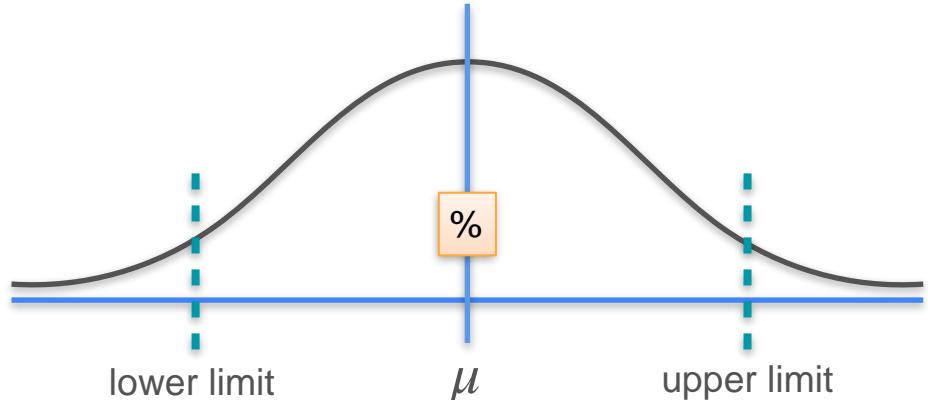


$1 - \alpha$

**Central Limit Theorem**

population standard deviation ( $\sigma$ )

$$\bar{X} \sim \mathcal{N}(\mu, \sigma^2)$$



# Confidence Interval - Intuition

$n = 1$



$\bar{x}$

$\alpha = 0.05$   
significance level

Confidence level

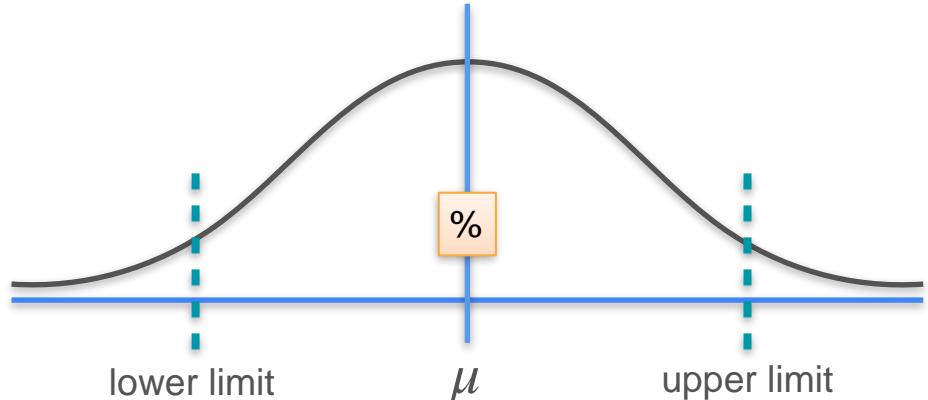


$1 - \alpha$

**Central Limit Theorem**

population standard deviation ( $\sigma$ )

$$\bar{X} \sim \mathcal{N}(\mu, \sigma^2)$$



# Confidence Interval - Intuition

$$n = 1$$



Confidence level



$$\bar{x}$$

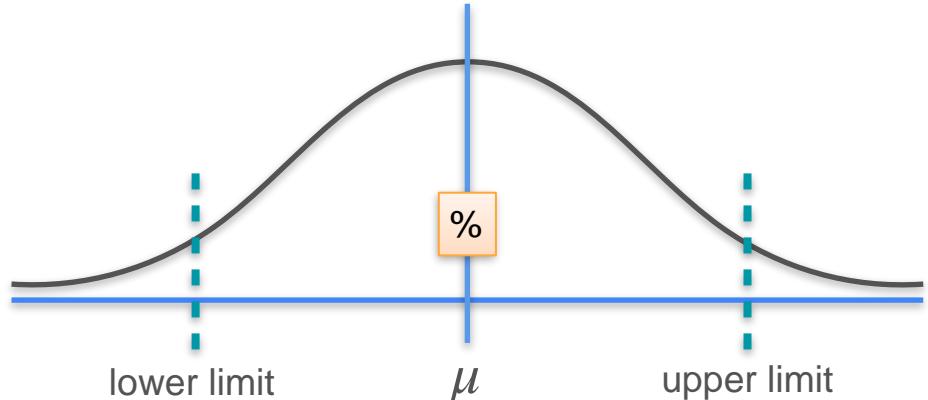
$$\alpha = 0.05 \text{ significance level}$$

$$1 - \alpha = 0.95$$

**Central Limit Theorem**

population standard deviation ( $\sigma$ )

$$\bar{X} \sim \mathcal{N}(\mu, \sigma^2)$$



# Confidence Interval - Intuition

$$n = 1$$



Confidence level

95%

$$\bar{x}$$

$$\alpha = 0.05$$

significance level

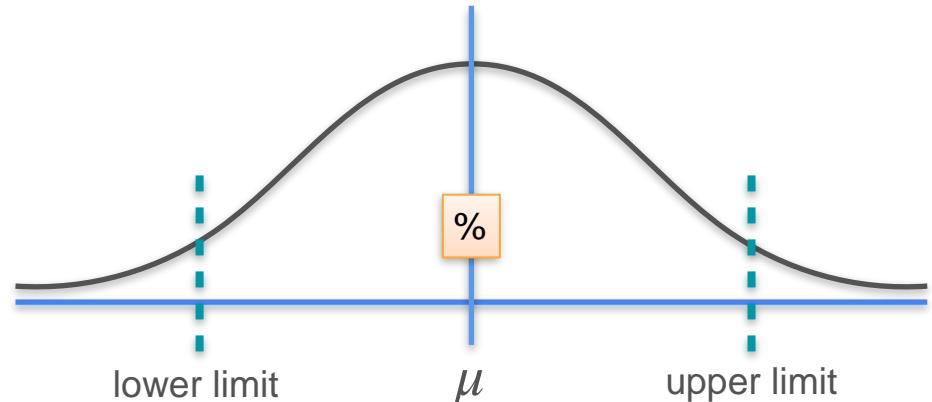
$$1 - \alpha = 0.95$$

$$1 - 0.05 = 0.95$$

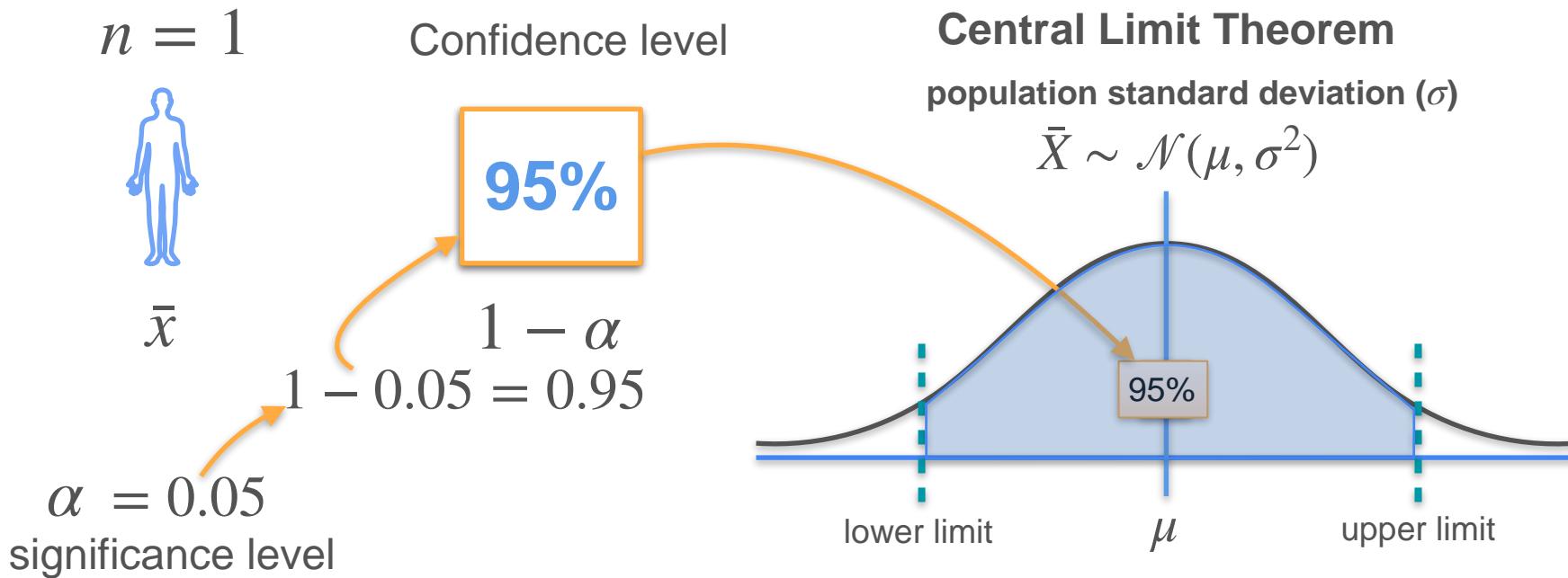
Central Limit Theorem

population standard deviation ( $\sigma$ )

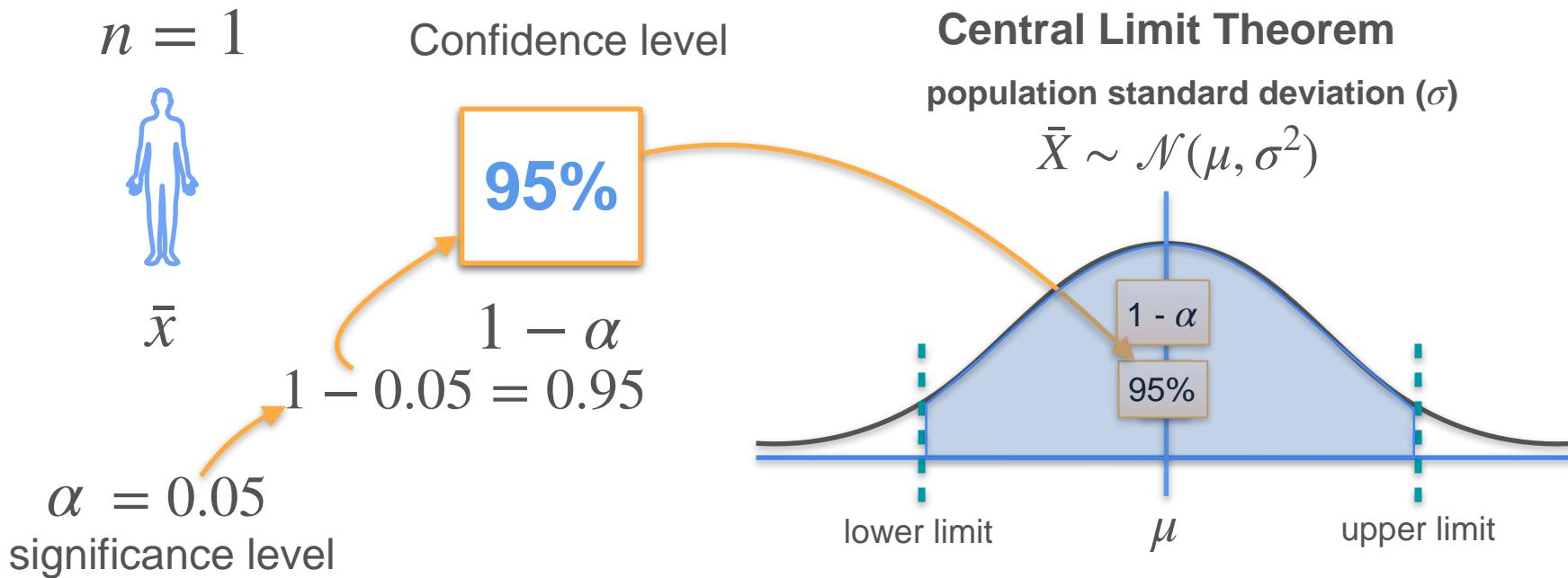
$$\bar{X} \sim \mathcal{N}(\mu, \sigma^2)$$



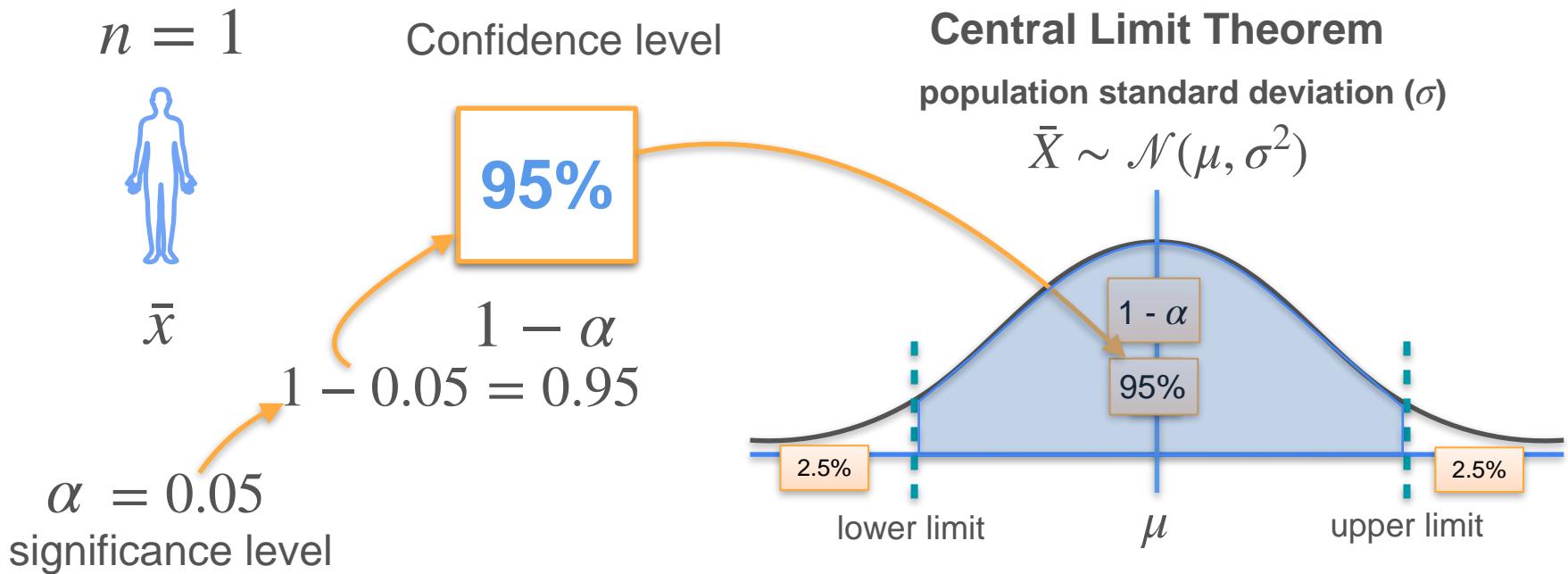
# Confidence Interval - Intuition



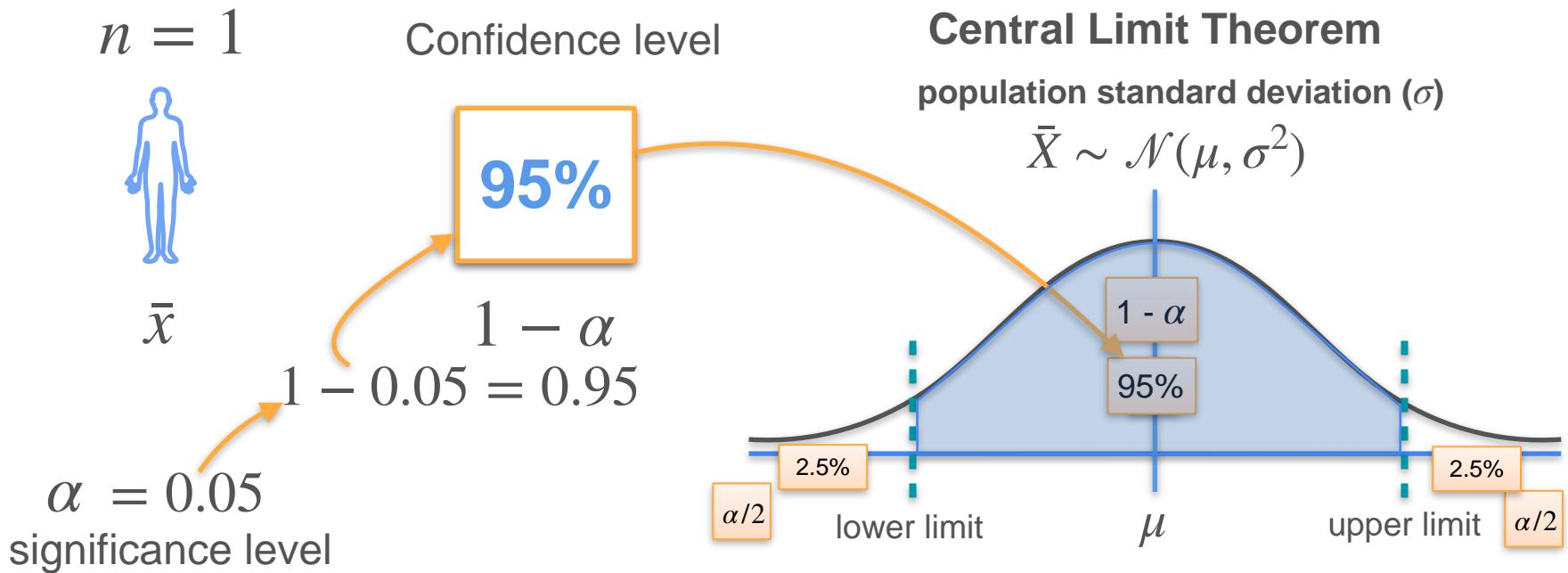
# Confidence Interval - Intuition



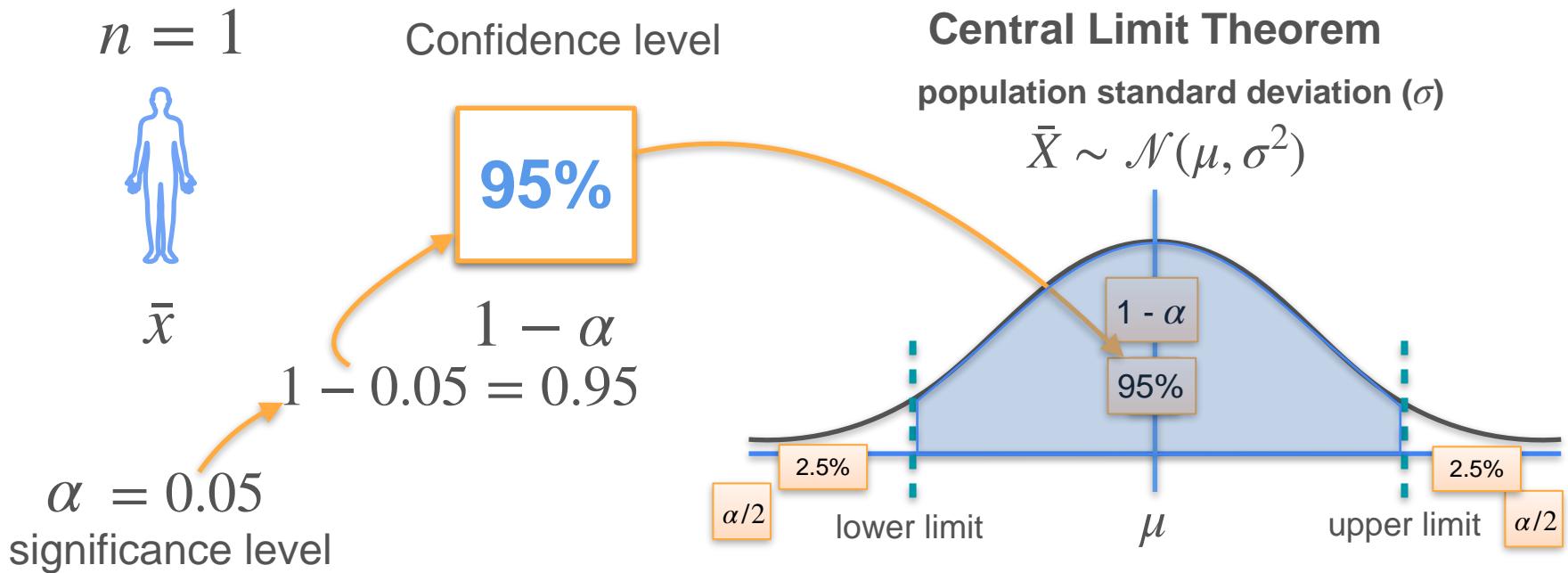
# Confidence Interval - Intuition



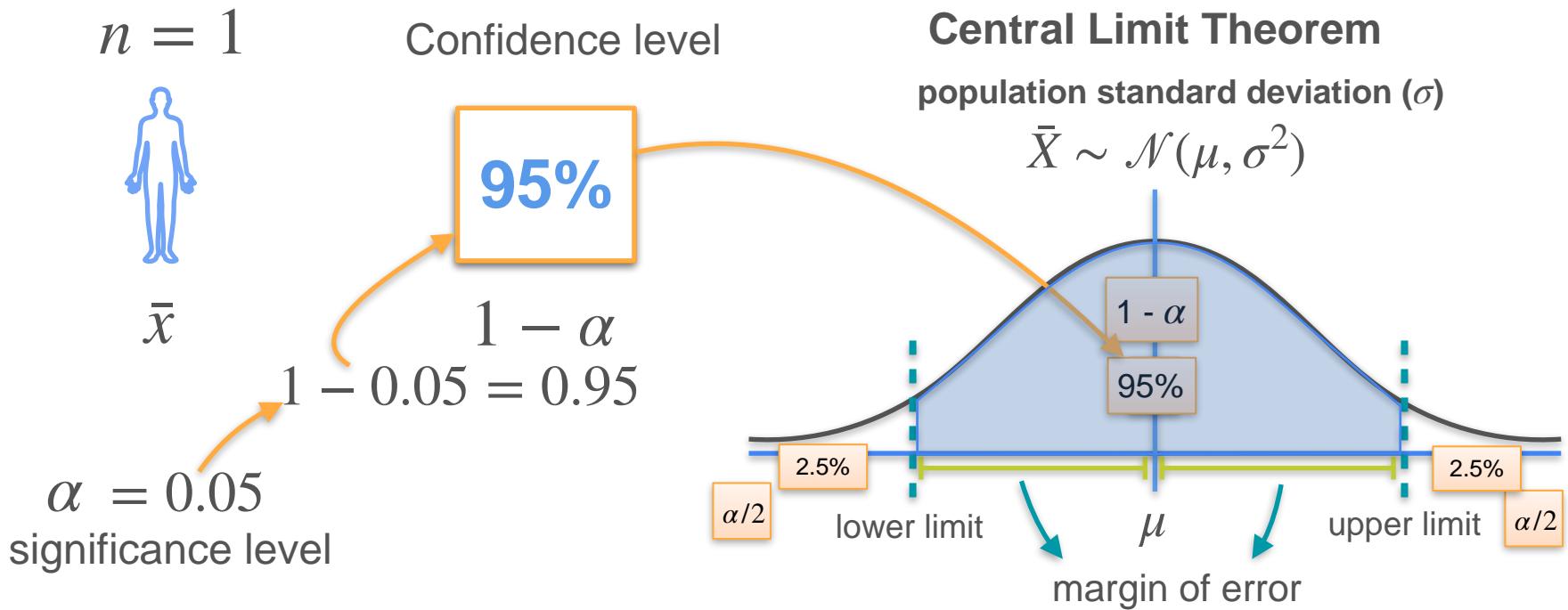
# Confidence Interval - Intuition



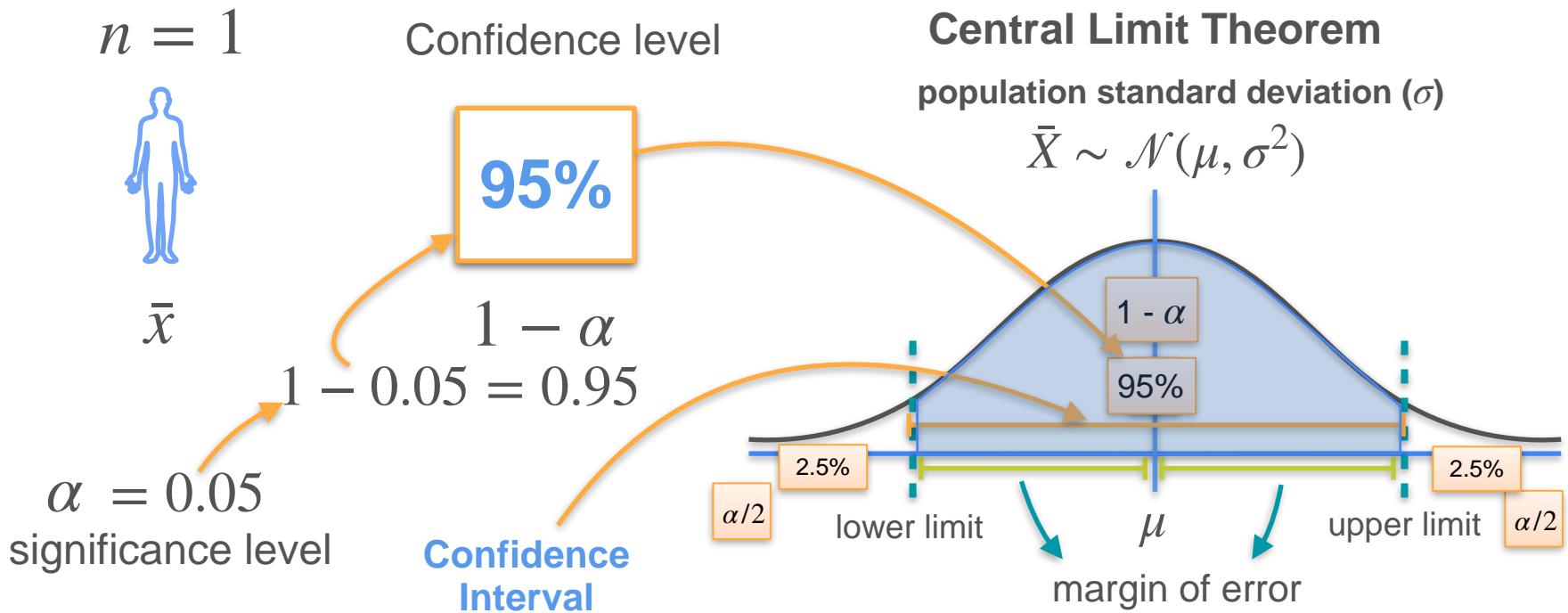
# Confidence Interval - Intuition



# Confidence Interval - Intuition



# Confidence Interval - Intuition



# Confidence Interval - Intuition

$$n = 1$$

Known  $\sigma$

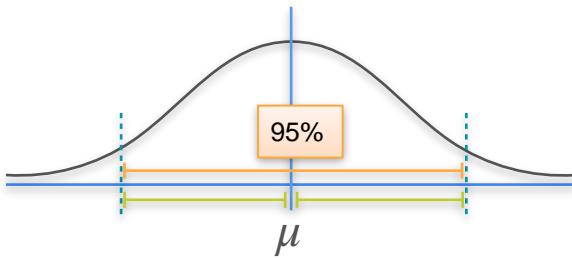
95%

# Confidence Interval - Intuition

$$n = 1$$

Known  $\sigma$

95%



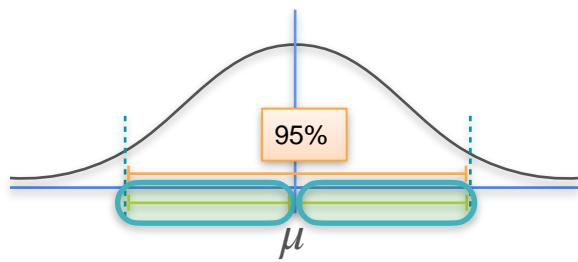
# Confidence Interval - Intuition

$$n = 1$$

Known  $\sigma$

95%

Margin of error



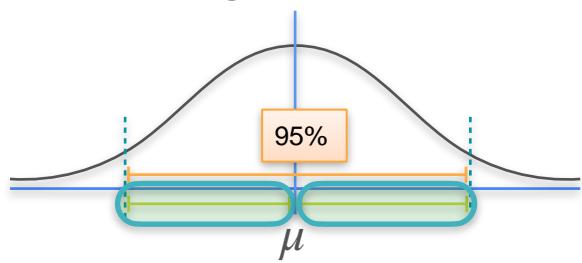
# Confidence Interval - Intuition

$n = 1$

Known  $\sigma$

95%

Margin of error



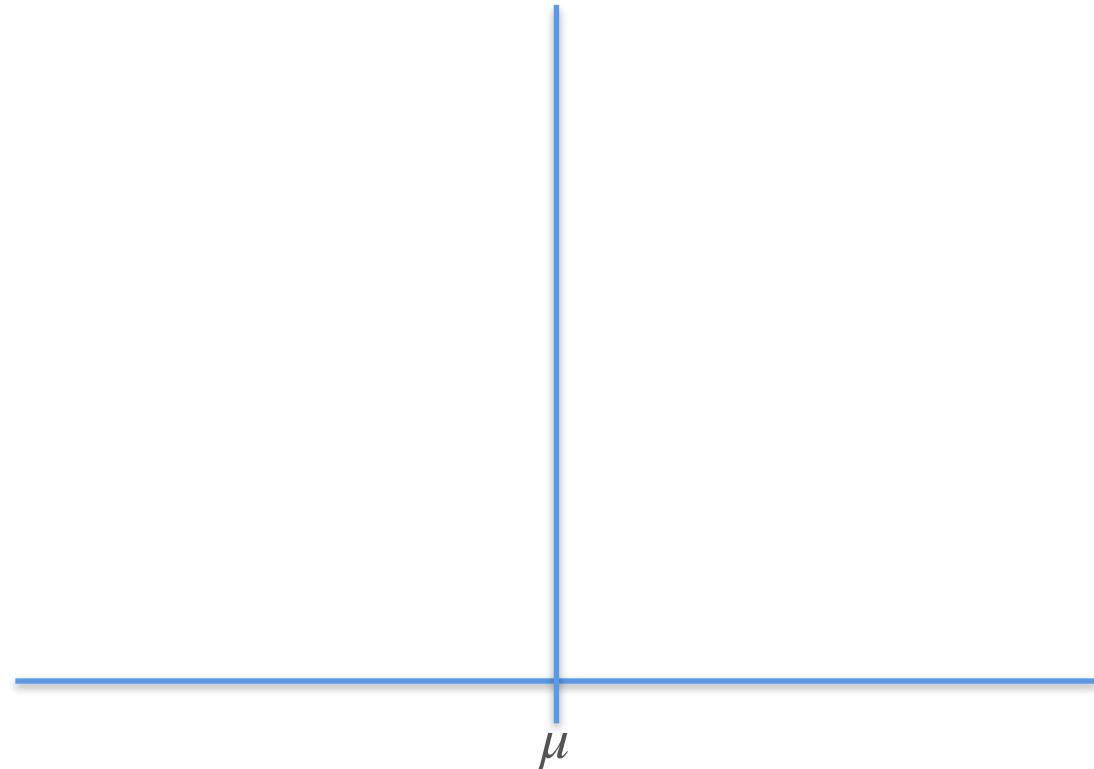
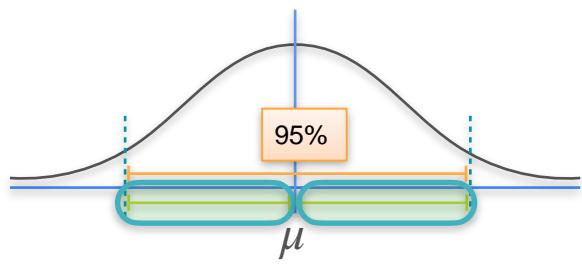
# Confidence Interval - Intuition

$n = 1$

Known  $\sigma$

95%

Margin of error



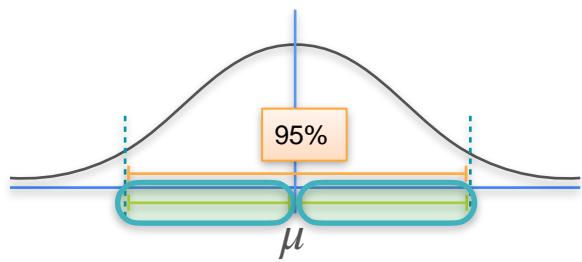
# Confidence Interval - Intuition

$n = 1$

Known  $\sigma$

95%

Margin of error



$\bar{x}_1$



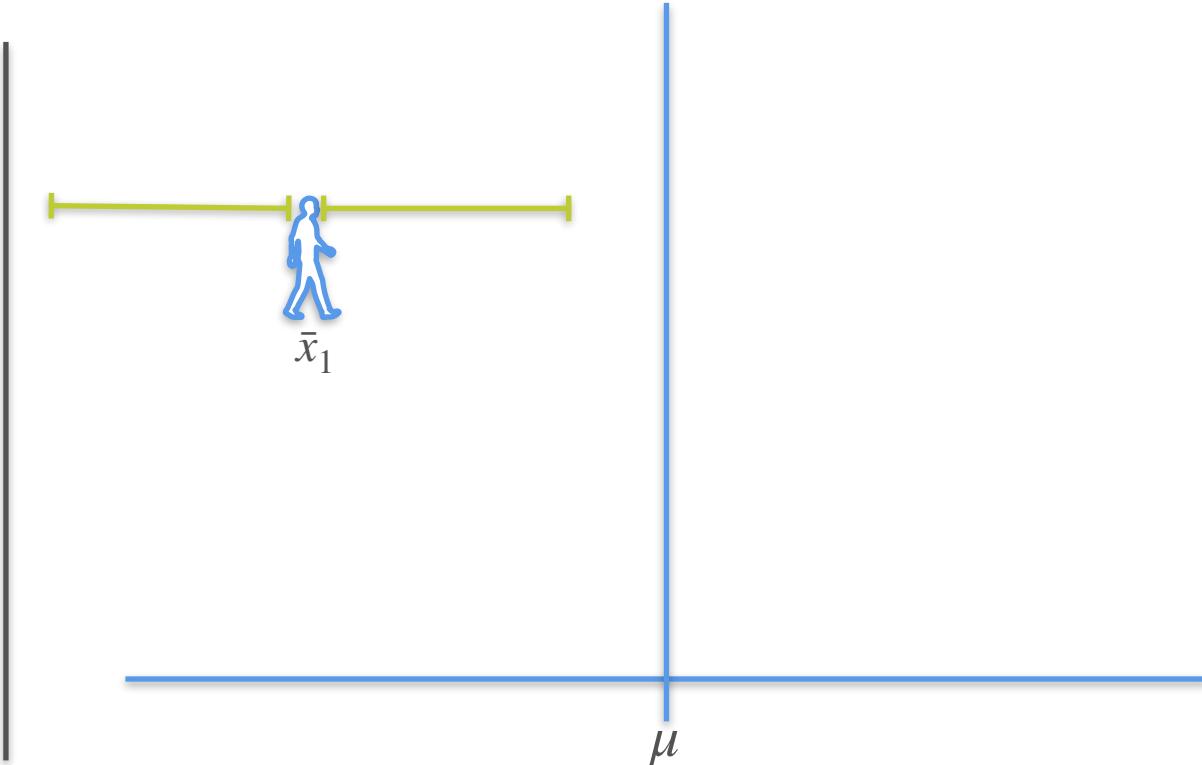
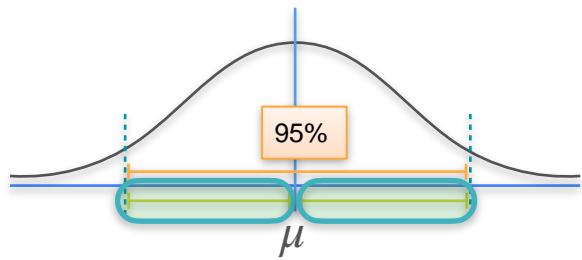
# Confidence Interval - Intuition

$n = 1$

Known  $\sigma$

95%

Margin of error



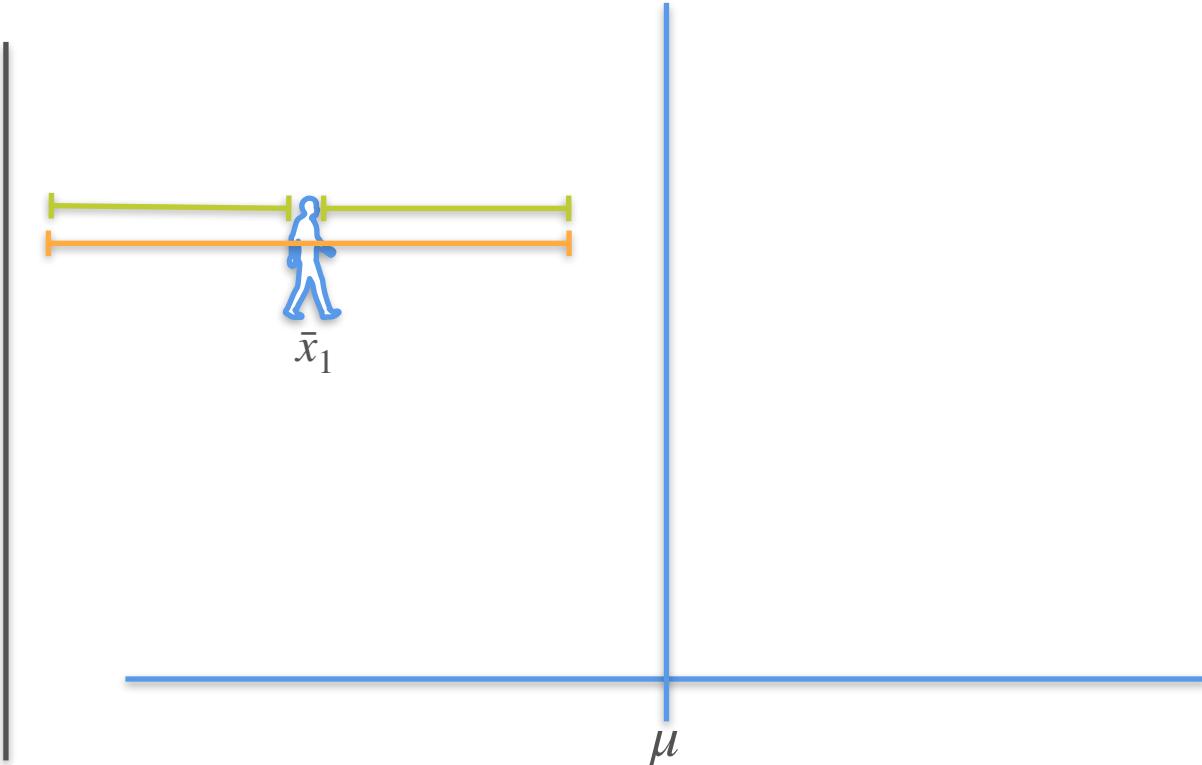
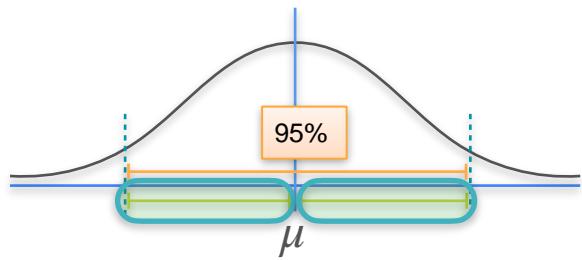
# Confidence Interval - Intuition

$n = 1$

Known  $\sigma$

95%

Margin of error



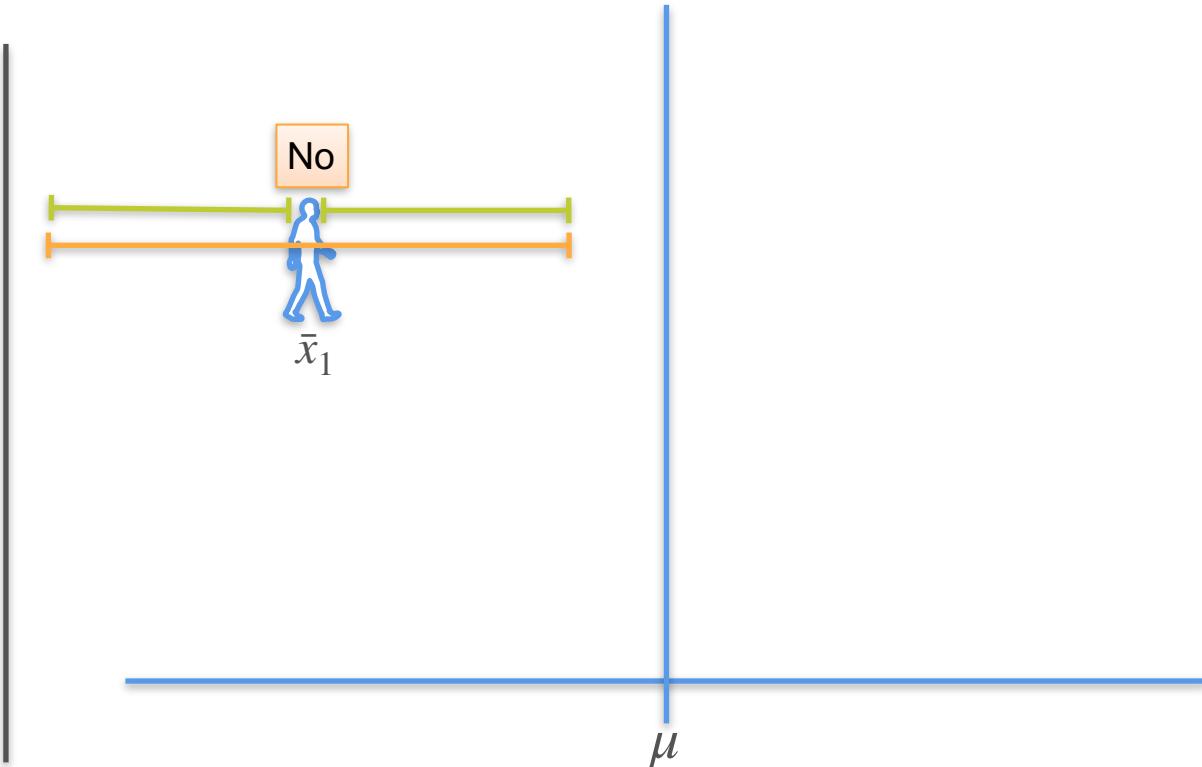
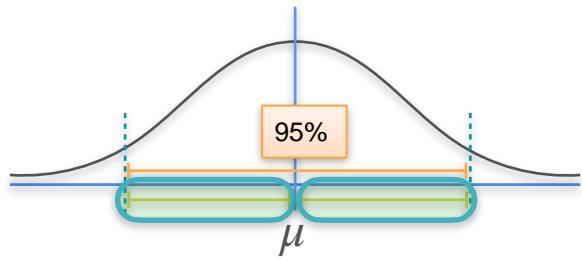
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$n = 1$

Known  $\sigma$

95%

Margin of error



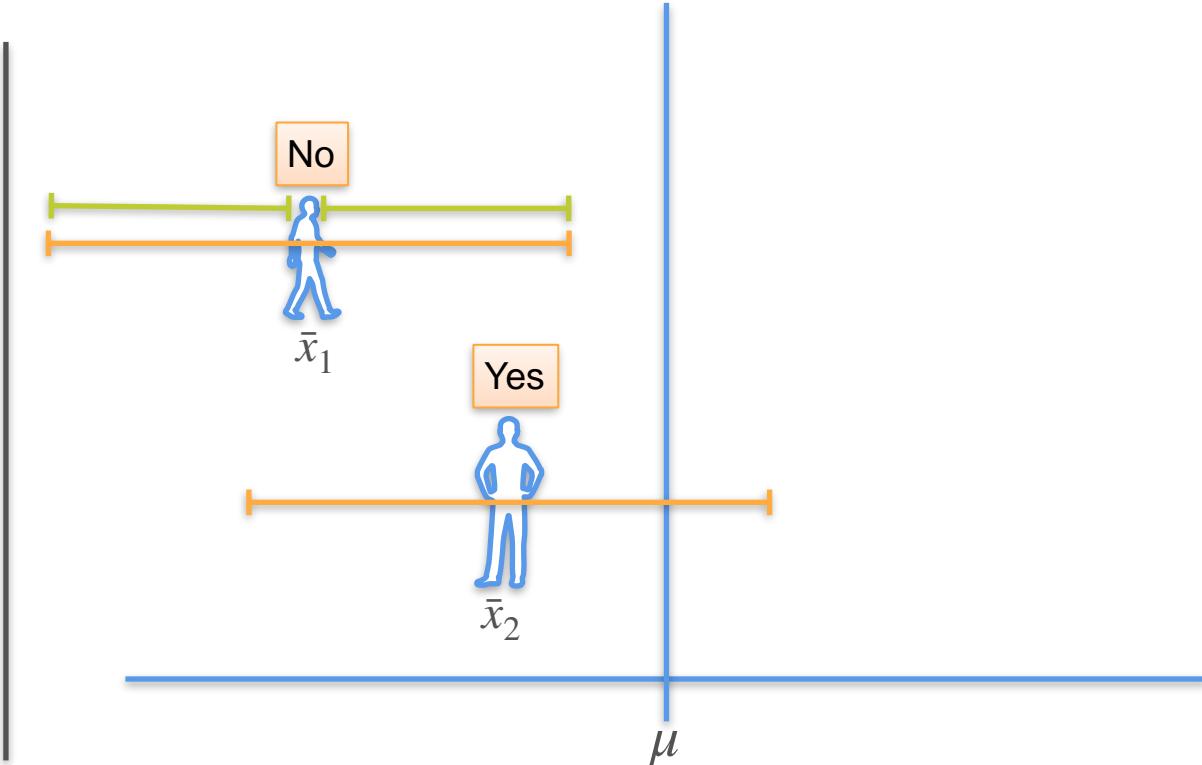
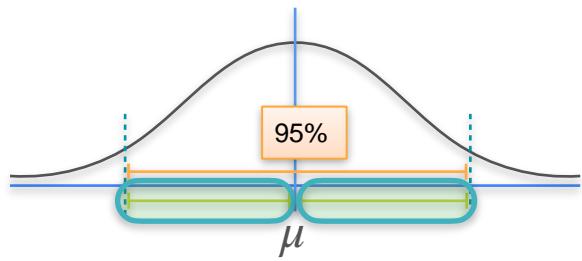
# Confidence Interval - Intuition

$n = 1$

Known  $\sigma$

95%

Margin of error



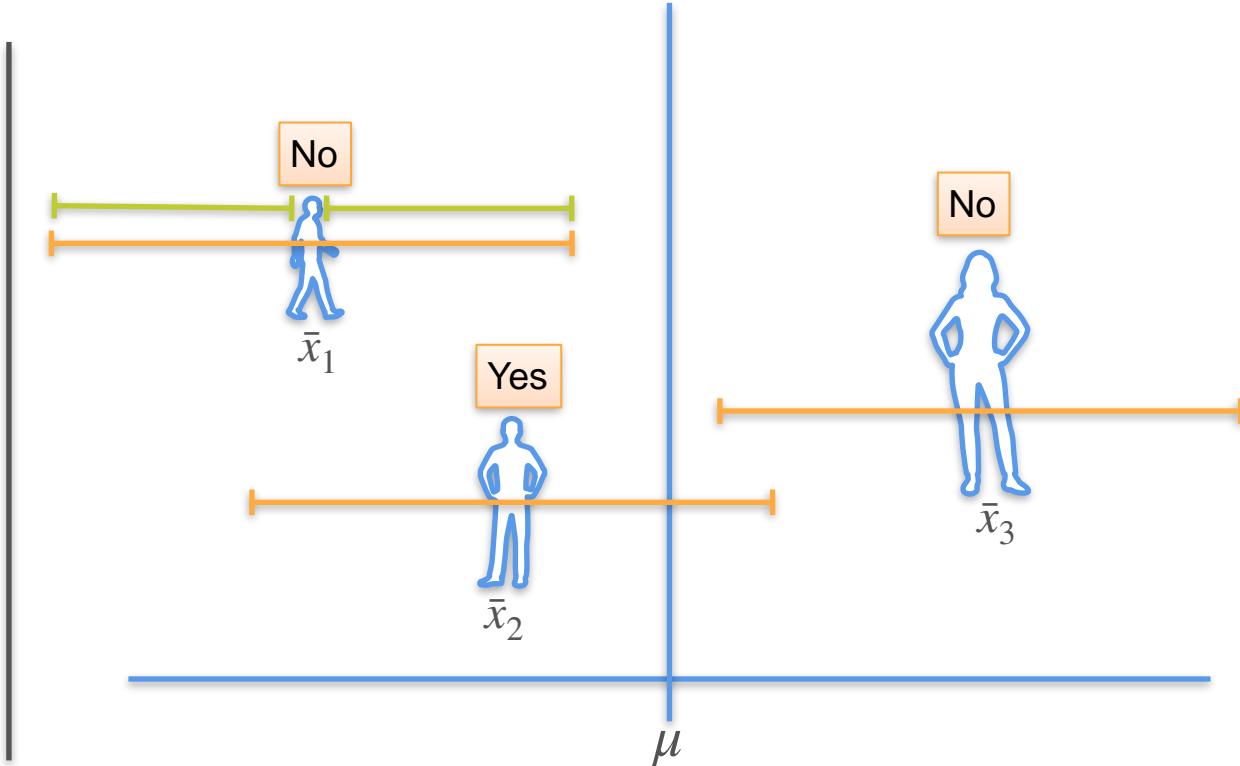
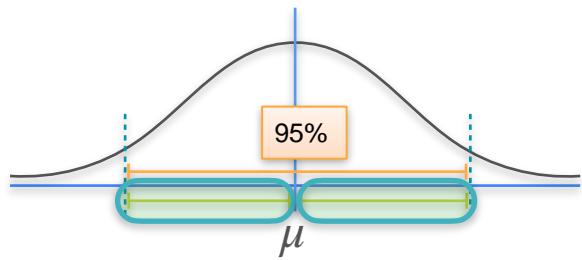
# Confidence Interval - Intuition

$n = 1$

Known  $\sigma$

95%

Margin of error



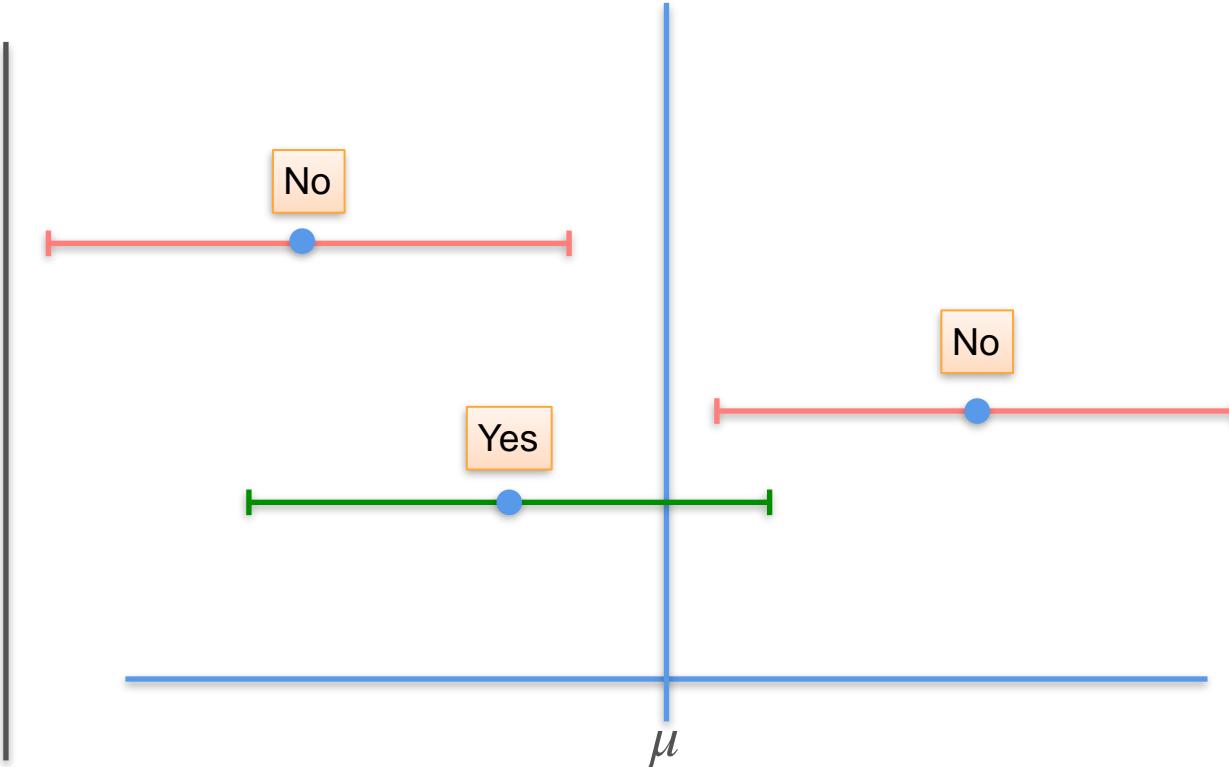
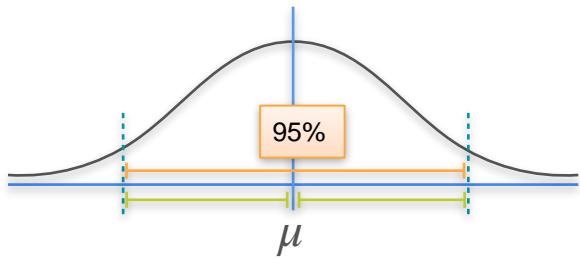
# Confidence Interval - Intuition

$n = 1$

Known  $\sigma$

95%

Margin of error



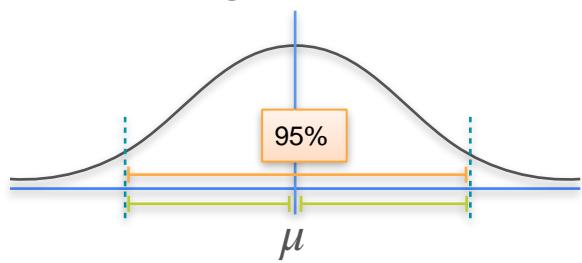
# Confidence Interval - Intuition

$n = 1$

Known  $\sigma$

95%

Margin of error



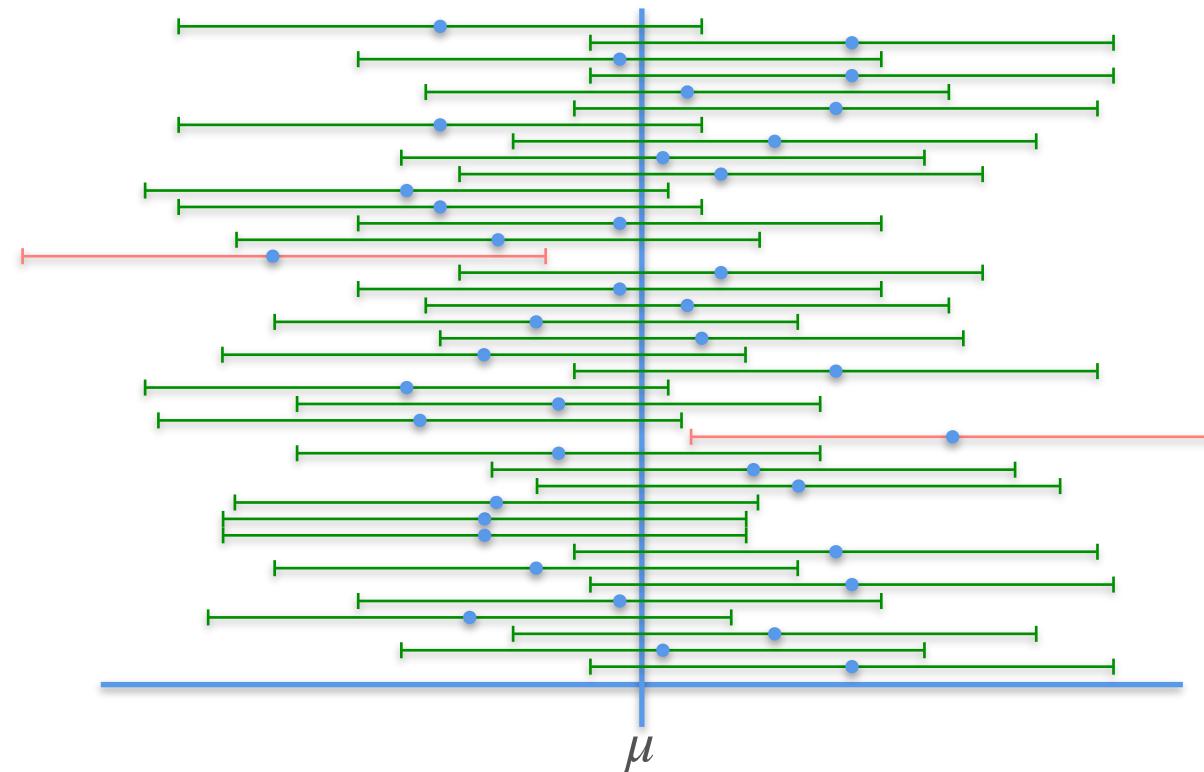
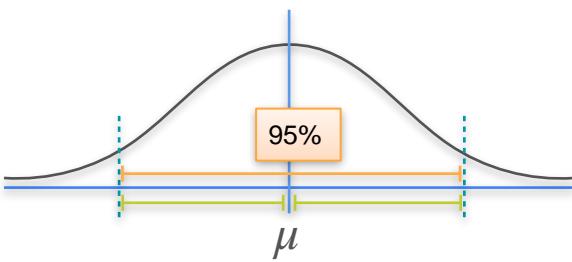
# Confidence Interval - Intuition

$n = 1$

Known  $\sigma$

95%

Margin of error



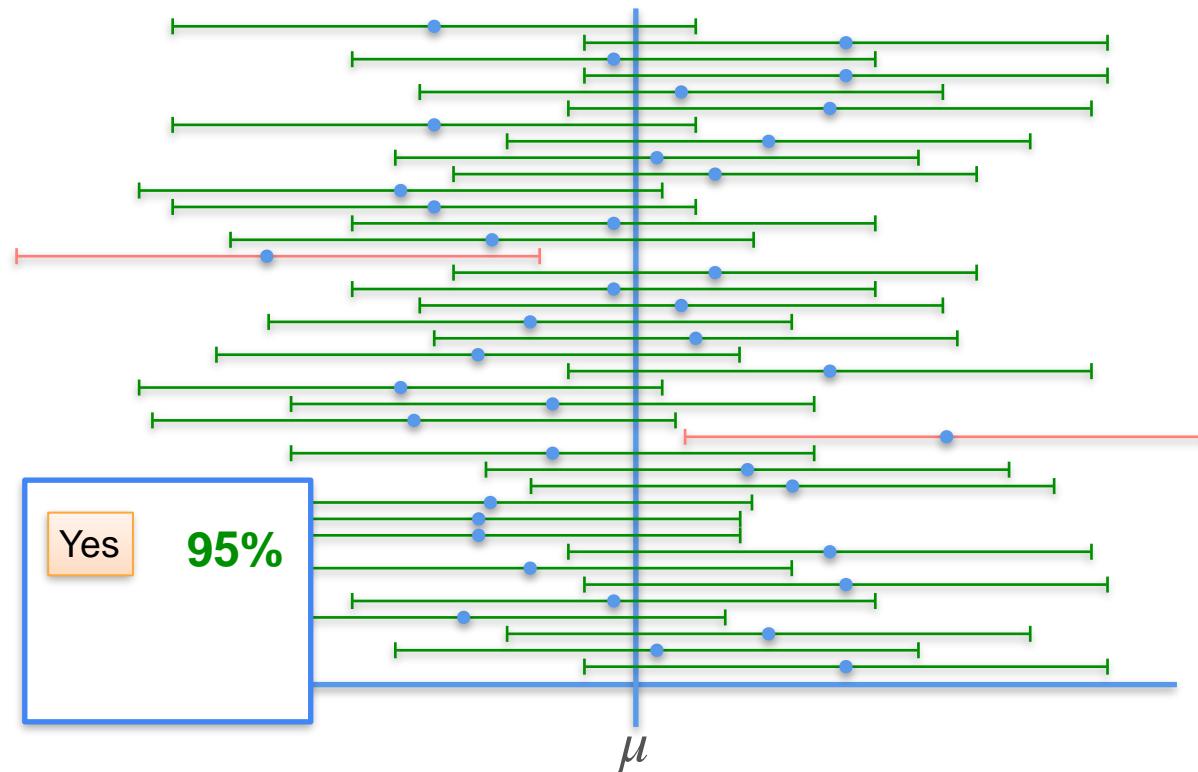
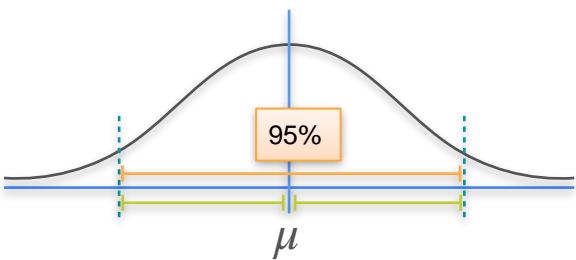
# Confidence Interval - Intuition

$n = 1$

Known  $\sigma$

95%

Margin of error



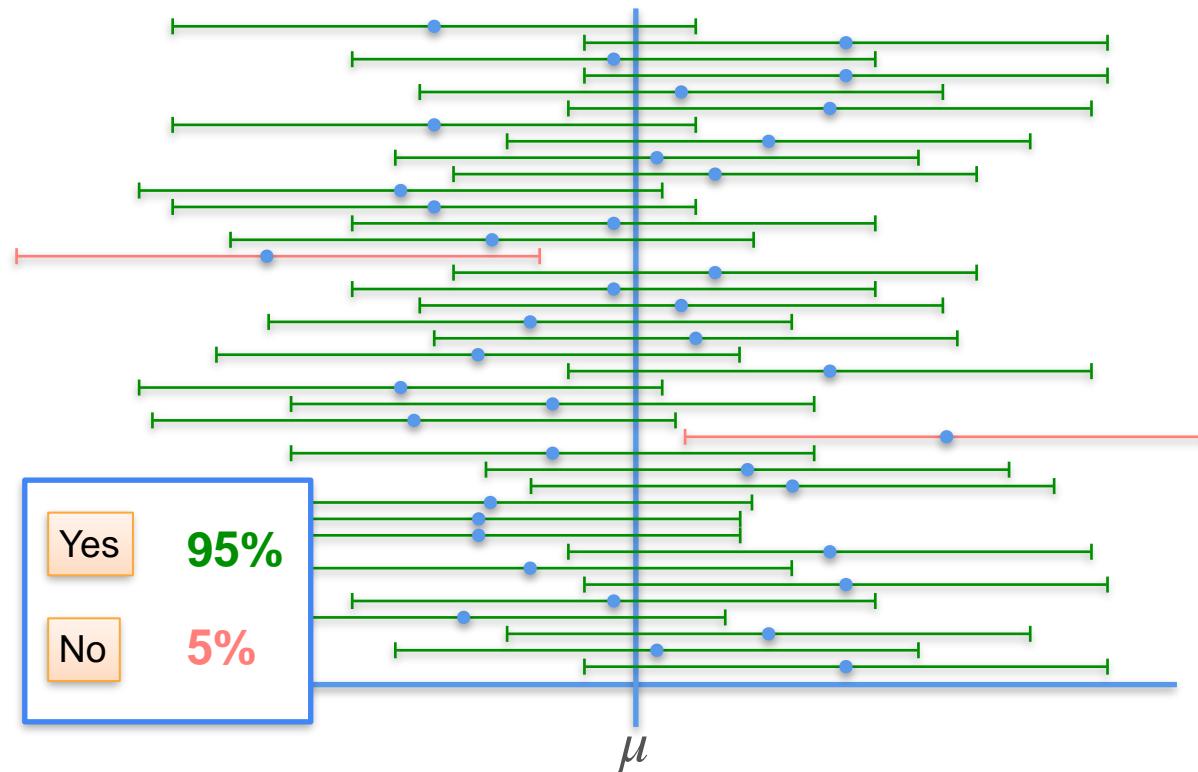
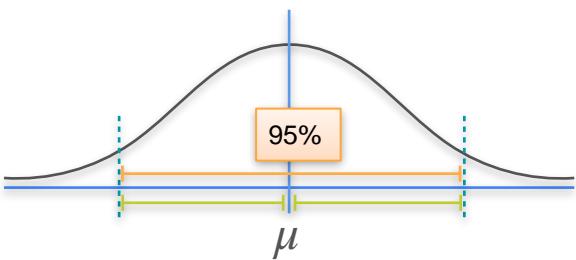
# Confidence Interval - Intuition

$n = 1$

Known  $\sigma$

95%

Margin of error



# Confidence Interval - Intuition

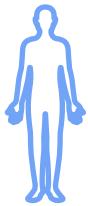
$$n = 1$$



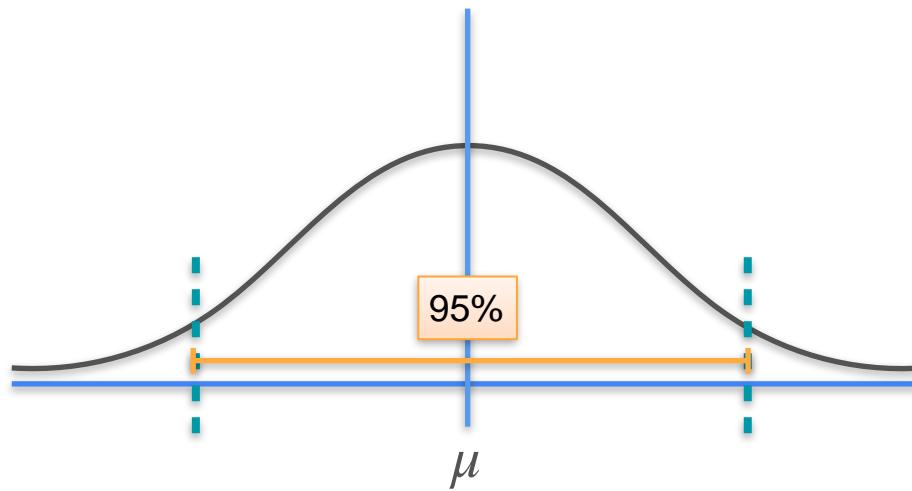
$$\bar{x}$$

# Confidence Interval - Intuition

$$n = 1$$

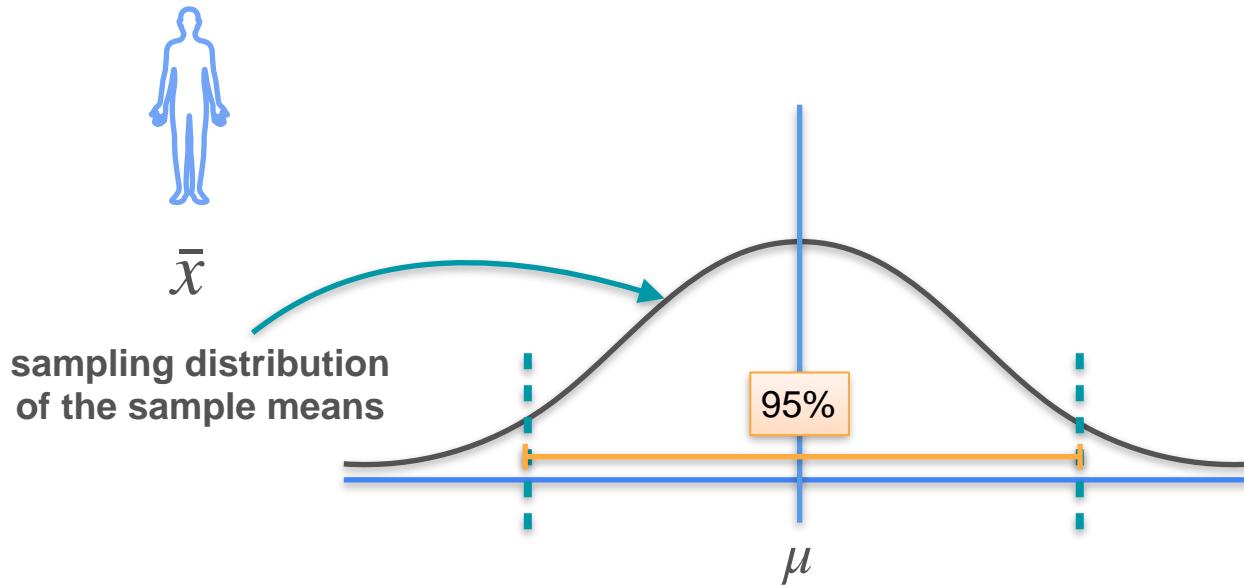


$$\bar{x}$$



# Confidence Interval - Intuition

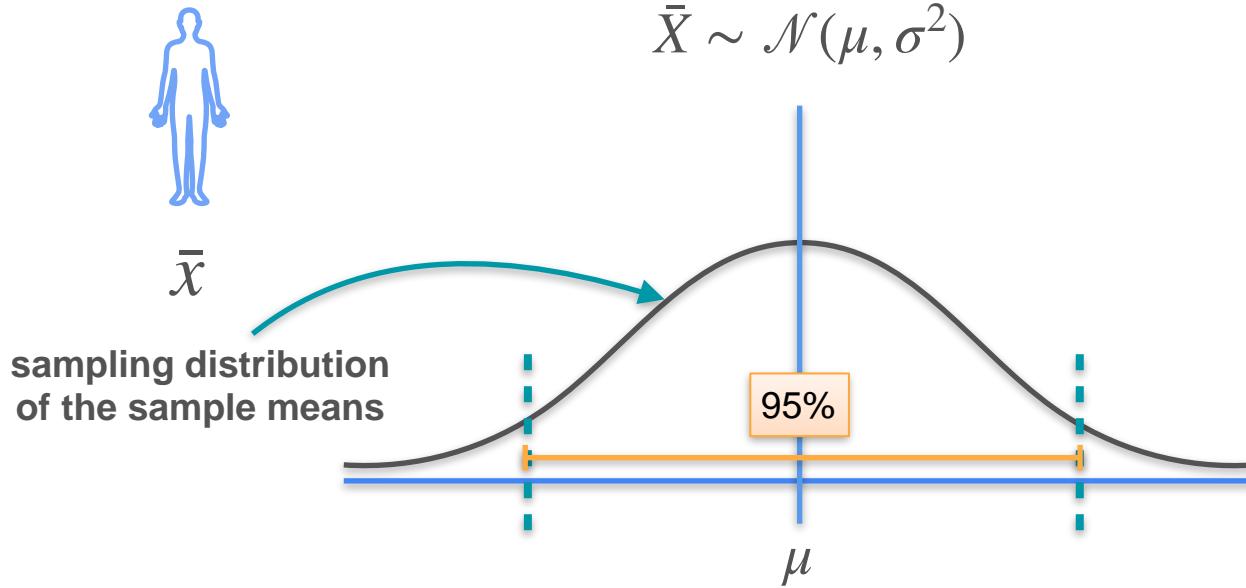
$$n = 1$$



# Confidence Interval - Intuition

$n = 1$

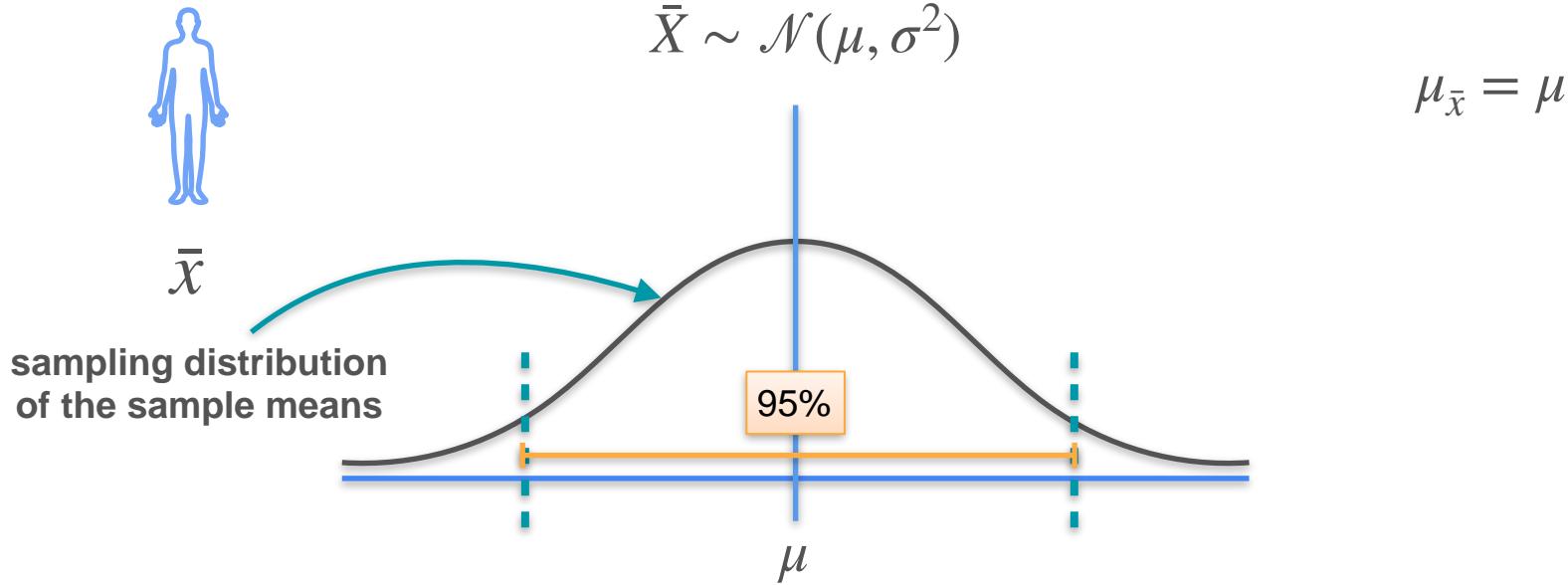
Central Limit Theorem



# Confidence Interval - Intuition

$$n = 1$$

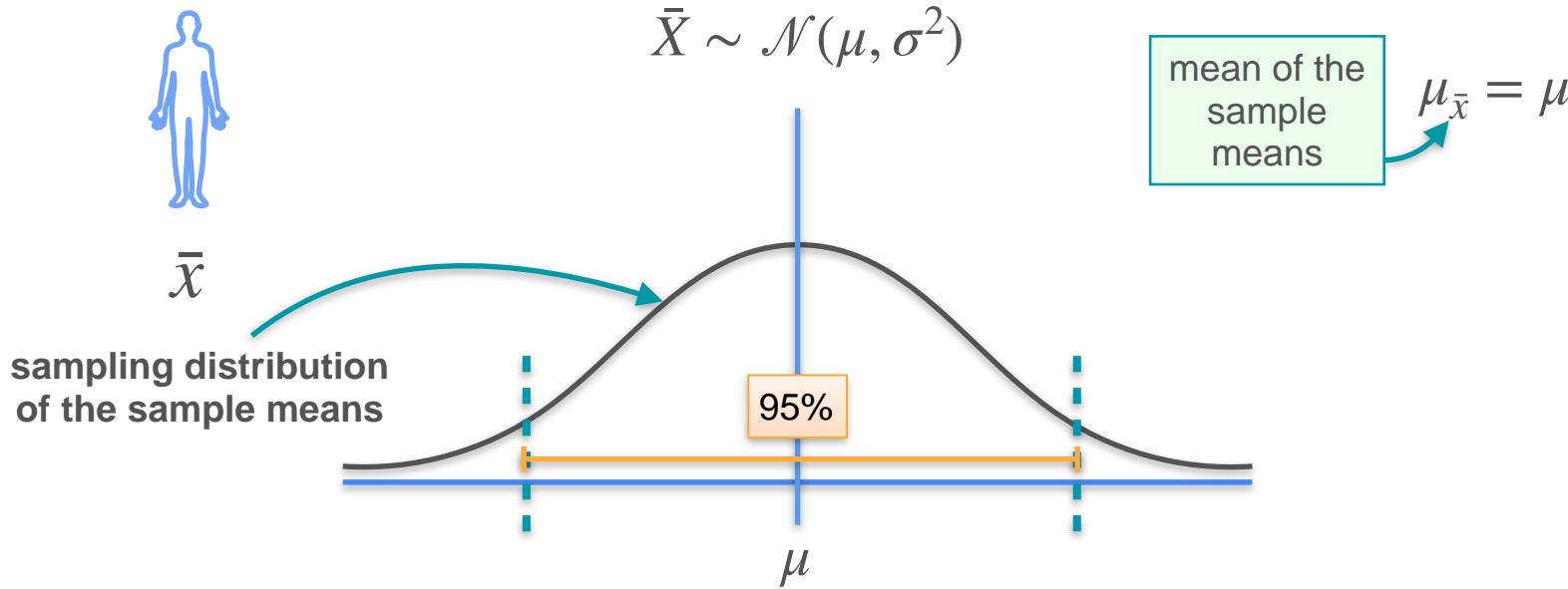
Central Limit Theorem



# Confidence Interval - Intuition

$$n = 1$$

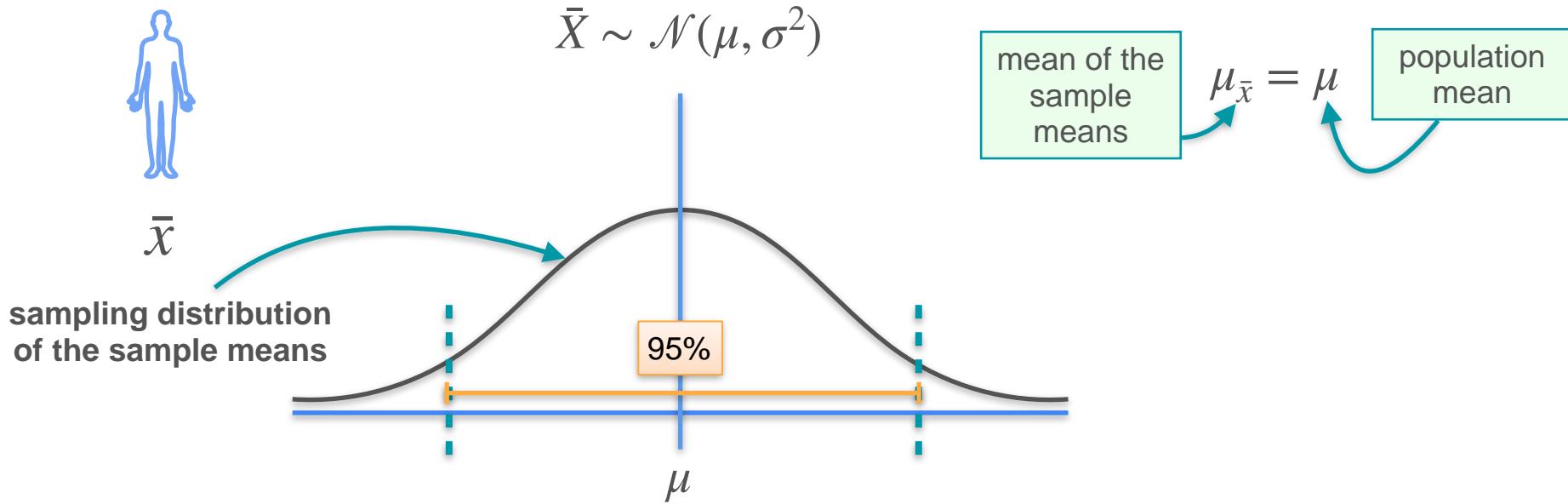
Central Limit Theorem



# Confidence Interval - Intuition

$$n = 1$$

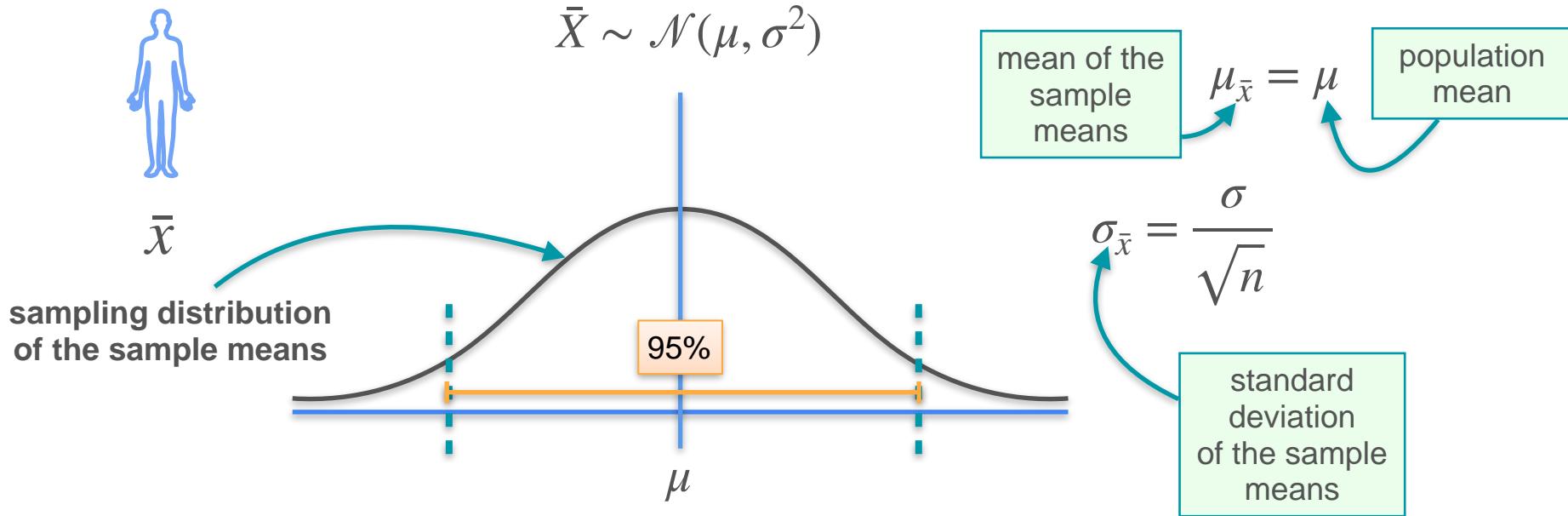
Central Limit Theorem



# Confidence Interval - Intuition

$$n = 1$$

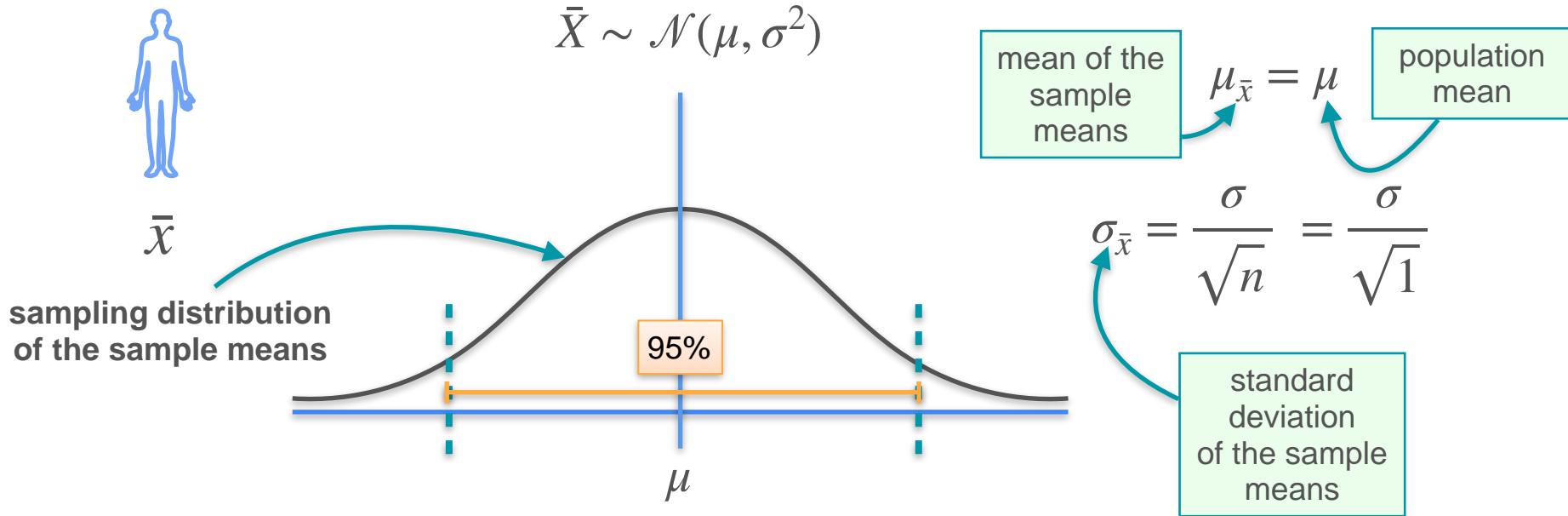
Central Limit Theorem



# Confidence Interval - Intuition

$n = 1$

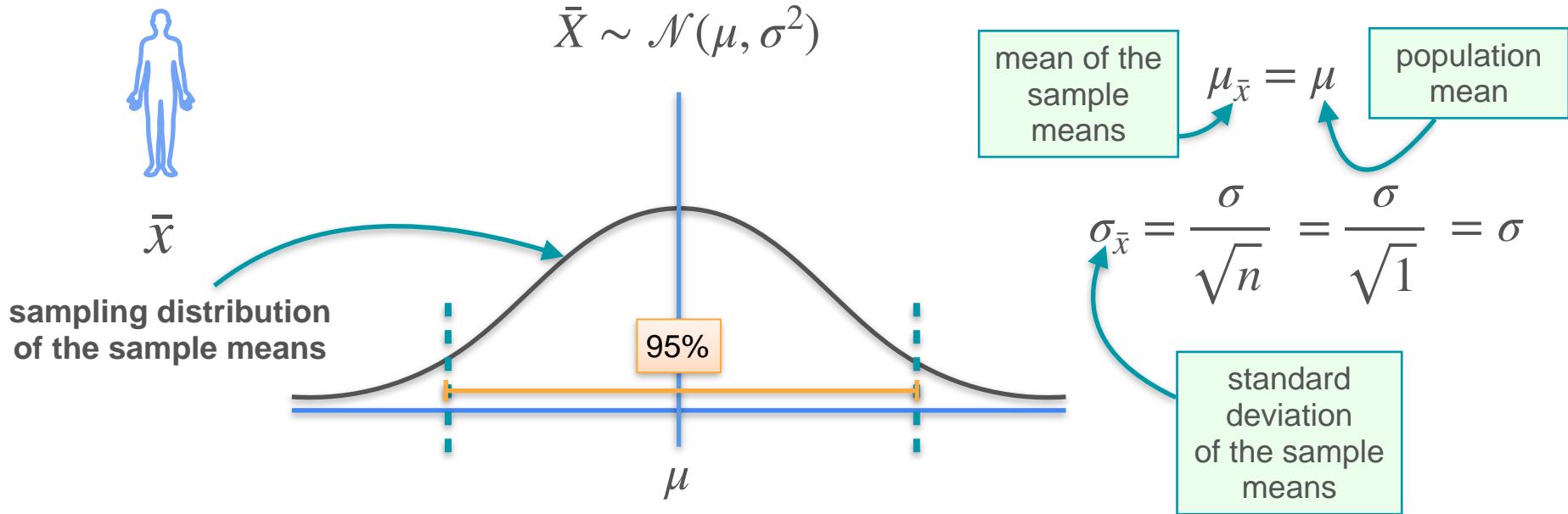
Central Limit Theorem



# Confidence Interval - Intuition

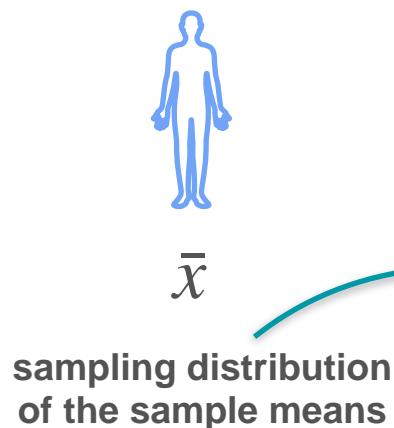
$n = 1$

Central Limit Theorem



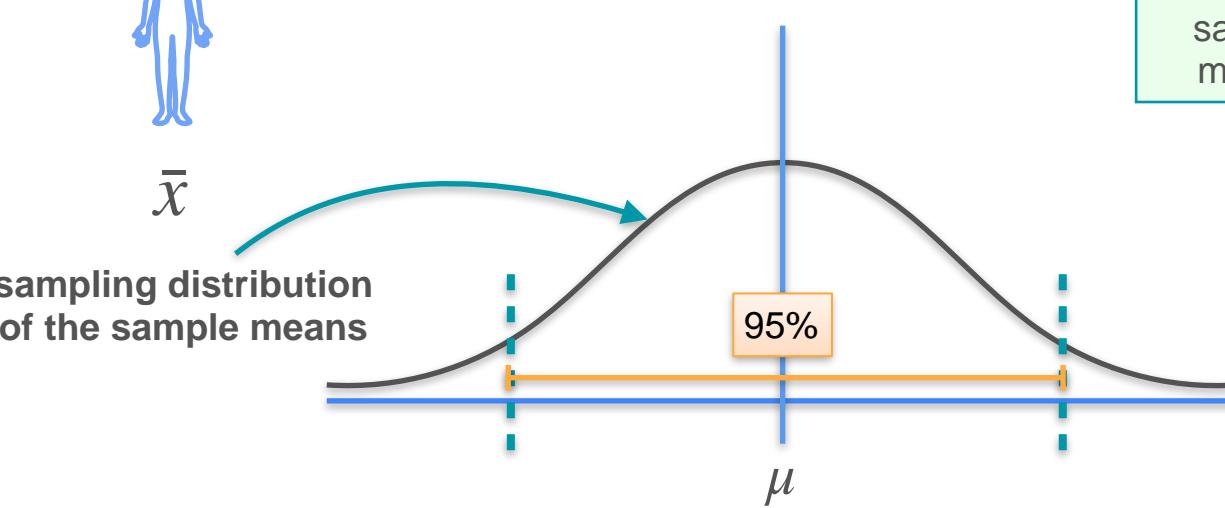
# Confidence Interval - Intuition

$$n = 1$$



Central Limit Theorem

$$\bar{X} \sim \mathcal{N}(\mu, \sigma^2)$$



Population standard deviation ( $\sigma$ )

mean of the  
sample  
means

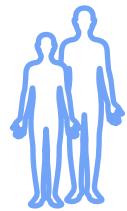
$$\mu_{\bar{x}} = \mu$$

population  
mean

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{\sigma}{\sqrt{1}} = \sigma$$

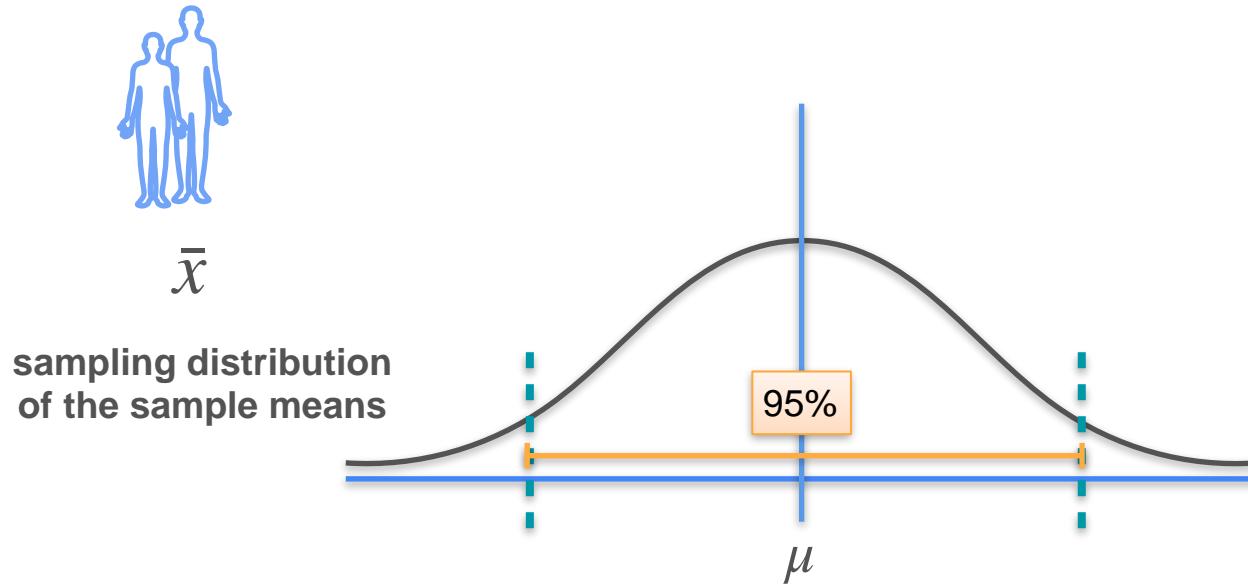
standard  
deviation  
of the sample  
means

# Confidence Interval - Intuition



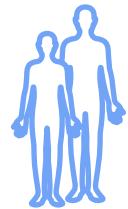
Central Limit Theorem

Population standard deviation ( $\sigma$ )



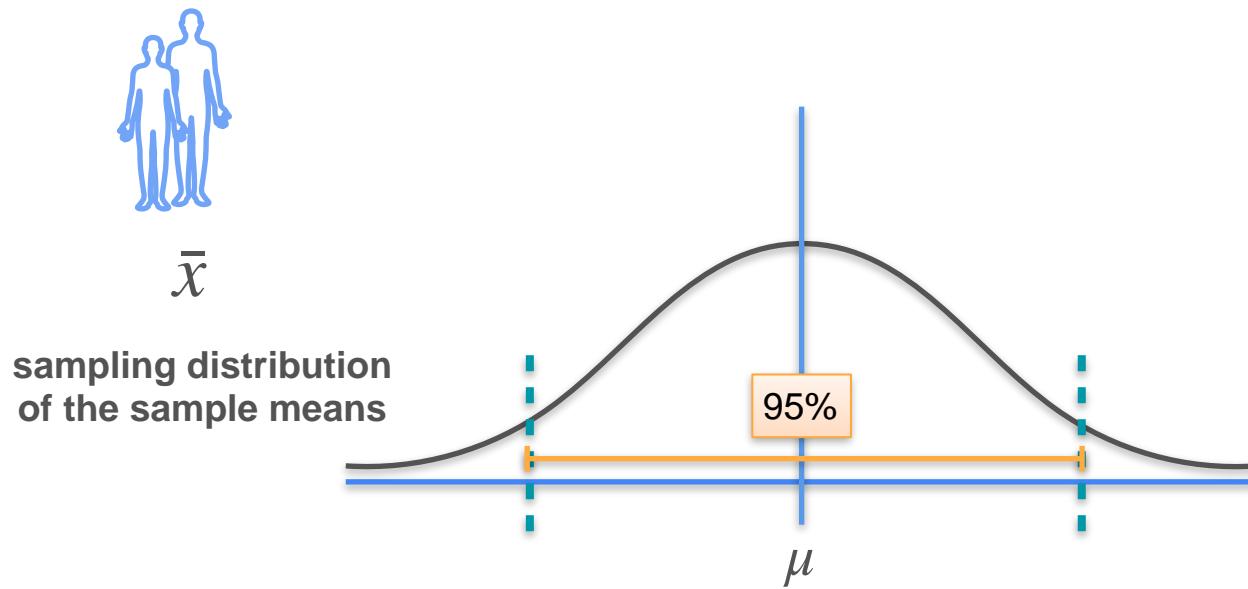
# Confidence Interval - Intuition

$n = 2$



Central Limit Theorem

Population standard deviation ( $\sigma$ )

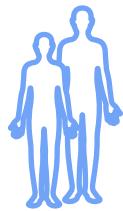


# Confidence Interval - Intuition

$n = 2$

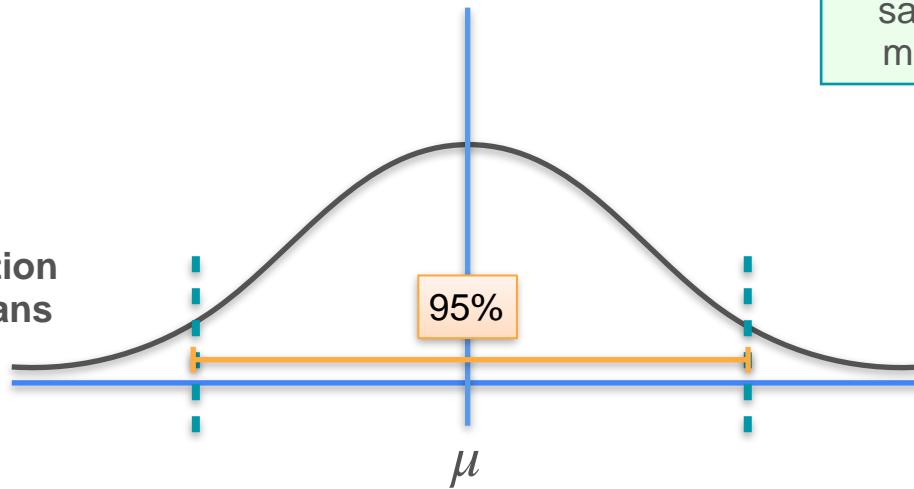
Central Limit Theorem

Population standard deviation ( $\sigma$ )



$\bar{x}$

sampling distribution  
of the sample means



mean of the  
sample  
means

$$\mu_{\bar{x}} = \mu$$

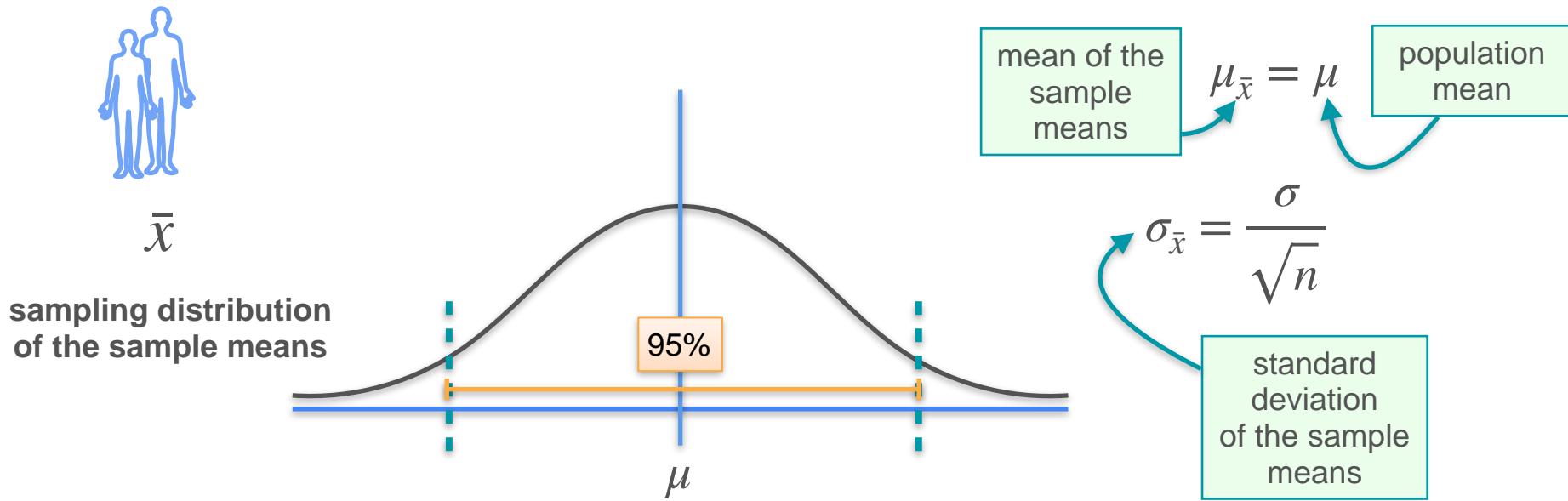
population  
mean

# Confidence Interval - Intuition

$n = 2$

Central Limit Theorem

Population standard deviation ( $\sigma$ )

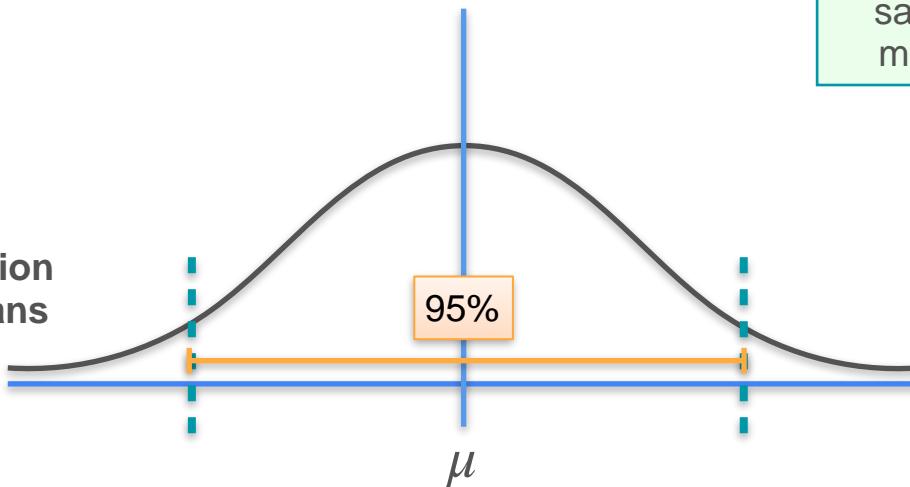


# Confidence Interval - Intuition

$n = 2$

  
 $\bar{x}$   
sampling distribution  
of the sample means

Central Limit Theorem



Population standard deviation ( $\sigma$ )

mean of the  
sample  
means

$$\mu_{\bar{x}} = \mu$$

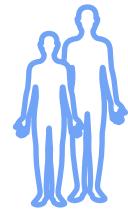
population  
mean

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{\sigma}{\sqrt{2}}$$

standard  
deviation  
of the sample  
means

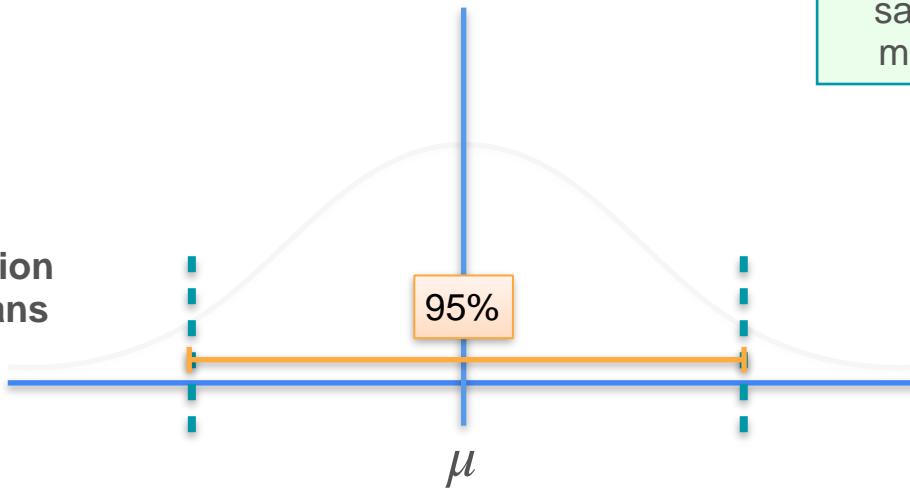
# Confidence Interval - Intuition

$n = 2$



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sampling distribution  
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Central Limit Theorem



Population standard deviation ( $\sigma$ )

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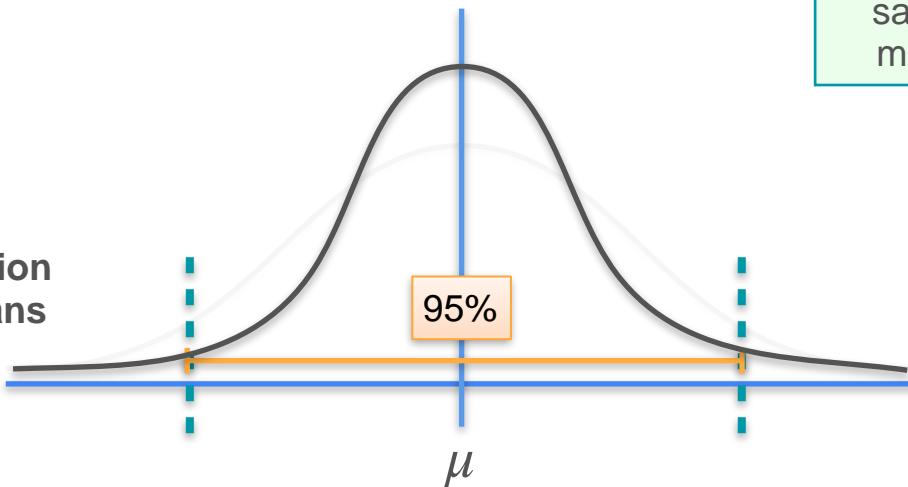
# Confidence Interval - Intuition

$n = 2$

Central Limit Theorem

Population standard deviation ( $\sigma$ )

  
 $\bar{x}$   
sampling distribution  
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mean of the  
sample  
means

$$\mu_{\bar{x}} = \mu$$

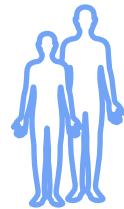
population  
mean

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{\sigma}{\sqrt{2}}$$

standard  
deviation  
of the sample  
means

# Confidence Interval - Intuition

$n = 2$



$\bar{x}$

sampling distribution  
of the sample means

Central Limit Theorem

$$\bar{X} \sim \mathcal{N}\left(\mu, \frac{\sigma^2}{n}\right)$$

95%

$\mu$

Population standard deviation ( $\sigma$ )

mean of the  
sample  
means

$$\mu_{\bar{x}} = \mu$$

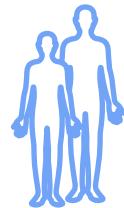
population  
mean

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{\sigma}{\sqrt{2}}$$

standard  
deviation  
of the sample  
means

# Confidence Interval - Intuition

$n = 2$



$\bar{x}$

sampling distribution  
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Central Limit Theorem

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95%

$\mu$

Population standard deviation ( $\sigma$ )

mean of the  
sample  
means

$$\mu_{\bar{x}} = \mu$$

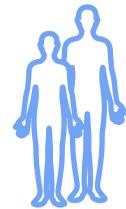
population  
mean

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{\sigma}{\sqrt{2}}$$

standard  
deviation  
of the sample  
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# Confidence Interval - Intuition

$n = 2$



$\bar{x}$

sampling distribution  
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95%

$\mu$

Population standard deviation ( $\sigma$ )

mean of the  
sample  
means

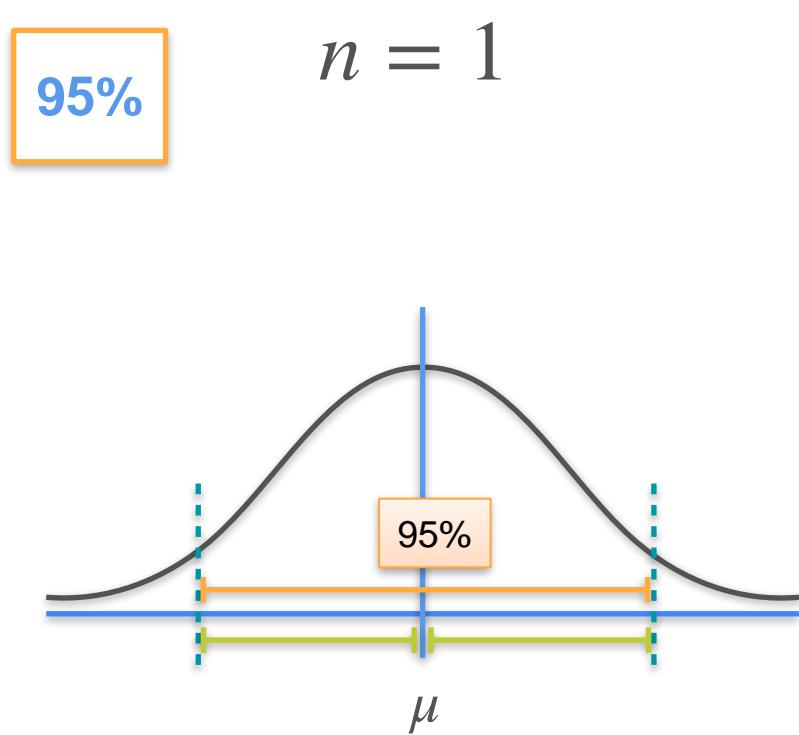
$$\mu_{\bar{x}} = \mu$$

population  
mean

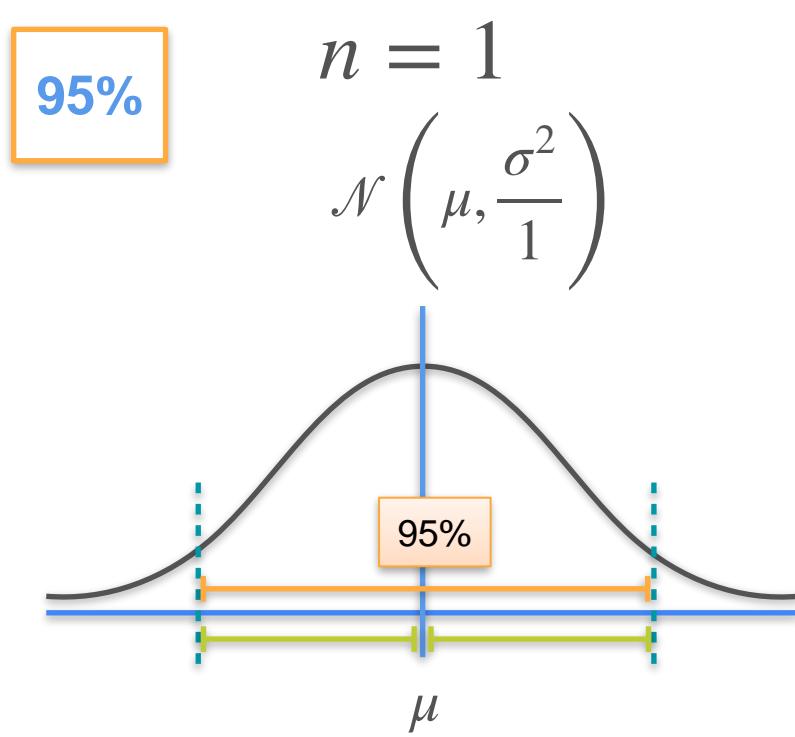
$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{\sigma}{\sqrt{2}}$$

standard  
deviation  
of the sample  
means

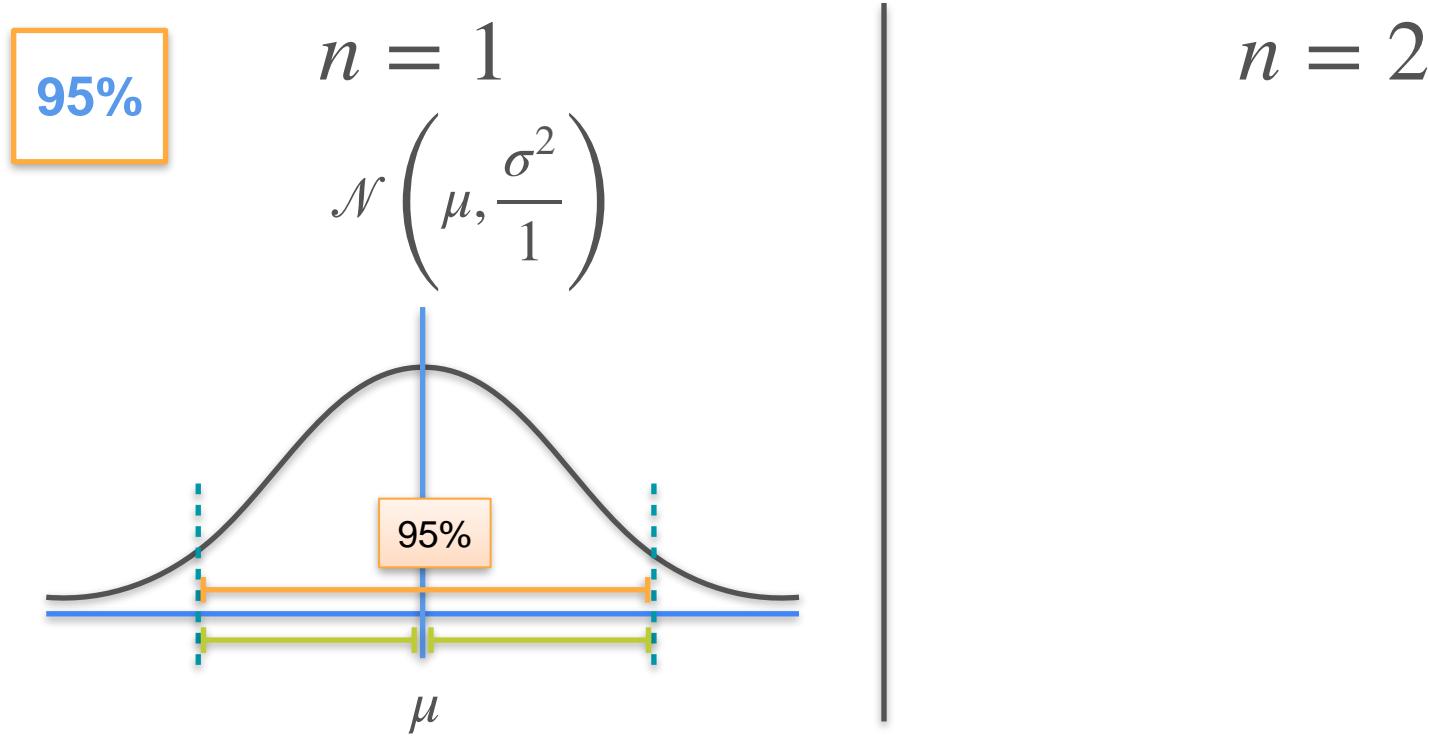
# Confidence Interval - Intuition



# Confidence Interval - Intuition



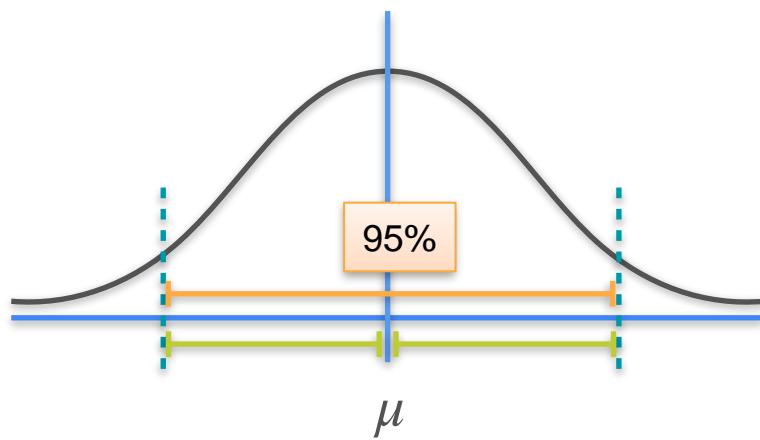
# Confidence Interval - Intuition



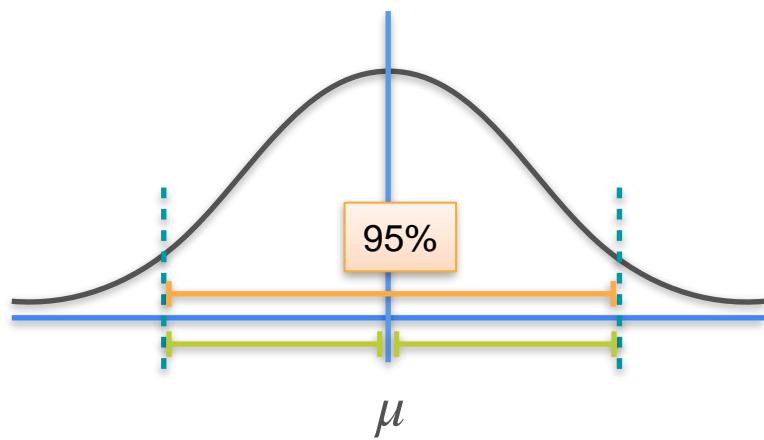
# Confidence Interval - Intuition

95%

$$n = 1$$
$$\mathcal{N}\left(\mu, \frac{\sigma^2}{1}\right)$$



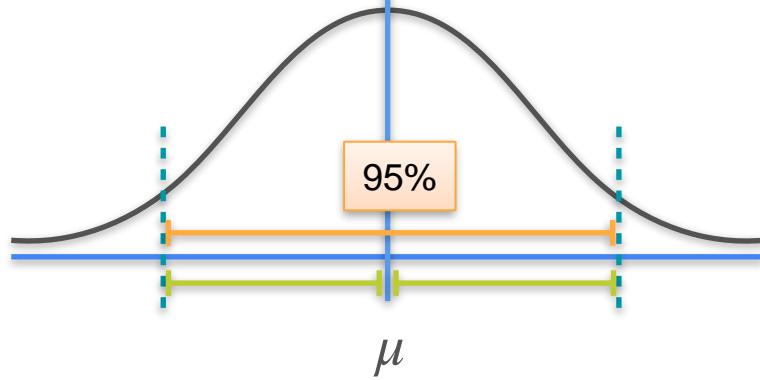
$n = 2$



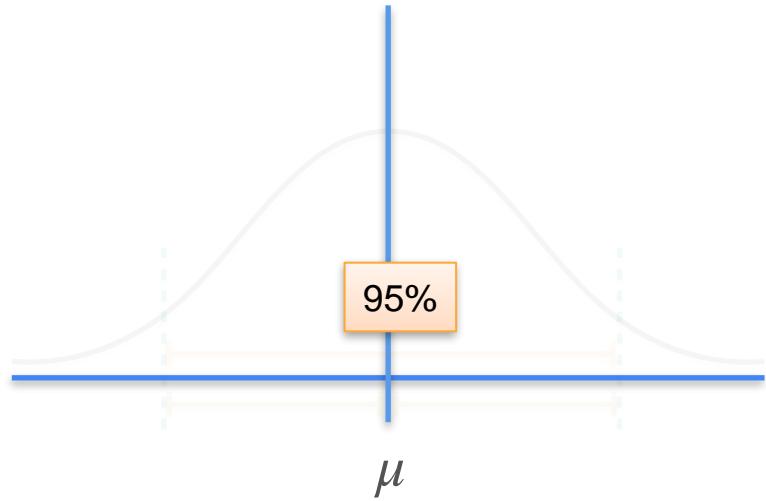
# Confidence Interval - Intuition

95%

$$n = 1$$
$$\mathcal{N}\left(\mu, \frac{\sigma^2}{1}\right)$$



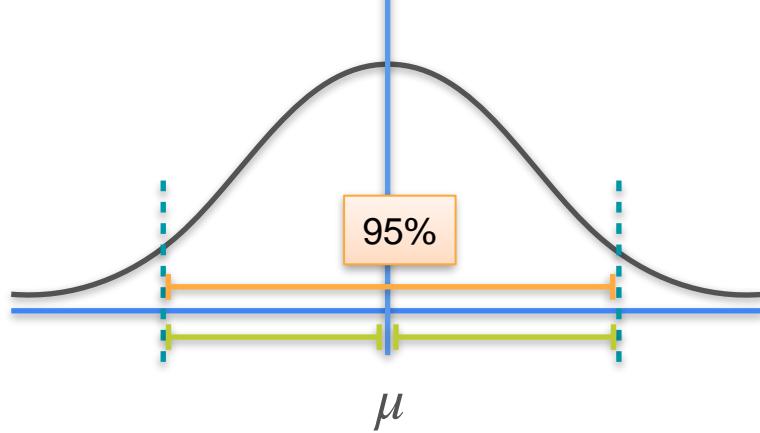
$n = 2$



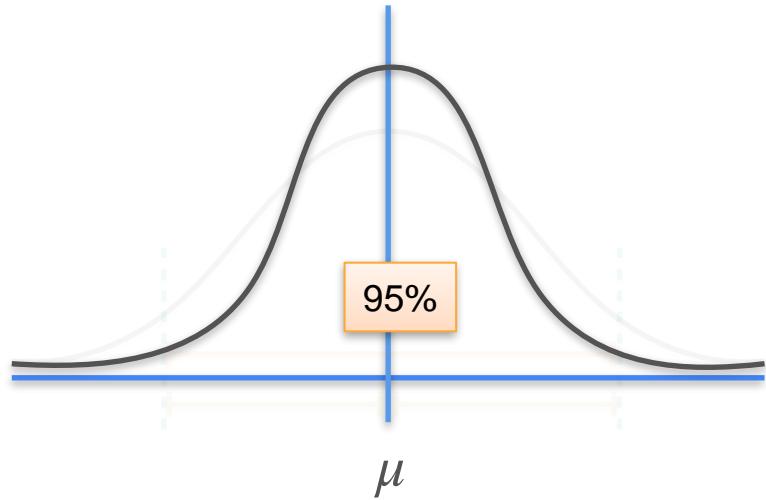
# Confidence Interval - Intuition

95%

$$n = 1$$
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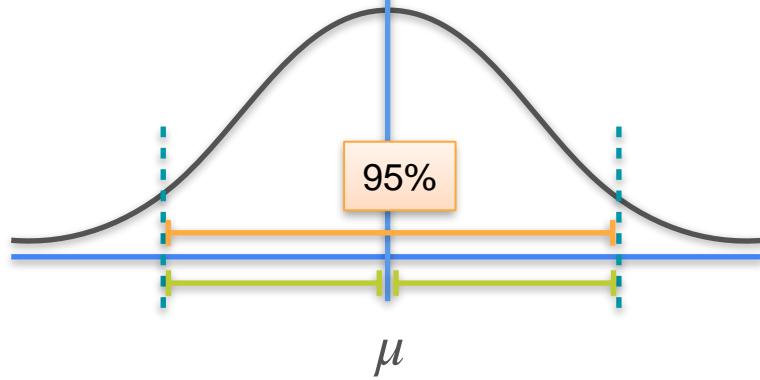
$n = 2$



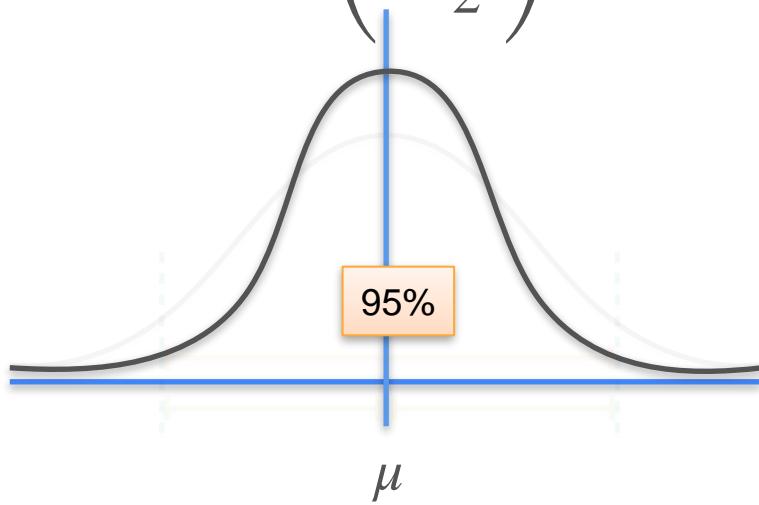
# Confidence Interval - Intuition

95%

$$n = 1$$
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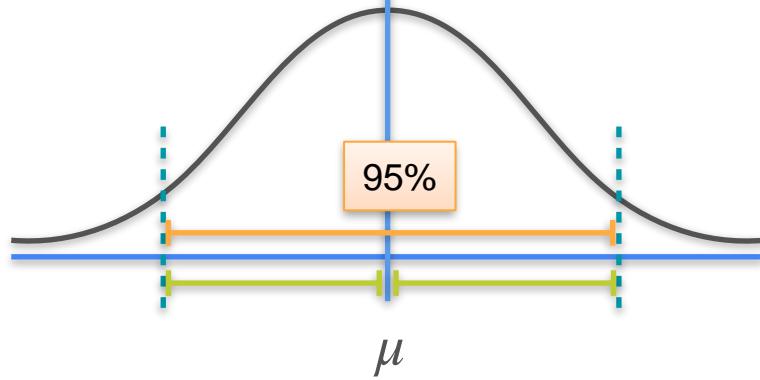
$$n = 2$$
$$\mathcal{N}\left(\mu, \frac{\sigma^2}{2}\right)$$



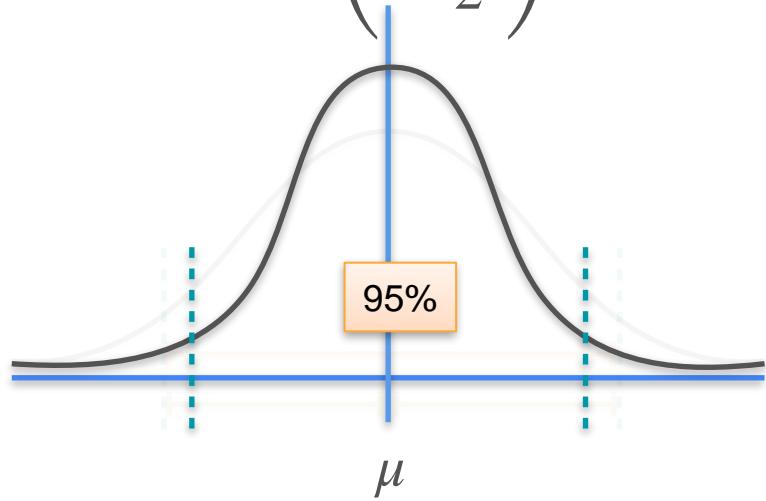
# Confidence Interval - Intuition

95%

$$n = 1$$
$$\mathcal{N}\left(\mu, \frac{\sigma^2}{1}\right)$$



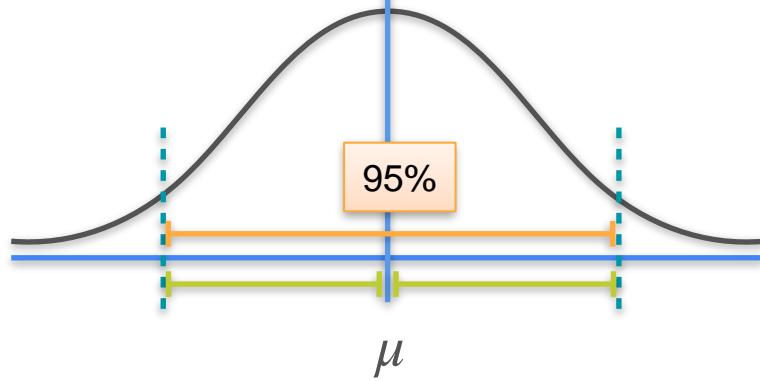
$$n = 2$$
$$\mathcal{N}\left(\mu, \frac{\sigma^2}{2}\right)$$



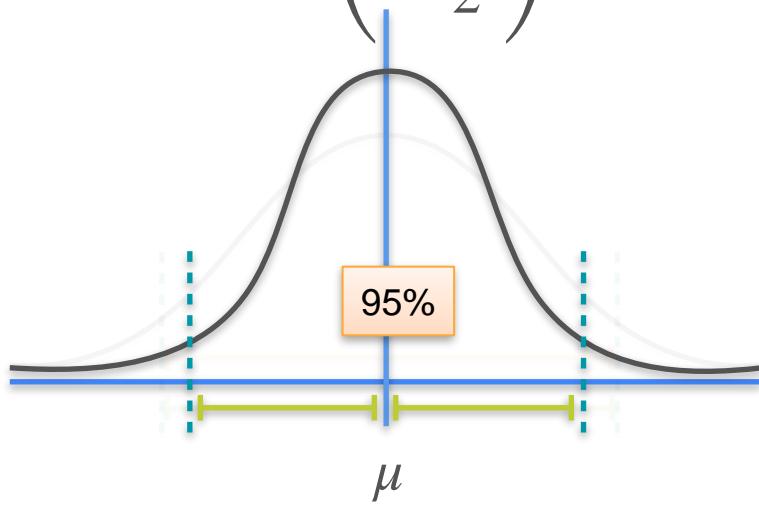
# Confidence Interval - Intuition

95%

$$n = 1$$
$$\mathcal{N}\left(\mu, \frac{\sigma^2}{1}\right)$$



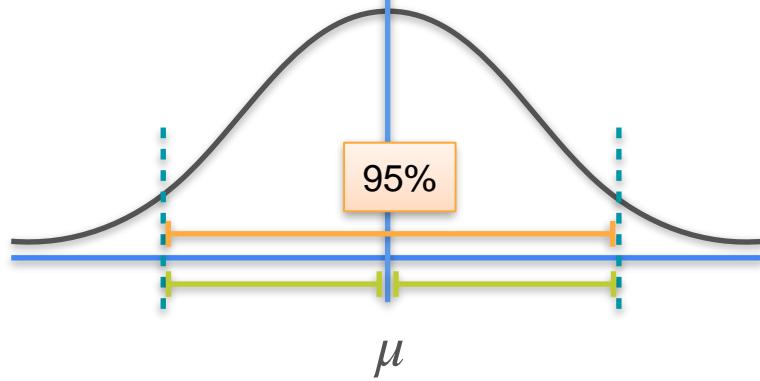
$$n = 2$$
$$\mathcal{N}\left(\mu, \frac{\sigma^2}{2}\right)$$



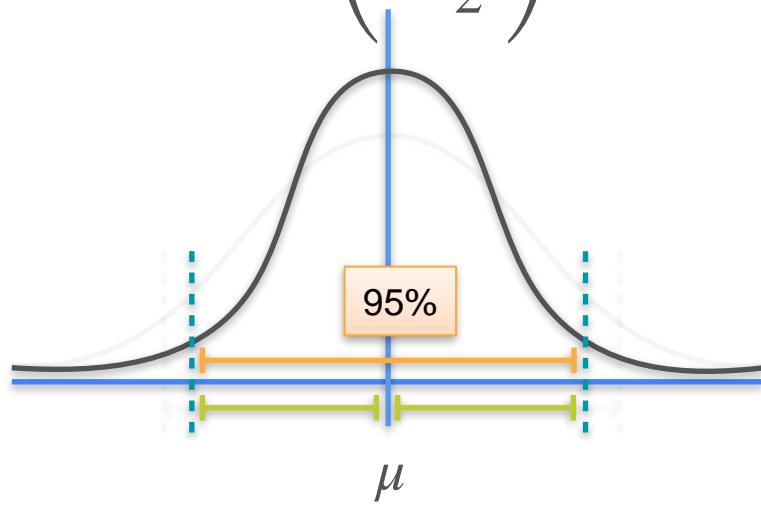
# Confidence Interval - Intuition

95%

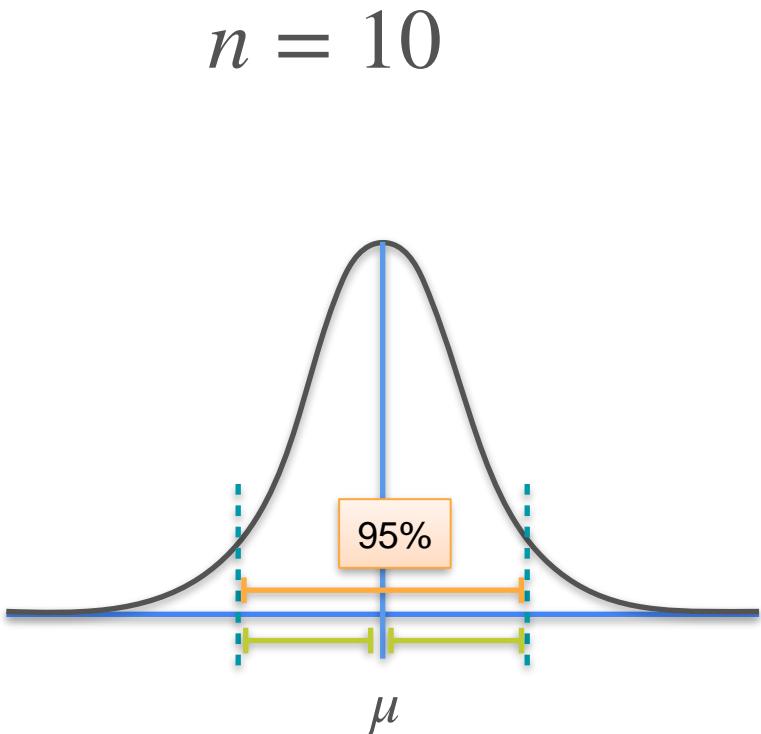
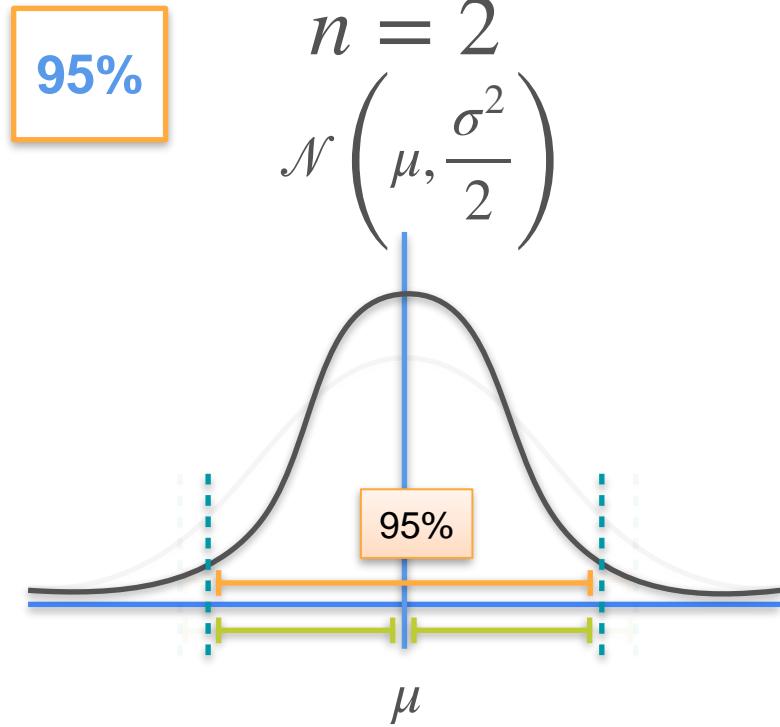
$$n = 1$$
$$\mathcal{N}\left(\mu, \frac{\sigma^2}{1}\right)$$



$$n = 2$$
$$\mathcal{N}\left(\mu, \frac{\sigma^2}{2}\right)$$



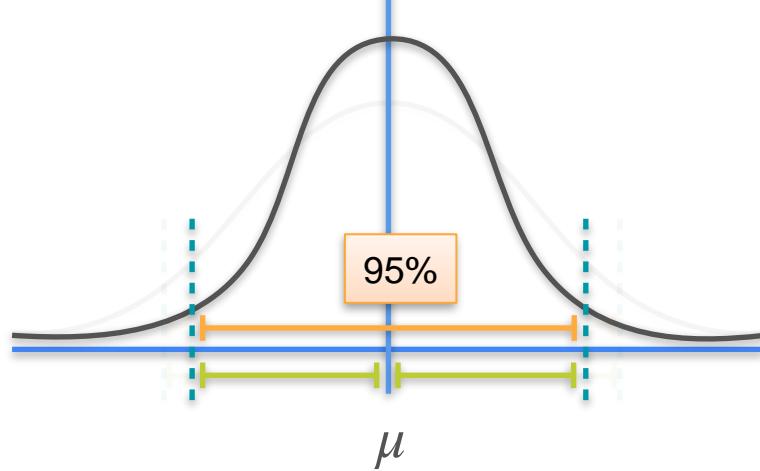
# Confidence Interval - Intuition



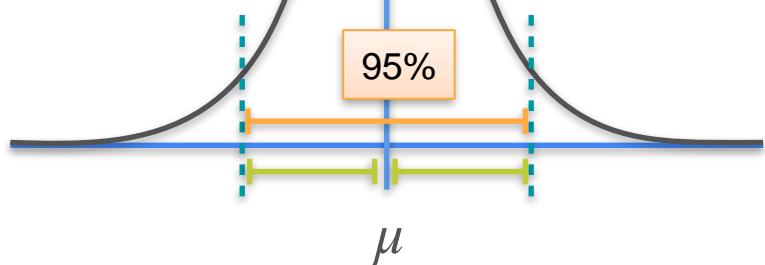
# Confidence Interval - Intuition

95%

$$n = 2$$
$$\mathcal{N}\left(\mu, \frac{\sigma^2}{2}\right)$$



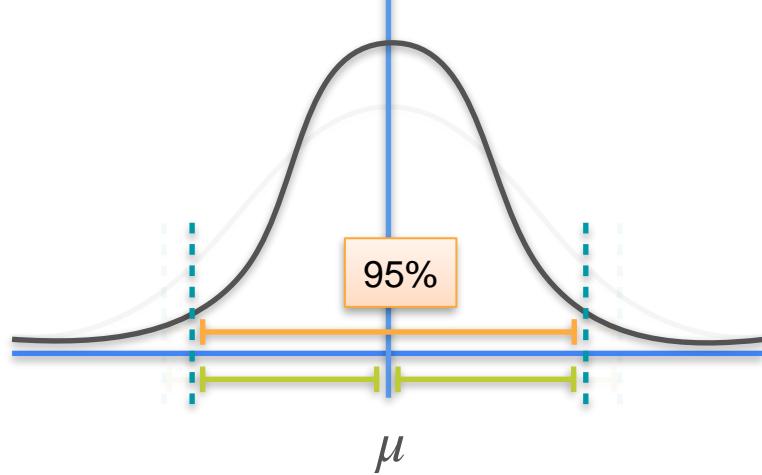
$$n = 10$$
$$\mathcal{N}\left(\mu, \frac{\sigma^2}{10}\right)$$



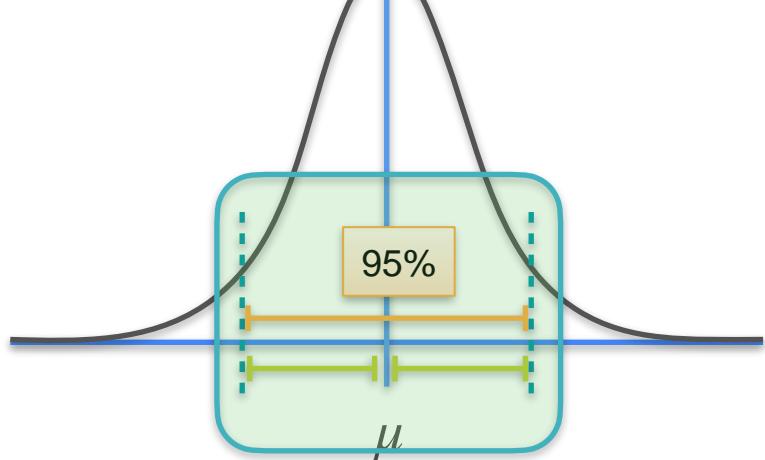
# Confidence Interval - Intuition

95%

$$n = 2$$
$$\mathcal{N}\left(\mu, \frac{\sigma^2}{2}\right)$$



$$n = 10$$
$$\mathcal{N}\left(\mu, \frac{\sigma^2}{10}\right)$$



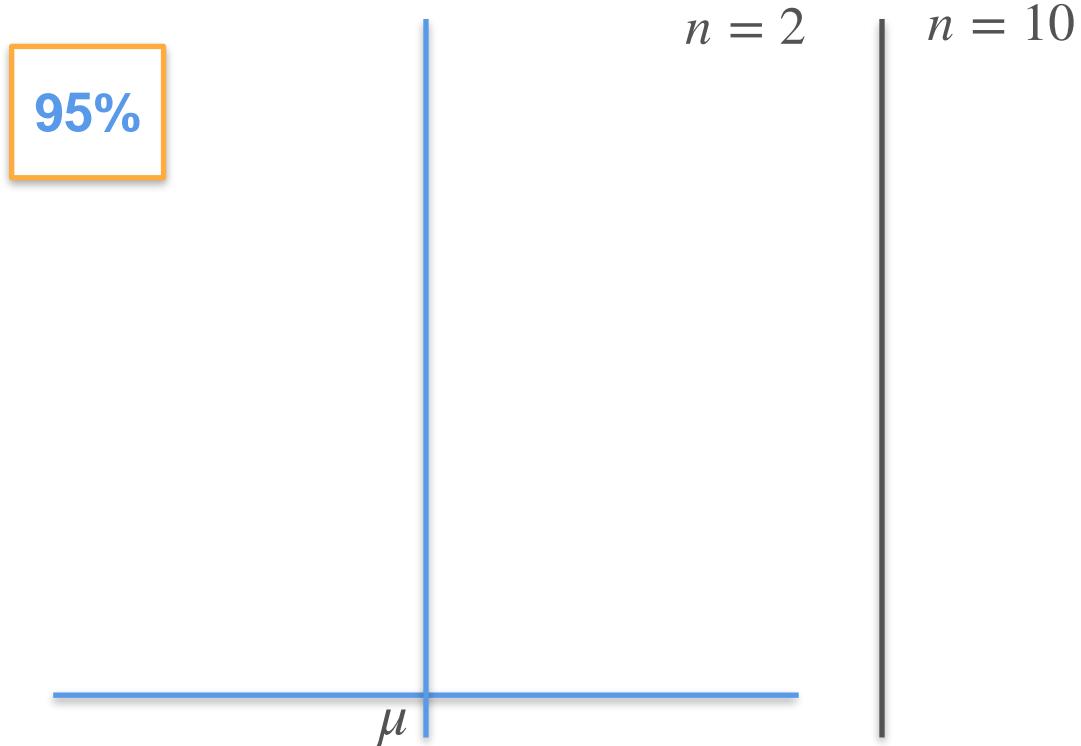
# Confidence Interval - Intuition

95%

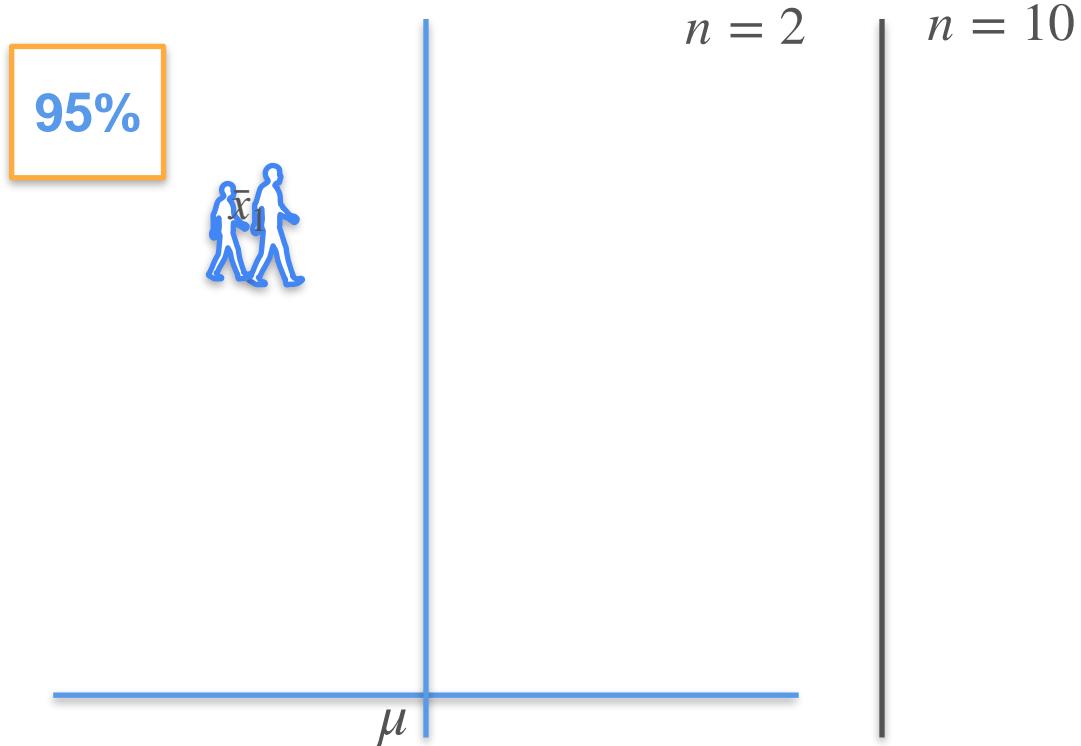
$n = 2$

$n = 10$

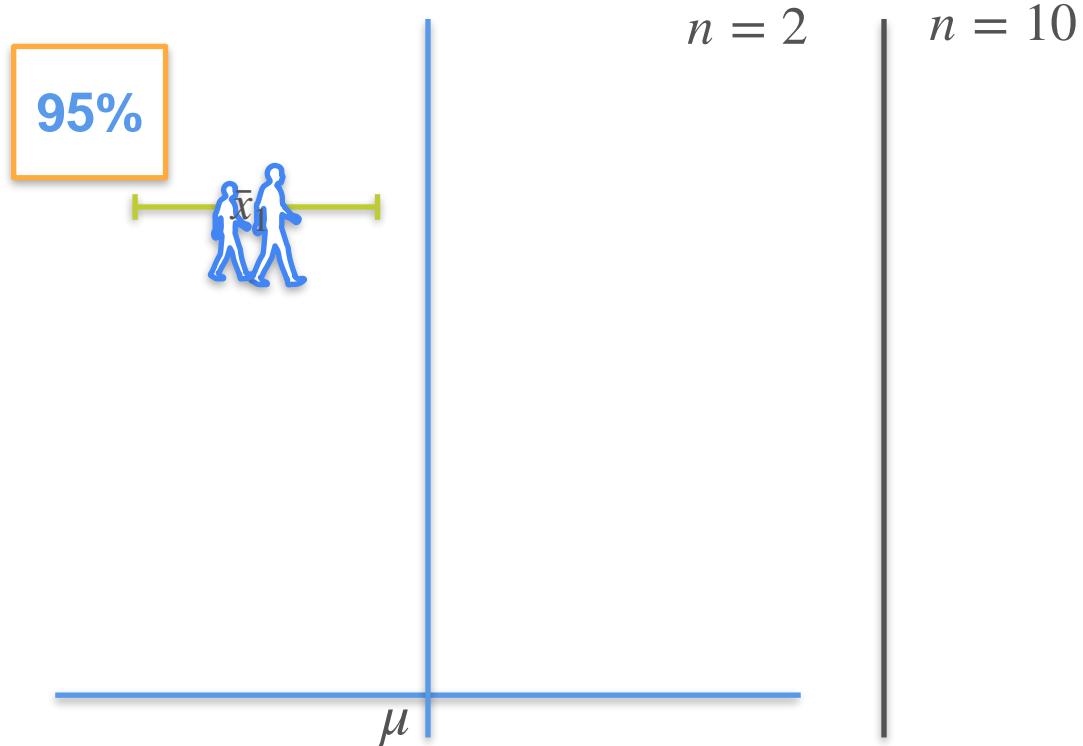
# Confidence Interval - Intuition



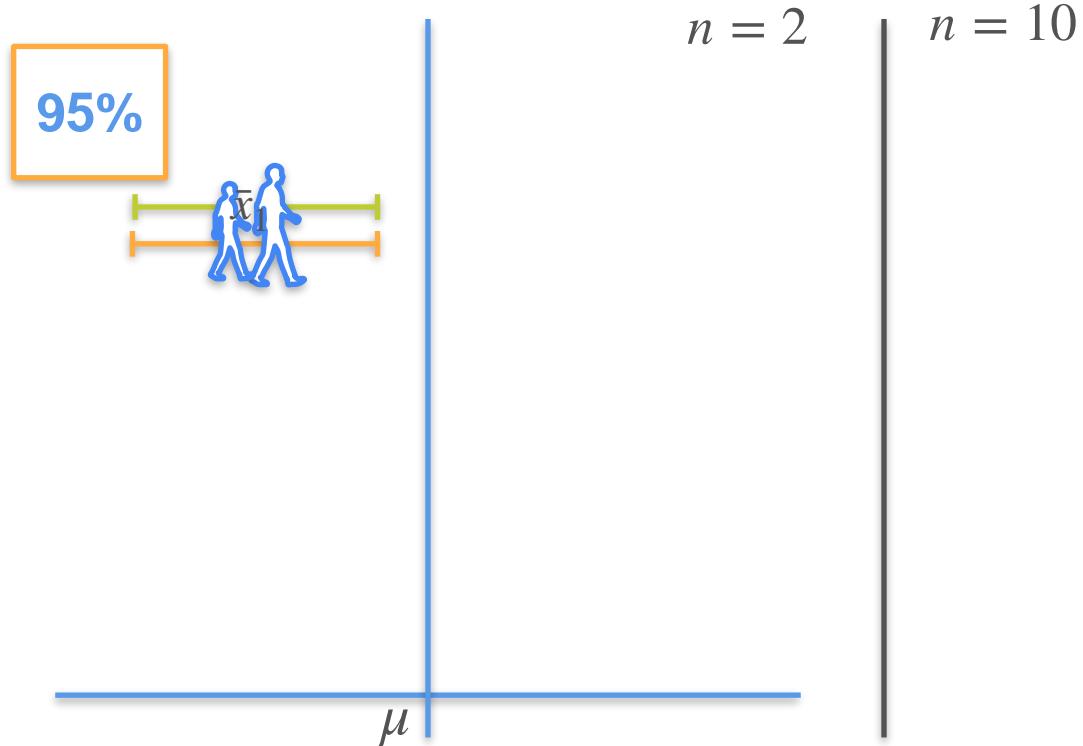
# Confidence Interval - Intuition



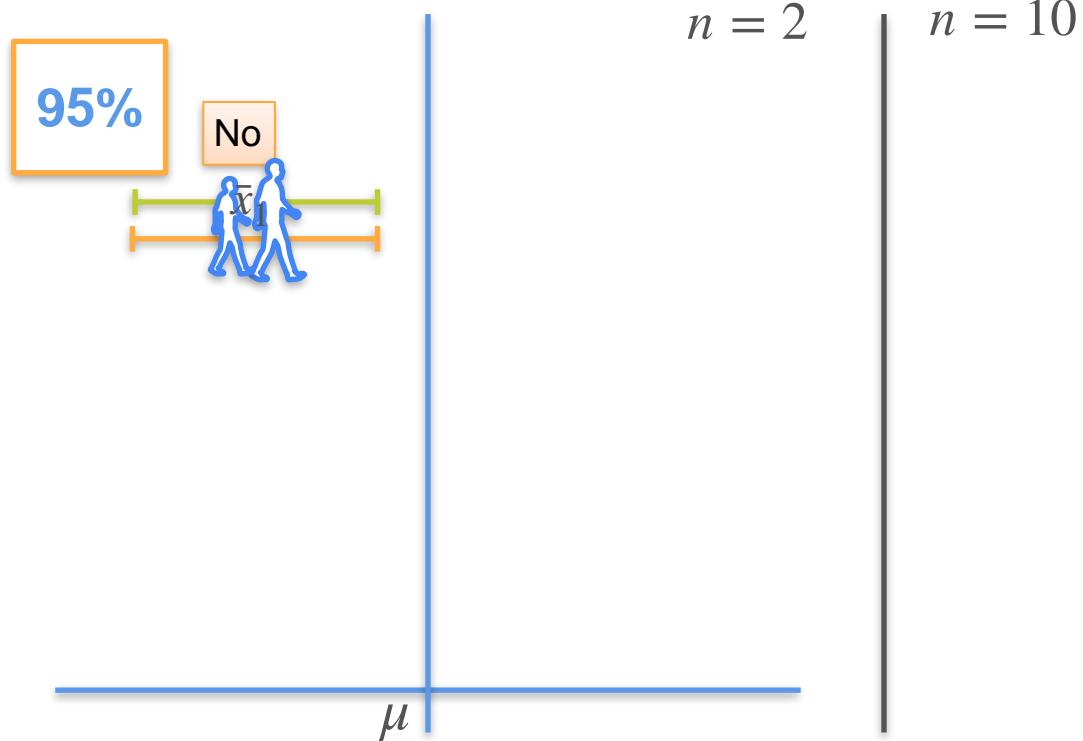
# Confidence Interval - Intuition



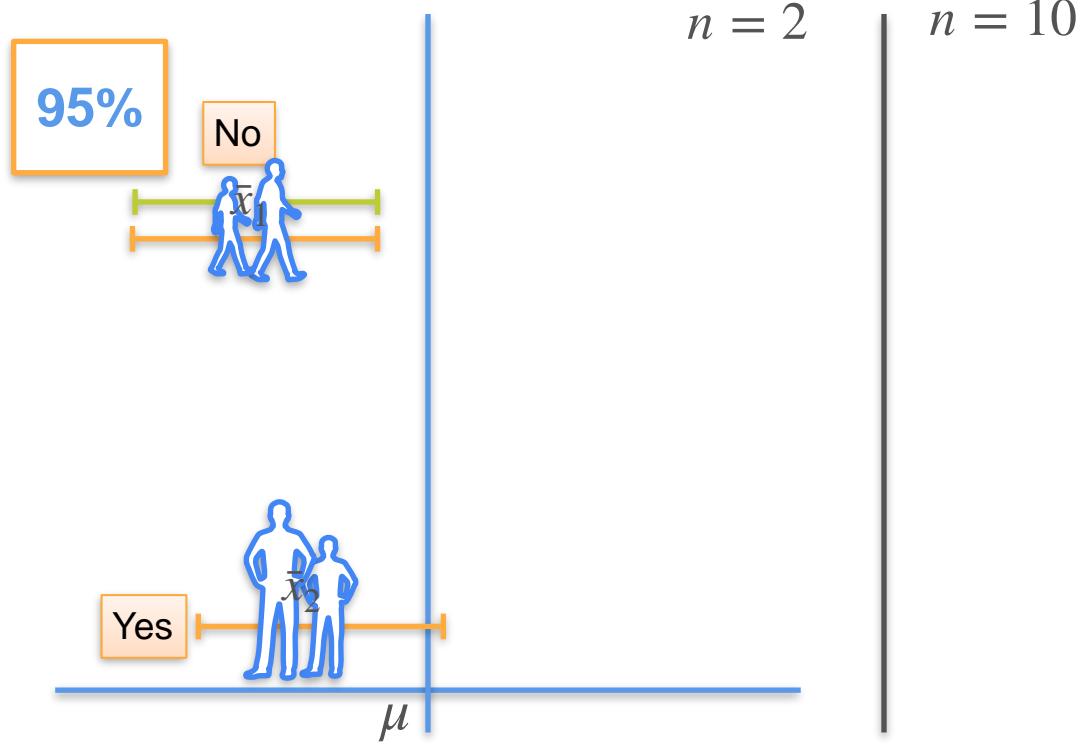
# Confidence Interval - Intuition



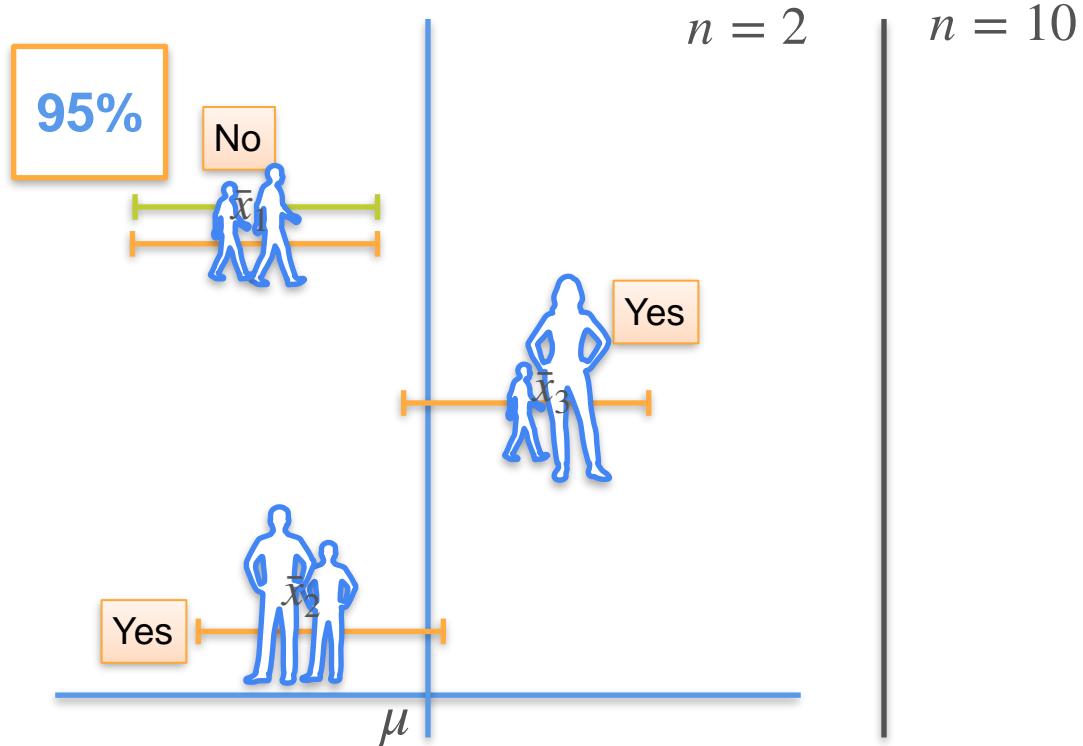
# Confidence Interval - Intuition



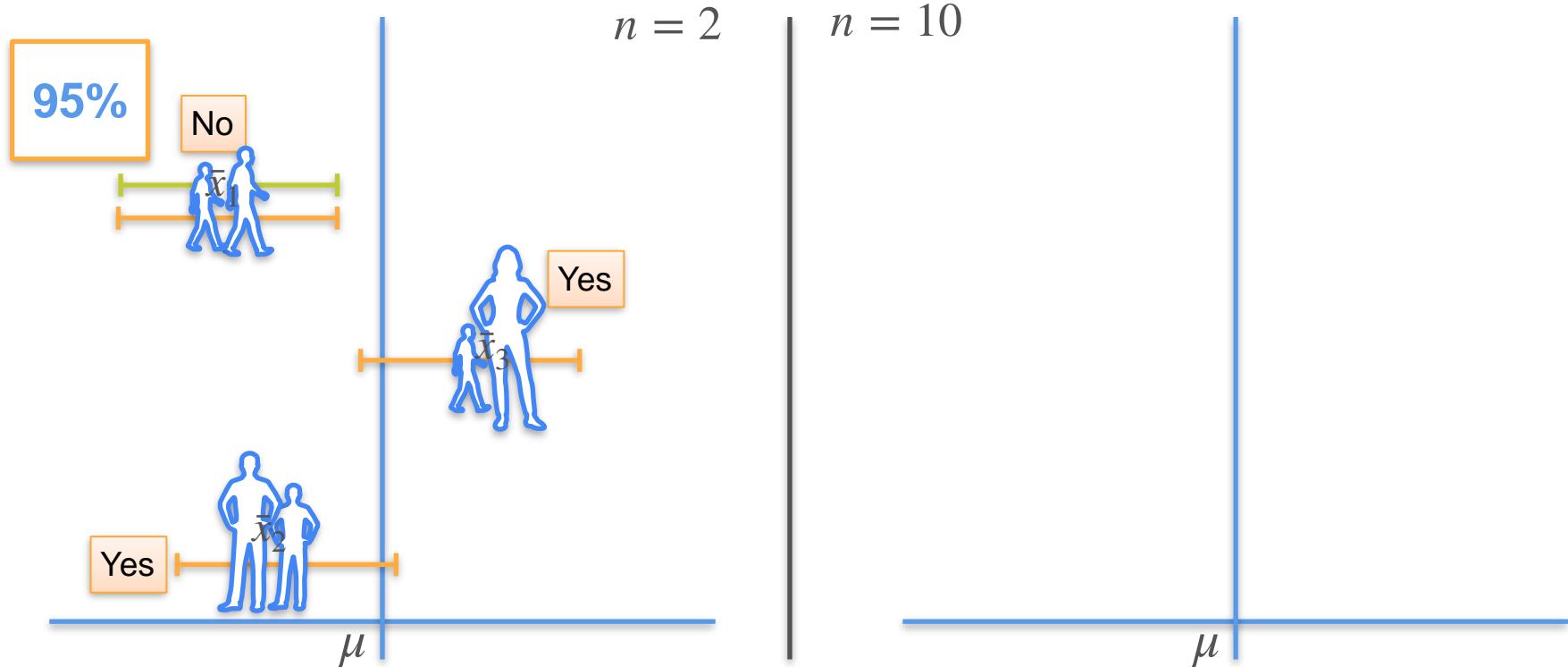
# Confidence Interval - Intuition



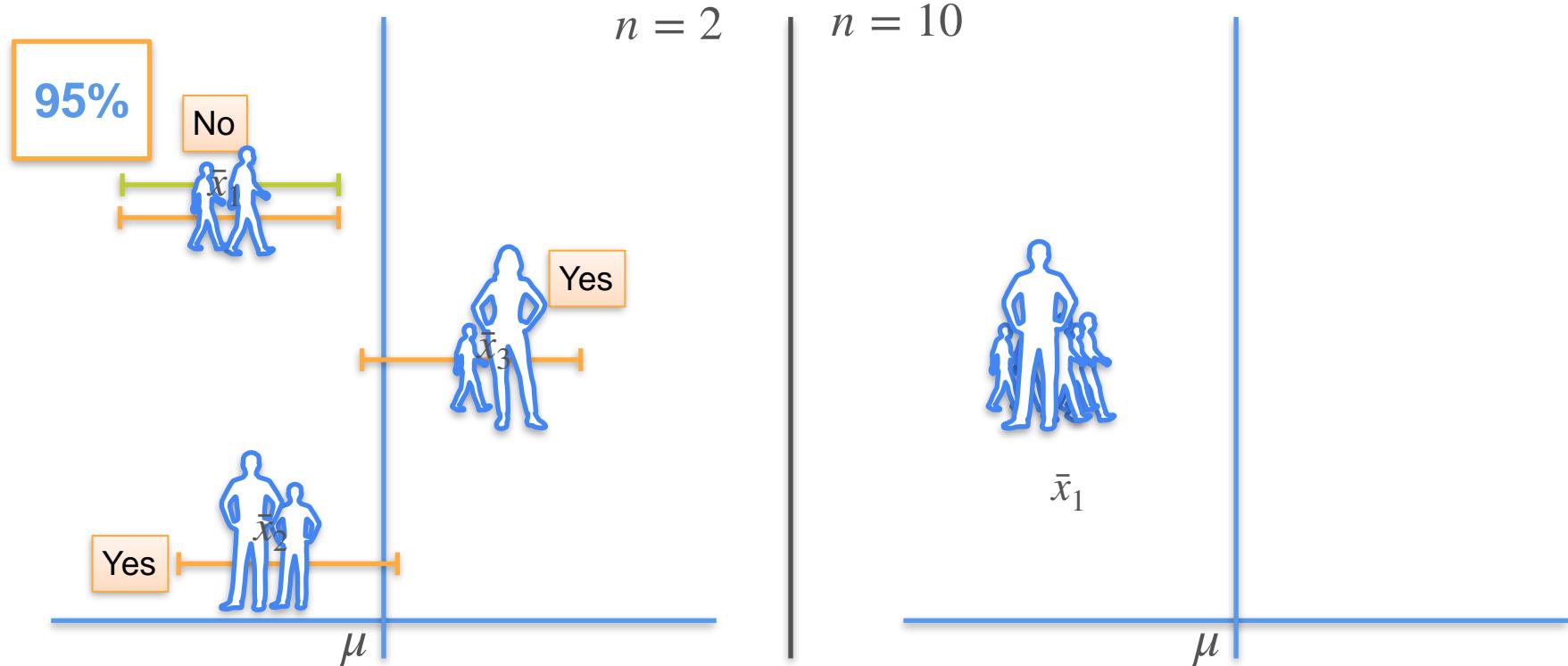
# Confidence Interval - Intuition



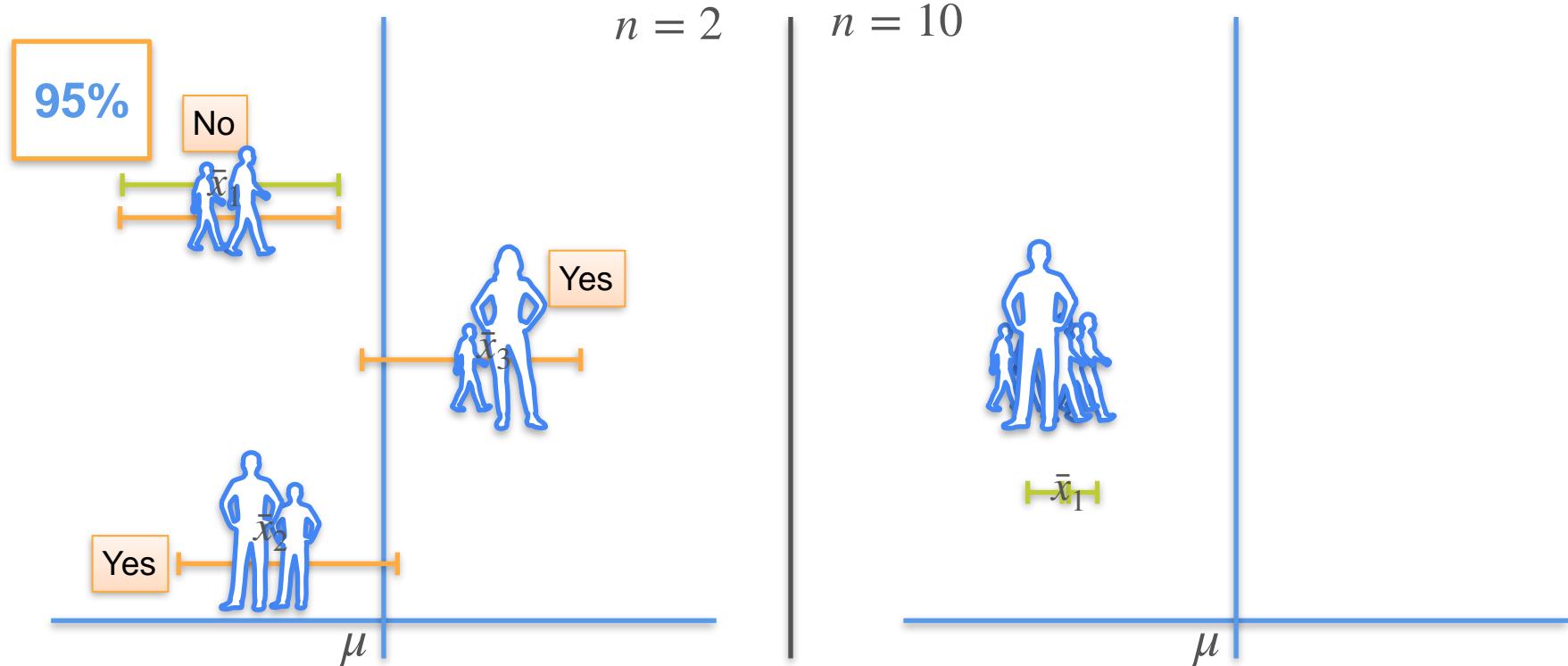
# Confidence Interval - Intuition



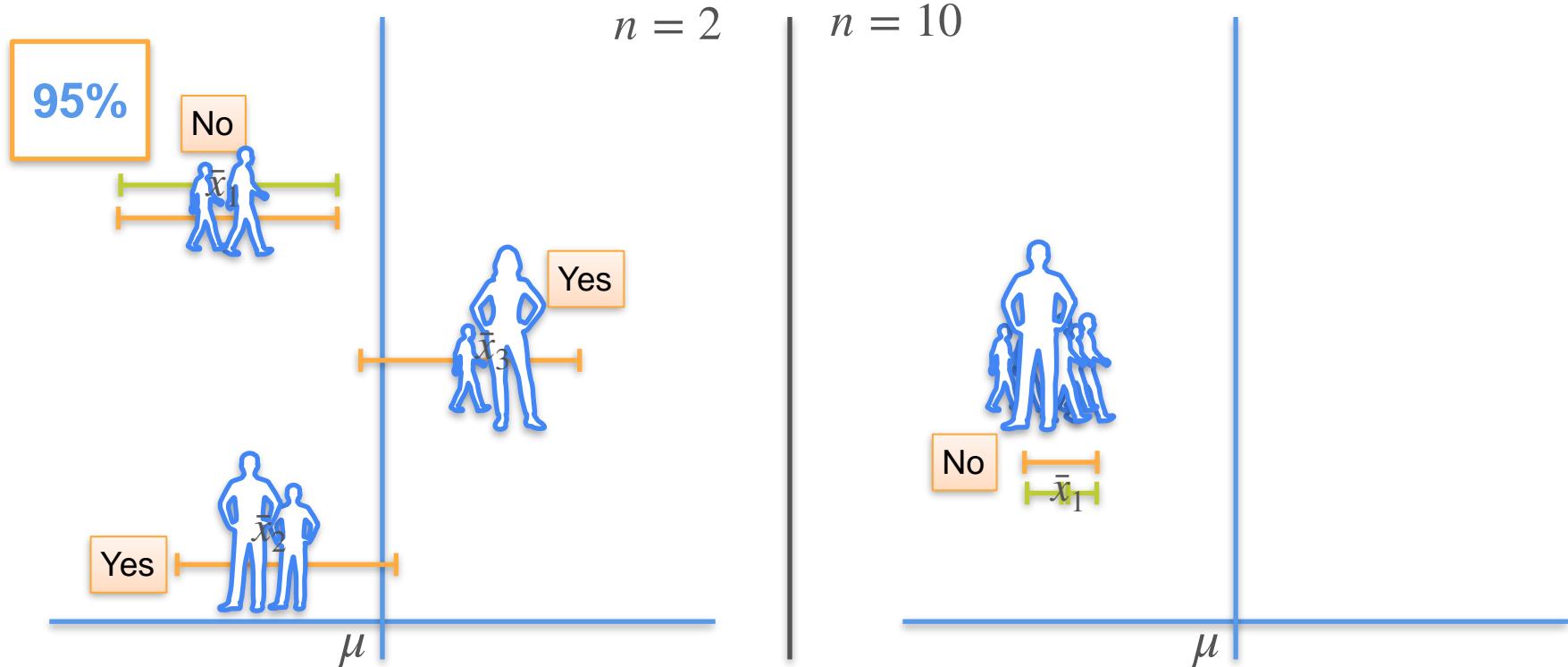
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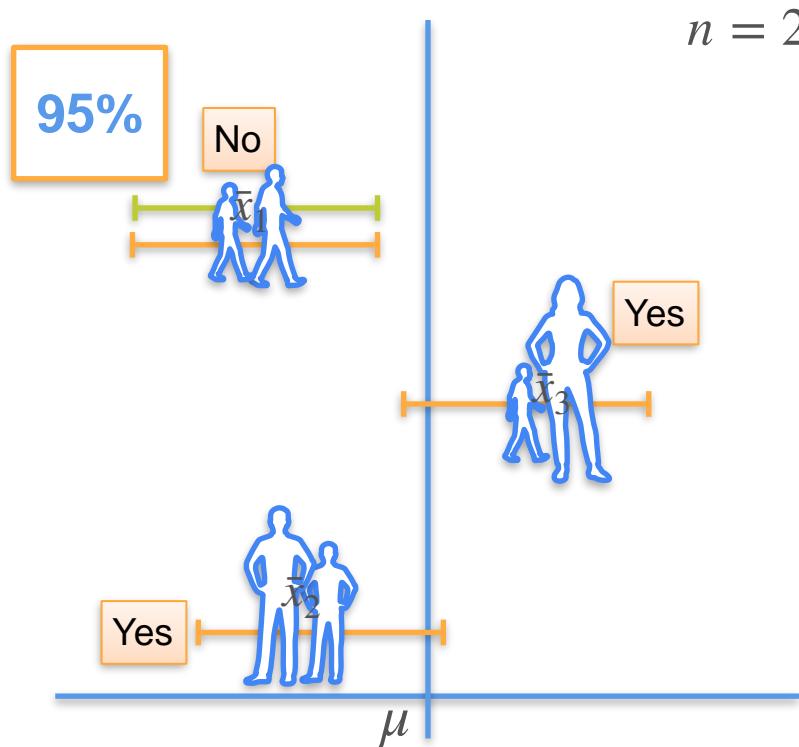
# Confidence Interval - Intuition



# Confidence Interval - Intuition

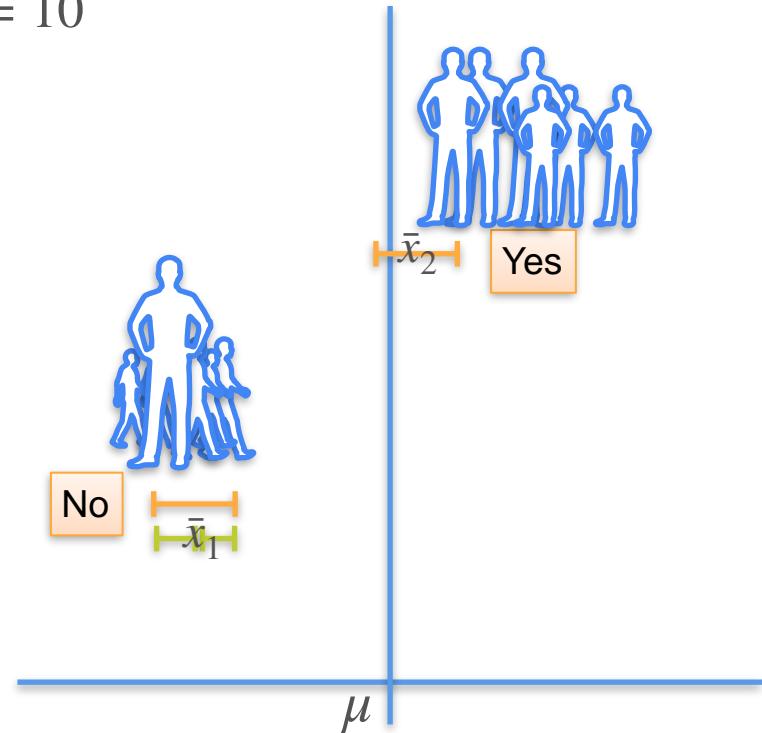


# Confidence Interval - Intuition

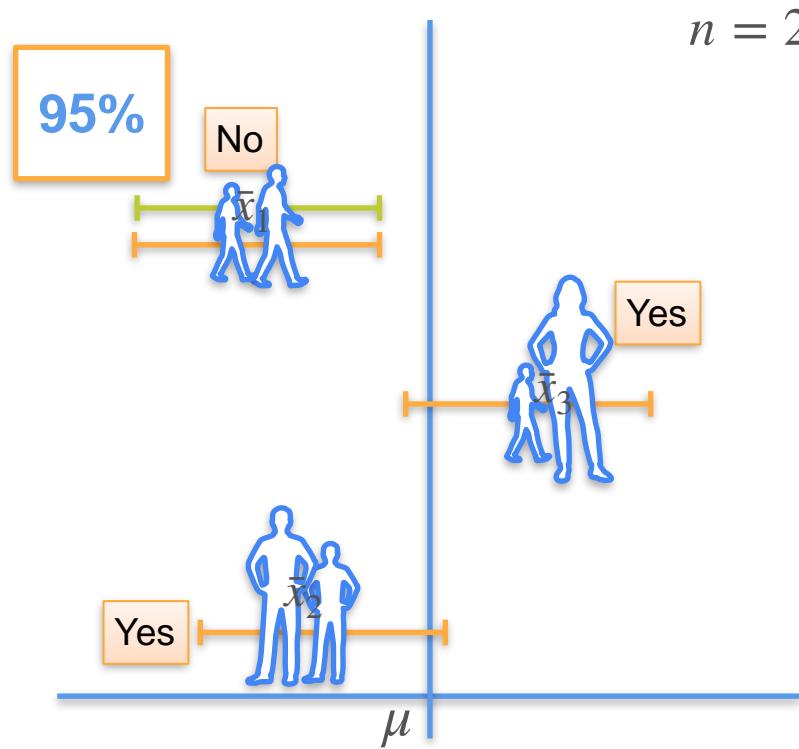


$n = 2$

$n = 10$

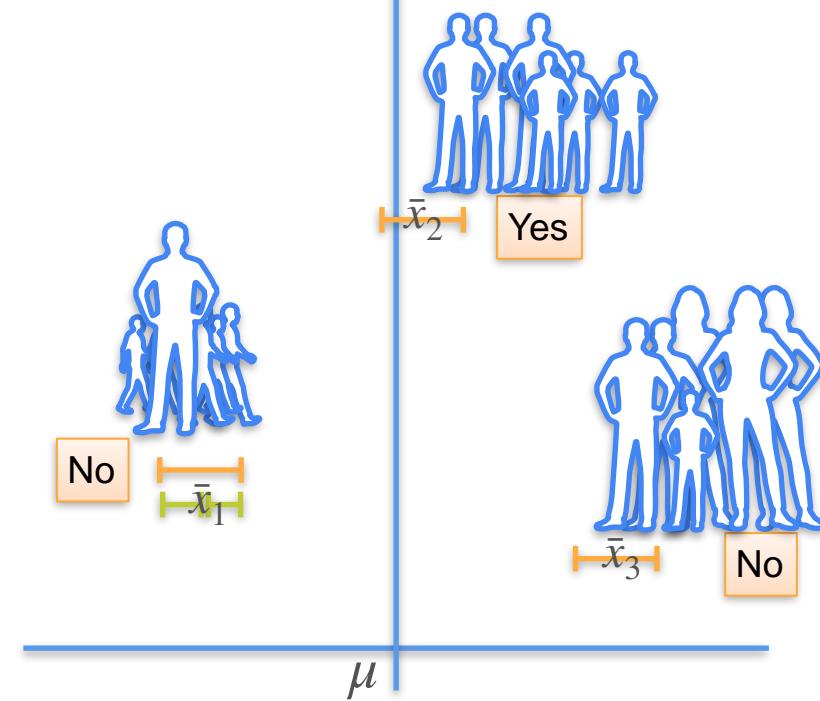


# Confidence Interval - Intuition

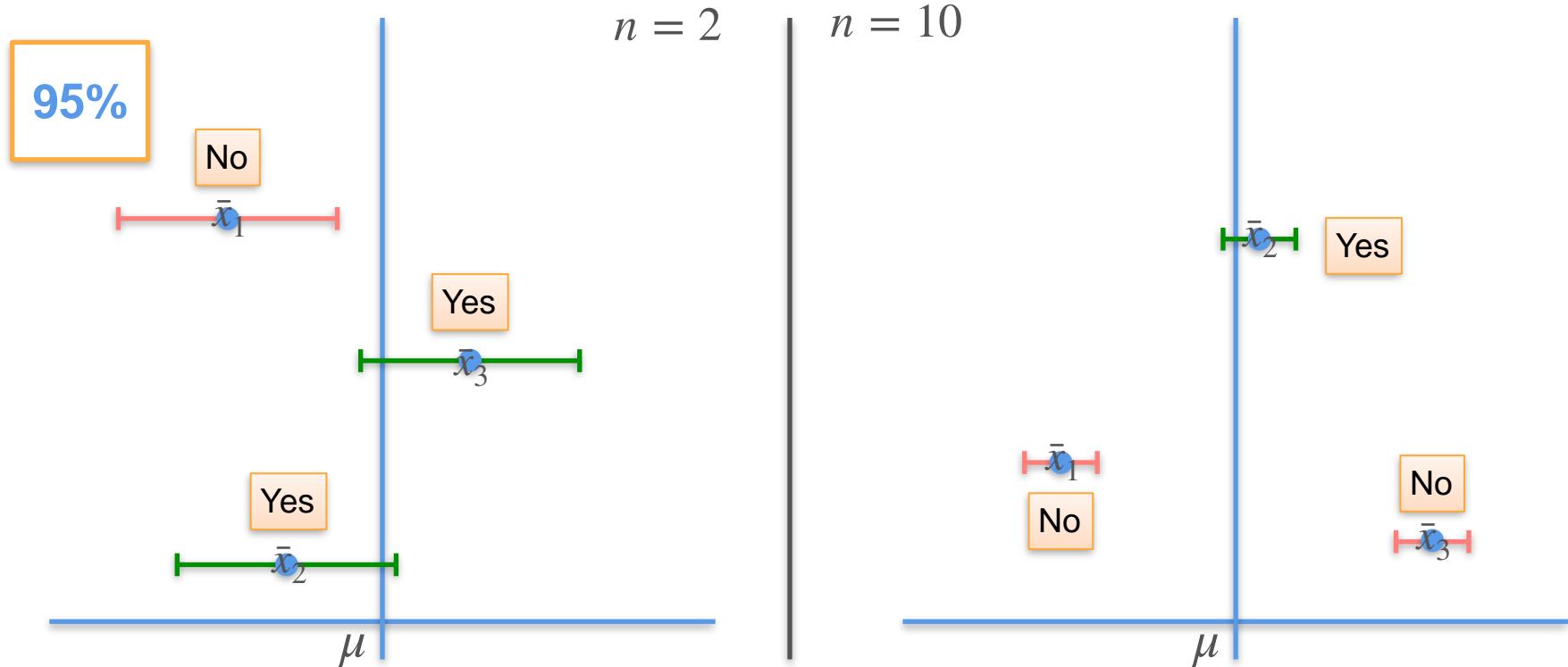


$n = 2$

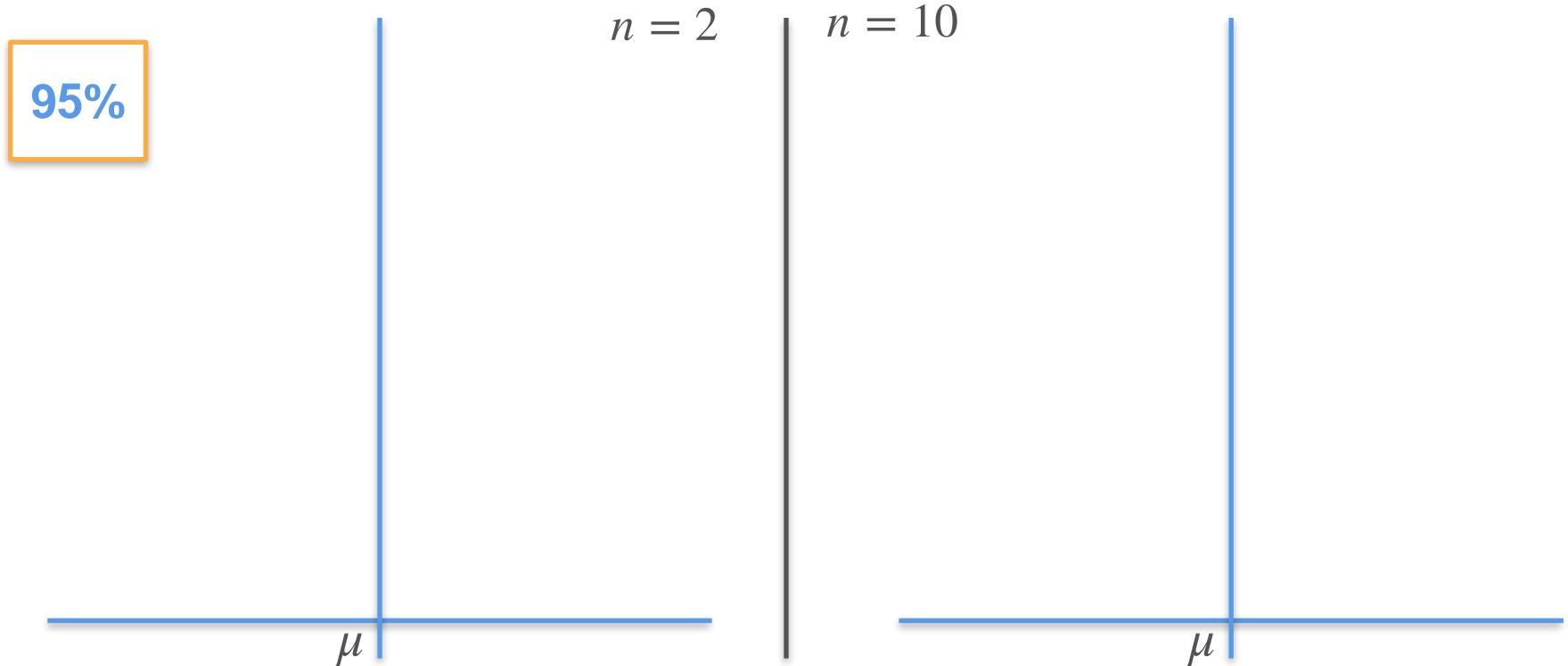
$n = 10$



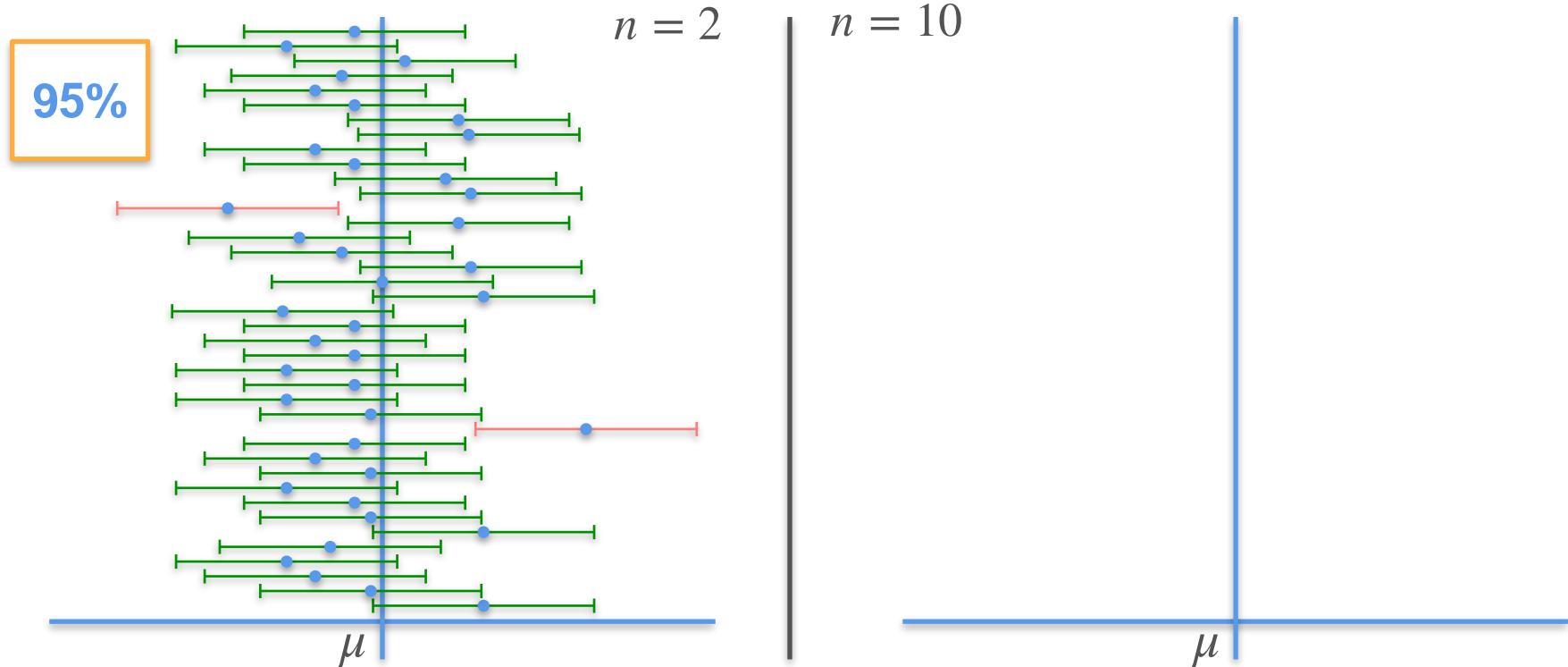
# Confidence Interval - Intuition



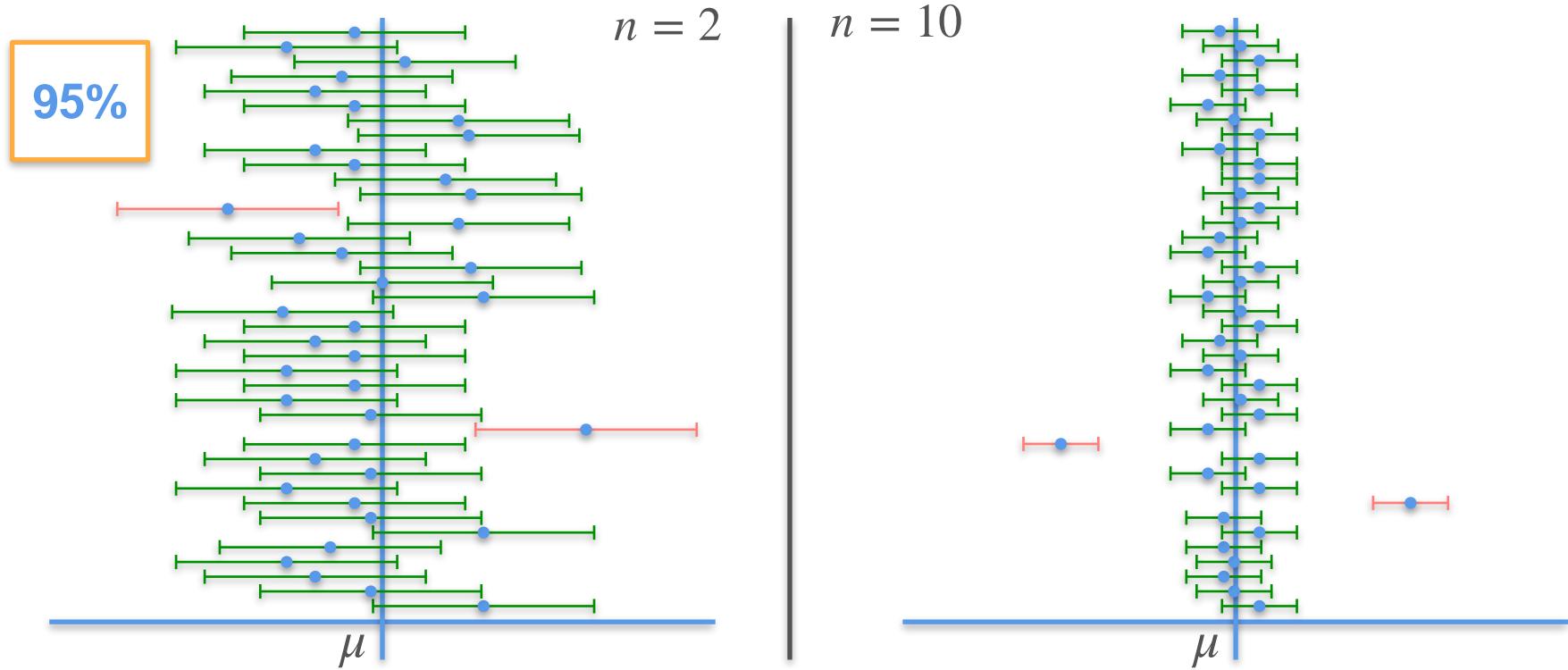
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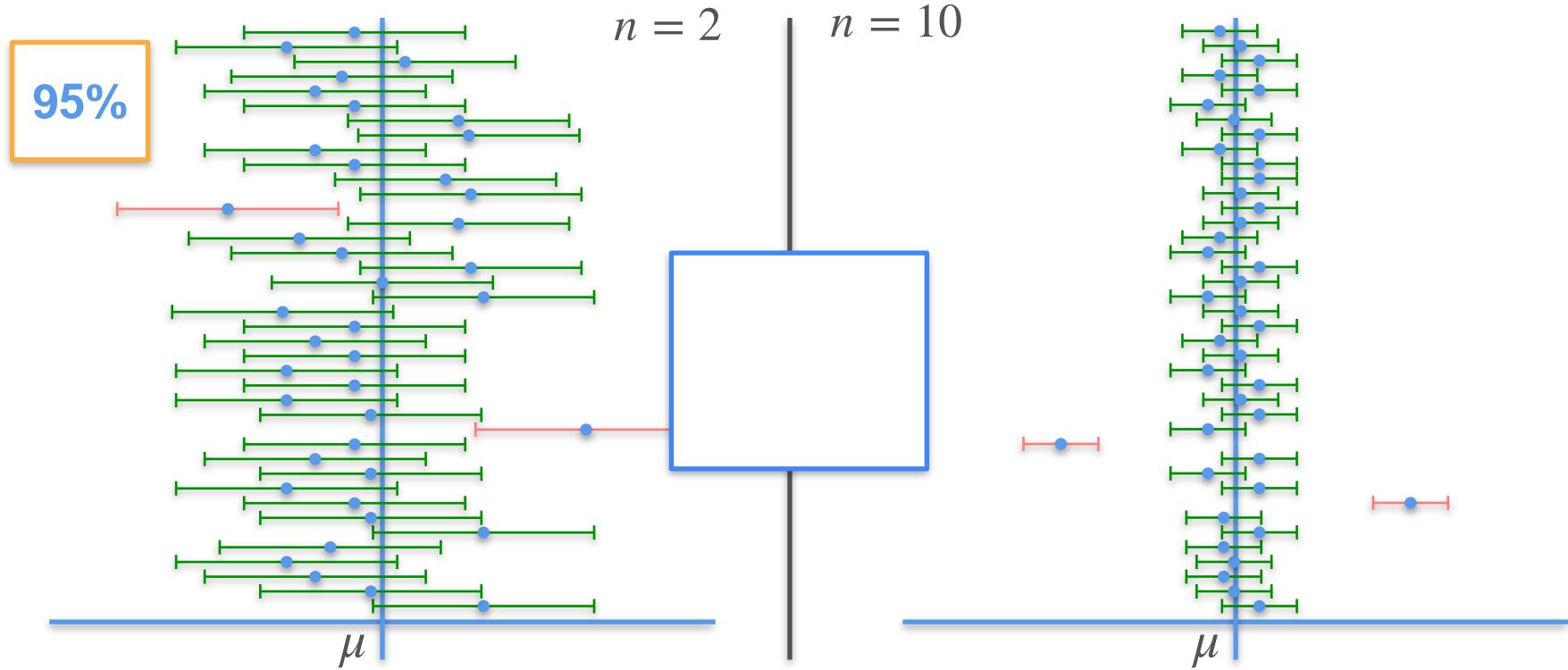
# Confidence Interval - Intuition



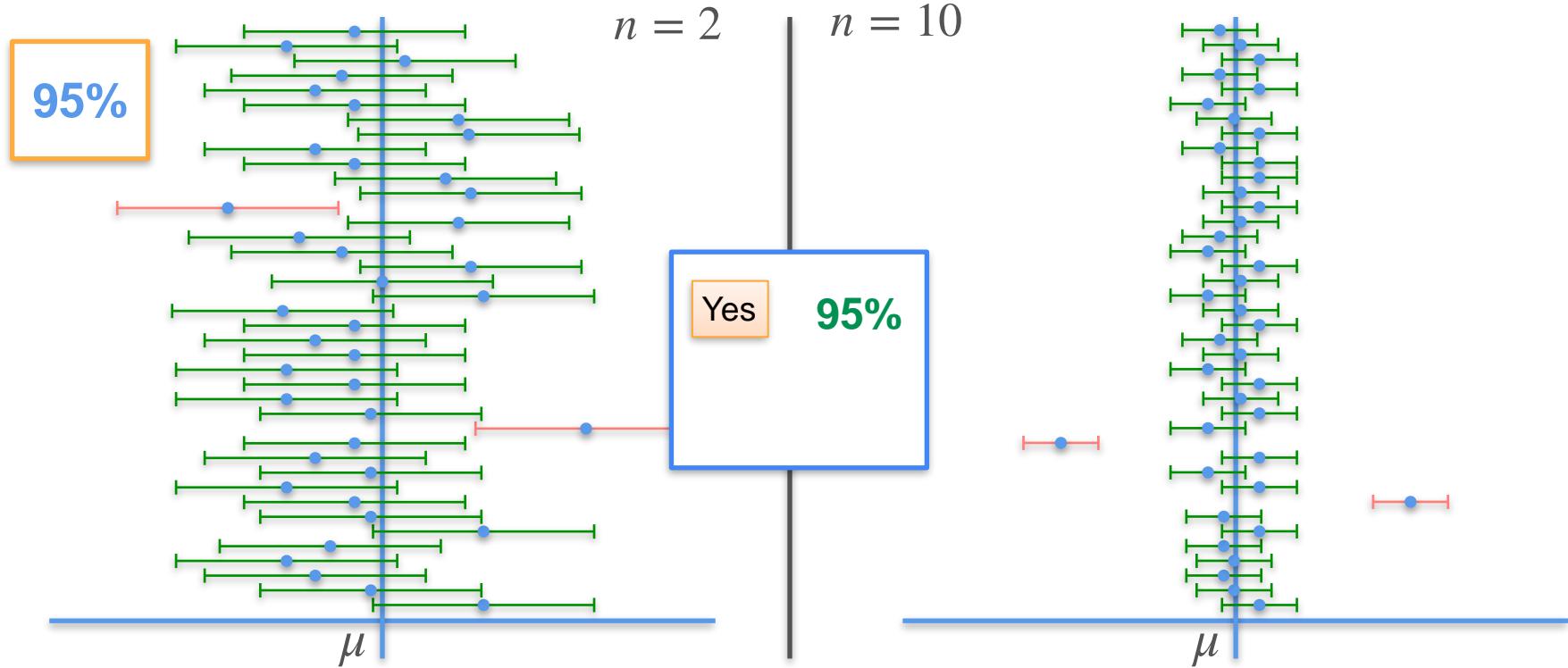
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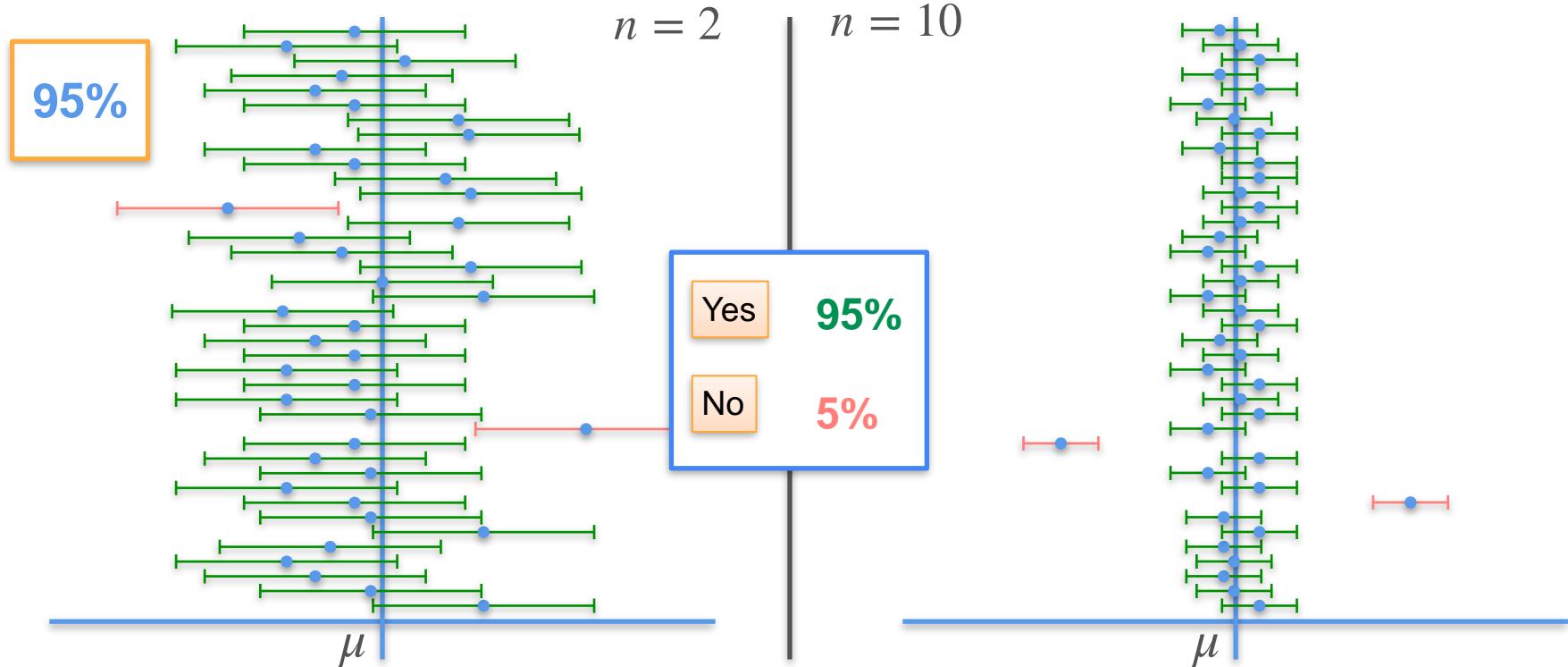
# Confidence Interval - Intuition



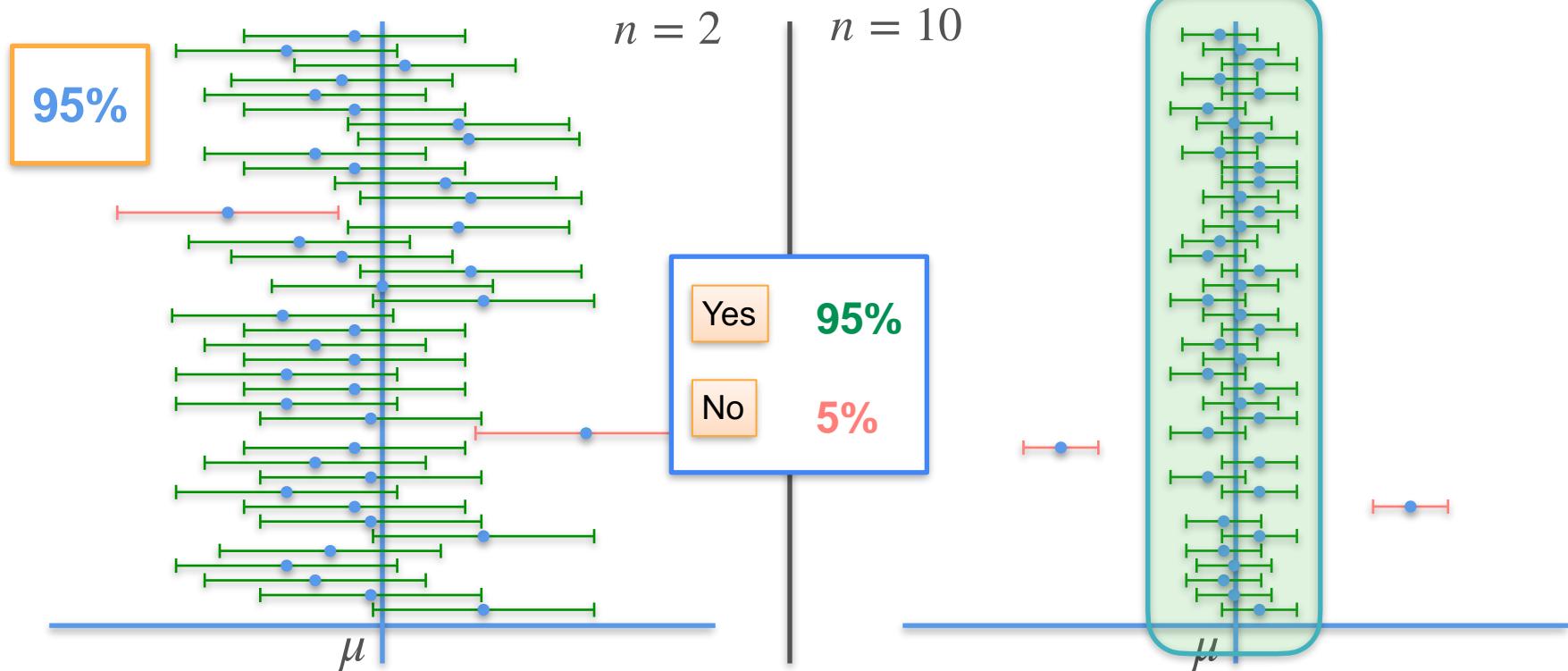
# Confidence Interval - Intuition



# Confidence Interval - Intuition



# Confidence Interval - Intuition

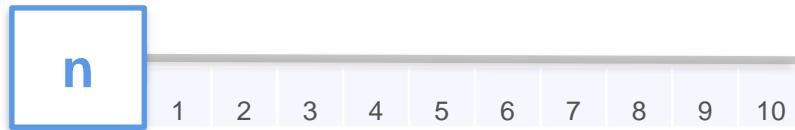


# Effect of the Sample Size

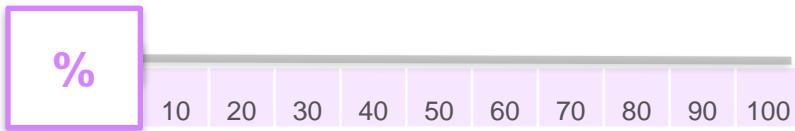


# Effect of the Sample Size

sample size

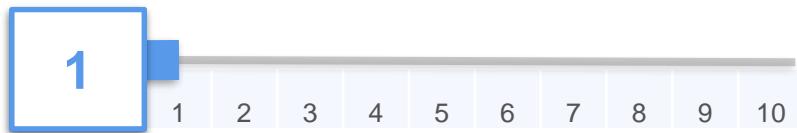


Confidence level

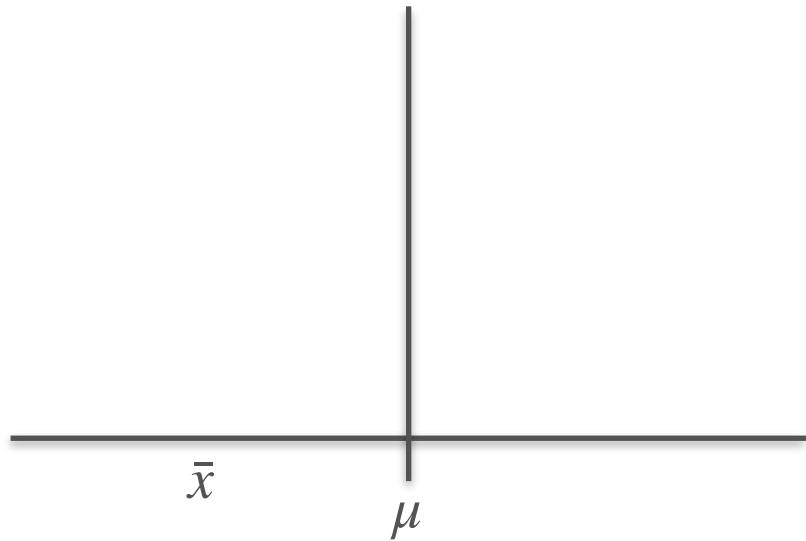
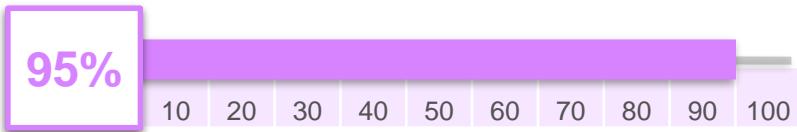


# Effect of the Sample Size

sample size

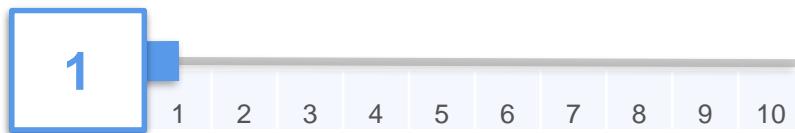


Confidence level

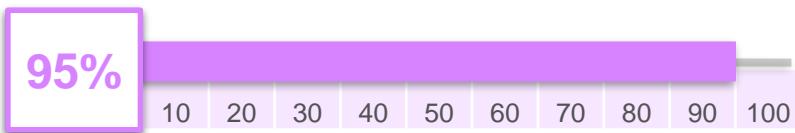


# Effect of the Sample Size

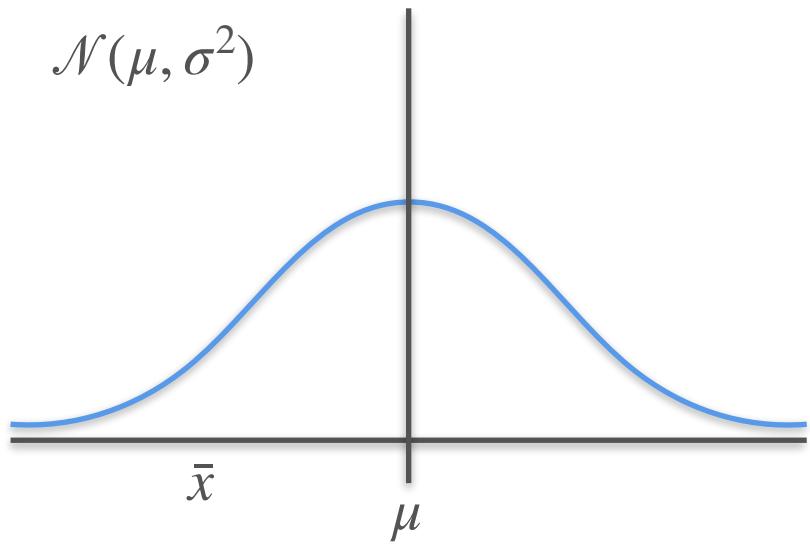
sample size



Confidence level



$$\mathcal{N}(\mu, \sigma^2)$$

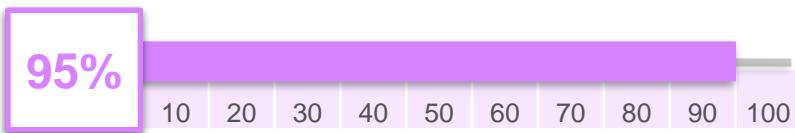


# Effect of the Sample Size

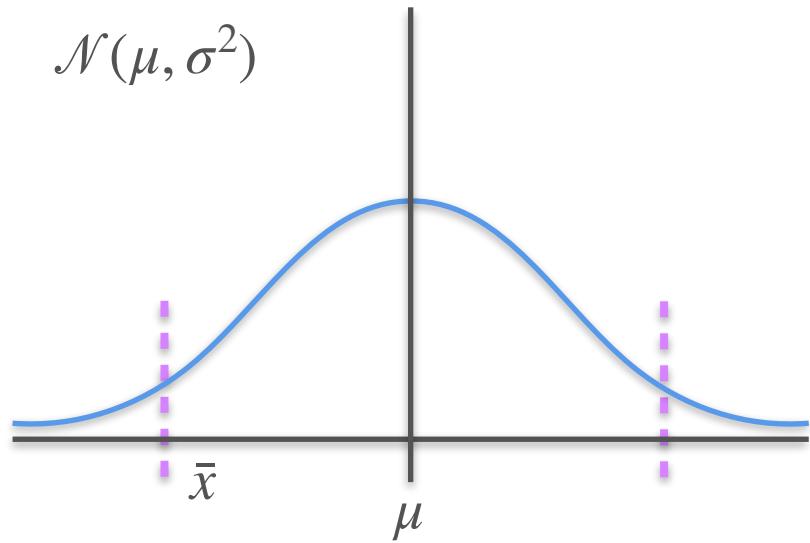
sample size



Confidence level

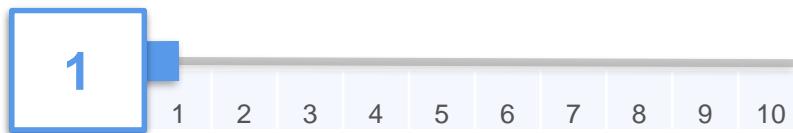


$$\mathcal{N}(\mu, \sigma^2)$$

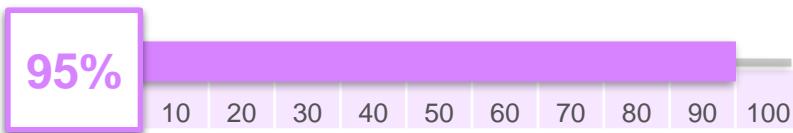


# Effect of the Sample Size

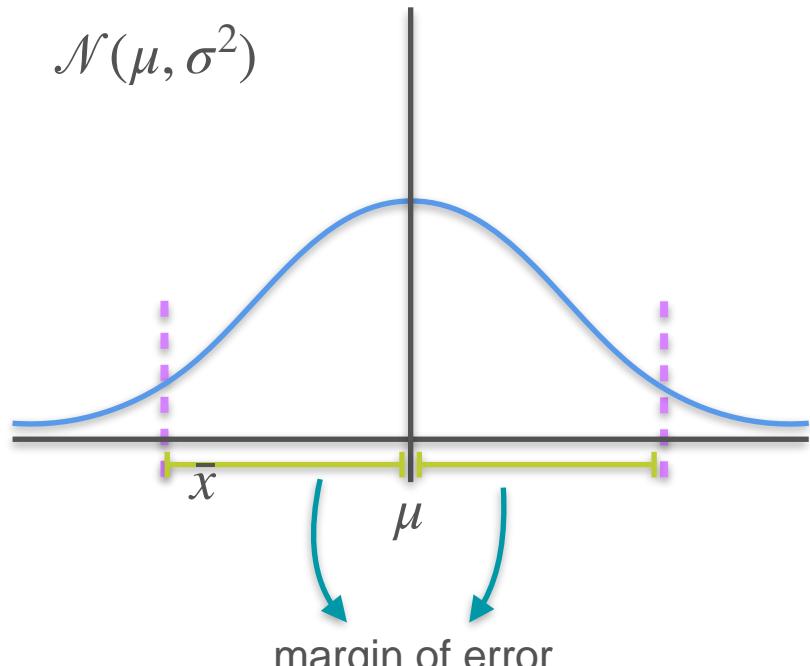
sample size



Confidence level

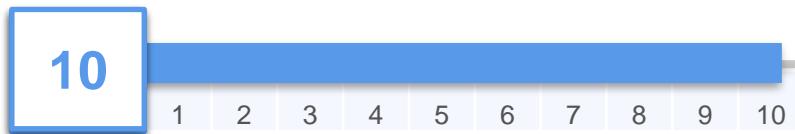


$$\mathcal{N}(\mu, \sigma^2)$$

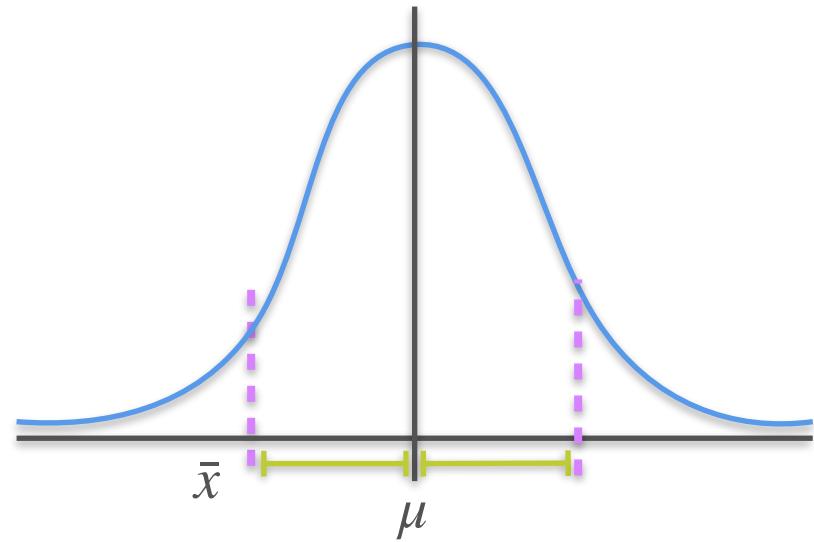
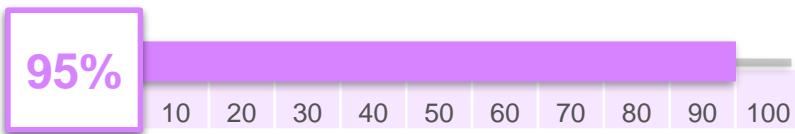


# Effect of the Sample Size

sample size

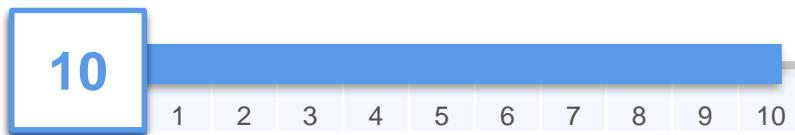


Confidence level

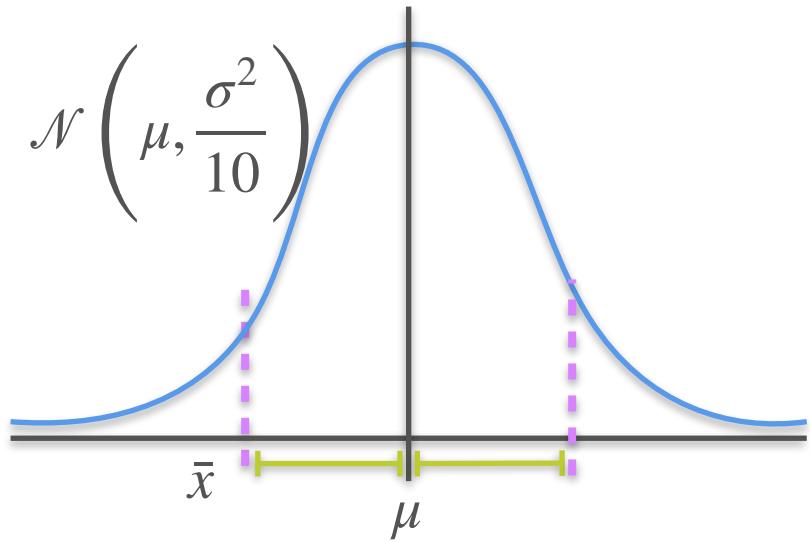
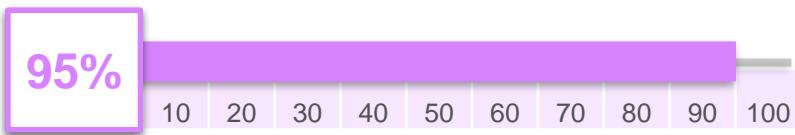


# Effect of the Sample Size

sample size

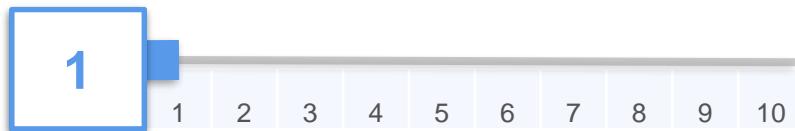


Confidence level

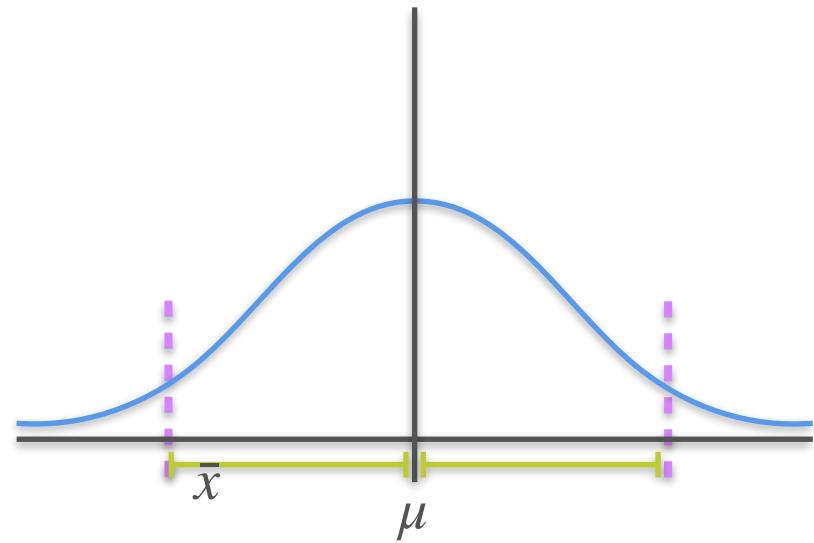
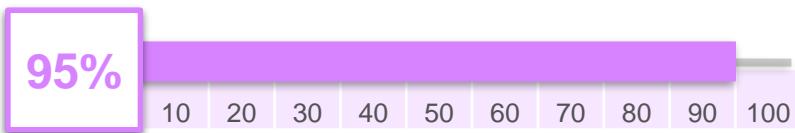


# Effect of the Sample Size

sample size

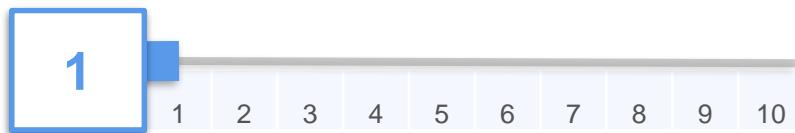


Confidence level

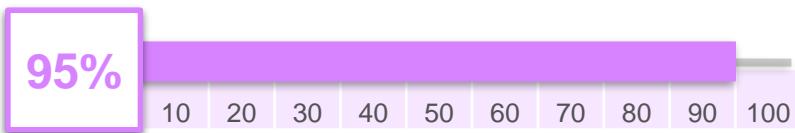


# Effect of the Sample Size

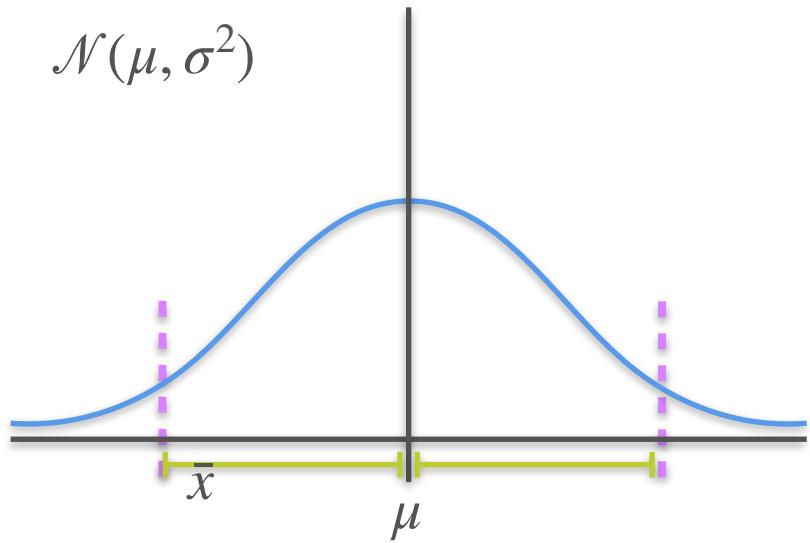
sample size



Confidence level

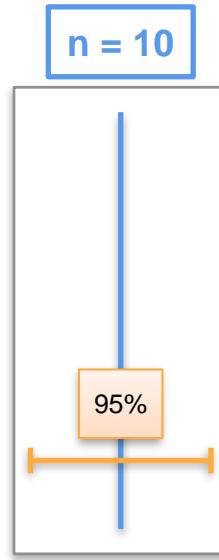
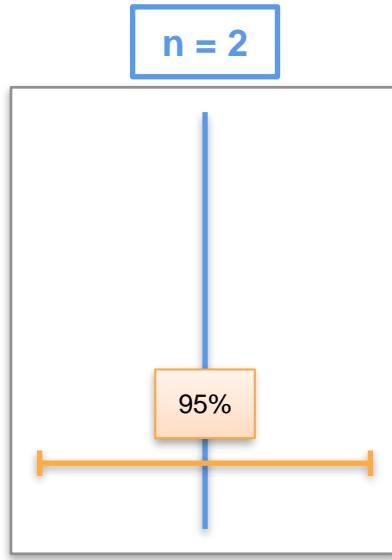
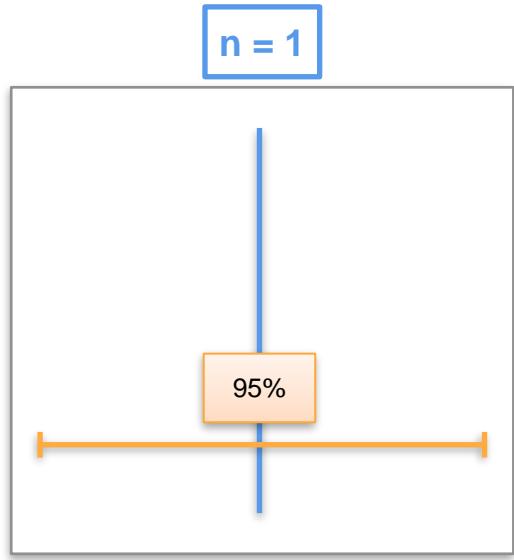


$$\mathcal{N}(\mu, \sigma^2)$$

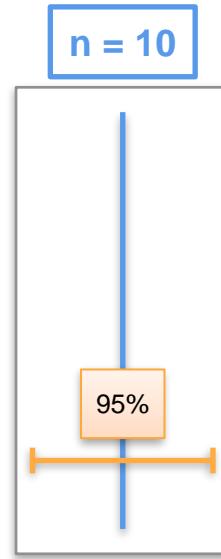
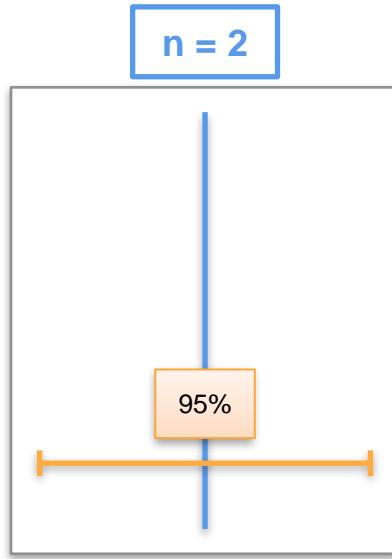
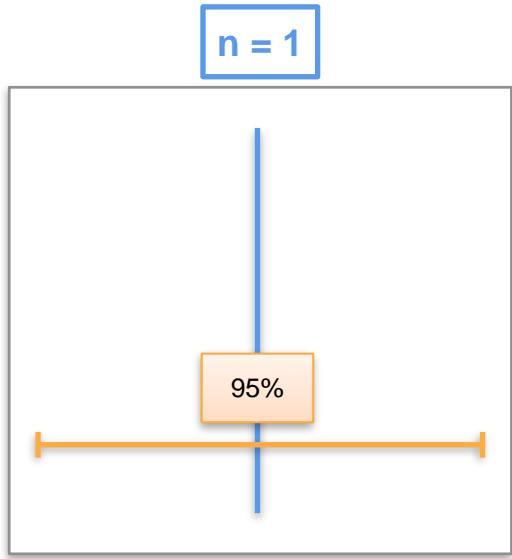


# Effect of the Sample Size

# Effect of the Sample Size



# Effect of the Sample Size



As  $n$  increases, the confidence interval shrinks

# Effect of the Confidence Level



# Effect of the Confidence Level

$n = 1$



# Effect of the Confidence Level

$n = 1$

$\mathcal{N}(\mu, \sigma^2)$



# Effect of the Confidence Level

$n = 1$

95%

$\mathcal{N}(\mu, \sigma^2)$

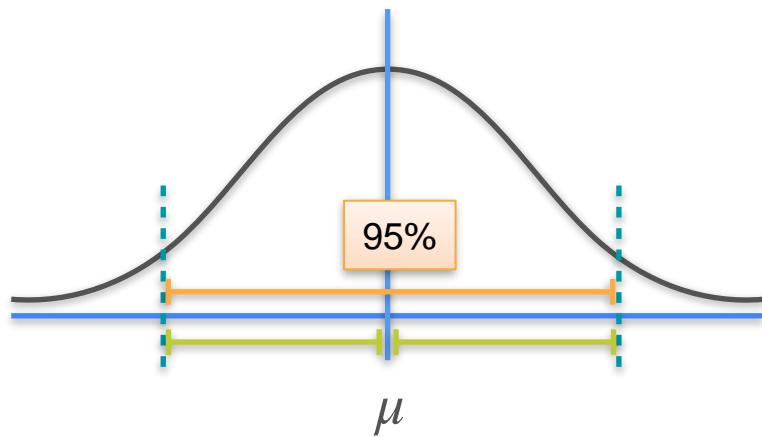


# Effect of the Confidence Level

$n = 1$

95%

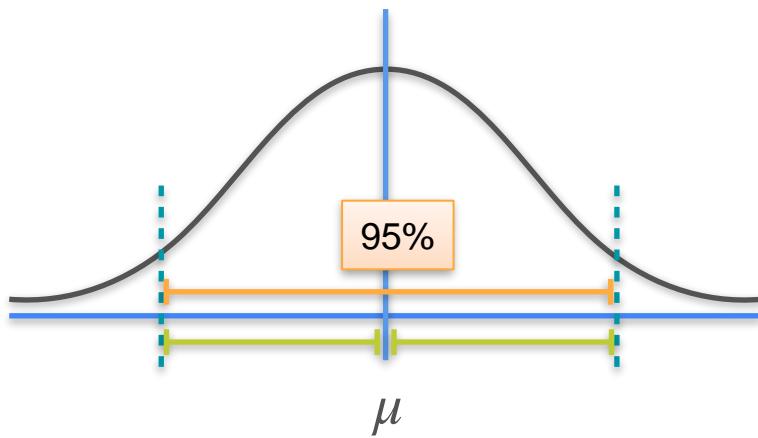
$\mathcal{N}(\mu, \sigma^2)$



# Effect of the Confidence Level

$n = 1$

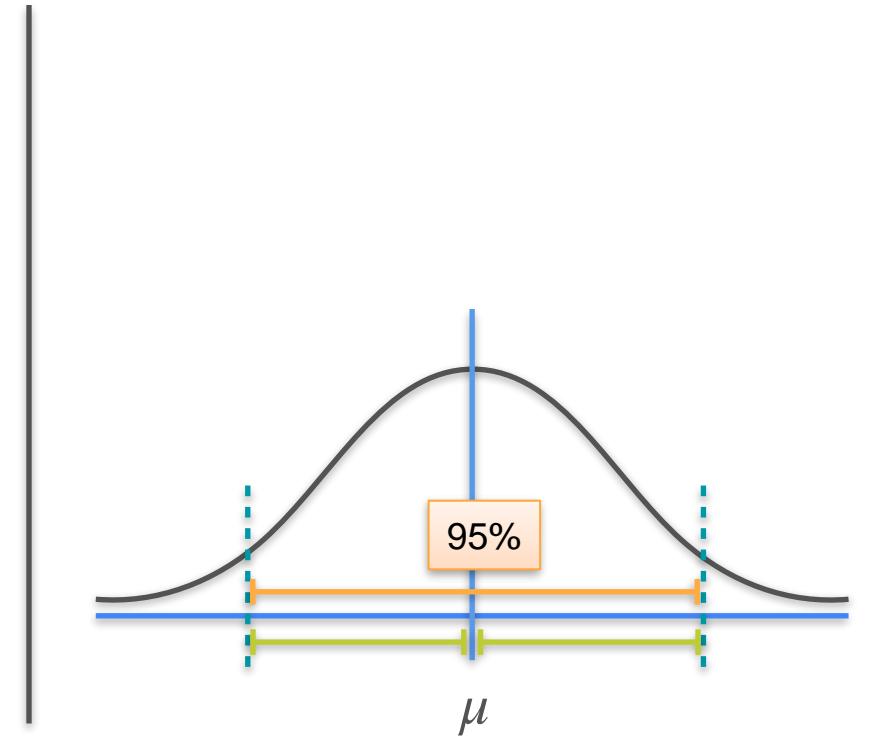
$\mathcal{N}(\mu, \sigma^2)$



95%

95%

$\mu$



95%

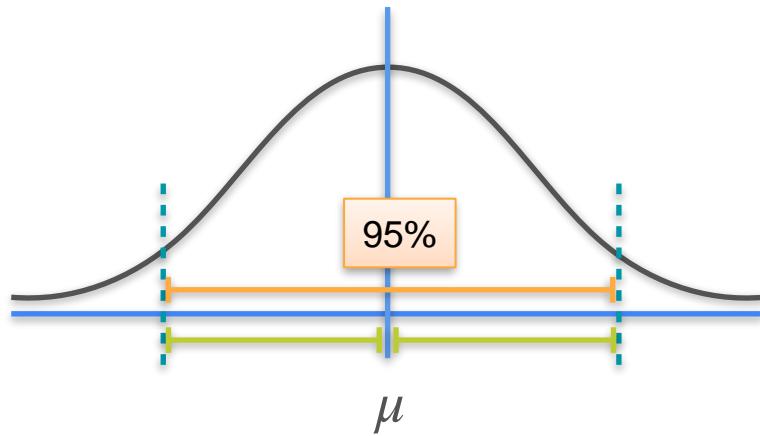
$\mu$

# Effect of the Confidence Level

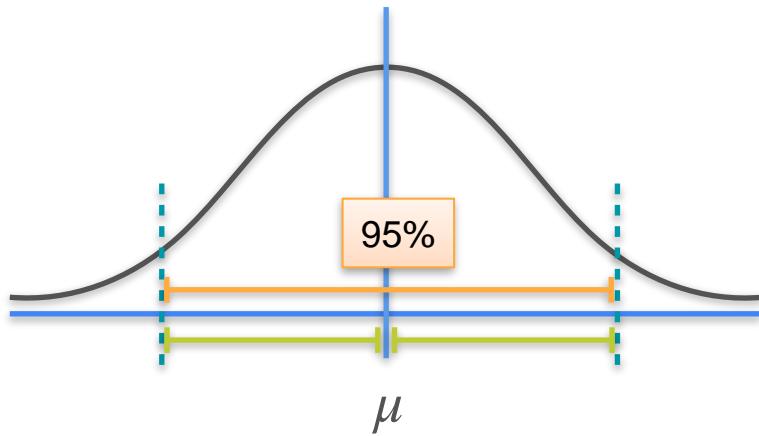
$$n = 1$$

$$\mathcal{N}(\mu, \sigma^2)$$

95%



70%

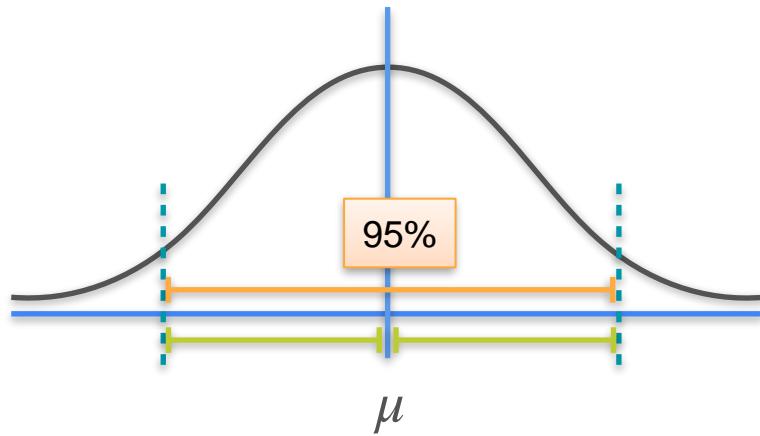


# Effect of the Confidence Level

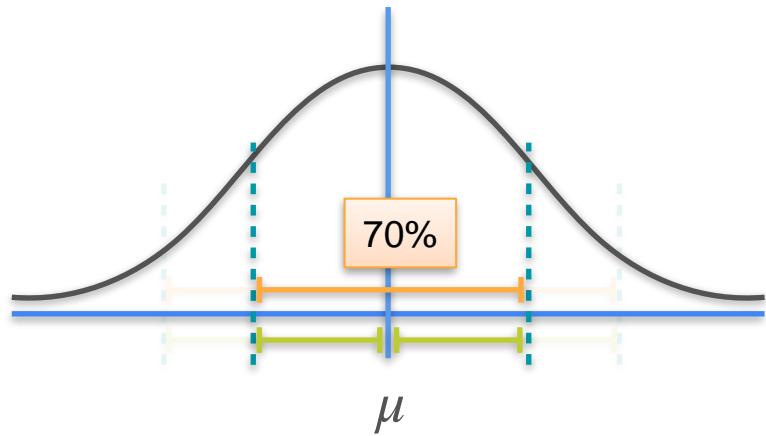
$$n = 1$$

$$\mathcal{N}(\mu, \sigma^2)$$

95%



70%

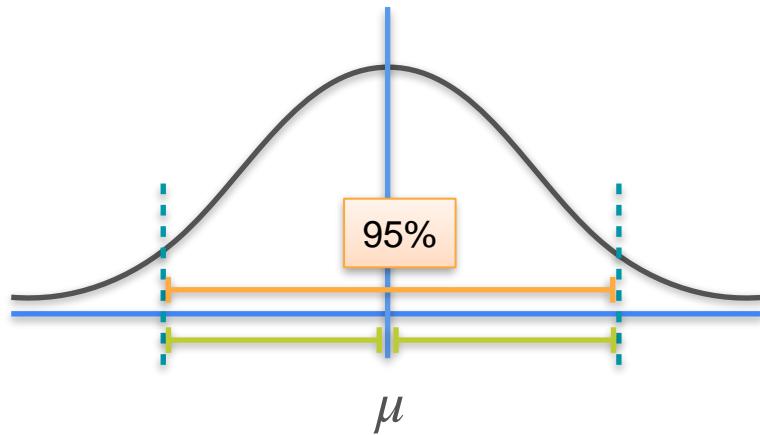


# Effect of the Confidence Level

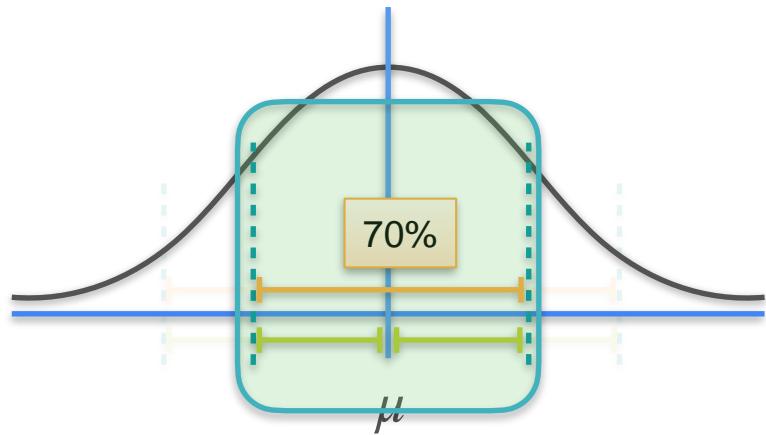
$$n = 1$$

$$\mathcal{N}(\mu, \sigma^2)$$

95%

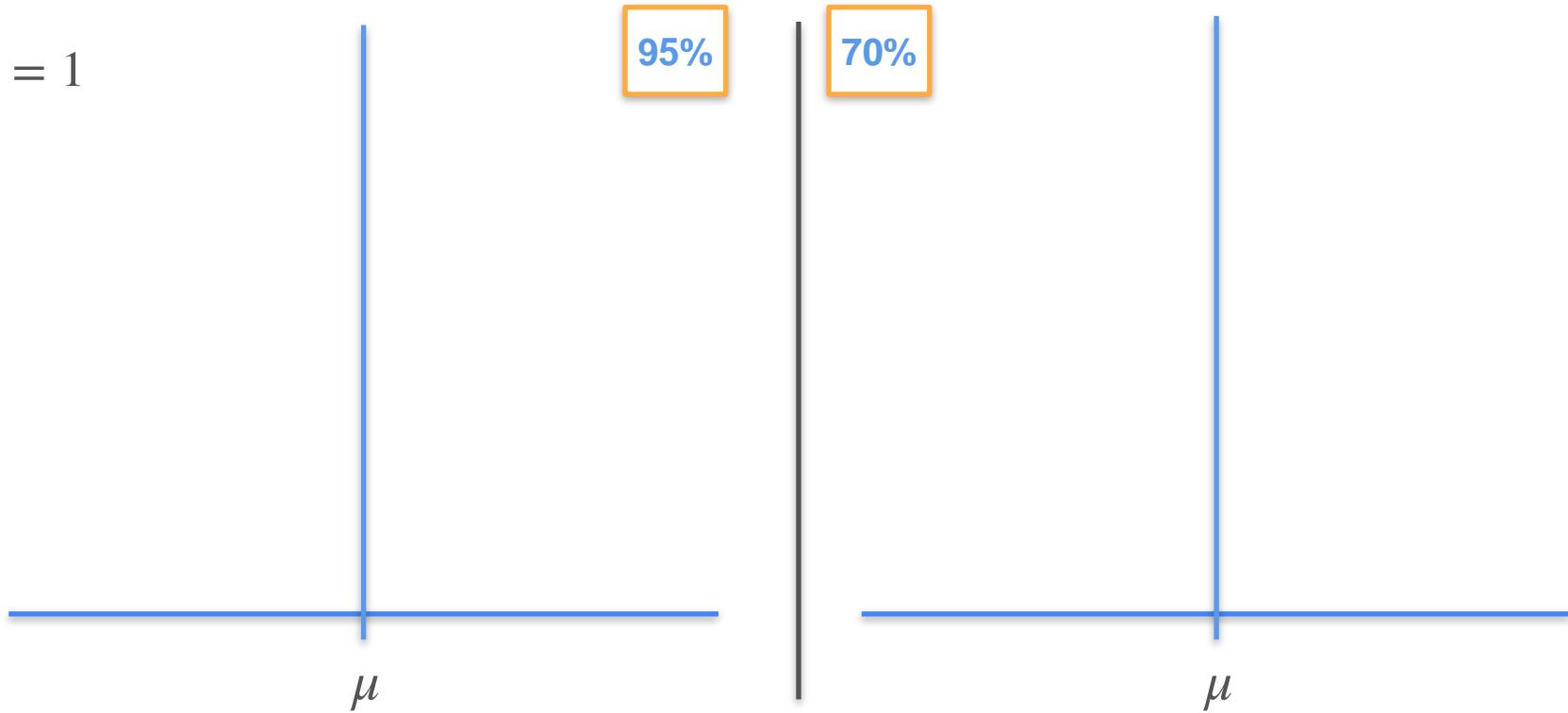


70%

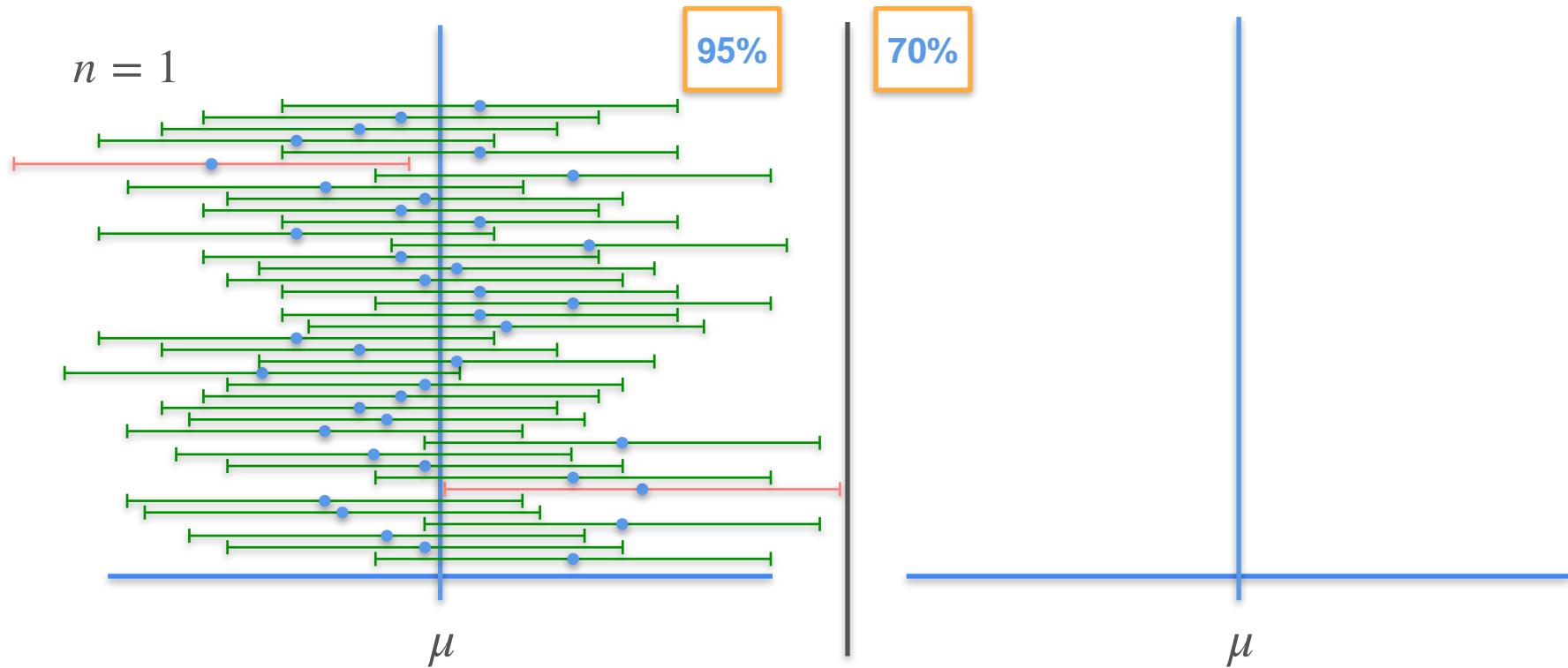


# Effect of the Confidence Level

$n = 1$

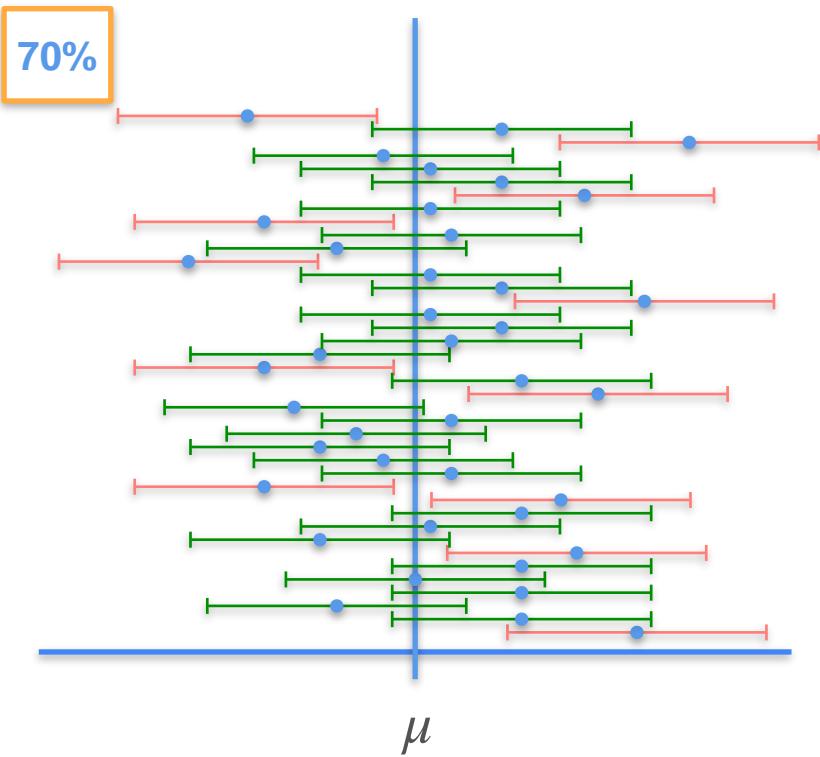
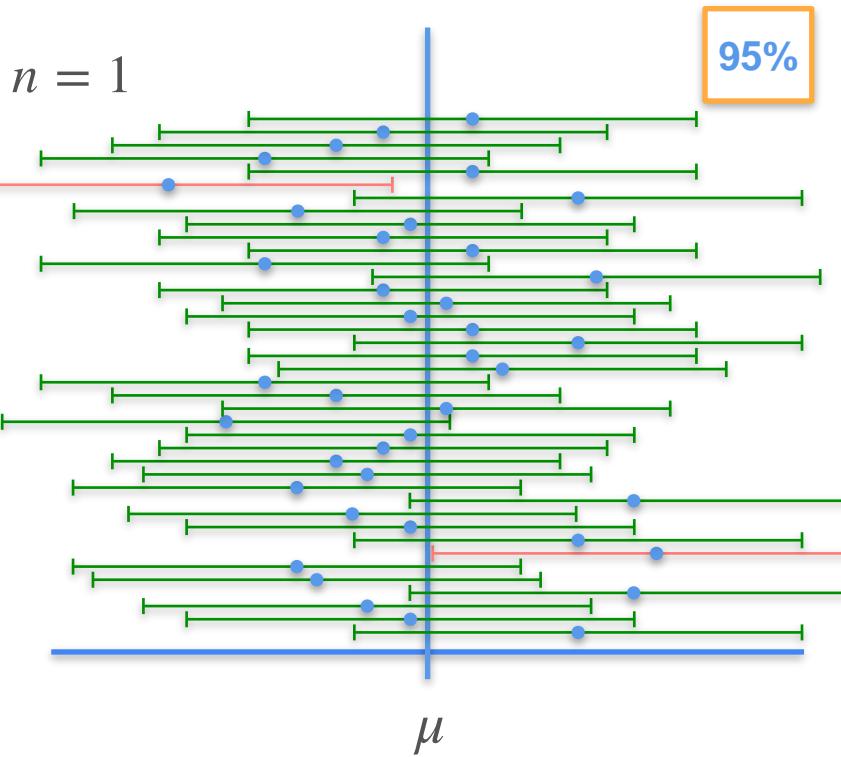


# Effect of the Confidence Level

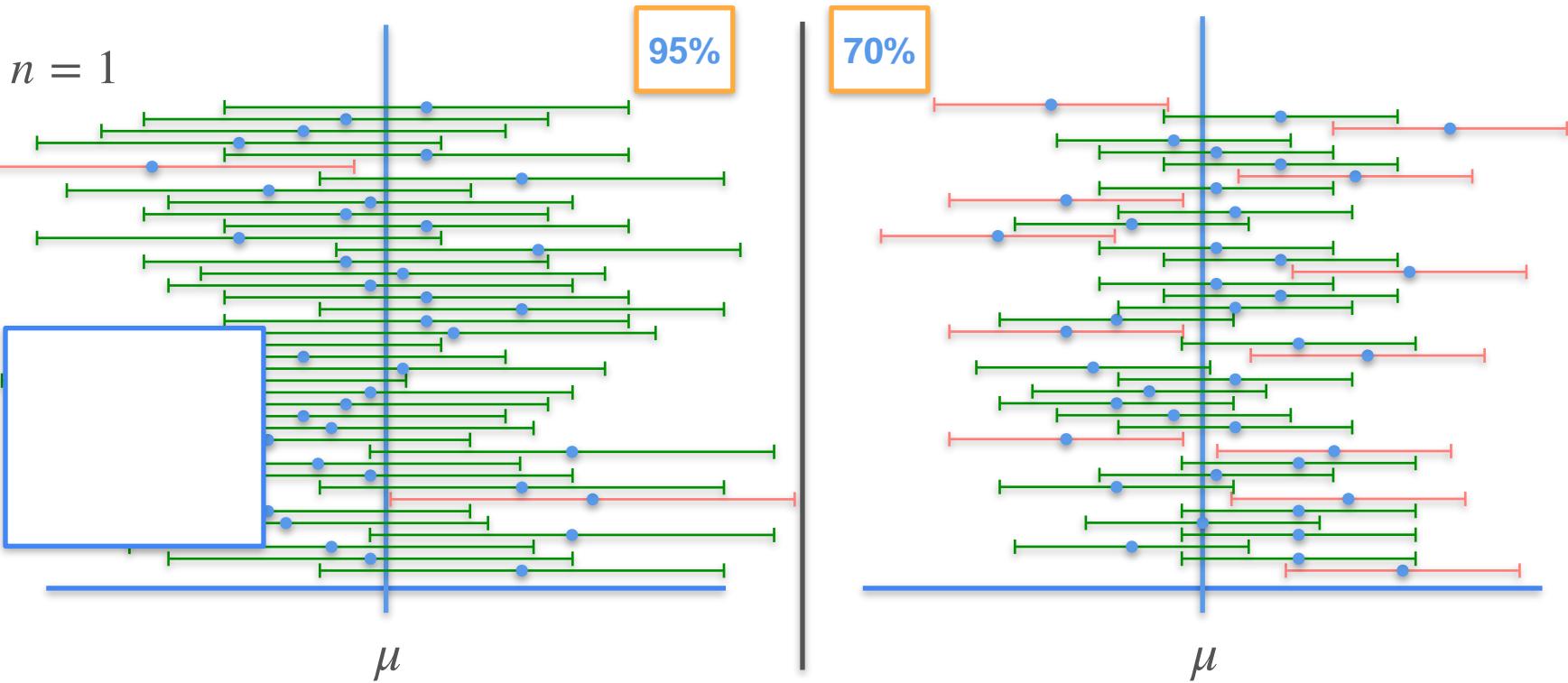


# Effect of the Confidence Level

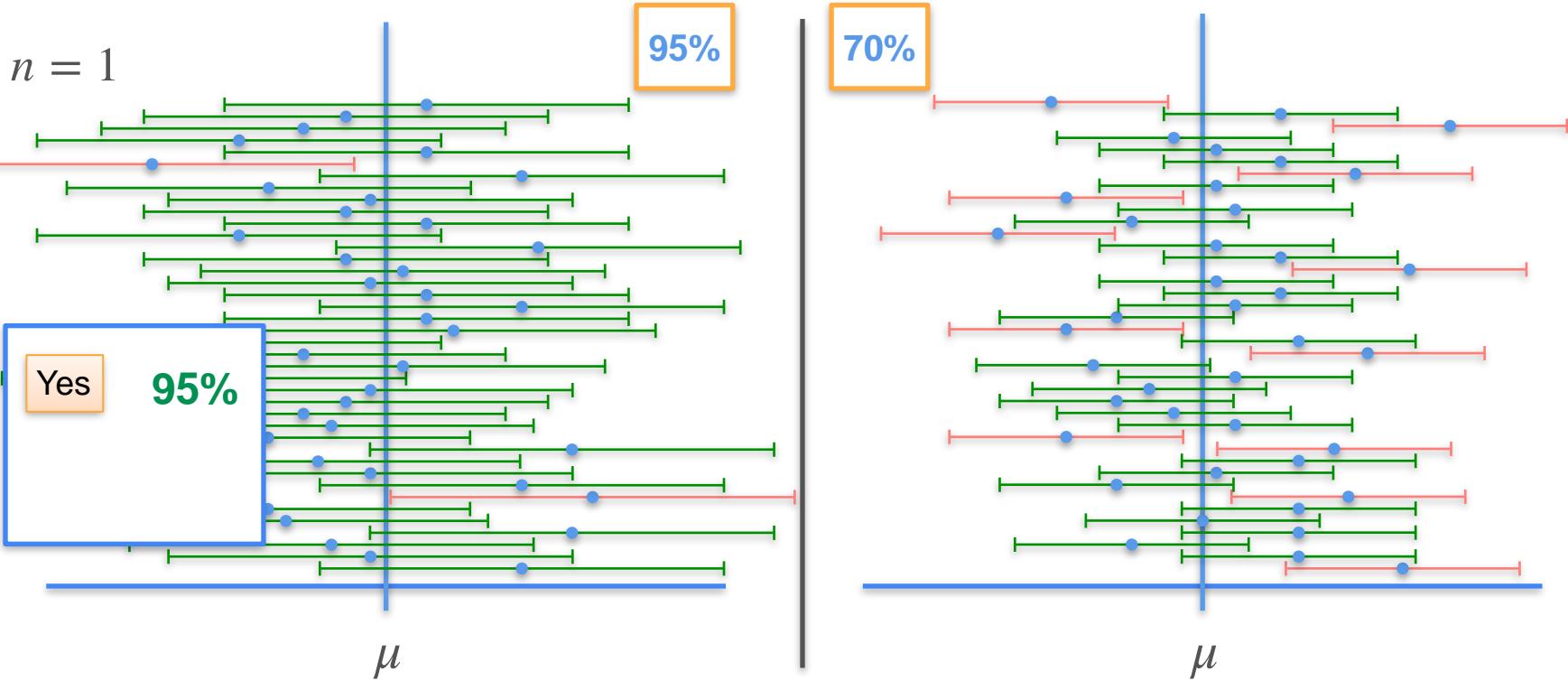
$n = 1$



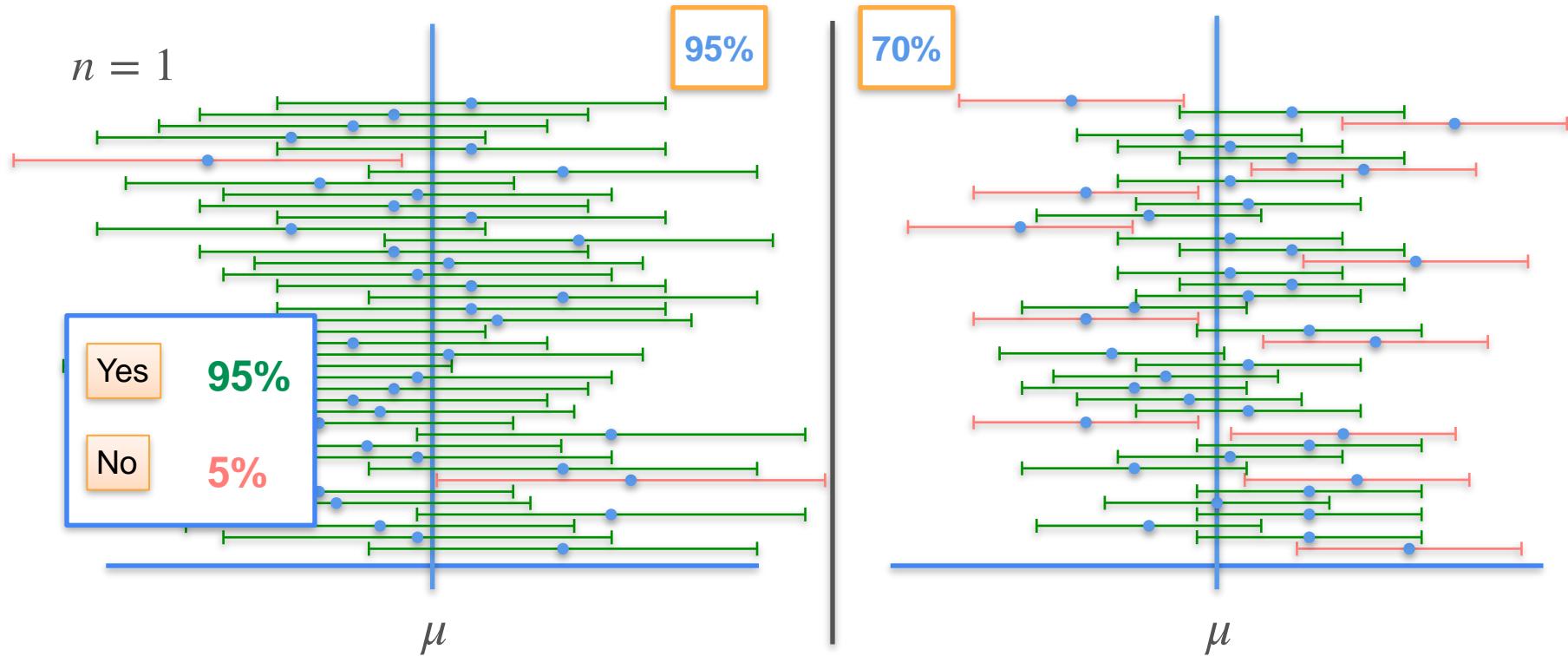
# Effect of the Confidence Level



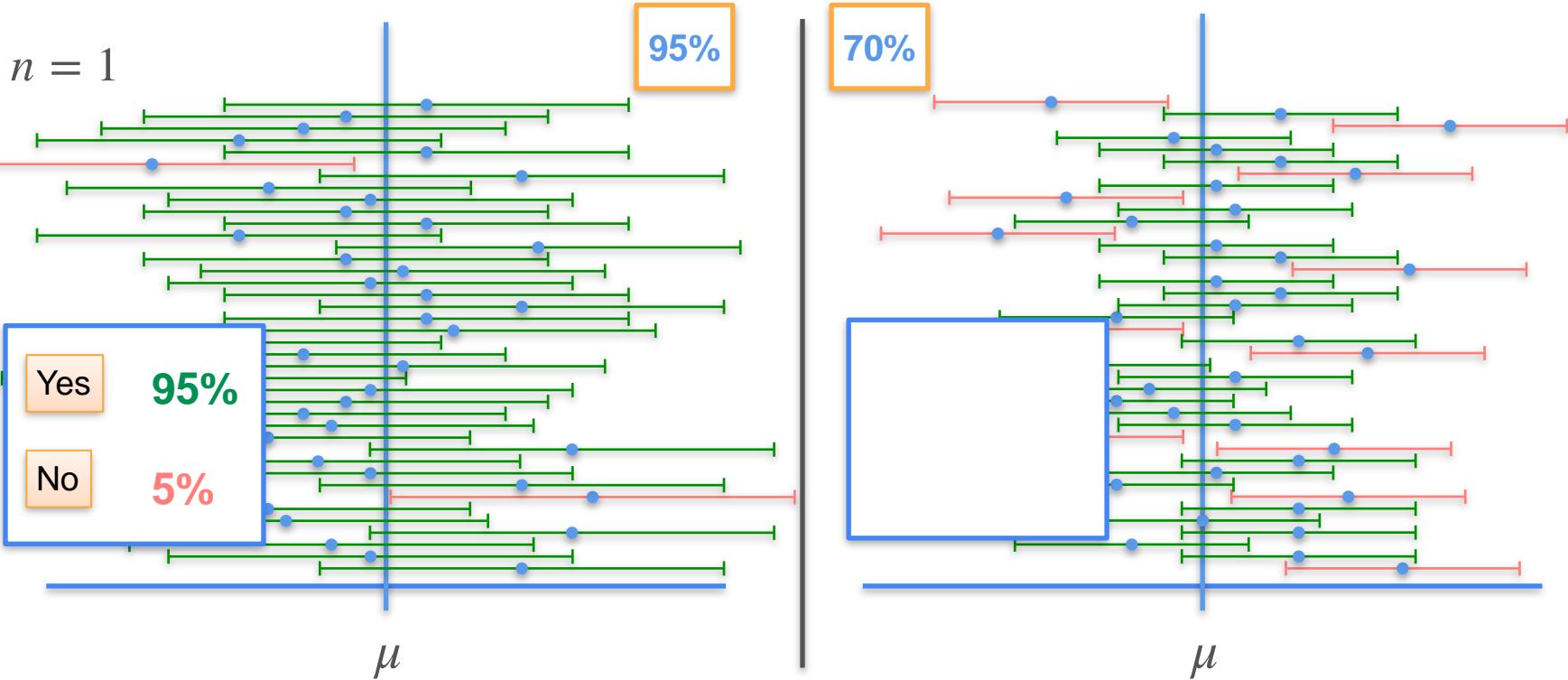
# Effect of the Confidence Level



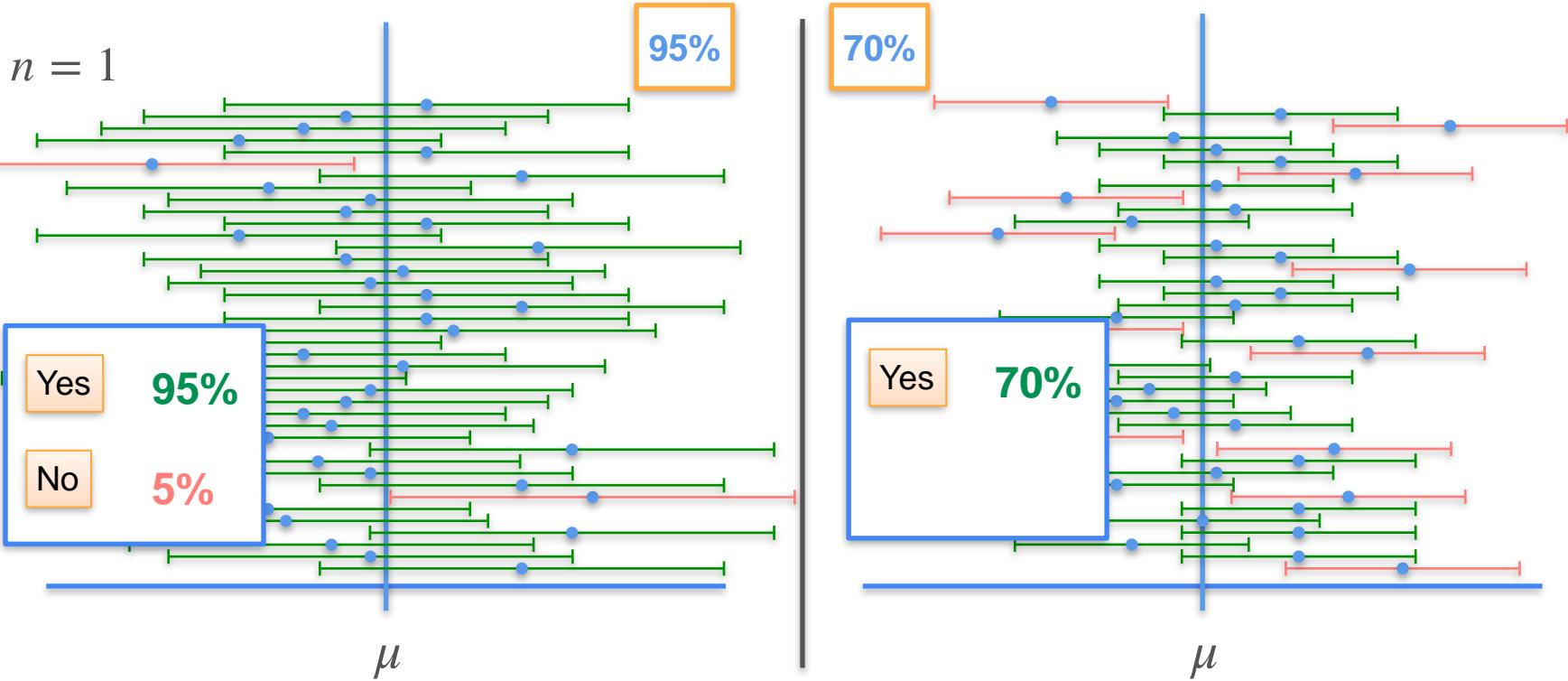
# Effect of the Confidence Level



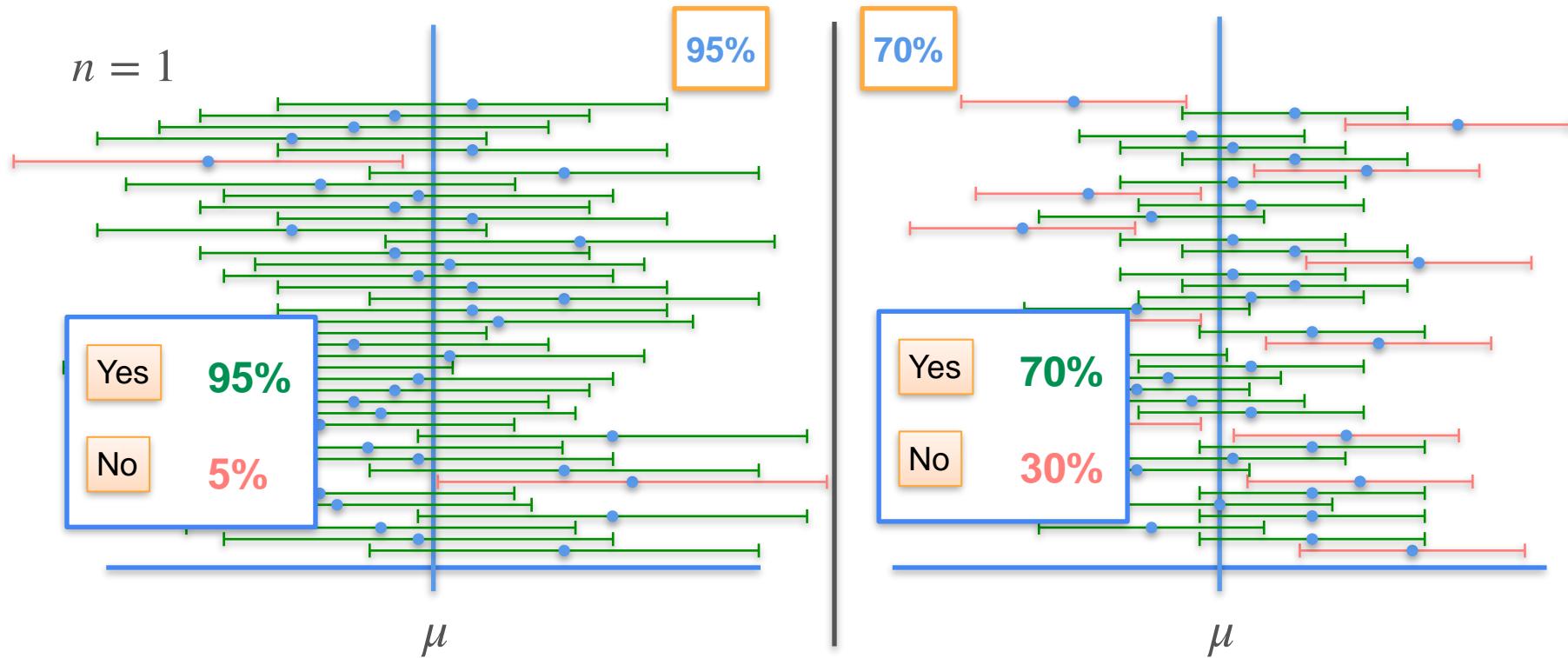
# Effect of the Confidence Level



# Effect of the Confidence Level

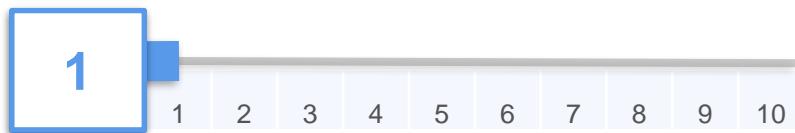


# Effect of the Confidence Level

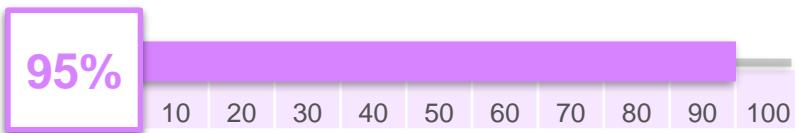


# Effect of the Confidence Level

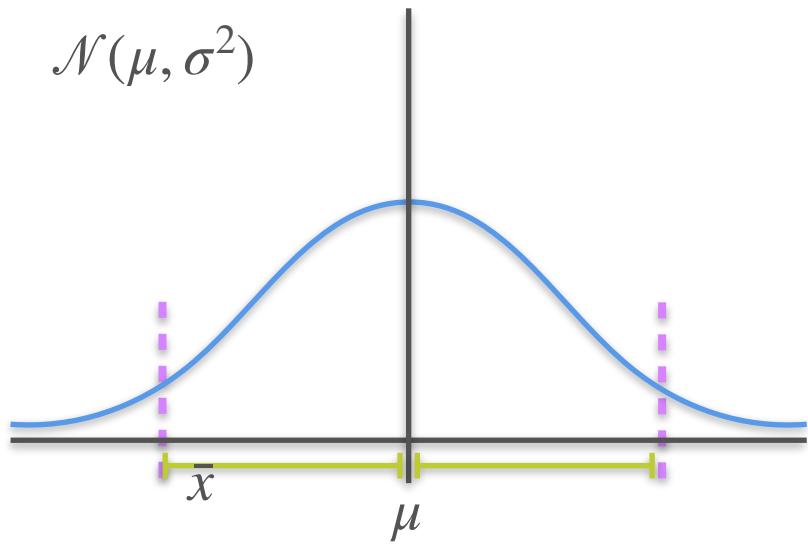
sample size



Confidence level

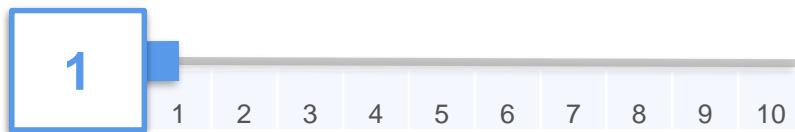


$$\mathcal{N}(\mu, \sigma^2)$$

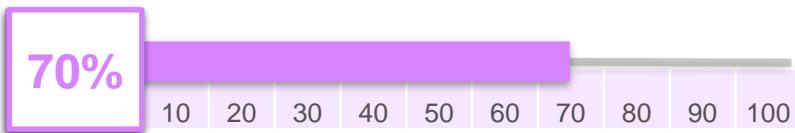


# Effect of the Confidence Level

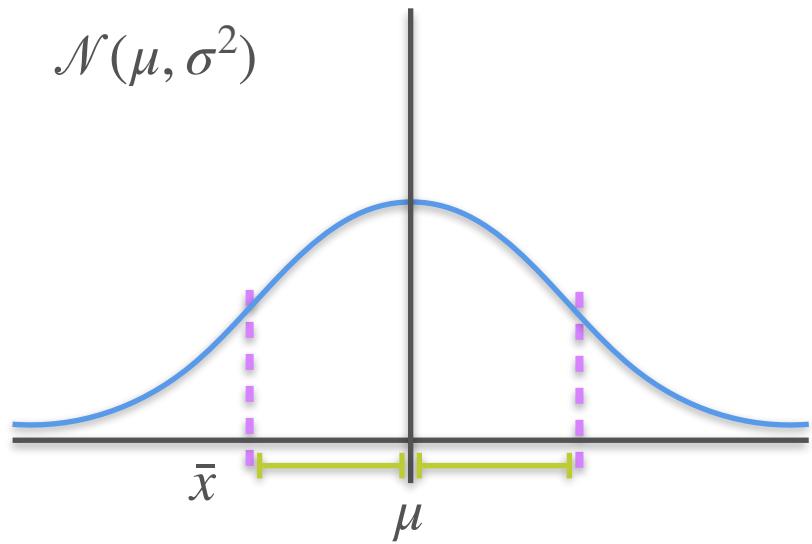
sample size



Confidence level

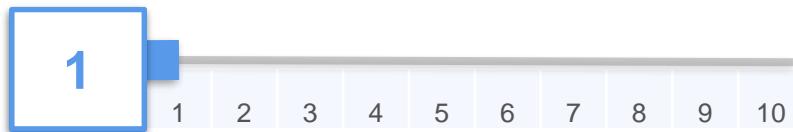


$$\mathcal{N}(\mu, \sigma^2)$$

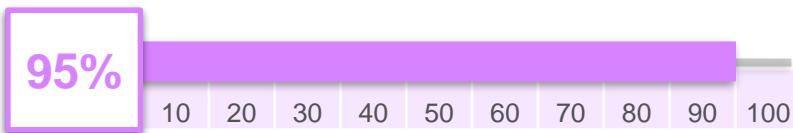


# Effect of the Confidence Level

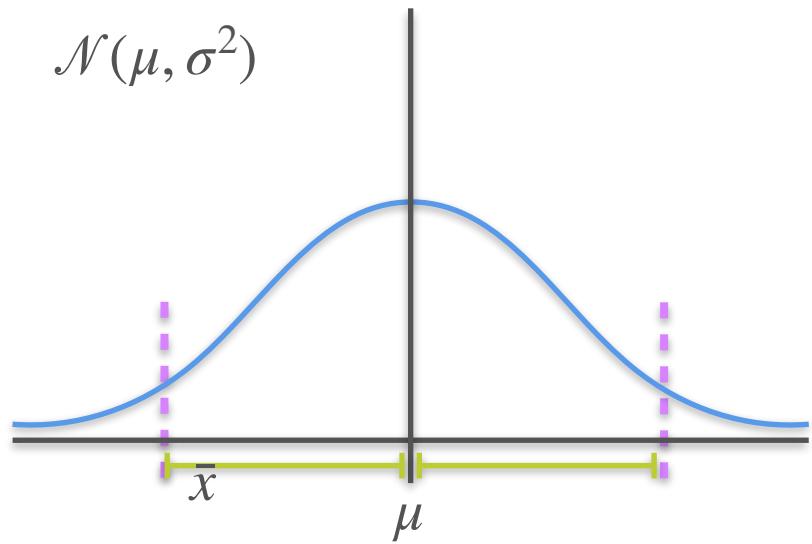
sample size



Confidence level

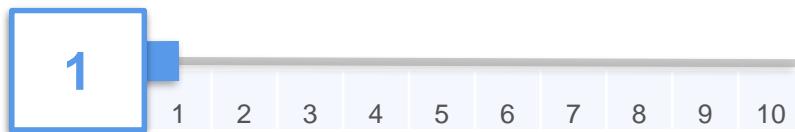


$$\mathcal{N}(\mu, \sigma^2)$$

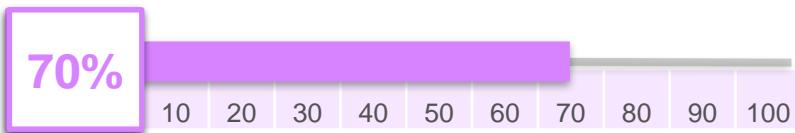


# Effect of the Confidence Level

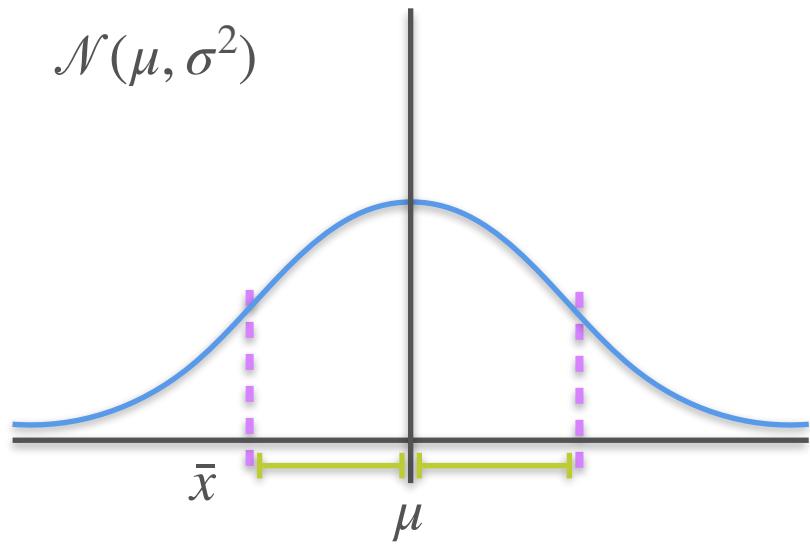
sample size



Confidence level



$$\mathcal{N}(\mu, \sigma^2)$$





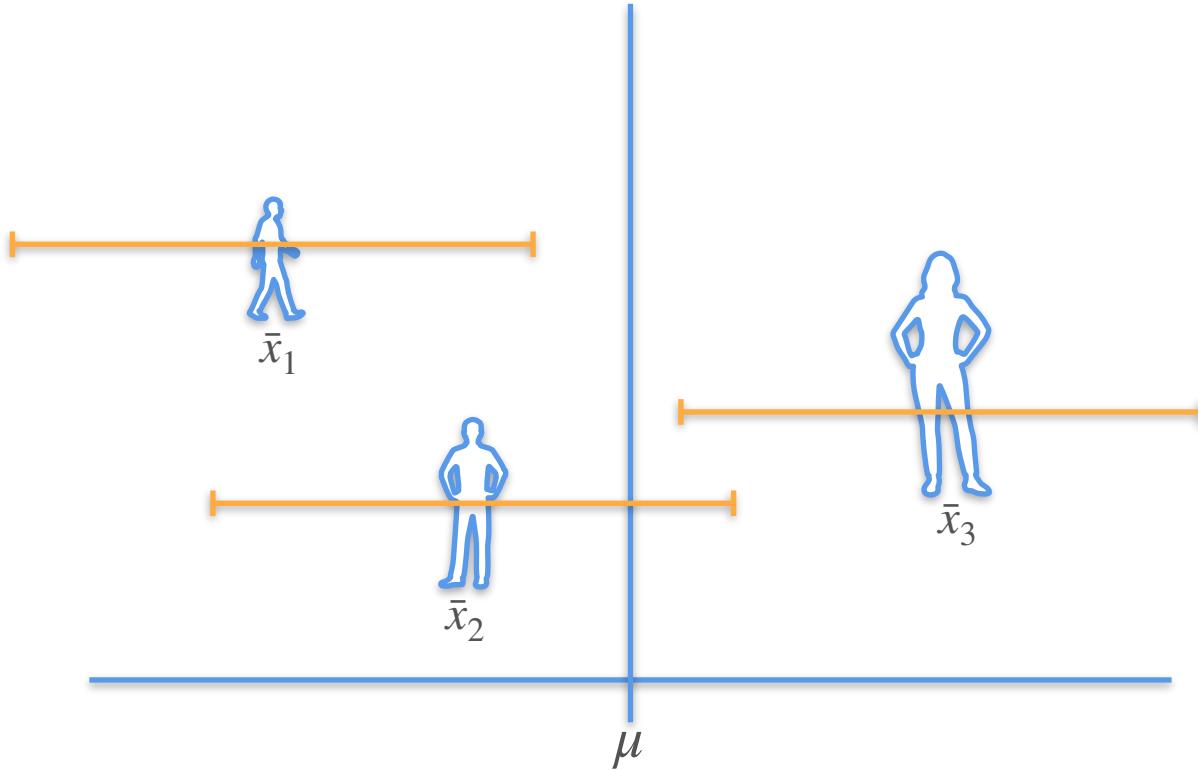
DeepLearning.AI

## Confidence Interval

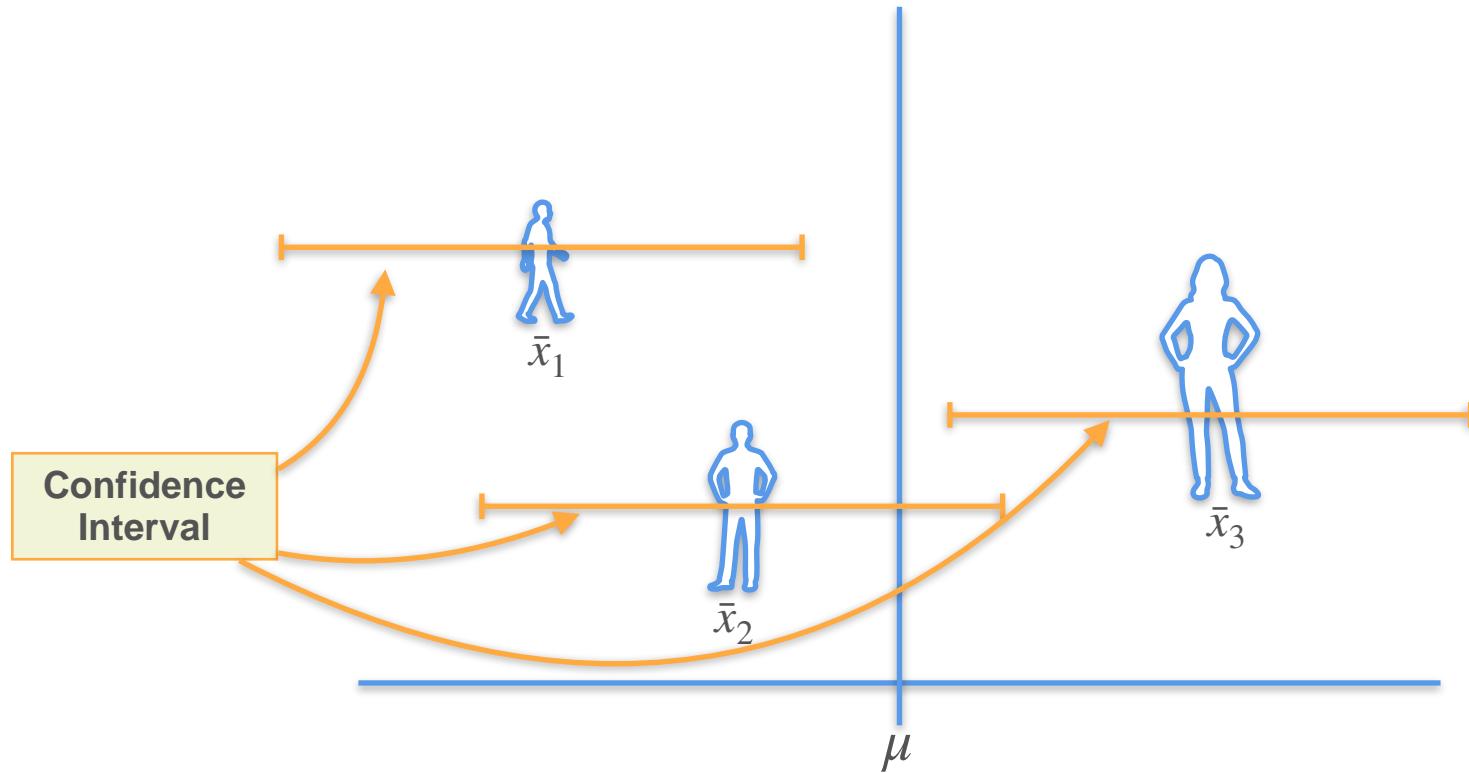
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## Margin of Error

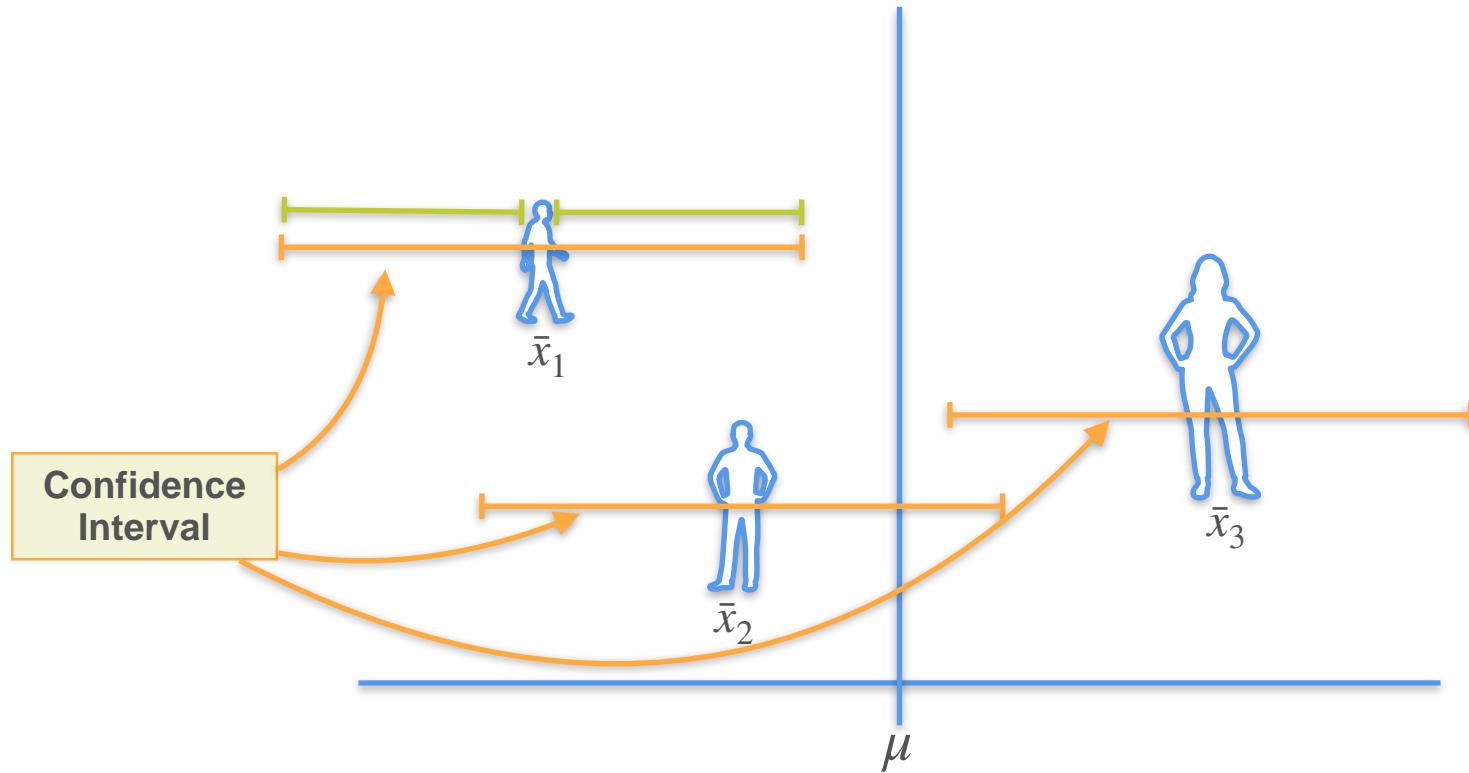
# Margin of Error - Introduction



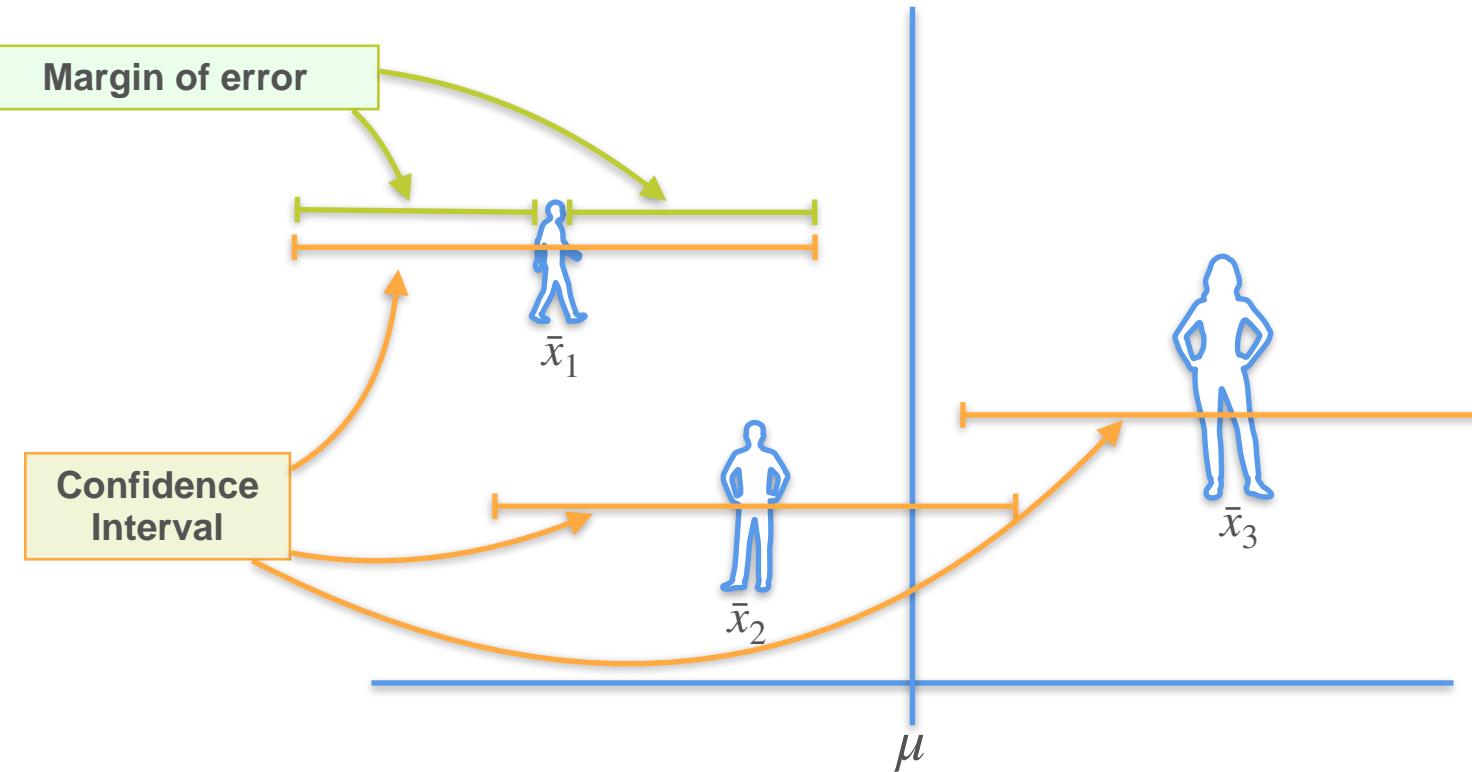
# Margin of Error - Introduction



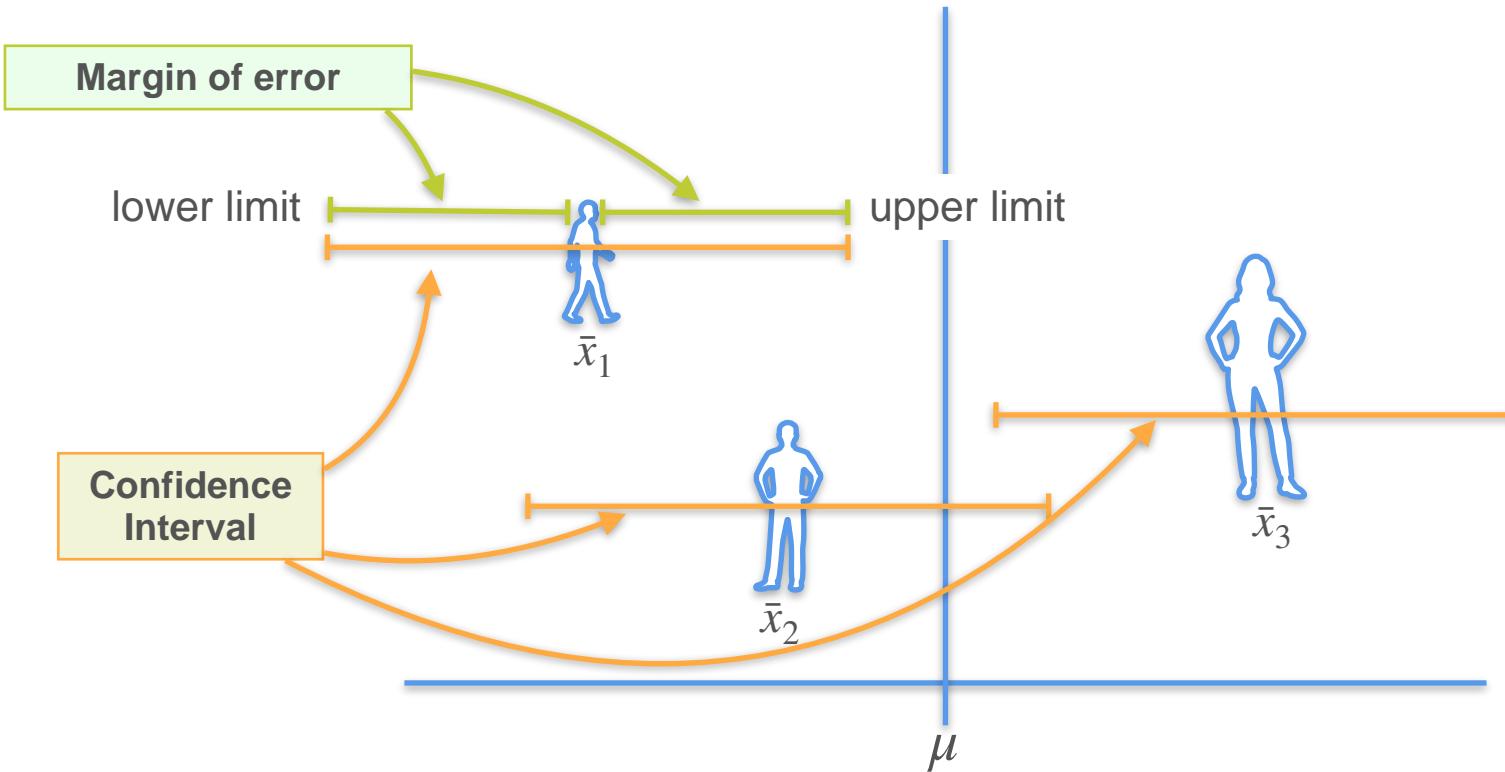
# Margin of Error - Introduction



# Margin of Error - Introduction



# Margin of Error - Introduction



# Margin of Error

# Margin of Error

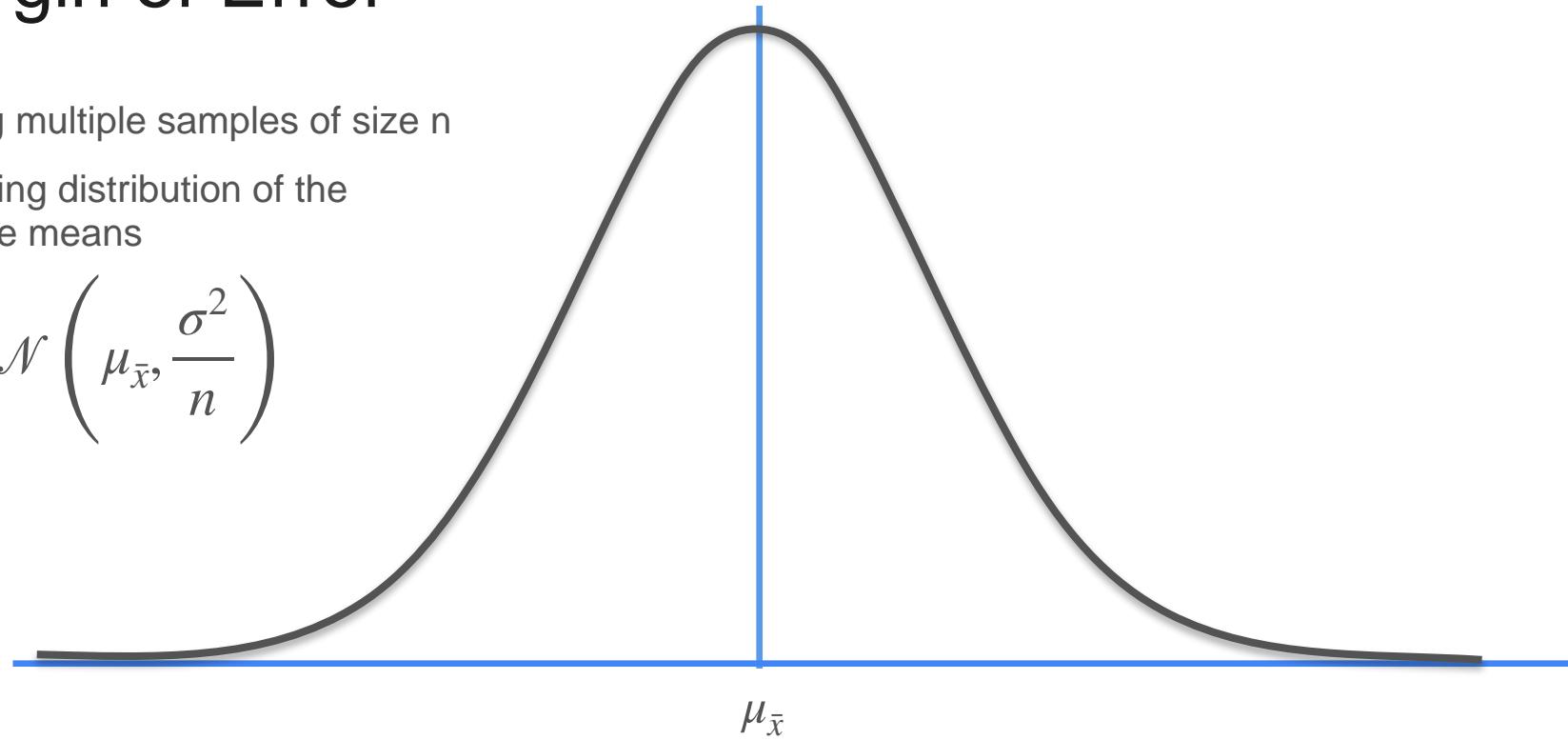
Taking multiple samples of size n  
sampling distribution of the  
sample means

$$\bar{X} \sim \mathcal{N}\left(\mu_{\bar{x}}, \frac{\sigma^2}{n}\right)$$

# Margin of Error

Taking multiple samples of size n  
sampling distribution of the  
sample means

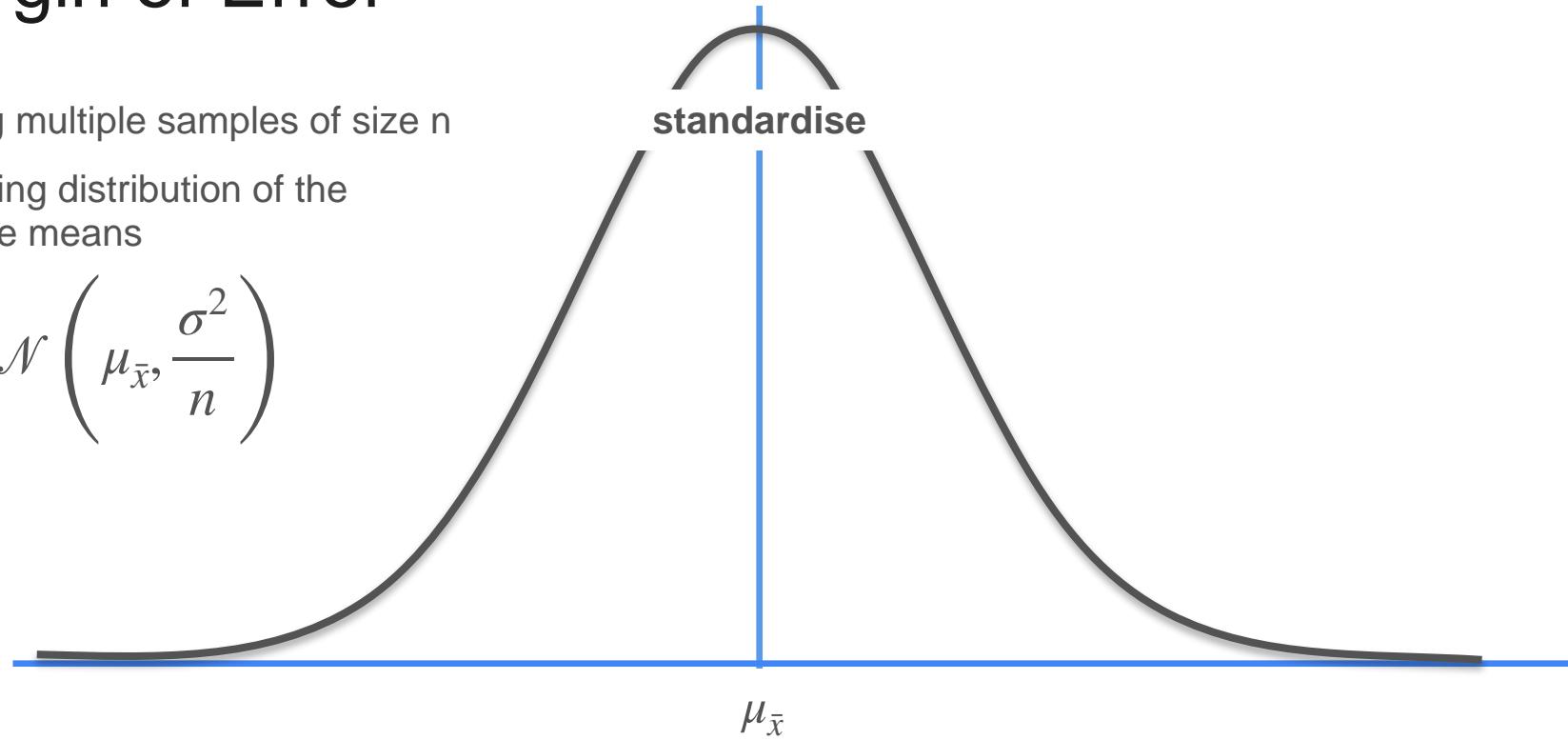
$$\bar{X} \sim \mathcal{N}\left(\mu_{\bar{x}}, \frac{\sigma^2}{n}\right)$$



# Margin of Error

Taking multiple samples of size n  
sampling distribution of the  
sample means

$$\bar{X} \sim \mathcal{N}\left(\mu_{\bar{x}}, \frac{\sigma^2}{n}\right)$$



# Margin of Error

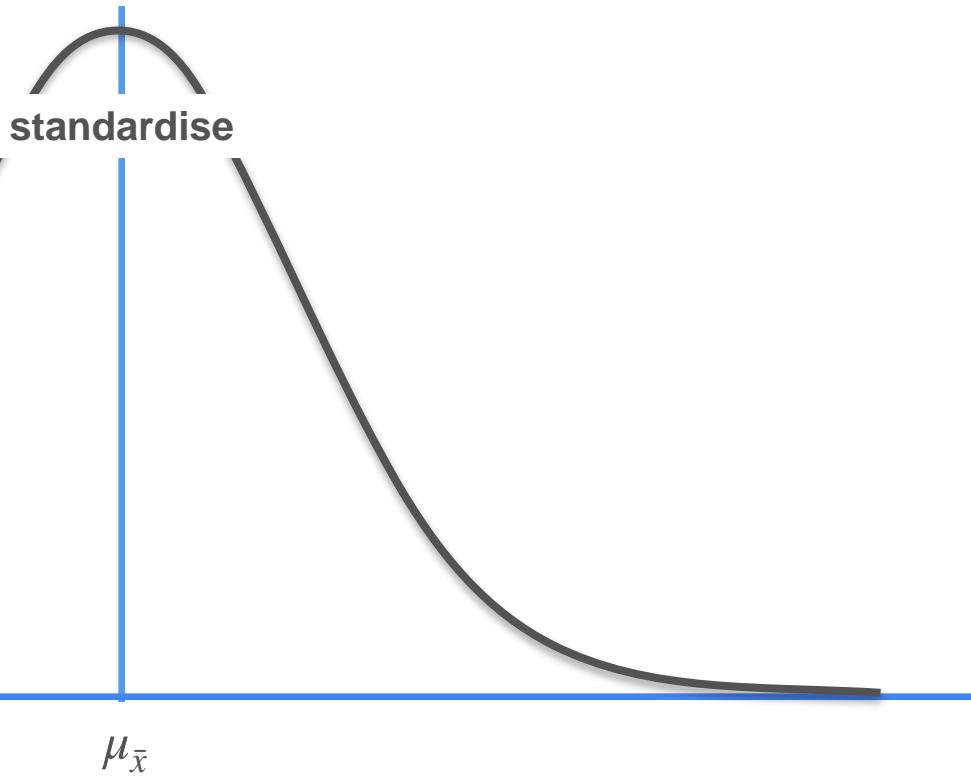
Taking multiple samples of size n  
sampling distribution of the  
sample means

$$\bar{X} \sim \mathcal{N}\left(\mu_{\bar{x}}, \frac{\sigma^2}{n}\right)$$

$$\mu = 0$$

$$\sigma = 1$$

$$\bar{X} \sim \mathcal{N}(0,1)$$



# Margin of Error

Taking multiple samples of size n

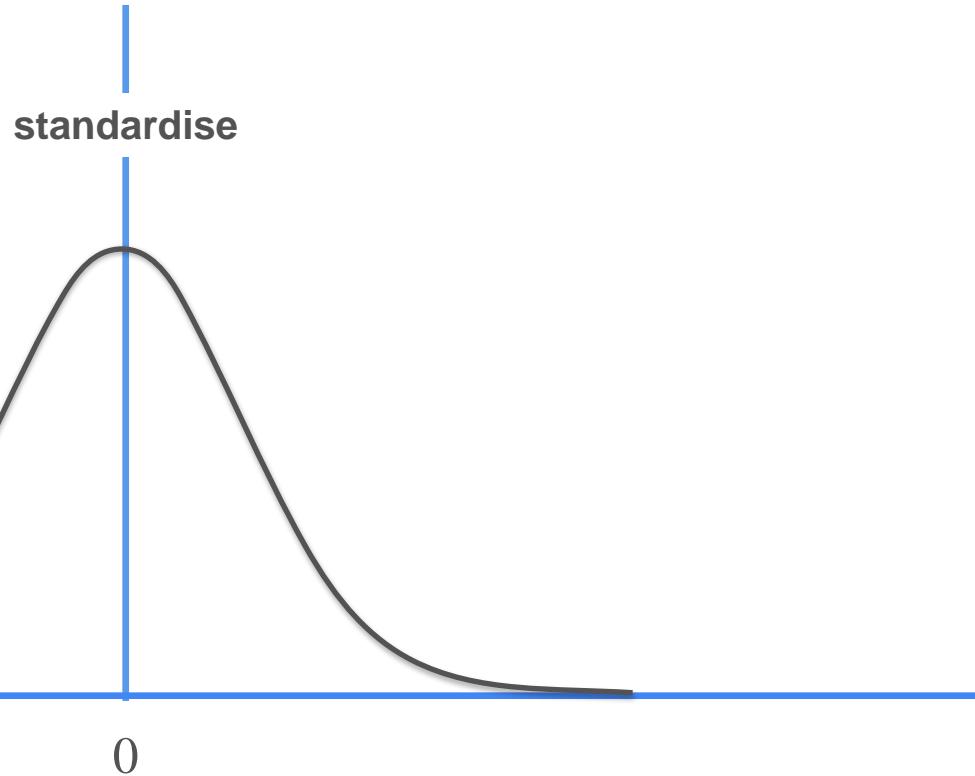
sampling distribution of the sample means

$$\bar{X} \sim \mathcal{N}\left(\mu_{\bar{x}}, \frac{\sigma^2}{n}\right)$$

$$\mu = 0$$

$$\sigma = 1$$

$$\bar{X} \sim \mathcal{N}(0,1)$$



# Margin of Error

Taking multiple samples of size n

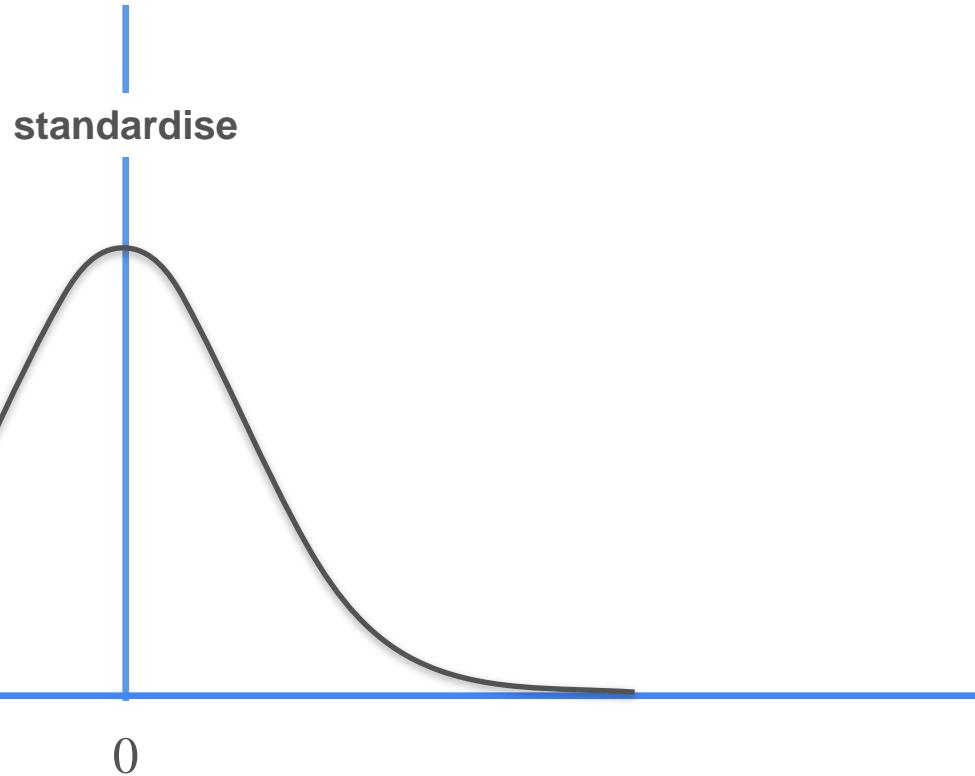
sampling distribution of the sample means

$$\bar{X} \sim \mathcal{N}\left(\mu_{\bar{x}}, \frac{\sigma^2}{n}\right)$$

$$\mu = 0$$

$$\sigma = 1$$

$$\bar{X} \sim \mathcal{N}(0, 1^2)$$

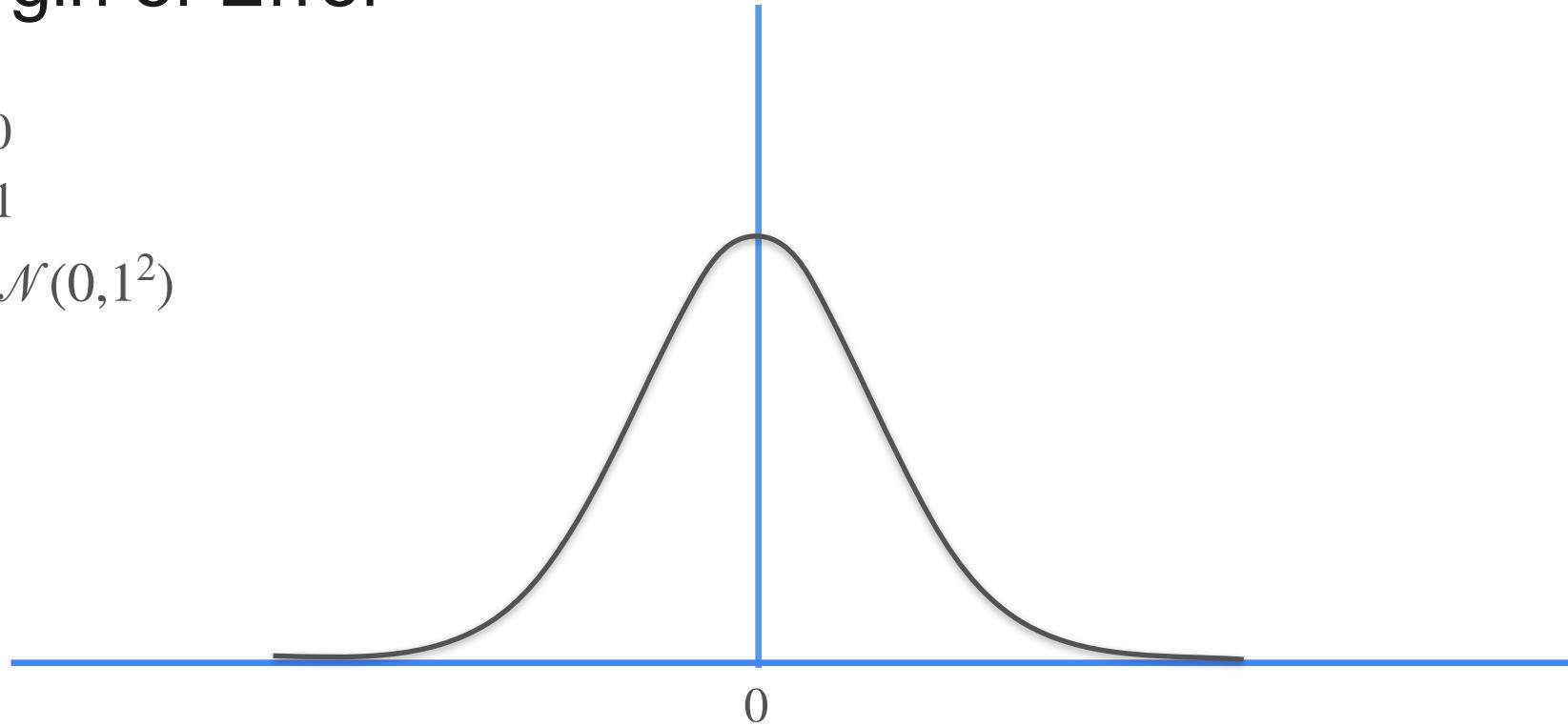


# Margin of Error

$$\mu = 0$$

$$\sigma = 1$$

$$\bar{X} \sim \mathcal{N}(0, 1^2)$$

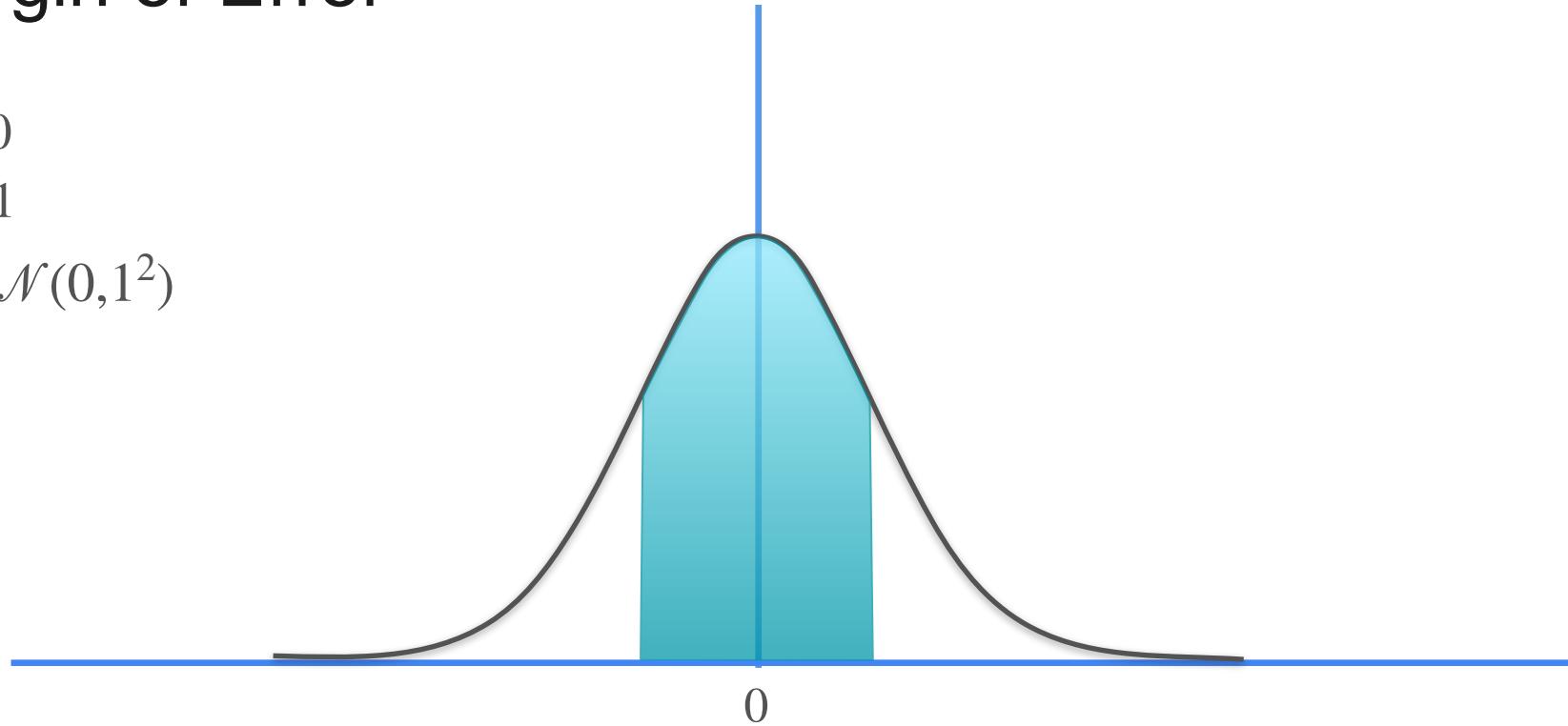


# Margin of Error

$$\mu = 0$$

$$\sigma = 1$$

$$\bar{X} \sim \mathcal{N}(0, 1^2)$$

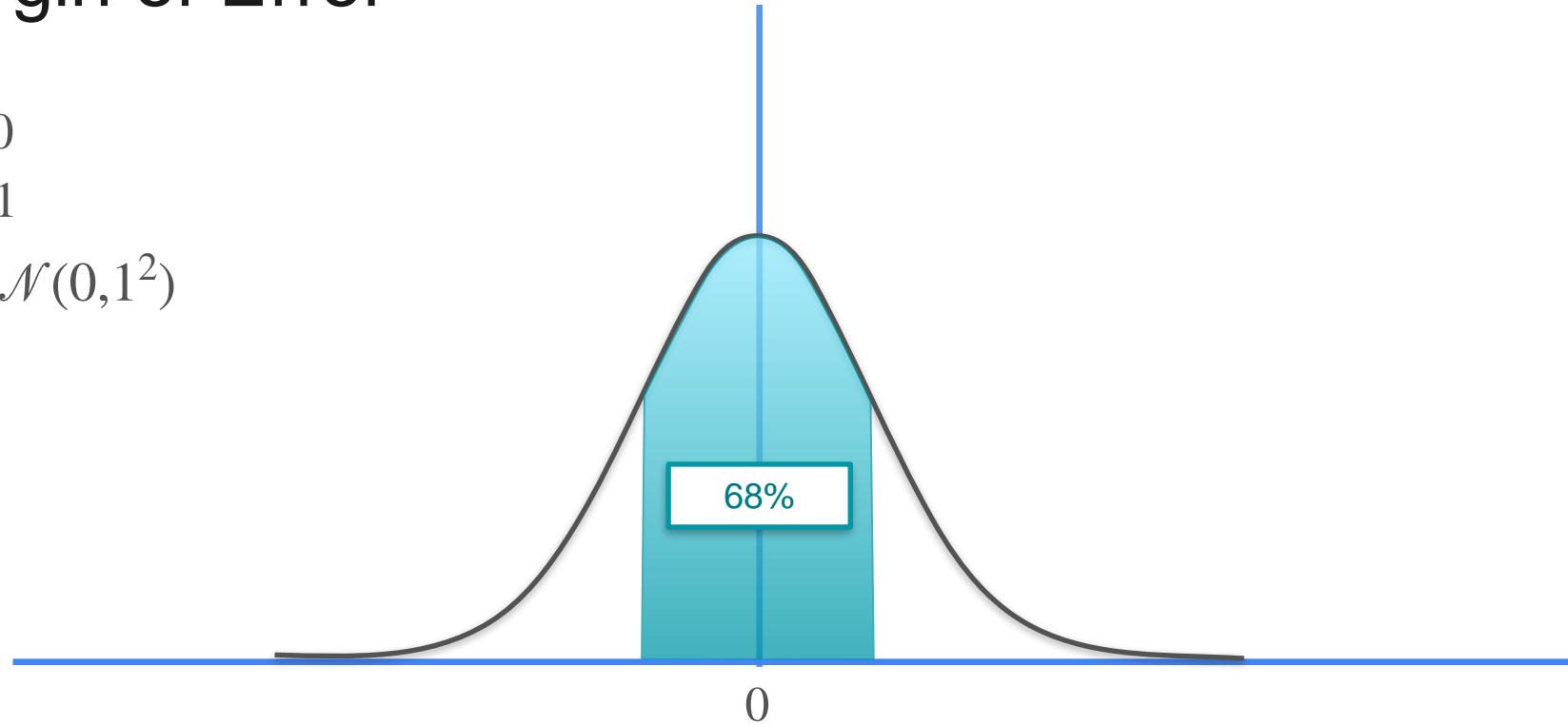


# Margin of Error

$$\mu = 0$$

$$\sigma = 1$$

$$\bar{X} \sim \mathcal{N}(0, 1^2)$$

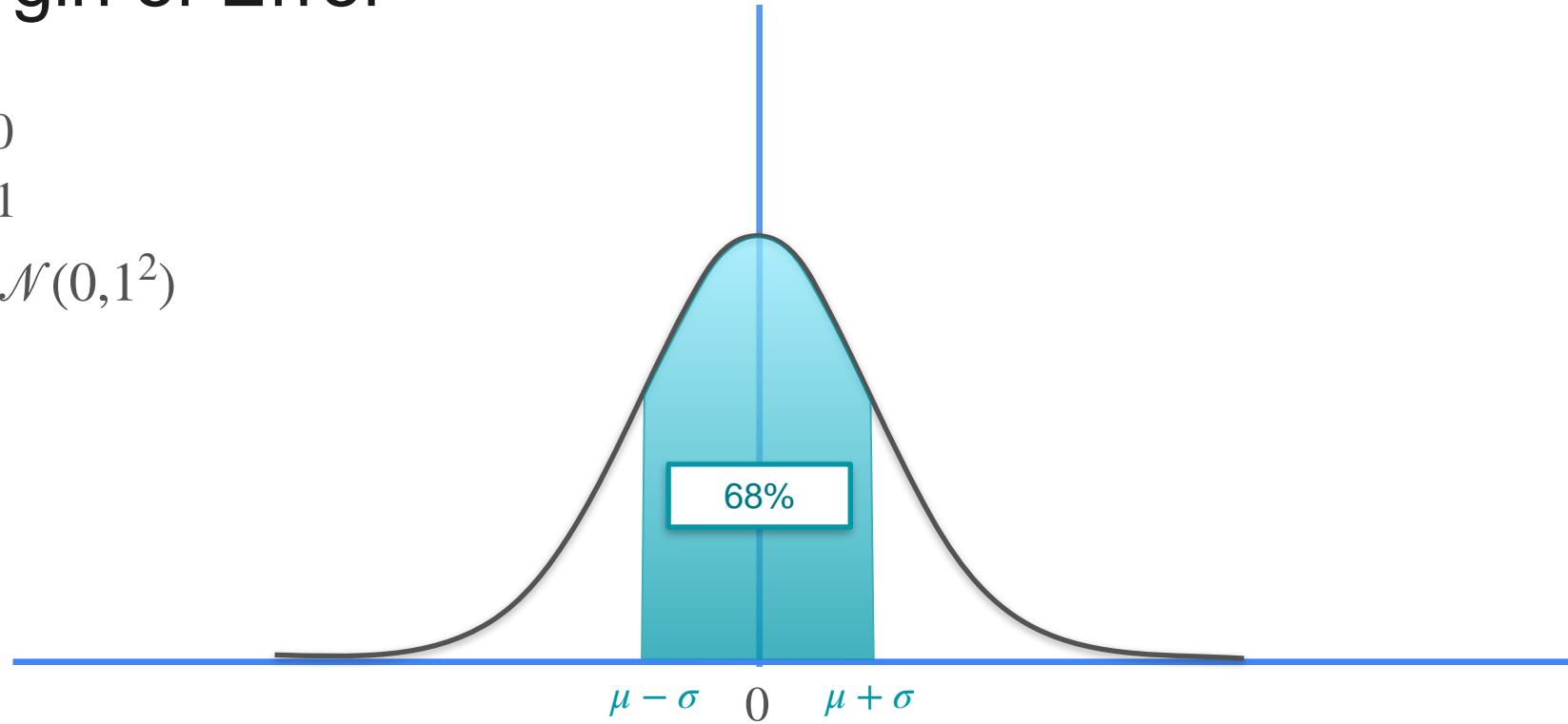


# Margin of Error

$$\mu = 0$$

$$\sigma = 1$$

$$\bar{X} \sim \mathcal{N}(0, 1^2)$$

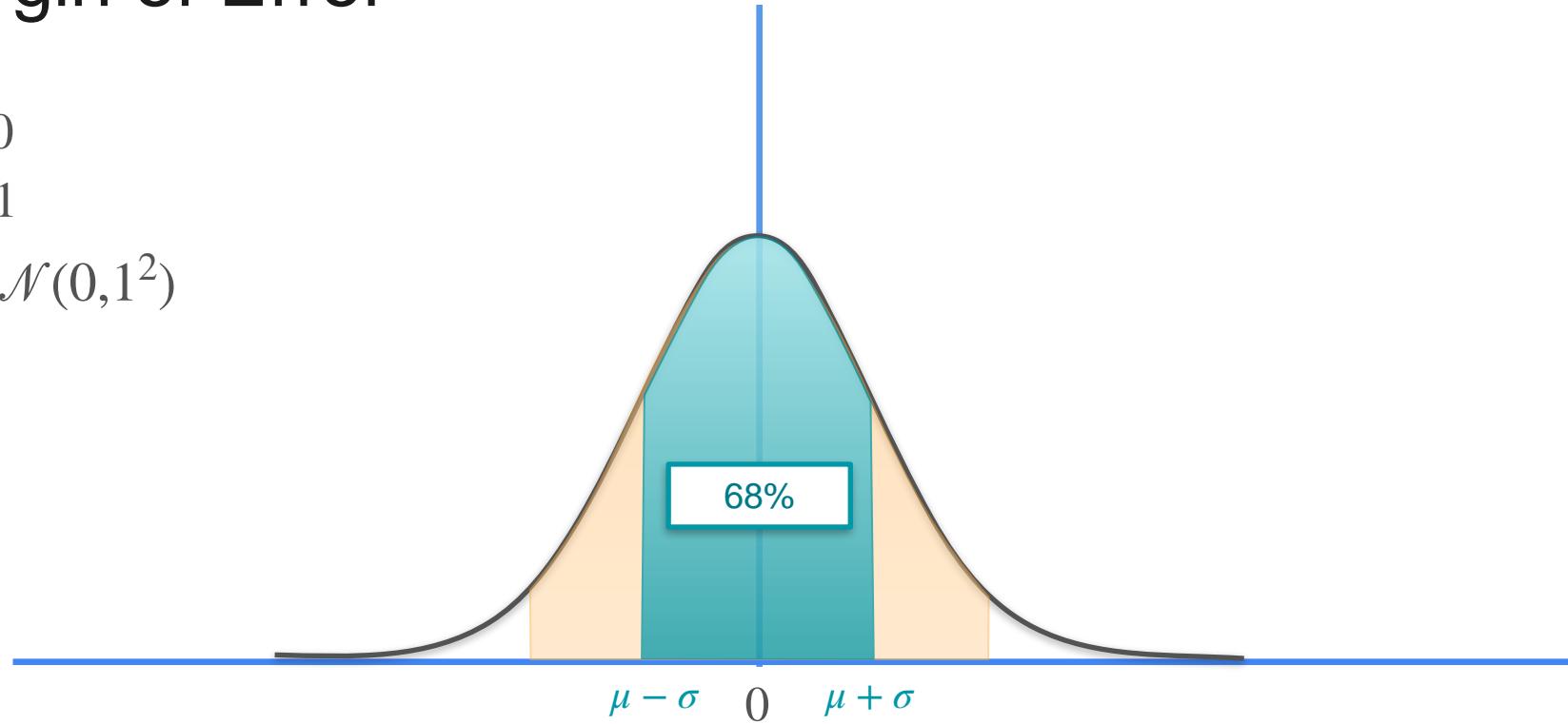


# Margin of Error

$$\mu = 0$$

$$\sigma = 1$$

$$\bar{X} \sim \mathcal{N}(0, 1^2)$$

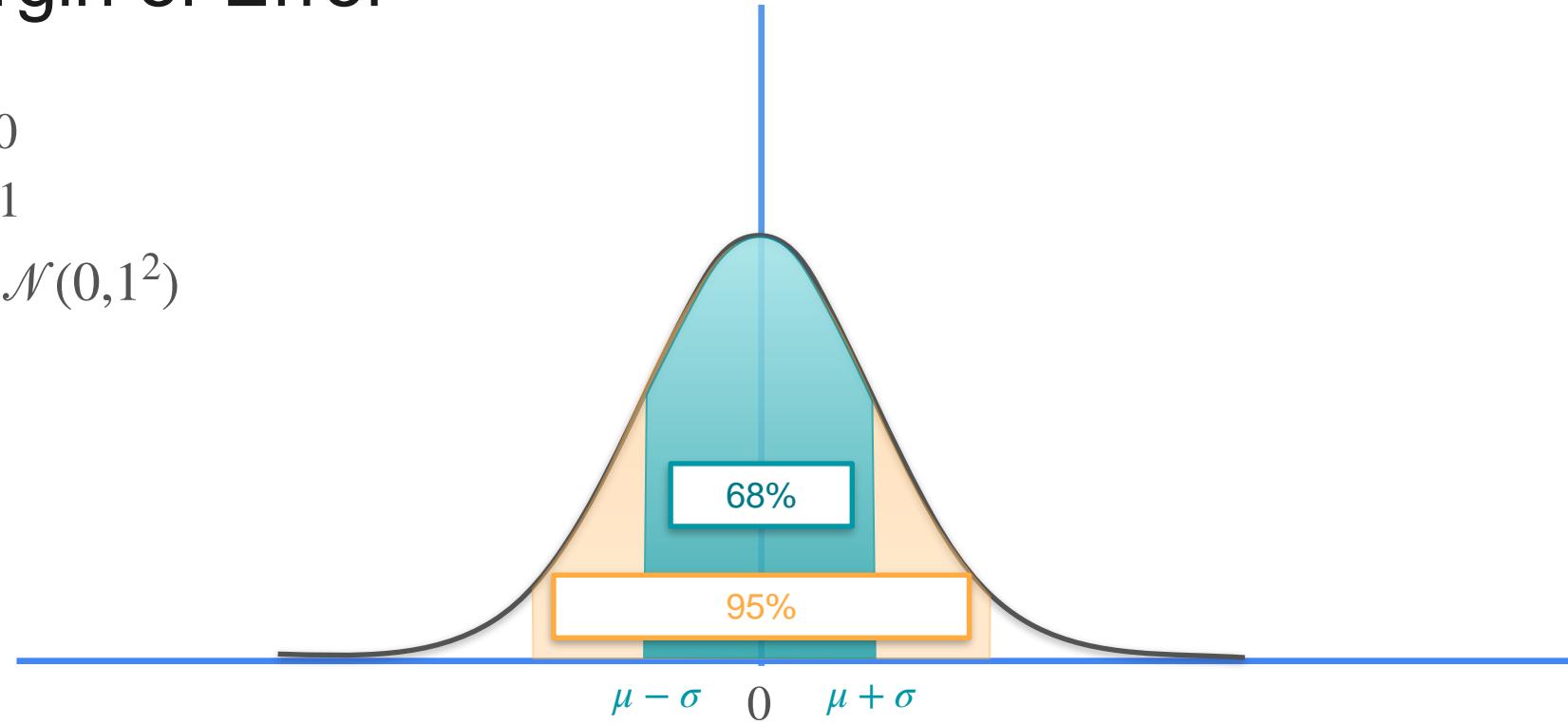


# Margin of Error

$$\mu = 0$$

$$\sigma = 1$$

$$\bar{X} \sim \mathcal{N}(0, 1^2)$$

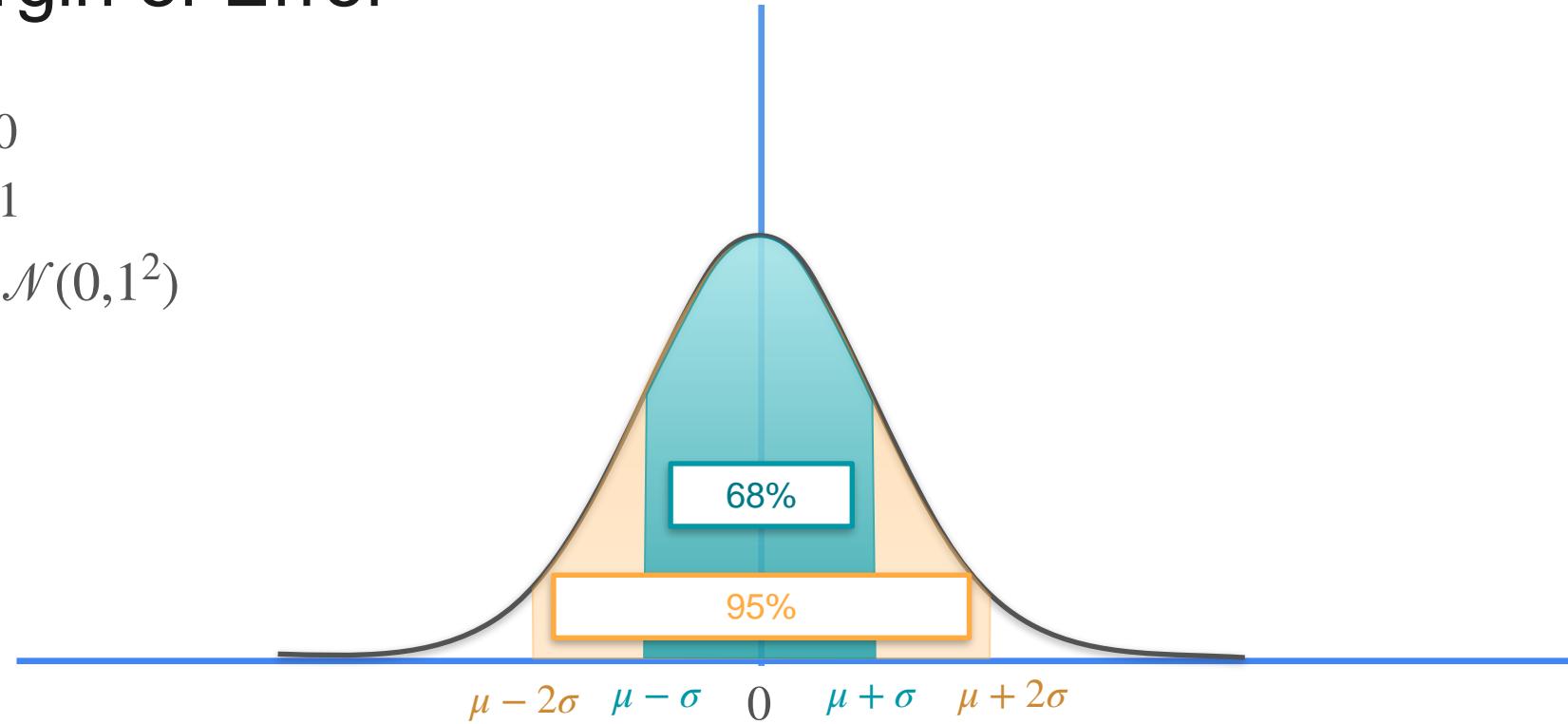


# Margin of Error

$$\mu = 0$$

$$\sigma = 1$$

$$\bar{X} \sim \mathcal{N}(0, 1^2)$$

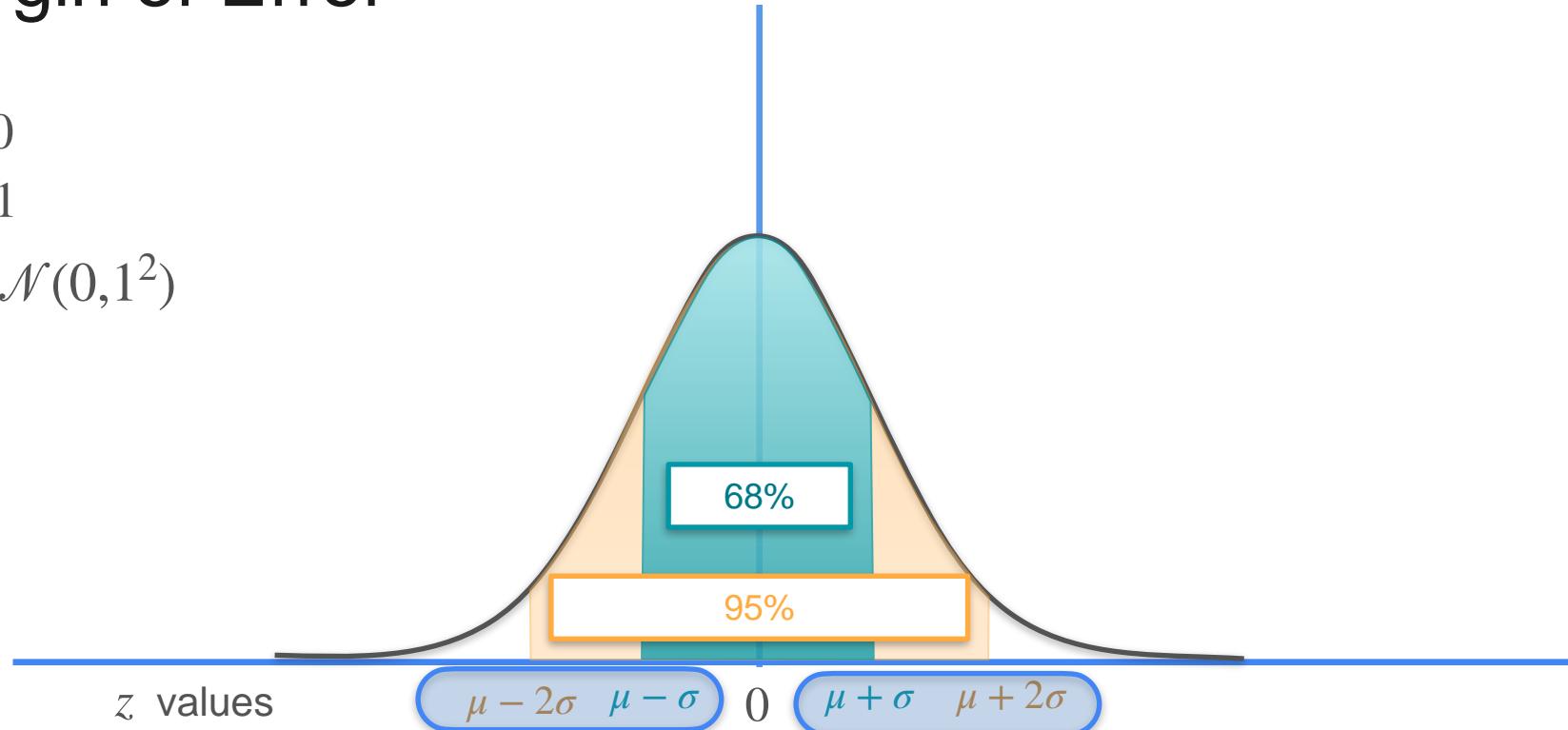


# Margin of Error

$$\mu = 0$$

$$\sigma = 1$$

$$\bar{X} \sim \mathcal{N}(0, 1^2)$$

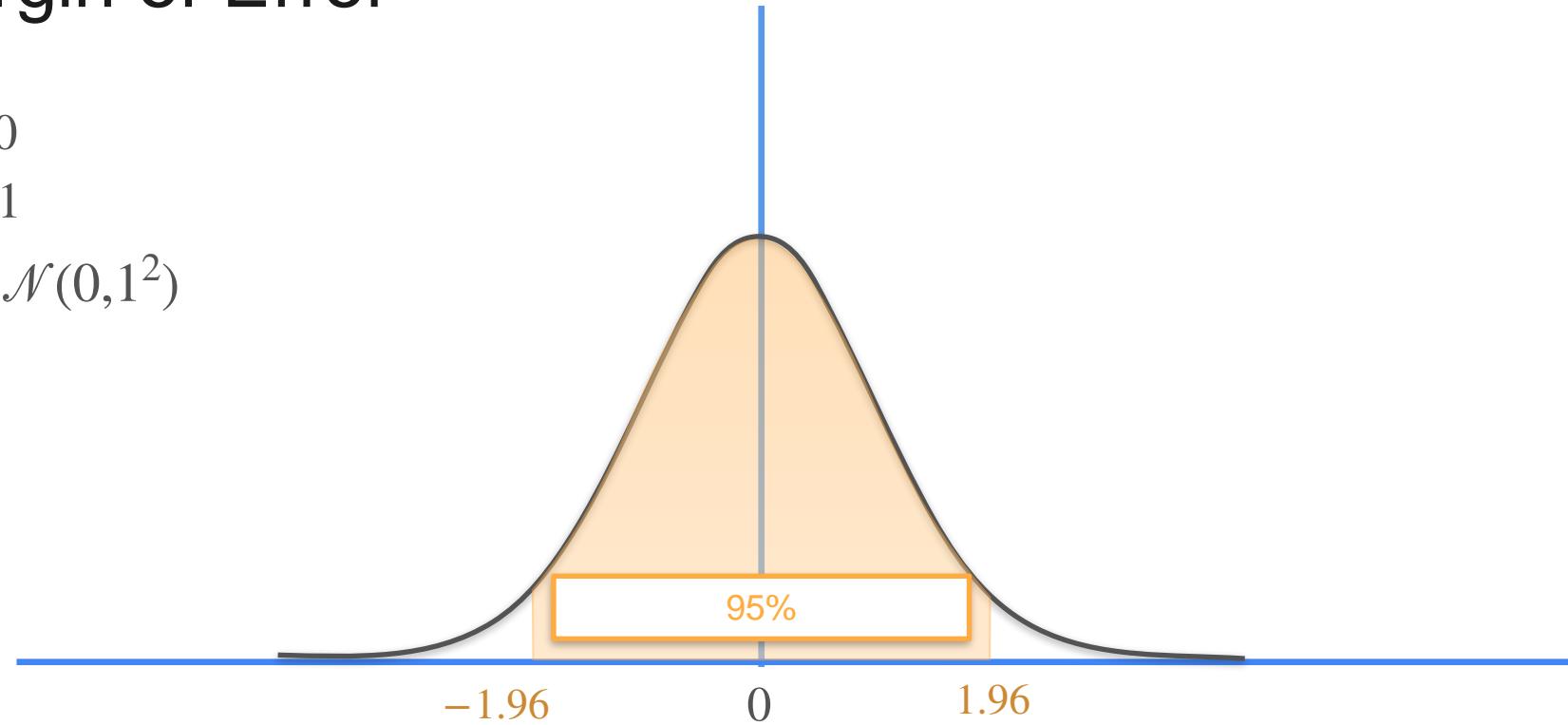


# Margin of Error

$$\mu = 0$$

$$\sigma = 1$$

$$\bar{X} \sim \mathcal{N}(0, 1^2)$$

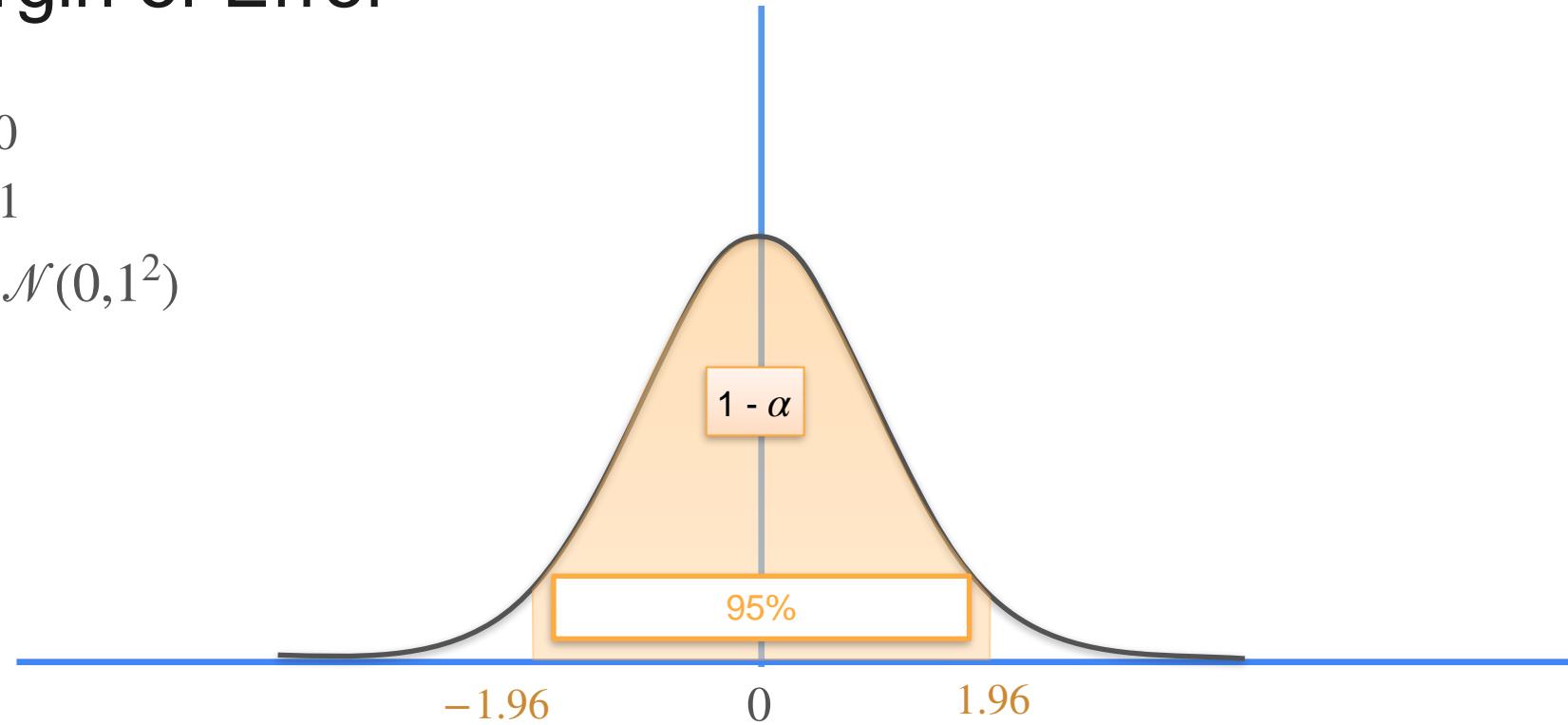


# Margin of Error

$$\mu = 0$$

$$\sigma = 1$$

$$\bar{X} \sim \mathcal{N}(0, 1^2)$$

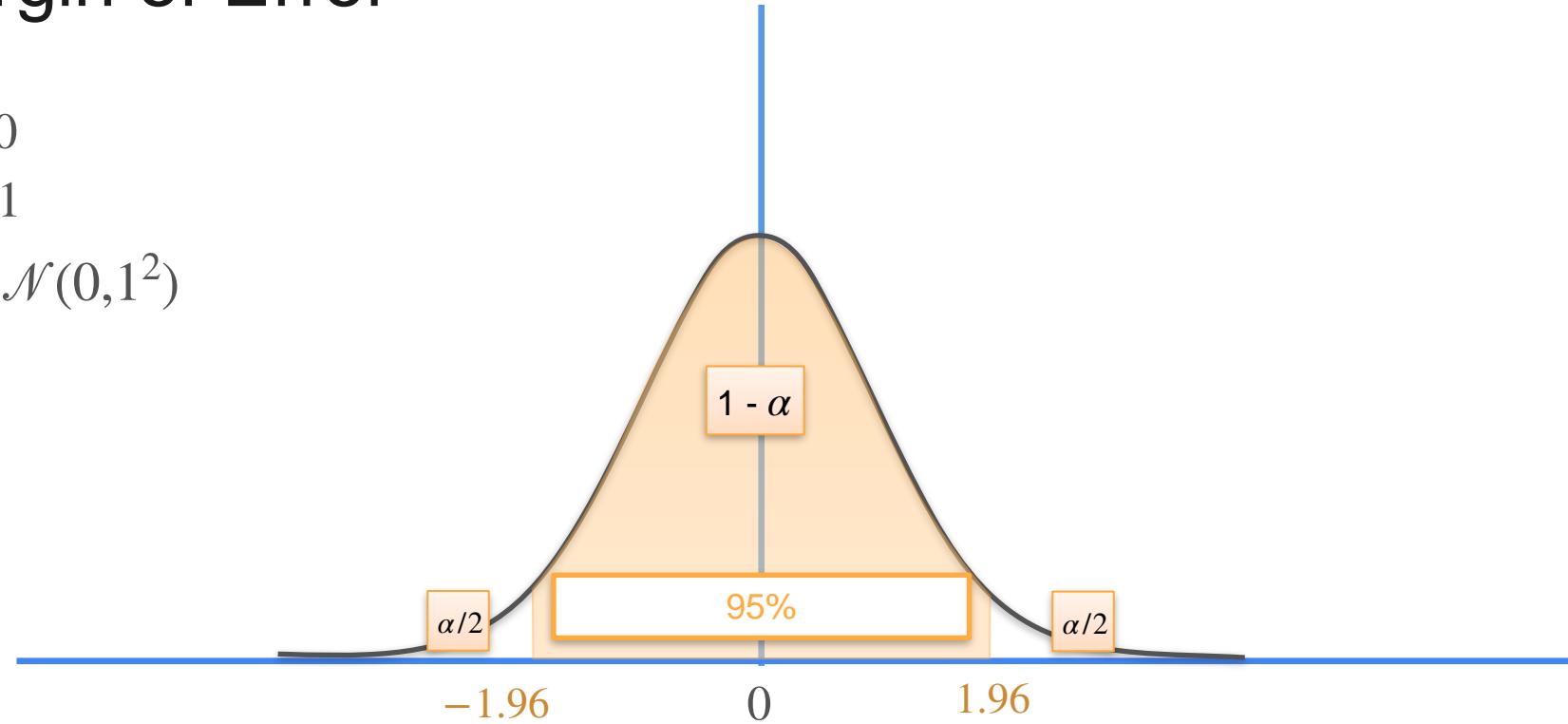


# Margin of Error

$$\mu = 0$$

$$\sigma = 1$$

$$\bar{X} \sim \mathcal{N}(0, 1^2)$$



# Margin of Error

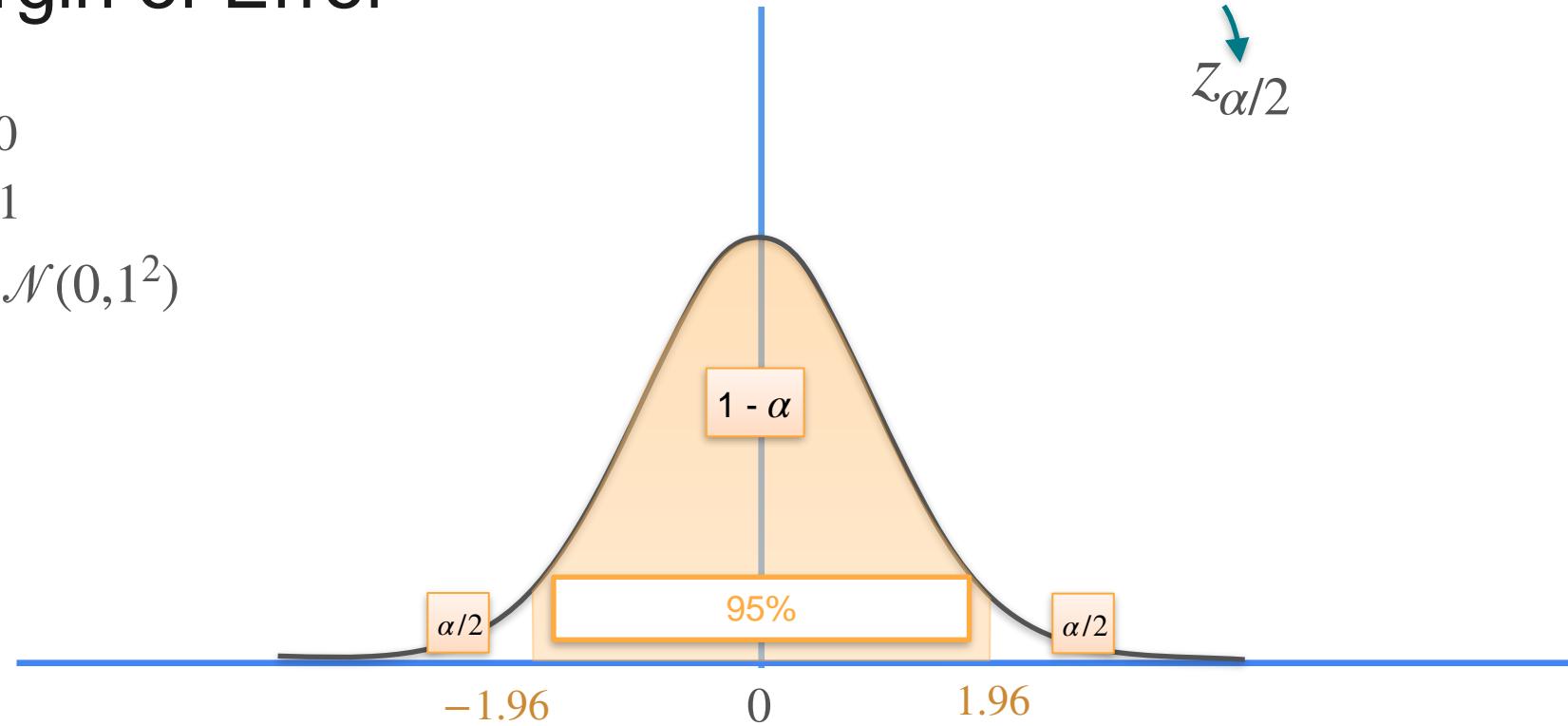
$$\mu = 0$$

$$\sigma = 1$$

$$\bar{X} \sim \mathcal{N}(0, 1^2)$$

Critical Value

$$z_{\alpha/2}$$



# Margin of Error

$$\mu = 0$$

$$\sigma = 1$$

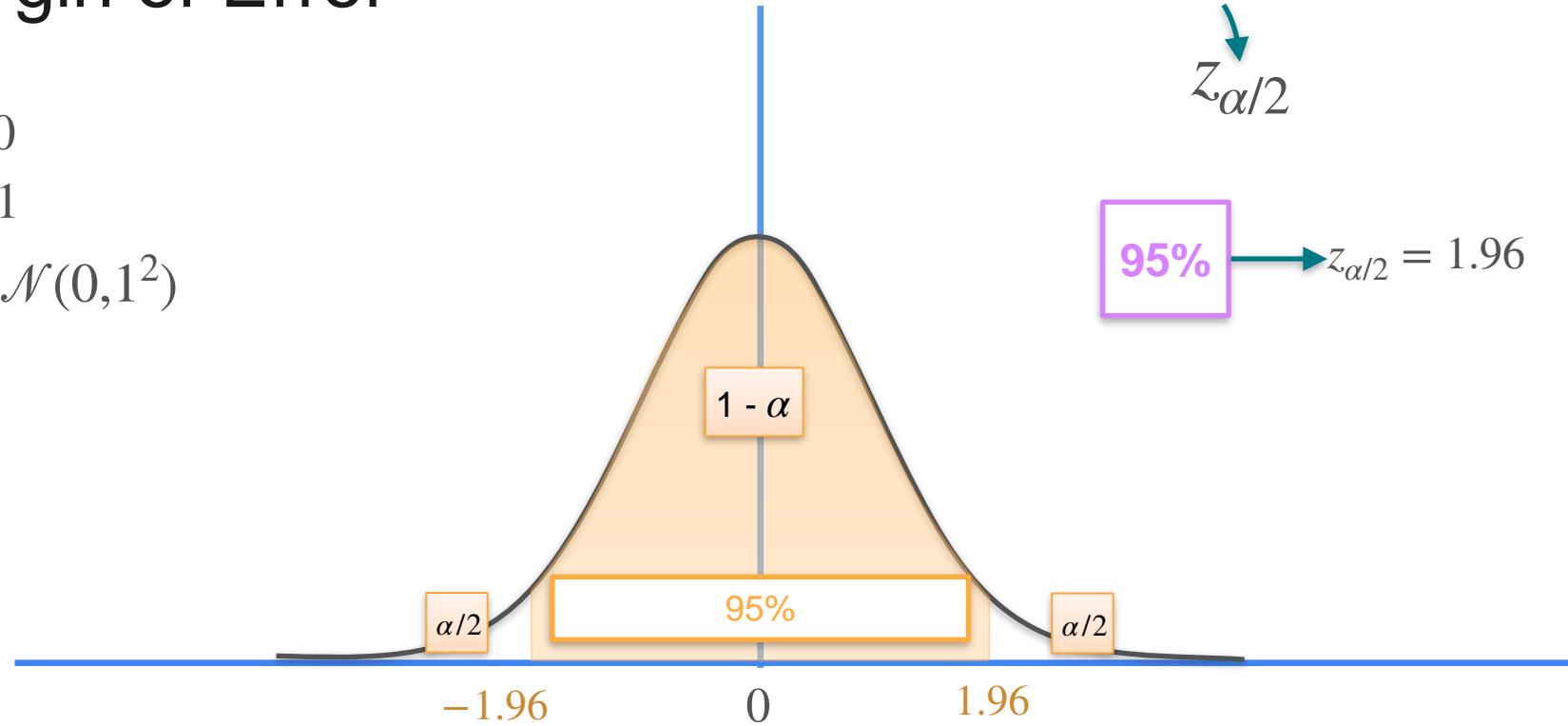
$$\bar{X} \sim \mathcal{N}(0, 1^2)$$

Critical Value

$$z_{\alpha/2}$$

95%

$$z_{\alpha/2} = 1.96$$



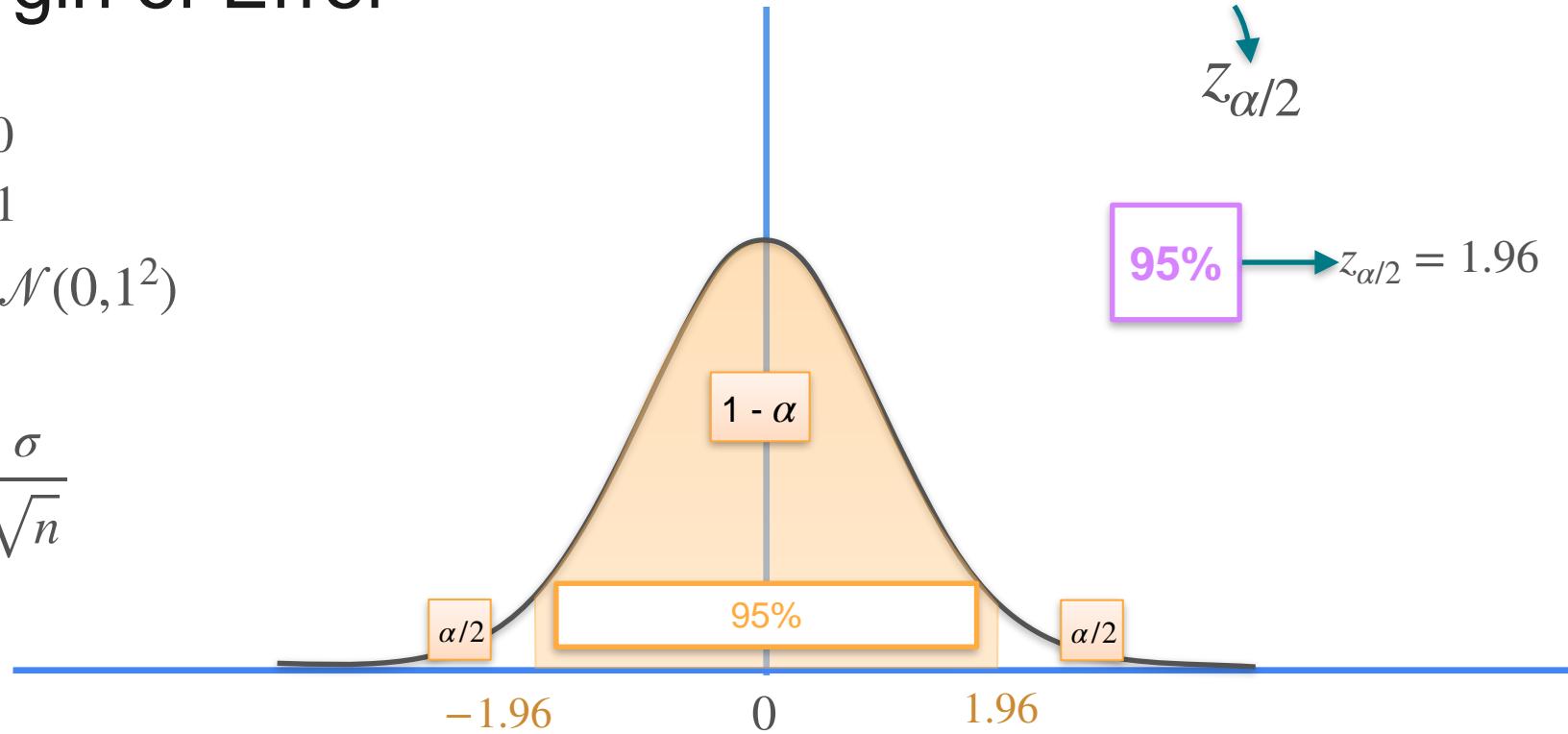
# Margin of Error

$$\mu = 0$$

$$\sigma = 1$$

$$\bar{X} \sim \mathcal{N}(0, 1^2)$$

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$



# Margin of Error

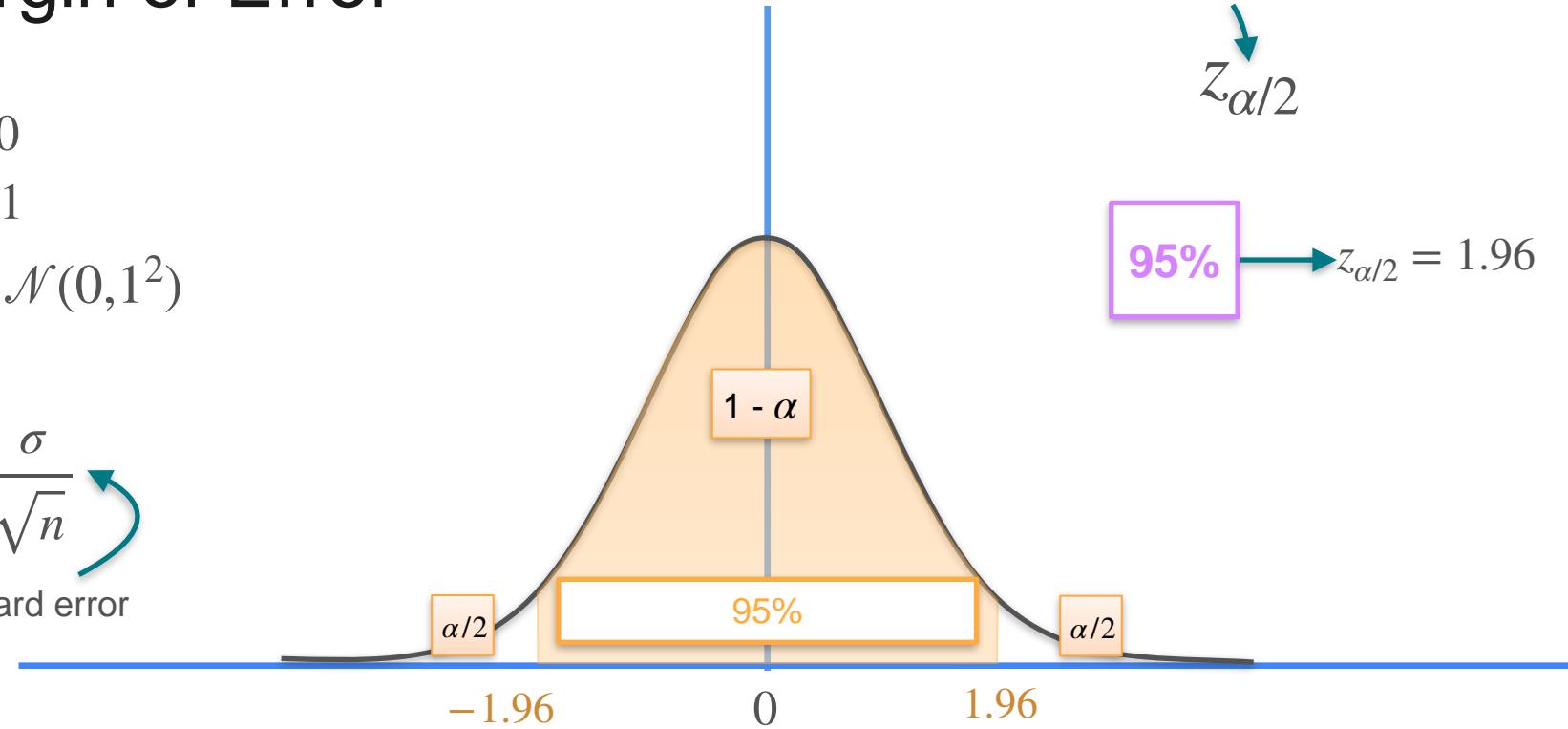
$$\mu = 0$$

$$\sigma = 1$$

$$\bar{X} \sim \mathcal{N}(0, 1^2)$$

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

Standard error



# Margin of Error

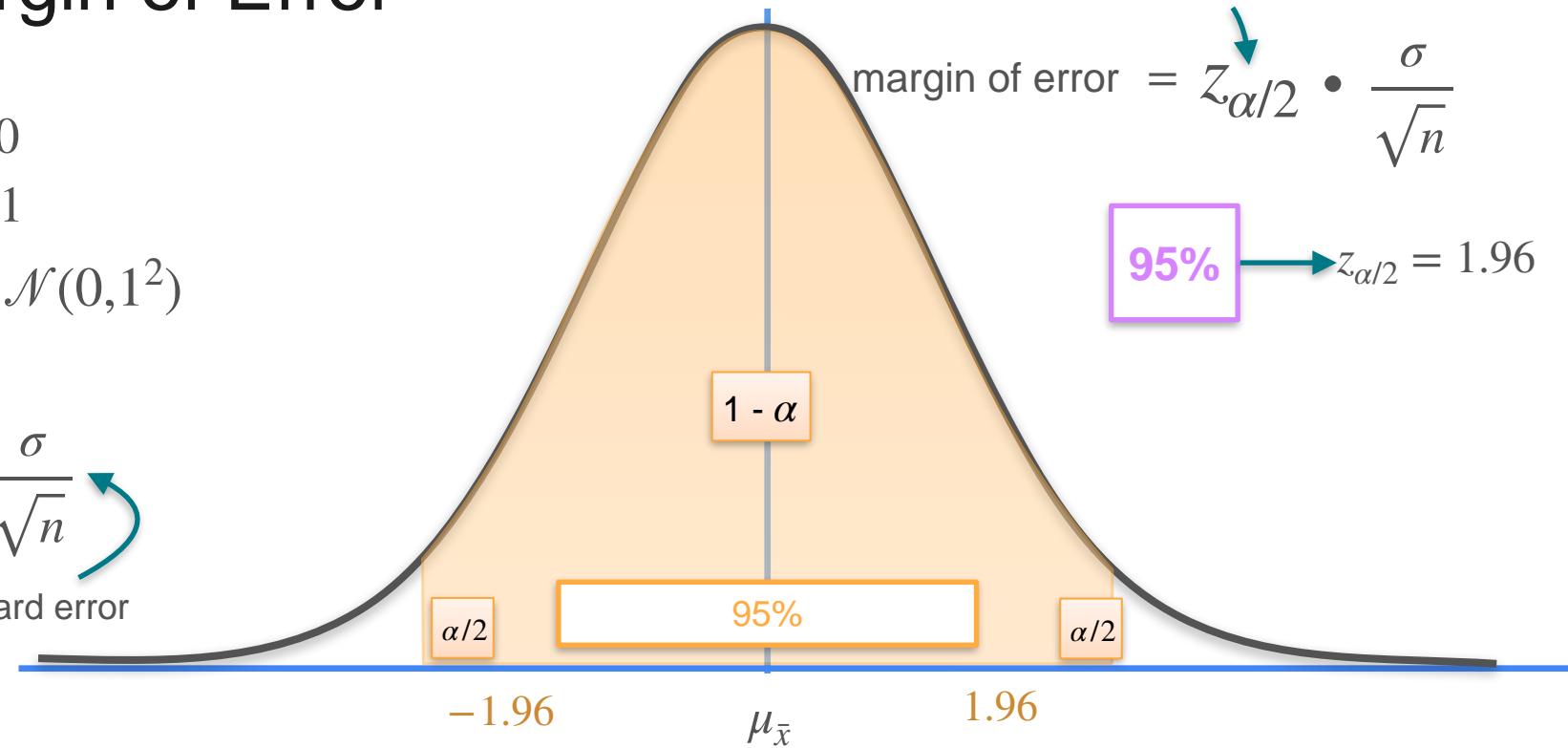
$$\mu = 0$$

$$\sigma = 1$$

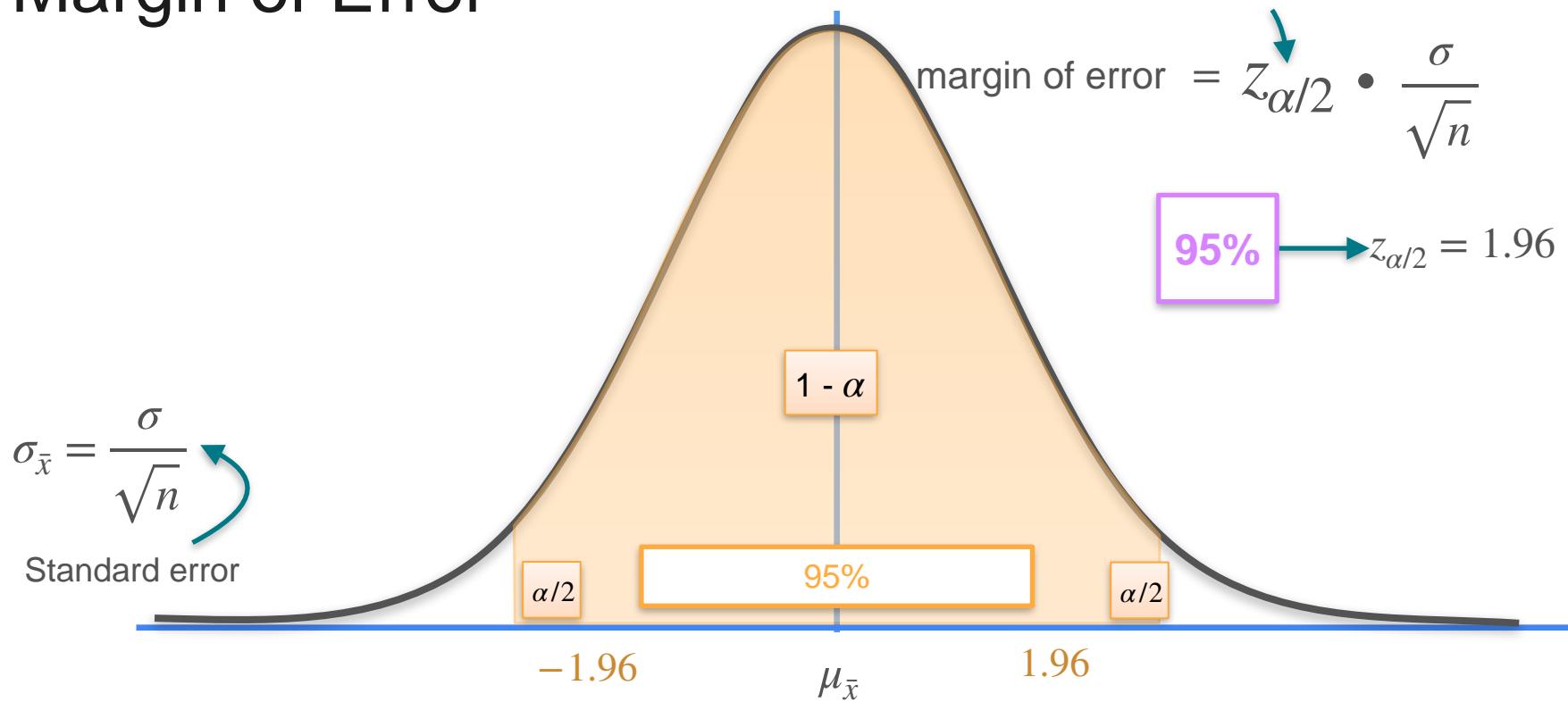
$$\bar{X} \sim \mathcal{N}(0, 1^2)$$

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

Standard error



# Margin of Error



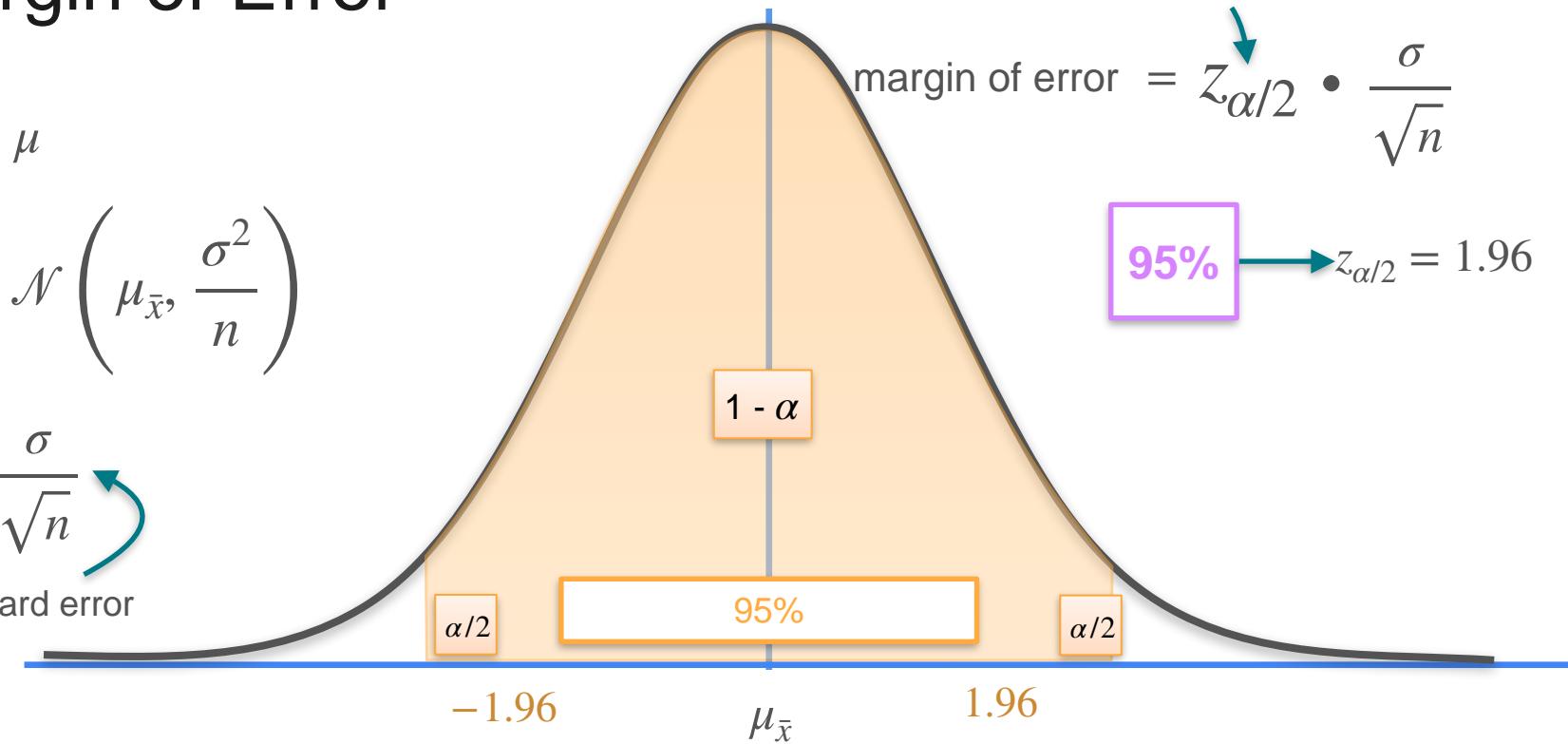
# Margin of Error

$$\mu_{\bar{x}} = \mu$$

$$\bar{X} \sim \mathcal{N}\left(\mu_{\bar{x}}, \frac{\sigma^2}{n}\right)$$

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

Standard error



# Margin of Error

$$\mu_{\bar{x}} = \mu$$

$$\bar{X} \sim \mathcal{N}\left(\mu_{\bar{x}}, \frac{\sigma^2}{n}\right)$$

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

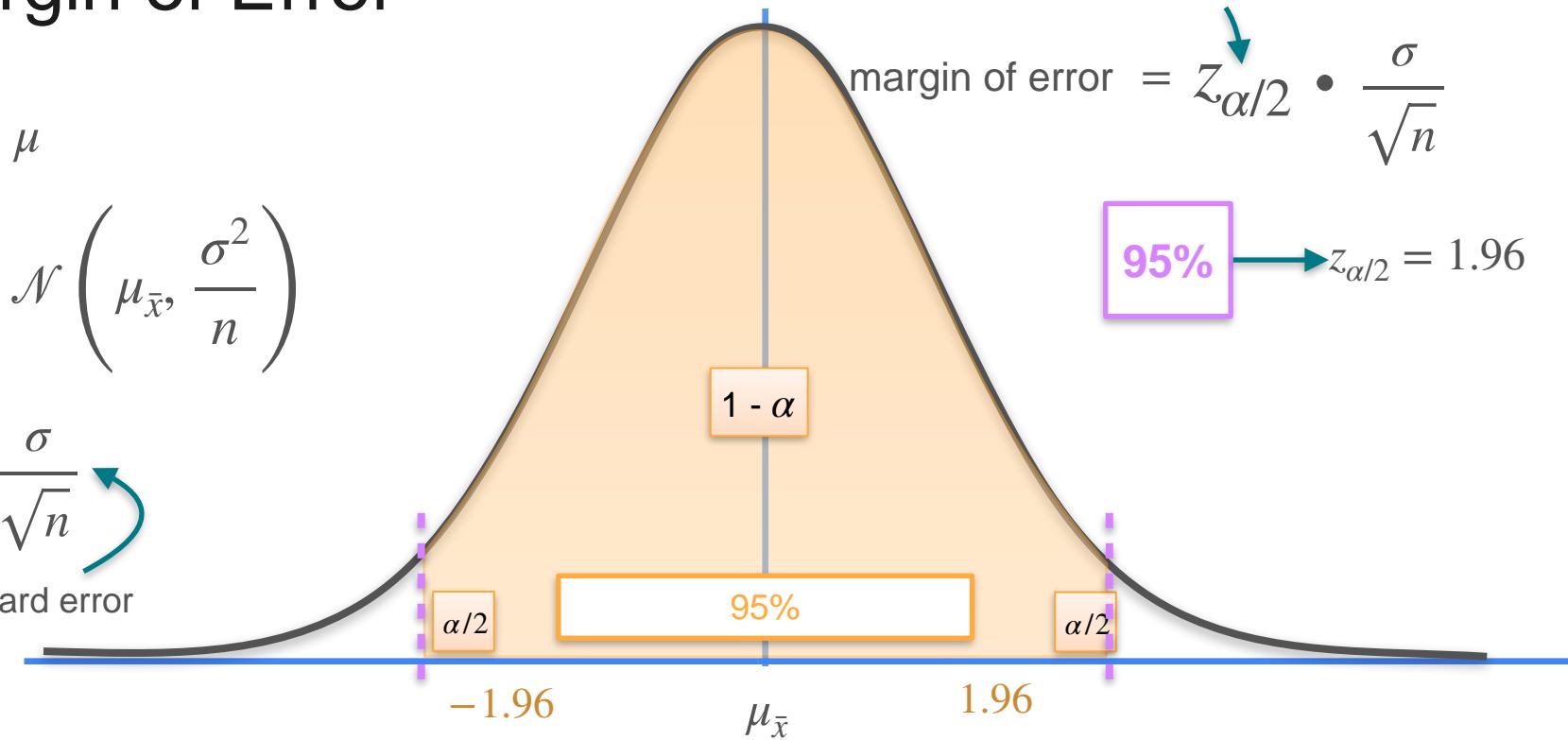
Standard error

Critical Value

$$\text{margin of error} = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

95%

$$z_{\alpha/2} = 1.96$$



# Margin of Error

$$\mu_{\bar{x}} = \mu$$

$$\bar{X} \sim \mathcal{N}\left(\mu_{\bar{x}}, \frac{\sigma^2}{n}\right)$$

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

Standard error

Critical Value

$$\text{margin of error} = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

95%

$$z_{\alpha/2} = 1.96$$

$1 - \alpha$

95%

$$z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

$\alpha/2$

-1.96

$\mu_{\bar{x}}$

1.96

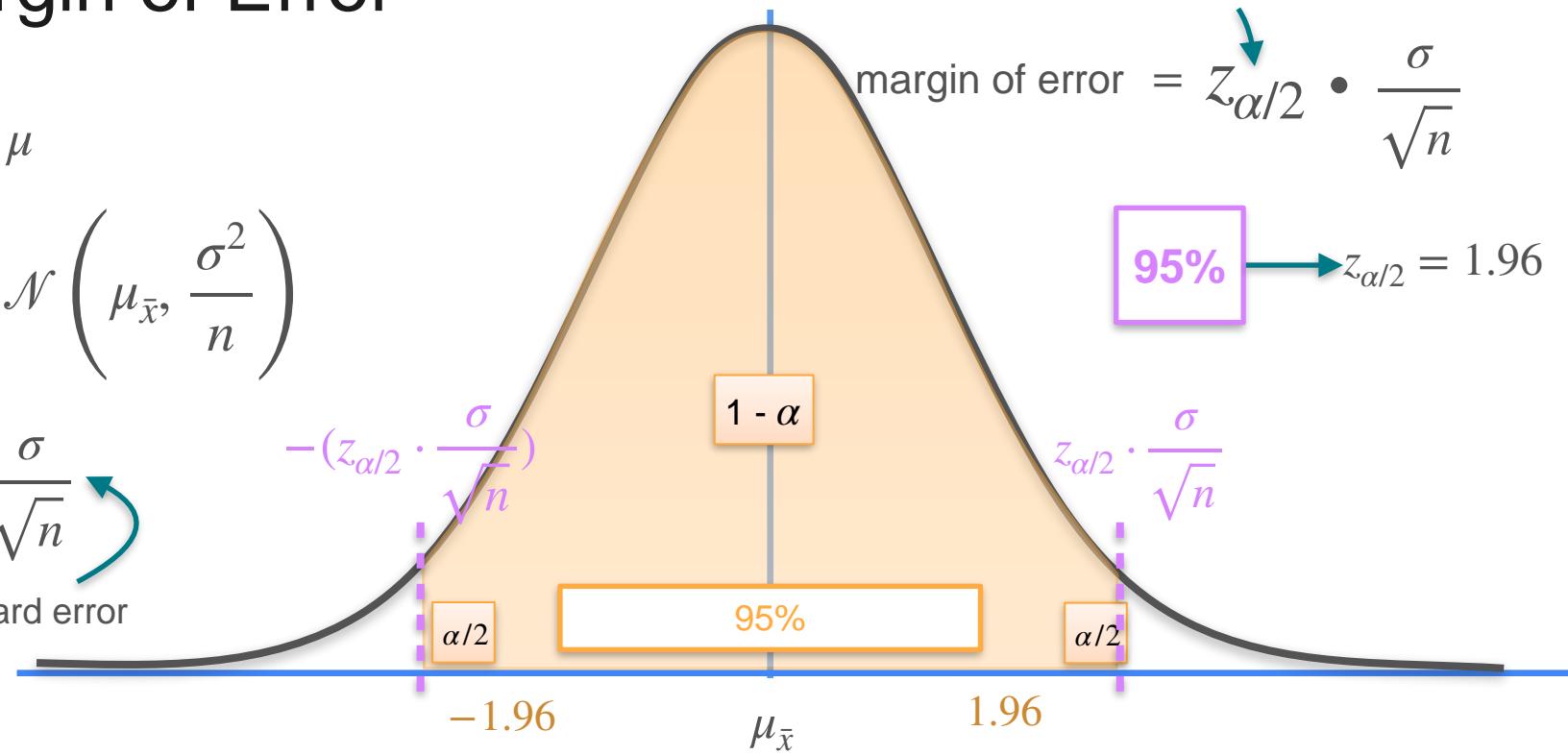
# Margin of Error

$$\mu_{\bar{x}} = \mu$$

$$\bar{X} \sim \mathcal{N}\left(\mu_{\bar{x}}, \frac{\sigma^2}{n}\right)$$

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

Standard error



# Margin of Error

$$\mu_{\bar{x}} = \mu$$

$$\bar{X} \sim \mathcal{N}\left(\mu_{\bar{x}}, \frac{\sigma^2}{n}\right)$$

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

Standard error

Critical Value

$$\text{margin of error} = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

95%

$$z_{\alpha/2} = 1.96$$

$$-(z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}})$$

$1 - \alpha$

$$z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

-1.96

$\mu_{\bar{x}}$

1.96



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## Confidence Interval

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**Confidence Interval -  
Calculation Steps**

# Confidence Interval - Calculation Steps

STEPS:

# Confidence Interval - Calculation Steps

## STEPS:

- Find the sample mean

# Confidence Interval - Calculation Steps

## STEPS:

- Find the sample mean

$$\bar{x}$$

# Confidence Interval - Calculation Steps

## STEPS:

- Find the sample mean
- Define a desired confidence level ( $1 - \alpha$ )

$\bar{x}$

# Confidence Interval - Calculation Steps

## STEPS:

- Find the sample mean
- Define a desired confidence level ( $1 - \alpha$ )

$\bar{x}$

95%

# Confidence Interval - Calculation Steps

## STEPS:

- Find the sample mean
- Define a desired confidence level ( $1 - \alpha$ )
- Get the critical value ( $z_{\alpha/2}$ )

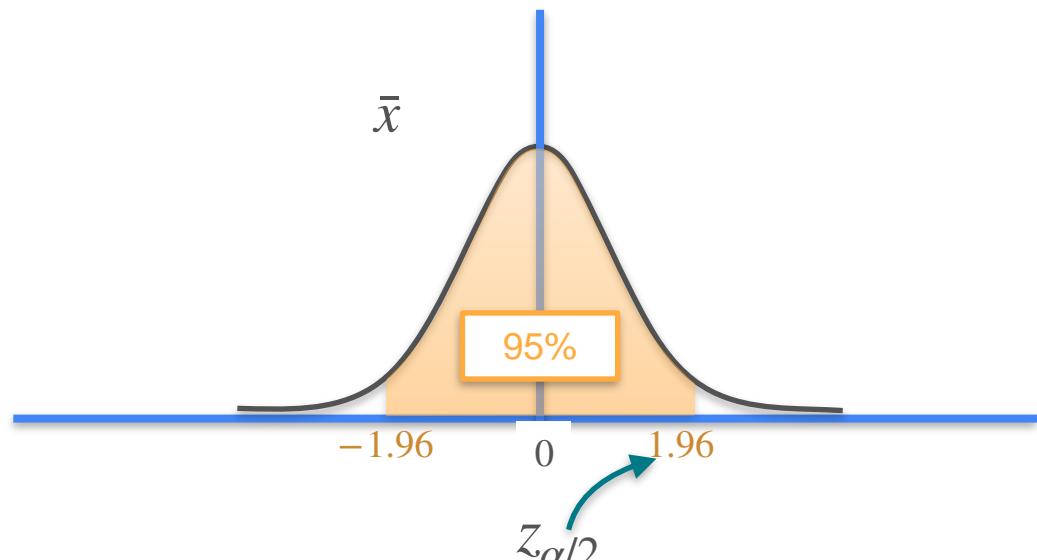
$\bar{x}$

95%

# Confidence Interval - Calculation Steps

## STEPS:

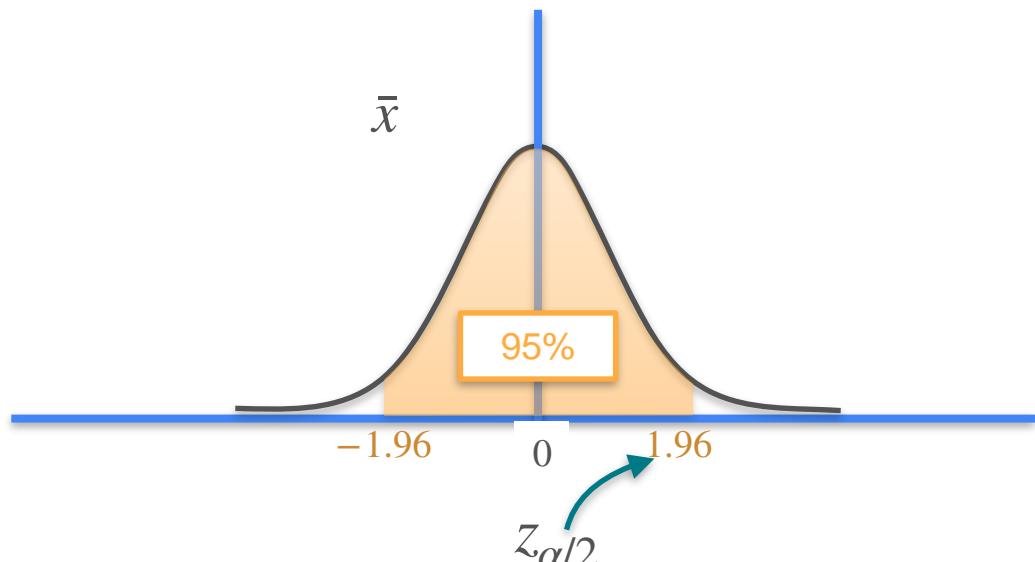
- Find the sample mean
- Define a desired confidence level ( $1 - \alpha$ )
- Get the critical value ( $z_{\alpha/2}$ )



# Confidence Interval - Calculation Steps

## STEPS:

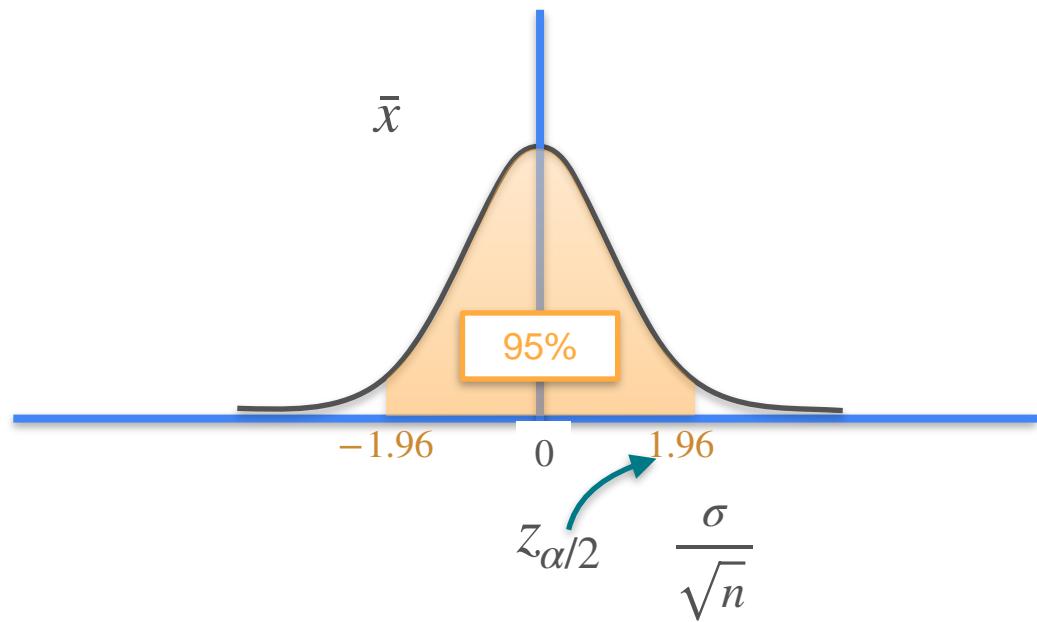
- Find the sample mean
- Define a desired confidence level ( $1 - \alpha$ )
- Get the critical value ( $z_{\alpha/2}$ )
- Find the standard error ( $\frac{\sigma}{\sqrt{n}}$ )



# Confidence Interval - Calculation Steps

## STEPS:

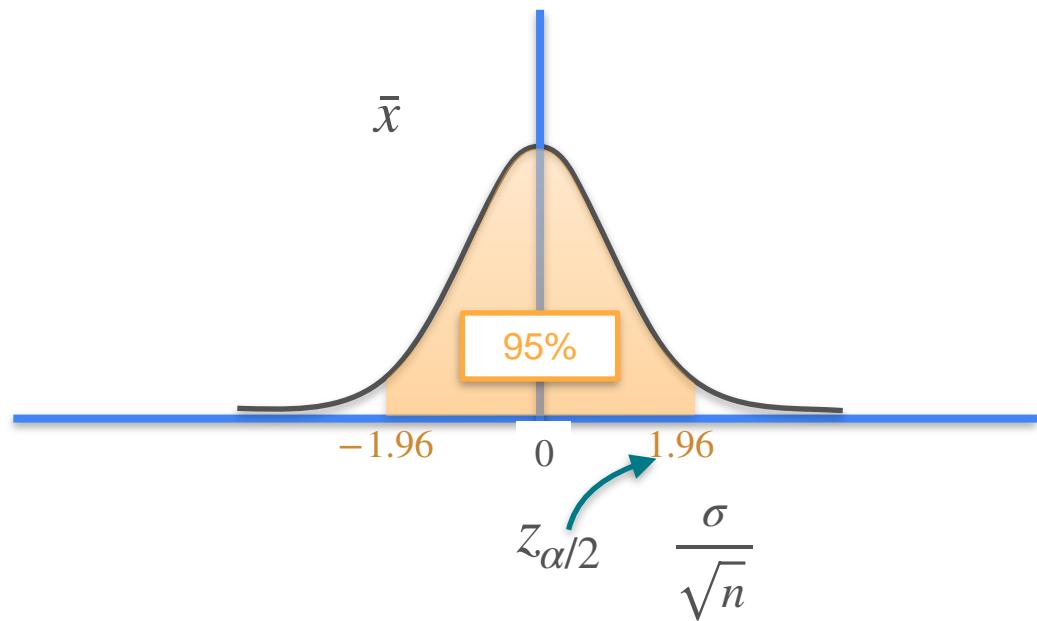
- Find the sample mean
- Define a desired confidence level ( $1 - \alpha$ )
- Get the critical value ( $z_{\alpha/2}$ )
- Find the standard error ( $\frac{\sigma}{\sqrt{n}}$ )



# Confidence Interval - Calculation Steps

## STEPS:

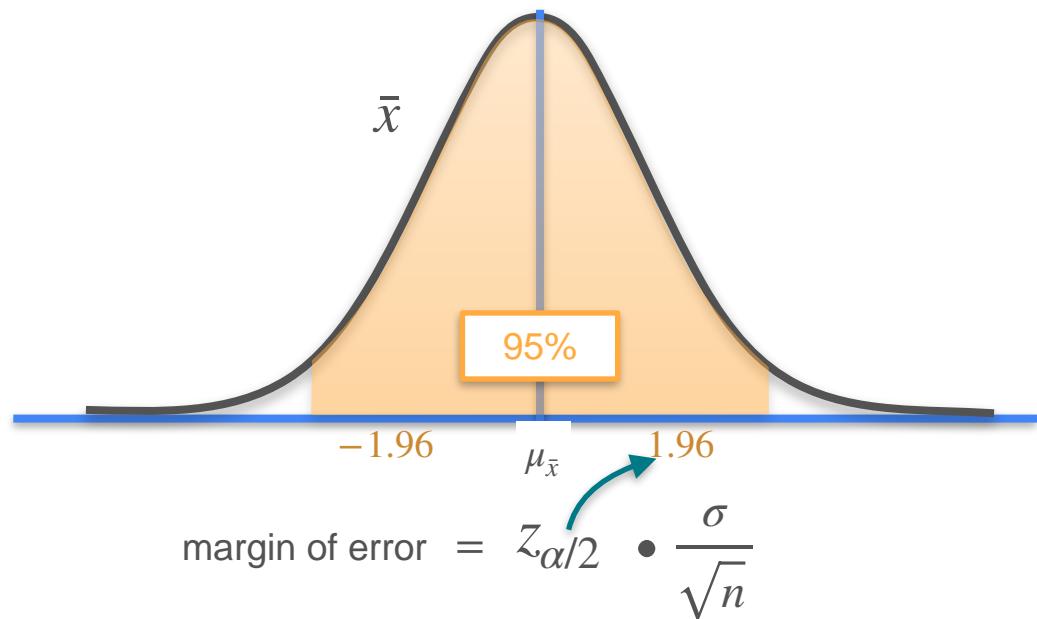
- Find the sample mean
- Define a desired confidence level ( $1 - \alpha$ )
- Get the critical value ( $z_{\alpha/2}$ )
- Find the standard error ( $\frac{\sigma}{\sqrt{n}}$ )
- Find the margin of error



# Confidence Interval - Calculation Steps

## STEPS:

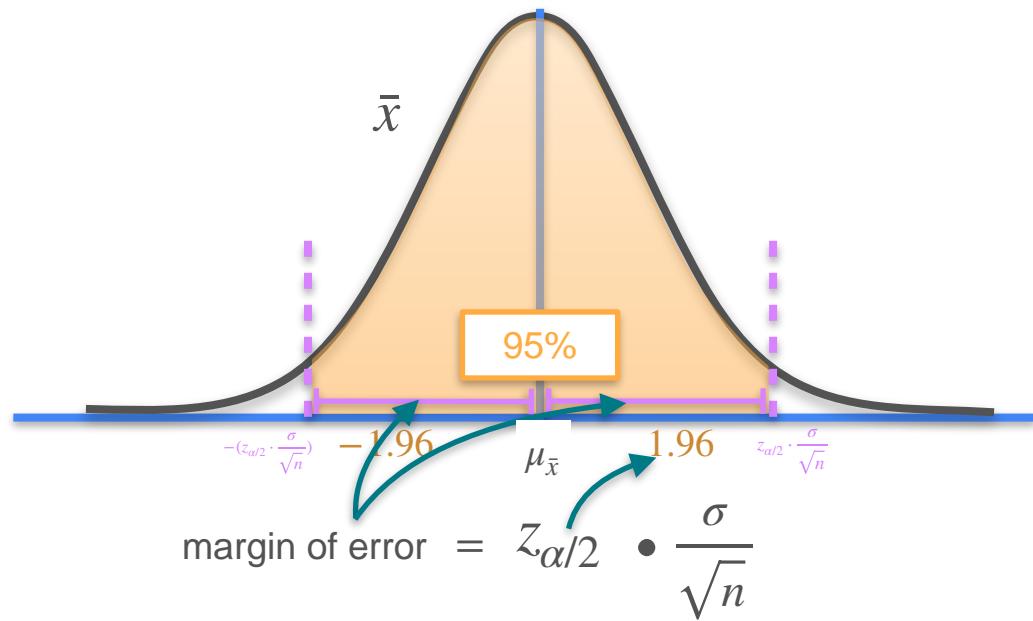
- Find the sample mean
- Define a desired confidence level ( $1 - \alpha$ )
- Get the critical value ( $z_{\alpha/2}$ )
- Find the standard error ( $\frac{\sigma}{\sqrt{n}}$ )
- Find the margin of error



# Confidence Interval - Calculation Steps

## STEPS:

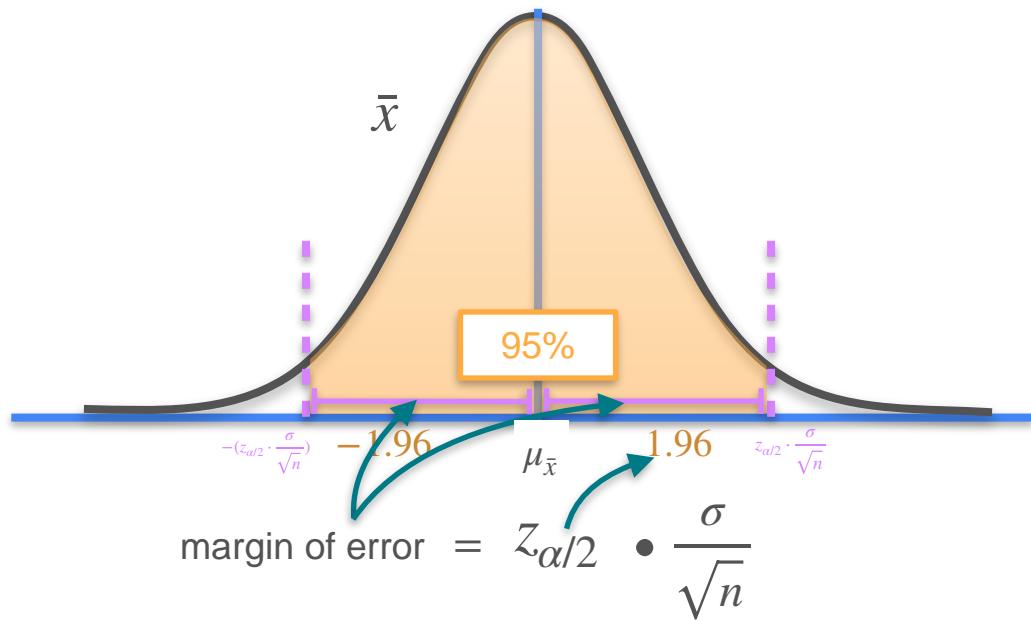
- Find the sample mean
- Define a desired confidence level ( $1 - \alpha$ )
- Get the critical value ( $z_{\alpha/2}$ )
- Find the standard error ( $\frac{\sigma}{\sqrt{n}}$ )
- Find the margin of error



# Confidence Interval - Calculation Steps

## STEPS:

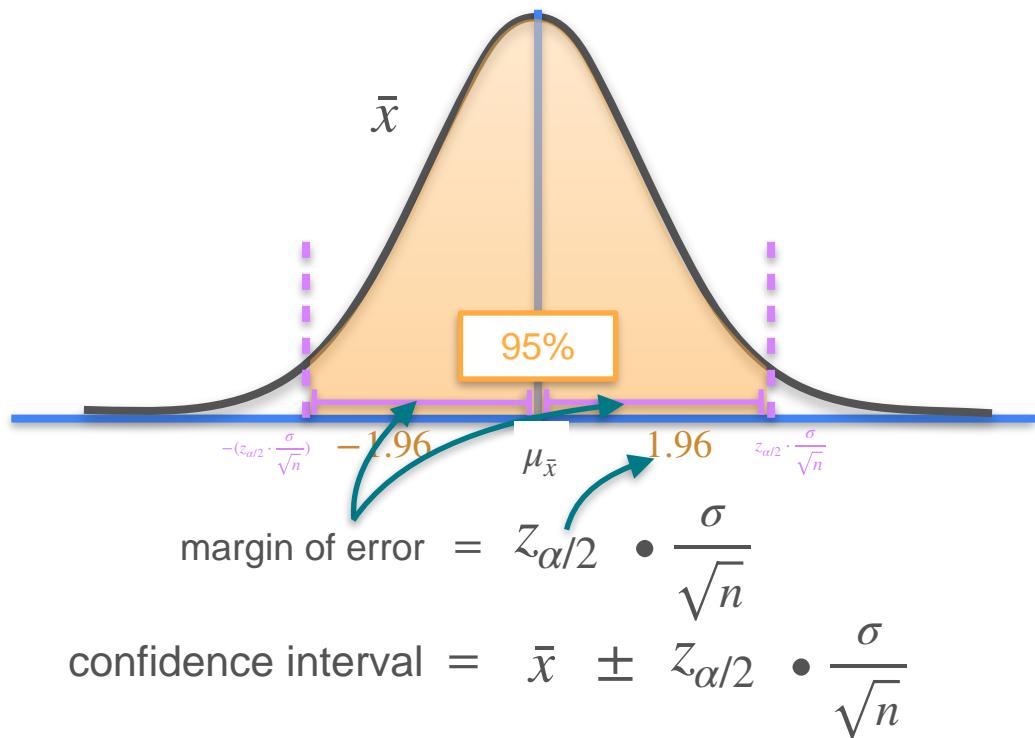
- Find the sample mean
- Define a desired confidence level ( $1 - \alpha$ )
- Get the critical value ( $z_{\alpha/2}$ )
- Find the standard error ( $\frac{\sigma}{\sqrt{n}}$ )
- Find the margin of error
- Add/subtract the margin of error to the sample mean



# Confidence Interval - Calculation Steps

## STEPS:

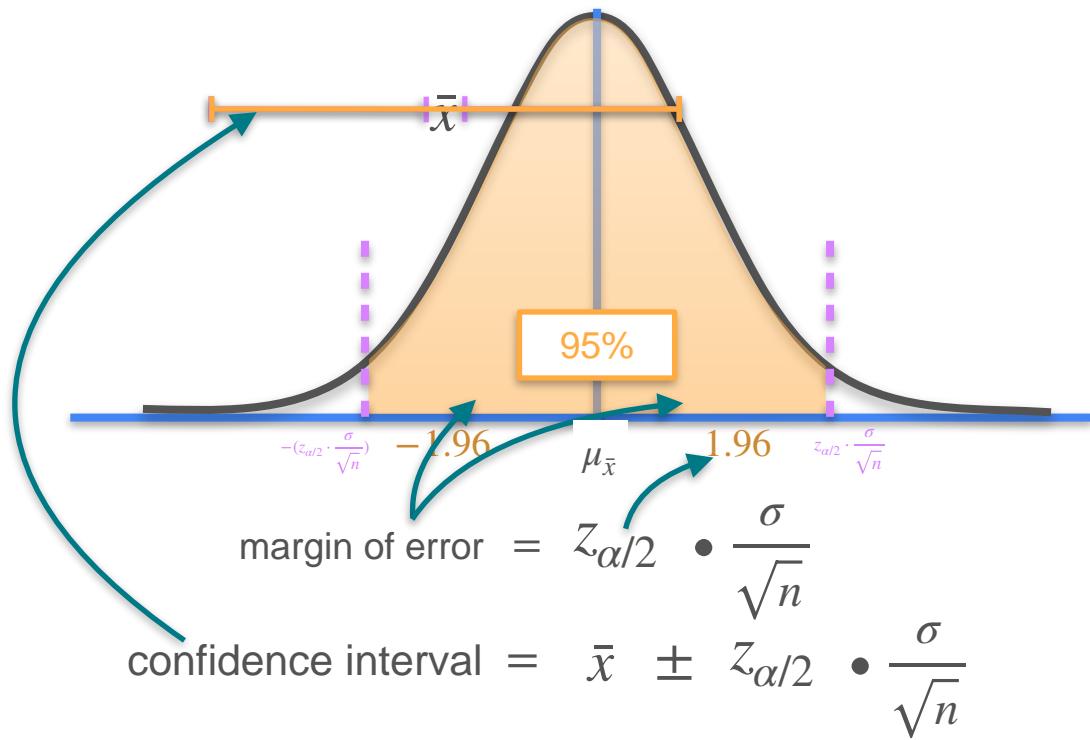
- Find the sample mean
- Define a desired confidence level ( $1 - \alpha$ )
- Get the critical value ( $z_{\alpha/2}$ )
- Find the standard error ( $\frac{\sigma}{\sqrt{n}}$ )
- Find the margin of error
- Add/subtract the margin of error to the sample mean



# Confidence Interval - Calculation Steps

## STEPS:

- Find the sample mean
- Define a desired confidence level ( $1 - \alpha$ )
- Get the critical value ( $z_{\alpha/2}$ )
- Find the standard error ( $\frac{\sigma}{\sqrt{n}}$ )
- Find the margin of error
- Add/subtract the margin of error to the sample mean



# Confidence Interval - Calculation Steps

## STEPS:

- Find the sample mean
- Define a desired confidence level ( $1 - \alpha$ )
- Get the critical value ( $z_{\alpha/2}$ )
- Find the standard error ( $\frac{\sigma}{\sqrt{n}}$ )
- Find the margin of error
- Add/subtract the margin of error to the sample mean

$$\text{confidence interval} = \bar{x} \pm z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

# Confidence Interval - Calculation Steps

## STEPS:

- Find the sample mean
- Define a desired confidence level ( $1 - \alpha$ )
- Get the critical value ( $z_{\alpha/2}$ )
- Find the standard error ( $\frac{\sigma}{\sqrt{n}}$ )
- Find the margin of error
- Add/subtract the margin of error to the sample mean

$$\text{confidence interval} = \bar{x} \pm z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

## Assumptions

# Confidence Interval - Calculation Steps

## STEPS:

- Find the sample mean
- Define a desired confidence level ( $1 - \alpha$ )
- Get the critical value ( $z_{\alpha/2}$ )
- Find the standard error ( $\frac{\sigma}{\sqrt{n}}$ )
- Find the margin of error
- Add/subtract the margin of error to the sample mean

$$\text{confidence interval} = \bar{x} \pm z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

## Assumptions

- Simple random sample

# Confidence Interval - Calculation Steps

## STEPS:

- Find the sample mean
- Define a desired confidence level ( $1 - \alpha$ )
- Get the critical value ( $z_{\alpha/2}$ )
- Find the standard error ( $\frac{\sigma}{\sqrt{n}}$ )
- Find the margin of error
- Add/subtract the margin of error to the sample mean

$$\text{confidence interval} = \bar{x} \pm z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

## Assumptions

- Simple random sample
- Sample size  $> 30$  or population is approximately normal



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## Confidence Interval

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**Confidence Interval -  
Example**

# Confidence Interval - Example

# Confidence Interval - Example

Statistopia

6,000 adults

# Confidence Interval - Example

Statistopia

6,000 adults

Random Selection

49



# Confidence Interval - Example

Statistopia

6,000 adults

Random Selection



$$\bar{x} = 170\text{cm}$$

# Confidence Interval - Example

Statistopia

6,000 adults

Random Selection



$$\bar{x} = 170\text{cm}$$

$$\sigma = 25\text{cm}$$

# Confidence Interval - Example

Statistopia

6,000 adults

Random Selection



$$\bar{x} = 170\text{cm}$$

$$\sigma = 25\text{cm}$$

Calculate a 95% confidence interval for the average height of adults on Statistopia.

# Confidence Interval - Example

Statistopia

6,000 adults

Random Selection



$$\bar{x} = 170\text{cm}$$

$$\sigma = 25\text{cm}$$

95%

$$\rightarrow z_{\alpha/2} = 1.96$$

Calculate a 95% confidence interval for the average height of adults on Statistopia.

# Confidence Interval - Example

Random Selection

49



$$\sigma = 25\text{cm}$$

95%

$$\rightarrow z_{\alpha/2} = 1.96$$

# Confidence Interval - Example

Random Selection

49



margin of error =

$$\sigma = 25\text{cm}$$

95%

$$\rightarrow z_{\alpha/2} = 1.96$$

# Confidence Interval - Example

Random Selection

49



$$\text{margin of error} = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

$$\sigma = 25\text{cm}$$

95%

$$z_{\alpha/2} = 1.96$$

# Confidence Interval - Example

Random Selection

49



$$\sigma = 25\text{cm}$$

$$\begin{aligned}\text{margin of error} &= z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} \\ &= 1.96 \cdot \frac{25}{\sqrt{49}}\end{aligned}$$

95%

$$z_{\alpha/2} = 1.96$$

# Confidence Interval - Example

Random Selection

49



$$\sigma = 25\text{cm}$$

95%

$$z_{\alpha/2} = 1.96$$

$$\begin{aligned}\text{margin of error} &= z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} \\ &= 1.96 \cdot \frac{25}{\sqrt{49}} \\ &= 1.96 \cdot \frac{25}{7}\end{aligned}$$

# Confidence Interval - Example

Random Selection

49



$$\sigma = 25\text{cm}$$

95%

$$z_{\alpha/2} = 1.96$$

$$\begin{aligned}\text{margin of error} &= z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} \\ &= 1.96 \cdot \frac{25}{\sqrt{49}} \\ &= 1.96 \cdot \frac{25}{7} \\ &= 7\end{aligned}$$

# Confidence Interval - Example

Random Selection



$$\sigma = 25\text{cm}$$

95%

$$z_{\alpha/2} = 1.96$$

Confidence Interval

$$\begin{aligned}\text{margin of error} &= z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} \\ &= 1.96 \cdot \frac{25}{\sqrt{49}} \\ &= 1.96 \cdot \frac{25}{7} \\ &= 7\end{aligned}$$

# Confidence Interval - Example

Random Selection



$$\sigma = 25\text{cm}$$

95%

$$z_{\alpha/2} = 1.96$$

Confidence Interval

$$170\text{cm} \pm \text{margin of error}$$

$$\text{margin of error} = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

$$= 1.96 \cdot \frac{25}{\sqrt{49}}$$

$$= 1.96 \cdot \frac{25}{7}$$

$$= 7$$

# Confidence Interval - Example

Random Selection

2500



$$\sigma = 10cm$$

**Confidence Interval**

$$170cm \pm \text{margin of error}$$

$$\text{margin of error} = 7$$

95%

$$\rightarrow z_{\alpha/2} = 1.96$$

# Confidence Interval - Example

Random Selection

2500



$$\sigma = 10cm$$

**Confidence Interval**

$$170cm \pm \text{margin of error}$$

$$\text{margin of error} = 7$$

**Confidence Interval**

$$\boxed{95\%} \rightarrow z_{\alpha/2} = 1.96$$

# Confidence Interval - Example

Random Selection

2500



$$\sigma = 10\text{cm}$$

**Confidence Interval**

$$170\text{cm} \pm \text{margin of error}$$

$$\text{margin of error} = 7$$

**Confidence Interval**

$$170\text{cm} - 7 = 163\text{cm}$$

95%

$$\rightarrow z_{\alpha/2} = 1.96$$

# Confidence Interval - Example

Random Selection

2500



$$\sigma = 10\text{cm}$$

95%

$$z_{\alpha/2} = 1.96$$

**Confidence Interval**

$$170\text{cm} \pm \text{margin of error}$$

$$\text{margin of error} = 7$$

**Confidence Interval**

$$170\text{cm} - 7 = 163\text{cm}$$

$$170\text{cm} + 7 = 177\text{cm}$$

# Confidence Interval - Example

Random Selection

2500



$$\sigma = 10\text{cm}$$

95%

$$z_{\alpha/2} = 1.96$$

**Confidence Interval**

$$170\text{cm} \pm \text{margin of error}$$

$$\text{margin of error} = 7$$

**Confidence Interval**

$$170\text{cm} - 7 = 163\text{cm}$$

$$170\text{cm} + 7 = 177\text{cm}$$

$$163\text{cm} < \mu < 177\text{cm}$$



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## Confidence Interval

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## Calculating Sample Size

# Calculating Sample Size

6,000 adults

95%

$$\rightarrow z_{\alpha/2} = 1.96$$

$$\bar{x} = 170\text{cm} \quad \sigma = 25\text{cm}$$

# Calculating Sample Size

6,000 adults

Random Selection

49



95%

$$\rightarrow z_{\alpha/2} = 1.96$$

$$\bar{x} = 170\text{cm} \quad \sigma = 25\text{cm}$$

# Calculating Sample Size

6,000 adults

Random Selection

49

Margin of error: 7cm

95%

$$\rightarrow z_{\alpha/2} = 1.96$$

$$\bar{x} = 170\text{cm} \quad \sigma = 25\text{cm}$$



# Calculating Sample Size

6,000 adults

Random Selection

49

Margin of error: 7cm

$$\bar{x} \pm 7\text{cm}$$



95% →  $z_{\alpha/2} = 1.96$

$$\bar{x} = 170\text{cm} \quad \sigma = 25\text{cm}$$

# Calculating Sample Size

6,000 adults

95%

$$\rightarrow z_{\alpha/2} = 1.96$$

$$\bar{x} = 170cm \quad \sigma = 25cm$$

Random Selection

49



Margin of error: 7cm

$$\bar{x} \pm 7cm$$

$$163cm < \mu < 177cm$$

# Calculating Sample Size

6,000 adults

Random Selection

49

Margin of error: 7cm

$$\bar{x} \pm 7\text{cm}$$

$$163\text{cm} < \mu < 177\text{cm}$$



$$95\% \rightarrow z_{\alpha/2} = 1.96$$

$$\bar{x} = 170\text{cm} \quad \sigma = 25\text{cm}$$

# Calculating Sample Size

6,000 adults

95%

$$\rightarrow z_{\alpha/2} = 1.96$$

$$\bar{x} = 170\text{cm} \quad \sigma = 25\text{cm}$$

Random Selection

49



Margin of error: 7cm

$$\bar{x} \pm 7\text{cm}$$

$$163\text{cm} < \mu < 177\text{cm}$$

Margin of error: 3 cm

$$\bar{x} \pm 3\text{cm}$$

# Calculating Sample Size

6,000 adults

95% →  $z_{\alpha/2} = 1.96$

$$\bar{x} = 170\text{cm} \quad \sigma = 25\text{cm}$$

Random Selection



Margin of error: 7cm

$$\bar{x} \pm 7\text{cm}$$

$$163\text{cm} < \mu < 177\text{cm}$$

Margin of error: 3 cm

$$\bar{x} \pm 3\text{cm}$$

# Calculating Sample Size

6,000 adults

95%

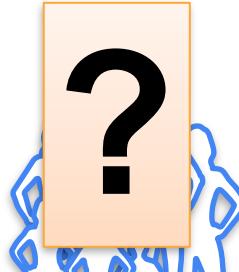
$$\rightarrow z_{\alpha/2} = 1.96$$

$$\bar{x} = 170\text{cm}$$

$$\sigma$$

What is the smallest sample size to obtain the  
desired margin of error?

Random Selection



Margin of error: 7cm

$$\bar{x} \pm 7\text{cm}$$

$$163\text{cm} < \mu < 177\text{cm}$$

Margin of error: 3 cm

$$\bar{x} \pm 3\text{cm}$$

# Calculating Sample Size

6,000 adults

95%

$$\rightarrow z_{\alpha/2} = 1.96$$

$$\bar{x} = 170\text{cm} \quad \sigma = 25\text{cm}$$

Margin of error: 3 cm

# Calculating Sample Size

6,000 adults

$$\text{margin of error} = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

95%

$$z_{\alpha/2} = 1.96$$

$$\bar{x} = 170\text{cm} \quad \sigma = 25\text{cm}$$

Margin of error: 3 cm

# Calculating Sample Size

6,000 adults

$$\text{margin of error} = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

95%

$$z_{\alpha/2} = 1.96$$

$$\bar{x} = 170\text{cm} \quad \sigma = 25\text{cm}$$

Margin of error: 3 cm

# Calculating Sample Size

6,000 adults

$$\text{margin of error} = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

95%

$$z_{\alpha/2} = 1.96$$

$$\bar{x} = 170\text{cm} \quad \sigma = 25\text{cm}$$

3

Margin of error: 3 cm

# Calculating Sample Size

6,000 adults

$$\text{margin of error} = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

95%

$$z_{\alpha/2} = 1.96$$

$$3 =$$

$$\bar{x} = 170\text{cm} \quad \sigma = 25\text{cm}$$

Margin of error: 3 cm

# Calculating Sample Size

6,000 adults

95%

$$z_{\alpha/2} = 1.96$$

$$\bar{x} = 170\text{cm} \quad \sigma = 25\text{cm}$$

Margin of error: 3 cm

$$\text{margin of error} = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

$$3 = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

# Calculating Sample Size

6,000 adults

95%

$$z_{\alpha/2} = 1.96$$

$$\bar{x} = 170\text{cm} \quad \sigma = 25\text{cm}$$

Margin of error: 3 cm

$$\text{margin of error} = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

$$3 = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

# Calculating Sample Size

6,000 adults

95%

$$z_{\alpha/2} = 1.96$$

$$\bar{x} = 170\text{cm} \quad \sigma = 25\text{cm}$$

Margin of error: 3 cm

$$\text{margin of error} = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

$$3 = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

$$3 \geq$$

# Calculating Sample Size

6,000 adults

95%

$$z_{\alpha/2} = 1.96$$

$$\bar{x} = 170\text{cm} \quad \sigma = 25\text{cm}$$

Margin of error: 3 cm

$$\text{margin of error} = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

$$3 = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

$$3 \geq z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

# Calculating Sample Size

6,000 adults

95%

$$z_{\alpha/2} = 1.96$$

$$\bar{x} = 170\text{cm} \quad \sigma = 25\text{cm}$$

Margin of error: 3 cm

$$\text{margin of error} = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

$$3 = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

$$3 \geq z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

# Calculating Sample Size

6,000 adults

$$\text{margin of error} = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

95%

$$z_{\alpha/2} = 1.96$$

$$3 \geq z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

$$\bar{x} = 170\text{cm} \quad \sigma = 25\text{cm}$$

Margin of error: 3 cm

# Calculating Sample Size

6,000 adults

$$\text{margin of error} = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

95%

$$z_{\alpha/2} = 1.96$$

$$\bar{x} = 170\text{cm} \quad \sigma = 25\text{cm}$$

Margin of error: 3 cm

$$3 \geq z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

$$3 \geq 1.96 \times \frac{25}{\sqrt{n}}$$

# Calculating Sample Size

6,000 adults

$$\text{margin of error} = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

95%

$$z_{\alpha/2} = 1.96$$

$$\bar{x} = 170\text{cm} \quad \sigma = 25\text{cm}$$

Margin of error: 3 cm

$$3 \geq z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

$$\frac{3}{1.96} \geq \frac{25}{\sqrt{n}}$$

# Calculating Sample Size

6,000 adults

$$\text{margin of error} = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

95%   $z_{\alpha/2} = 1.96$

$$\bar{x} = 170\text{cm} \quad \sigma = 25\text{cm}$$

Margin of error: 3 cm

$$3 \geq z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

$$\frac{3}{1.96 \times 25} \geq \frac{1}{\sqrt{n}}$$

# Calculating Sample Size

6,000 adults

95%

$$z_{\alpha/2} = 1.96$$

$$\bar{x} = 170\text{cm} \quad \sigma = 25\text{cm}$$

Margin of error: 3 cm

$$\text{margin of error} = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

$$3 \geq z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

$$\frac{1.96 \times 25}{3} \leq \frac{\sqrt{n}}{1}$$

# Calculating Sample Size

6,000 adults

$$\text{margin of error} = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

95%   $z_{\alpha/2} = 1.96$

$$\frac{1.96 \times 25}{3} \leq \frac{\sqrt{n}}{1}$$

$$\bar{x} = 170\text{cm} \quad \sigma = 25\text{cm}$$

Margin of error: 3 cm

# Calculating Sample Size

6,000 adults

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$$\left( \frac{1.96 \times 25}{3} \right)^2 \leq n$$

# Calculating Sample Size

6,000 adults

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6,000 adults

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$$\bar{x} = 170\text{cm} \quad \sigma = 25\text{cm}$$

$$n \geq \left( \frac{1.96 \times 25}{3} \right)^2$$

$$n \geq 266.78 \approx 267$$

Margin of error: 3 cm

# Calculating Sample Size

$$\text{margin of error} = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

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$$\text{margin of error} = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

$$n \geq \left( \frac{1.96 \times 25}{3} \right)^2$$

$$n \geq \left( \frac{z_{\alpha/2} \cdot \sigma}{MOE} \right)^2$$



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## Confidence Interval

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**Difference Between  
Confidence and Probability**

# Difference Between Confidence and Probability

# Difference Between Confidence and Probability

$\bar{x}$

# Difference Between Confidence and Probability

95%  
Confidence  
Level



# Difference Between Confidence and Probability

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The confidence interval contains the true population parameter approximately 95% of the time.

# Difference Between Confidence and Probability

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# Difference Between Confidence and Probability

95%  
Confidence  
Level



The confidence interval contains the true population parameter approximately 95% of the time.



There's a 95% probability that the population parameter falls within the confidence interval.



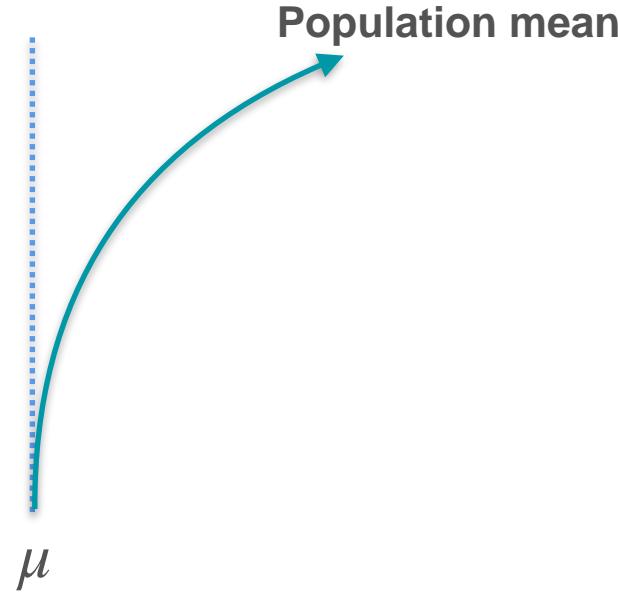
# Difference Between Confidence and Probability

95%  
Confidence  
Level



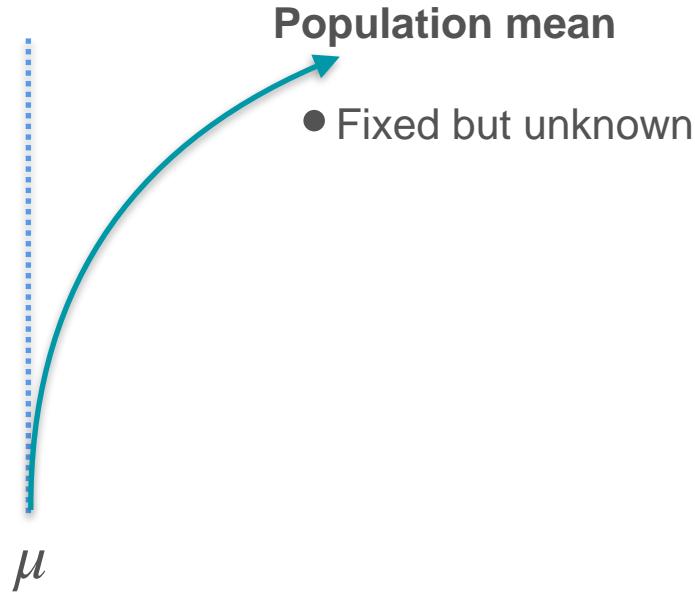
# Difference Between Confidence and Probability

95%  
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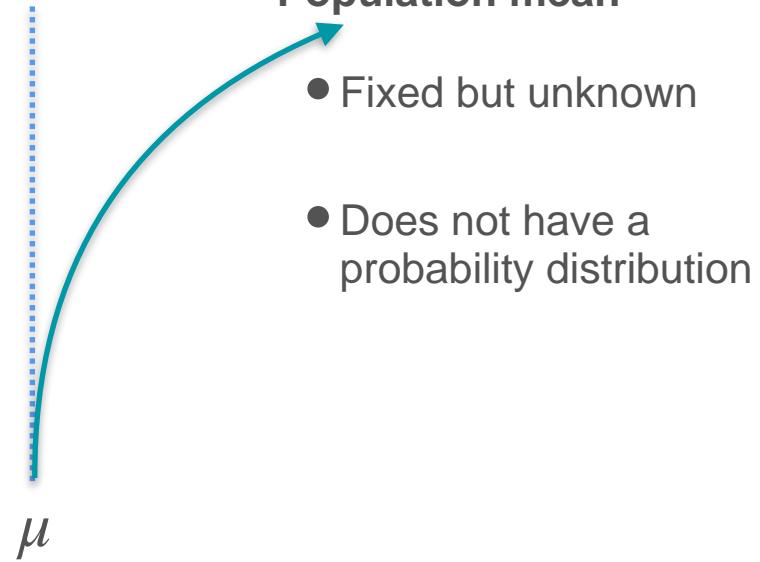
# Difference Between Confidence and Probability

95%  
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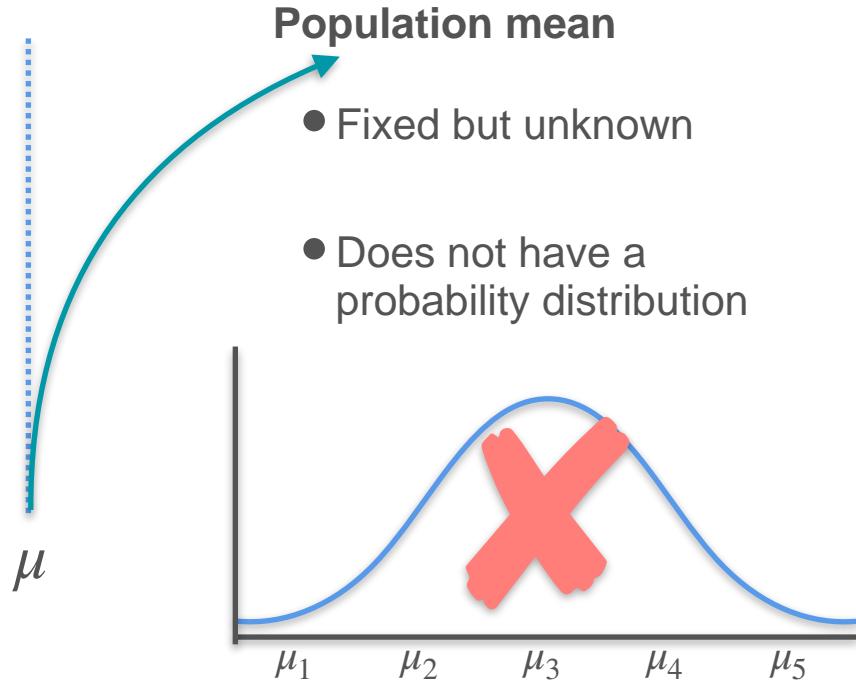
# Difference Between Confidence and Probability

95%  
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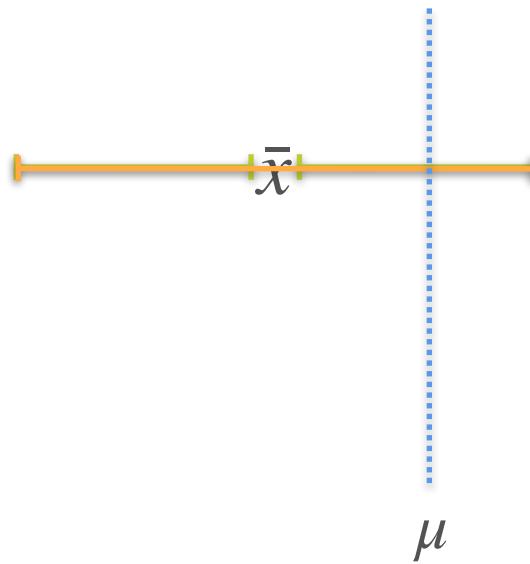
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95%  
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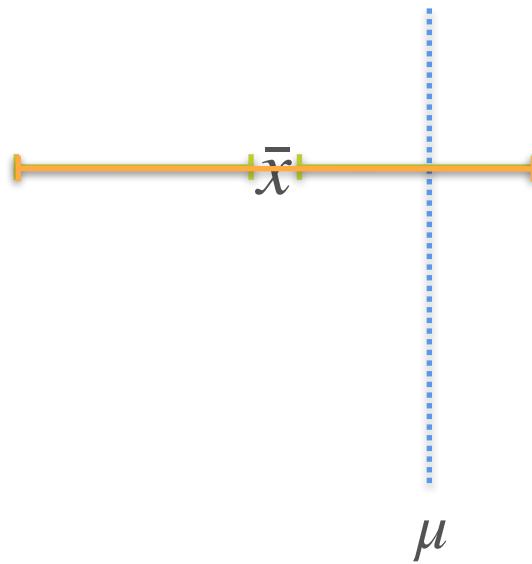


## Population mean

- Fixed but unknown
- Does not have a probability distribution
- In the interval

# Difference Between Confidence and Probability

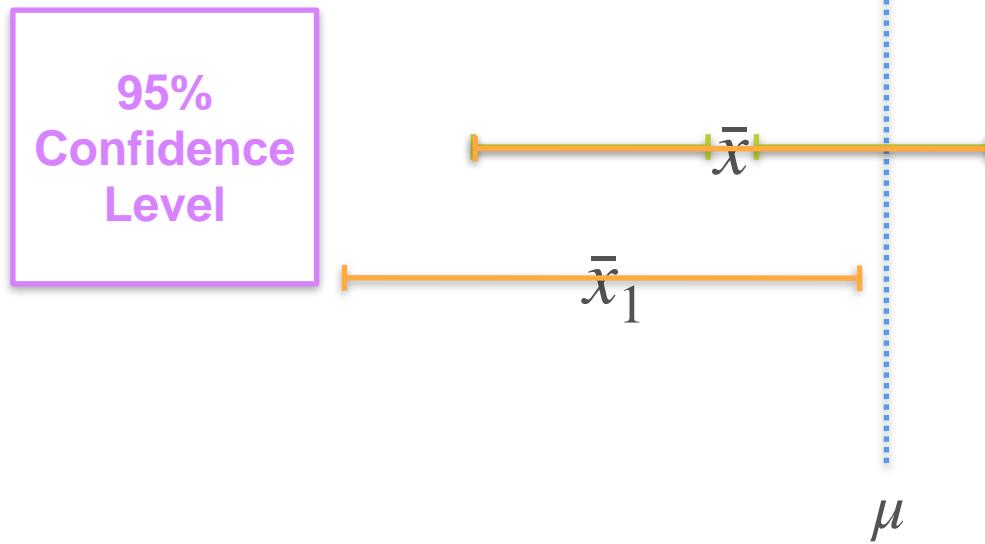
95%  
Confidence  
Level



## Population mean

- Fixed but unknown
- Does not have a probability distribution
- In the interval or not

# Difference Between Confidence and Probability

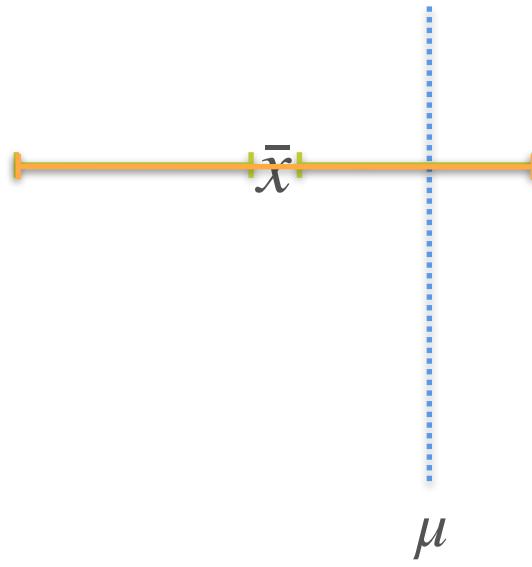


## Population mean

- Fixed but unknown
- Does not have a probability distribution
- In the interval or not

# Difference Between Confidence and Probability

95%  
Confidence  
Level

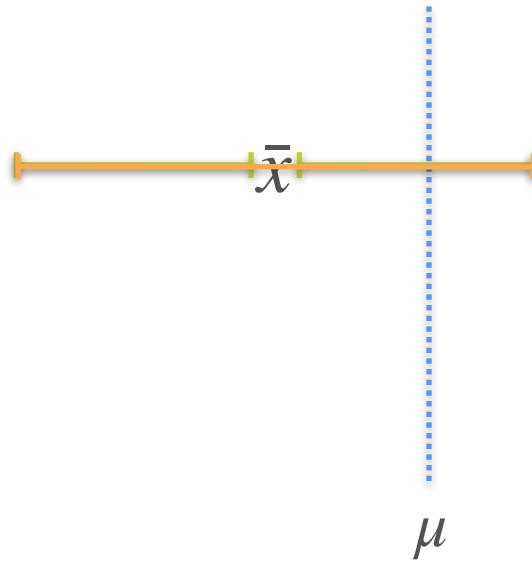


## Population mean

- Fixed but unknown
- Does not have a probability distribution
- In the interval or not
- Does not fall within a specific interval 95% of the time

# Difference Between Confidence and Probability

95%  
Confidence  
Level



## Population mean

- Fixed but unknown
- Does not have a probability distribution
- In the interval or not
- Does not fall within a specific interval 95% of the time

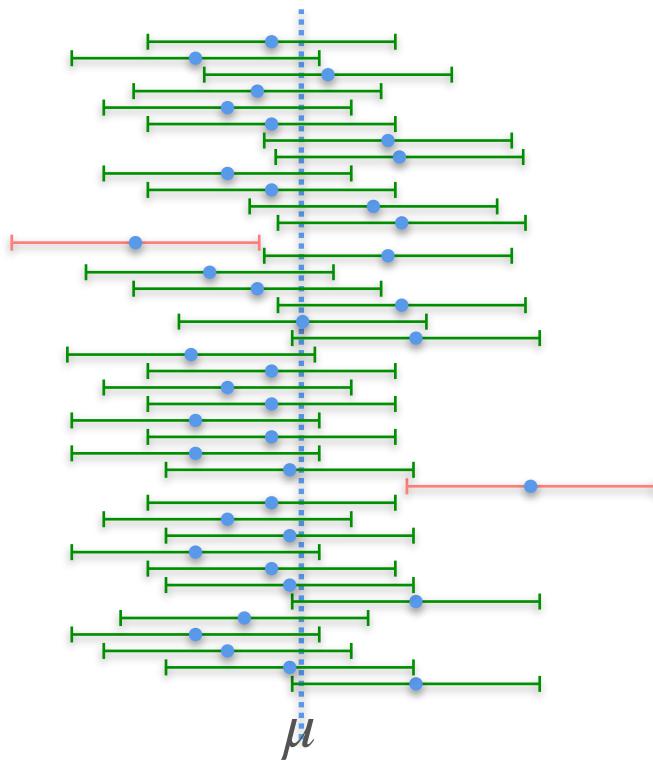
# Difference Between Confidence and Probability

95%  
Confidence  
Level



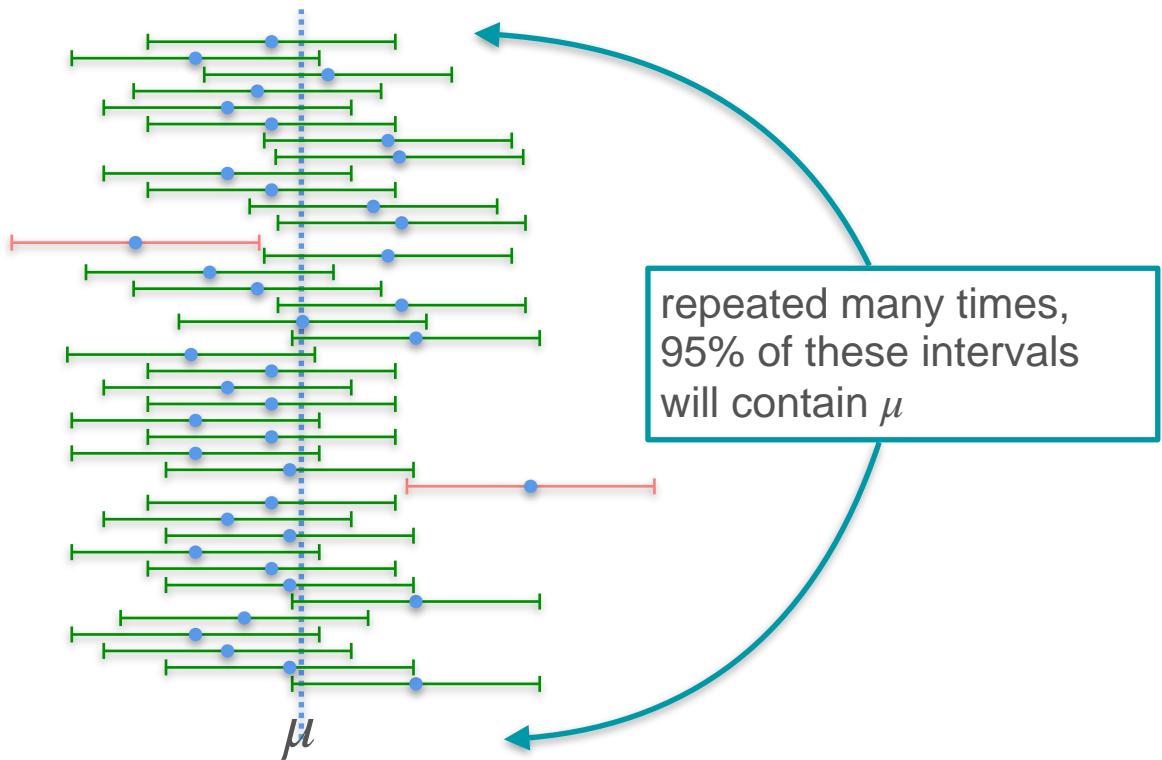
# Difference Between Confidence and Probability

95%  
Confidence  
Level



# Difference Between Confidence and Probability

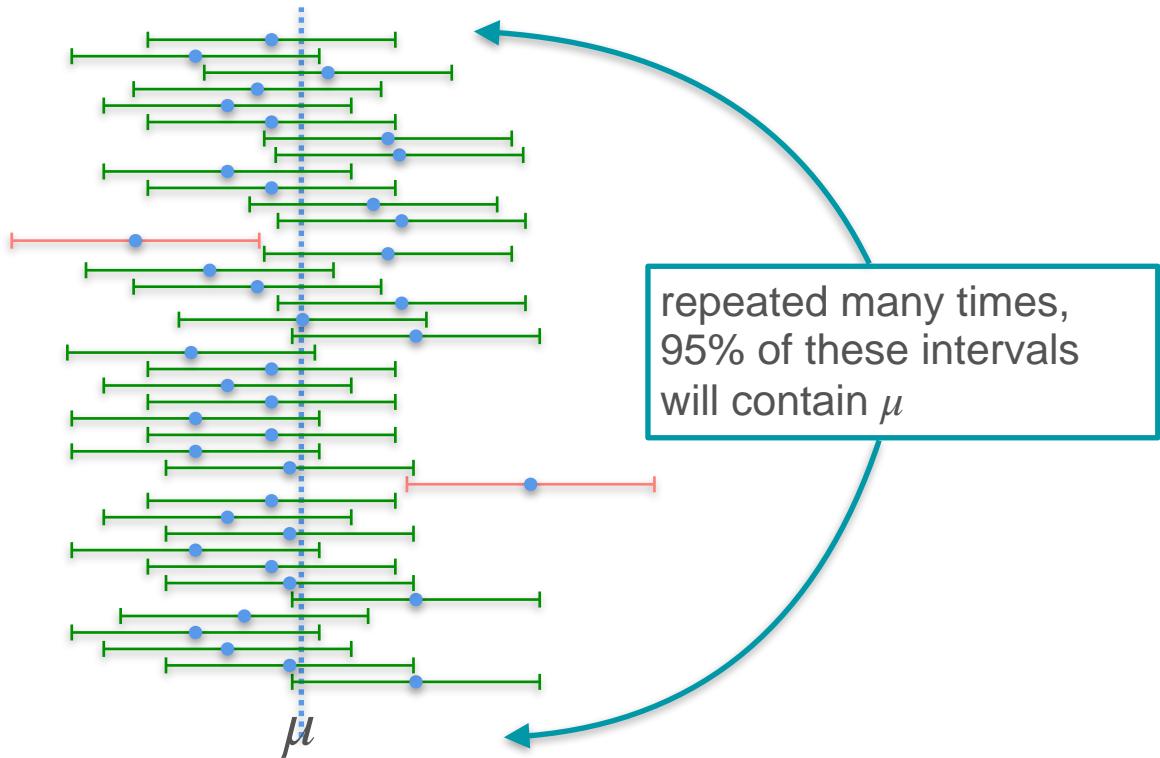
95%  
Confidence  
Level



# Difference Between Confidence and Probability

95%  
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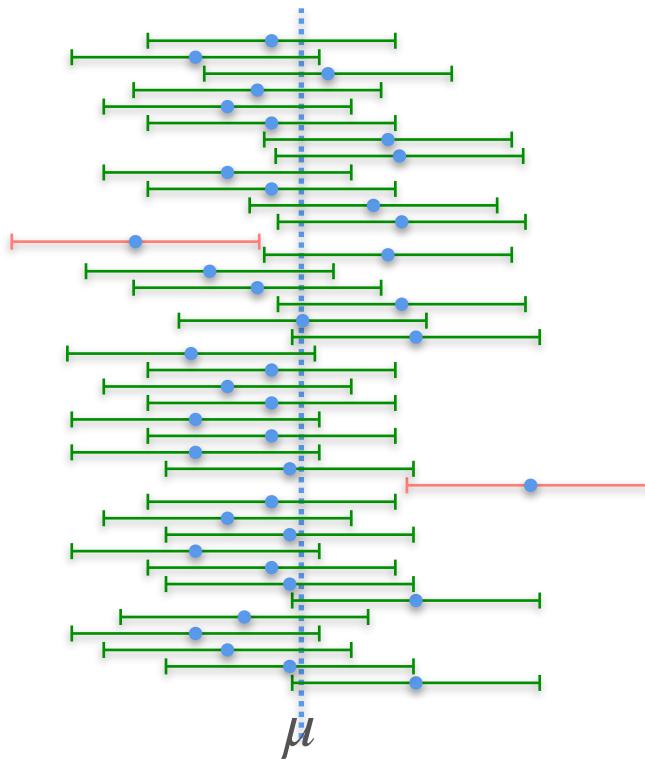
success rate for constructing  
the confidence interval



# Difference Between Confidence and Probability

95%  
Confidence  
Level

success rate for constructing  
the confidence interval

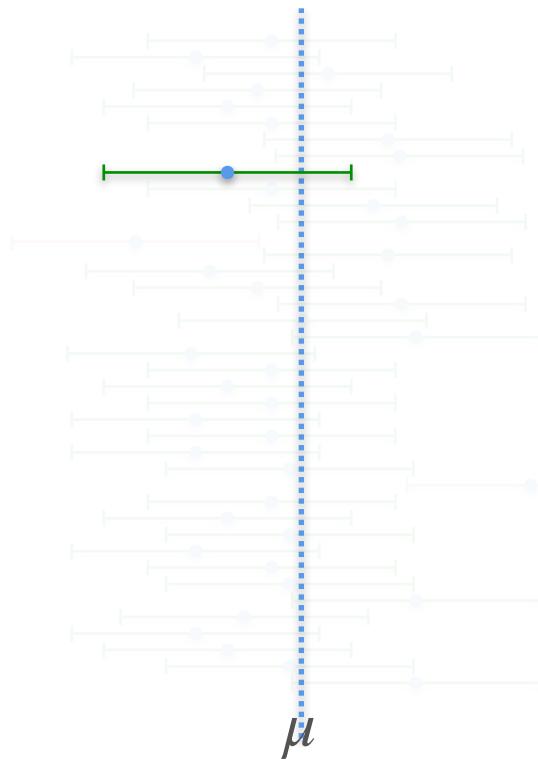


repeated many times,  
95% of these intervals  
will contain  $\mu$

# Difference Between Confidence and Probability

95%  
Confidence  
Level

success rate for constructing  
the confidence interval

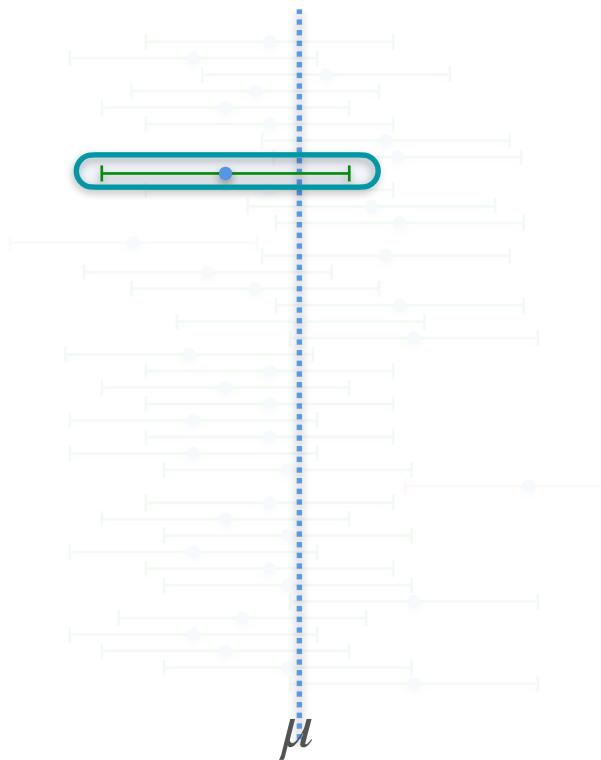


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# Difference Between Confidence and Probability

95%  
Confidence  
Level

success rate for constructing  
the confidence interval



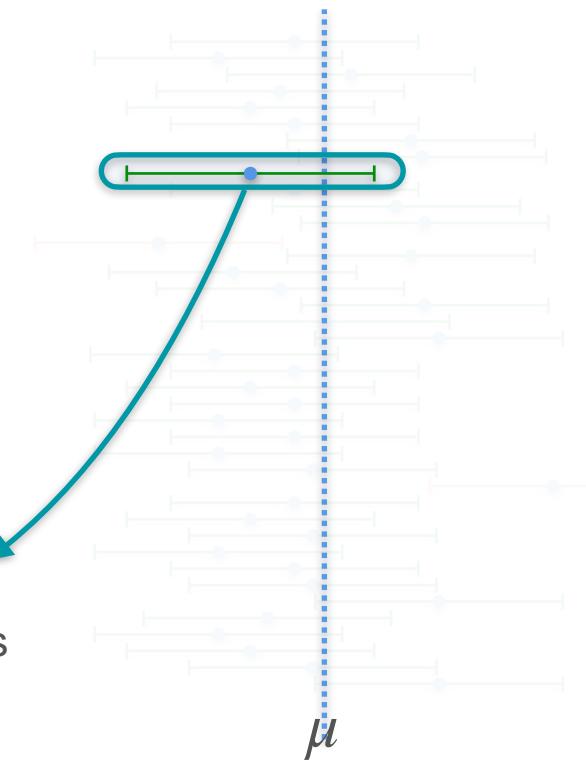
repeated many times,  
95% of these intervals  
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# Difference Between Confidence and Probability

95%  
Confidence  
Level

success rate for constructing  
the confidence interval

not the probability that  
one specific intervals contains  
the population mean



repeated many times,  
95% of these intervals  
will contain  $\mu$



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## Confidence Interval

---

**Confidence Interval  
(Unknown Standard Deviation)**

# Confidence Interval - $t$ Distribution



# Confidence Interval - $t$ Distribution

$$\bar{x} \pm z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$



# Confidence Interval - $t$ Distribution

known  $\sigma$

$$\bar{x} \pm z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$



# Confidence Interval - $t$ Distribution

known  $\sigma$

$$\bar{x} \pm z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

$$\frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$



# Confidence Interval - $t$ Distribution

known  $\sigma$

$$\bar{x} \pm z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

$$\frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

normal  
distribution



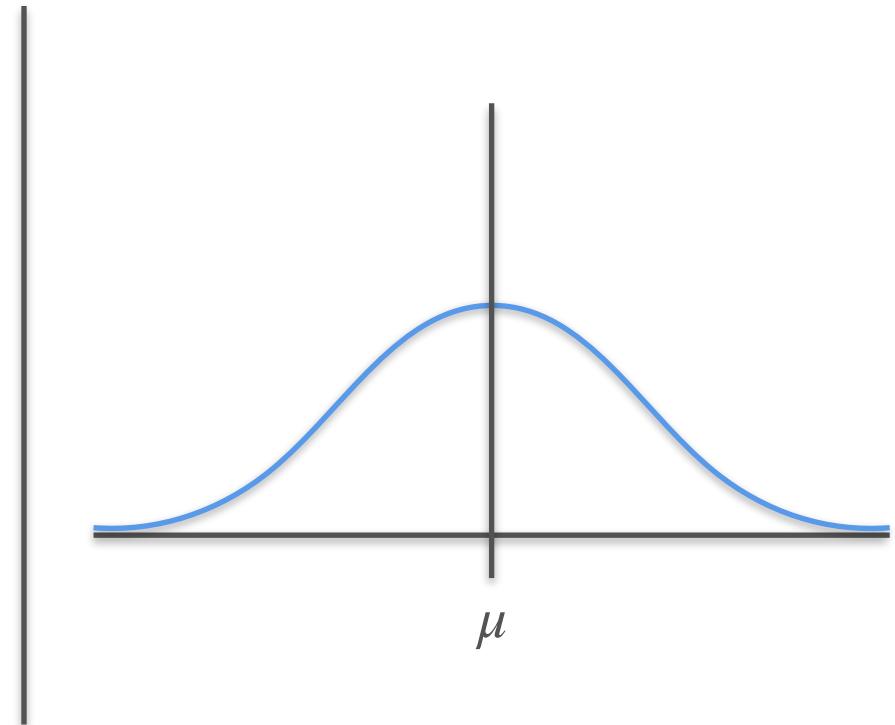
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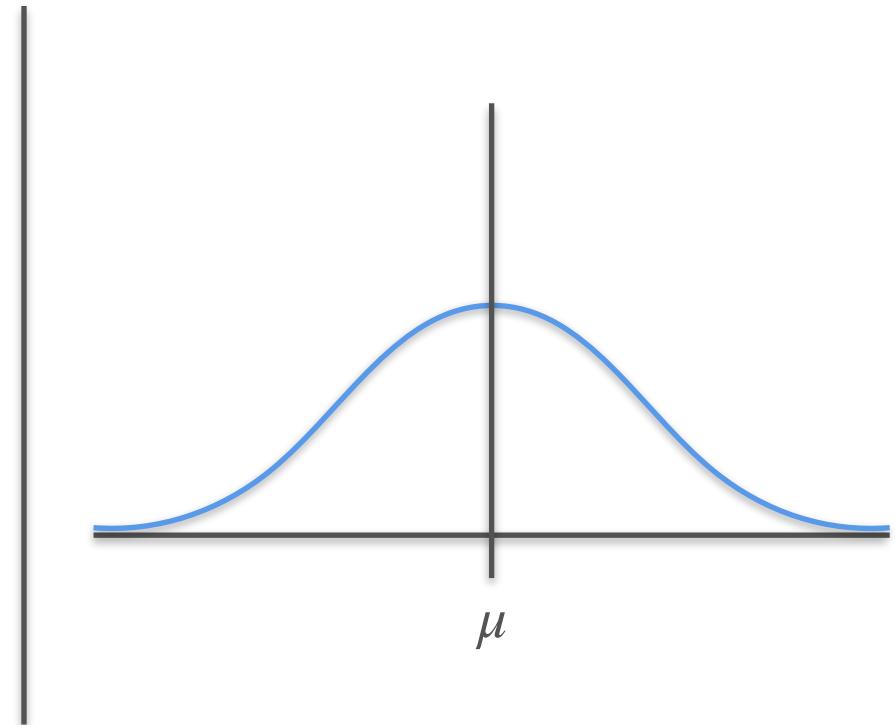
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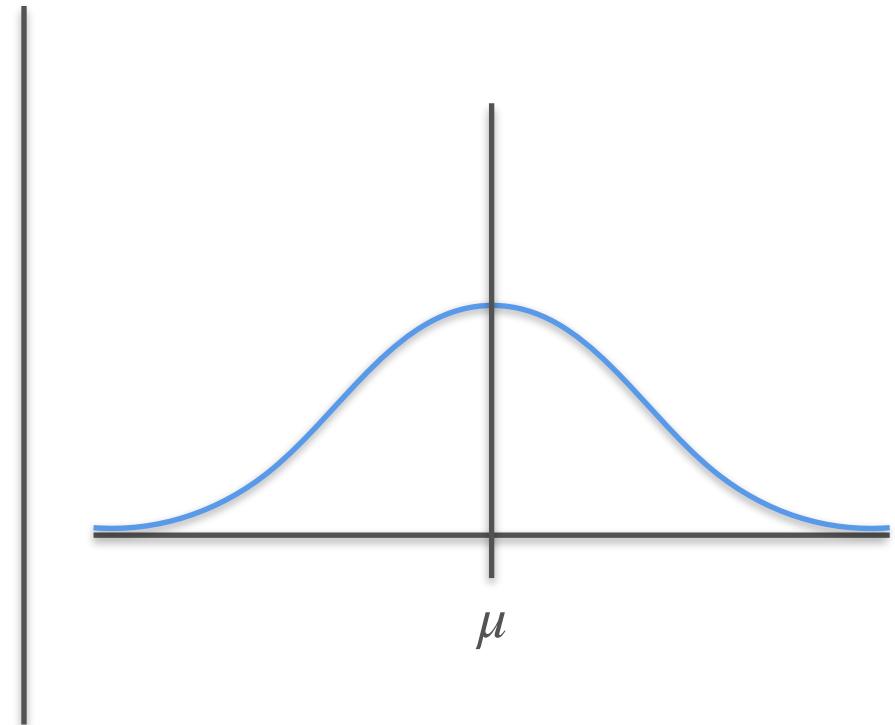
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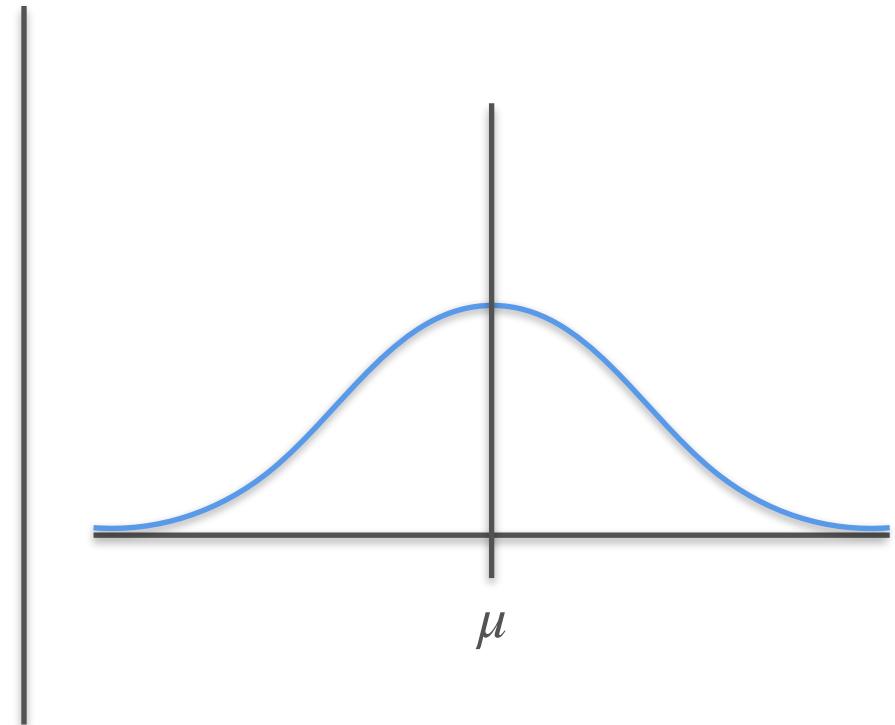
# Confidence Interval - $t$ Distribution

known  $\sigma$  ?

$$\bar{x} \pm z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

$$\frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

normal  
distribution



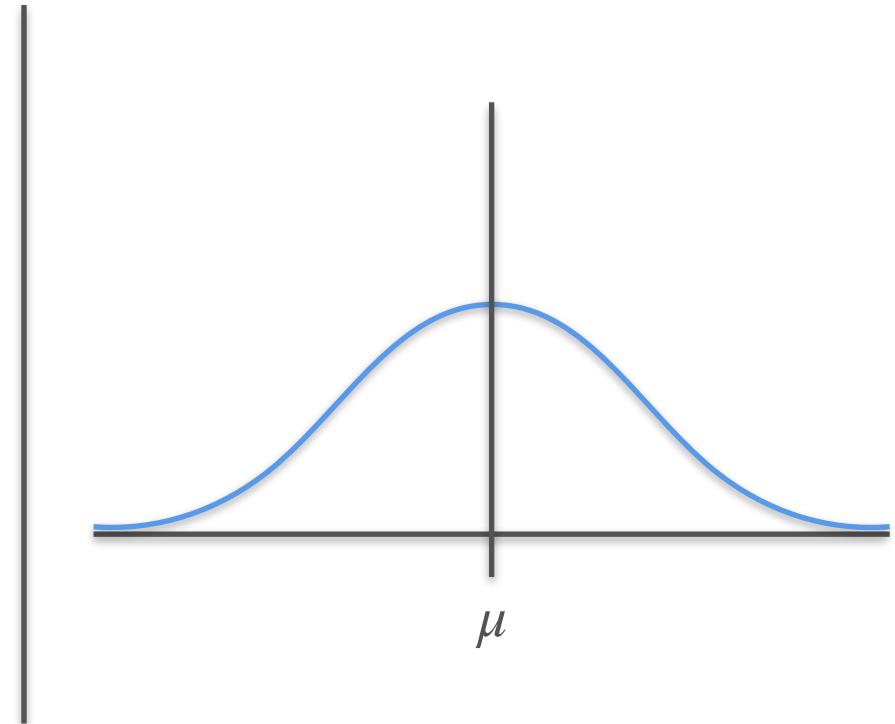
# Confidence Interval - $t$ Distribution

known  $\sigma$  ?  $\rightarrow s$

$$\bar{x} \pm z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

$$\frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

normal  
distribution



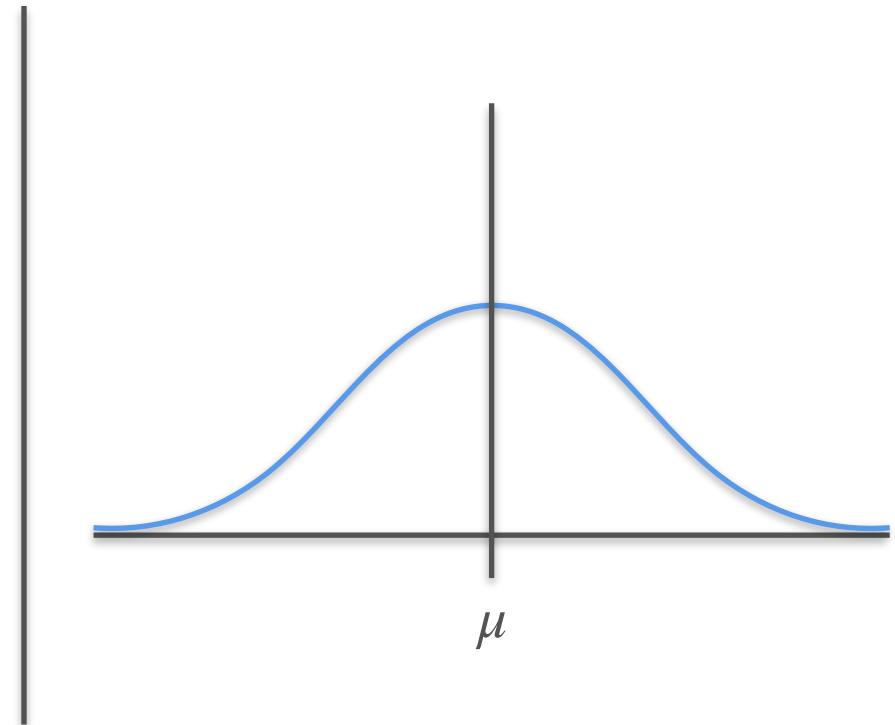
# Confidence Interval - $t$ Distribution

known  $\sigma$  ?  $\rightarrow s$

$$\bar{x} \pm z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

$$\frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} \xrightarrow{\text{dotted arrow}} \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$$

normal  
distribution



# Confidence Interval - $t$ Distribution

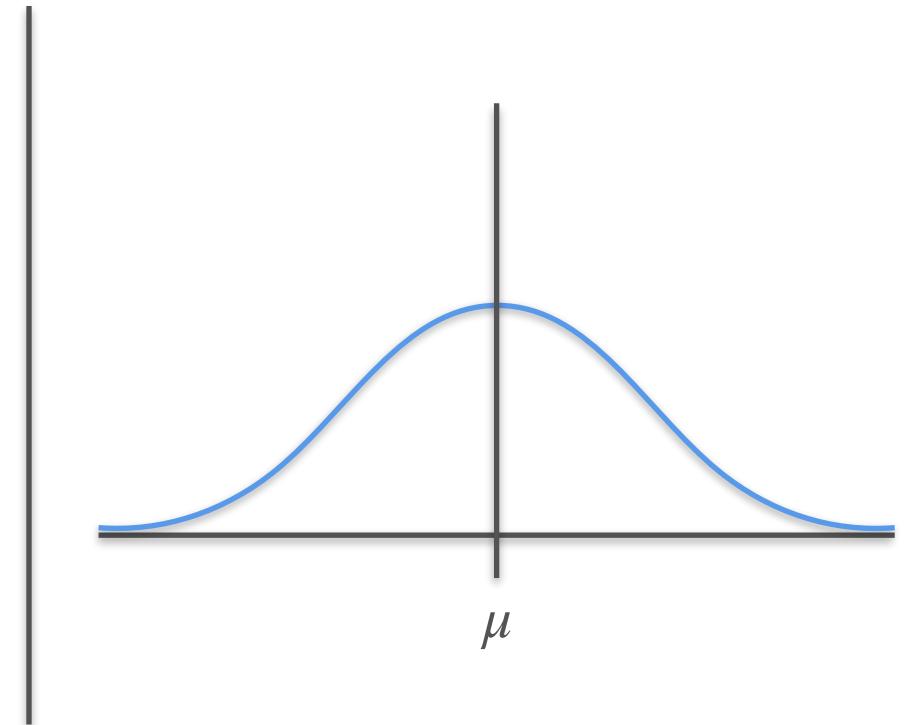
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normal  
distribution

not a normal  
distribution



# Confidence Interval - $t$ Distribution

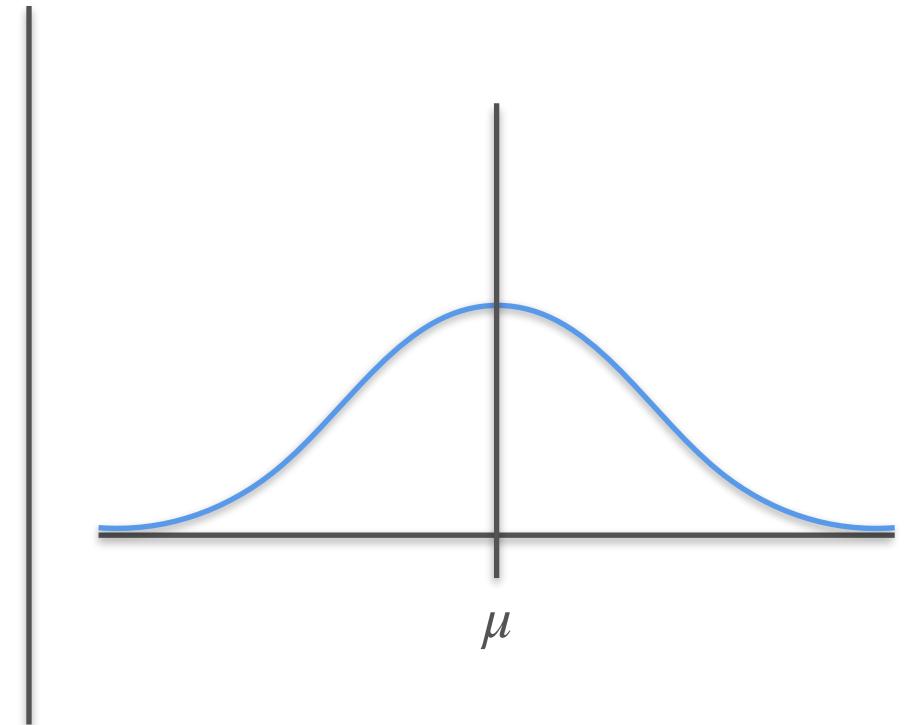
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normal  
distribution

not a normal  
distribution  
**student's t  
distribution**



# Confidence Interval - $t$ Distribution

known  $\sigma$  ?  $\rightarrow s$

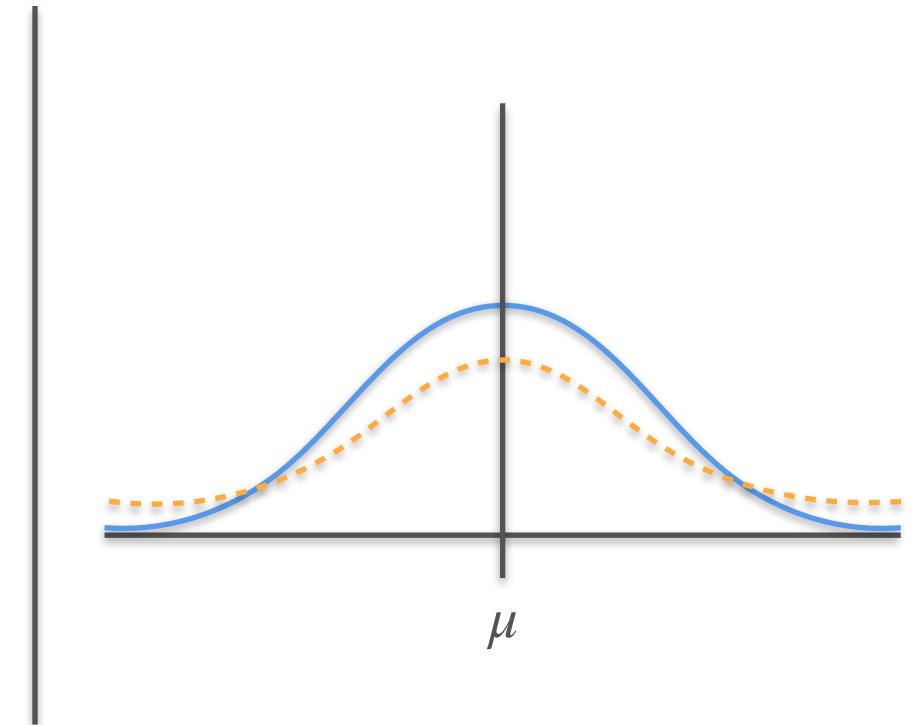
$$\bar{x} \pm z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

$$\frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} \xrightarrow{\text{dotted arrow}} \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$$

normal  
distribution

not a normal  
distribution

**student's t  
distribution**



# Confidence Interval - $t$ Distribution

known  $\sigma$  ?  $\rightarrow s$

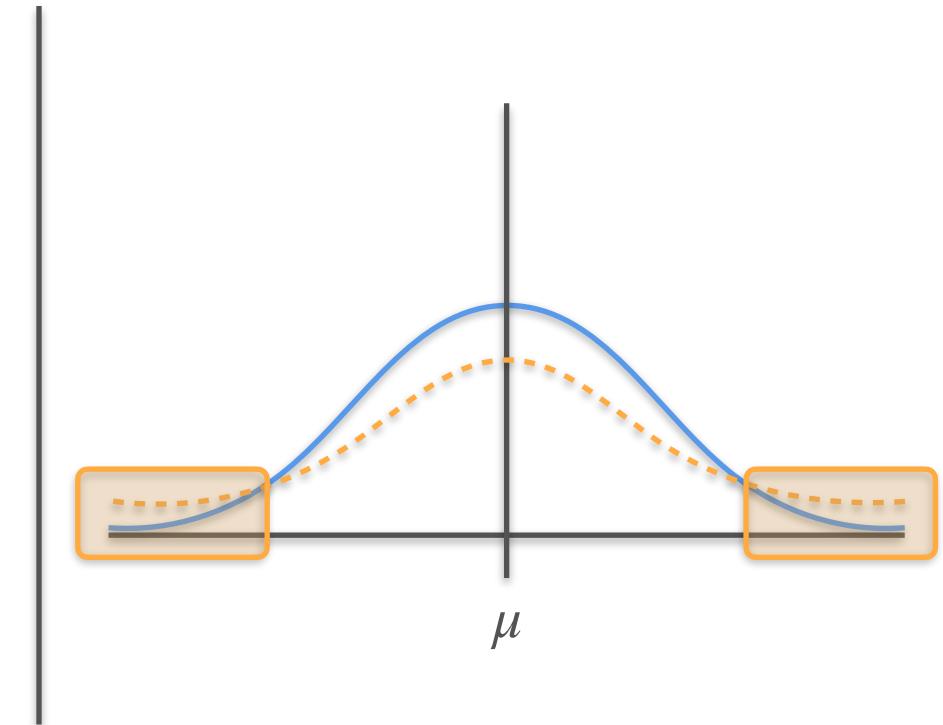
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normal distribution

not a normal distribution

**student's t distribution**



# Confidence Interval - $t$ Distribution

known  $\sigma$

$$\frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

normal  
distribution

$$\bar{x} \pm z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$



# Confidence Interval - $t$ Distribution

known  $\sigma$



$$\frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

normal  
distribution

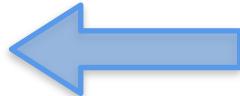
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# Confidence Interval - $t$ Distribution

known  $\sigma$

$$\frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$



normal  
distribution

$$\bar{x} \pm z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

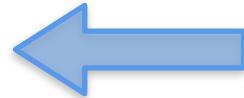


# Confidence Interval - $t$ Distribution

known  $\sigma$

$$\frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

normal  
distribution



$$\bar{x} \pm z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$



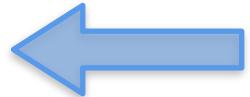
# Confidence Interval - $t$ Distribution

known  $\sigma$

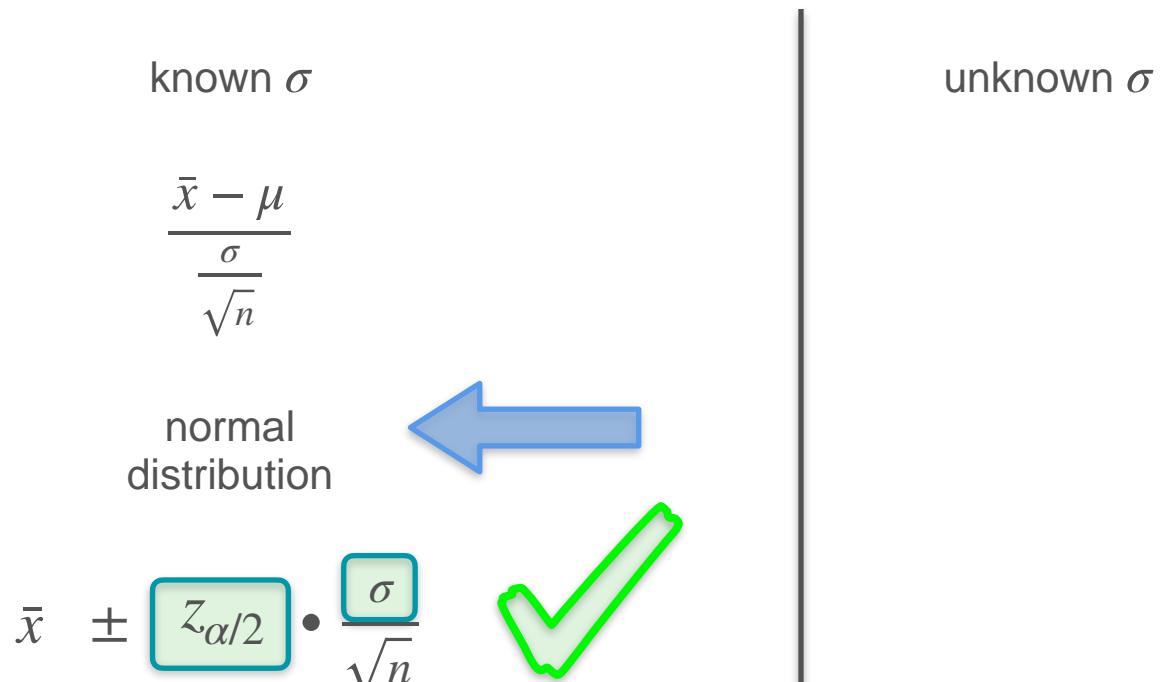
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normal  
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# Confidence Interval - $t$ Distribution



# Confidence Interval - $t$ Distribution

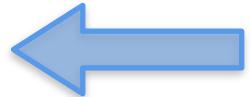
known  $\sigma$

$$\frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

normal distribution

$$\bar{x} \pm z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

unknown  $\sigma$   replace with  $s$



# Confidence Interval - $t$ Distribution

known  $\sigma$

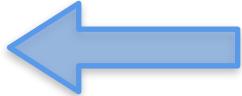
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normal distribution

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unknown  $\sigma$   replace with  $s$

$$\frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$$



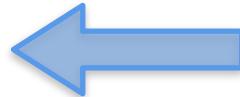
# Confidence Interval - $t$ Distribution

known  $\sigma$

$$\frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

normal distribution

$$\bar{x} \pm z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$



unknown  $\sigma$  replace with  $s$

$$\frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} \quad \xrightarrow{\text{dotted arrow}} \quad \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$$

# Confidence Interval - $t$ Distribution

known  $\sigma$

$$\frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

normal distribution

$$\bar{x} \pm z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$



unknown  $\sigma$   replace with  $s$

$$\frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$$

student's t distribution



# Confidence Interval - $t$ Distribution

known  $\sigma$

$$\frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

normal distribution

$$\bar{x} \pm z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$



unknown  $\sigma$  replace with  $s$

$$\frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} \quad \xrightarrow{\text{dotted arrow}} \quad \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$$

student's t distribution

$$\bar{x} \pm z_{\alpha/2} \cdot \frac{s}{\sqrt{n}}$$

# Confidence Interval - $t$ Distribution

known  $\sigma$

$$\frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

normal distribution

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student's t distribution

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# Confidence Interval - $t$ Distribution

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unknown  $\sigma$

$$\bar{x} \pm z_{\alpha/2} \cdot \frac{s}{\sqrt{n}}$$



# Confidence Interval - $t$ Distribution

known  $\sigma$

$$\bar{x} \pm z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$



unknown  $\sigma$

$$\bar{x} \pm z_{\alpha/2} \cdot \frac{s}{\sqrt{n}}$$



$$\bar{x} \pm t_{\alpha/2} \cdot \frac{s}{\sqrt{n}}$$

# Confidence Interval - $t$ Distribution

known  $\sigma$

$$\bar{x} \pm z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$



unknown  $\sigma$

$$\bar{x} \pm z_{\alpha/2} \cdot \frac{s}{\sqrt{n}}$$



$$\bar{x} \pm t_{\alpha/2} \cdot \frac{s}{\sqrt{n}}$$



# Confidence Interval - $t$ Distribution

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# Confidence Interval - $t$ Distribution

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# Confidence Interval - $t$ Distribution

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# Confidence Interval - $t$ Distribution

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# Confidence Interval - $t$ Distribution

unknown  $\sigma$

$$\bar{x} \pm t_{\alpha/2} \cdot \frac{s}{\sqrt{n}}$$

**degrees of freedom**

# Confidence Interval - $t$ Distribution

unknown  $\sigma$

$$\bar{x} \pm t_{\alpha/2} \cdot \frac{s}{\sqrt{n}}$$

**degrees of freedom**

$$n - 1$$

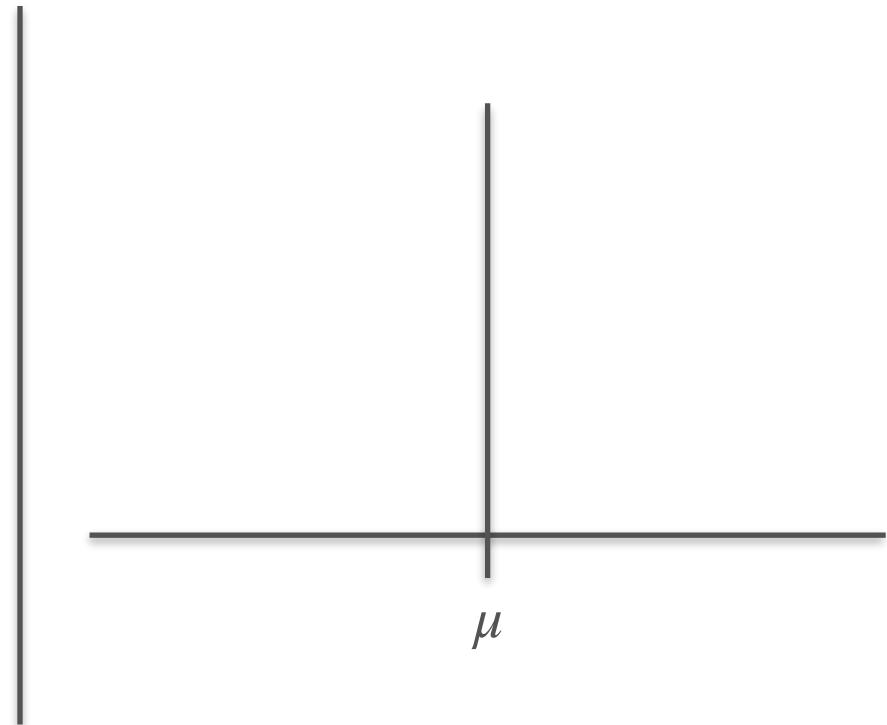
# Confidence Interval - $t$ Distribution

unknown  $\sigma$

$$\bar{x} \pm t_{\alpha/2} \cdot \frac{s}{\sqrt{n}}$$

degrees of freedom

$$n - 1$$



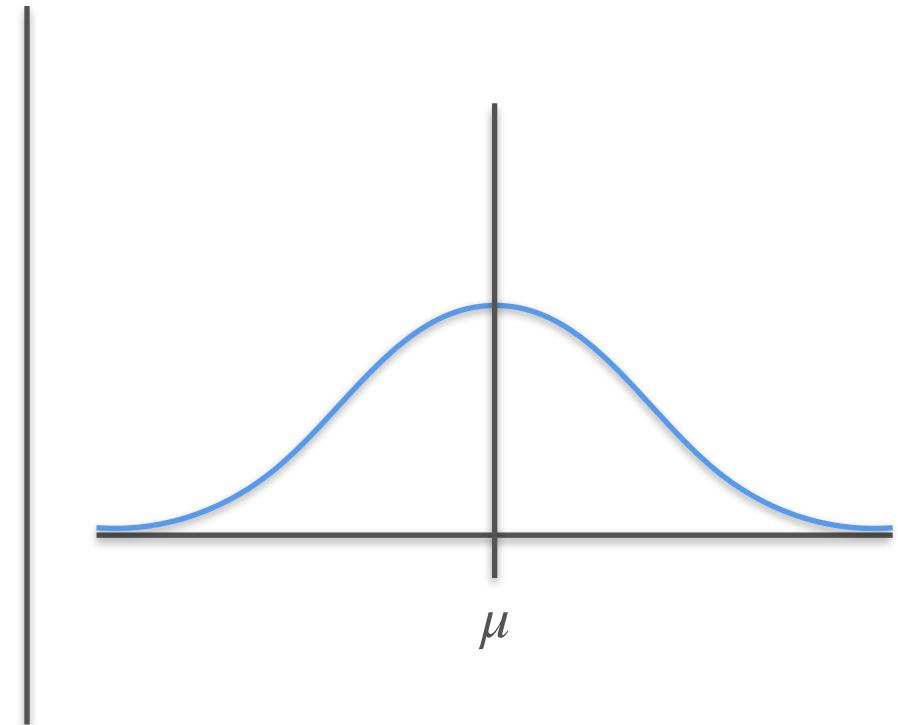
# Confidence Interval - $t$ Distribution

unknown  $\sigma$

$$\bar{x} \pm t_{\alpha/2} \cdot \frac{s}{\sqrt{n}}$$

degrees of freedom

$$n - 1$$



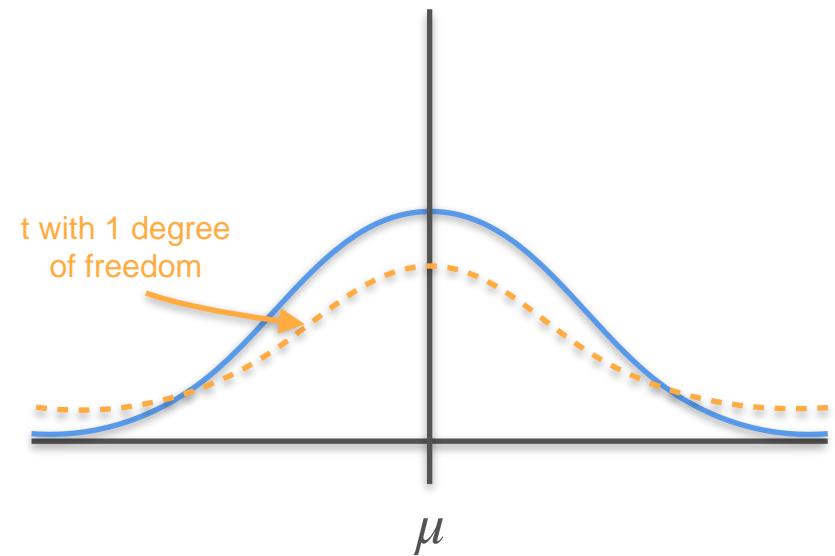
# Confidence Interval - $t$ Distribution

unknown  $\sigma$

$$\bar{x} \pm t_{\alpha/2} \cdot \frac{s}{\sqrt{n}}$$

degrees of freedom

$$n - 1$$



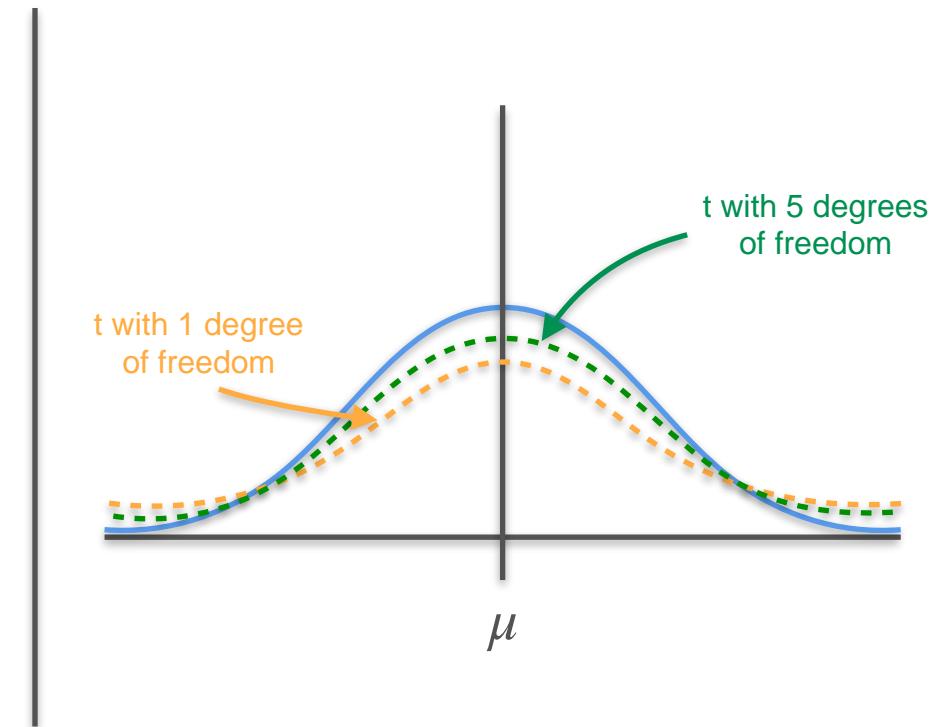
# Confidence Interval - $t$ Distribution

unknown  $\sigma$

$$\bar{x} \pm t_{\alpha/2} \cdot \frac{s}{\sqrt{n}}$$

degrees of freedom

$$n - 1$$



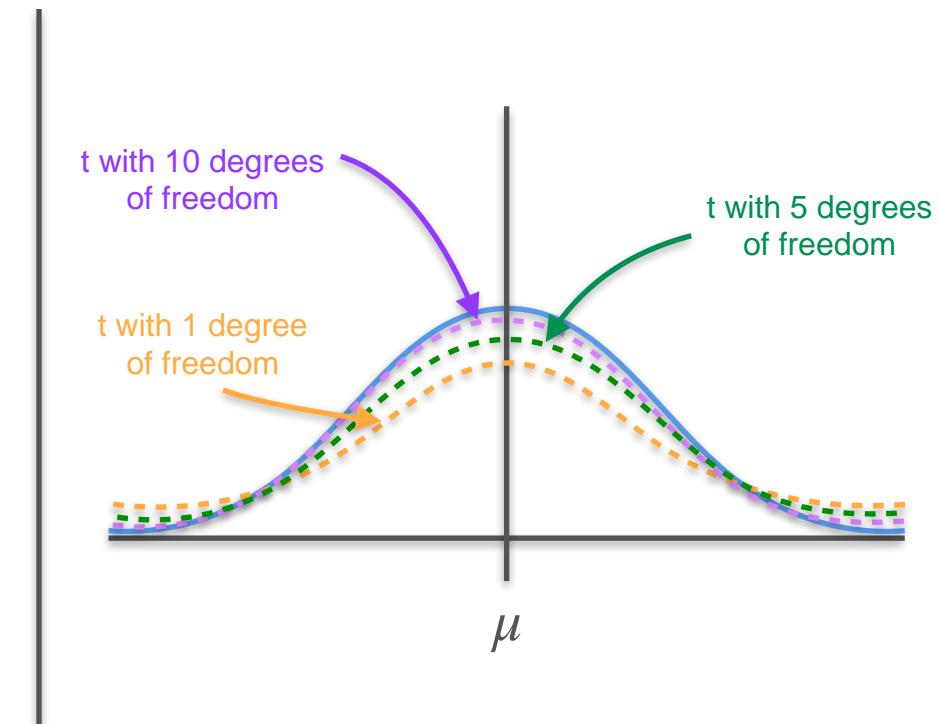
# Confidence Interval - $t$ Distribution

unknown  $\sigma$

$$\bar{x} \pm t_{\alpha/2} \cdot \frac{s}{\sqrt{n}}$$

degrees of freedom

$$n - 1$$





DeepLearning.AI

## Confidence Interval

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# Confidence Intervals for Proportion

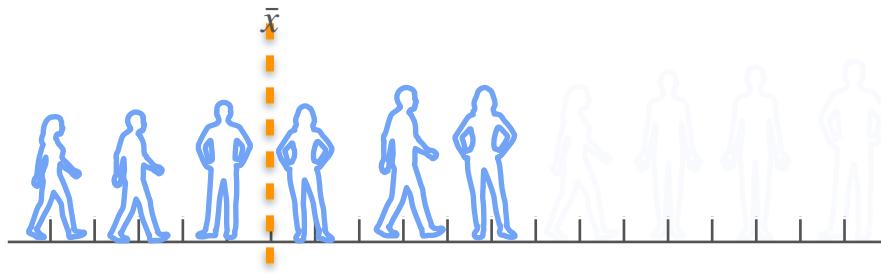
# Confidence Interval for Proportions

# Confidence Interval for Proportions

## Confidence Interval for Means

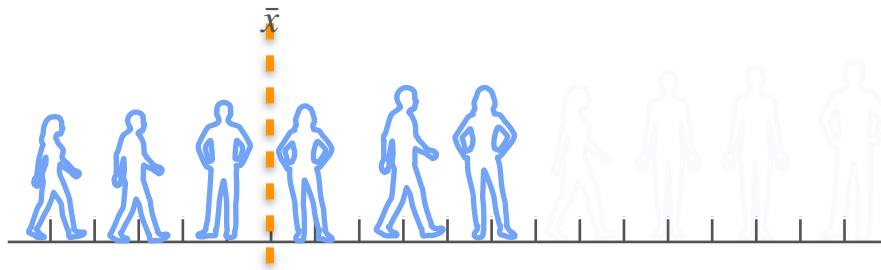
# Confidence Interval for Proportions

## Confidence Interval for Means



# Confidence Interval for Proportions

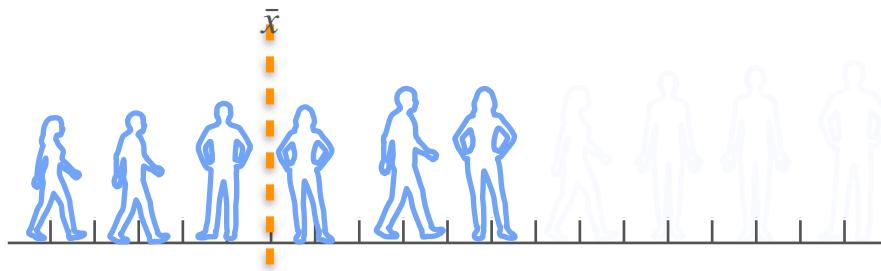
## Confidence Interval for Means



$$\text{confidence interval} = \bar{x} \pm \text{margin of error}$$

# Confidence Interval for Proportions

## Confidence Interval for Means

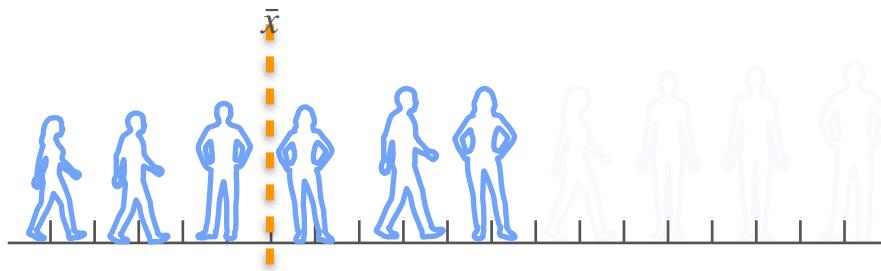


confidence interval =  $\bar{x} \pm$  margin of error

margin of error =

# Confidence Interval for Proportions

## Confidence Interval for Means

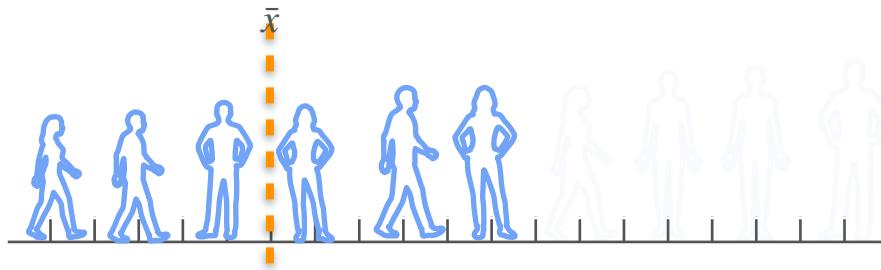


confidence interval =  $\bar{x} \pm$  margin of error

$$\text{margin of error} = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

# Confidence Interval for Proportions

## Confidence Interval for Means

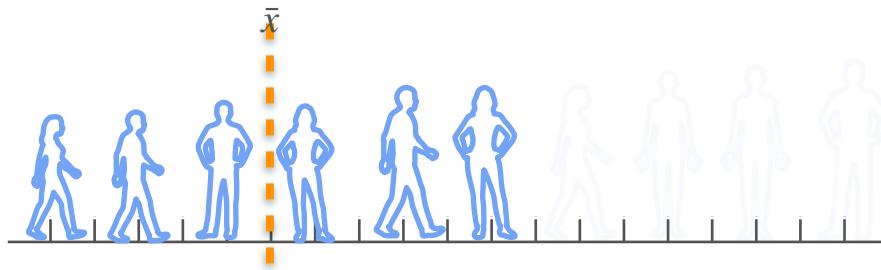


$$\text{confidence interval} = \bar{x} \pm \text{margin of error}$$

$$\text{margin of error} = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

# Confidence Interval for Proportions

## Confidence Interval for Means



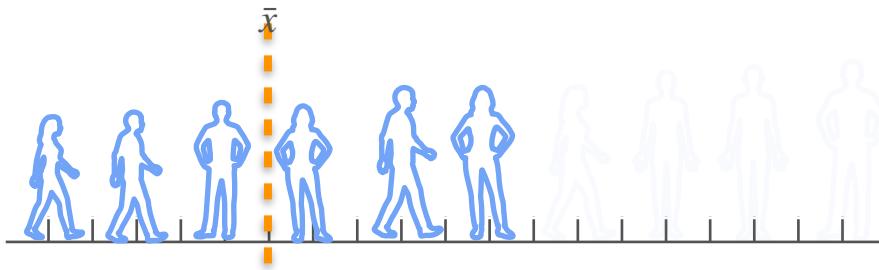
$$\text{confidence interval} = \bar{x} \pm \text{margin of error}$$

## Confidence Interval for Proportions

$$\text{margin of error} = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

# Confidence Interval for Proportions

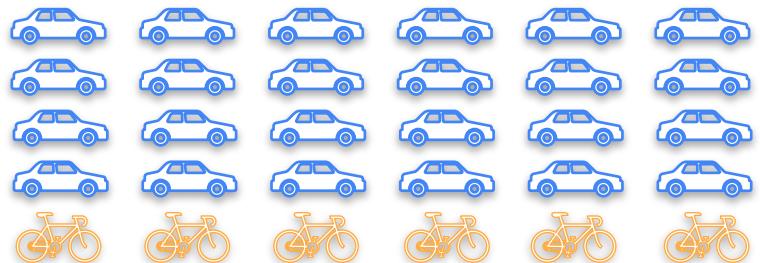
## Confidence Interval for Means



$$\text{confidence interval} = \bar{x} \pm \text{margin of error}$$

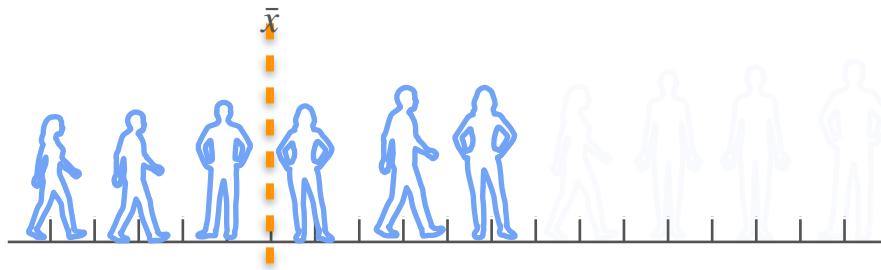
$$\text{margin of error} = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

## Confidence Interval for Proportions



# Confidence Interval for Proportions

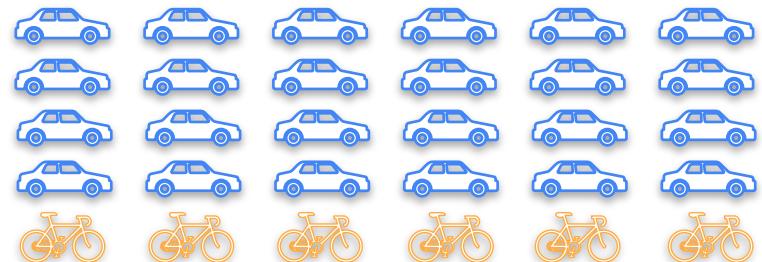
## Confidence Interval for Means



$$\text{confidence interval} = \bar{x} \pm \text{margin of error}$$

$$\text{margin of error} = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

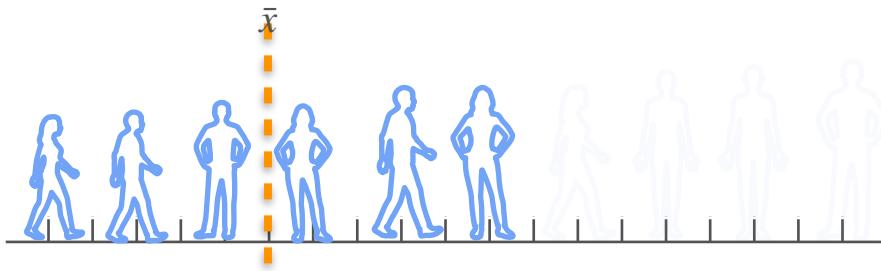
## Confidence Interval for Proportions



$$n = 30$$

# Confidence Interval for Proportions

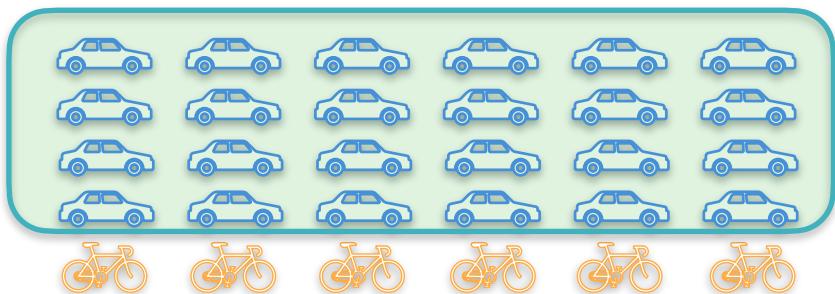
## Confidence Interval for Means



$$\text{confidence interval} = \bar{x} \pm \text{margin of error}$$

$$\text{margin of error} = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

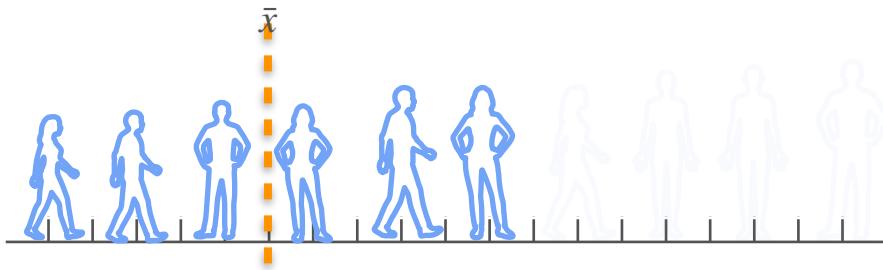
## Confidence Interval for Proportions



$$n = 30$$

# Confidence Interval for Proportions

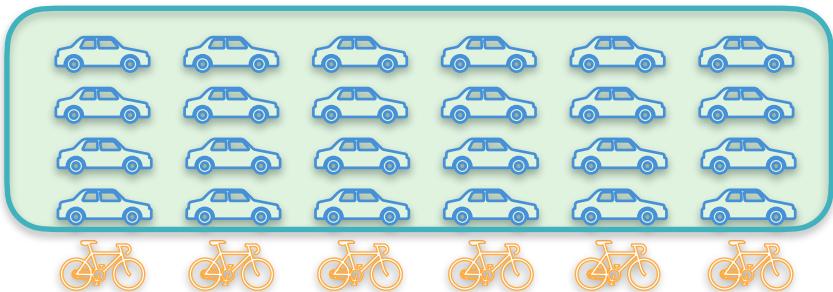
## Confidence Interval for Means



$$\text{confidence interval} = \bar{x} \pm \text{margin of error}$$

$$\text{margin of error} = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

## Confidence Interval for Proportions

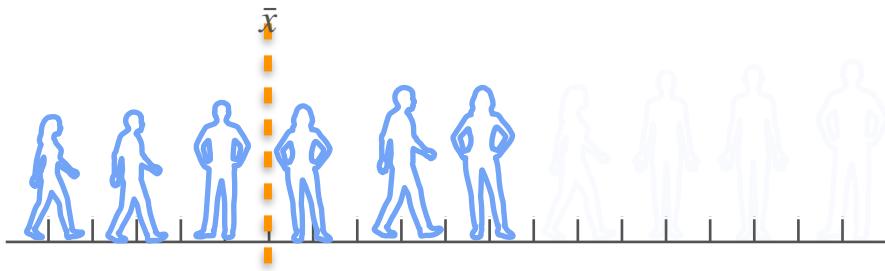


$$x = 24$$

$$n = 30$$

# Confidence Interval for Proportions

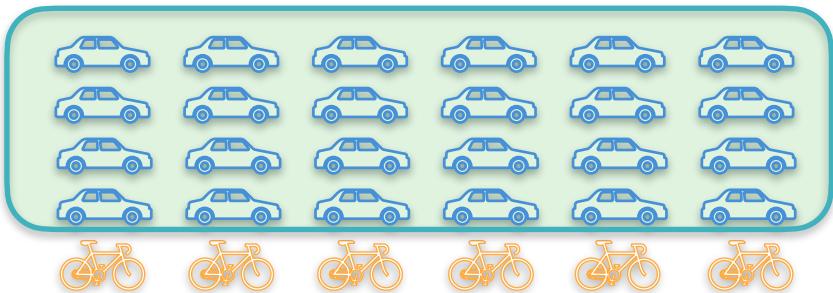
## Confidence Interval for Means



$$\text{confidence interval} = \bar{x} \pm \text{margin of error}$$

$$\text{margin of error} = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

## Confidence Interval for Proportions



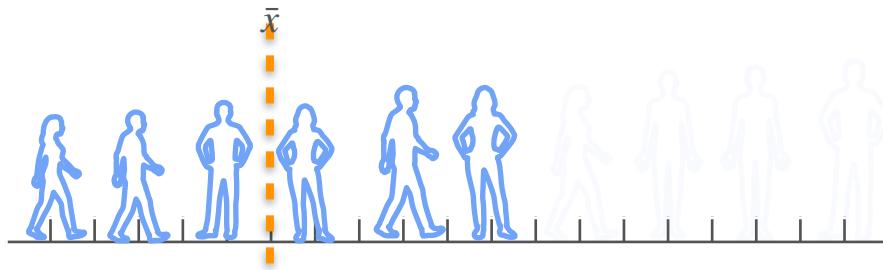
$$x = 24$$

$$n = 30$$

$$\hat{p} = \frac{x}{n} = \frac{24}{30} = 80\%$$

# Confidence Interval for Proportions

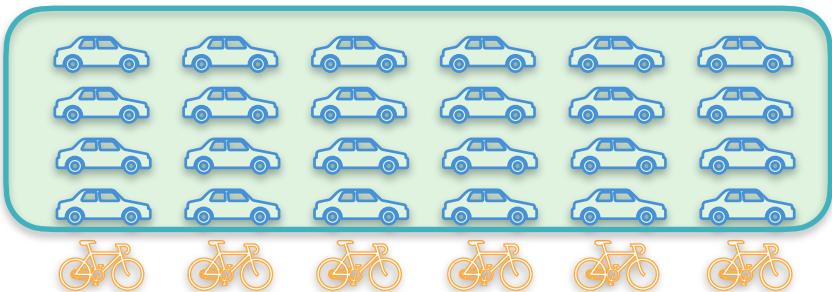
## Confidence Interval for Means



$$\text{confidence interval} = \bar{x} \pm \text{margin of error}$$

$$\text{margin of error} = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

## Confidence Interval for Proportions



$$x = 24$$

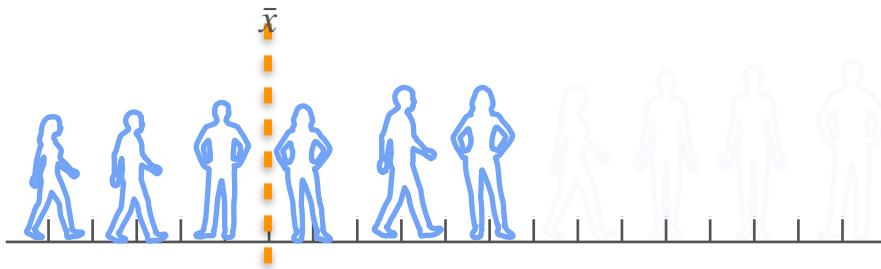
$$n = 30$$

$$\hat{p} = \frac{x}{n} = \frac{24}{30} = 80\%$$

How do you calculate a 95% confidence interval for this sample proportion?

# Confidence Interval for Proportions

## Confidence Interval for Means



confidence interval =  $\bar{x} \pm$  margin of error

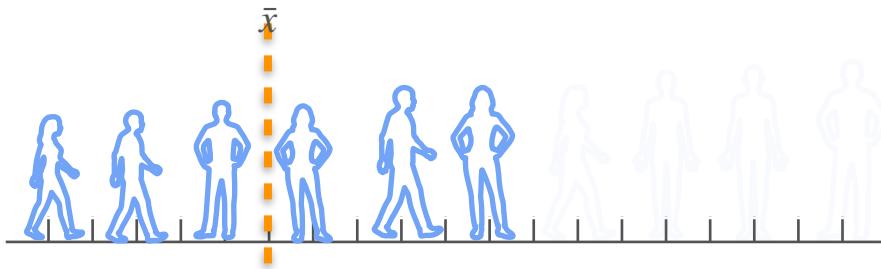
$$\text{margin of error} = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

## Confidence Interval for Proportions

$$\hat{p} = \frac{x}{n} = \frac{24}{30} = 80\%$$

# Confidence Interval for Proportions

## Confidence Interval for Means



confidence interval =  $\bar{x} \pm$  margin of error

$$\text{margin of error} = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

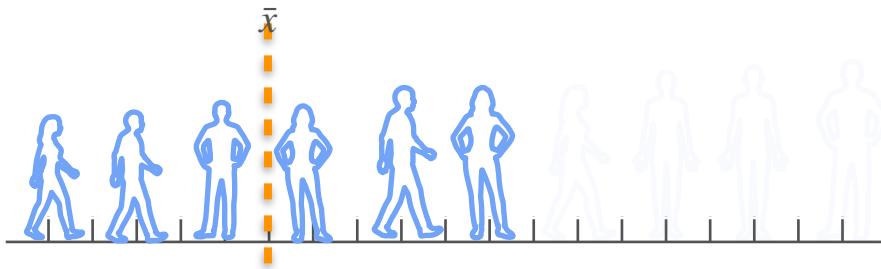
## Confidence Interval for Proportions

$$\hat{p} = \frac{x}{n} = \frac{24}{30} = 80\%$$

confidence interval =

# Confidence Interval for Proportions

## Confidence Interval for Means



confidence interval =  $\bar{x} \pm$  margin of error

$$\text{margin of error} = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

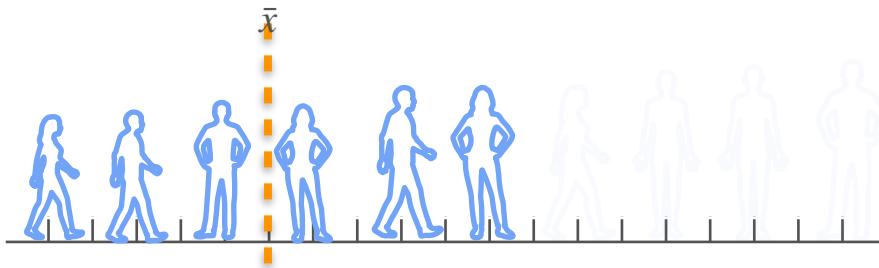
## Confidence Interval for Proportions

$$\hat{p} = \frac{x}{n} = \frac{24}{30} = 80\%$$

confidence interval =  $\hat{p}$

# Confidence Interval for Proportions

## Confidence Interval for Means



confidence interval =  $\bar{x} \pm$  margin of error

$$\text{margin of error} = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

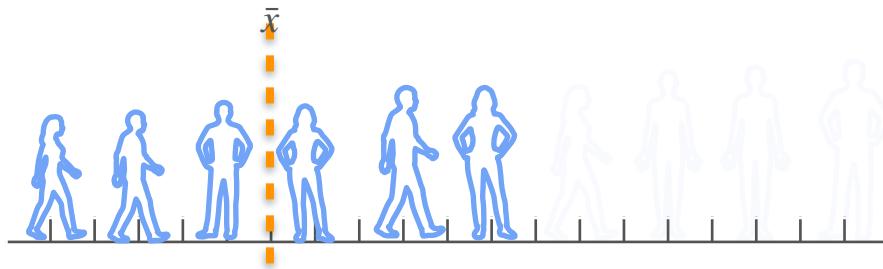
## Confidence Interval for Proportions

$$\hat{p} = \frac{x}{n} = \frac{24}{30} = 80\%$$

confidence interval =  $\hat{p} \pm$  margin of error

# Confidence Interval for Proportions

## Confidence Interval for Means



$$\text{confidence interval} = \bar{x} \pm \text{margin of error}$$

$$\text{margin of error} = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

## Confidence Interval for Proportions

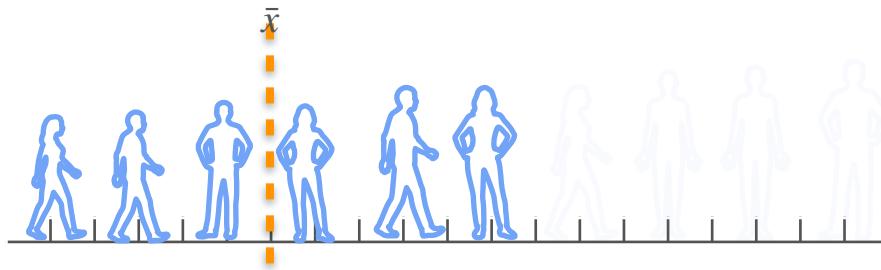
$$\hat{p} = \frac{x}{n} = \frac{24}{30} = 80\%$$

$$\text{confidence interval} = \hat{p} \pm \text{margin of error}$$

$$\text{margin of error} =$$

# Confidence Interval for Proportions

## Confidence Interval for Means



confidence interval =  $\bar{x} \pm$  margin of error

$$\text{margin of error} = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

## Confidence Interval for Proportions

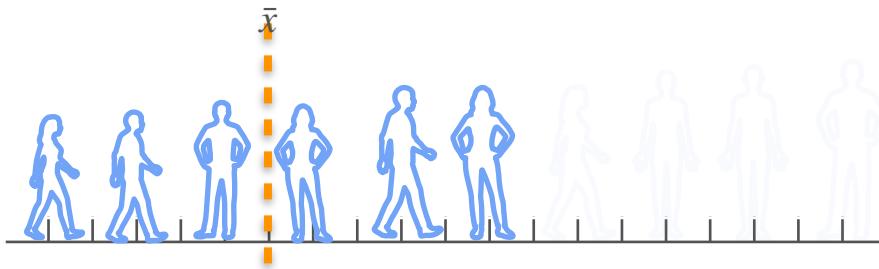
$$\hat{p} = \frac{x}{n} = \frac{24}{30} = 80\%$$

confidence interval =  $\hat{p} \pm$  margin of error

$$\text{margin of error} = z_{\alpha/2} \cdot$$

# Confidence Interval for Proportions

## Confidence Interval for Means



$$\text{confidence interval} = \bar{x} \pm \text{margin of error}$$

$$\text{margin of error} = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

## Confidence Interval for Proportions

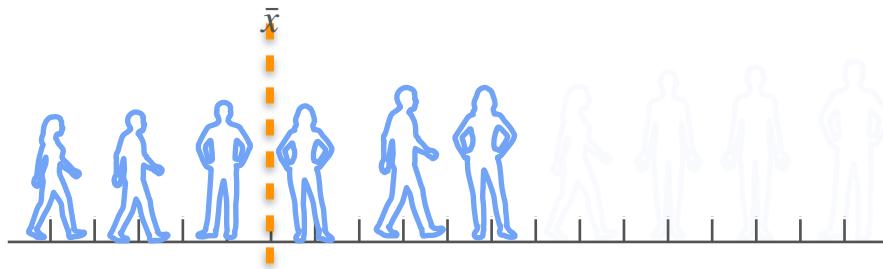
$$\hat{p} = \frac{x}{n} = \frac{24}{30} = 80\%$$

$$\text{confidence interval} = \hat{p} \pm \text{margin of error}$$

$$\text{margin of error} = z_{\alpha/2} \cdot \sqrt{\frac{\hat{p}(1 - \hat{p})}{n}}$$

# Confidence Interval for Proportions

## Confidence Interval for Means



confidence interval =  $\bar{x} \pm$  margin of error

$$\text{margin of error} = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

## Confidence Interval for Proportions

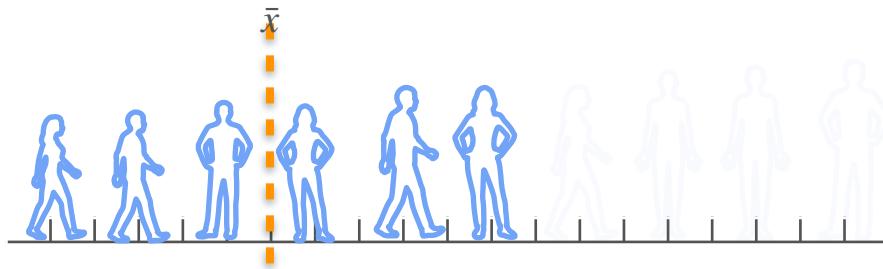
$$\hat{p} = \frac{x}{n} = \frac{24}{30} = 80\%$$

confidence interval =  $\hat{p} \pm$  margin of error

$$\text{margin of error} = z_{\alpha/2} \cdot \sqrt{\frac{\hat{p}(1 - \hat{p})}{n}}$$

# Confidence Interval for Proportions

## Confidence Interval for Means



$$\text{confidence interval} = \bar{x} \pm \text{margin of error}$$

$$\text{margin of error} = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

## Confidence Interval for Proportions

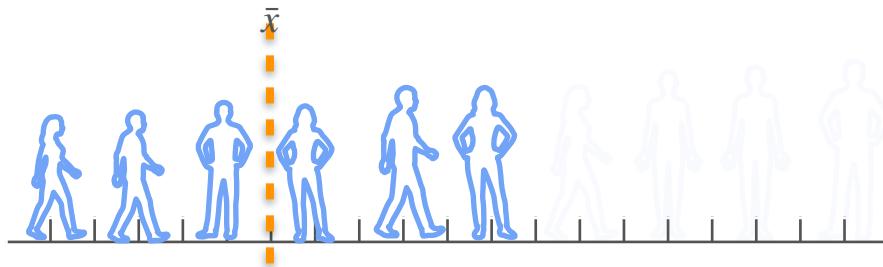
$$\hat{p} = \frac{x}{n} = \frac{24}{30} = 80\%$$

$$\text{confidence interval} = \hat{p} \pm \text{margin of error}$$

$$\text{margin of error} = z_{\alpha/2} \cdot \sqrt{\frac{\hat{p}(1 - \hat{p})}{n}}$$

# Confidence Interval for Proportions

## Confidence Interval for Means



$$\text{confidence interval} = \bar{x} \pm \text{margin of error}$$

$$\text{margin of error} = z_{\alpha/2} \cdot$$

$$\frac{\sigma}{\sqrt{n}}$$

## Confidence Interval for Proportions

$$\hat{p} = \frac{x}{n} = \frac{24}{30} = 80\%$$

$$\text{confidence interval} = \hat{p} \pm \text{margin of error}$$

$$\text{margin of error} = z_{\alpha/2} \cdot$$

$$\sqrt{\frac{\hat{p}(1 - \hat{p})}{n}}$$

**standard error**

# Confidence Interval for Proportions

## Confidence Interval for Proportions

# Confidence Interval for Proportions

## Confidence Interval for Proportions

confidence interval =

# Confidence Interval for Proportions

## Confidence Interval for Proportions

confidence interval =  $\hat{p}$

# Confidence Interval for Proportions

## Confidence Interval for Proportions

confidence interval =  $\hat{p} \pm$  margin of error

# Confidence Interval for Proportions

## Confidence Interval for Proportions

confidence interval =  $\hat{p} \pm$  margin of error

margin of error =

# Confidence Interval for Proportions

## Confidence Interval for Proportions

confidence interval =  $\hat{p} \pm$  margin of error

margin of error =  $z_{\alpha/2} \cdot$

# Confidence Interval for Proportions

## Confidence Interval for Proportions

confidence interval =  $\hat{p} \pm$  margin of error

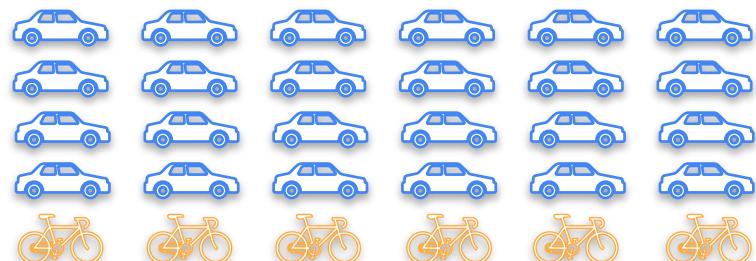
$$\text{margin of error} = z_{\alpha/2} \cdot \sqrt{\frac{\hat{p}(1 - \hat{p})}{n}}$$

# Confidence Interval for Proportions

## Confidence Interval for Proportions

confidence interval =  $\hat{p} \pm$  margin of error

$$\text{margin of error} = z_{\alpha/2} \cdot \sqrt{\frac{\hat{p}(1 - \hat{p})}{n}}$$

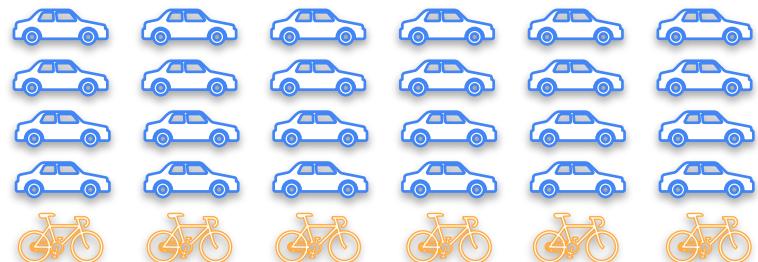


# Confidence Interval for Proportions

## Confidence Interval for Proportions

confidence interval =  $\hat{p} \pm$  margin of error

$$\text{margin of error} = z_{\alpha/2} \cdot \sqrt{\frac{\hat{p}(1 - \hat{p})}{n}}$$



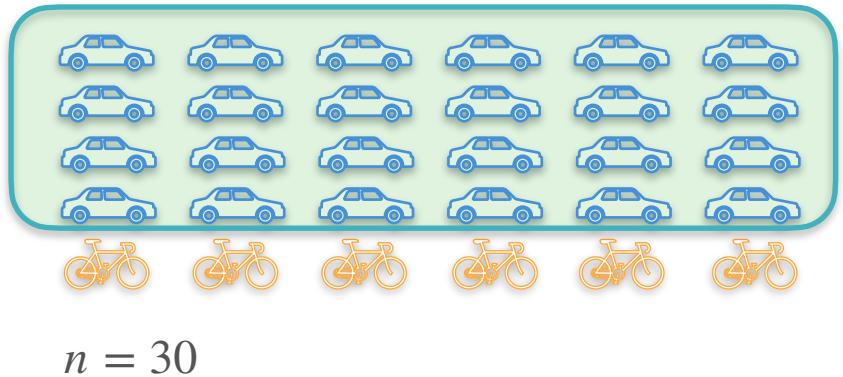
$$n = 30$$

# Confidence Interval for Proportions

## Confidence Interval for Proportions

$$\text{confidence interval} = \hat{p} \pm \text{margin of error}$$

$$\text{margin of error} = z_{\alpha/2} \cdot \sqrt{\frac{\hat{p}(1 - \hat{p})}{n}}$$

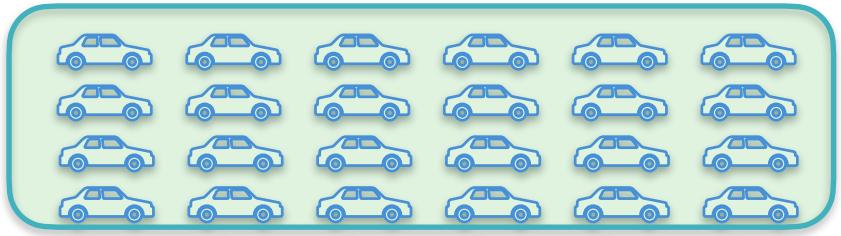


# Confidence Interval for Proportions

## Confidence Interval for Proportions

$$\text{confidence interval} = \hat{p} \pm \text{margin of error}$$

$$\text{margin of error} = z_{\alpha/2} \cdot \sqrt{\frac{\hat{p}(1 - \hat{p})}{n}}$$



$$n = 30$$

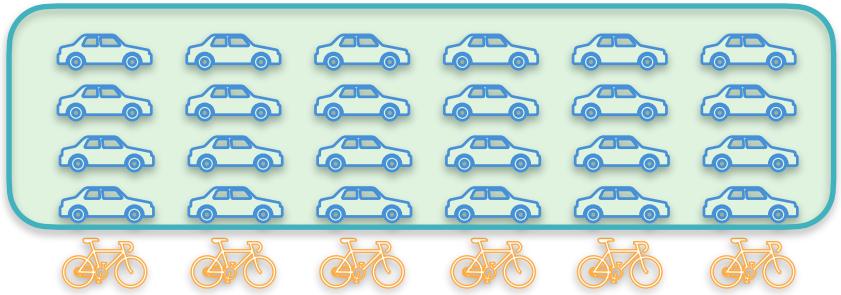
$$\hat{p} = 80\% = 0.8$$

# Confidence Interval for Proportions

## Confidence Interval for Proportions

$$\text{confidence interval} = \hat{p} \pm \text{margin of error}$$

$$\text{margin of error} = z_{\alpha/2} \cdot \sqrt{\frac{\hat{p}(1 - \hat{p})}{n}}$$



$$n = 30 \quad \hat{p} = 80 \% = 0.8$$

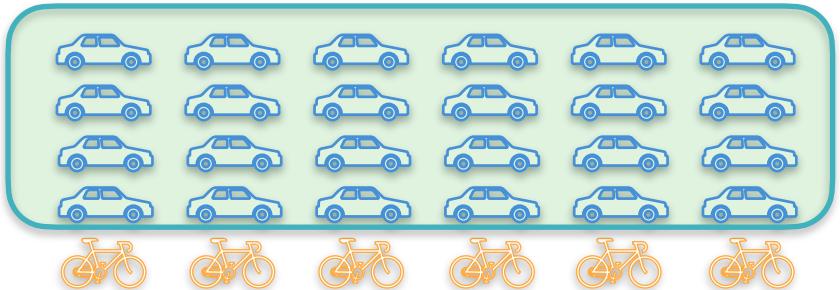
Calculate a 95% confidence interval for this sample proportion

# Confidence Interval for Proportions

## Confidence Interval for Proportions

confidence interval =  $\hat{p} \pm$  margin of error

$$\text{margin of error} = z_{\alpha/2} \cdot \sqrt{\frac{\hat{p}(1 - \hat{p})}{n}}$$



$$n = 30 \quad \hat{p} = 80 \% = 0.8$$

Calculate a 95% confidence interval for this sample proportion

95%

$$\rightarrow z_{\alpha/2} = 1.96$$

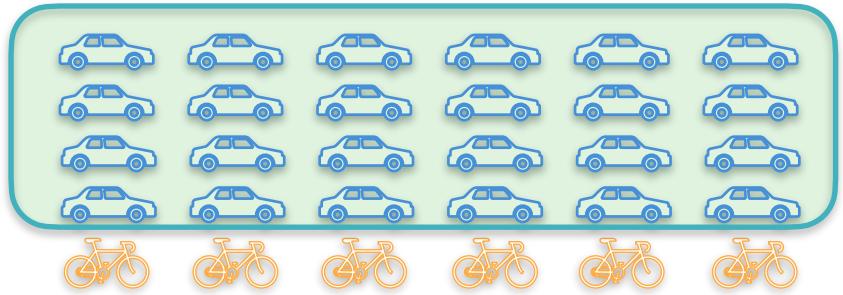
# Confidence Interval for Proportions

## Confidence Interval for Proportions

confidence interval =  $\hat{p} \pm$  margin of error

$$\text{margin of error} = z_{\alpha/2} \cdot \sqrt{\frac{\hat{p}(1 - \hat{p})}{n}}$$

margin of error =



$$n = 30 \quad \hat{p} = 80\% = 0.8$$

Calculate a 95% confidence interval for this sample proportion

$$95\% \rightarrow z_{\alpha/2} = 1.96$$

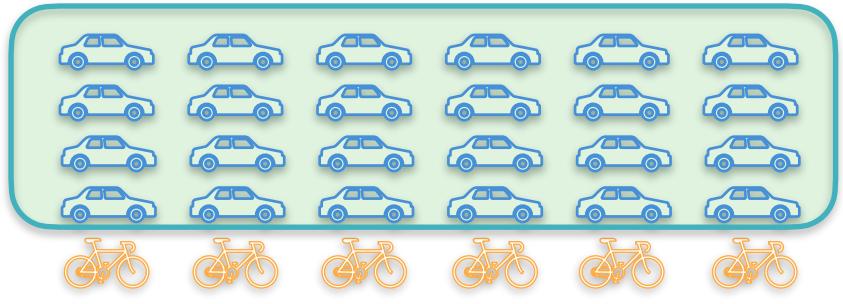
# Confidence Interval for Proportions

## Confidence Interval for Proportions

$$\text{confidence interval} = \hat{p} \pm \text{margin of error}$$

$$\text{margin of error} = z_{\alpha/2} \cdot \sqrt{\frac{\hat{p}(1 - \hat{p})}{n}}$$

$$\text{margin of error} = 1.96$$



$$n = 30 \quad \hat{p} = 80\% = 0.8$$

Calculate a 95% confidence interval for this sample proportion

95%

$$z_{\alpha/2} = 1.96$$

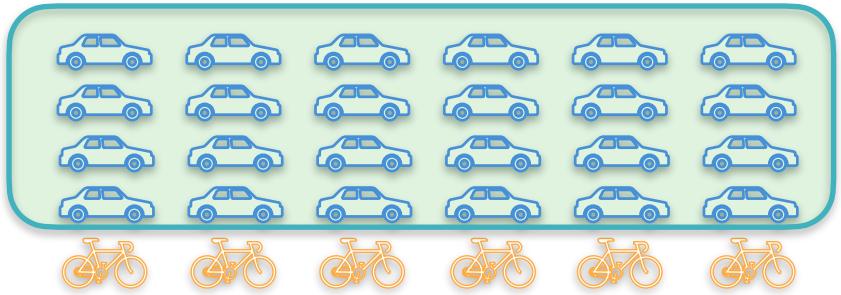
# Confidence Interval for Proportions

## Confidence Interval for Proportions

$$\text{confidence interval} = \hat{p} \pm \text{margin of error}$$

$$\text{margin of error} = z_{\alpha/2} \cdot \sqrt{\frac{\hat{p}(1 - \hat{p})}{n}}$$

$$\text{margin of error} = 1.96 \cdot \sqrt{\frac{0.8(1 - 0.8)}{30}}$$



$$n = 30 \quad \hat{p} = 80\% = 0.8$$

Calculate a 95% confidence interval for this sample proportion

95%  $\rightarrow z_{\alpha/2} = 1.96$

# Confidence Interval for Proportions

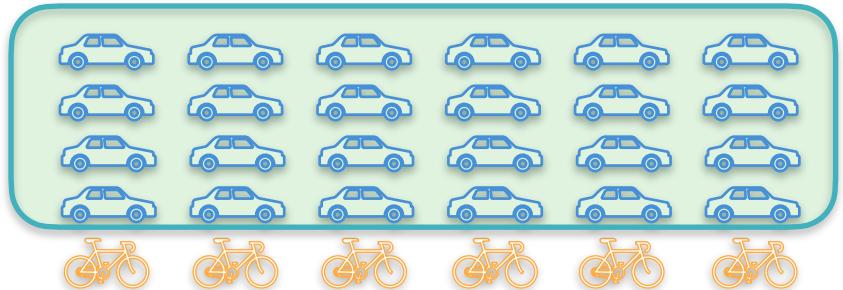
## Confidence Interval for Proportions

$$\text{confidence interval} = \hat{p} \pm \text{margin of error}$$

$$\text{margin of error} = z_{\alpha/2} \cdot \sqrt{\frac{\hat{p}(1 - \hat{p})}{n}}$$

$$\text{margin of error} = 1.96 \cdot \sqrt{\frac{0.8(1 - 0.8)}{30}}$$

$$\text{margin of error} = 0.14$$



$$n = 30 \quad \hat{p} = 80\% = 0.8$$

Calculate a 95% confidence interval for this sample proportion

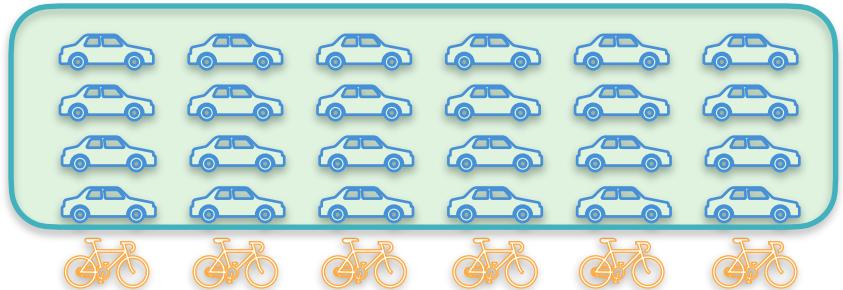
$$95\% \rightarrow z_{\alpha/2} = 1.96$$

# Confidence Interval for Proportions

## Confidence Interval for Proportions

$$\text{confidence interval} = \hat{p} \pm \text{margin of error}$$

$$\text{margin of error} = 0.14$$



$$n = 30 \quad \hat{p} = 80\% = 0.8$$

Calculate a 95% confidence interval for this sample proportion

95%

$$\rightarrow z_{\alpha/2} = 1.96$$

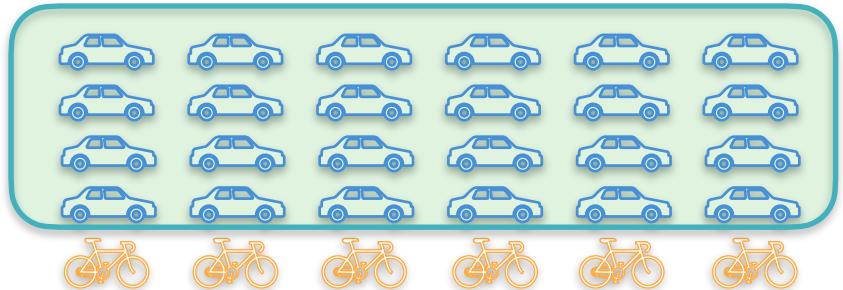
# Confidence Interval for Proportions

## Confidence Interval for Proportions

$$\text{confidence interval} = \hat{p} \pm \text{margin of error}$$

$$\text{margin of error} = 0.14$$

$$\text{confidence interval} =$$



$$n = 30 \quad \hat{p} = 80\% = 0.8$$

Calculate a 95% confidence interval for this sample proportion

$$95\% \rightarrow z_{\alpha/2} = 1.96$$

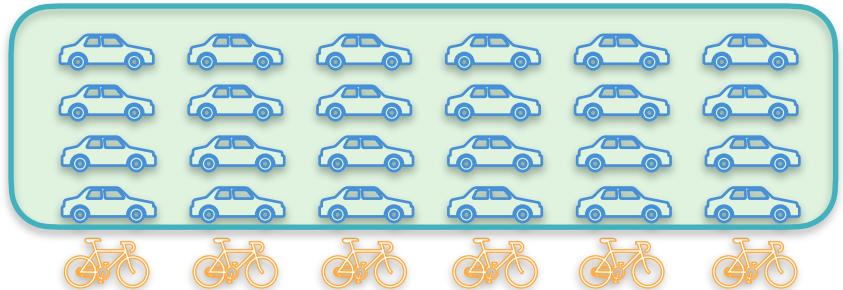
# Confidence Interval for Proportions

## Confidence Interval for Proportions

$$\text{confidence interval} = \hat{p} \pm \text{margin of error}$$

$$\text{margin of error} = 0.14$$

$$\text{confidence interval} = 0.8 \pm 0.14$$



$$n = 30 \quad \hat{p} = 80\% = 0.8$$

Calculate a 95% confidence interval for this sample proportion

95%

$$\rightarrow z_{\alpha/2} = 1.96$$

# Confidence Interval for Proportions

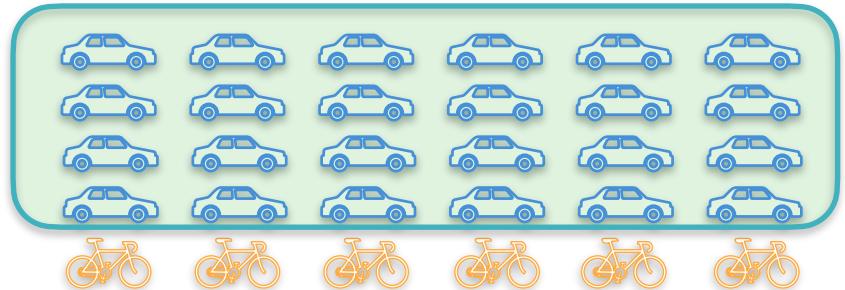
## Confidence Interval for Proportions

$$\text{confidence interval} = \hat{p} \pm \text{margin of error}$$

$$\text{margin of error} = 0.14$$

$$\text{confidence interval} = 0.8 \pm 0.14$$

$$0.66 < p < 0.94$$



$$n = 30 \quad \hat{p} = 80\% = 0.8$$

Calculate a 95% confidence interval for this sample proportion

$$95\% \rightarrow z_{\alpha/2} = 1.96$$

# Confidence Interval for Proportions

## Confidence Interval for Proportions

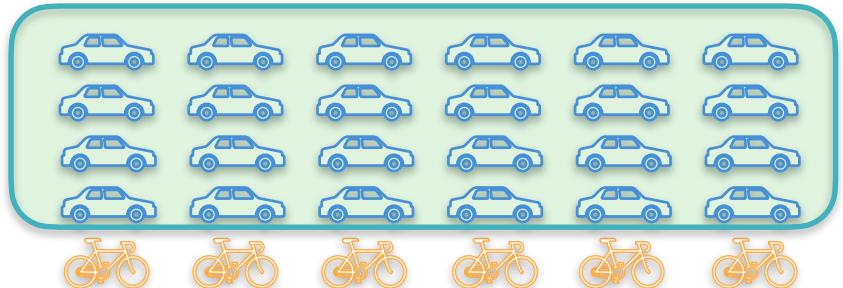
$$\text{confidence interval} = \hat{p} \pm \text{margin of error}$$

$$\text{margin of error} = 0.14$$

$$\text{confidence interval} = 0.8 \pm 0.14$$

$$0.66 < p < 0.94$$

$$66\% < p < 94\%$$



$$n = 30 \quad \hat{p} = 80\% = 0.8$$

Calculate a 95% confidence interval for this sample proportion

$$95\% \rightarrow z_{\alpha/2} = 1.96$$

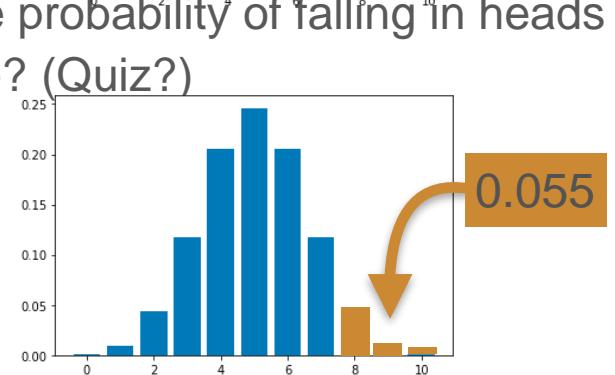
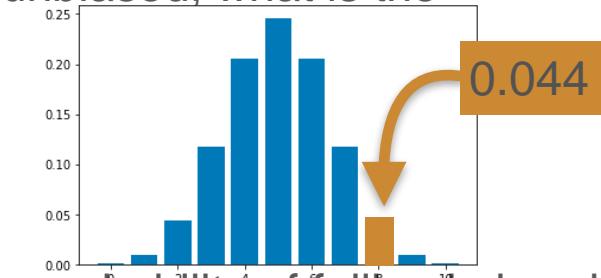
# W4 Lesson 2

# Hypothesis Testing

- Data Science interview
  - I throw a coin 10 times. I get heads 8 times.
  - Quiz 1: Do you think the coin is biased, or not?
    - Keep both answers correct. Specify that we don't have enough information.
  - More information: We agree to say that something is *unlikely* if the probability of it happening is less than 5%.

# Hypothesis Testing

- Quiz: If we were to assume that the coin is unbiased, what is the probability that it falls in heads 8 times?
  - Answer:  $(10 \text{ choose } 8) * 0.5^{10}$   
*this is for unbiased*
  - However, should we instead look at the probability of falling in heads 8 or more times? How much is this one? (Quiz?)
  - Answer:  
 $((10 \text{ choose } 8) +$   
 $(10 \text{ choose } 9) +$   
 $(10 \text{ choose } 10)) * 0.5^{10}$

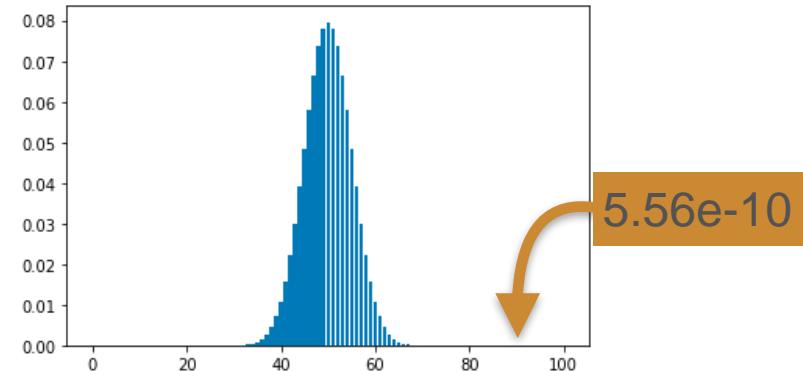


# Hypothesis Testing

- If we assume that the coin is fair (**null hypothesis**)
- Then the probability that it lands in heads 8 times (**outcome**) is 0.055
- That probability is higher than 5%
  - Therefore the coin *could* potentially be fair (i.e., **we can't reject the null hypothesis**)

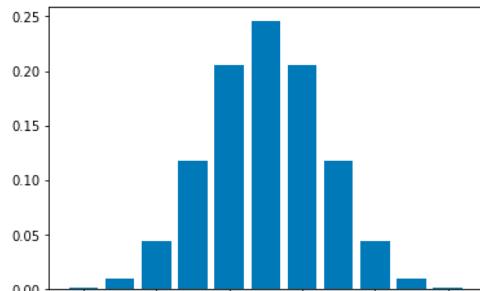
# Hypothesis Testing

- Now, same problem except that we toss the coin 100 times and get heads 80 times.
- Probability of 80 or more heads:
  - Sum from  $i=80$  to  $100$  of  $(100 \text{ choose } i) * 0.5^{100}$
  - The probability is TINY
- Therefore, we reject the null hypothesis
  - We conclude the coin is not biased
- Problem: A huge sum of tiny numbers to calculate
- Solution: Approximate with a Gaussian!

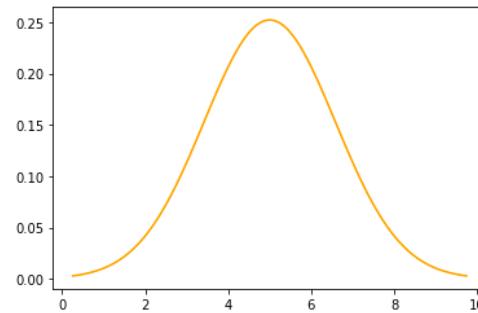


# Central Limits Theorem (-Ish)

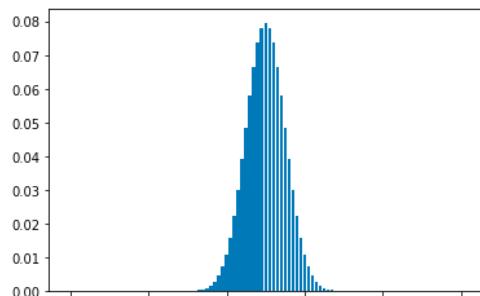
This is called on W<sub>3</sub>



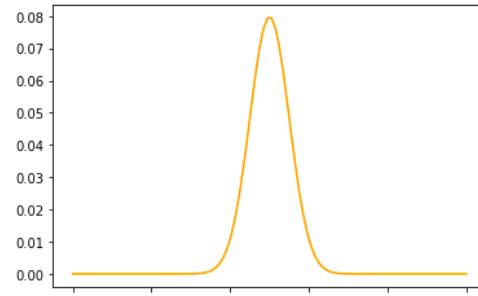
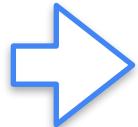
Binomial(10, 0.5)



Gaussian( $5, \sqrt{2.5}$ )

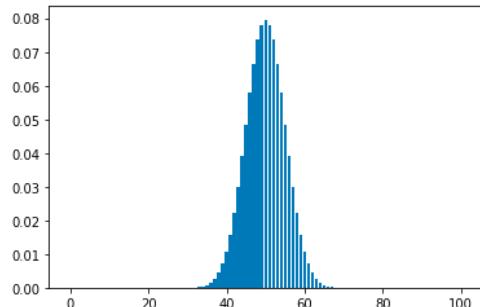


Binomial(100, 0.5)

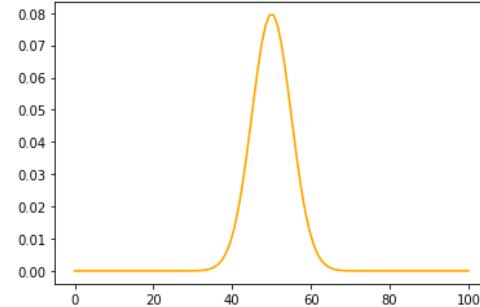


Gaussian( $50, 5$ )

# Central Limits Theorem (-Ish)



Binomial( $n, p$ )



Gaussian(?, ?)

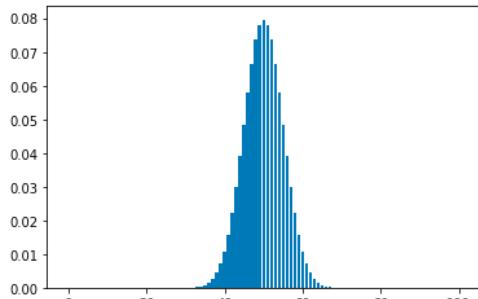
$\text{Gaussian}(np, \sqrt{np(1 - p)})$



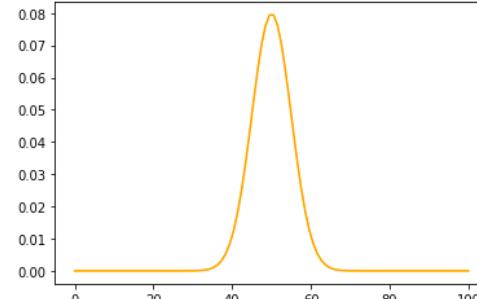
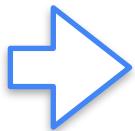
Mean =  $np$   
Variance =  $np(1-p)$

Gaussian with:  
mean= $np$   
Variance =  $np(1-p)$

# Approximating the Binomial With a Gaussian



Binomial(100, 0.5)



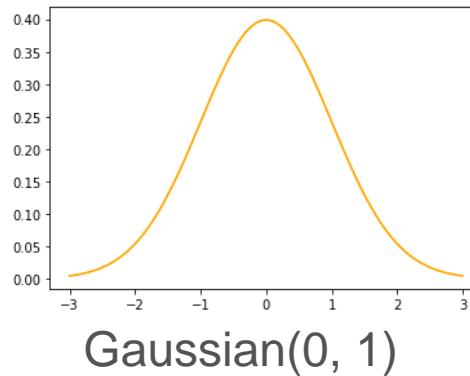
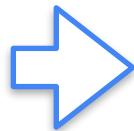
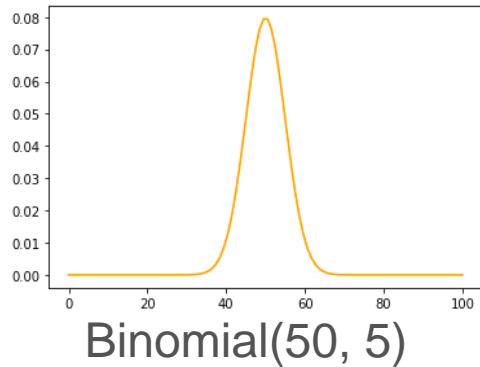
Gaussian(50, 5)

Sum of binomials from 80 to 100

Area under the curve from 80 to 100

Can be calculated using CDF!

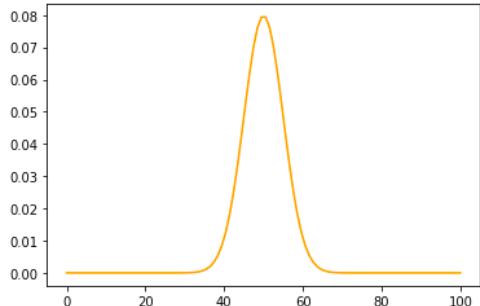
# How To Do It? Normalize



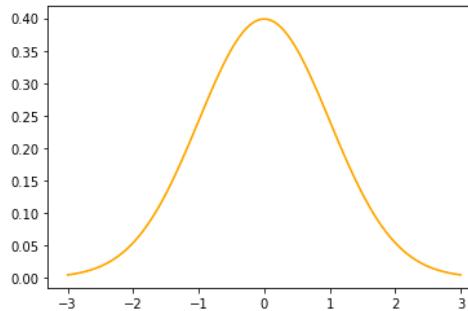
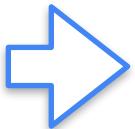
X

$$\frac{X - \text{mean}}{\text{std}}$$
$$\frac{X - 50}{5}$$

# How To Do It? Normalize



Binomial( $n$ ,  $p$ )



Gaussian(0, 1)

$X$

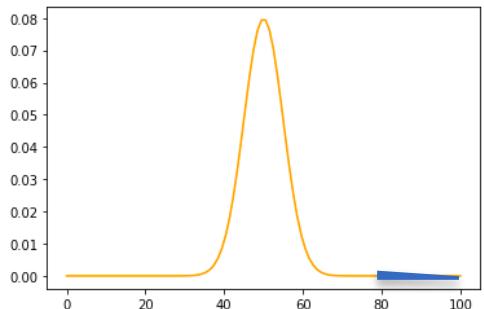
$$\mu = np$$

$$\sigma = np(1 - p)$$

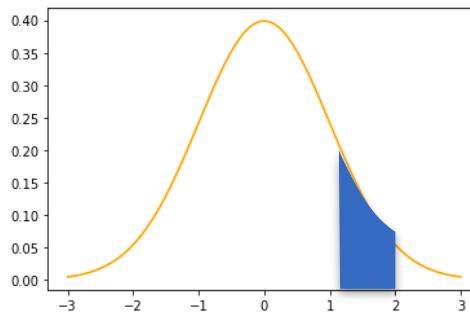
$$\frac{X - \text{mean}}{\text{std}}$$

$$\frac{X - \mu}{\sigma}$$

# How To Do It? Normalize



$\text{Binomial}(n, p)$



$\text{Gaussian}(0, 1)$

$P(80 < X < 100)$  

$$P\left(\frac{80 - \mu}{\sigma} < \frac{X - \mu}{\sigma} < \frac{100 - \mu}{\sigma}\right)$$

$$P\left(\frac{80 - 50}{5} < \frac{X - 50}{5} < \frac{100 - 50}{5}\right)$$

$$= P(12 < Z < 20)$$

# How To Do It? Normalize

$$\begin{aligned} P\left(\frac{80 - 50}{25} < \frac{X - 50}{25} < \frac{100 - 50}{25}\right) &= P(12 < Z < 20) \\ &= \Phi(20) - \Phi(12) \\ &= \text{almost } 0 \end{aligned}$$

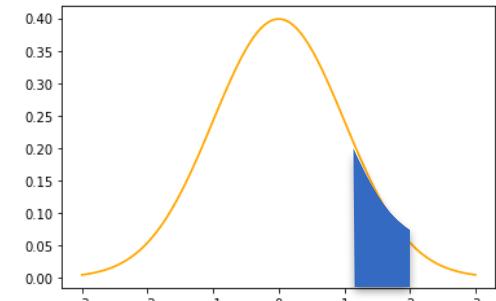
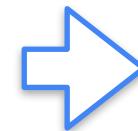
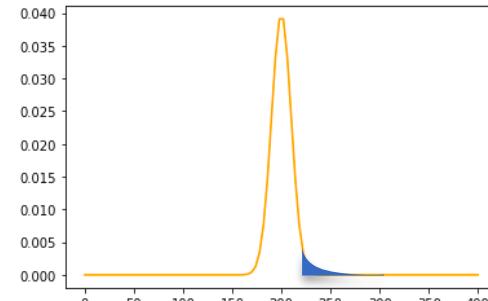
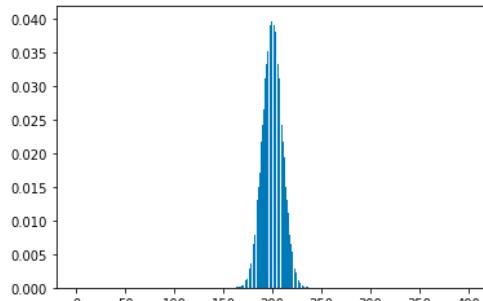
Conclusion: The coin must be biased

Why?

Because we reject the **null hypothesis** which says the coin is not biased.

# Another Example

I toss a fair coin 400 times. What is the probability that the number of heads is between 250 and 350?



We can use this  
on top of this  
one way

$$0.9332 - 0.6915$$

$$P\left(\frac{250 - 200}{100} < \frac{X - 200}{100} < \frac{350 - 200}{100}\right)$$

$$P\left(0.5 < \frac{X - 200}{100} < 1.5\right)$$

$$= P(0.5 < Z < 1.5)$$

$$= 0.2417$$



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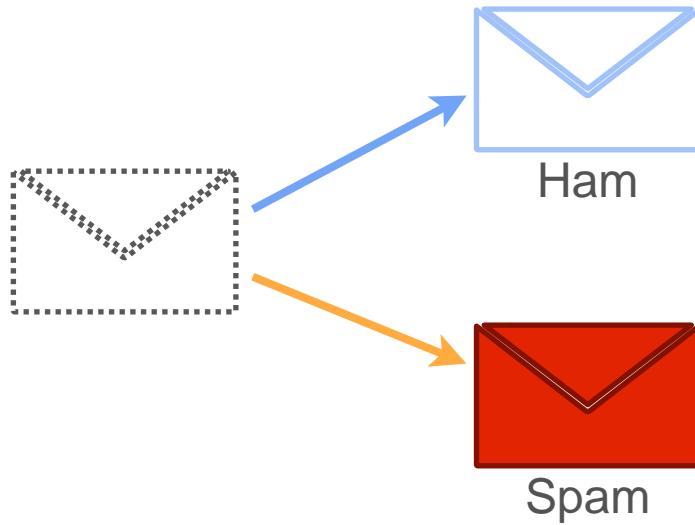
# Hypothesis Testing

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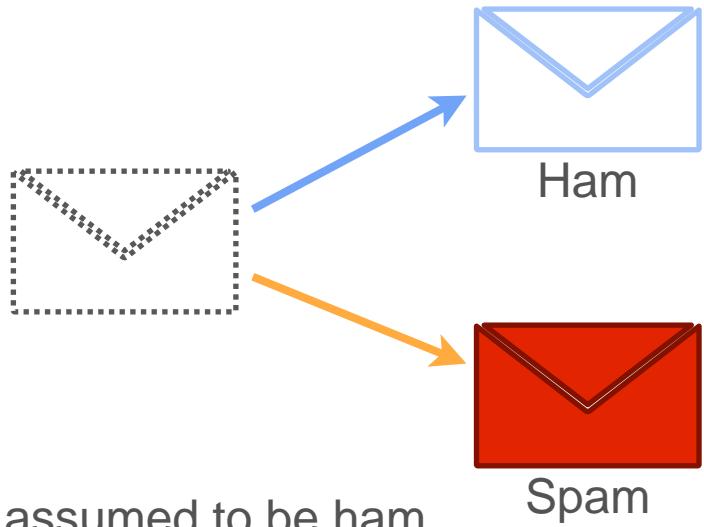
## Defining hypothesis

# Motivation

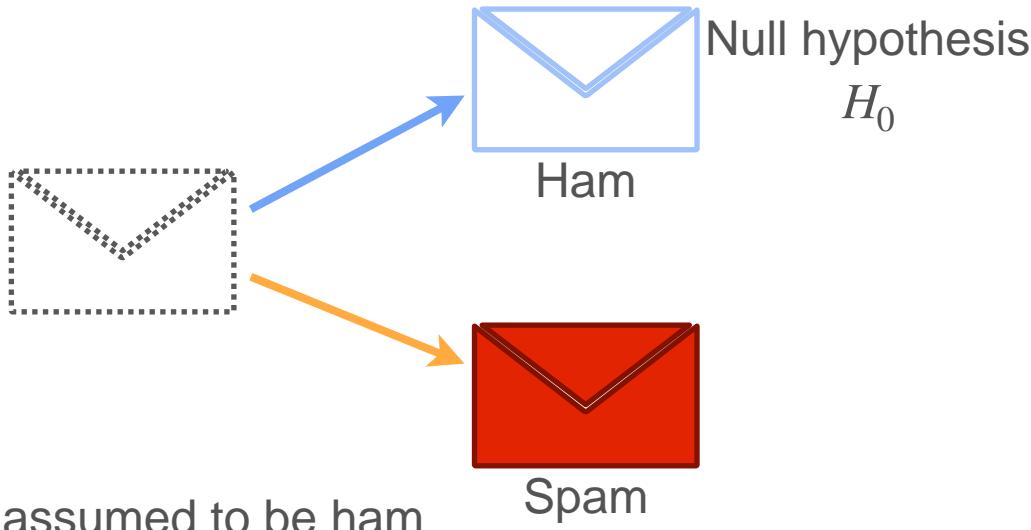
# Motivation



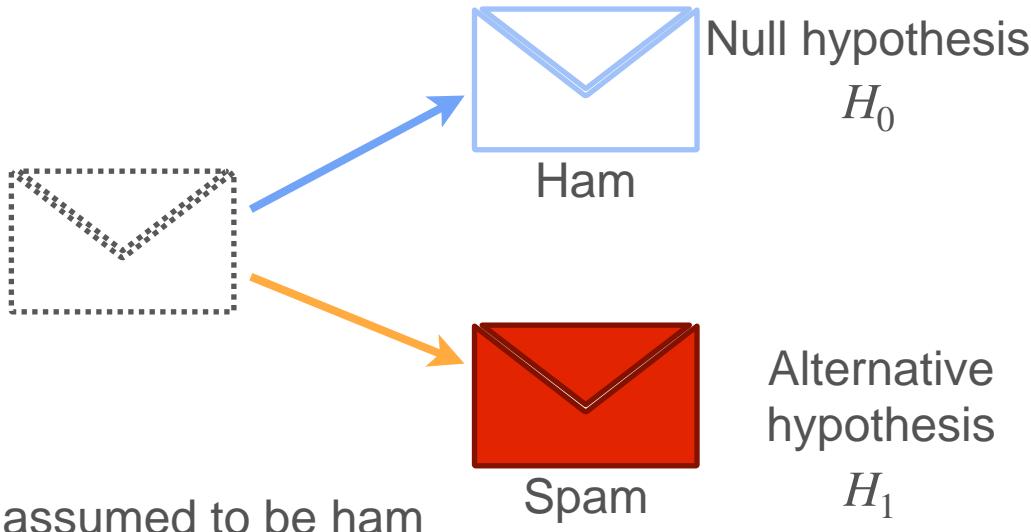
# Motivation



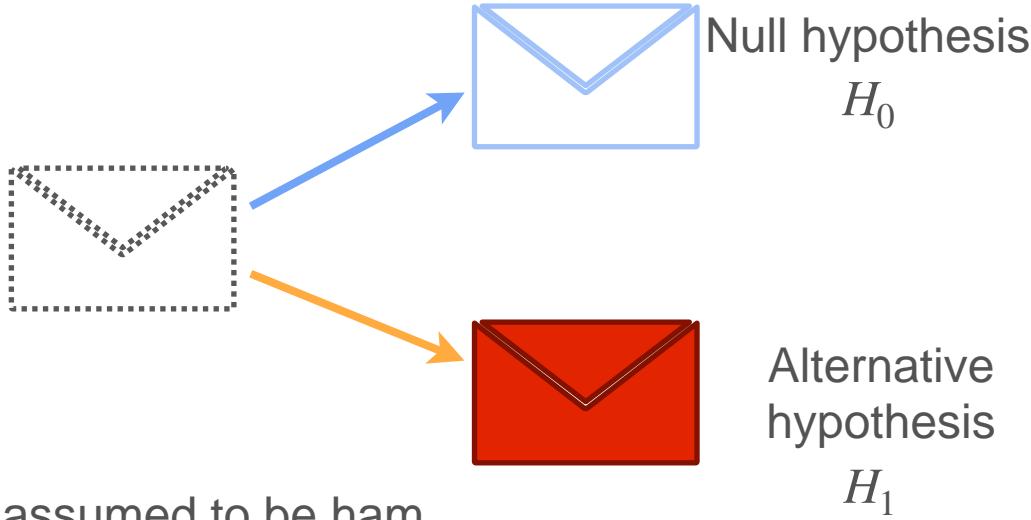
# Motivation



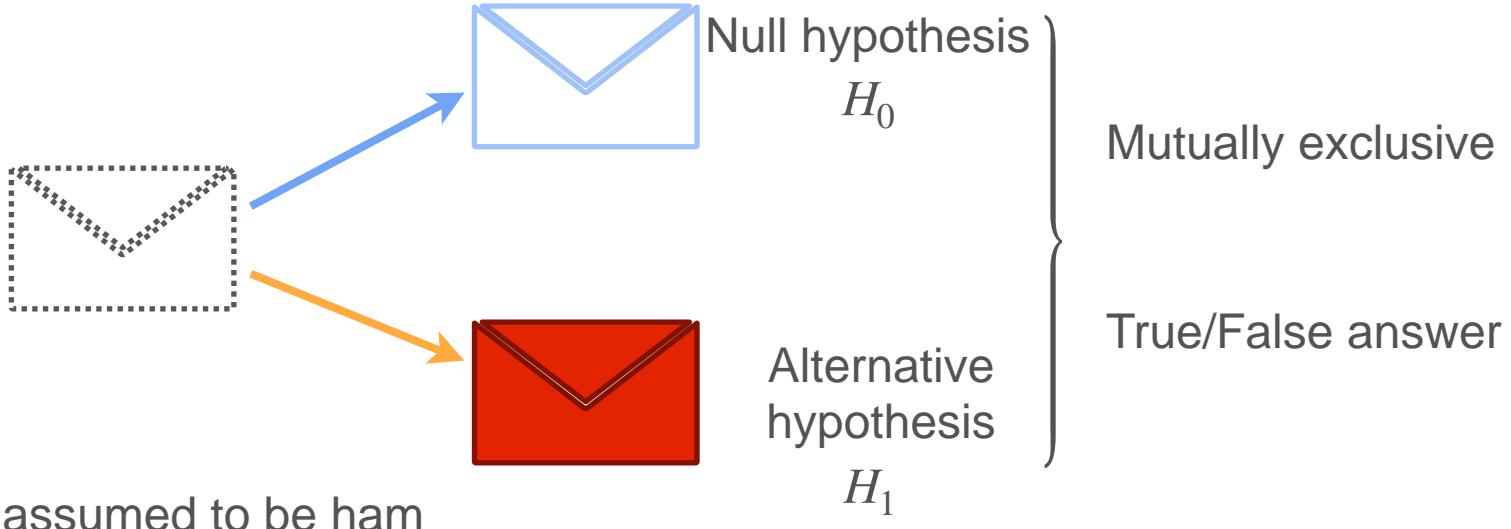
# Motivation



# Motivation

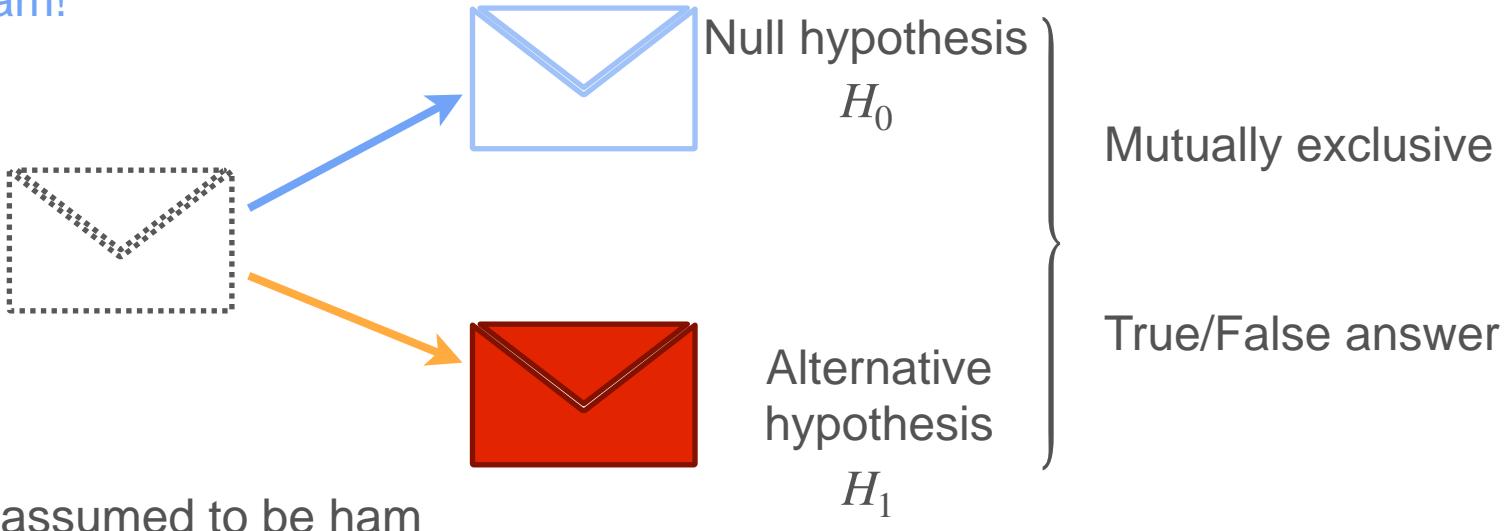


# Motivation



# Motivation

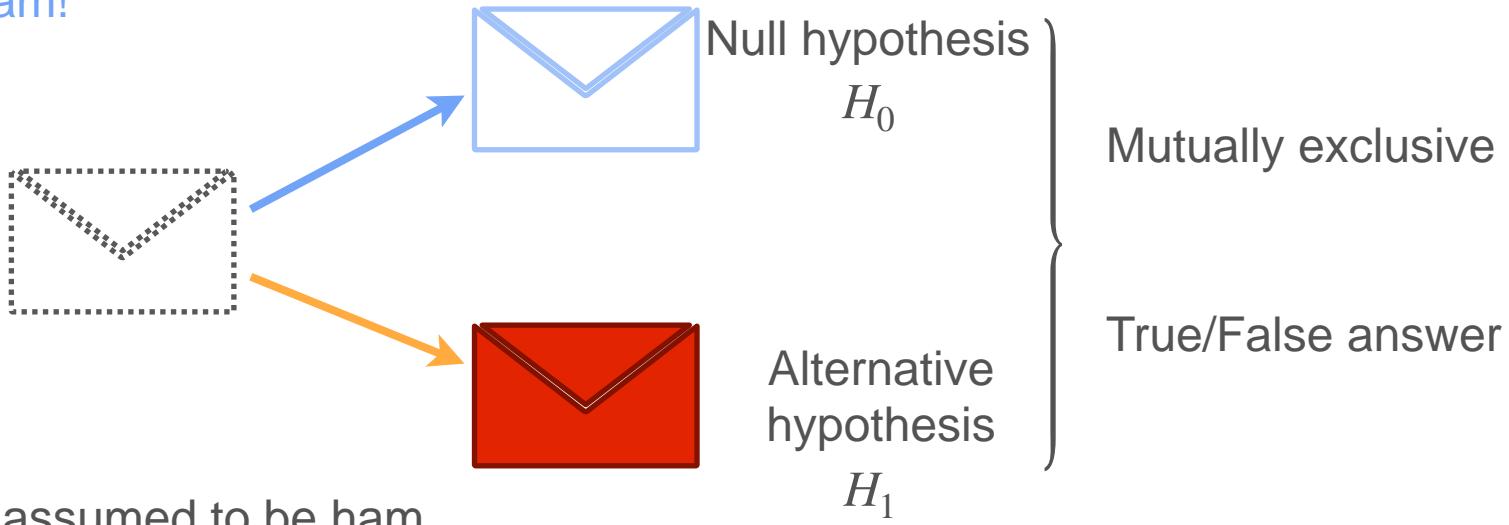
When rejecting that the email is not spam, you are accepting that the email is spam!



# Motivation

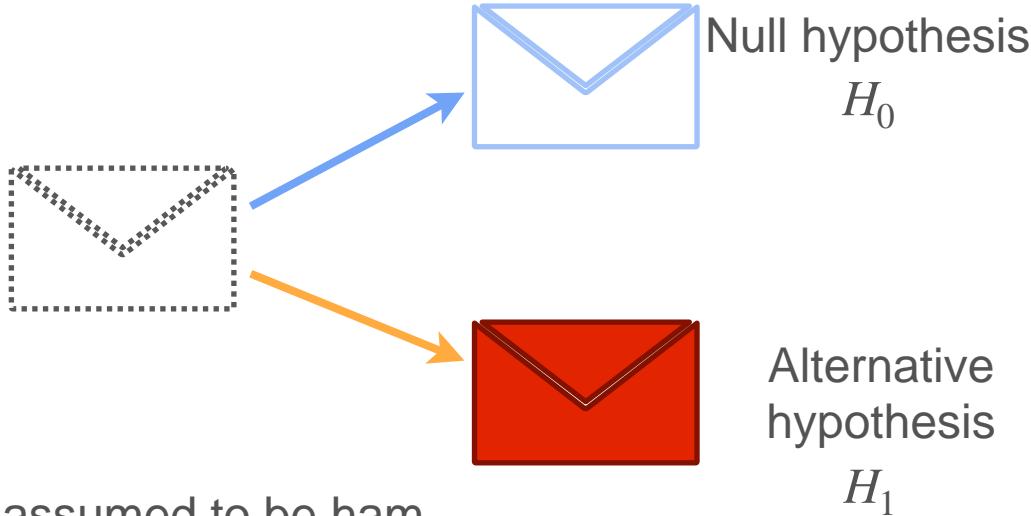
When rejecting that the email is not spam, you are accepting that the email is spam!

By failing to reject that the email IS spam, you are **not** accepting that it's ham



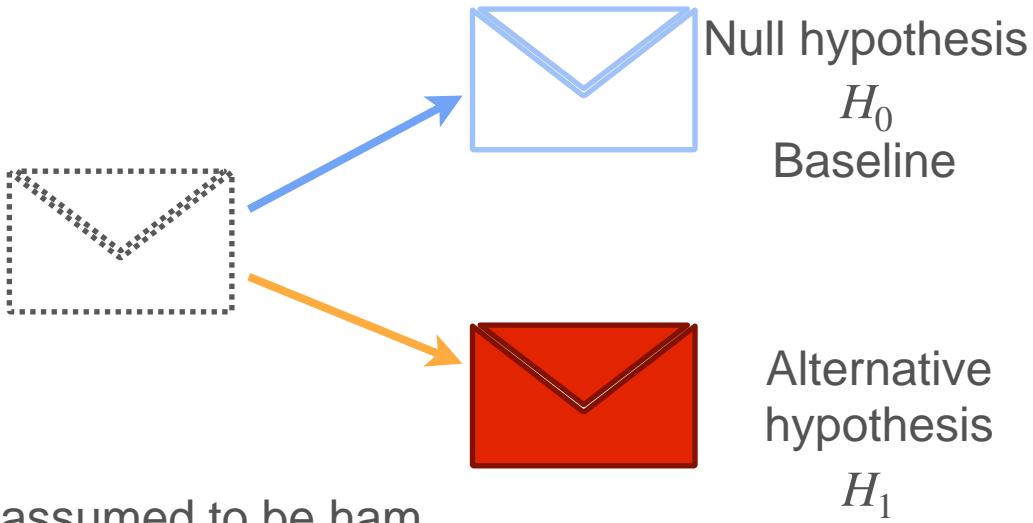
# Motivation

Not labeling the email spam, doesn't mean the email is ham!



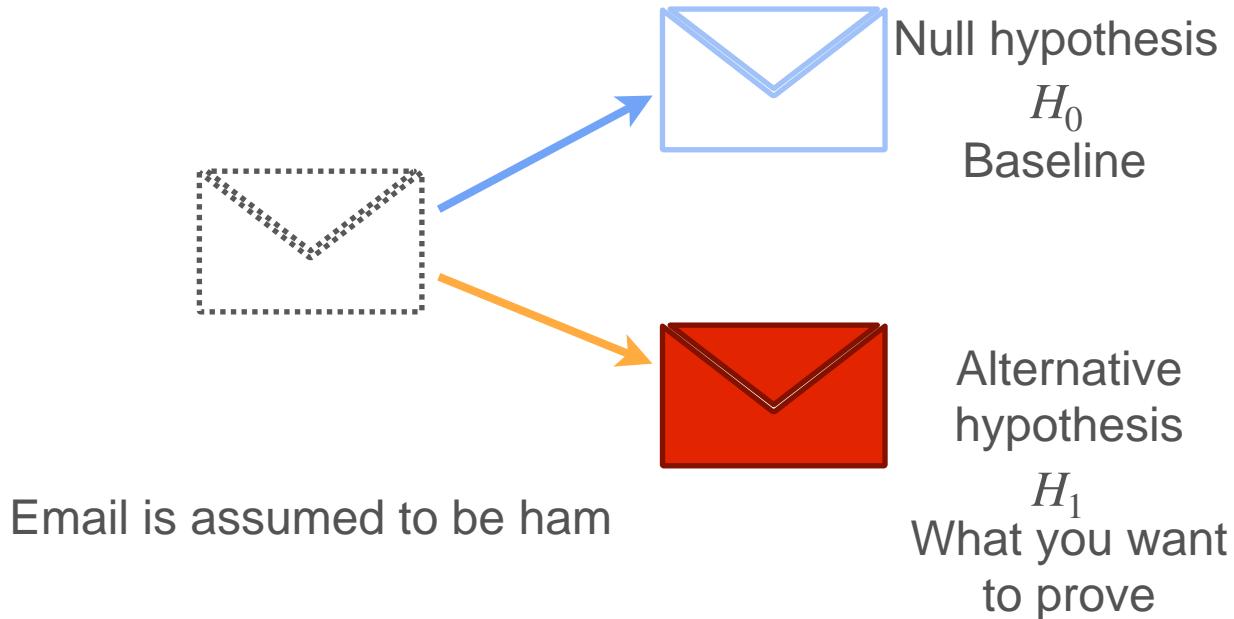
# Motivation

Not labeling the email spam, doesn't mean the email is ham!



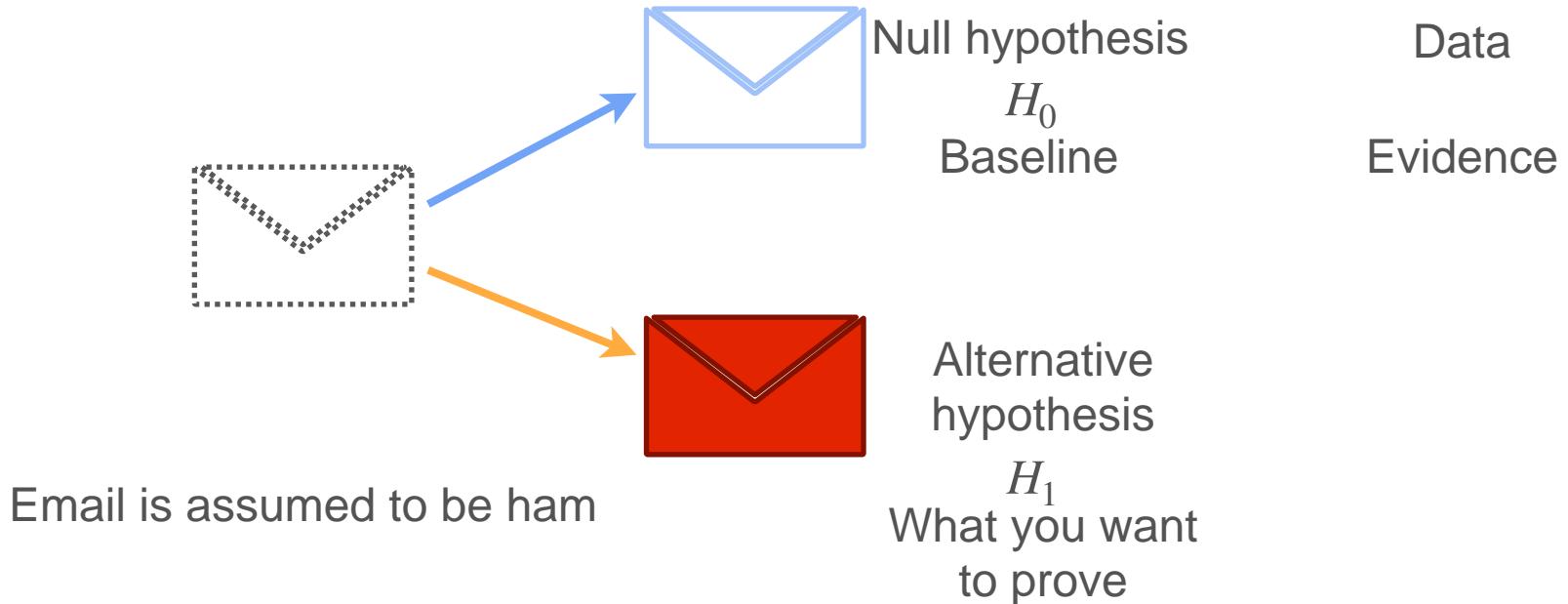
# Motivation

Not labeling the email spam, doesn't mean the email is ham!



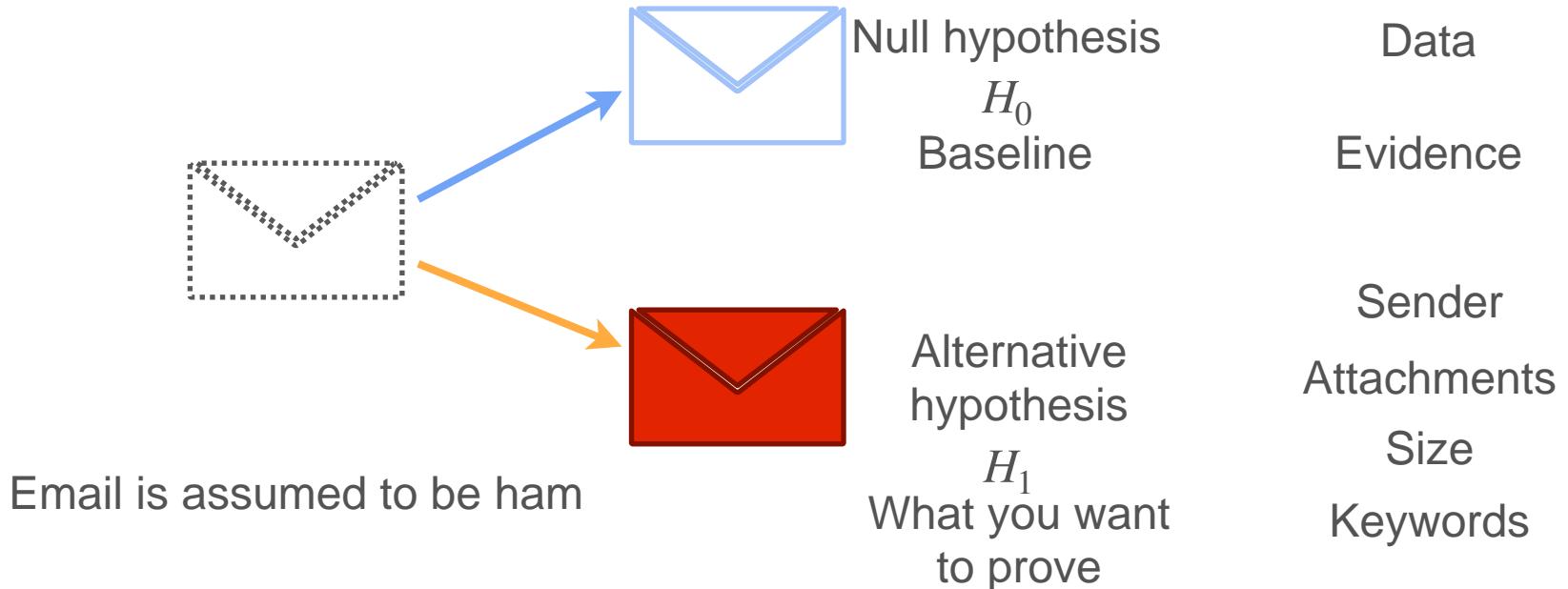
# Motivation

Not labeling the email spam, doesn't mean the email is ham!



# Motivation

Not labeling the email spam, doesn't mean the email is ham!



# How To Determine the Result of the Test

# How To Determine the Result of the Test

Plenty of evidence  
against  $H_0$



Reject  $H_0$  (and accept  $H_1$ )

# How To Determine the Result of the Test

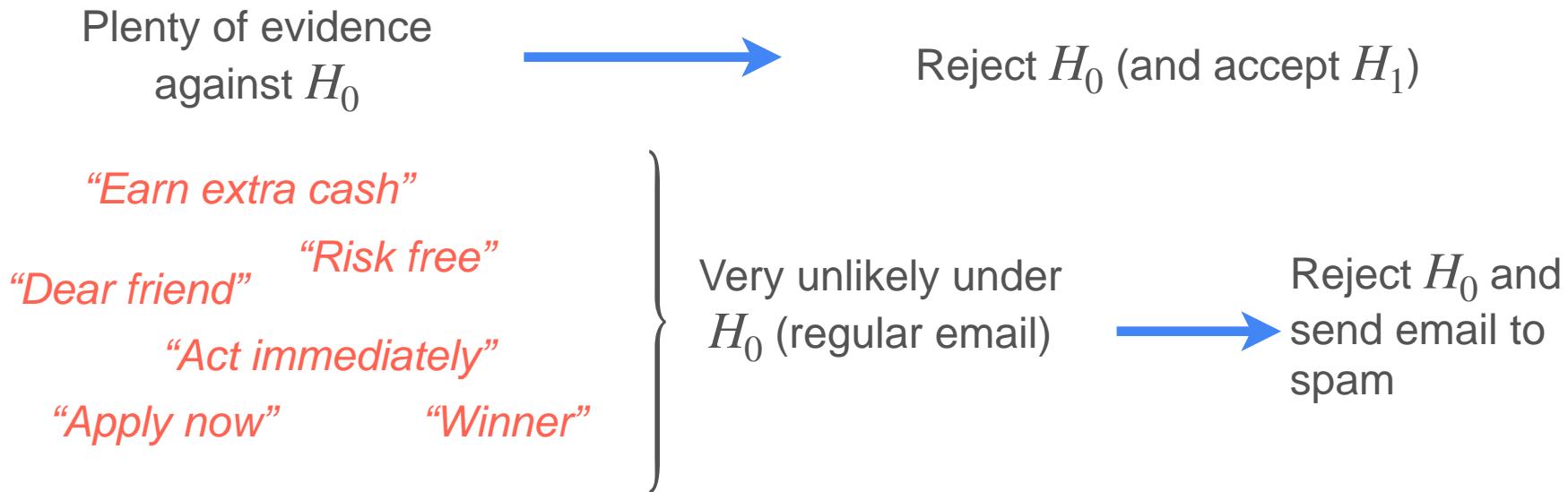
Plenty of evidence  
against  $H_0$



Reject  $H_0$  (and accept  $H_1$ )

*“Earn extra cash”*  
*“Dear friend”*      *“Risk free”*  
*“Act immediately”*  
*“Apply now”*      *“Winner”*

# How To Determine the Result of the Test





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# Hypothesis Testing

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**Type I and Type II errors**

# Sometimes Things Go Wrong...

# Sometimes Things Go Wrong...

What if I make the wrong decision?

# Sometimes Things Go Wrong...

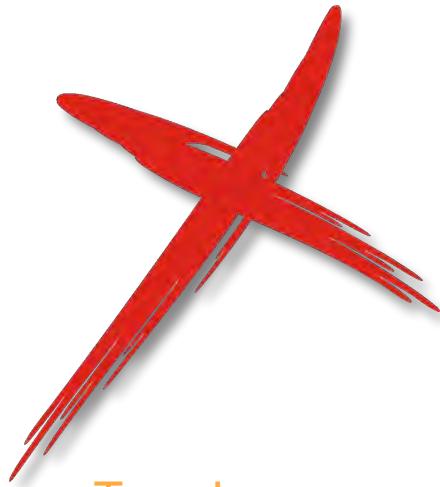
What if I make the wrong decision?



Type I error  
(False positive)

# Sometimes Things Go Wrong...

What if I make the wrong decision?



Type I error  
(False positive)



Type II error  
(False negative)

# Sometimes Things Go Wrong...

What if I make the wrong decision?



Type I error  
(False positive)



Type II error  
(False negative)

# Sometimes Things Go Wrong...

What if I make the wrong decision?



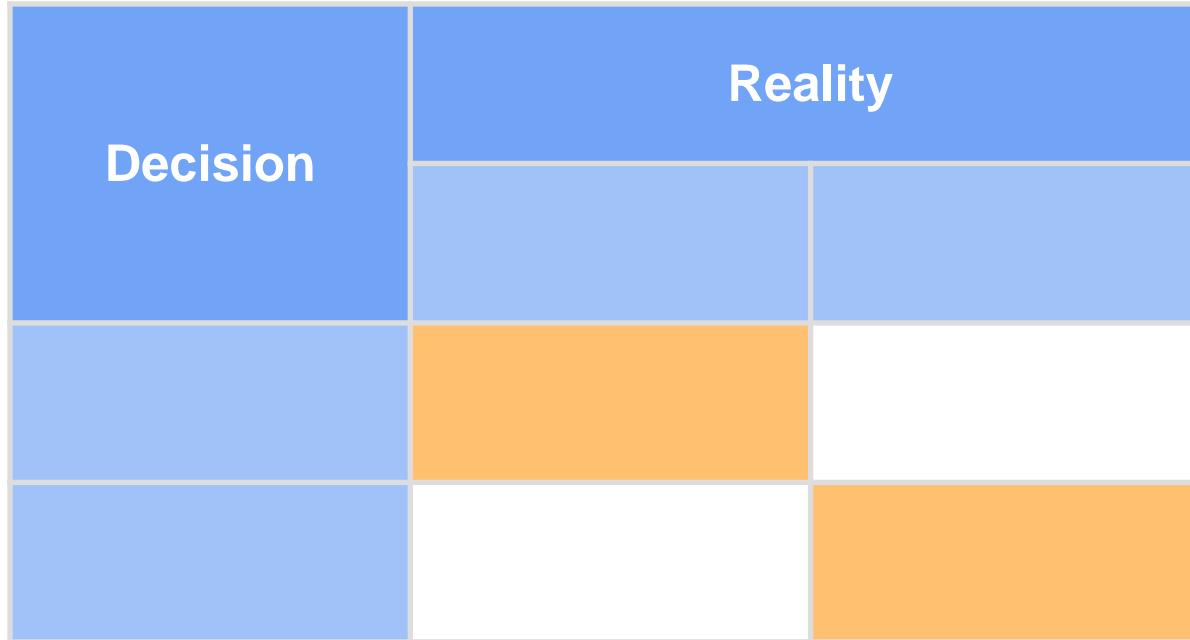
Type I error  
(False positive)



Type II error  
(False negative)

# Type I and Type II Errors

# Type I and Type II Errors



# Type I and Type II Errors

Decision	Reality	
	$H_0$ True (Innocent)	
Reject $H_0$ (Decide Guilty)	Type I error	

# Type I and Type II Errors

Decision	Reality	
	$H_0$ True (Innocent)	
Reject $H_0$ (Decide Guilty)	Type I error	
Don't reject $H_0$ (Decide not guilty)	Correct	

# Type I and Type II Errors

Decision	Reality	
	$H_0$ True (Innocent)	$H_0$ False (Guilty)
Reject $H_0$ (Decide Guilty)	Type I error	Correct
Don't reject $H_0$ (Decide not guilty)	Correct	

# Type I and Type II Errors

Decision	Reality	
	$H_0$ True (Innocent)	$H_0$ False (Guilty)
Reject $H_0$ (Decide Guilty)	Type I error	Correct
Don't reject $H_0$ (Decide not guilty)	Correct	Type II error

# Significance Level

# Significance Level

Type I error

Type II error

# Significance Level

The presumption of innocence implies that sending an innocent person to prison is worse than letting a criminal walk

Type I error



Type II error



# Significance Level

The presumption of innocence implies that sending an innocent person to prison is worse than letting a criminal walk

Type I error



Type II error



What is the greatest probability of type I error you are willing to tolerate?

# Significance Level

What is the greatest probability of type I error you are willing to tolerate?

Type I error



# Significance Level

What is the greatest probability of type I error you are willing to tolerate?

Type I error



Innocent person determined guilty

# Significance Level

What is the greatest probability of type I error you are willing to tolerate?

Type I error



↓  
Significance level

Innocent person determined guilty

# Significance Level

What is the greatest probability of type I error you are willing to tolerate?

Type I error



Significance level ( $\alpha$ )

Innocent person determined guilty

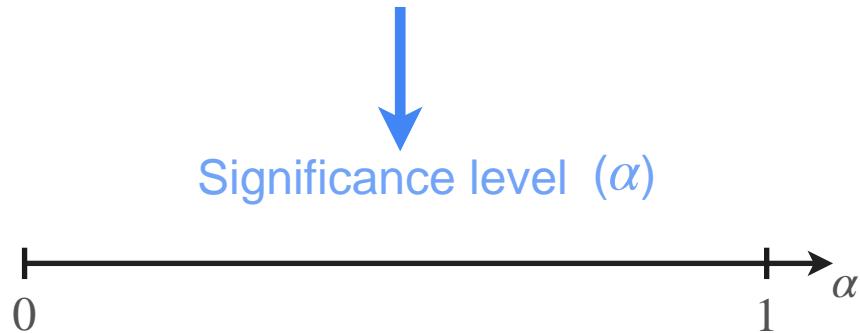
# Significance Level

What is the greatest probability of type I error you are willing to tolerate?

Type I error



Innocent person determined guilty



# Significance Level

What is the greatest probability of type I error you are willing to tolerate?

Type I error



Innocent person determined guilty



Significance level ( $\alpha$ )

0

1

Defendants are  
always considered →  
'not guilty'



No Type I error

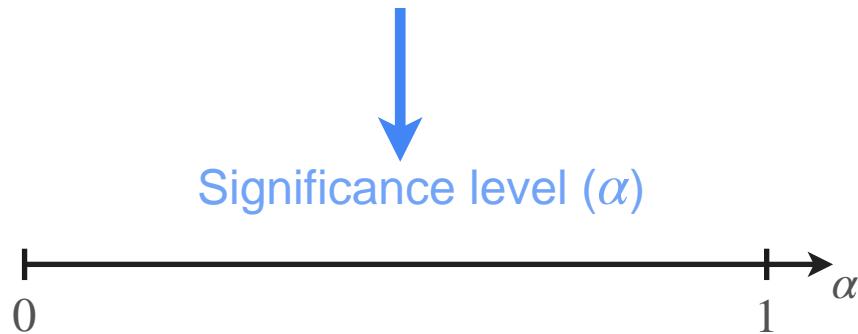
# Significance Level

What is the greatest probability of type I error you are willing to tolerate?

Type I error



Innocent person determined guilty



# Significance Level

What is the greatest probability of type I error you are willing to tolerate?

Type I error



Innocent person determined guilty you make a Type I error

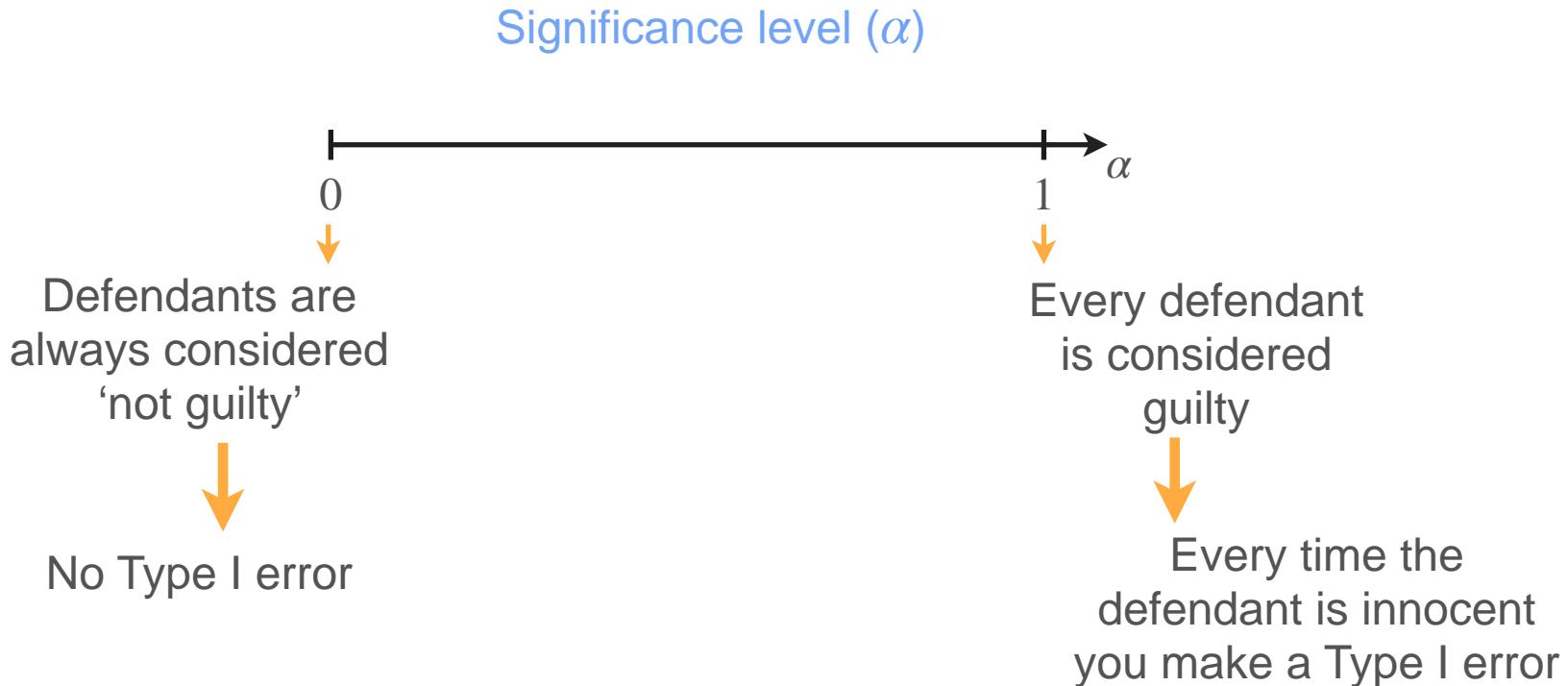


Significance level ( $\alpha$ )

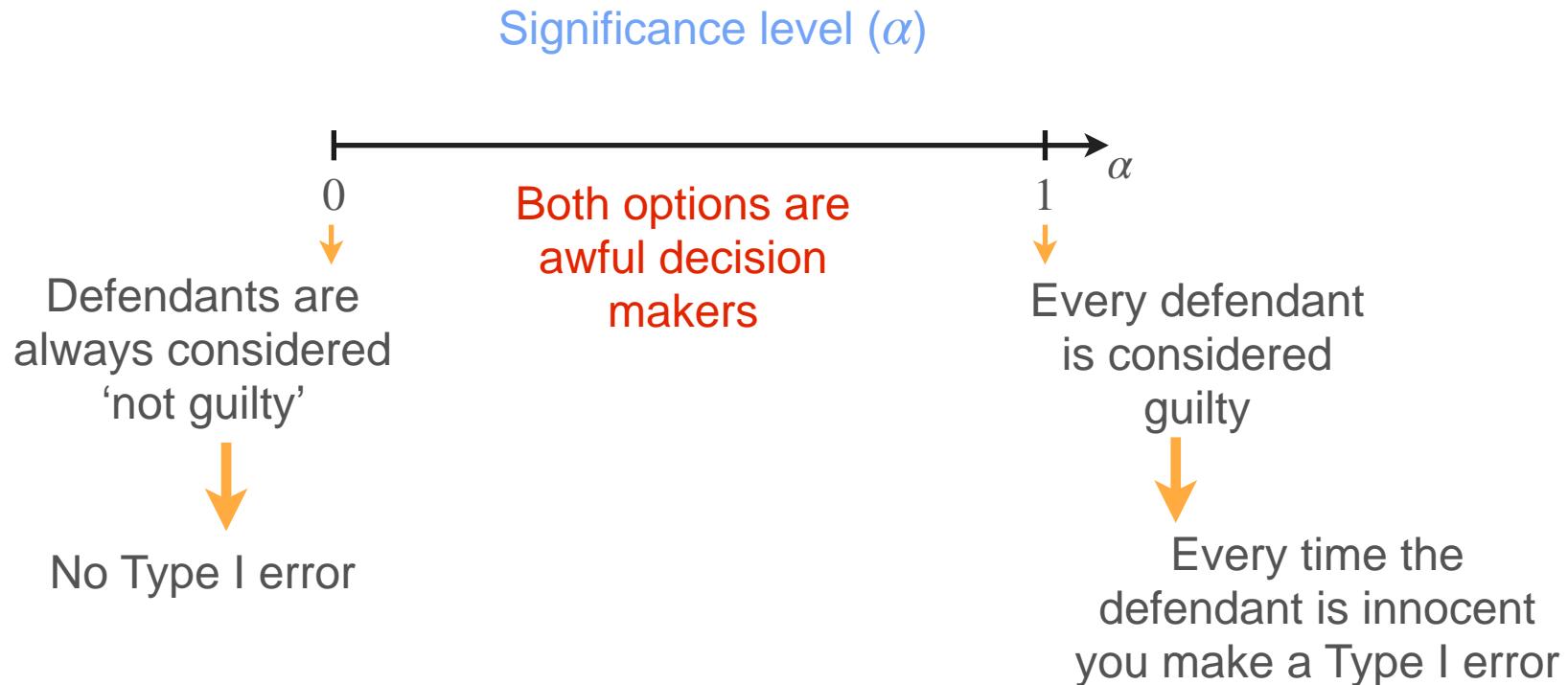
Every time the defendant is innocent

Every defendant is considered guilty

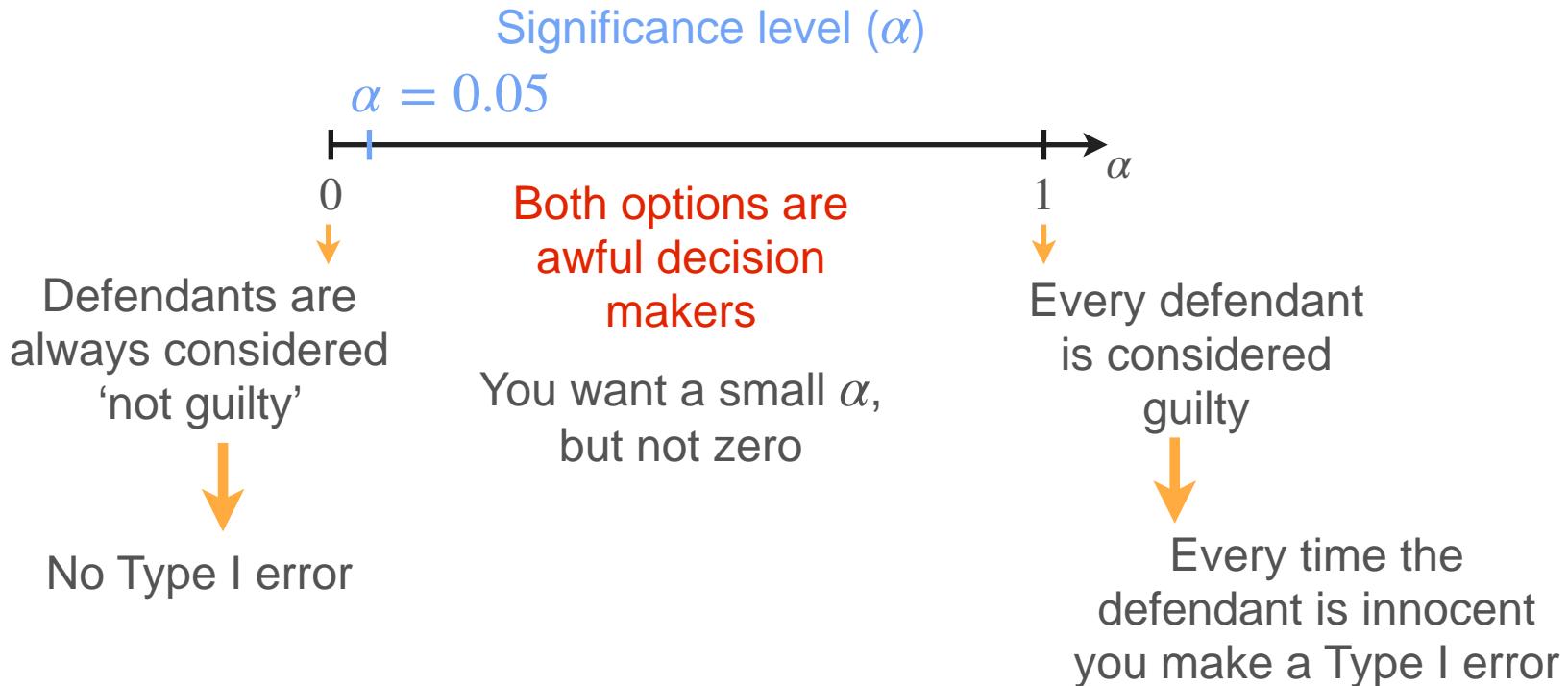
# Significance Level



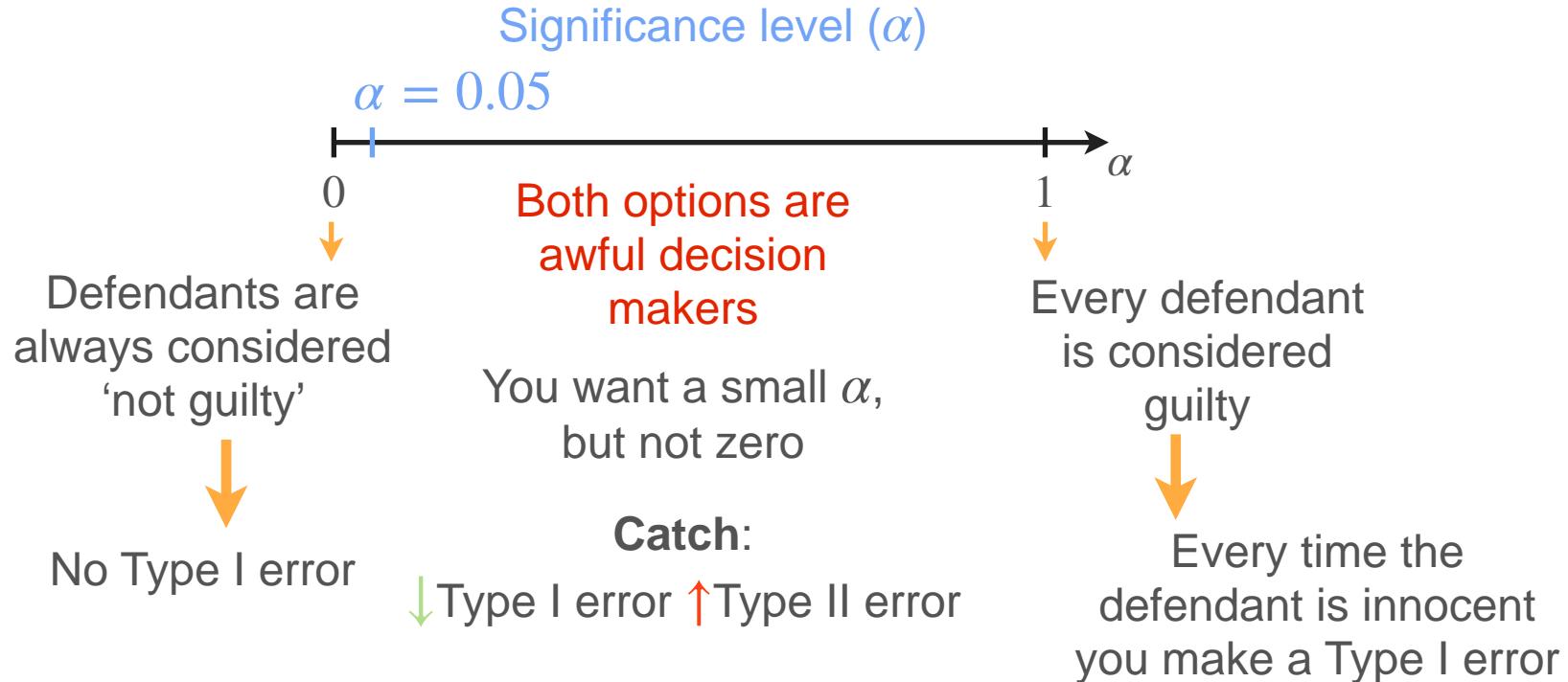
# Significance Level



# Significance Level



# Significance Level



# Significance Level

# Significance Level

Type I error



Innocent person determined guilty

# Significance Level

Type I error



$$\alpha = \max P(\text{Type I error})$$

Innocent person determined guilty

# Significance Level

Type I error



$$\begin{aligned}\alpha &= \max \mathbf{P}(\text{Type I error}) \\ &= \max \mathbf{P}(\text{Reject } H_0 | H_0)\end{aligned}$$

Innocent person determined guilty

# Significance Level

Type I error



Innocent person determined guilty

$$\begin{aligned}\alpha &= \max \mathbf{P}(\text{Type I error}) \\ &= \max \mathbf{P}(\text{Reject } H_0 | H_0)\end{aligned}$$

The value of  $\alpha$  is your criteria for designing your test

# Significance Level

Type I error



Innocent person determined guilty

$$\begin{aligned}\alpha &= \max \mathbf{P}(\text{Type I error}) \\ &= \max \mathbf{P}(\text{Reject } H_0 | H_0)\end{aligned}$$

The value of  $\alpha$  is your criteria for designing your test

For a given sample,  $\alpha$  will determine if you reject  $H_0$  or not



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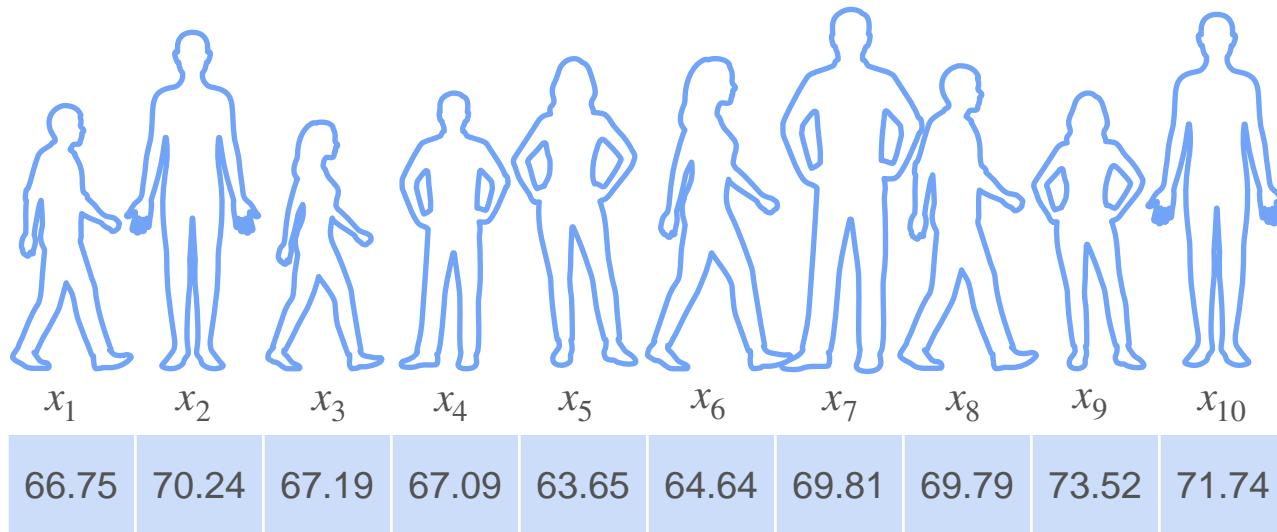
# Hypothesis Testing

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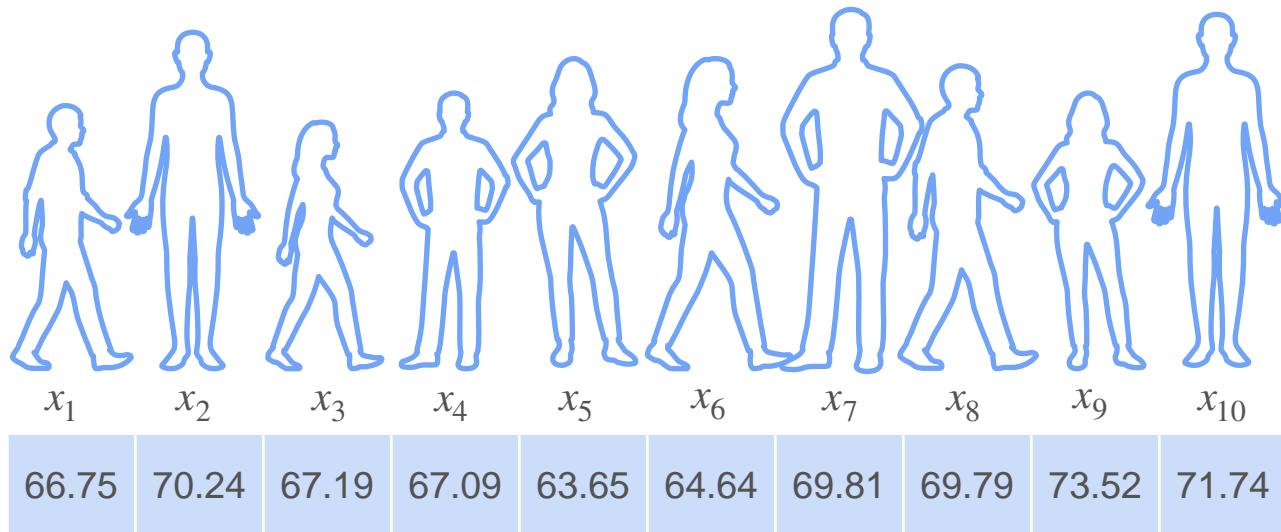
**Right-Tailed, Left-Tailed and  
Two-Tailed tests**

# Example: Heights

# Example: Heights

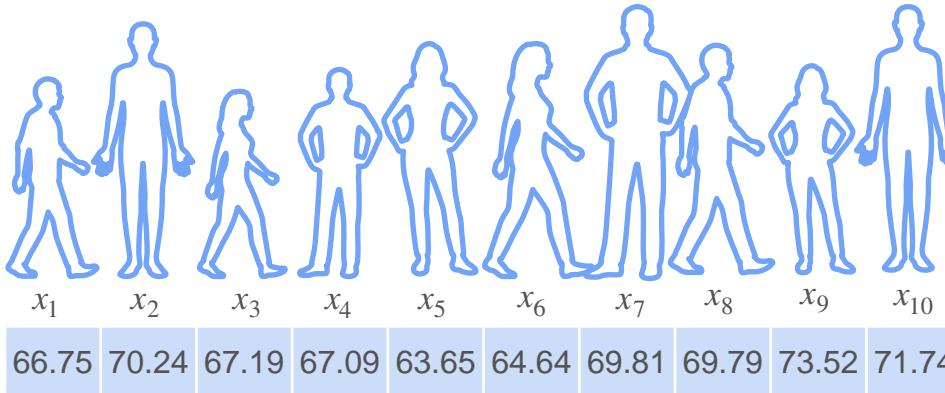


# Example: Heights



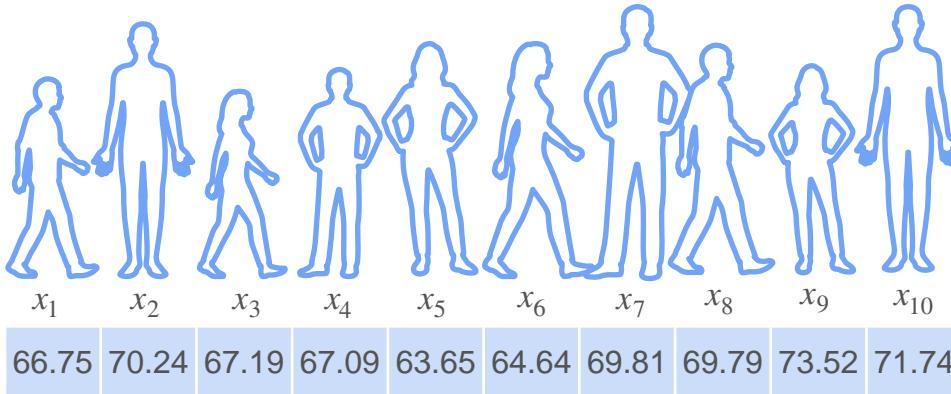
$$\bar{x} = 68.442$$

# Data Quality



$$\bar{x} = 68.442$$

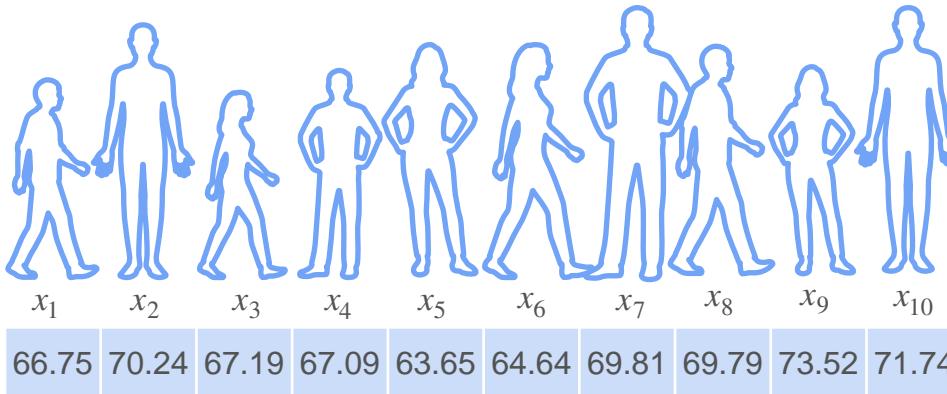
# Data Quality



$$\bar{x} = 68.442$$

**Reliable**

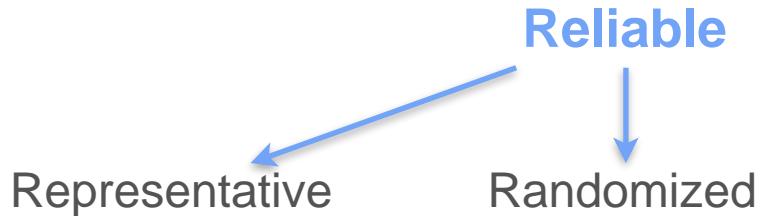
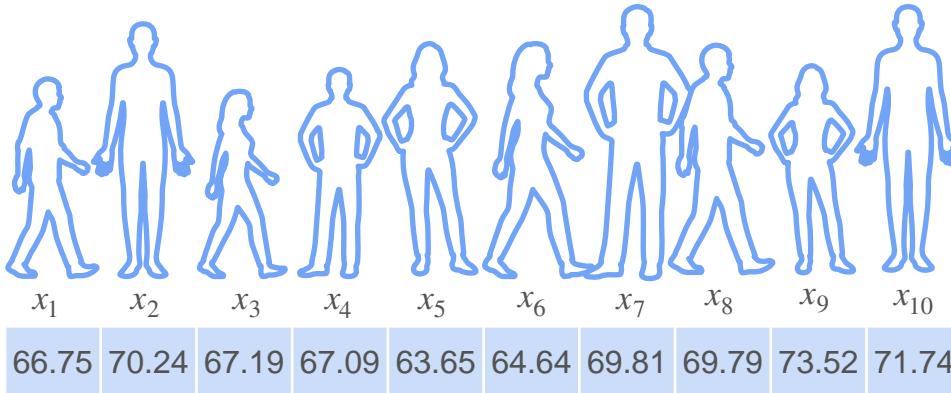
# Data Quality



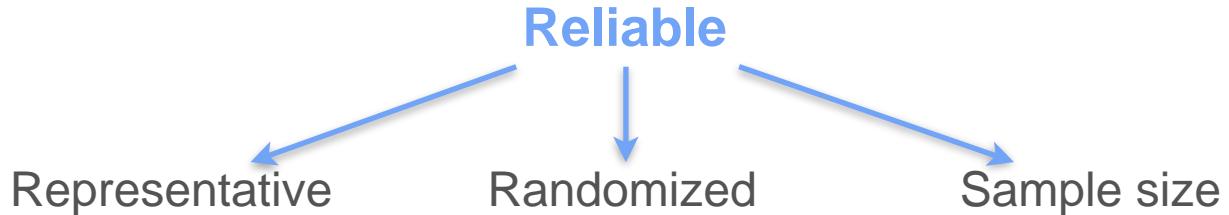
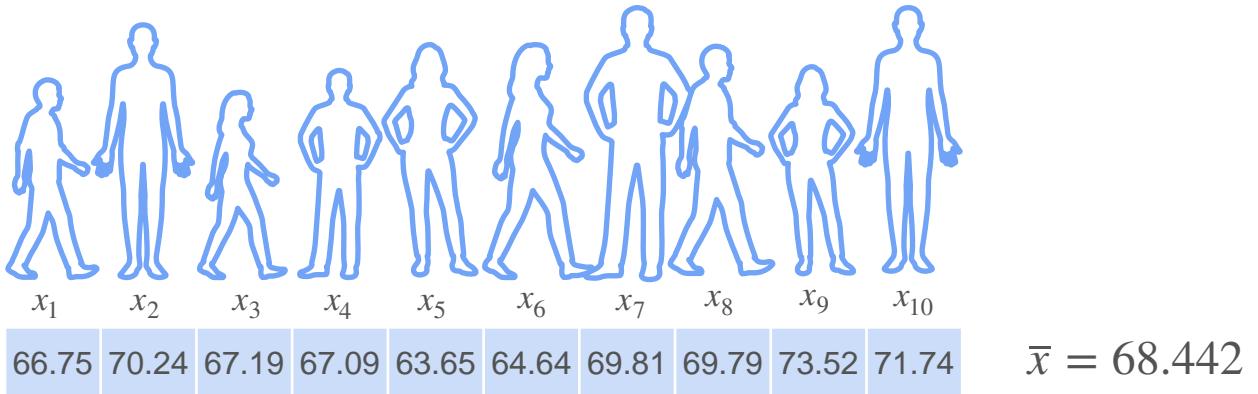
Reliable

Representative

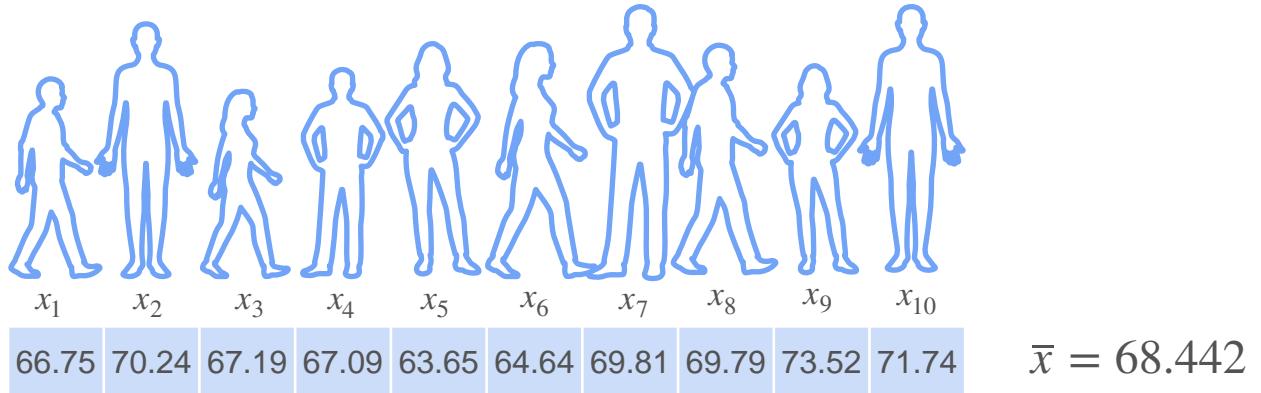
# Data Quality



# Data Quality

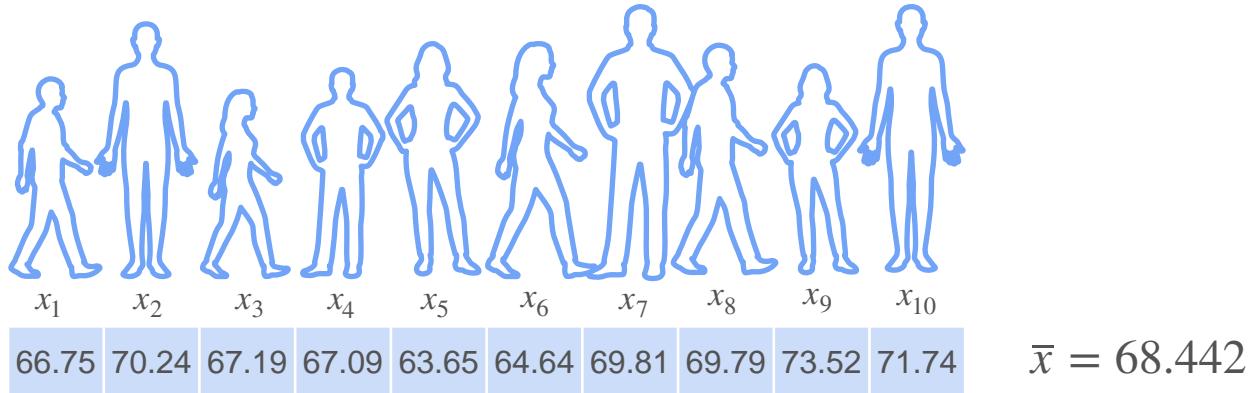


# Determining the Hypothesis



Population vs.  $H_1 : \mu > 66.7$

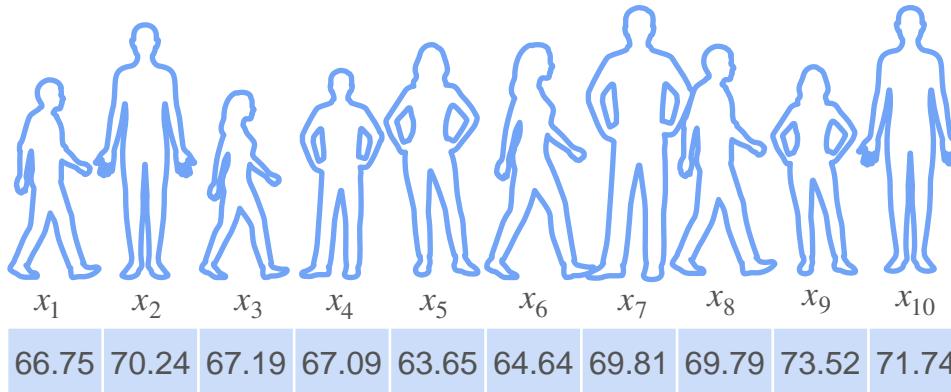
# Determining the Hypothesis



The **mean** height for 18 y/o in the US in the 70s was **66.7 in.**

Population  $H_0 : \mu = 66.7$  vs.  $H_1 : \mu > 66.7$

# Test Statistic



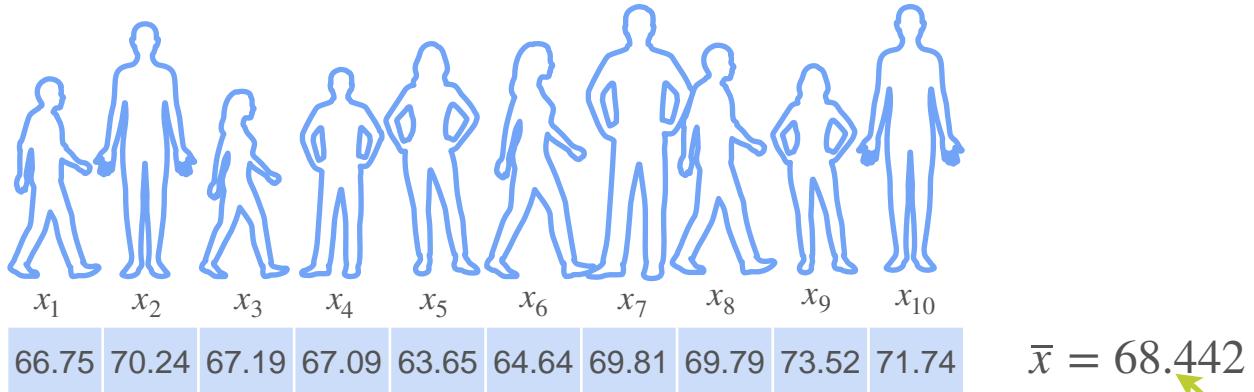
$$\bar{x} = 68.442$$

Observed statistic

$$\text{vs. } H_1 : \mu > 66.7$$

Test statistic  $\longrightarrow \bar{X} = \frac{1}{10} \sum_{i=1}^{10} X_i$

# Test Statistic



The **mean** height for 18 y/o in the US in the 70s was **66.7 in.**

$$H_0 : \mu = 66.7 \text{ vs. } H_1 : \mu > 66.7$$

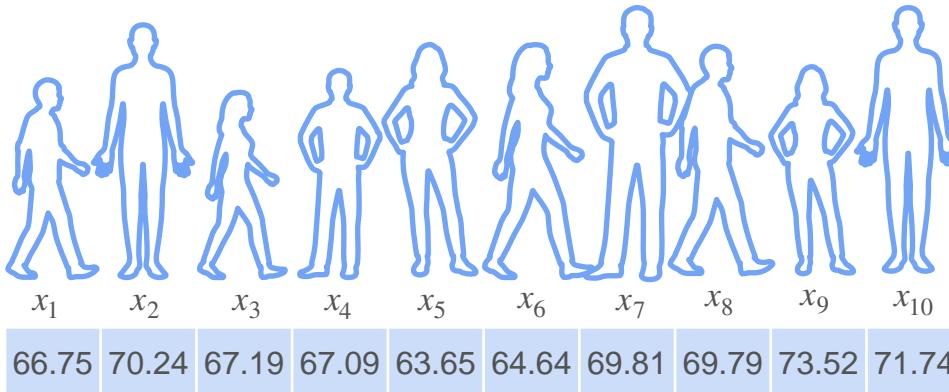
Observed statistic

Test statistic  $\longrightarrow \bar{X} = \frac{1}{10} \sum_{i=1}^{10} X_i$

# Test Statistic

Test statistic

$$\bar{X} = \frac{1}{10} \sum_{i=1}^{10} X_i$$

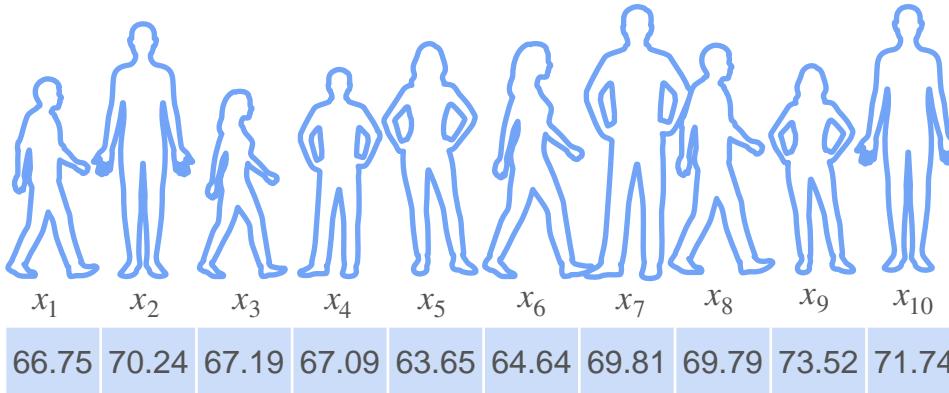


$$\bar{x} = 68.442$$

# Test Statistic

Test statistic

$$\bar{X} = \frac{1}{10} \sum_{i=1}^{10} X_i$$



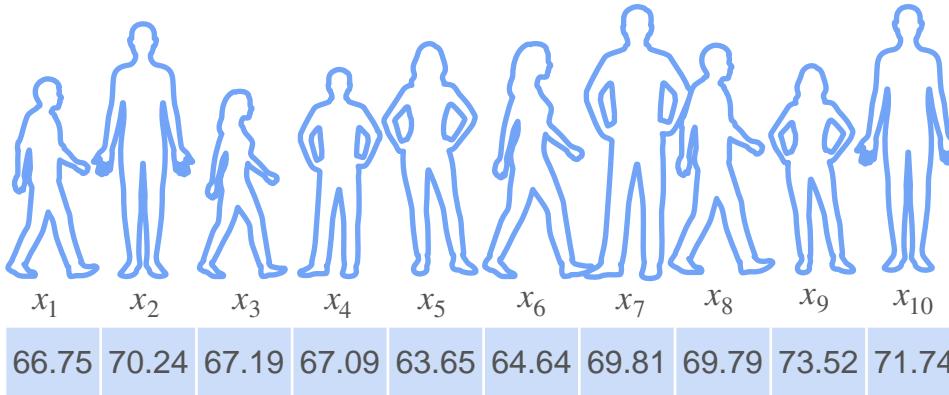
$$\bar{x} = 68.442$$

Test statistic:  $T(X)$      $X = (X_1, \dots, X_n)$

# Test Statistic

Test statistic

$$\bar{X} = \frac{1}{10} \sum_{i=1}^{10} X_i$$



$$\bar{x} = 68.442$$

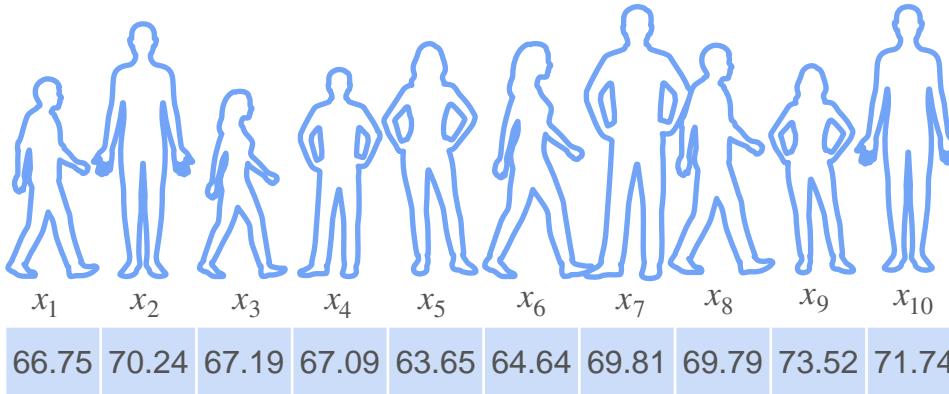
Test statistic:  $T(X)$     $X = (X_1, \dots, X_n)$

Information about the population parameter under study

# Test Statistic

Test statistic

$$\bar{X} = \frac{1}{10} \sum_{i=1}^{10} X_i$$



$$\bar{x} = 68.442$$

Test statistic:  $T(X)$     $X = (X_1, \dots, X_n)$

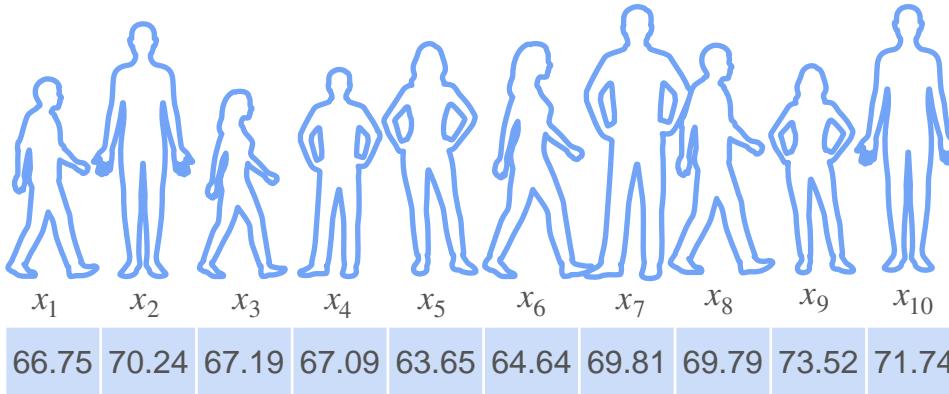
Information about the population parameter under study

$$\mu \rightarrow \bar{X}$$

# Test Statistic

Test statistic

$$\bar{X} = \frac{1}{10} \sum_{i=1}^{10} X_i$$



$$\bar{x} = 68.442$$

Test statistic:  $T(X)$      $X = (X_1, \dots, X_n)$

Information about the population parameter under study

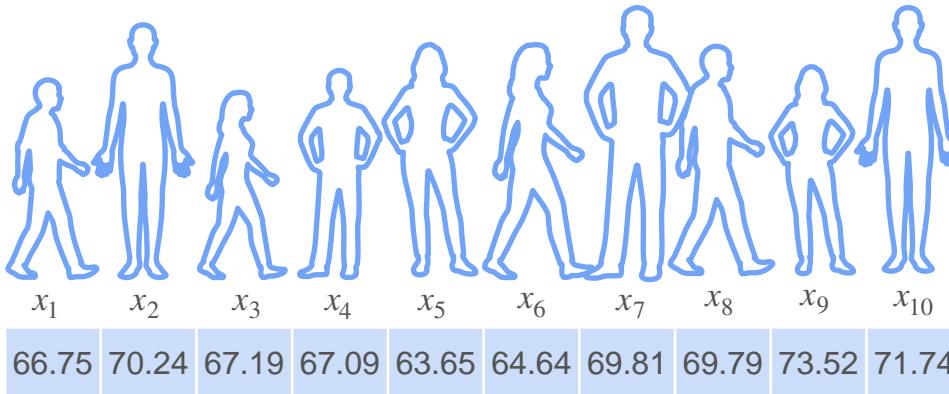
$$\mu \rightarrow \bar{X}$$

$$p \rightarrow \bar{X}$$

# Test Statistic

Test statistic

$$\bar{X} = \frac{1}{10} \sum_{i=1}^{10} X_i$$



$$\bar{x} = 68.442$$

Test statistic:  $T(X) \quad X = (X_1, \dots, X_n)$

Information about the population parameter under study

$$\mu \rightarrow \bar{X}$$

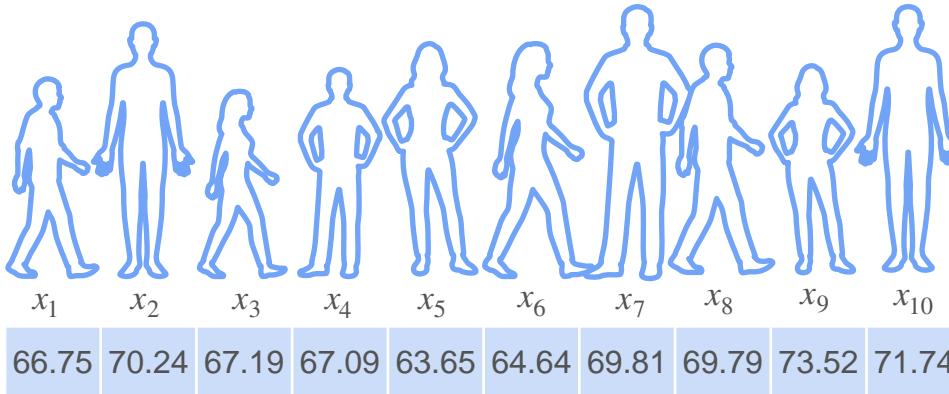
$$p \rightarrow \bar{X}$$

$$\sigma^2 \rightarrow S^2 = \frac{1}{n-1} \sum_{i=1}^n (X_i - \bar{X})^2$$

# Test Statistic

Test statistic

$$\bar{X} = \frac{1}{10} \sum_{i=1}^{10} X_i$$



$$\bar{x} = 68.442$$

Test statistic:  $T(X)$      $X = (X_1, \dots, X_n)$

Information about the population parameter under study

$$\mu \rightarrow \bar{X}$$

$$p \rightarrow \bar{X}$$

Not unique!

$$\sigma^2 \rightarrow S^2 = \frac{1}{n-1} \sum_{i=1}^n (X_i - \bar{X})^2$$

# Example: Heights

# Example: Heights

The **mean** height for 18 y/o in the US in the 70s was **66.7 in.**

# Example: Heights

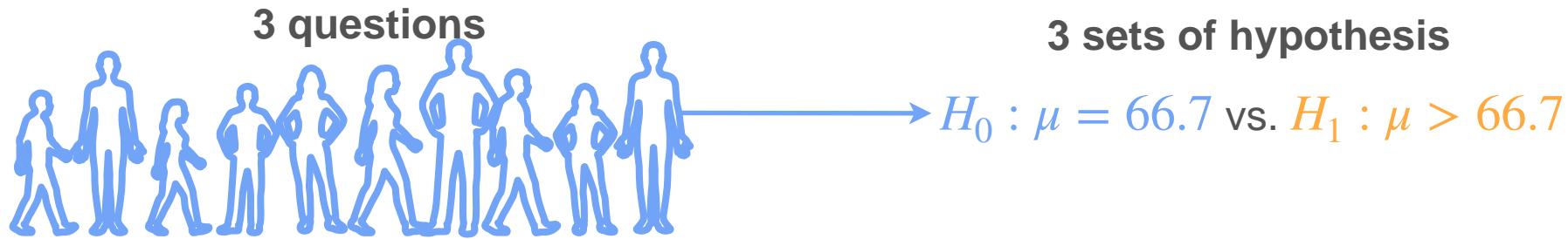
The **mean** height for 18 y/o in the US in the 70s was **66.7 in.**

**3 questions**

**3 sets of hypothesis**

# Example: Heights

The **mean** height for 18 y/o in the US in the 70s was **66.7 in.**



# Example: Heights

The **mean** height for 18 y/o in the US in the 70s was **66.7 in.**

**3 questions**

Right-Tailed Test

**3 sets of hypothesis**

$$\longrightarrow H_0 : \mu = 66.7 \text{ vs. } H_1 : \mu > 66.7$$

# Example: Heights

The **mean** height for 18 y/o in the US in the 70s was **66.7 in.**

**3 questions**

Right-Tailed Test

**3 sets of hypothesis**

$$\longrightarrow H_0 : \mu = 66.7 \text{ vs. } H_1 : \mu > 66.7$$



$$\longrightarrow H_0 : \mu = 66.7 \text{ vs. } H_1 : \mu < 66.7$$

# Example: Heights

The **mean** height for 18 y/o in the US in the 70s was **66.7 in.**

**3 questions**

Right-Tailed Test

**3 sets of hypothesis**

$$\longrightarrow H_0 : \mu = 66.7 \text{ vs. } H_1 : \mu > 66.7$$

Left Tailed Test

$$\longrightarrow H_0 : \mu = 66.7 \text{ vs. } H_1 : \mu < 66.7$$

# Example: Heights

The **mean** height for 18 y/o in the US in the 70s was **66.7 in.**

**3 questions**

Right-Tailed Test

**3 sets of hypothesis**

$$H_0 : \mu = 66.7 \text{ vs. } H_1 : \mu > 66.7$$

Left Tailed Test

$$H_0 : \mu = 66.7 \text{ vs. } H_1 : \mu < 66.7$$



$$H_0 : \mu = 66.7 \text{ vs. } H_1 : \mu \neq 66.7$$

# Example: Heights

The **mean** height for 18 y/o in the US in the 70s was **66.7 in.**

**3 questions**

Right-Tailed Test

**3 sets of hypothesis**

$$H_0 : \mu = 66.7 \text{ vs. } H_1 : \mu > 66.7$$

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$$H_0 : \mu = 66.7 \text{ vs. } H_1 : \mu < 66.7$$

Two-Tailed Test

$$H_0 : \mu = 66.7 \text{ vs. } H_1 : \mu \neq 66.7$$

# Example: Heights

The **mean** height for 18 y/o in the US in the 70s was **66.7 in.**

$$H_0 : \mu = 66.7 \text{ vs. } H_1 : \mu > 66.7$$

# Example: Heights

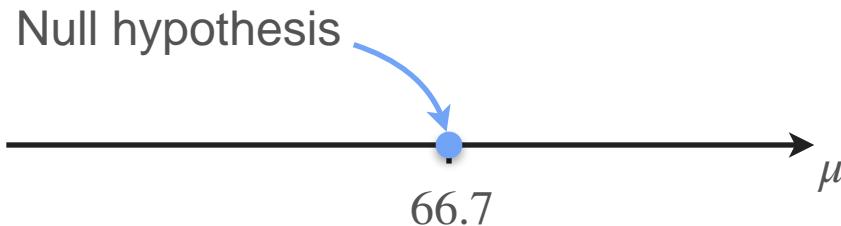
The **mean** height for 18 y/o in the US in the 70s was **66.7 in.**

Right-tailed test   $H_0 : \mu = 66.7$  vs.  $H_1 : \mu > 66.7$

# Example: Heights

The **mean** height for 18 y/o in the US in the 70s was **66.7 in.**

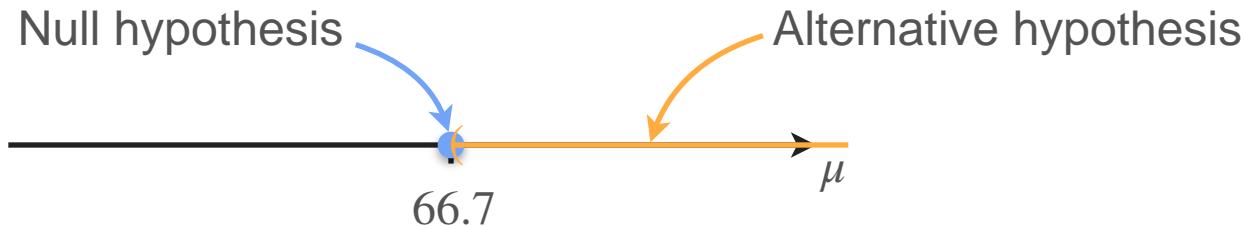
Right-tailed test  $\longrightarrow H_0 : \mu = 66.7$  vs.  $H_1 : \mu > 66.7$



# Example: Heights

The **mean** height for 18 y/o in the US in the 70s was **66.7 in.**

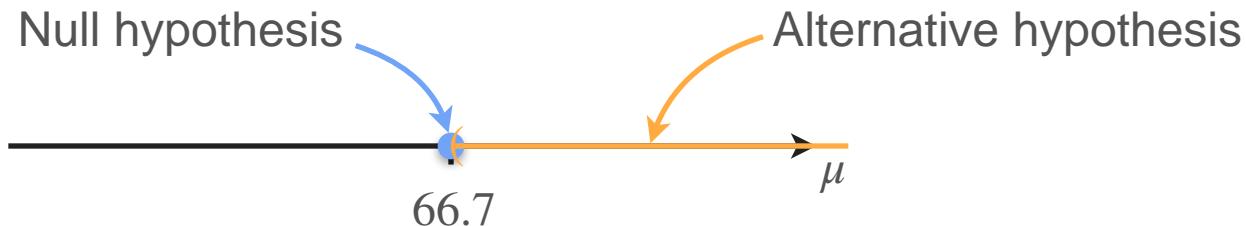
Right-tailed test  $\longrightarrow H_0 : \mu = 66.7$  vs.  $H_1 : \mu > 66.7$



# Example: Heights

The **mean** height for 18 y/o in the US in the 70s was **66.7 in.**

Right-tailed test  $\longrightarrow H_0 : \mu \leq 66.7$  vs.  $H_1 : \mu > 66.7$

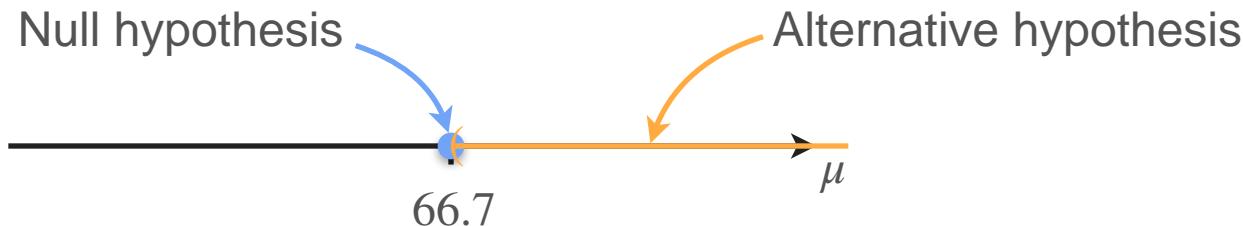


# Example: Heights

The **mean** height for 18 y/o in the US in the 70s was **66.7 in.**

$\bar{X}$  Test statistic

Right-tailed test  $\longrightarrow H_0 : \mu = 66.7$  vs.  $H_1 : \mu > 66.7$

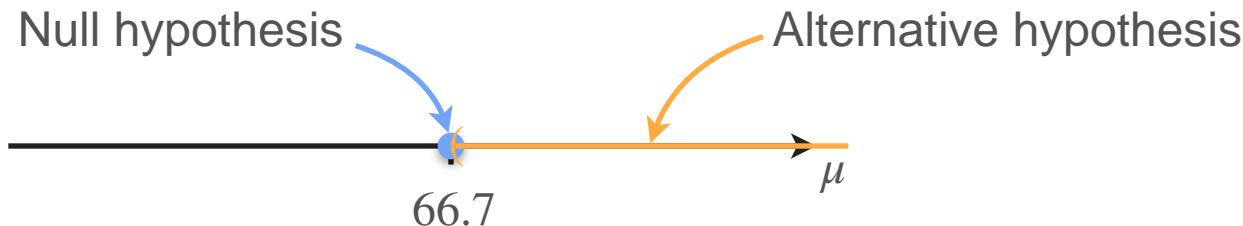


# Example: Heights

The **mean** height for 18 y/o in the US in the 70s was **66.7 in.**

$\bar{X}$  Test statistic

Right-tailed test  $\longrightarrow H_0 : \mu = 66.7$  vs.  $H_1 : \mu > 66.7$



If  $\bar{x} \gg 66.7 \Rightarrow$  Reject  $H_0$

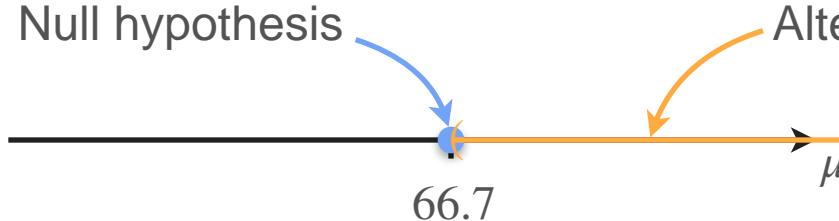
# Example: Heights

The mean height for 18 y/o in the US in the 70s was 66.7 in.

$\bar{X}$  Test statistic

Right-tailed test  $\longrightarrow H_0 : \mu = 66.7$  vs.  $H_1 : \mu > 66.7$

Null hypothesis



Type I error: Determine  $\mu > 66.7$ , when population mean did not change

If  $\bar{x} \gg 66.7 \Rightarrow$  Reject  $H_0$

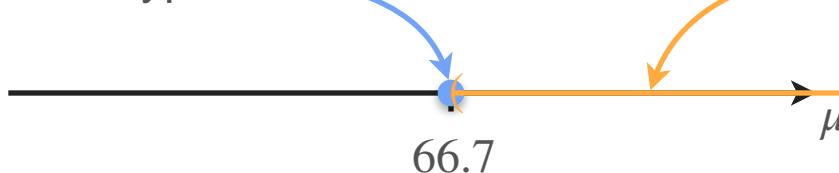
# Example: Heights

The mean height for 18 y/o in the US in the 70s was **66.7 in.**

$\bar{X}$  Test statistic

Right-tailed test  $\longrightarrow H_0 : \mu = 66.7$  vs.  $H_1 : \mu > 66.7$

Null hypothesis      Alternative hypothesis



Type I error: Determine  $\mu > 66.7$ , when population mean did not change

If  $\bar{x} \gg 66.7 \Rightarrow$  Reject  $H_0$

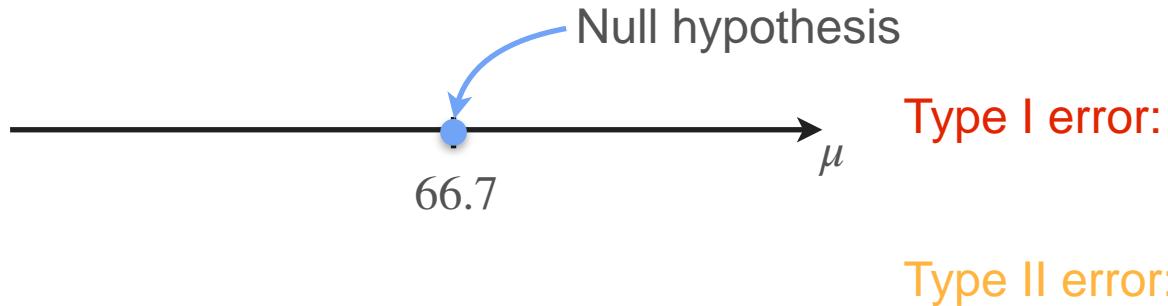
Type II error: Do not reject that  $\mu = 66.7$  when in true  $\mu > 66.7$

# Example: Heights

The **mean** height for 18 y/o in the US in the 70s was **66.7 in.**

$$\bar{X}$$

$$H_0 : \mu = 66.7 \text{ vs. } H_1 : \mu < 66.7$$



# Example: Heights

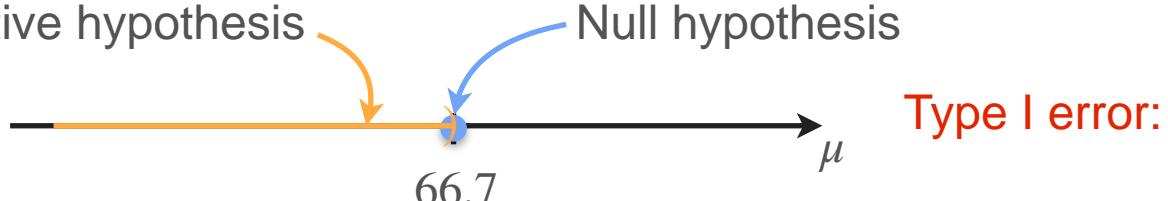
The mean height for 18 y/o in the US in the 70s was **66.7 in.**

$$\bar{X}$$

Left tailed test  $\longrightarrow H_0 : \mu = 66.7$  vs.  $H_1 : \mu < 66.7$

Alternative hypothesis

Null hypothesis



Type II error:

# Example: Heights

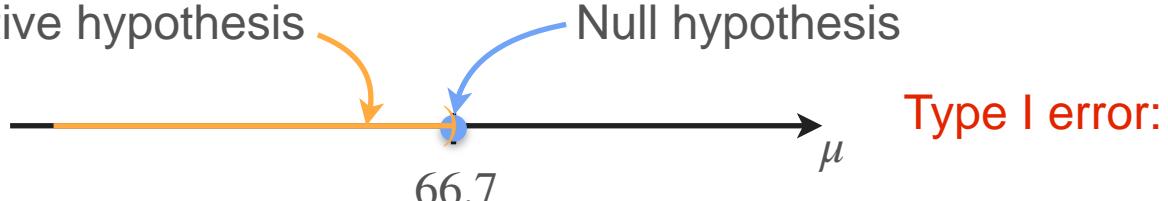
The mean height for 18 y/o in the US in the 70s was **66.7 in.**

$$\bar{X}$$

Left tailed test  $\longrightarrow H_0 : \mu \geq 66.7$  vs.  $H_1 : \mu < 66.7$

Alternative hypothesis

Null hypothesis



Type II error:

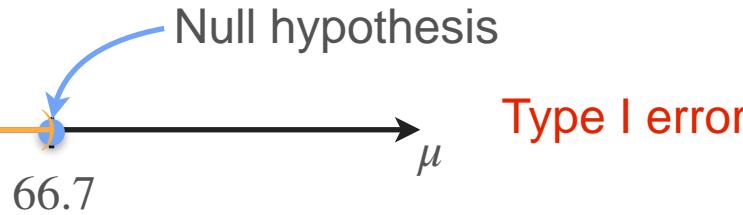
# Example: Heights

The mean height for 18 y/o in the US in the 70s was **66.7 in.**

$$\bar{X}$$

Left tailed test  $\longrightarrow H_0 : \mu = 66.7$  vs.  $H_1 : \mu < 66.7$

Alternative hypothesis



Type I error:

If  $\bar{x} \ll 66.7 \Rightarrow$  Reject  $H_0$

Type II error:

# Example: Heights

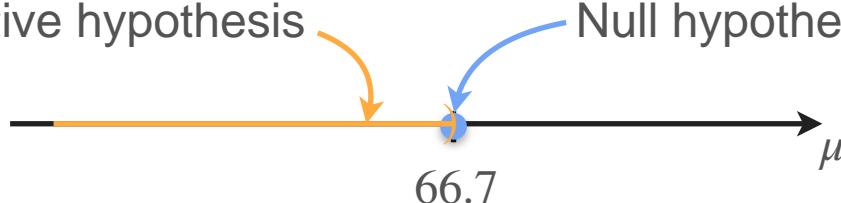
The mean height for 18 y/o in the US in the 70s was **66.7 in.**

$$\bar{X}$$

Left tailed test  $\longrightarrow H_0 : \mu = 66.7$  vs.  $H_1 : \mu < 66.7$

Alternative hypothesis

Null hypothesis



**Type I error:** Determine  $\mu < 66.7$ , when population mean did not change

If  $\bar{x} \ll 66.7 \Rightarrow$  Reject  $H_0$

**Type II error:**

# Example: Heights

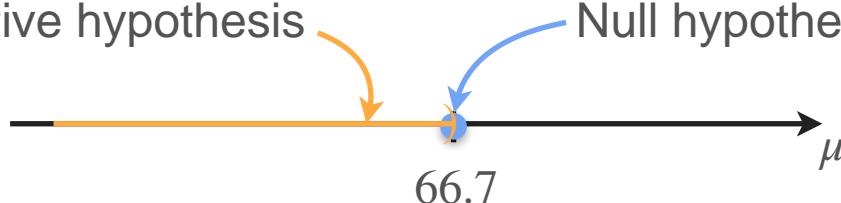
The mean height for 18 y/o in the US in the 70s was 66.7 in.

$$\bar{X}$$

Left tailed test  $\longrightarrow H_0 : \mu = 66.7$  vs.  $H_1 : \mu < 66.7$

Alternative hypothesis

Null hypothesis



If  $\bar{x} \ll 66.7 \Rightarrow$  Reject  $H_0$

Type I error: Determine  $\mu < 66.7$ , when population mean did not change

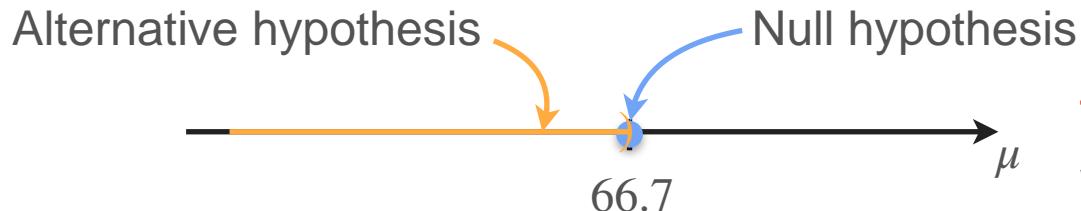
Type II error: Don't reject that  $\mu = 66.7$  when true  $\mu < 66.7$

# Example: Heights

The mean height for 18 y/o in the US in the 70s was 66.7 in.

$\bar{X}$  Test statistic

Left tailed test  $\longrightarrow H_0 : \mu = 66.7$  vs.  $H_1 : \mu < 66.7$



If  $\bar{x} \ll 66.7 \Rightarrow$  Reject  $H_0$

Type I error: Determine  $\mu < 66.7$ , when population mean did not change

Type II error: Don't reject that  $\mu = 66.7$  when true  $\mu < 66.7$

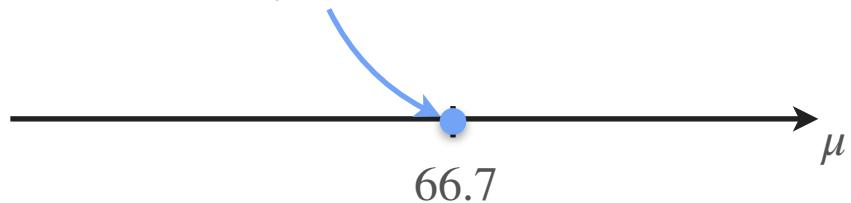
# Example: Heights

The **mean** height for 18 y/o in the US in the 70s was **66.7 in.**

$$\bar{X}$$

$$H_0 : \mu = 66.7 \text{ vs. } H_1 : \mu \neq 66.7$$

Null hypothesis



Type I error:

Type II error:

# Example: Heights

The mean height for 18 y/o in the US in the 70s was **66.7 in.**

$$\bar{X}$$

Two tailed test  $\longrightarrow H_0 : \mu = 66.7$  vs.  $H_1 : \mu \neq 66.7$



# Example: Heights

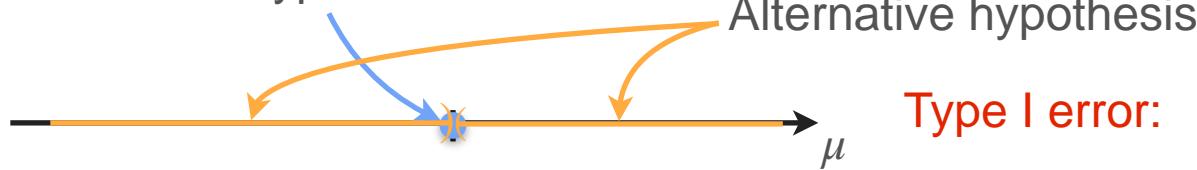
The mean height for 18 y/o in the US in the 70s was **66.7 in.**

$$\bar{X}$$

Two tailed test  $\longrightarrow H_0 : \mu = 66.7$  vs.  $H_1 : \mu \neq 66.7$

Null hypothesis

Alternative hypothesis



$$\bar{x} \gg 66.7$$

If or  $\Rightarrow$  Reject  $H_0$

$$\bar{x} \ll 66.7$$

Type I error:

Type II error:

# Example: Heights

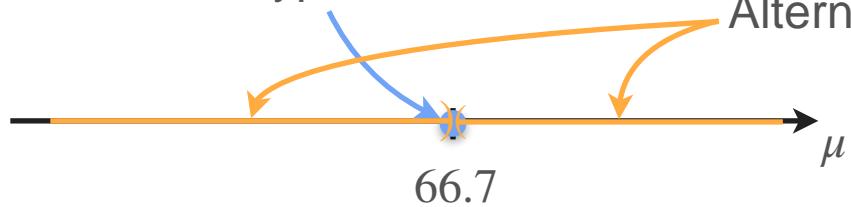
The mean height for 18 y/o in the US in the 70s was **66.7 in.**

$$\bar{X}$$

Two tailed test  $\longrightarrow H_0 : \mu = 66.7$  vs.  $H_1 : \mu \neq 66.7$

Null hypothesis

Alternative hypothesis



If    or       $\Rightarrow$  Reject  $H_0$

$\bar{x} \gg 66.7$

$\bar{x} \ll 66.7$

Type I error: Determine  $\mu \neq 66.7$ , when population mean did not change

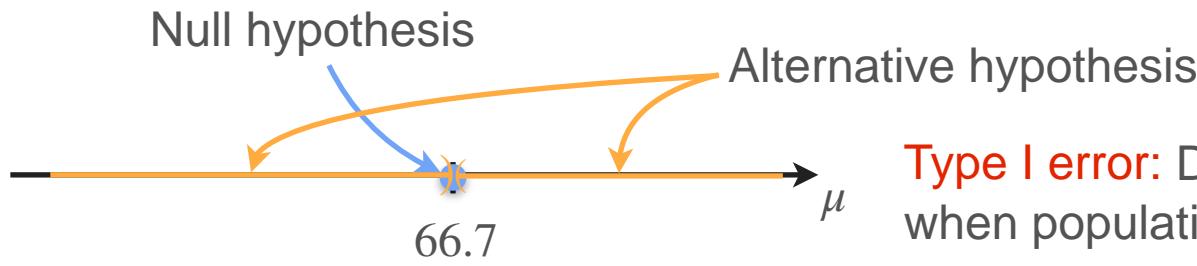
Type II error:

# Example: Heights

The mean height for 18 y/o in the US in the 70s was **66.7 in.**

$$\bar{X}$$

Two tailed test  $\longrightarrow H_0 : \mu = 66.7$  vs.  $H_1 : \mu \neq 66.7$



If  $\bar{x} \gg 66.7$  or  $\bar{x} \ll 66.7 \Rightarrow \text{Reject } H_0$

**Type I error:** Determine  $\mu \neq 66.7$ , when population mean did not change

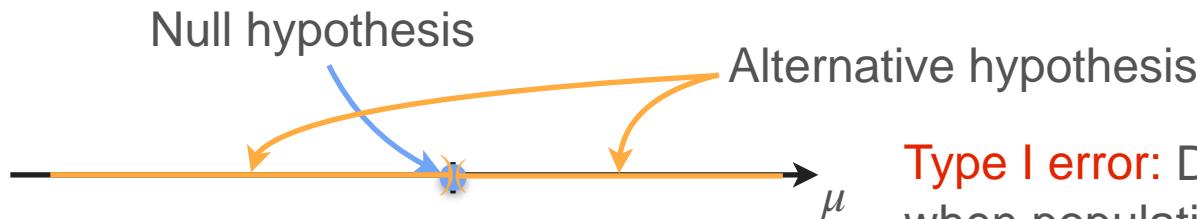
**Type II error:** Don't reject that  $\mu = 66.7$  when true  $\mu \neq 66.7$

# Example: Heights

The mean height for 18 y/o in the US in the 70s was 66.7 in.

$\bar{X}$  Test statistic

Two tailed test  $\longrightarrow H_0 : \mu = 66.7$  vs.  $H_1 : \mu \neq 66.7$



If  $\bar{x} \gg 66.7$  or  $\bar{x} \ll 66.7 \Rightarrow \text{Reject } H_0$

Type I error: Determine  $\mu \neq 66.7$ , when population mean did not change

Type II error: Don't reject that  $\mu = 66.7$  when true  $\mu \neq 66.7$



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# Hypothesis Testing

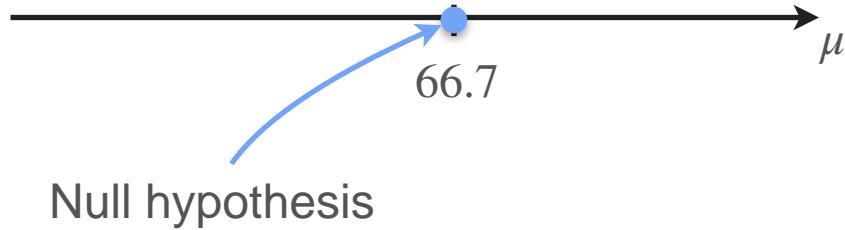
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**$p$ -Value**

# Example: Heights

# Example: Heights

The **mean** height for 18 y/o in the US in the 70s was **66.7 in.**

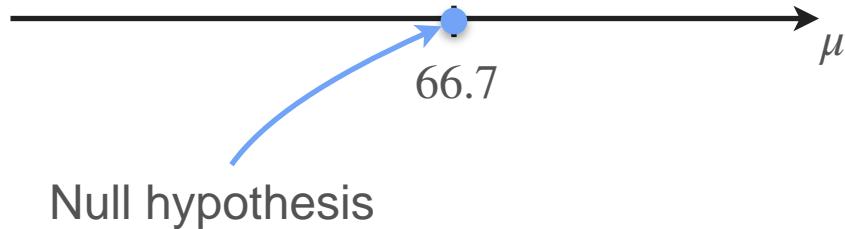


# Example: Heights

The **mean** height for 18 y/o in the US in the 70s was **66.7 in.**

$$\sigma = 3$$

$$n = 10$$



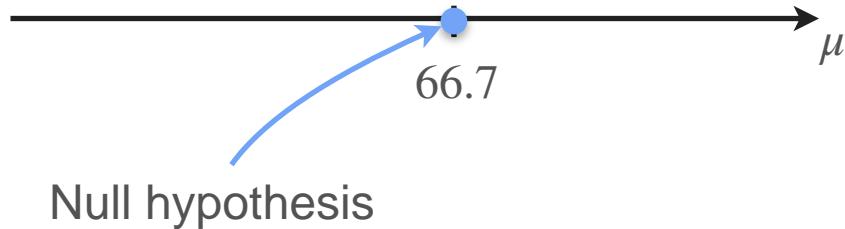
# Example: Heights

The **mean** height for 18 y/o in the US in the 70s was **66.7 in.**

$$\sigma = 3$$

$$n = 10$$

If  $H_0$  is true:

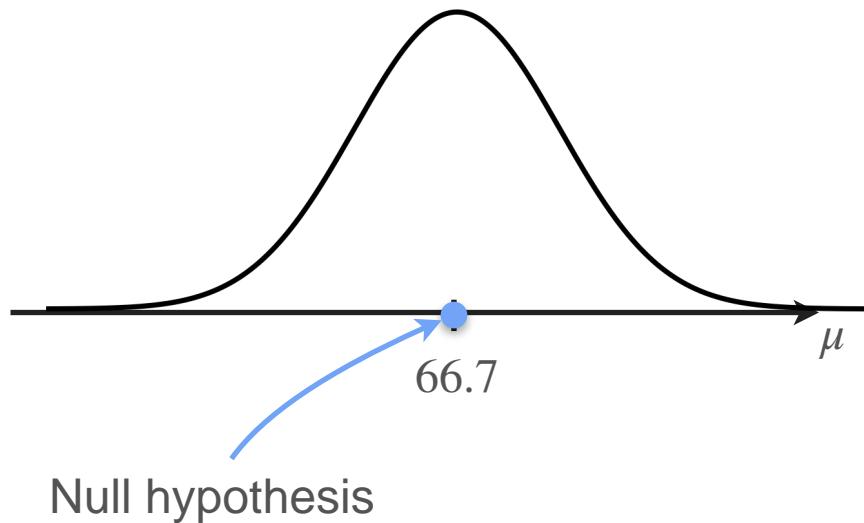


# Example: Heights

The mean height for 18 y/o in the US in the 70s was **66.7 in.**

$$\sigma = 3$$
$$n = 10$$

If  $H_0$  is true:  $\bar{X} \sim \mathcal{N}\left( \quad, \quad \right)$

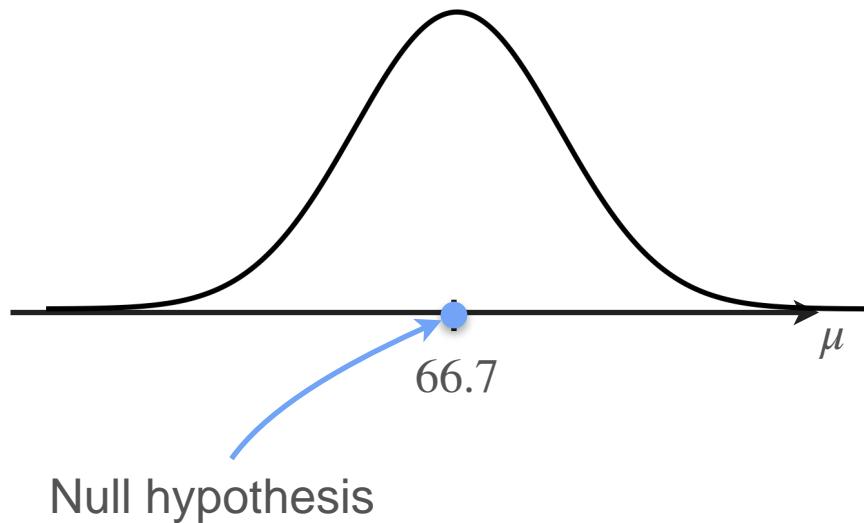


# Example: Heights

The mean height for 18 y/o in the US in the 70s was **66.7** in.

$$\begin{aligned}\sigma &= 3 \\ n &= 10\end{aligned}$$

If  $H_0$  is true:  $\bar{X} \sim \mathcal{N}\left(66.7, \frac{3^2}{10}\right)$

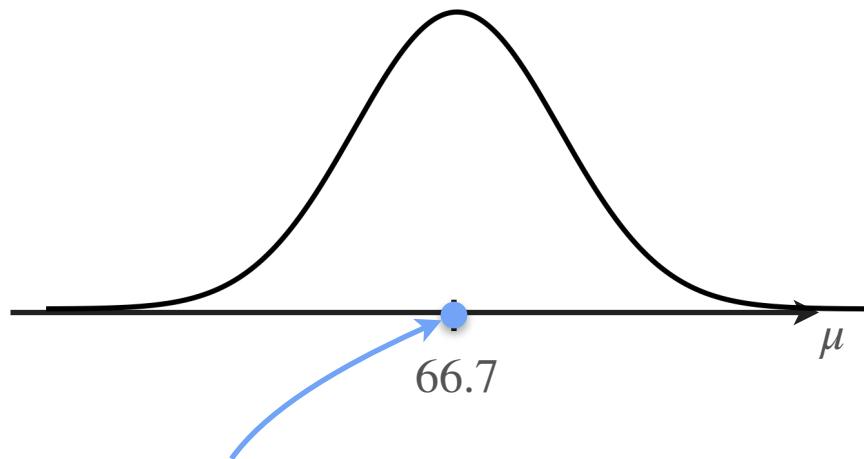


# Example: Heights

The mean height for 18 y/o in the US in the 70s was **66.7** in.

$$\begin{aligned}\sigma &= 3 \\ n &= 10\end{aligned}$$

If  $H_0$  is true:  $\bar{X} \sim \mathcal{N}\left(66.7, \frac{3^2}{10}\right)$



How likely was your sample if  $H_0$  is true?

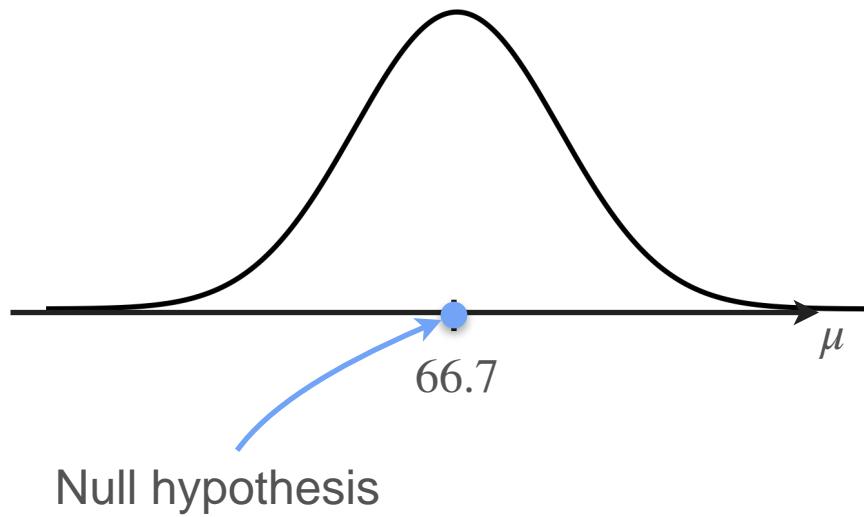
If the answer is very unlikely, then reject  $H_0$

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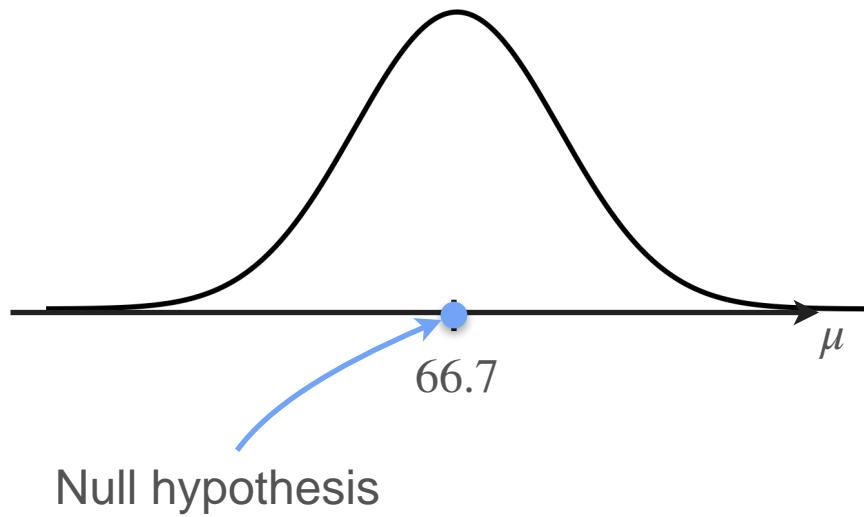
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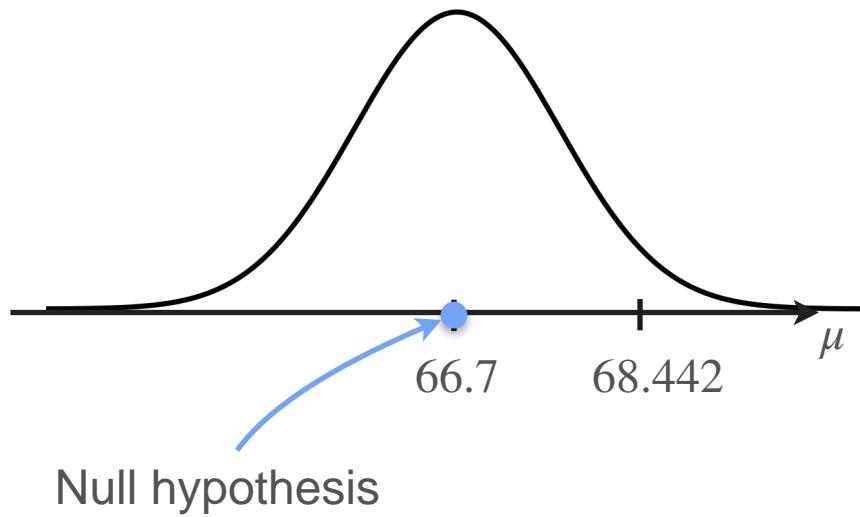
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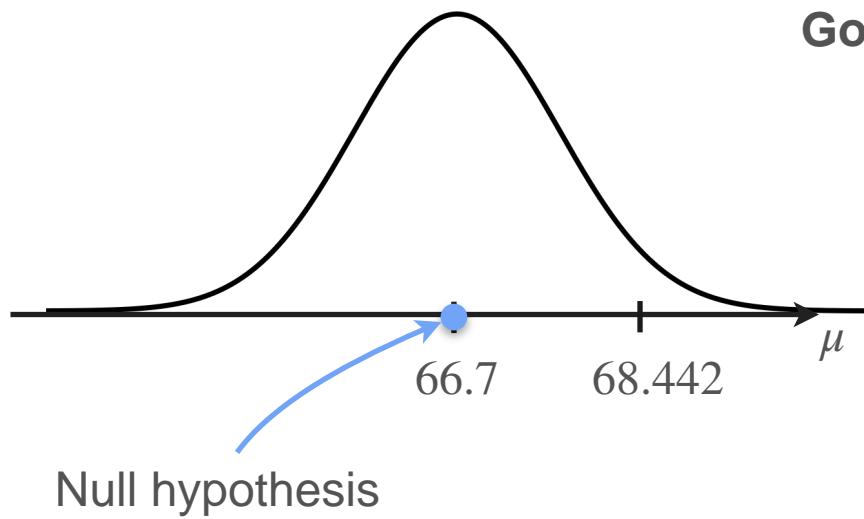
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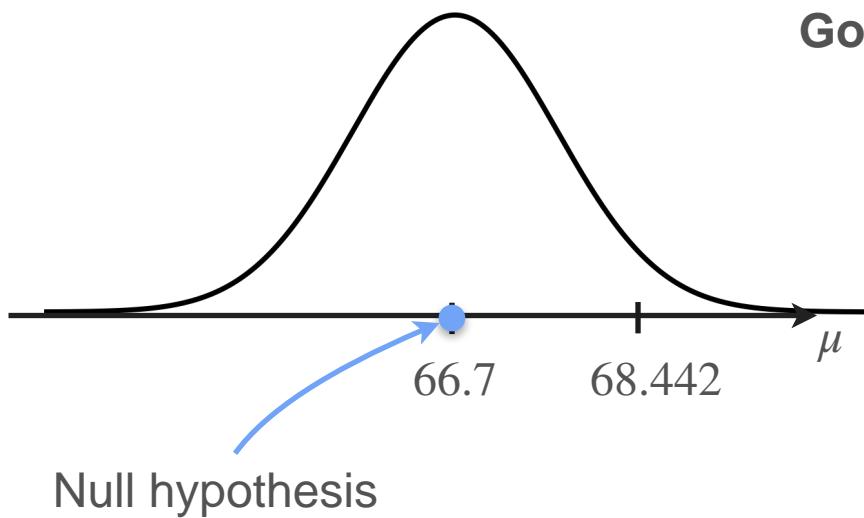
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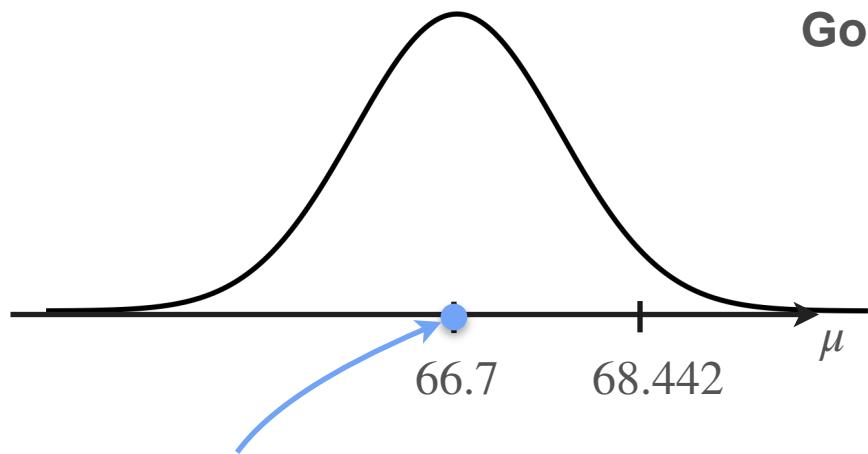
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Null hypothesis

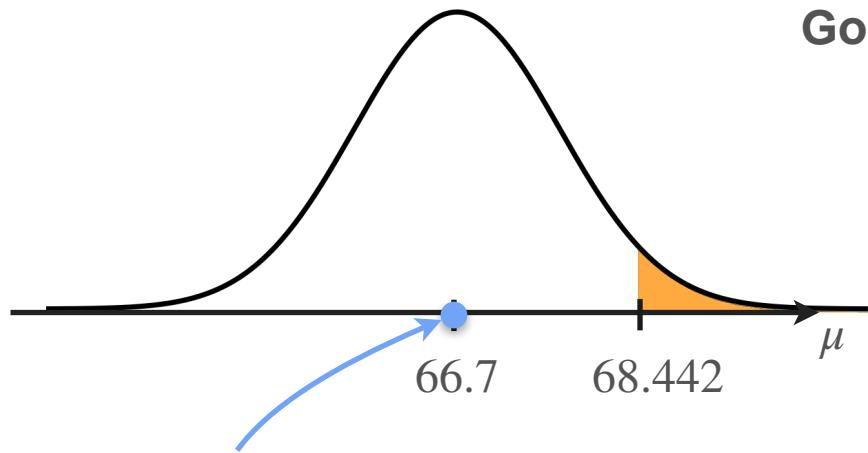
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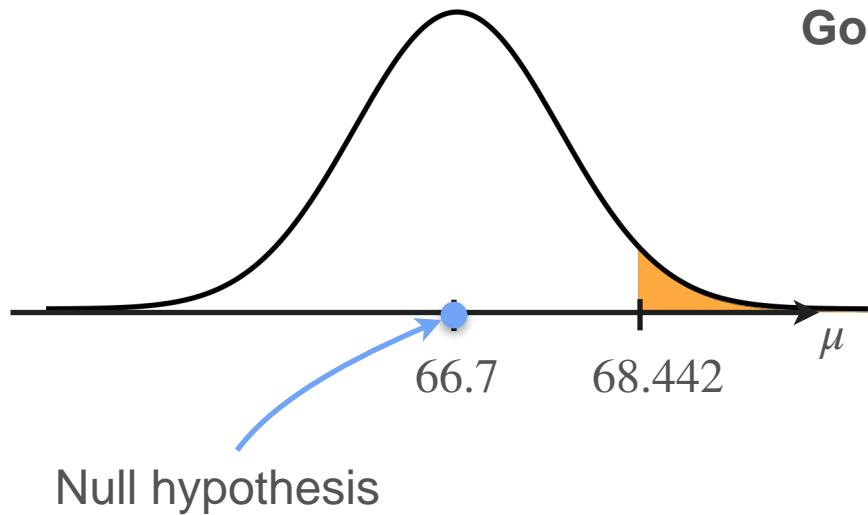
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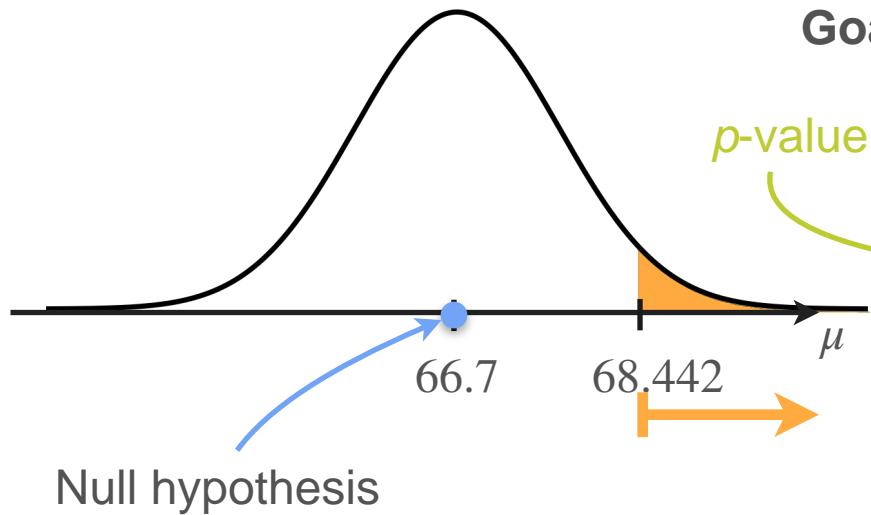
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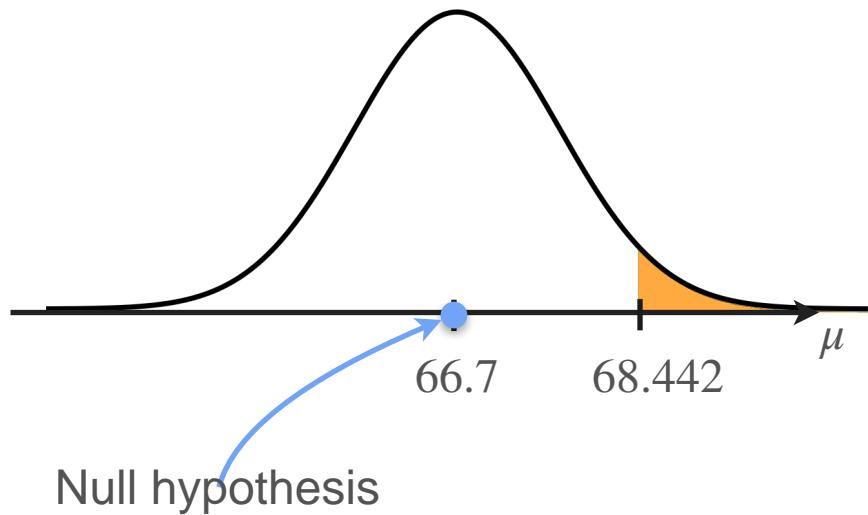
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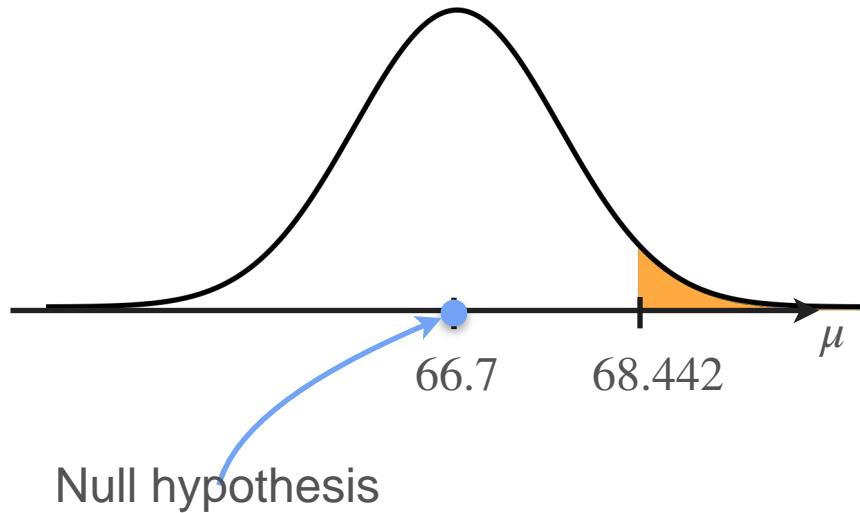
# P-Values



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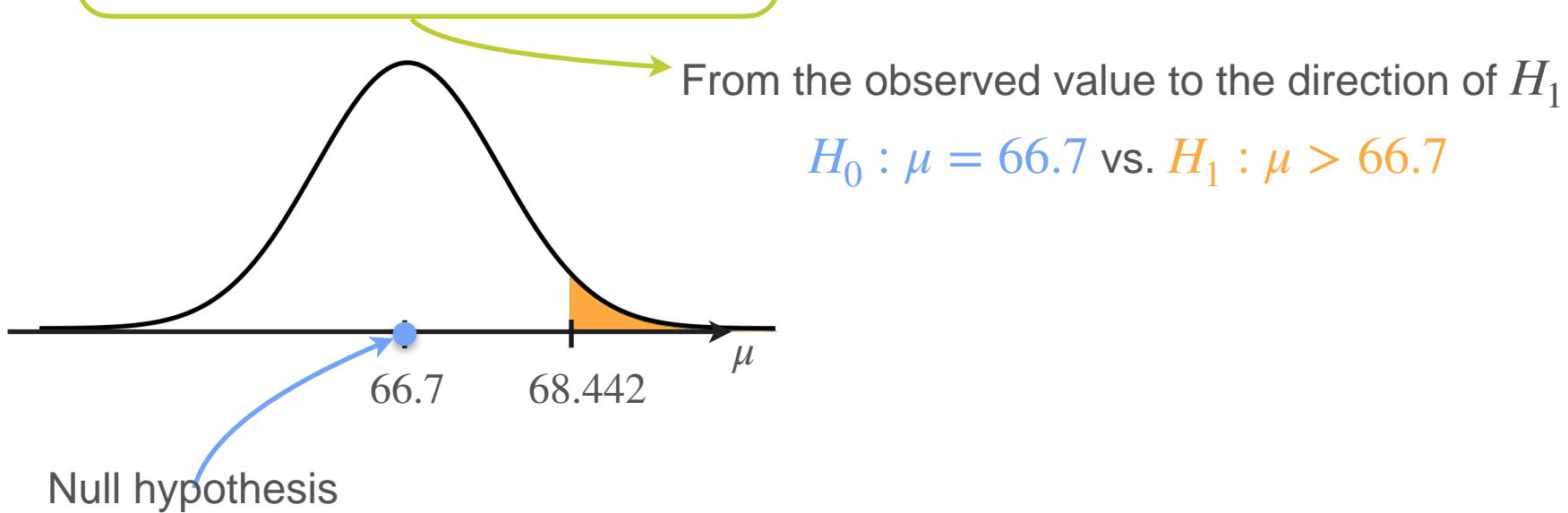
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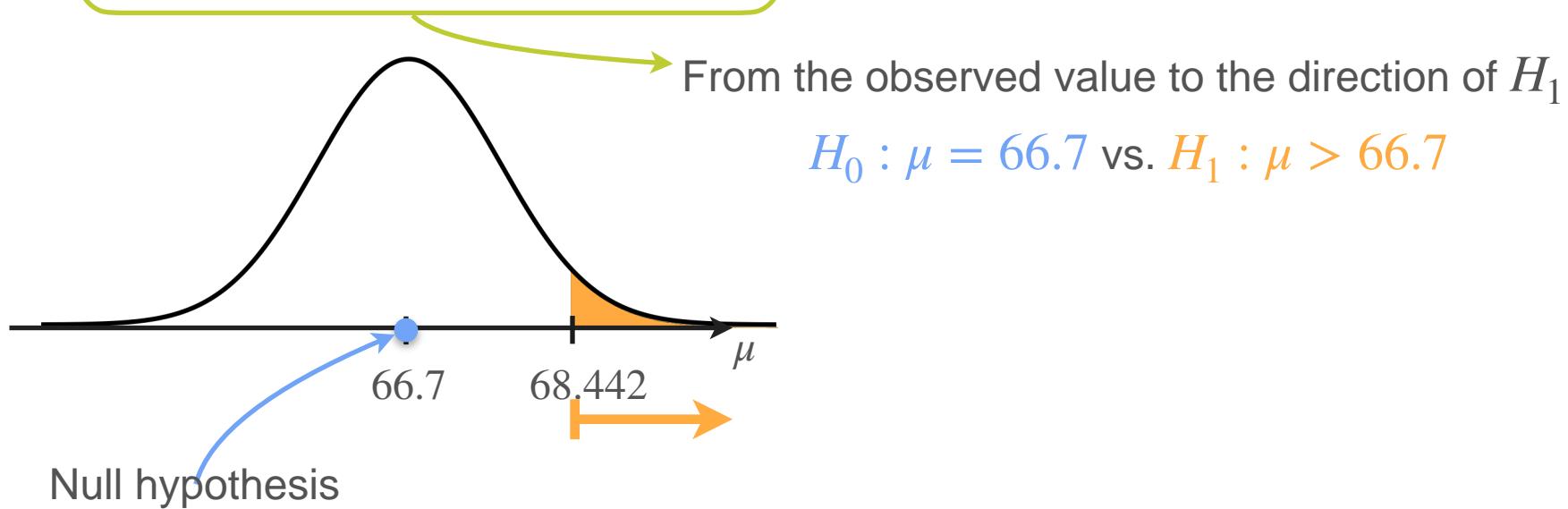
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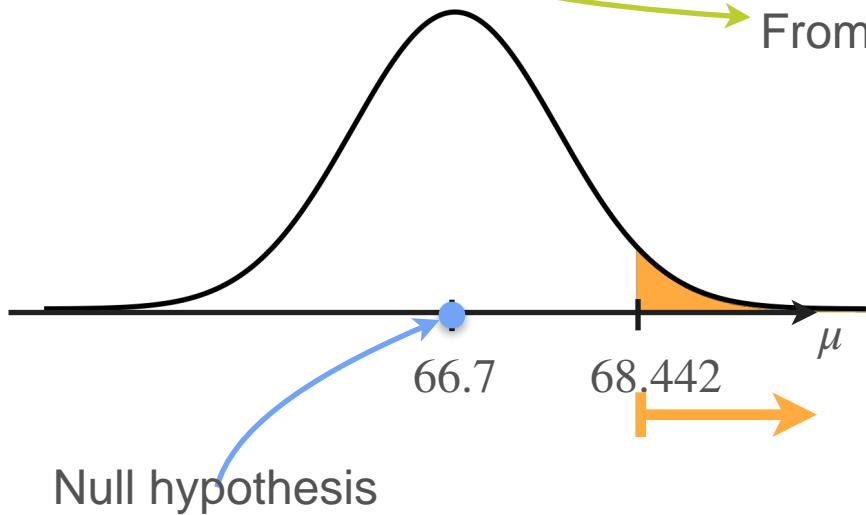
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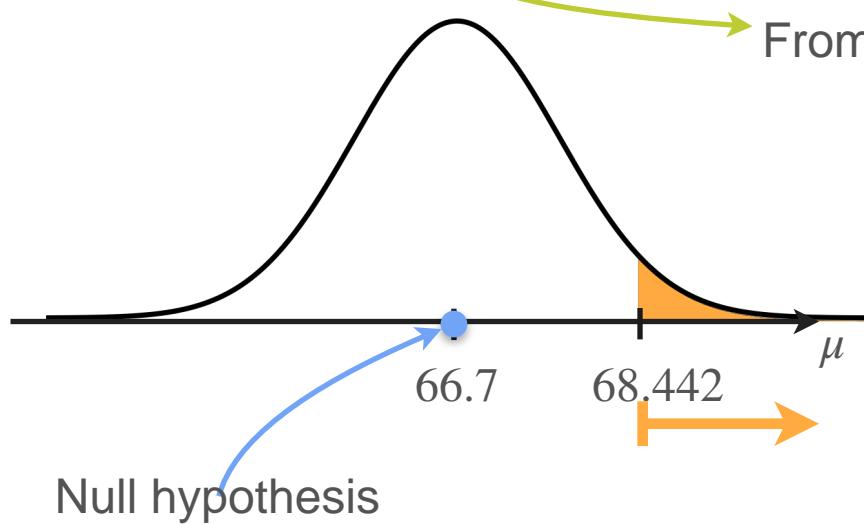
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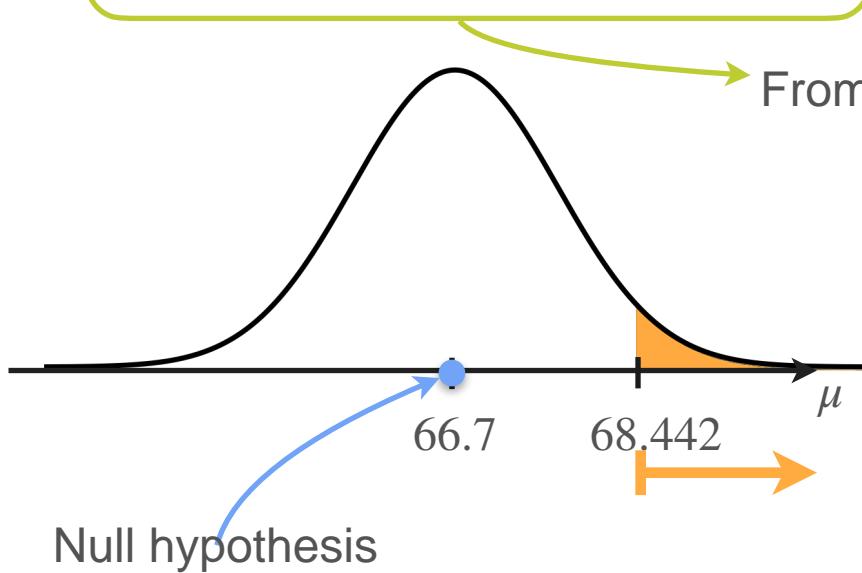
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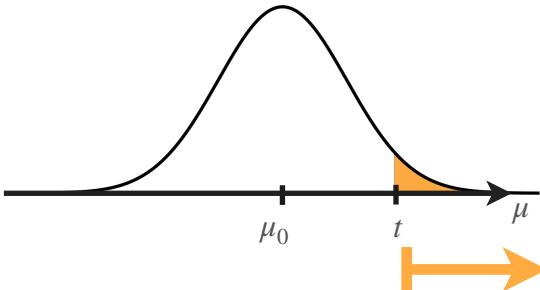
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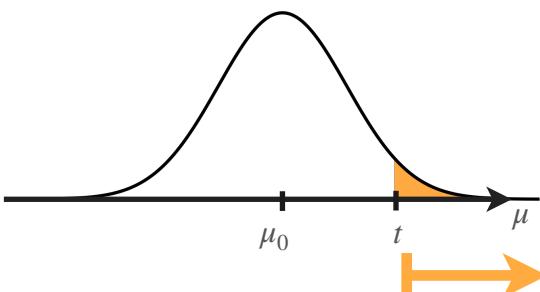
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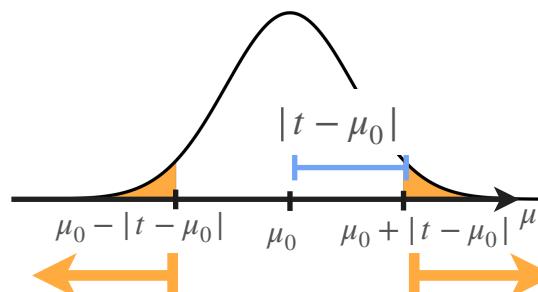
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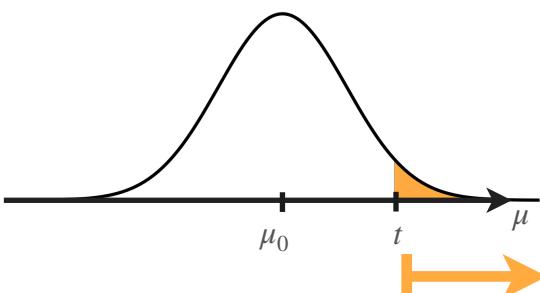
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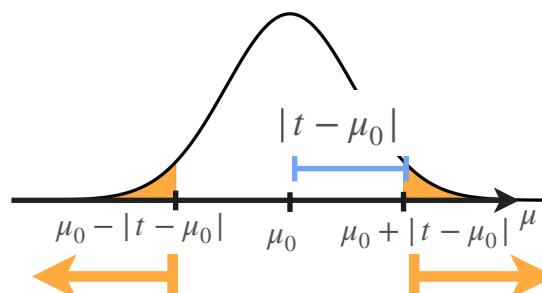
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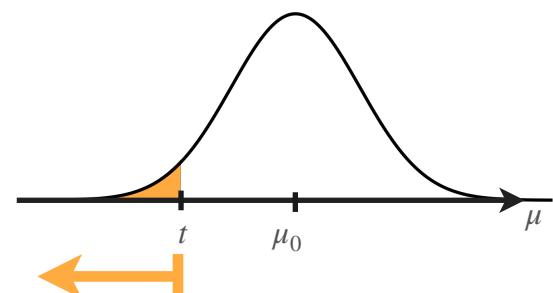
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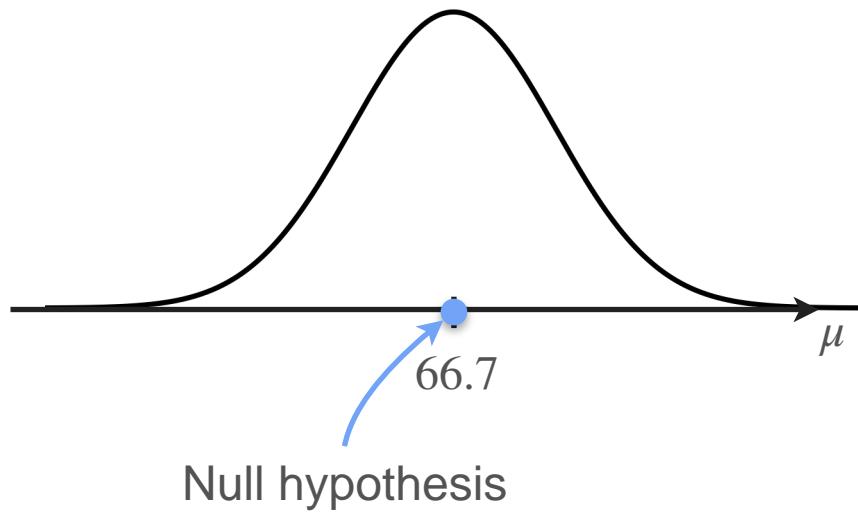
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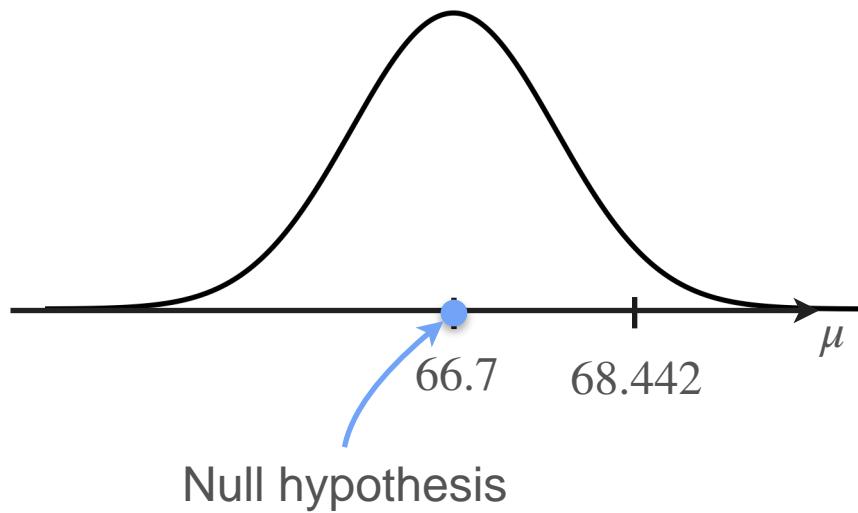
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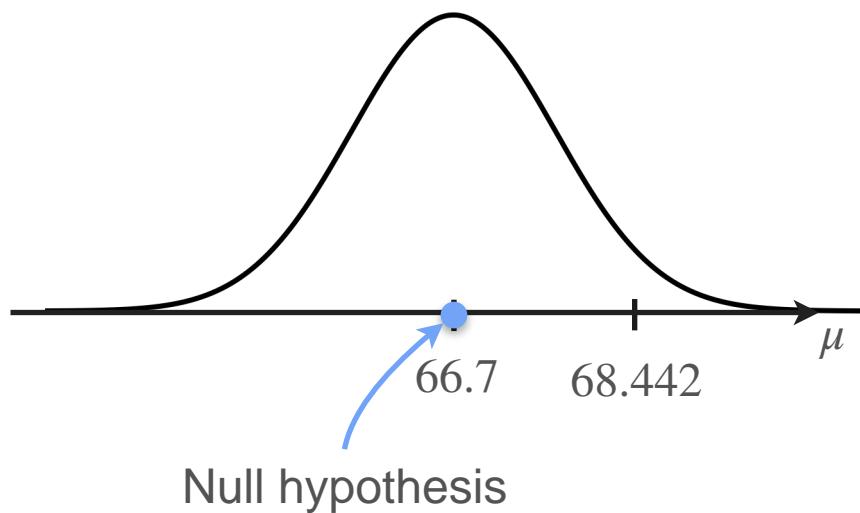
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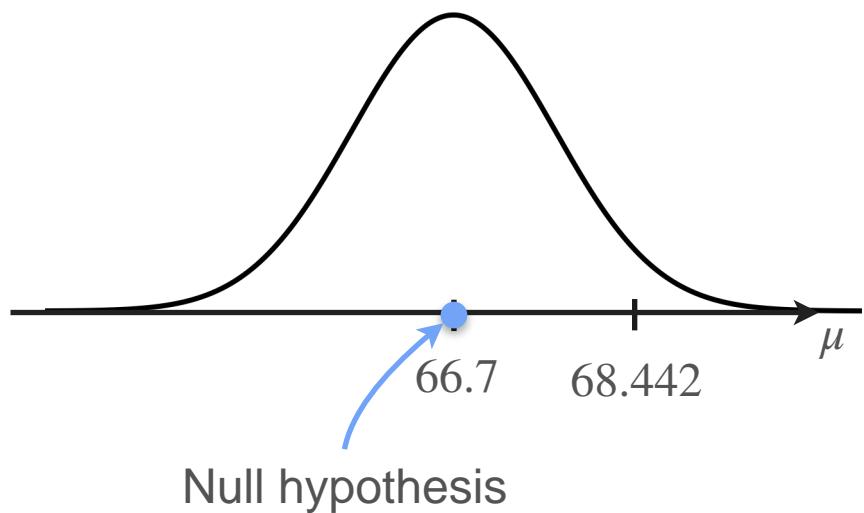
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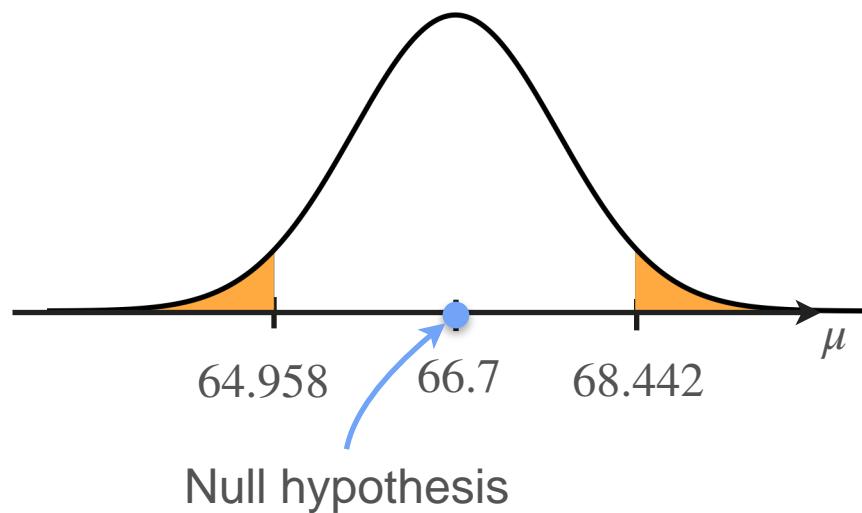
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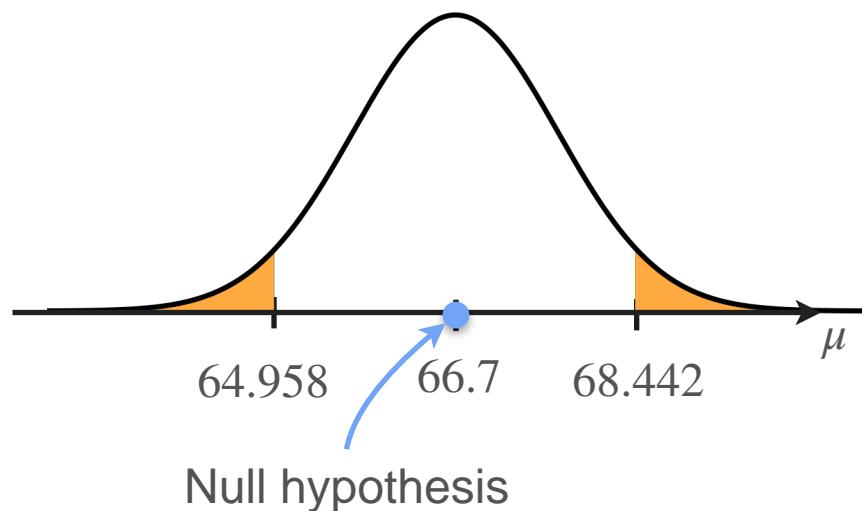
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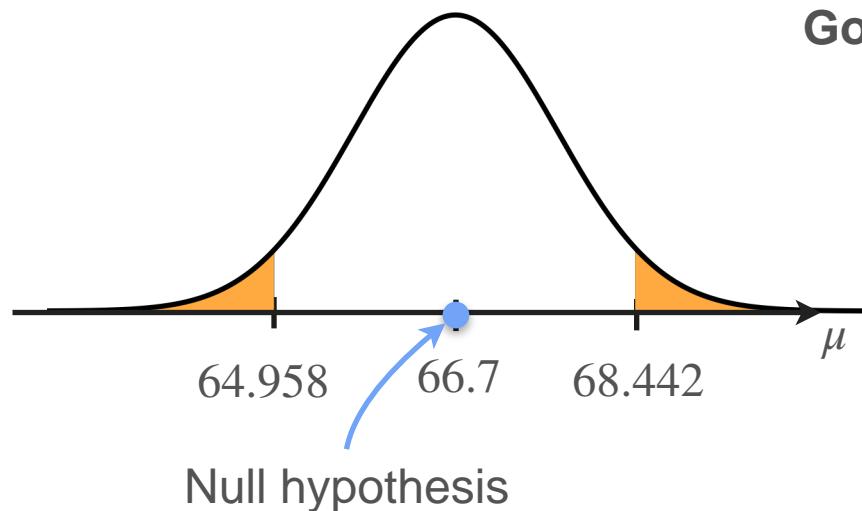
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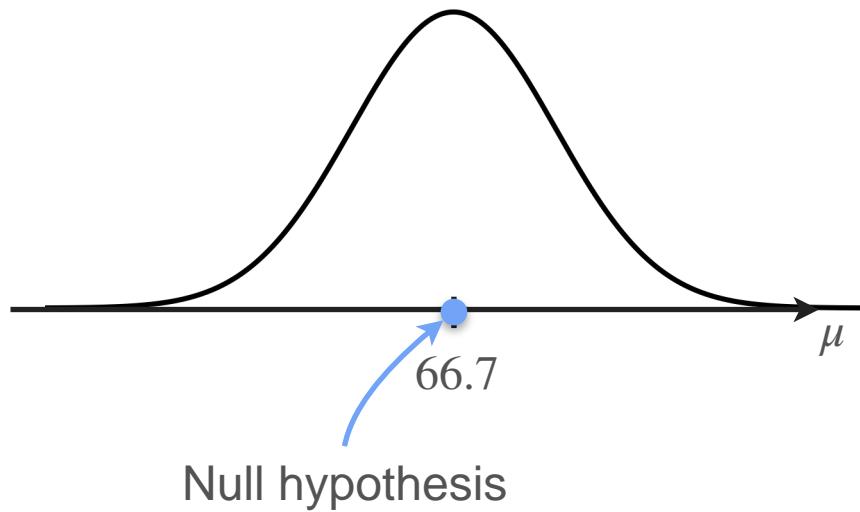
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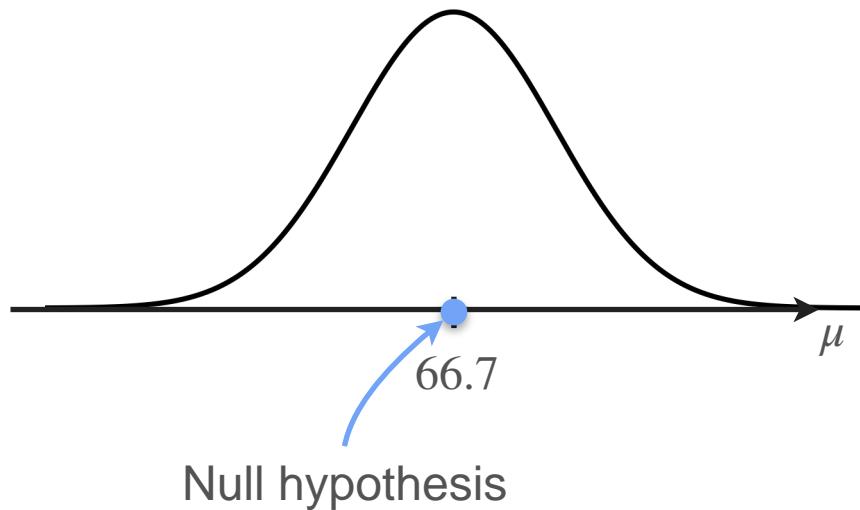
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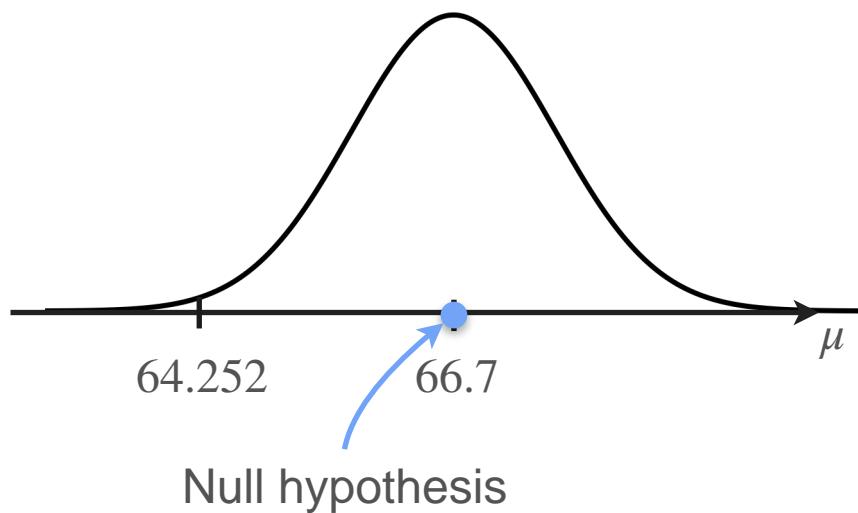
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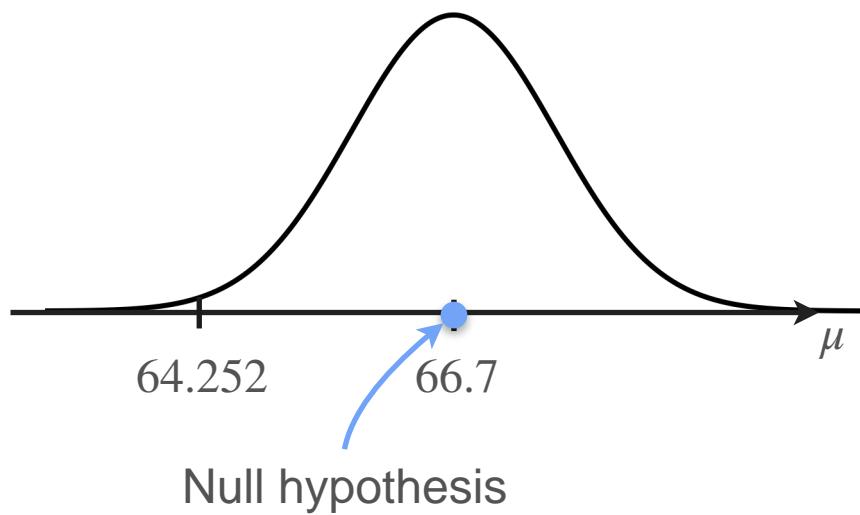
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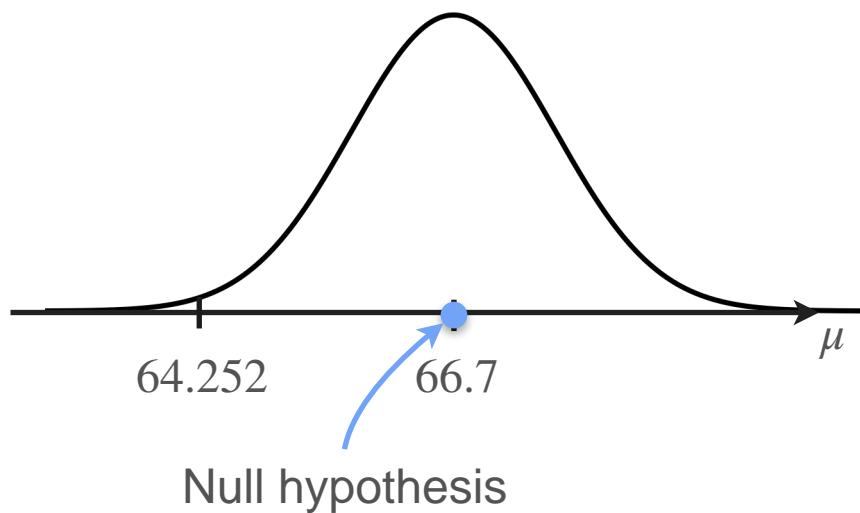
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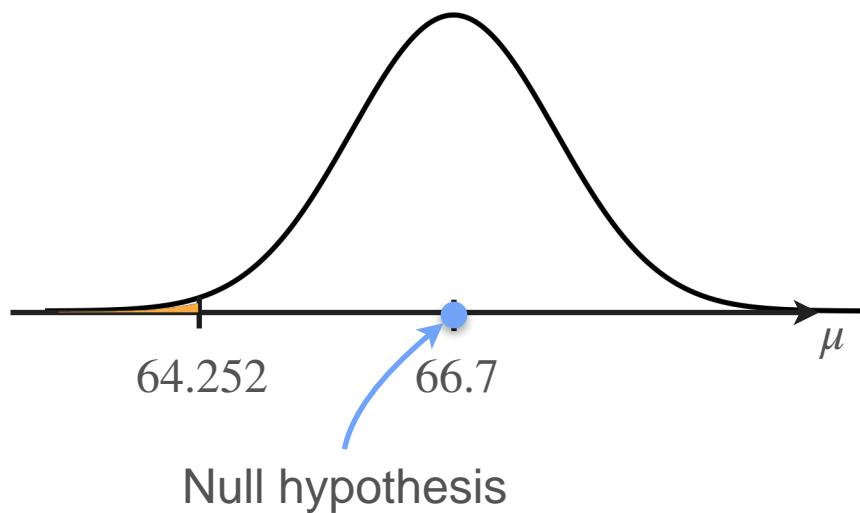
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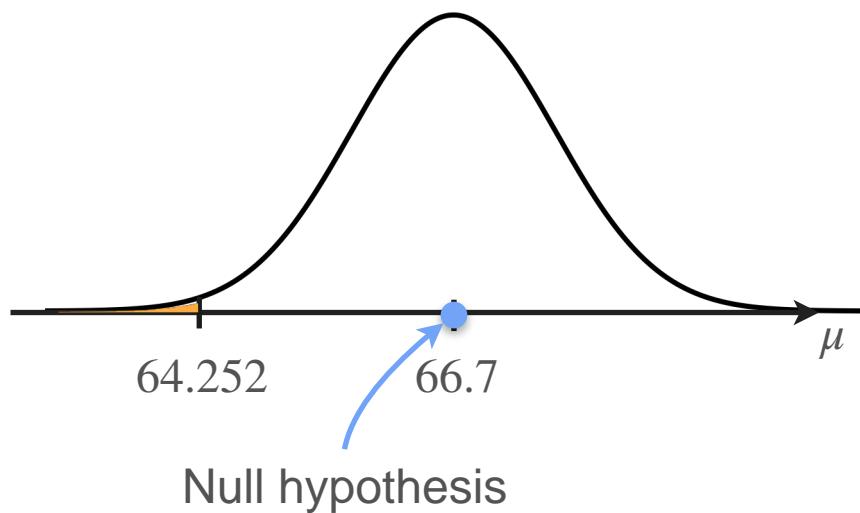
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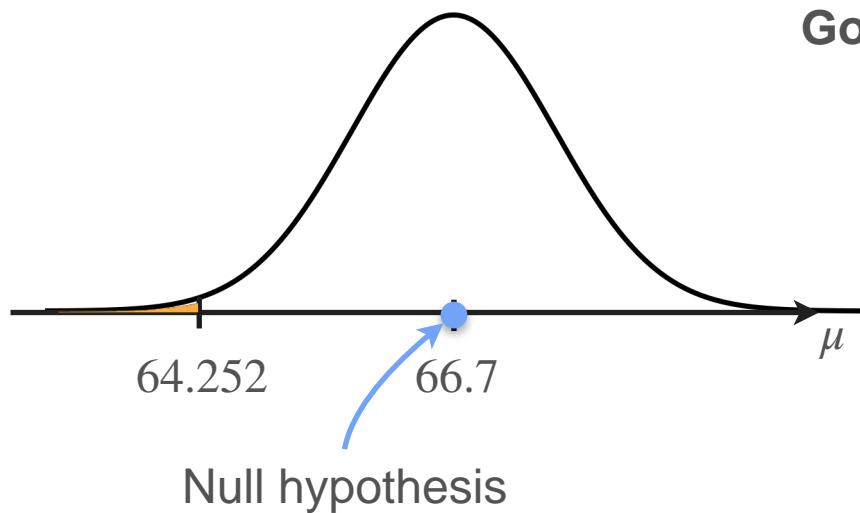
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$$\text{Z-statistic } Z = \frac{\bar{X} - \mu_0}{3/\sqrt{10}}$$

$$\text{If } H_0 \text{ is true, } Z = \frac{\bar{X} - \mu_0}{3/\sqrt{10}} \sim \mathcal{N}(0,1)$$

# Right-Tailed Test Using the Z Statistic

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The **mean** height for 18 y/o in the US in the 70s was **66.7** in.

$$H_0 : \mu = 66.7 \text{ vs. } H_1 : \mu > 66.7$$

**Goal:** Type I error probability  $< \alpha = 0.05$

**Type I error:** Determine  $\mu > 66.7$ ,  
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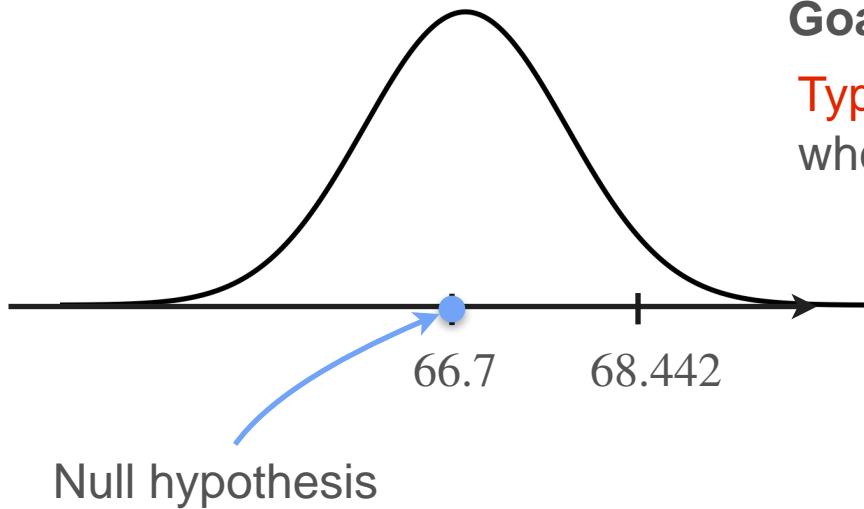
$$\sigma = 3$$
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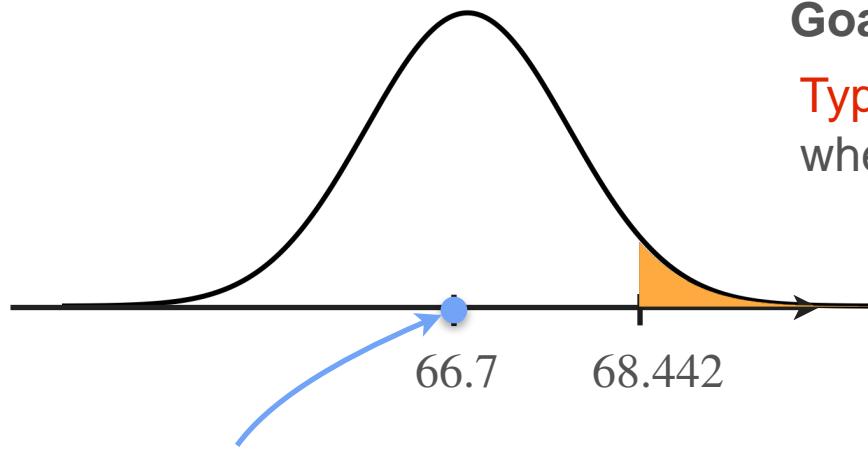
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$$P(\bar{X} > 68.442 \mid \mu = 66.7) \\ = 0.0407 < \alpha$$

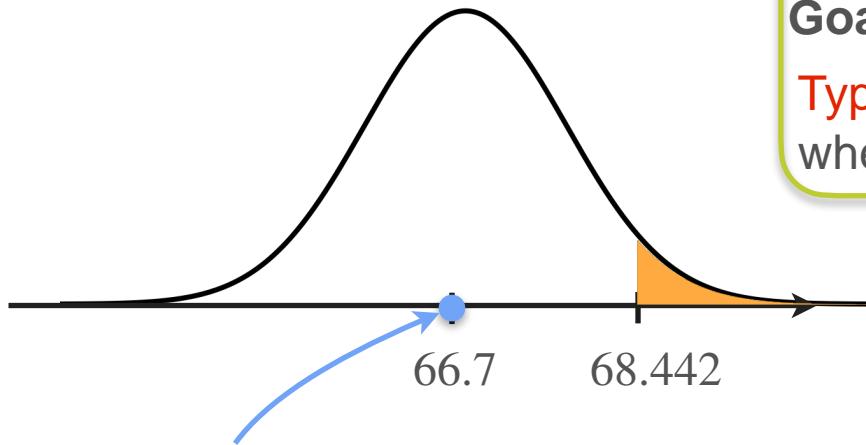
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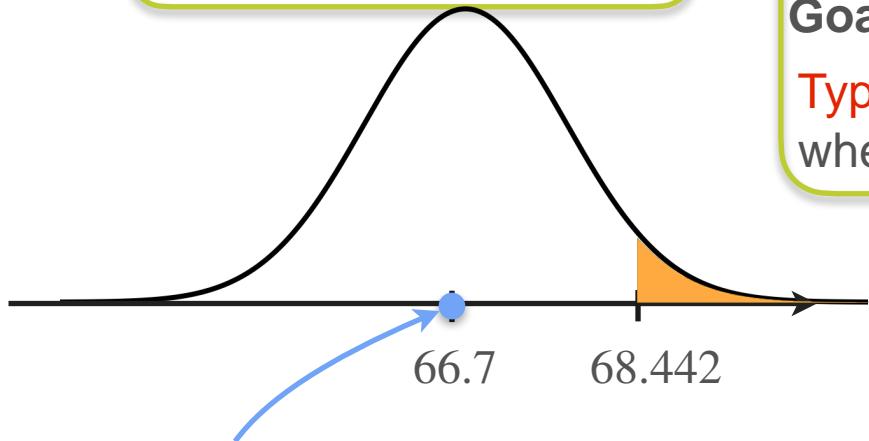
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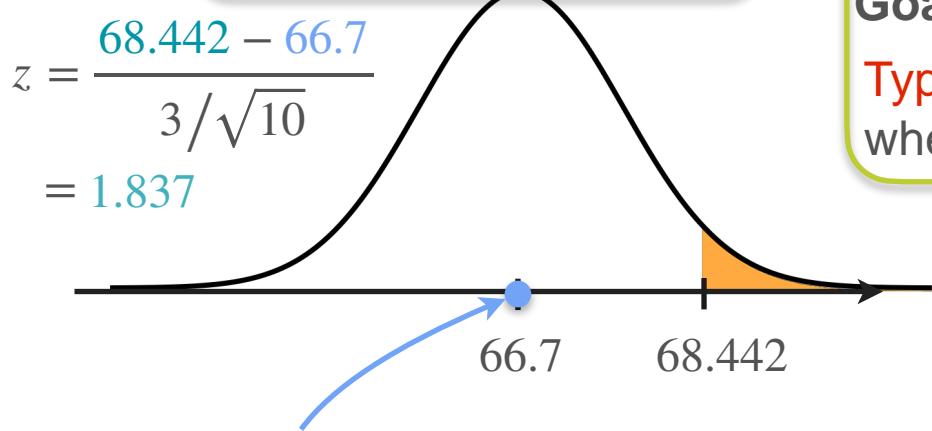
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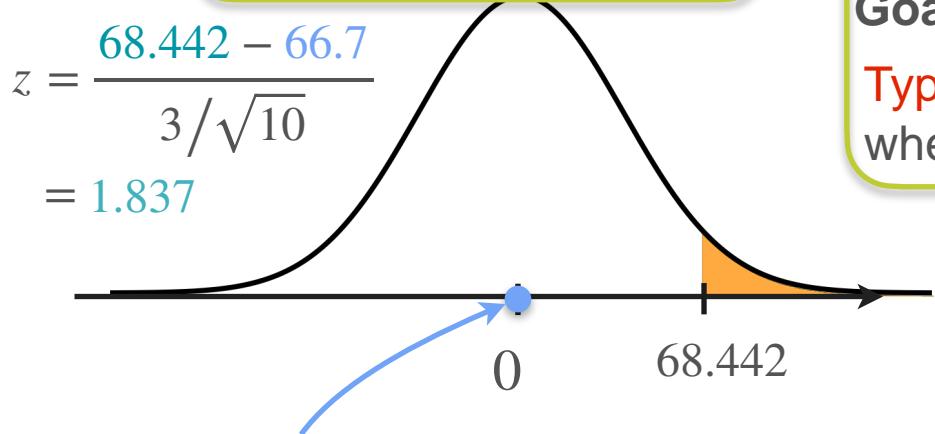
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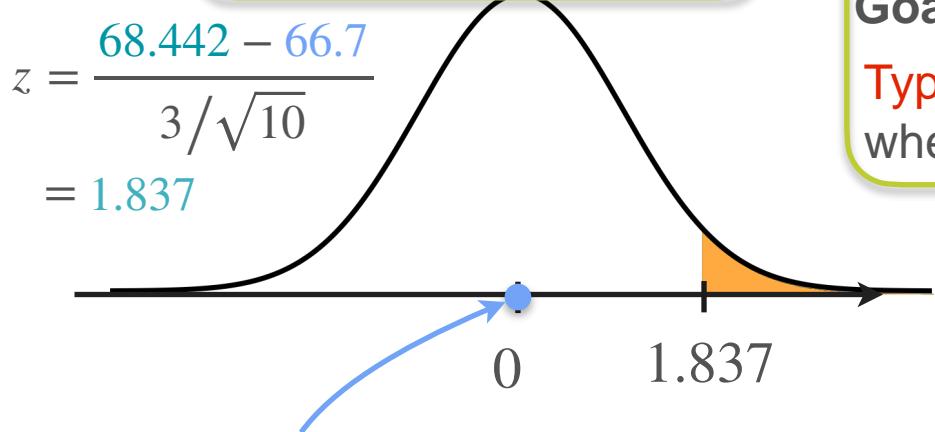
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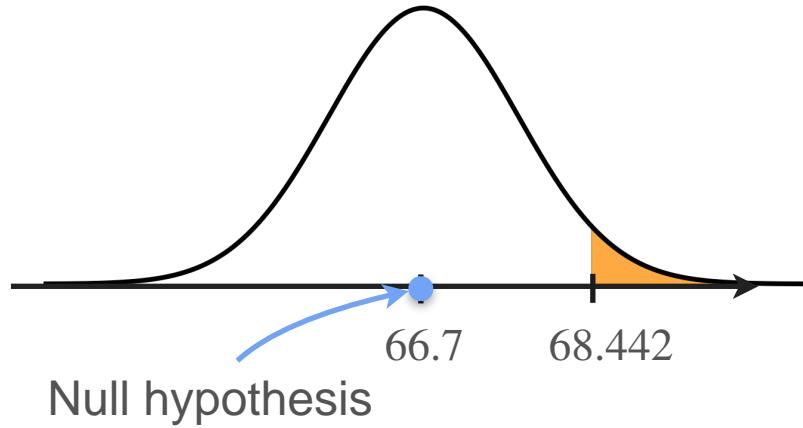
# Hypothesis Testing

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## Critical Values

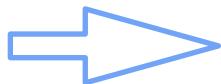
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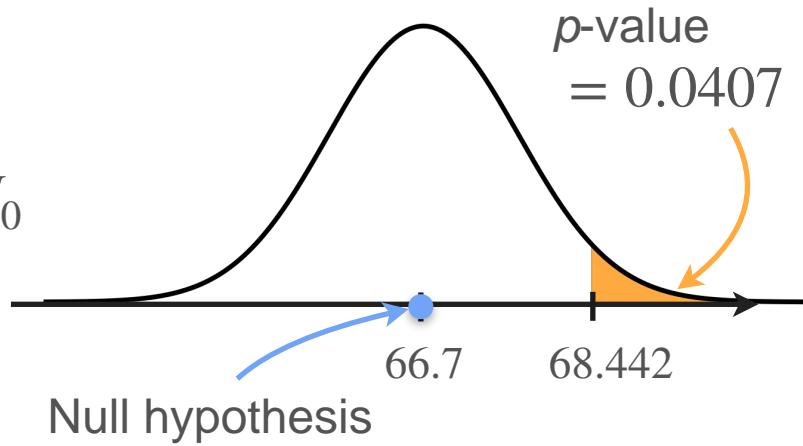


# P-Values and Critical Values

If  $p\text{-value} < \alpha$

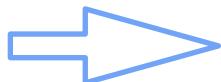


Reject  $H_0$



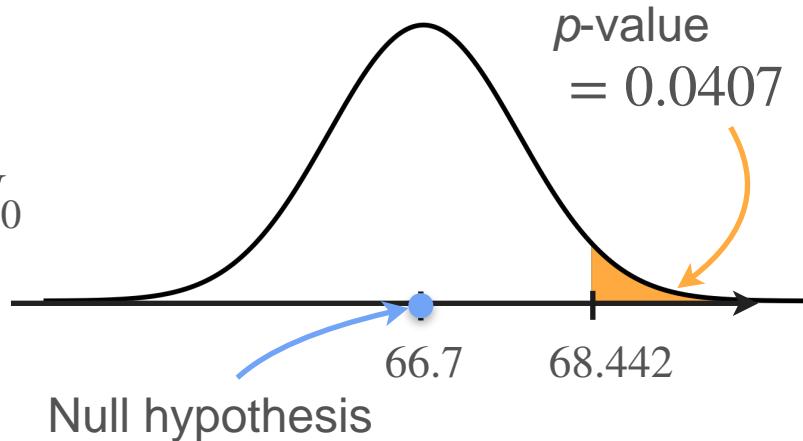
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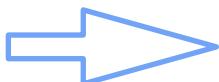
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What is the least extreme sample you could get that you would still reject  $H_0$ ?



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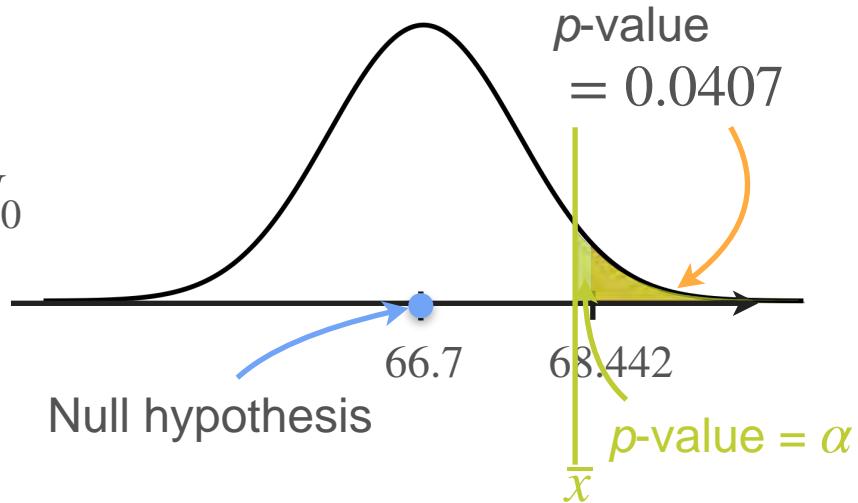
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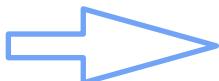
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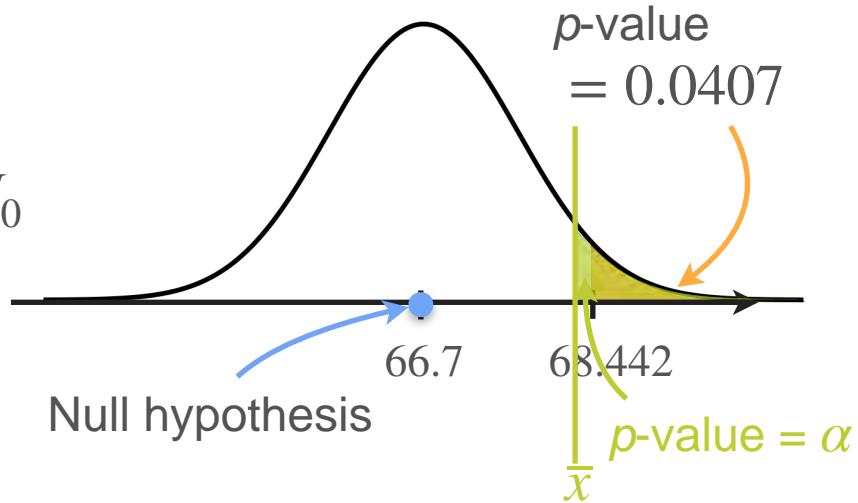


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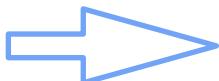
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Critical values



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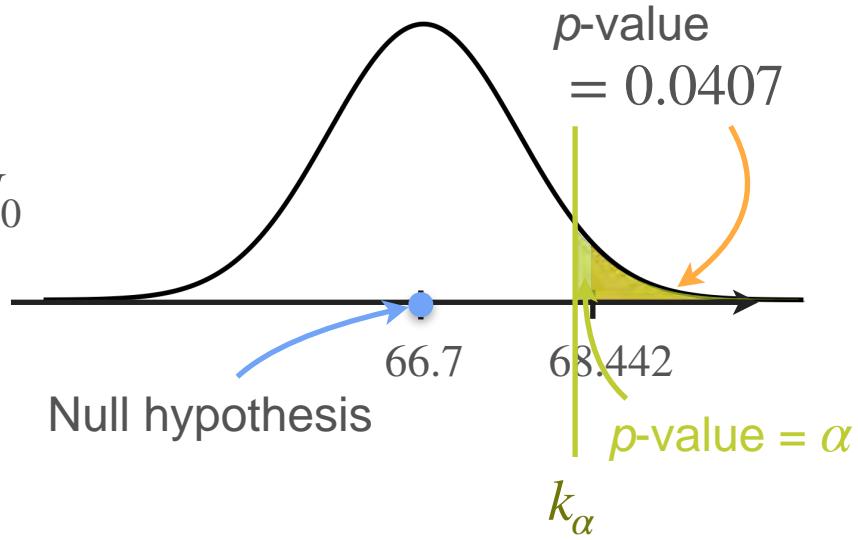


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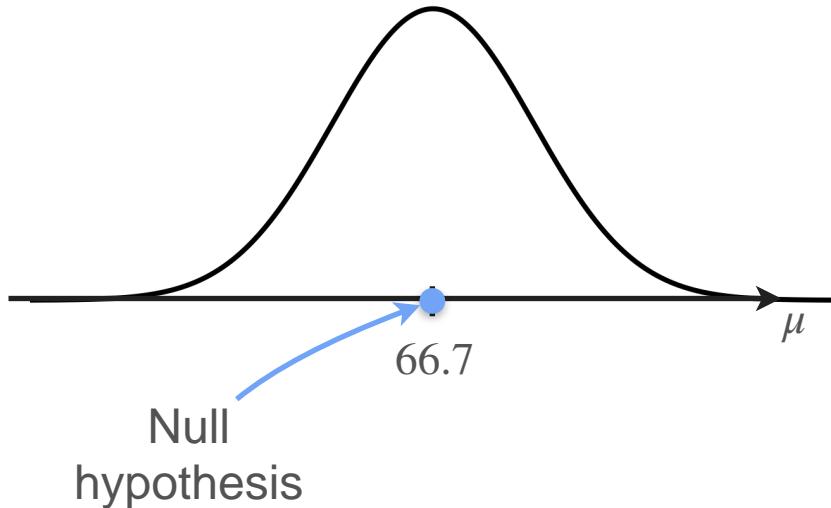


# Computing Critical Values

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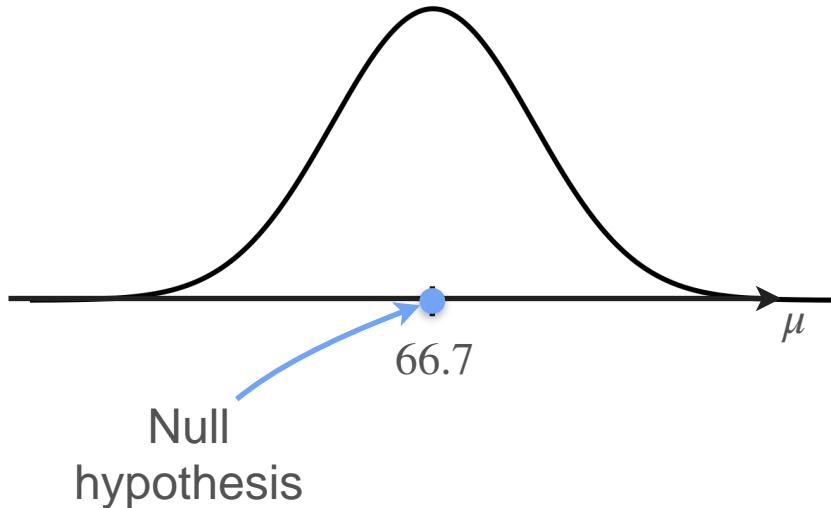
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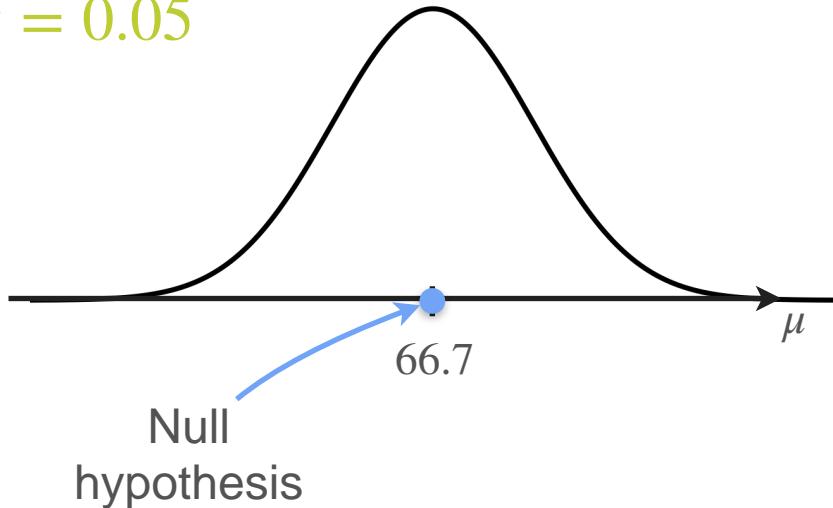
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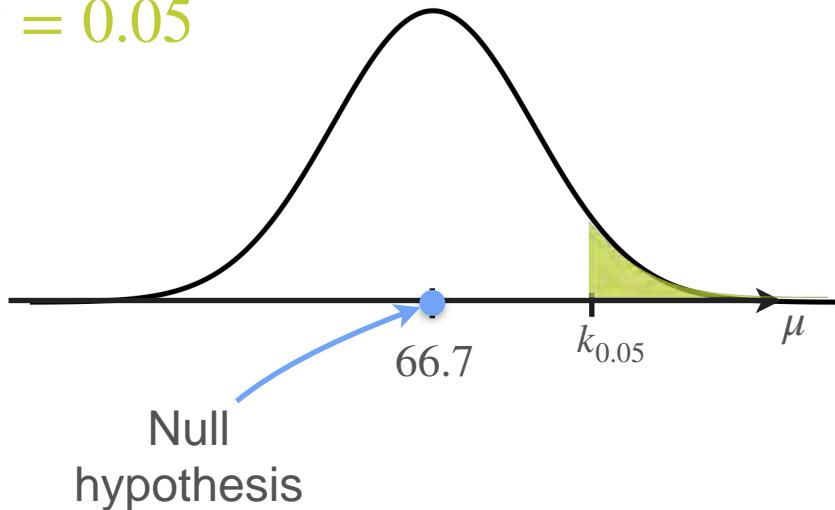
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$$0.05 = P(\bar{X} > k_{0.05} \mid \mu = 66.7)$$



# Computing Critical Values

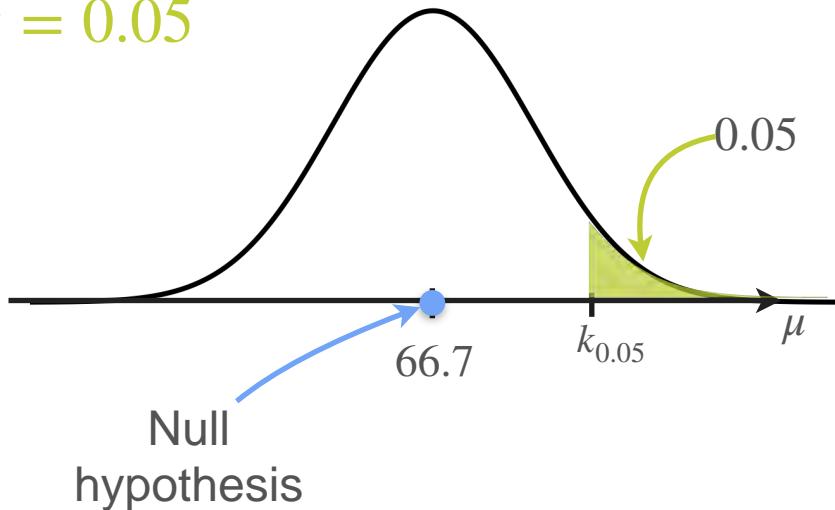
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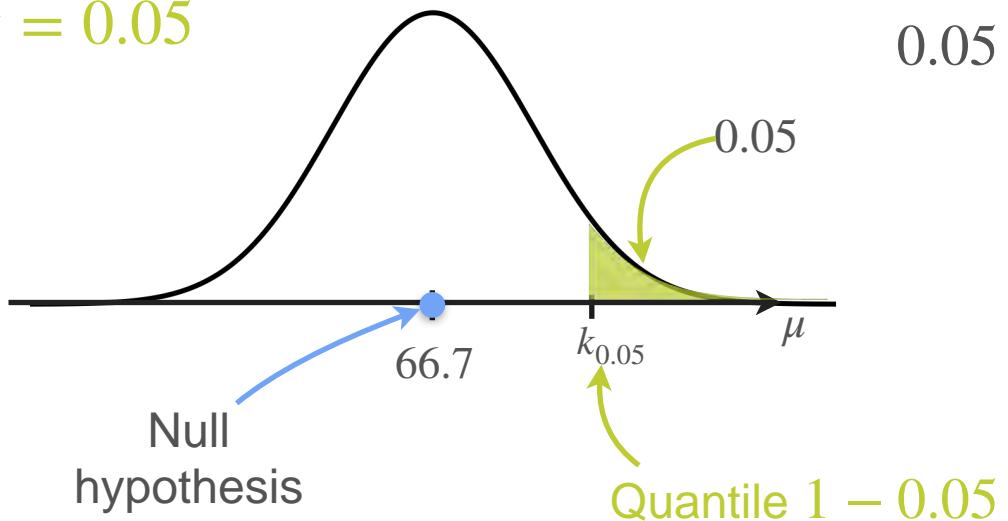
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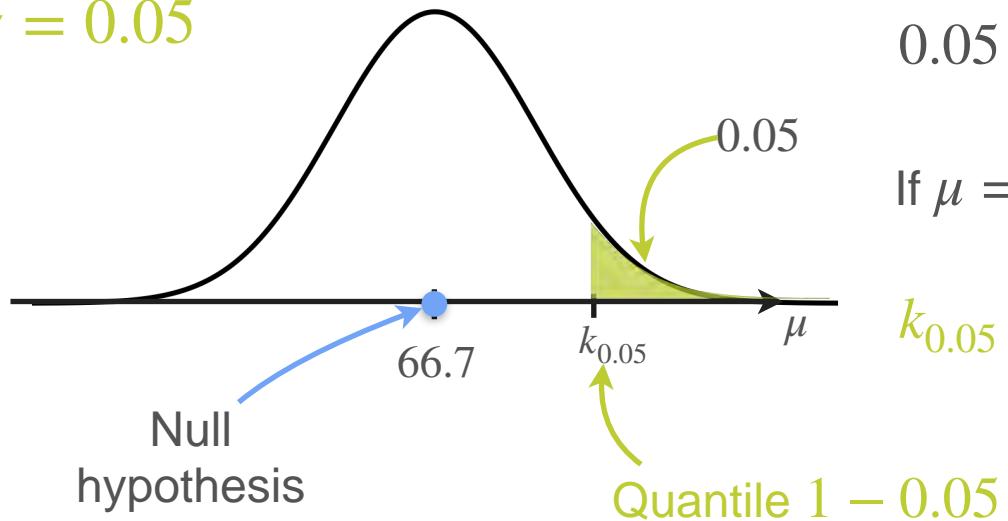


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If  $\mu = 66.7$   $\bar{X} \sim \mathcal{N}\left(66.7, \frac{3^2}{10}\right)$

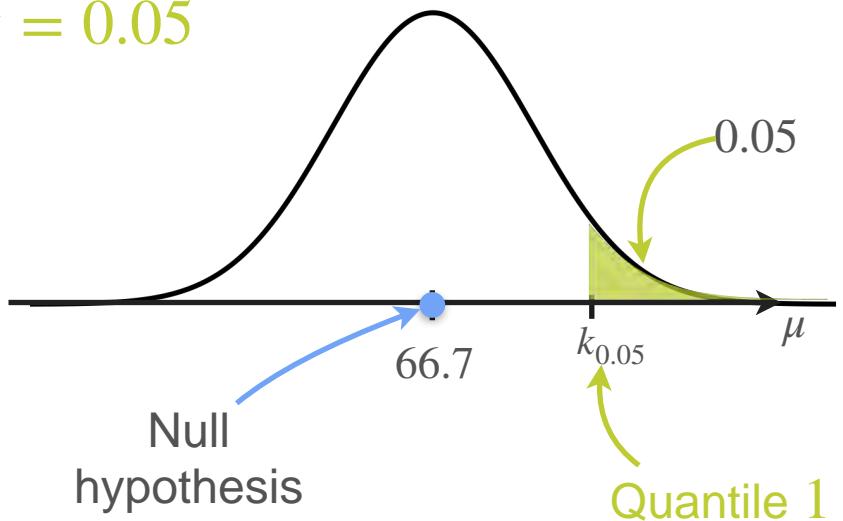
$$k_{0.05} = 68.26$$

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If  $\mu = 66.7$   $\bar{X} \sim \mathcal{N}\left(66.7, \frac{3^2}{10}\right)$

$$k_{0.05} = 68.26$$

**Decision rule:** Reject  $H_0$  if  $\bar{x} > 68.26$

# Computing Critical Values

The mean height for 18 y/o in the US in the 70s was **66.7 in.** Reject  $H_0$

$H_0 : \mu = 66.7$  vs.  $H_1 : \mu > 66.7$

$\alpha = 0.05$

$$n = 10 \quad \sigma = 3 \quad \bar{x} = 68.442$$

$$0.05 = P(\bar{X} > k_{0.05} \mid \mu = 66.7)$$

If  $\mu = 66.7$   $\bar{X} \sim \mathcal{N}\left(66.7, \frac{3^2}{10}\right)$

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Null hypothesis

66.7

$k_{0.05}$

Quantile  $1 - 0.05$

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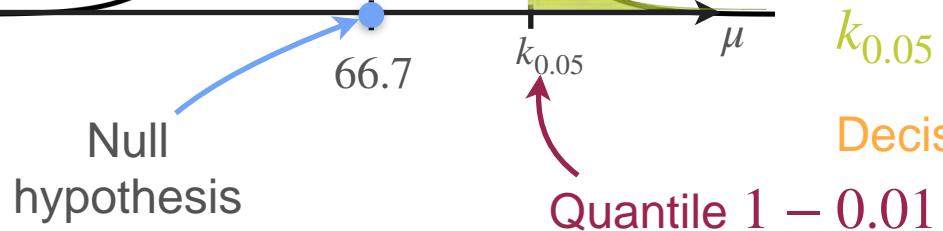
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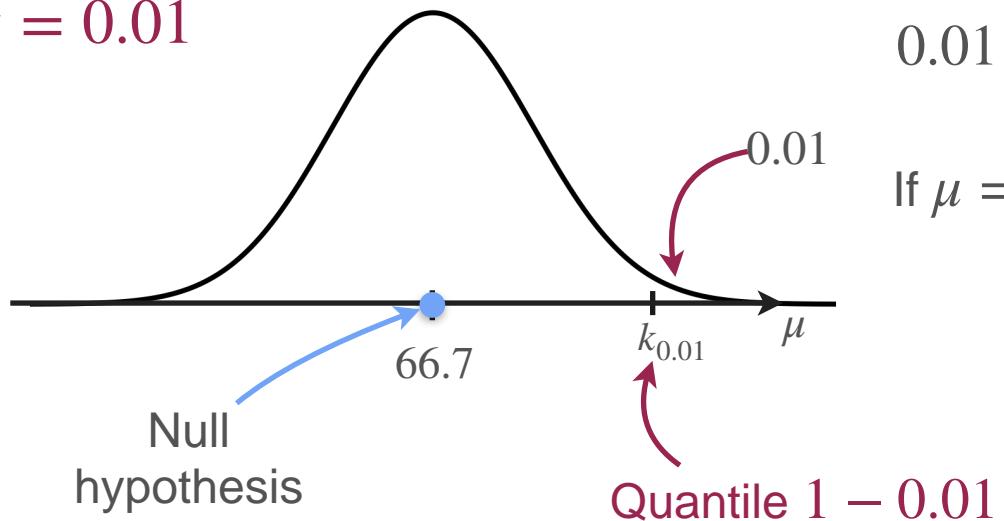


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The mean height for 18 y/o in the US in the 70s was **66.7 in.**

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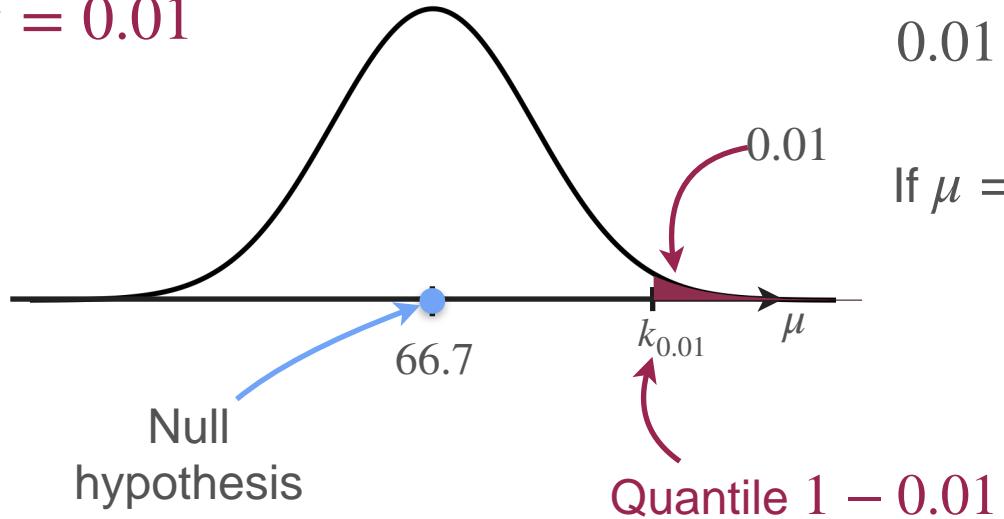
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$$n = 10 \quad \sigma = 3 \quad \bar{x} = 68.442$$

$$0.01 = P(\bar{X} > k_{0.01} \mid \mu = 66.7)$$

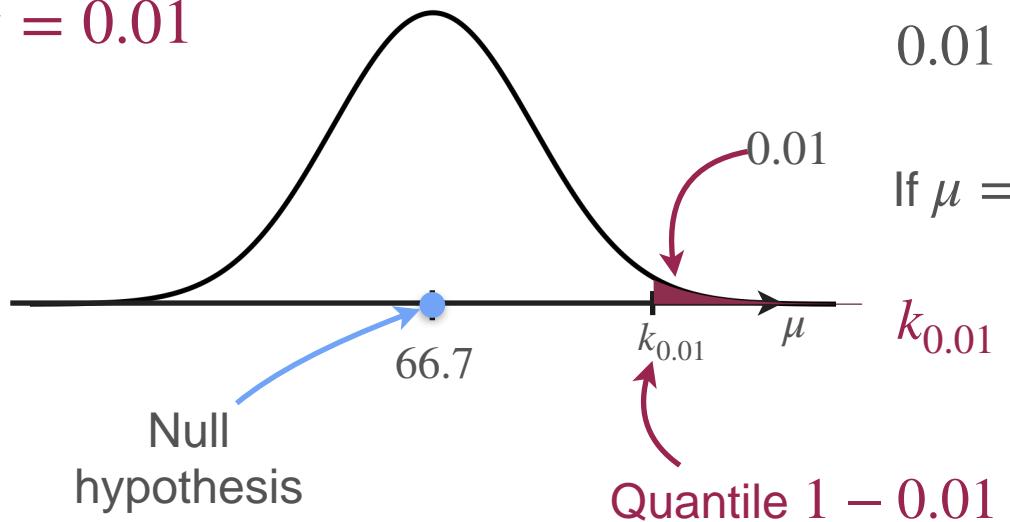
If  $\mu = 66.7$   $\bar{X} \sim \mathcal{N}\left(66.7, \frac{3^2}{10}\right)$

# Computing Critical Values

The mean height for 18 y/o in the US in the 70s was **66.7 in.**

$$H_0 : \mu = 66.7 \text{ vs. } H_1 : \mu > 66.7$$

$$\alpha = 0.01$$



$$n = 10 \quad \sigma = 3 \quad \bar{x} = 68.442$$

$$0.01 = P(\bar{X} > k_{0.01} \mid \mu = 66.7)$$

If  $\mu = 66.7$   $\bar{X} \sim \mathcal{N}\left(66.7, \frac{3^2}{10}\right)$

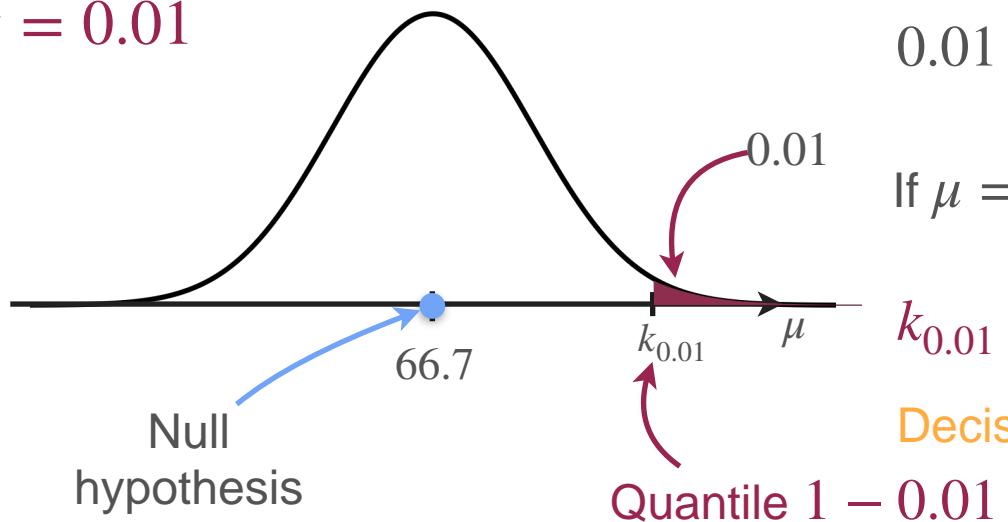
$$k_{0.01} = 68.91$$

# Computing Critical Values

The mean height for 18 y/o in the US in the 70s was **66.7 in.**

$$H_0 : \mu = 66.7 \text{ vs. } H_1 : \mu > 66.7$$

$$\alpha = 0.01$$



$$n = 10 \quad \sigma = 3 \quad \bar{x} = 68.442$$

$$0.01 = P(\bar{X} > k_{0.01} \mid \mu = 66.7)$$

If  $\mu = 66.7$   $\bar{X} \sim \mathcal{N}\left(66.7, \frac{3^2}{10}\right)$

$$k_{0.01} = 68.91$$

**Decision rule:** Reject  $H_0$  if  $\bar{x} > 68.91$

# Computing Critical Values

The mean height for 18 y/o in the US in the 70s was **66.7 in.**

Do not reject  $H_0$

$H_0 : \mu = 66.7$  vs.  $H_1 : \mu > 66.7$

$\alpha = 0.01$

$$n = 10 \quad \sigma = 3 \quad \bar{x} = 68.442$$

$$0.01 = P(\bar{X} > k_{0.01} \mid \mu = 66.7)$$

If  $\mu = 66.7$   $\bar{X} \sim \mathcal{N}\left(66.7, \frac{3^2}{10}\right)$

$$k_{0.01} = 68.91$$

Decision rule: Reject  $H_0$  if  $\bar{x} > 68.91$

Null hypothesis

66.7

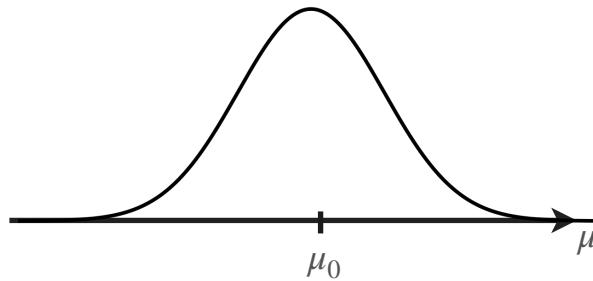
$k_{0.01}$

Quantile 1 – 0.01

# Critical Values

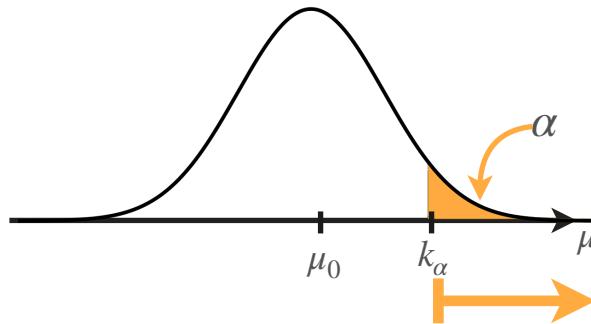
# Critical Values

$H_0 : \mu = \mu_0$  vs.  $H_1 : \mu > \mu_0$



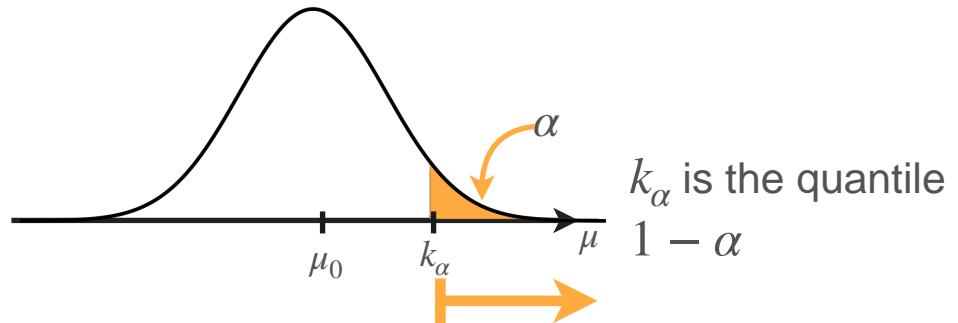
# Critical Values

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# Critical Values

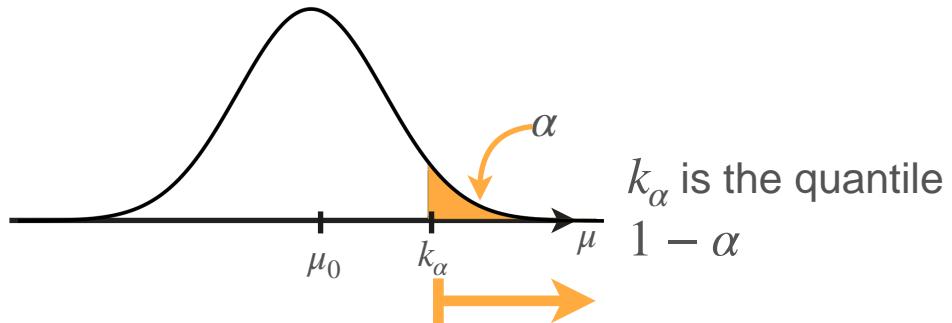
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# Critical Values

$H_0 : \mu = \mu_0$  vs.  $H_1 : \mu > \mu_0$

Decision rule: Reject  $H_0$  if  $t > k_\alpha$

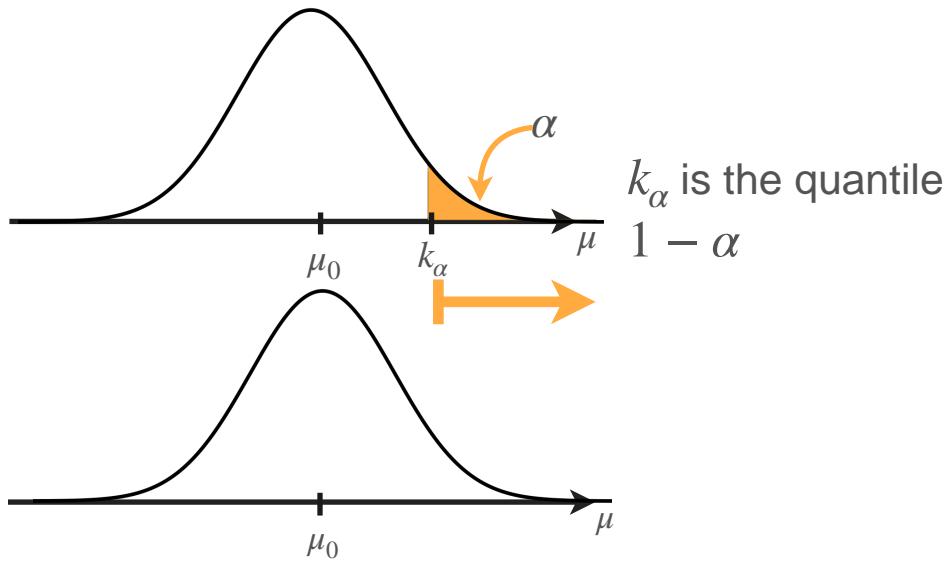


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$H_0 : \mu = \mu_0$  vs.  $H_1 : \mu > \mu_0$

Decision rule: Reject  $H_0$  if  $t > k_\alpha$

$H_0 : \mu = \mu_0$  vs.  $H_1 : \mu < \mu_0$

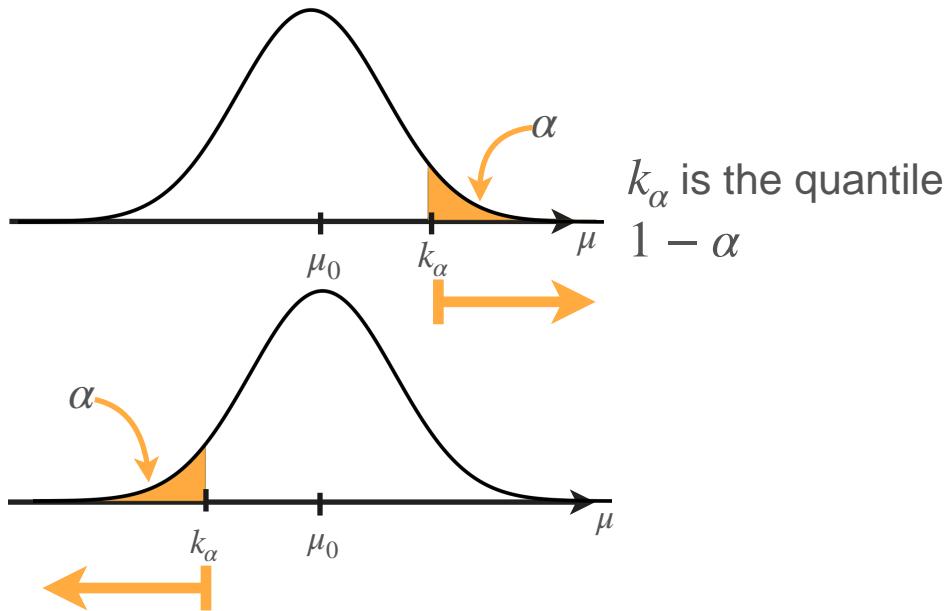


# Critical Values

$H_0 : \mu = \mu_0$  vs.  $H_1 : \mu > \mu_0$

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$H_0 : \mu = \mu_0$  vs.  $H_1 : \mu < \mu_0$

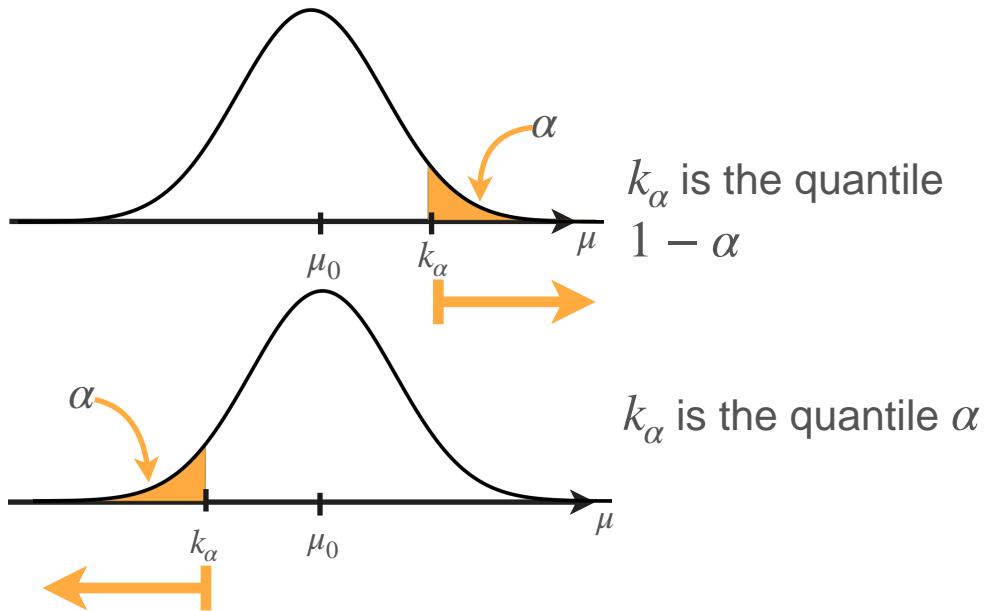


# Critical Values

$H_0 : \mu = \mu_0$  vs.  $H_1 : \mu > \mu_0$

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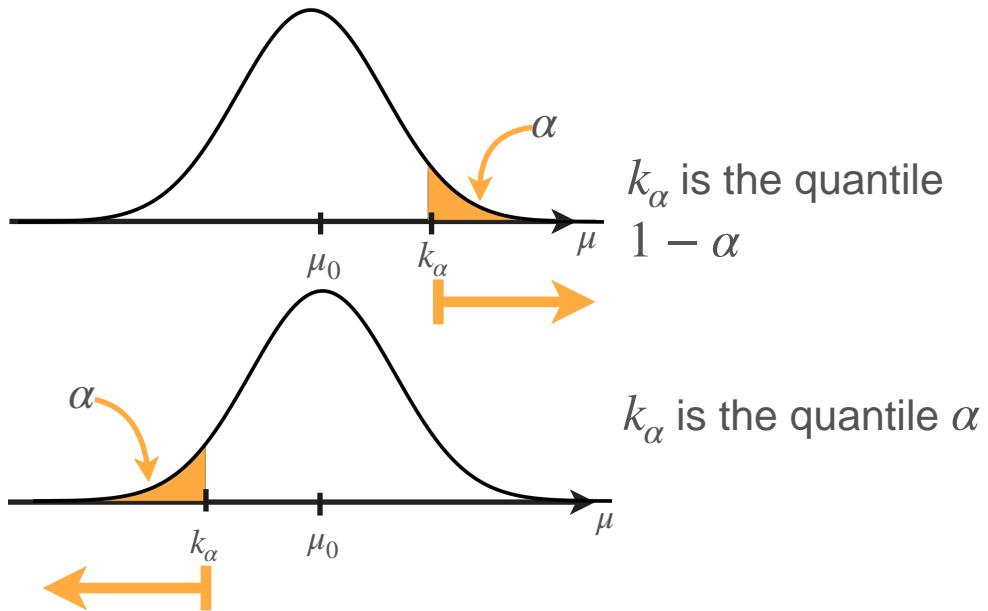
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# Critical Values

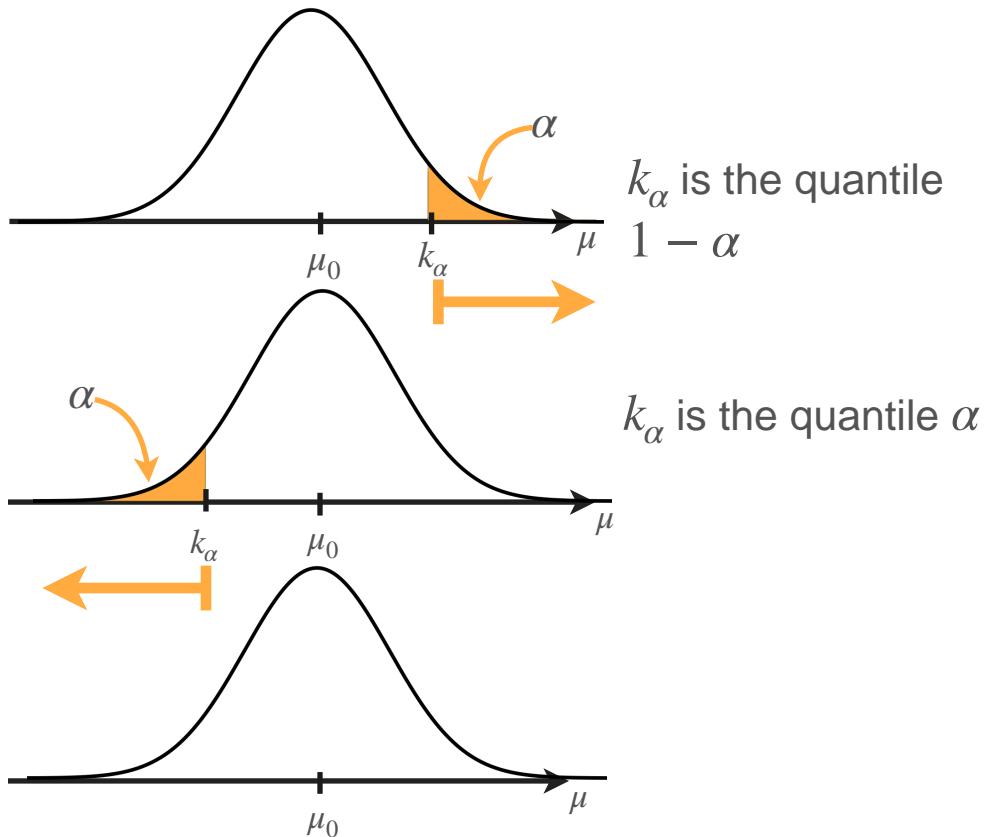
$H_0 : \mu = \mu_0$  vs.  $H_1 : \mu > \mu_0$

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# Critical Values

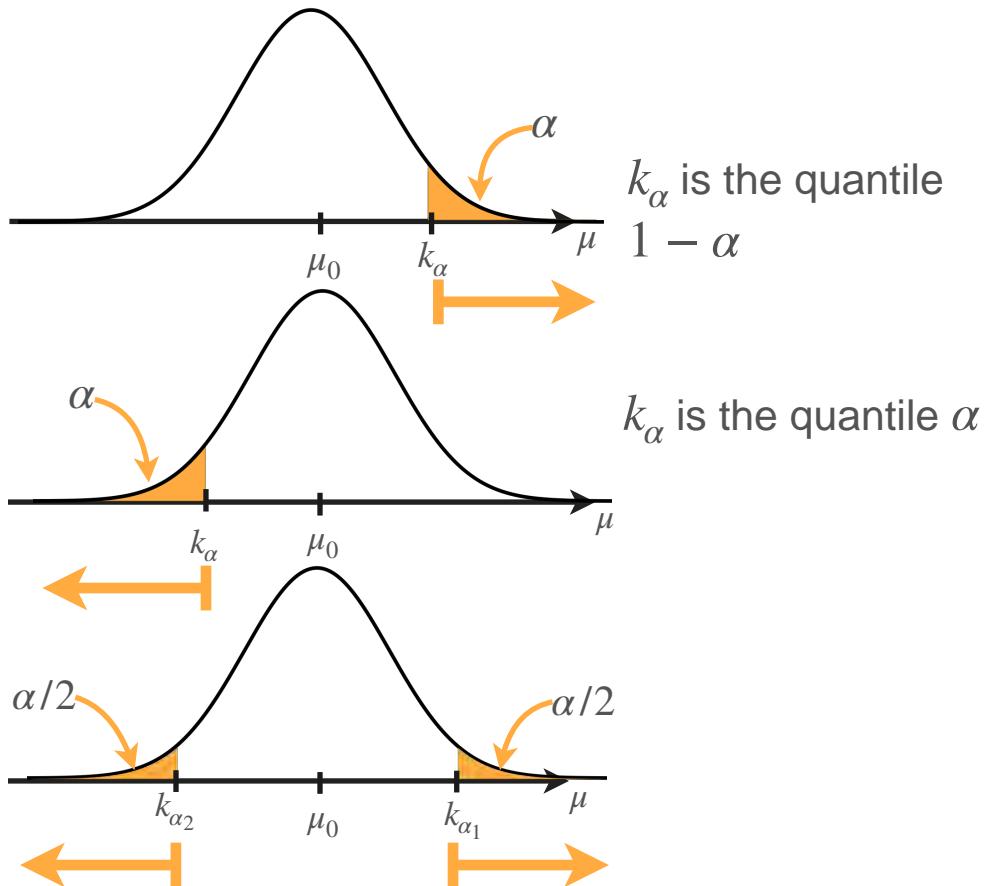
$H_0 : \mu = \mu_0$  vs.  $H_1 : \mu > \mu_0$

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# Critical Values

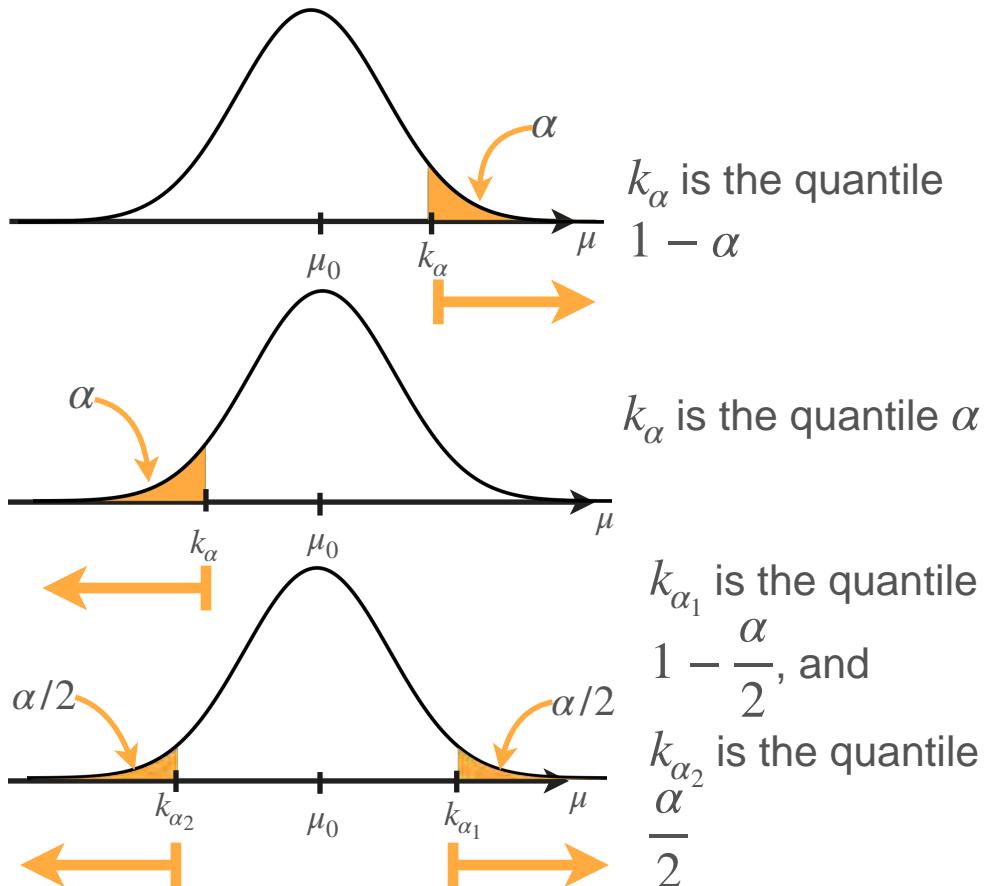
$H_0 : \mu = \mu_0$  vs.  $H_1 : \mu > \mu_0$

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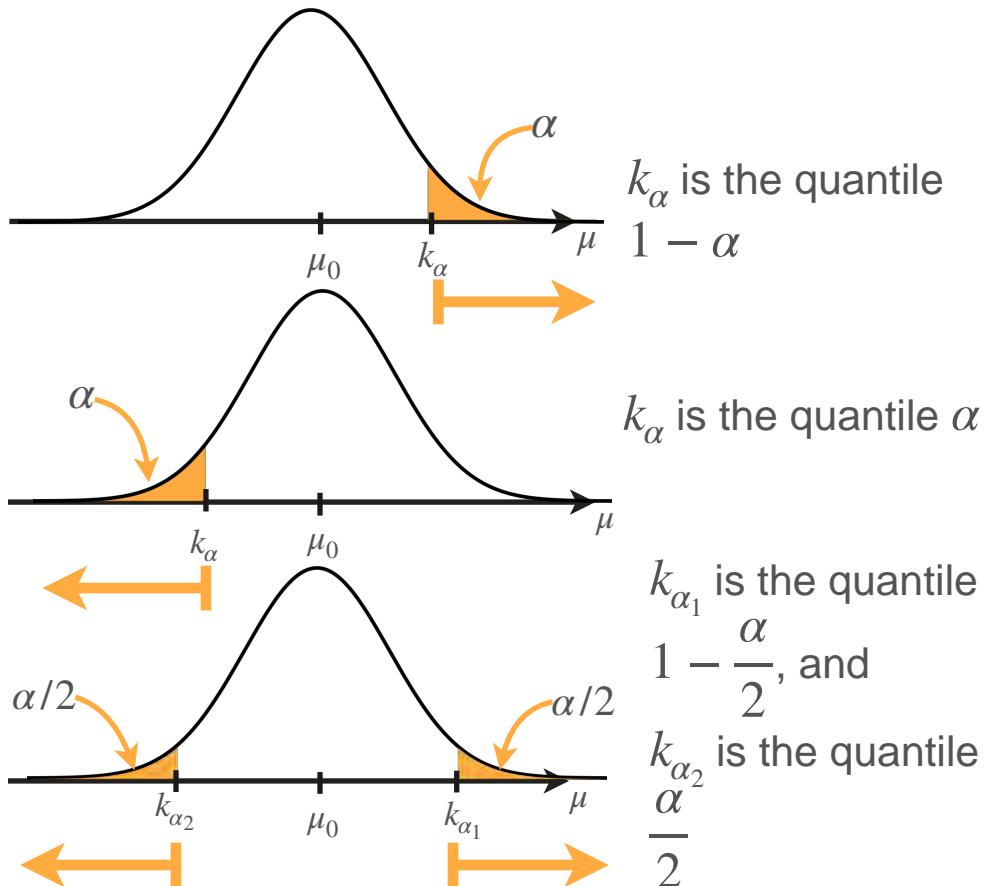
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$H_0 : \mu = \mu_0$  vs.  $H_1 : \mu < \mu_0$

Decision rule: Reject  $H_0$  if  $t < k_\alpha$

$H_0 : \mu = \mu_0$  vs.  $H_1 : \mu \neq \mu_0$

Decision rule: Reject  $H_0$  if  $t > k_{\alpha_1}$  or  
 $t < k_{\alpha_2}$



# Critical Values: Concluding Remarks

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- For a given sample, using  $p$ -value and critical value will lead to the same conclusion
- Defining a test in terms of critical values makes determining Type II error probabilities for the decision rule.



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# Hypothesis Testing

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**Power of a test**

# Type I and Type II Errors

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The **mean** height for 18 y/o in the US in the 70s was **66.7 in.**

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Decision	Reality	
	$H_0$ True ( $\mu = 66.7$ )	$H_0$ False ( $\mu > 66.7$ )
Reject $H_0$ (Decide $\mu > 66.7$ )	Type I error	Correct
Don't reject $H_0$ (Decide $\mu = 66.7$ )	Correct	Type II error

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# Finding the Type II Error Probabilities

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For  $\alpha = 0.05$ :  $k_\alpha = 68.26$

Decision rule: Reject  $H_0$  if  $\bar{x} > 68.26$

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For  $\alpha = 0.05$ :  $k_\alpha = 68.26$

Decision rule: Reject  $H_0$  if  $\bar{x} > 68.26$

What is the **Type II error probability** if the true value is  $\mu = 70$ ?

$$\mathbf{P}(\text{Do not reject } H_0 | \mu = 70) \longrightarrow \mathbf{P}(\bar{X} < 68.26 | \mu = 70)$$

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The **mean** height for 18 y/o in the US in the 70s was **66.7 in.**

$$H_0 : \mu = 66.7 \text{ vs. } H_1 : \mu > 66.7 \quad n = 10 \quad \sigma = 3$$

For  $\alpha = 0.05$ :  $k_\alpha = 68.2604$       Decision rule: Reject  $H_0$  if  $\bar{X} > 68.26$

What is the **Type II error probability** if the true value is  $\mu = 70$ ?

$$\mathbf{P}(\bar{X} < 68.26 | \mu = 70)$$

# Finding the Type II Error Probabilities

The mean height for 18 y/o in the US in the 70s was **66.7 in.**

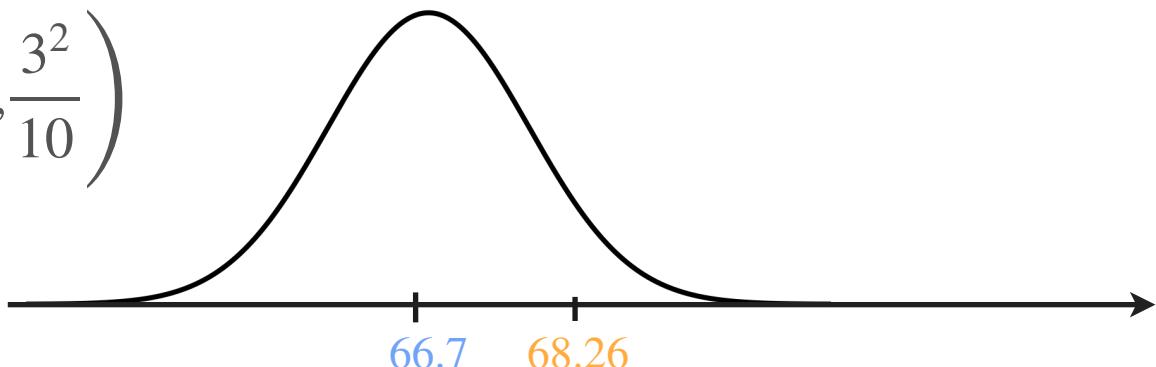
$$H_0 : \mu = 66.7 \text{ vs. } H_1 : \mu > 66.7 \quad n = 10 \quad \sigma = 3$$

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What is the **Type II error probability** if the true value is  $\mu = 70$ ?

If  $\mu = 66.7$   $\bar{X} \sim \mathcal{N}\left(66.7, \frac{3^2}{10}\right)$

$$\mathbf{P}(\bar{X} < 68.26 | \mu = 70)$$



# Finding the Type II Error Probabilities

The mean height for 18 y/o in the US in the 70s was **66.7 in.**

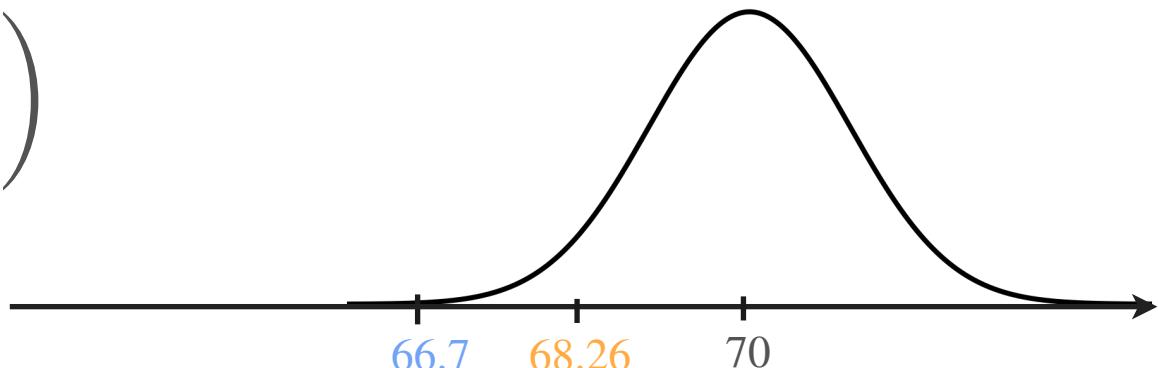
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For  $\alpha = 0.05$ :  $k_\alpha = 68.2604$       Decision rule: Reject  $H_0$  if  $\bar{X} > 68.26$

What is the **Type II error probability** if the true value is  $\mu = 70$ ?

$$\text{If } \mu = 70 \quad \bar{X} \sim \mathcal{N}\left(70, \frac{3^2}{10}\right)$$

$$\mathbf{P}(\bar{X} < 68.26 | \mu = 70)$$



# Finding the Type II Error Probabilities

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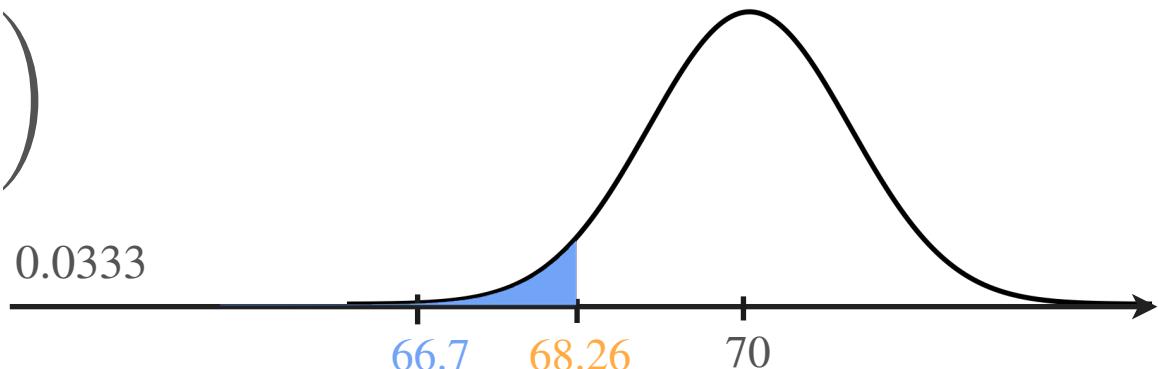
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What is the **Type II error probability** if the true value is  $\mu = 70$ ?

$$\text{If } \mu = 70 \quad \bar{X} \sim \mathcal{N}\left(70, \frac{3^2}{10}\right)$$

$$\mathbf{P}(\bar{X} < 68.26 | \mu = 70) = 0.0333$$



# Finding the Type II Error Probabilities

The mean height for 18 y/o in the US in the 70s was **66.7 in.**

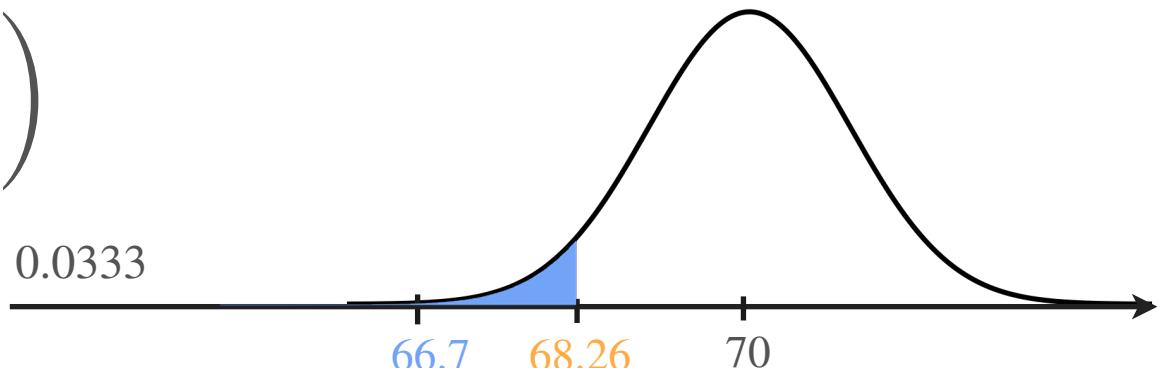
$$H_0 : \mu = 66.7 \text{ vs. } H_1 : \mu > 66.7 \quad n = 10 \quad \sigma = 3$$

For  $\alpha = 0.05$ :  $k_\alpha = 68.2604$       Decision rule: Reject  $H_0$  if  $\bar{X} > 68.26$

What is the **Type II error probability** if the true value is  $\mu = 70$ ?

$$\text{If } \mu = 70 \quad \bar{X} \sim \mathcal{N}\left(70, \frac{3^2}{10}\right)$$

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The mean height for 18 y/o in the US in the 70s was **66.7 in.**

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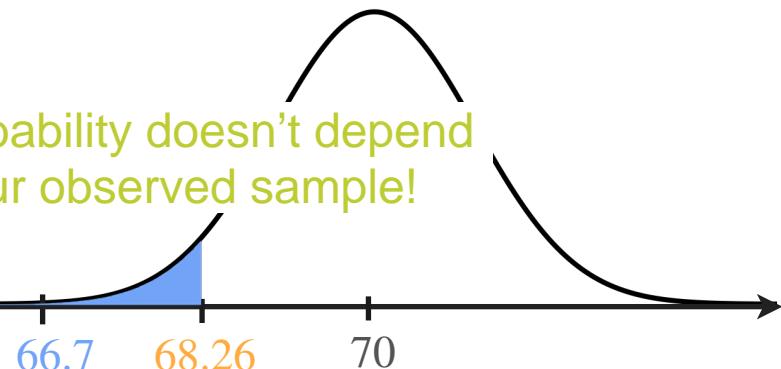
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What is the **Type II error probability** if the true value is  $\mu = 70$ ?

$$\text{If } \mu = 70 \quad \bar{X} \sim \mathcal{N}\left(70, \frac{3^2}{10}\right)$$

$$\beta = P(\bar{X} < 68.26 | \mu = 70) = 0.0333$$

This probability doesn't depend on your observed sample!



# Power of the Test

The **mean** height for 18 y/o in the US in the 70s was **66.7 in.**

$$H_0 : \mu = 66.7 \text{ vs. } H_1 : \mu > 66.7$$

Power of the test

$$\mathbf{P}(\text{Reject } H_0 | \mu \in H_1)$$

# Power of the Test

The **mean** height for 18 y/o in the US in the 70s was **66.7 in.**

$$H_0 : \mu = 66.7 \text{ vs. } H_1 : \mu > 66.7$$

Decision	Reality		Power of the test $P(\text{Reject } H_0   \mu \in H_1)$
	$H_0$ True ( $\mu = 66.7$ )	$H_0$ False ( $\mu > 66.7$ )	
Reject $H_0$ (Decide $\mu > 66.7$ )	Type I error	Correct	
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# Power of the Test



# Power of the Test

Type II error:  $\overbrace{\mathbf{P}(\text{Do not reject } H_0 | \mu \in H_1)}$

# Power of the Test

Type II error:  $\overbrace{\mathbf{P}(\text{Do not reject } H_0 | \mu \in H_1)}^{\beta}$

Power of the test:  $\overbrace{\mathbf{P}(\text{Reject } H_0 | \mu \in H_1)}^{1 - \beta}$

# Power of the Test

$$\left. \begin{array}{l} \text{Type II error: } \mathbf{P} \left( \text{Do not reject } H_0 \mid \mu \in H_1 \right) \\ \text{Power of the test: } \mathbf{P} \left( \text{Reject } H_0 \mid \mu \in H_1 \right) \end{array} \right\} \overset{\beta}{\overbrace{\phantom{\text{Type II error: } \mathbf{P} \left( \text{Do not reject } H_0 \mid \mu \in H_1 \right)}}^{\text{Complementary probabilities}}}$$

# Power of the Test

$$\left. \begin{array}{l} \text{Type II error: } \mathbf{P} \left( \text{Do not reject } H_0 \mid \mu \in H_1 \right) \\ \text{Power of the test: } \mathbf{P} \left( \text{Reject } H_0 \mid \mu \in H_1 \right) \end{array} \right\} \begin{array}{l} \beta \\ 1 - \beta \end{array}$$

Complementary probabilities

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Complementary probabilities

Power of the test = 1 – Type II error probability

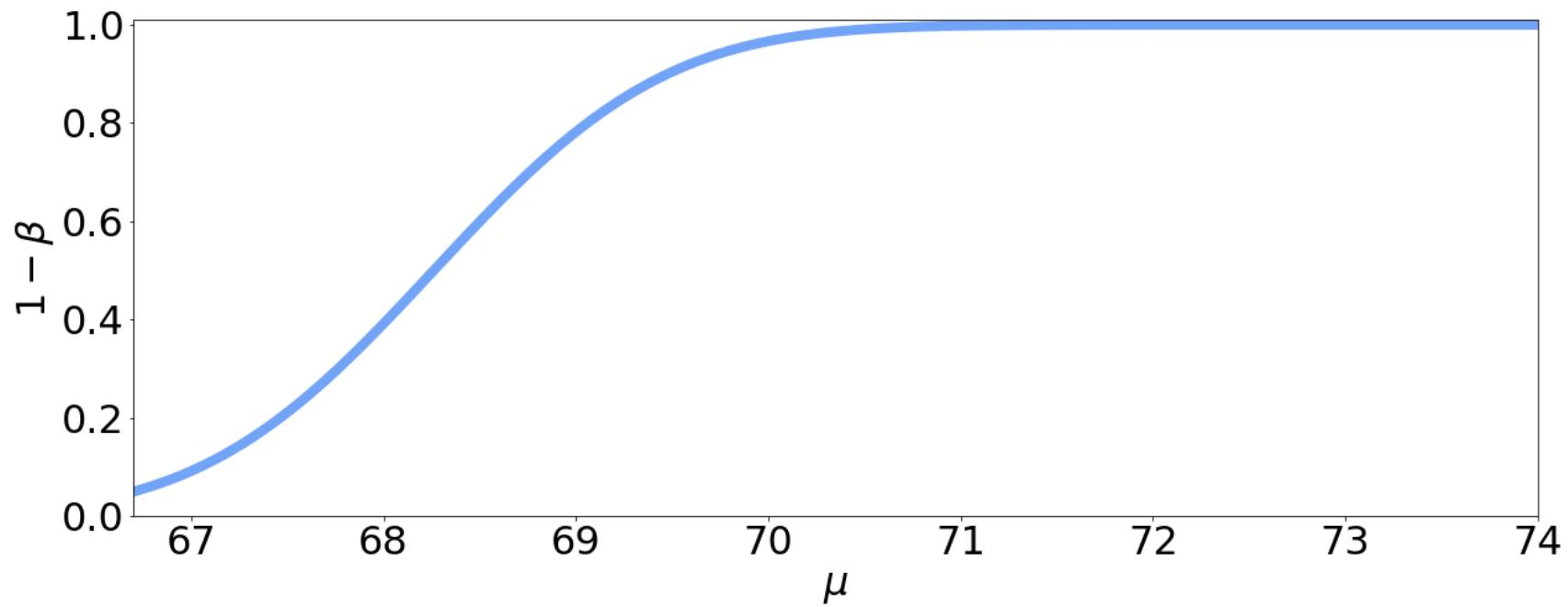
$$= 1 - \mathbf{P} \left( \text{Do not reject } H_0 \mid \mu \in H_1 \right)$$

# Power of the Test

$H_0 : \mu = 66.7$  vs.  $H_1 : \mu > 66.7$

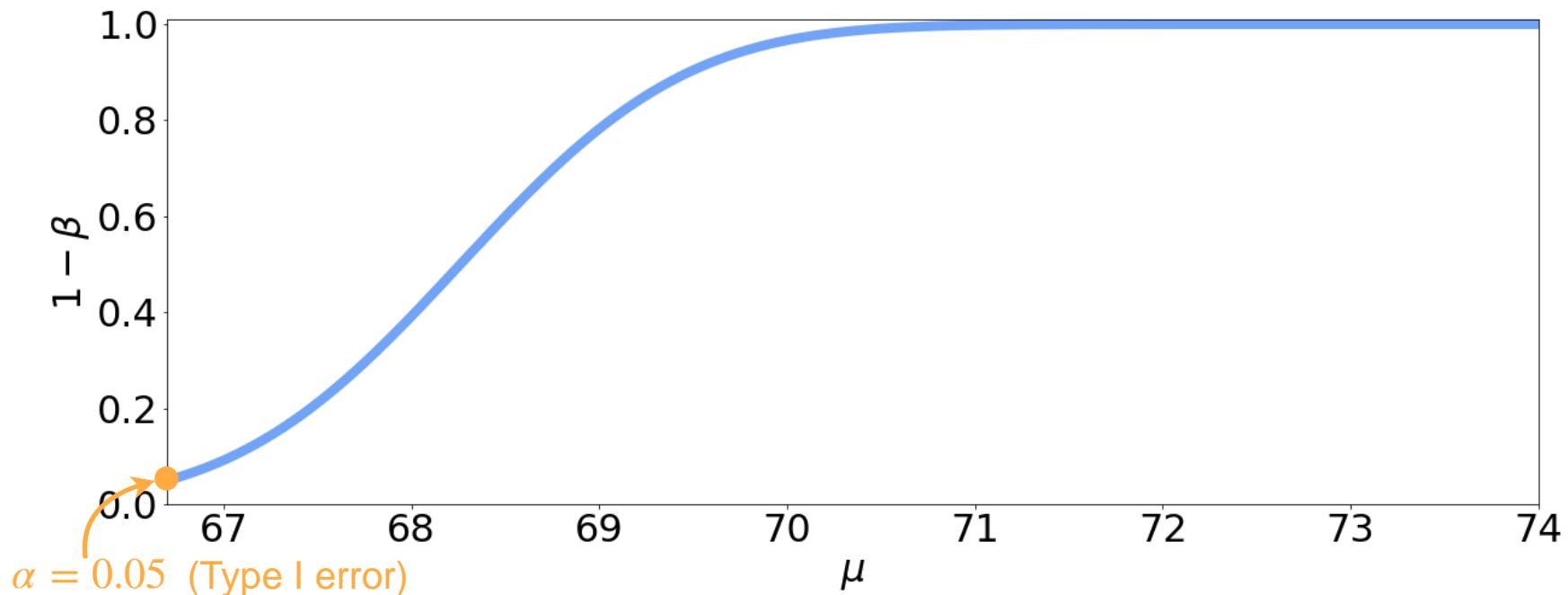
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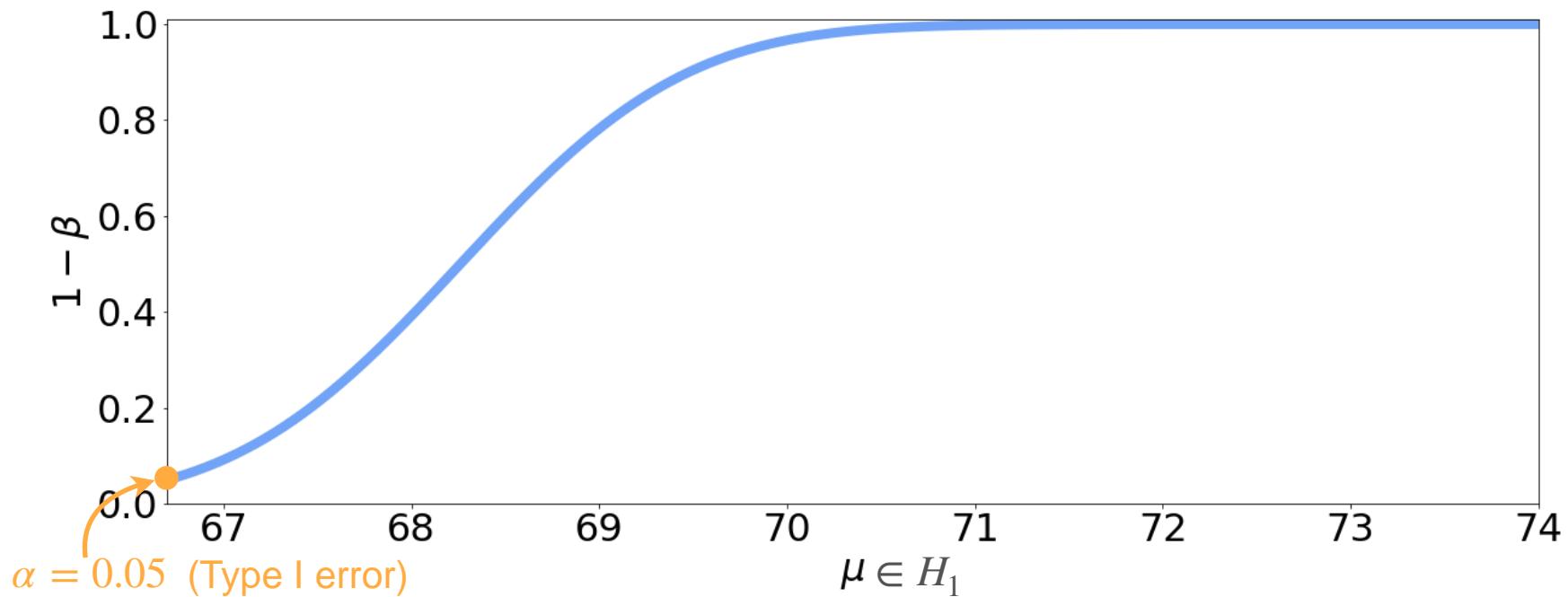
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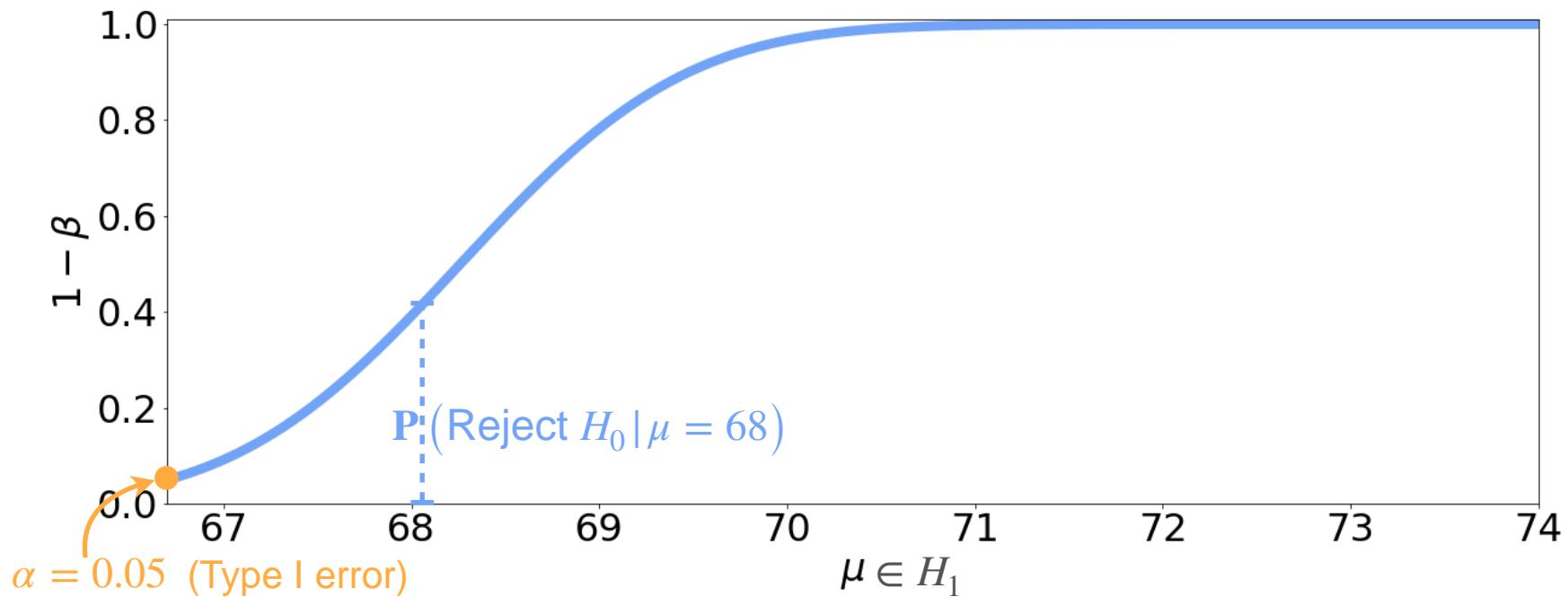
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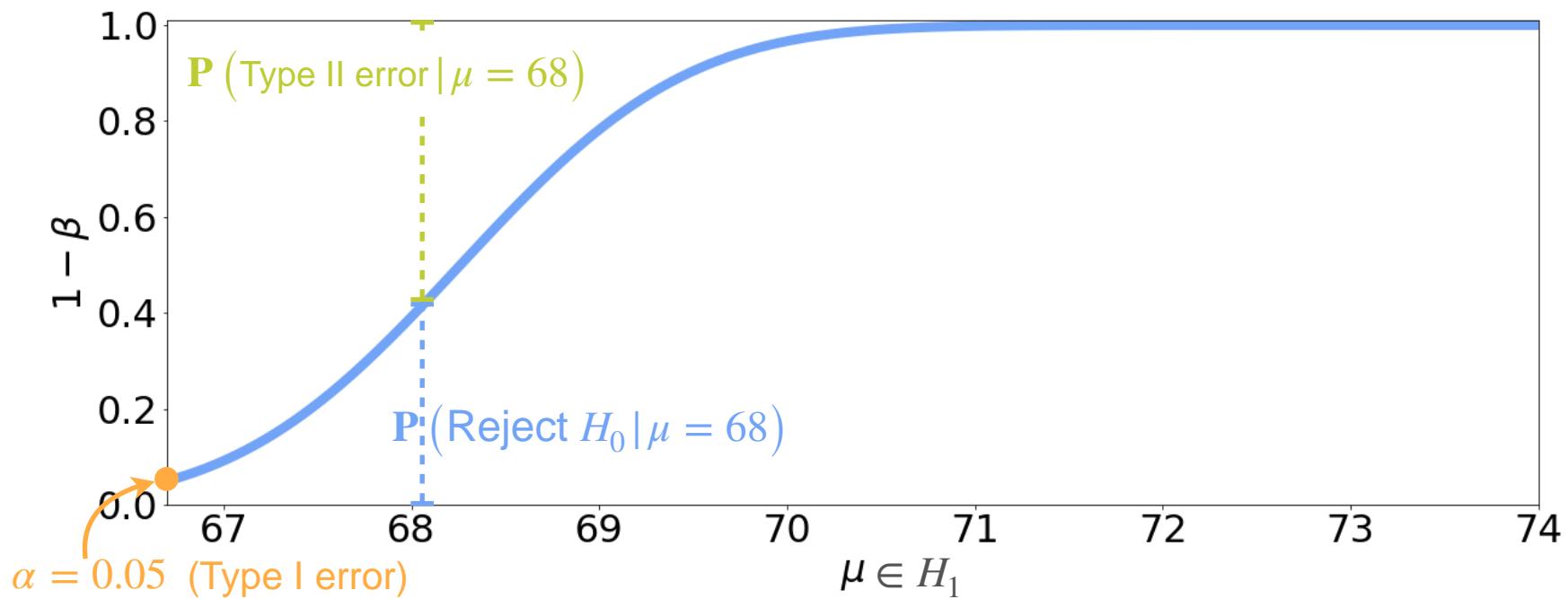
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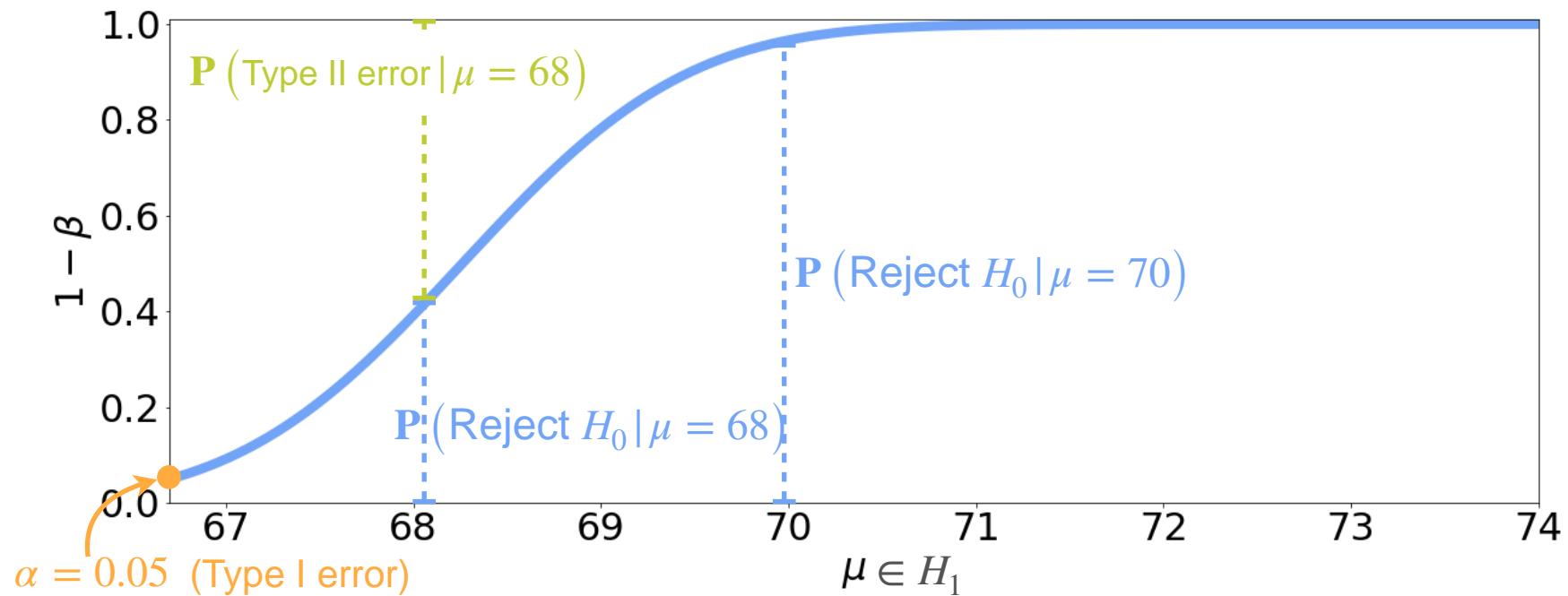
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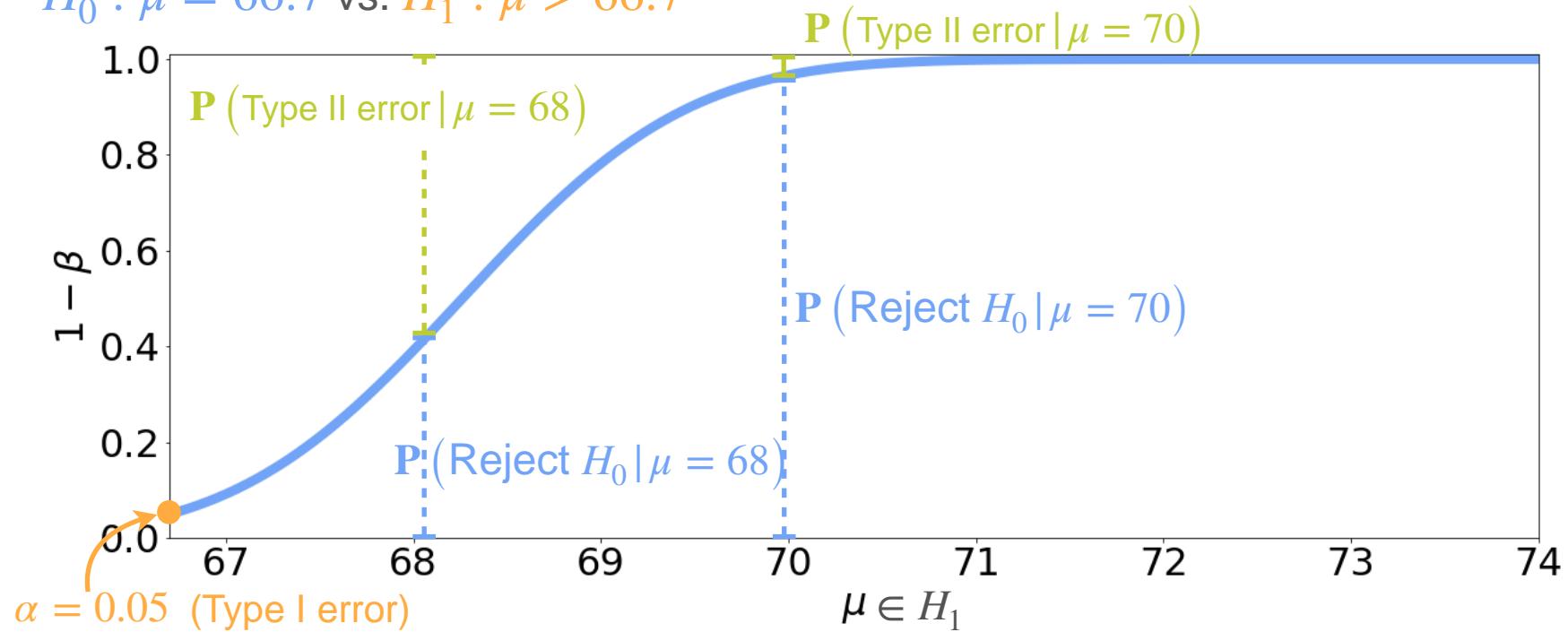
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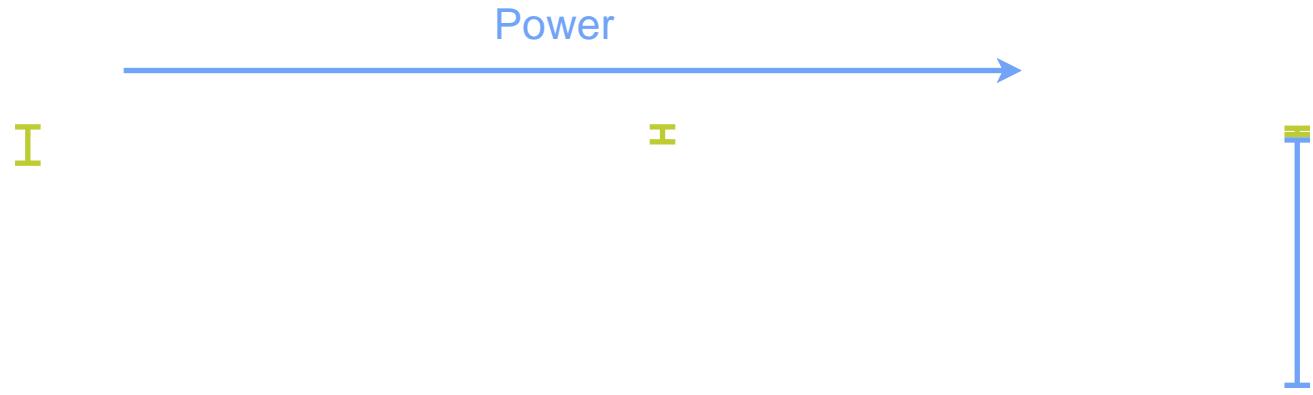


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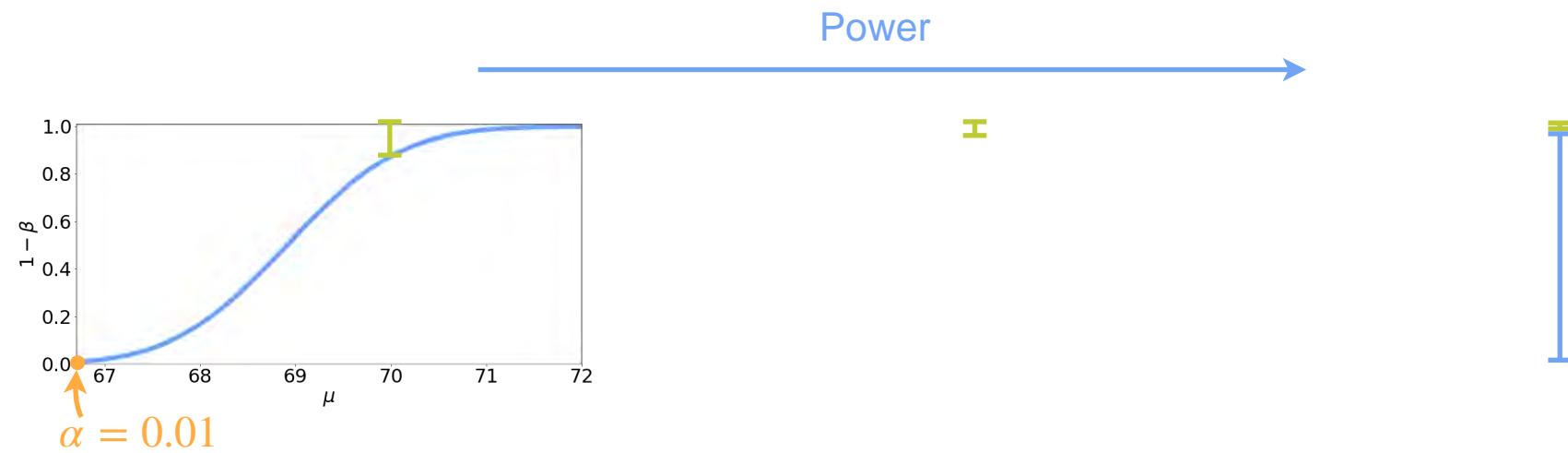
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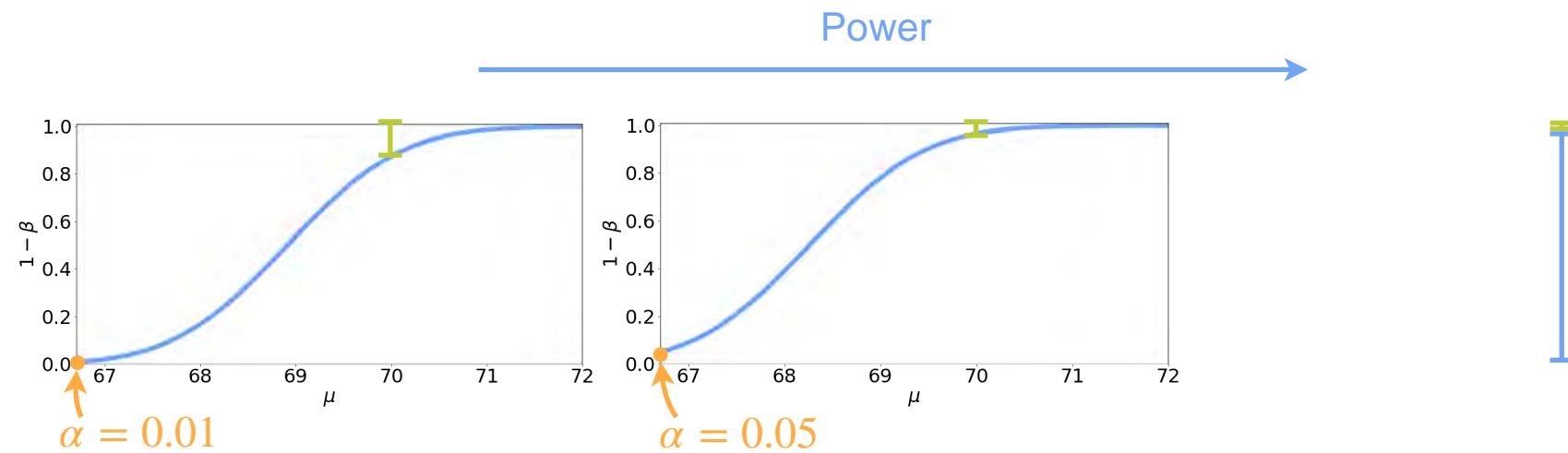
# Power of the Test



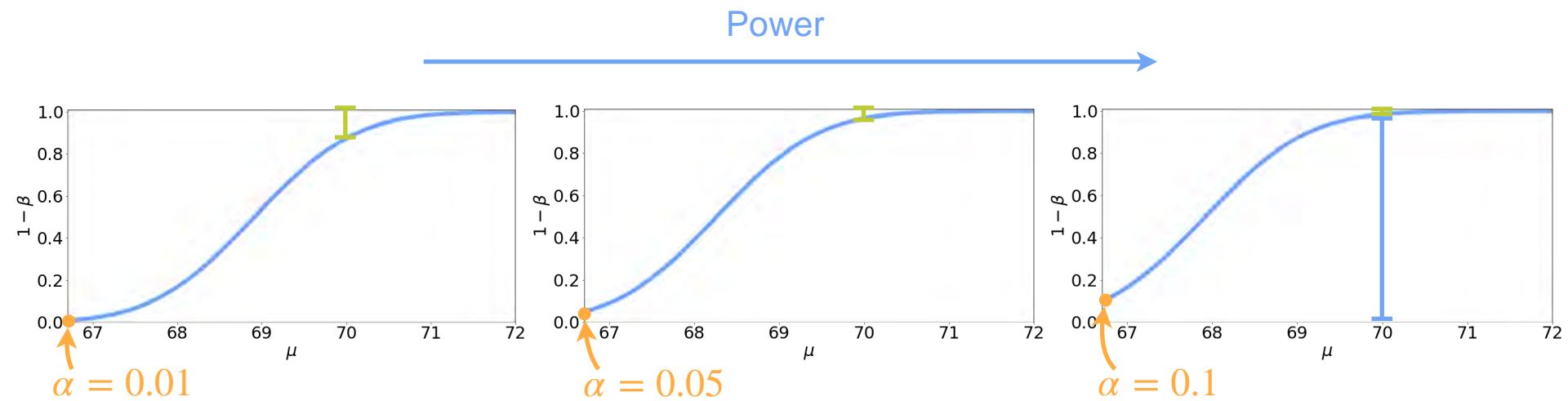
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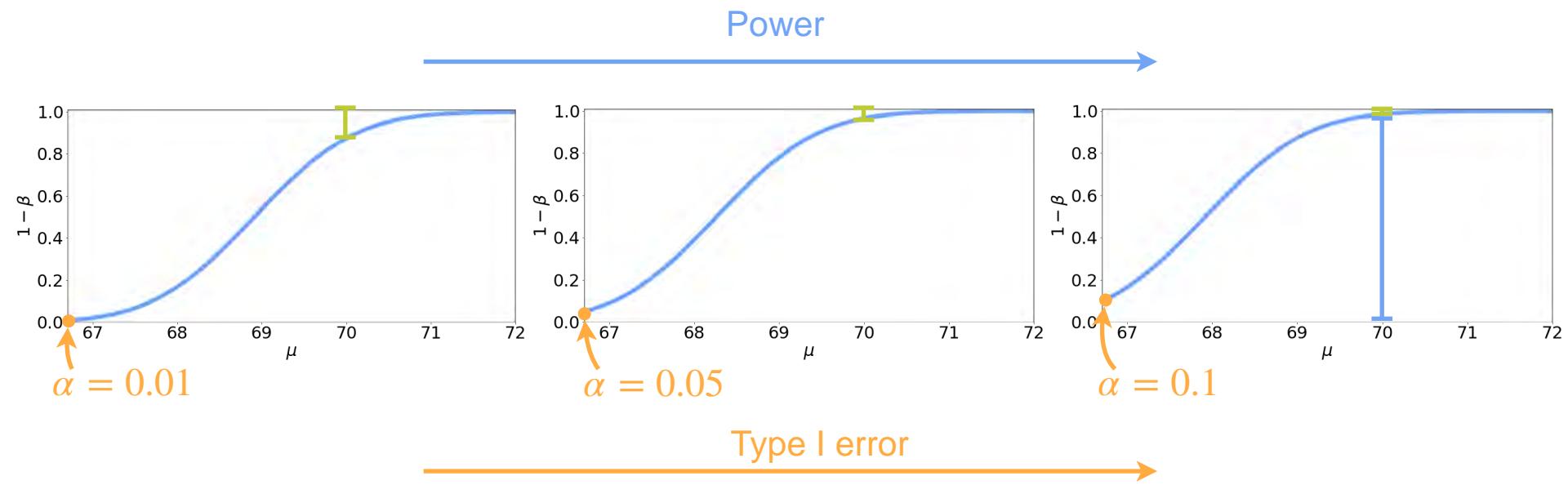
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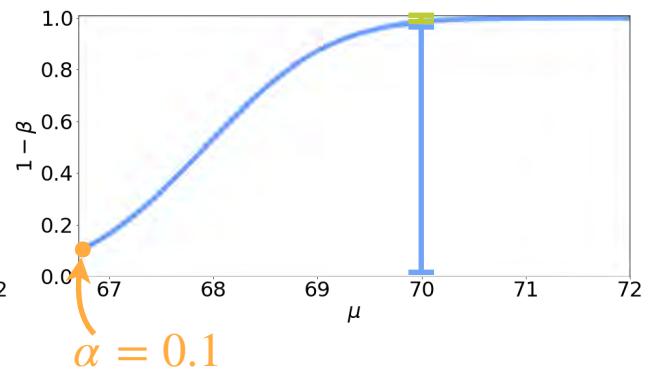
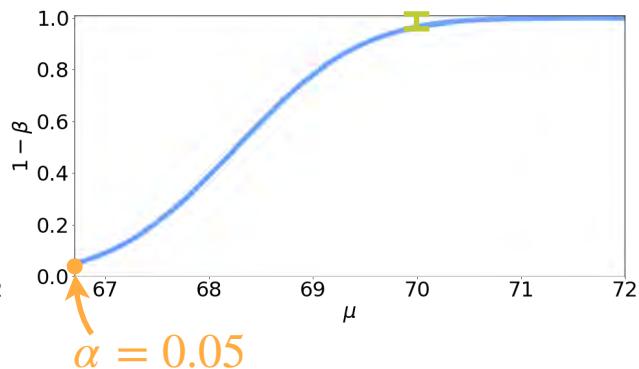
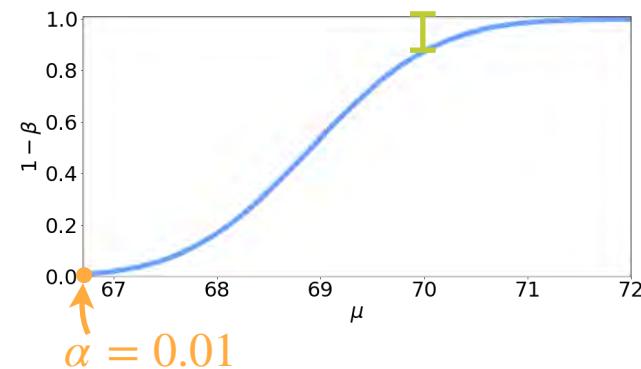
# Power of the Test



# Power of the Test

$$\mu = 70$$

Power

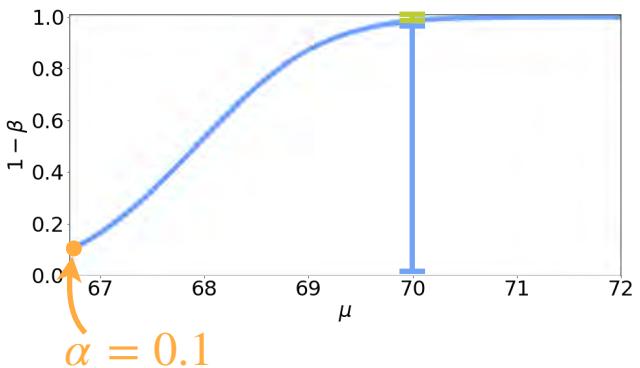
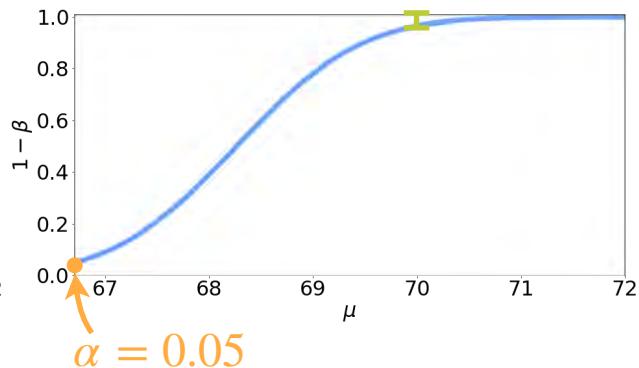
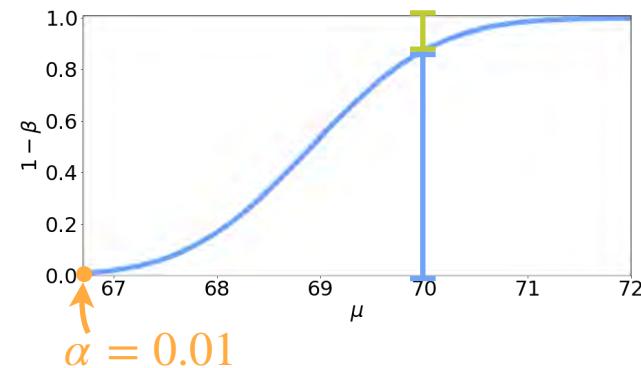


Type I error

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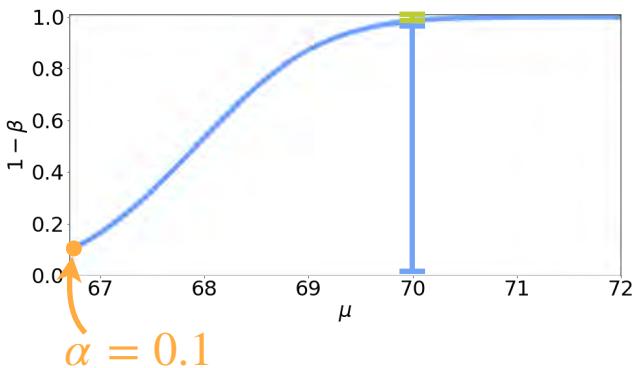
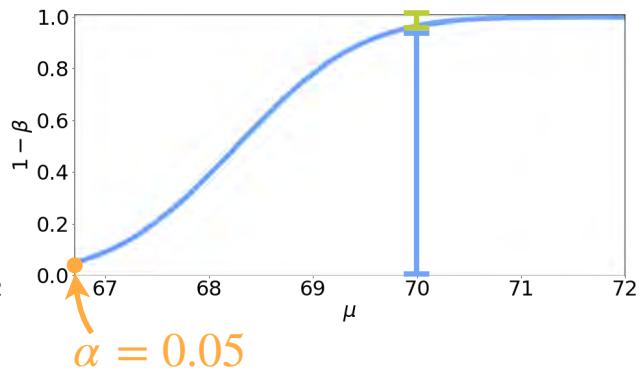
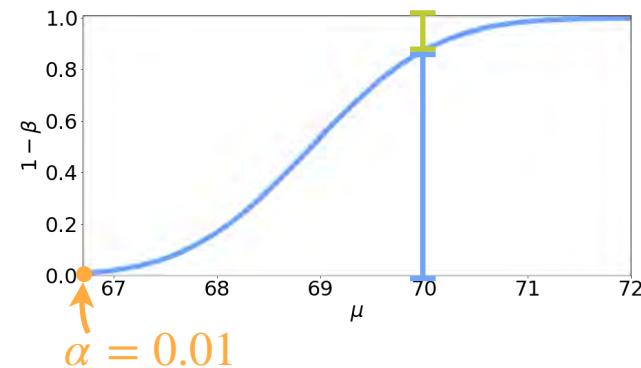


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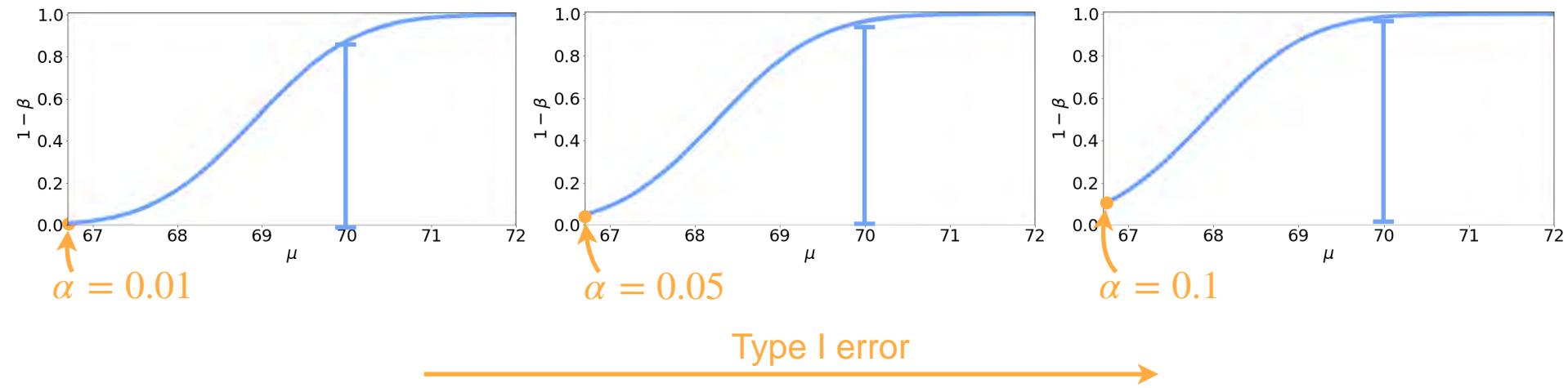
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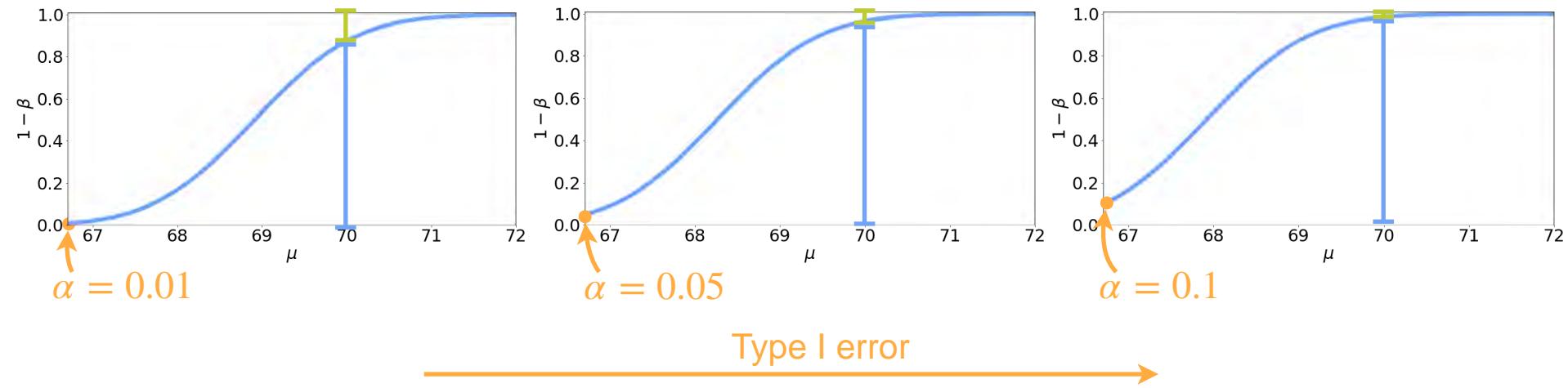
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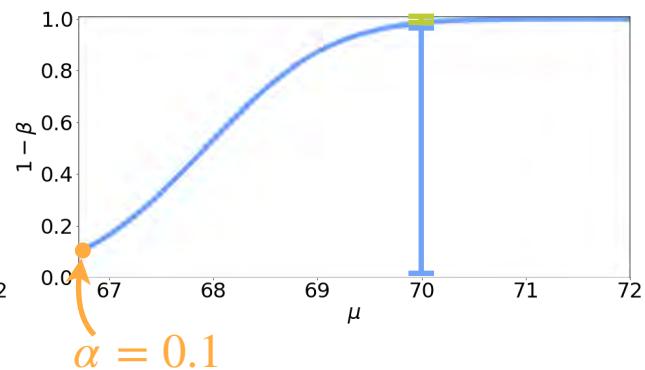
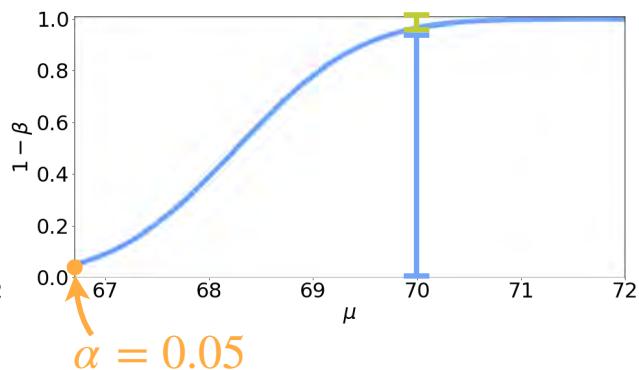
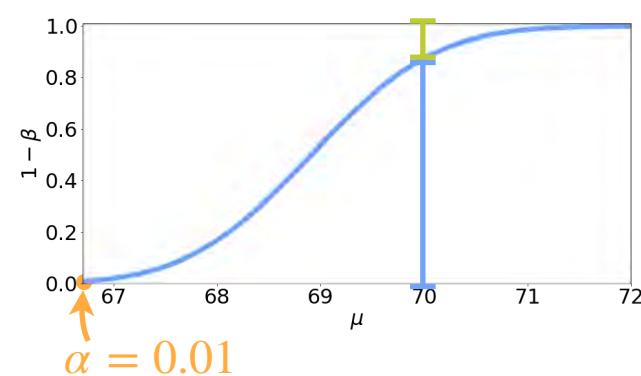
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# Power of the Test

$$\mu = 70$$

Type II error



Type I error



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## Hypothesis Testing

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### Interpreting results

# Steps for Performing Hypothesis Testing

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## 4. Reach a conclusion:

- If the  $p$ -value is less than the significance level reject  $H_0$   
 $\rightarrow P(\bar{X} > 68.442 | \mu = 66.7) > 0.05$

# Important Remarks - Interpreting Tests

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- $\downarrow \alpha \uparrow \beta$

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- *p*-values:

- If  $\mathbf{P}(\text{Reject } H_0 | H_0) < \alpha \longrightarrow \text{Reject } H_0 \text{ and then accept } H_1$

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You can only say that there is not enough evidence



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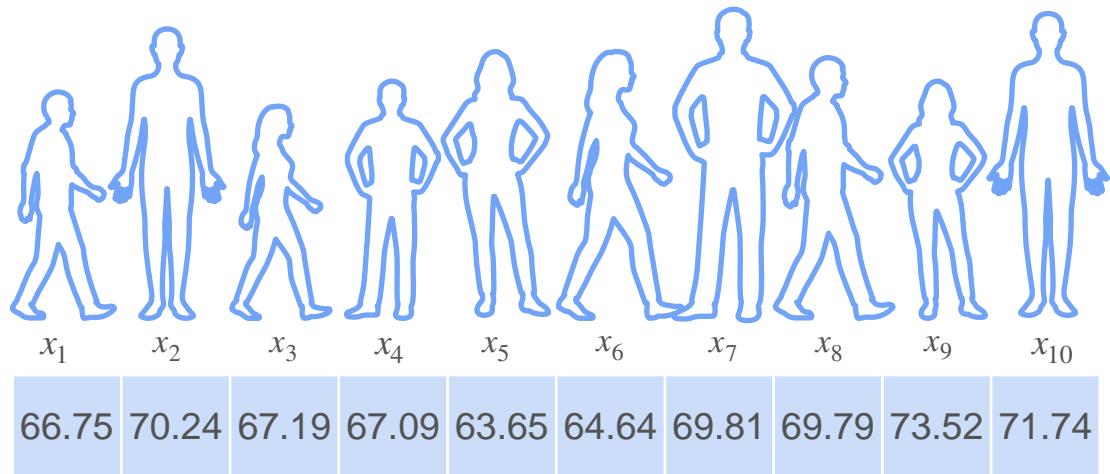
# Hypothesis Testing

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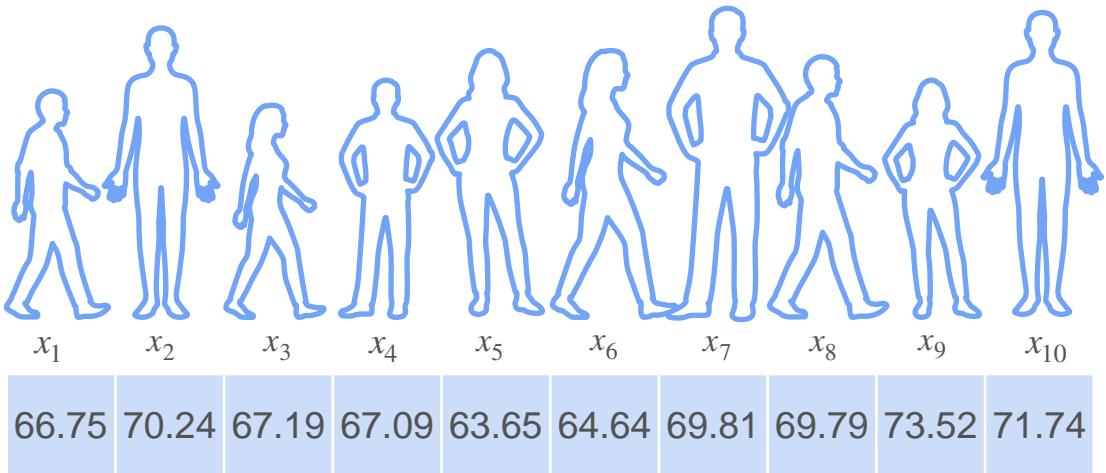
## t-Distribution

# $t$ -Distribution: Motivation

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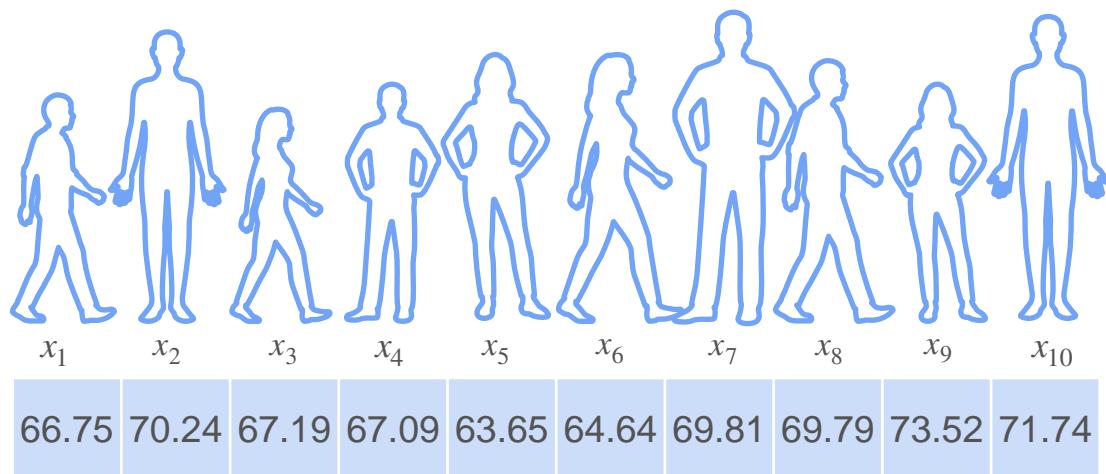


# $t$ -Distribution: Motivation



$$X_i \stackrel{i.i.d}{\sim} \mathcal{N}(\mu, \sigma^2)$$

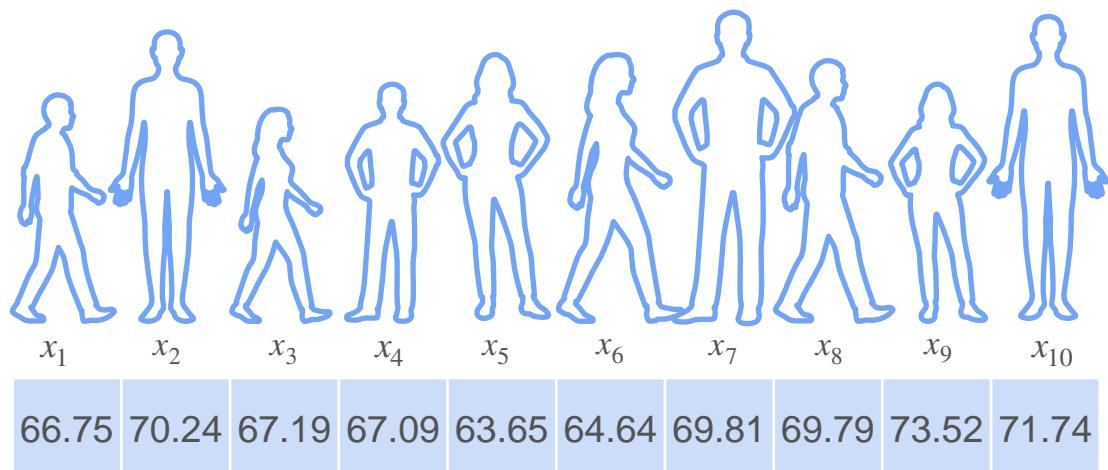
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This is fine if you know  $\mu$  and  $\sigma$

What if  $\sigma$  is unknown?

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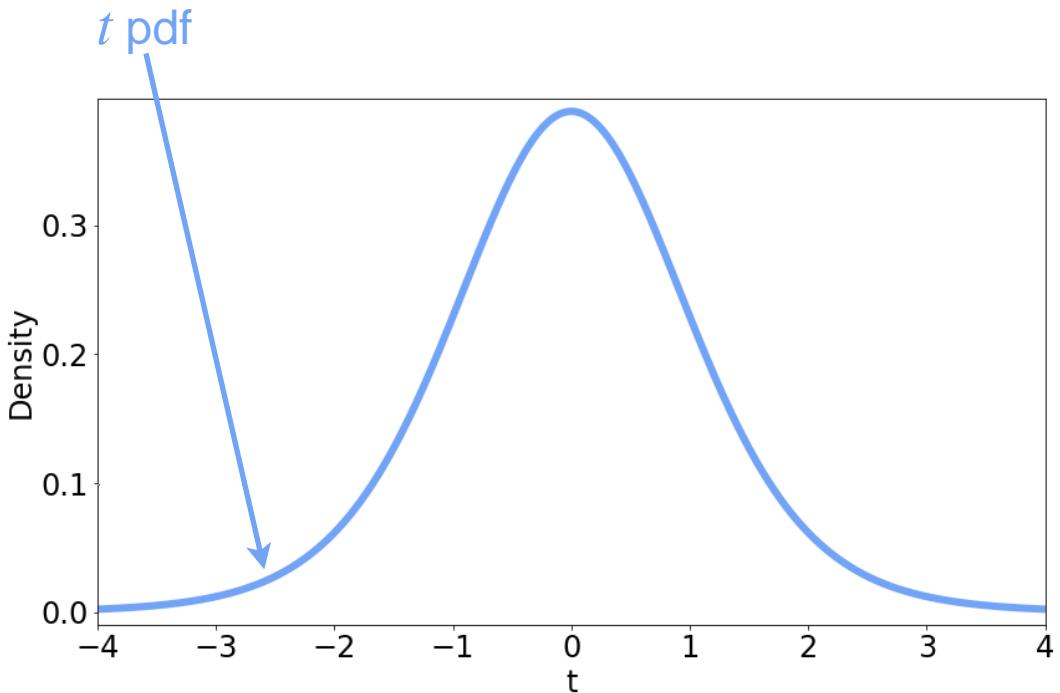
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$\frac{\bar{X} - \mu}{S/\sqrt{10}}$  follows a *t* distribution

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What does it look like?

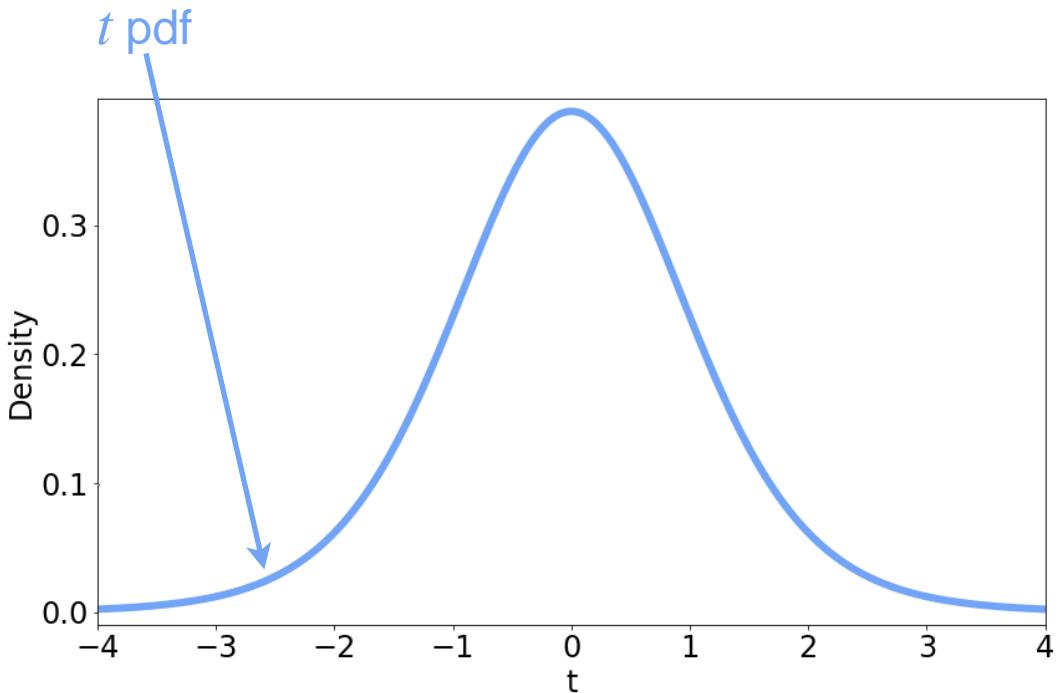


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$\frac{\bar{X} - \mu}{S/\sqrt{10}}$  follows a  $t$  distribution

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Still bell-shaped



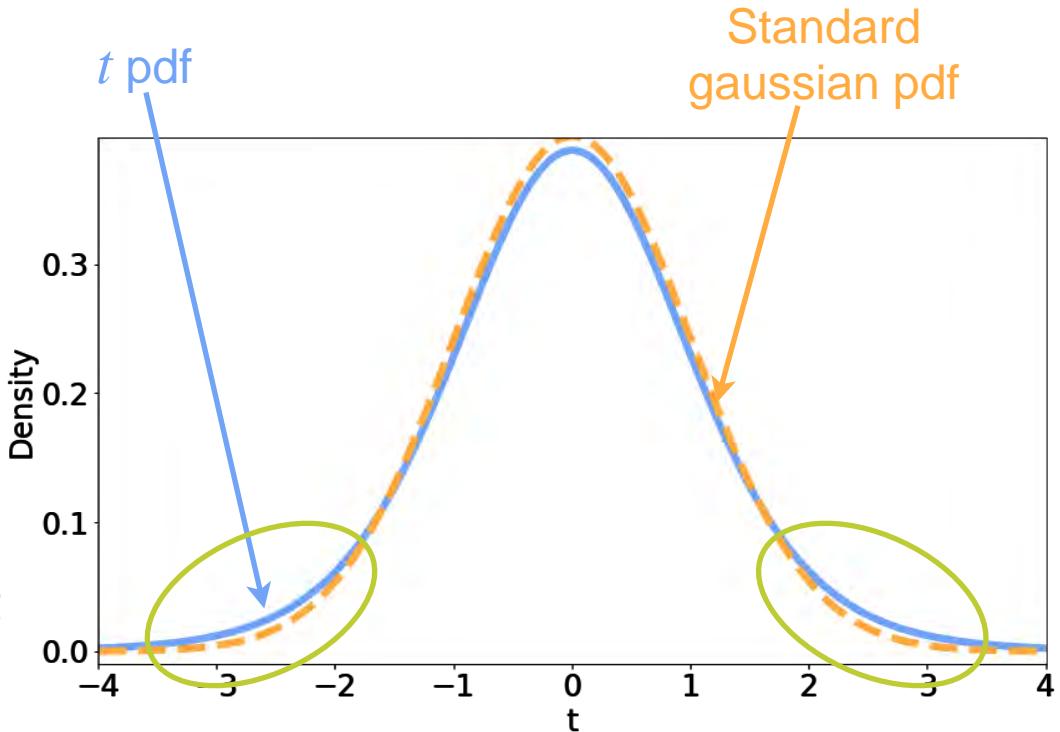
# $t$ -Distribution

$\frac{\bar{X} - \mu}{S/\sqrt{10}}$  follows a  $t$  distribution

What does it look like?

Still bell-shaped

It has heavier tails that account for the uncertainty introduced with the std estimation



# $t$ -Distribution

# *t*-Distribution

## Parameters:

- Degrees of freedom

# *t*-Distribution

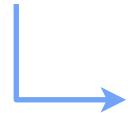
## Parameters:

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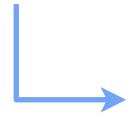


Controls how heavy  
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# *t*-Distribution

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Controls how heavy  
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$$X \sim t_{\nu}$$

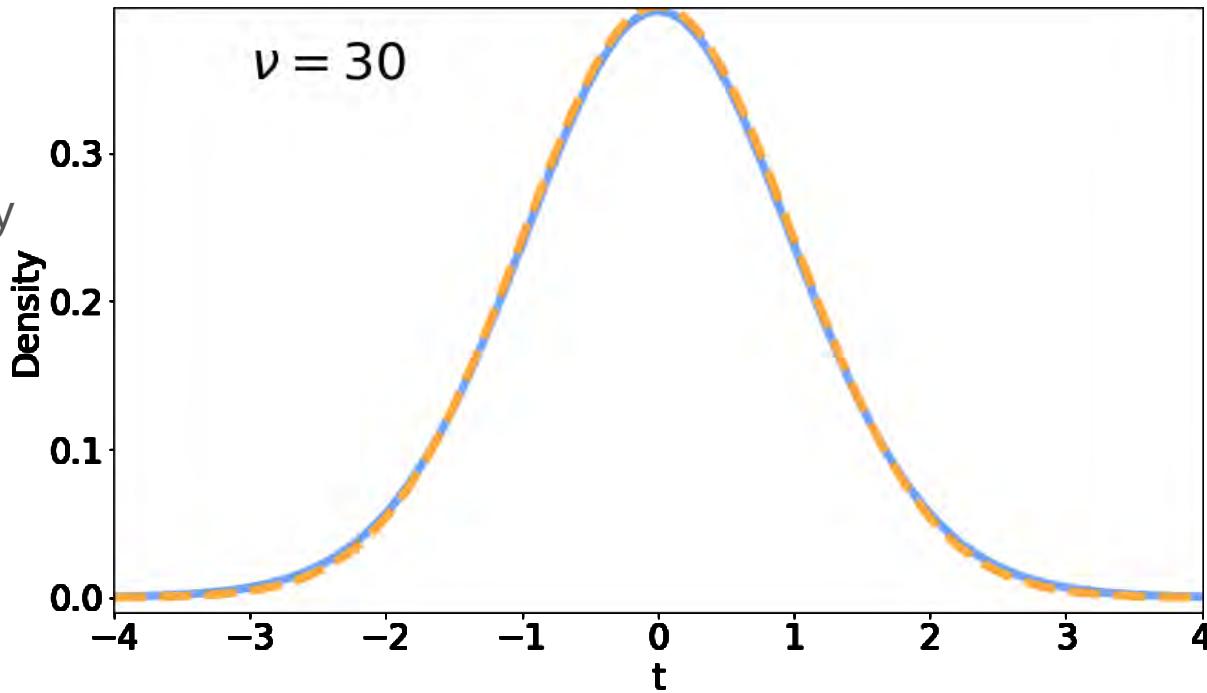
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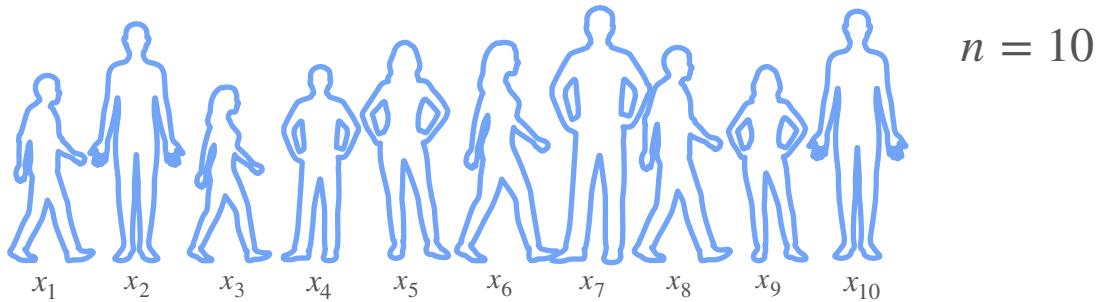
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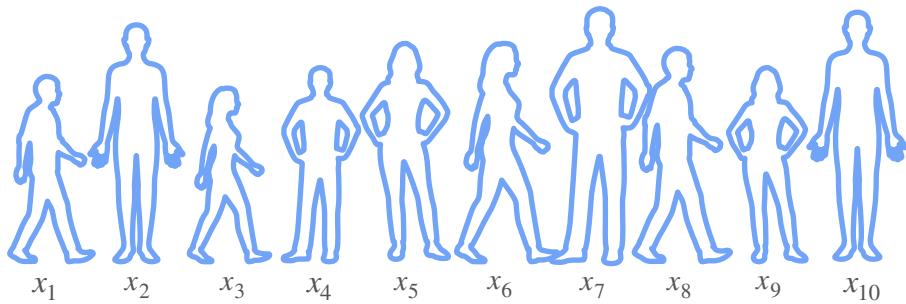


# $t$ -Distribution and $T$ -Statistic

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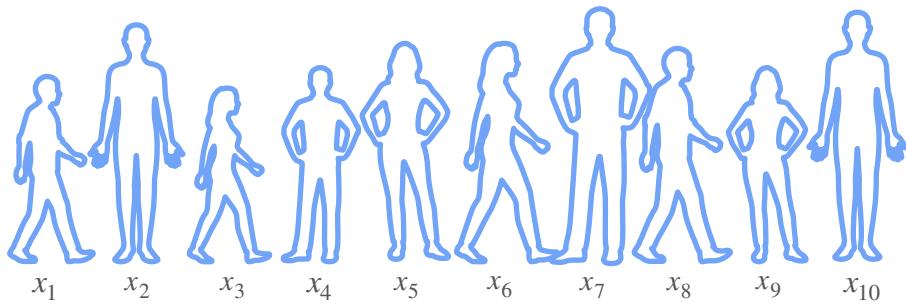
# $t$ -Distribution and $T$ -Statistic



$$n = 10$$

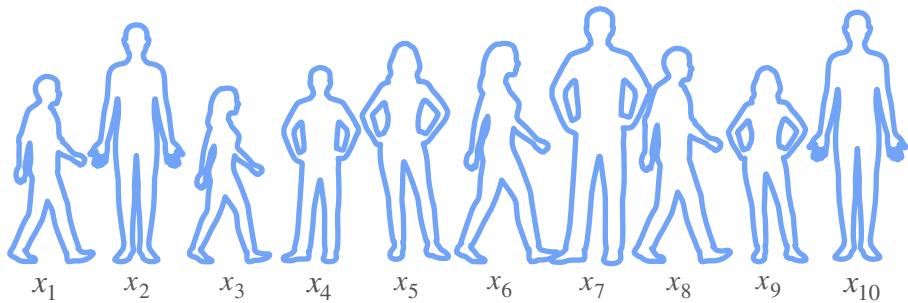
$$T = \frac{\bar{X} - \mu}{S/\sqrt{10}} \sim t_{\nu}$$

# $t$ -Distribution and $T$ -Statistic



$$n = 10$$
$$\nu = 10 - 1$$
$$T = \frac{\bar{X} - \mu}{S/\sqrt{10}} \sim t_9$$

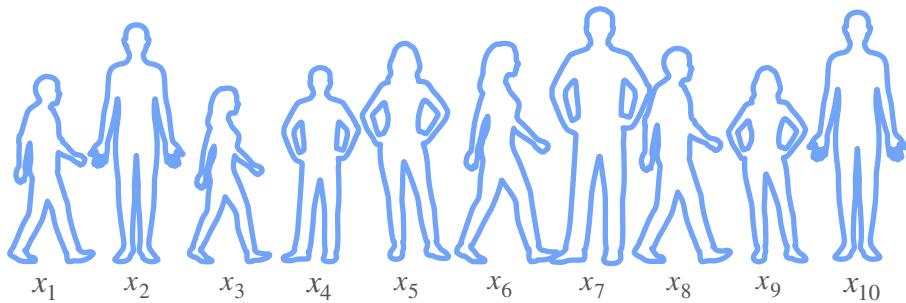
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Degrees of freedom ( $\nu$ ) = sample size - 1  
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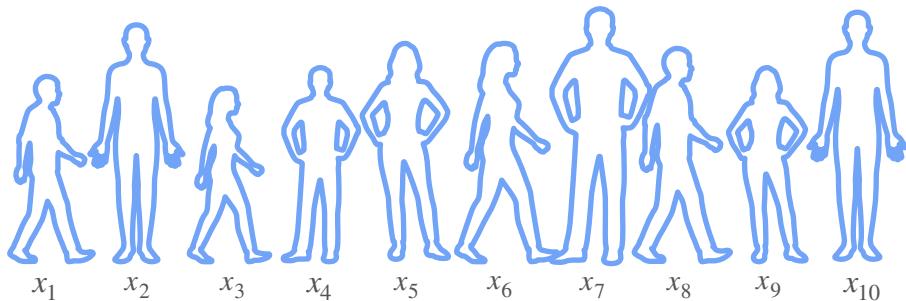


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$T$ -statistic is used when

- The population has a Gaussian distribution
- But you don't know the variance



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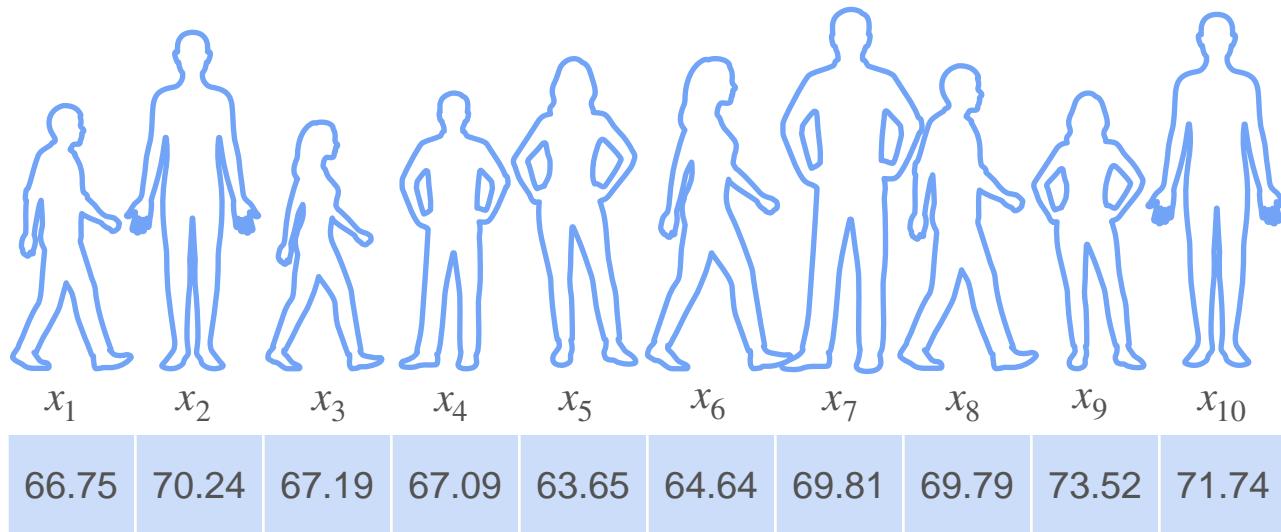
# Hypothesis Testing

---

## t-Tests

# Example: Heights

# Example: Heights



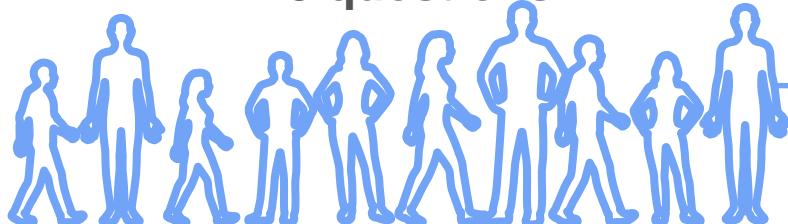
$$\bar{x} = 68.442$$

# Example: Heights

# Example: Heights

The **mean** height for 18 y/o in the US in the 70s was **66.7 in.**

**3 questions**



**3 sets of hypothesis**

$$H_0 : \mu = 66.7 \text{ vs. } H_1 : \mu > 66.7$$



$$H_0 : \mu = 66.7 \text{ vs. } H_1 : \mu < 66.7$$



$$H_0 : \mu = 66.7 \text{ vs. } H_1 : \mu \neq 66.7$$

# Example: Heights

The **mean** height for 18 y/o in the US in the 70s was **66.7 in.**

# Example: Heights

The **mean** height for 18 y/o in the US in the 70s was **66.7 in.**

$$\sigma = 3$$

$$n = 10$$

$$H_0 : \mu = 66.7$$

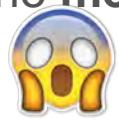
$$\text{If } H_0 \text{ is true: } \bar{X} \sim \mathcal{N}\left(66.7, \frac{3^2}{10}\right)$$

Null hypothesis



# Example: Heights

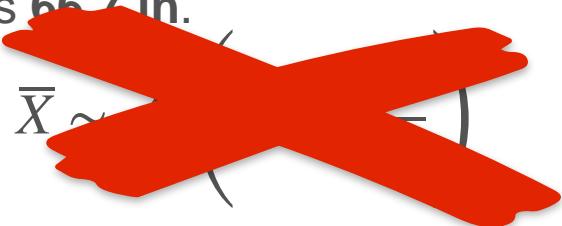
The **mean** height for 18 y/o in the US in the 70s was 66.7 in.



~~n = 10~~

$$H_0 : \mu = 66.7$$

If  $H_0$  is true:  $\bar{X} \sim$



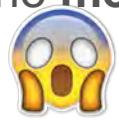
Null hypothesis

66.7

$\mu$

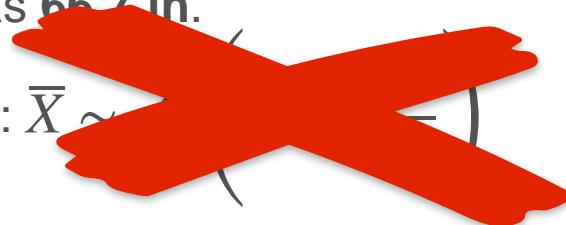
# Example: Heights

The **mean** height for 18 y/o in the US in the 70s was 66.7 in.



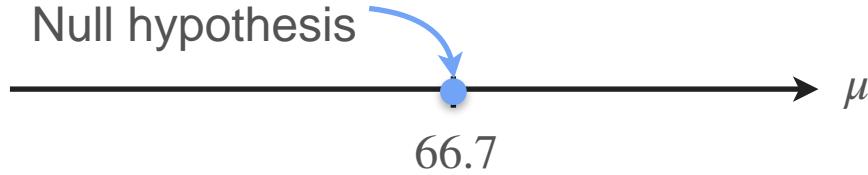
~~n = 10~~

$$H_0 : \mu = 66.7$$



If  $H_0$  is true:  $\bar{X} \sim$

Null hypothesis



$$\text{If } H_0 \text{ is true: } T = \frac{\bar{X} - 66.7}{S/\sqrt{10}} \sim t_9$$

# Example: Heights

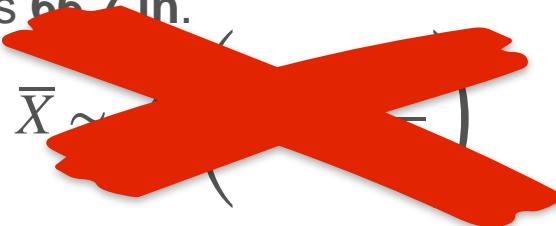
The **mean** height for 18 y/o in the US in the 70s was 66.7 in.



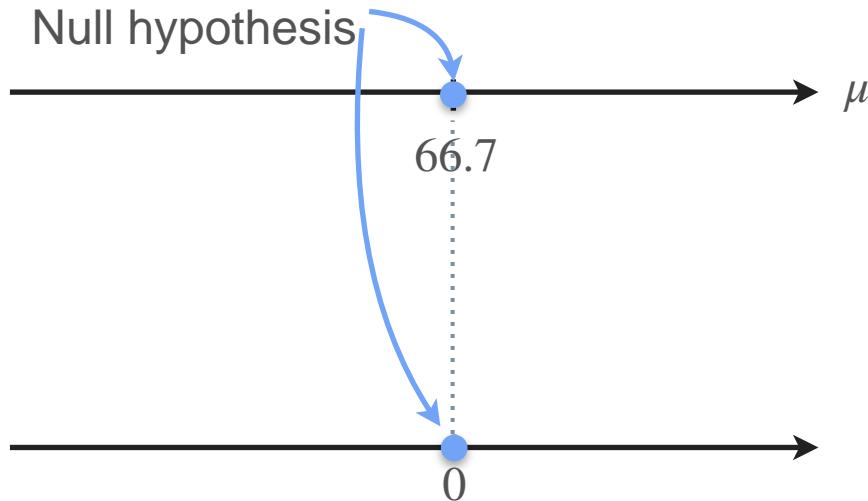
~~n = 10~~

$$H_0 : \mu = 66.7$$

If  $H_0$  is true:  $\bar{X} \sim$



Null hypothesis



$$\text{If } H_0 \text{ is true: } T = \frac{\bar{X} - 66.7}{S/\sqrt{10}} \sim t_9$$

# Example: Heights

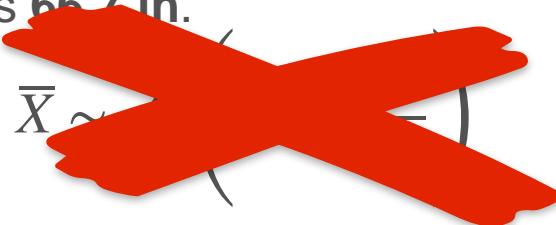
The mean height for 18 y/o in the US in the 70s was 66.7 in.



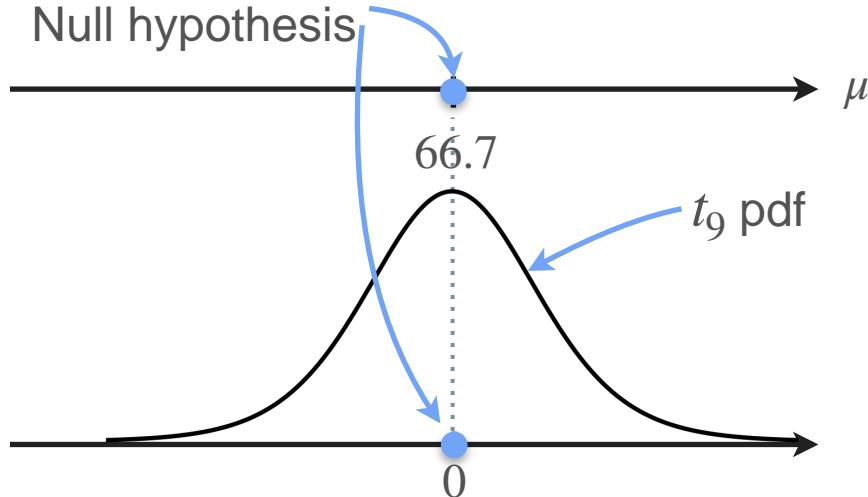
~~$n = 10$~~

$$H_0 : \mu = 66.7$$

If  $H_0$  is true:  $\bar{X} \sim$



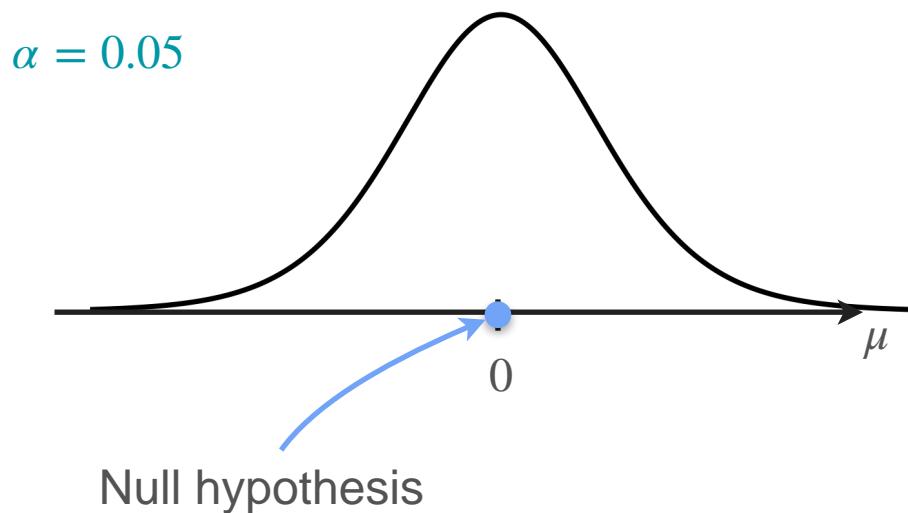
Null hypothesis



$$\text{If } H_0 \text{ is true: } T = \frac{\bar{X} - 66.7}{S/\sqrt{10}} \sim t_9$$

# Right-Tailed Test for Gaussian Data (Unknown $\sigma$ )

The mean height for 18 y/o in the US in the 70s was **66.7 in.**

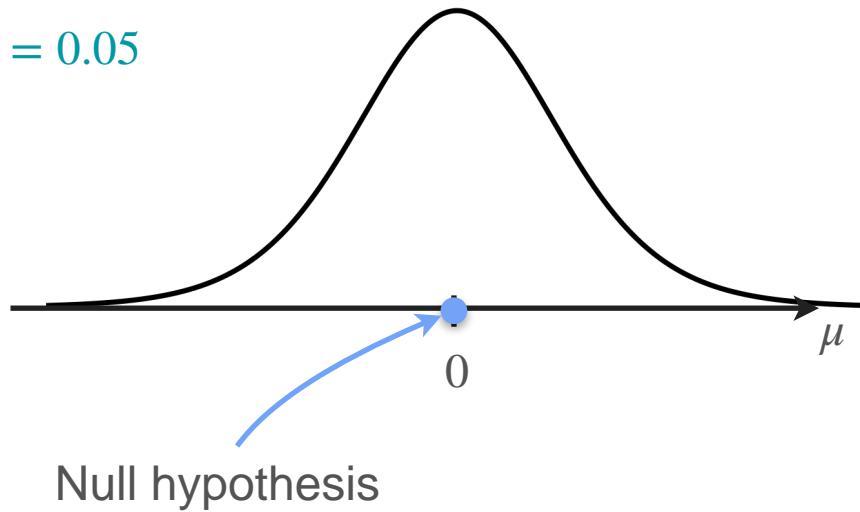


# Right-Tailed Test for Gaussian Data (Unknown $\sigma$ )

The mean height for 18 y/o in the US in the 70s was **66.7 in.**

$$H_0 : \mu = 66.7 \text{ vs. } H_1 : \mu > 66.7$$

$$\alpha = 0.05$$



# Right-Tailed Test for Gaussian Data (Unknown $\sigma$ )

The mean height for 18 y/o in the US in the 70s was **66.7** in.

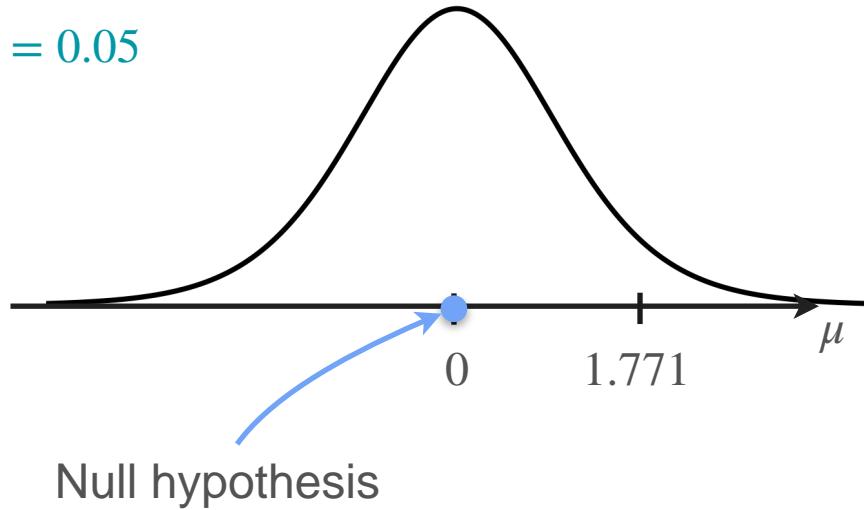
$$H_0 : \mu = 66.7 \text{ vs. } H_1 : \mu > 66.7 \quad n = 10$$

$$\alpha = 0.05$$

$$\bar{x} = 68.442$$

$$s = 3.113$$

$$t = \frac{68.442 - 66.7}{3.113/\sqrt{10}} = 1.771$$



# Right-Tailed Test for Gaussian Data (Unknown $\sigma$ )

The mean height for 18 y/o in the US in the 70s was **66.7** in.

$$H_0 : \mu = 66.7 \text{ vs. } H_1 : \mu > 66.7 \quad n = 10$$

$$\bar{x} = 68.442$$

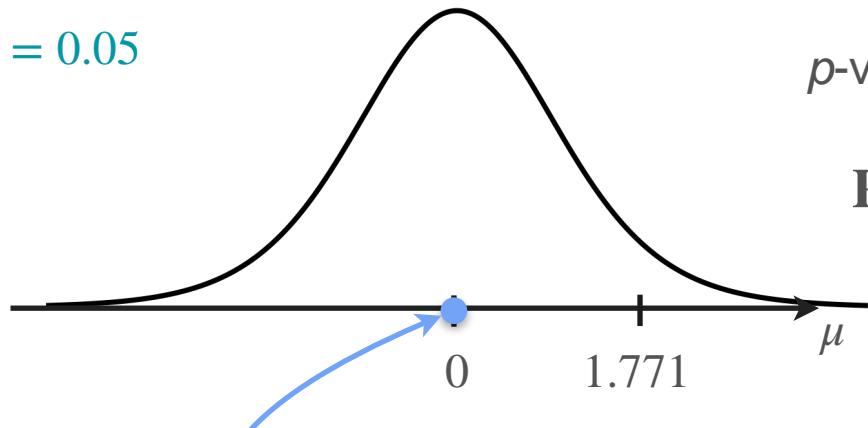
$$s = 3.113$$

$$t = \frac{68.442 - 66.7}{3.113/\sqrt{10}} = 1.771$$

$$\alpha = 0.05$$

*p*-value:

$$P\left(\frac{\bar{X} - 66.7}{S/\sqrt{10}} > 1.771 \mid \mu = 66.7\right) ?$$



Null hypothesis

# Right-Tailed Test for Gaussian Data (Unknown $\sigma$ )

The mean height for 18 y/o in the US in the 70s was **66.7** in.

$$H_0 : \mu = 66.7 \text{ vs. } H_1 : \mu > 66.7 \quad n = 10$$

$$\alpha = 0.05$$

$$\bar{x} = 68.442$$

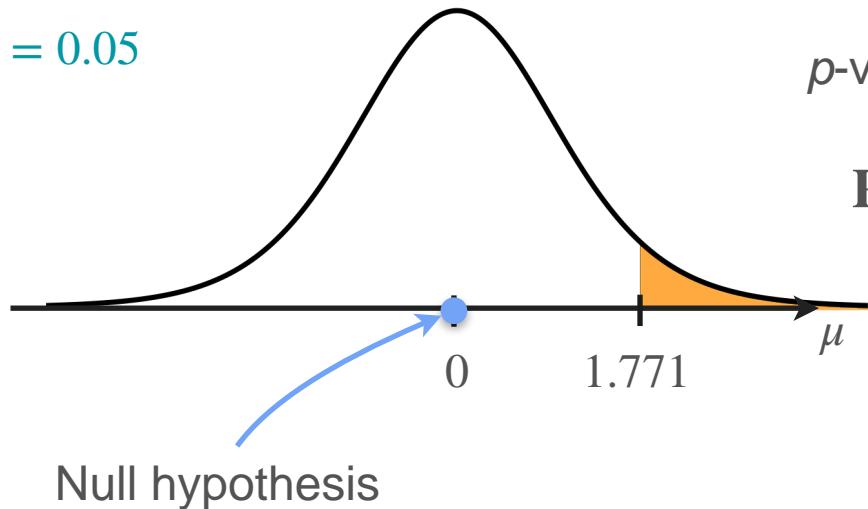
$$s = 3.113$$

$$t = \frac{68.442 - 66.7}{3.113/\sqrt{10}} = 1.771$$

p-value:

$$P\left(\frac{\bar{X} - 66.7}{S/\sqrt{10}} > 1.771 \mid \mu = 66.7\right)$$

$$= 0.0552$$



# Right-Tailed Test for Gaussian Data (Unknown $\sigma$ )

The mean height for 18 y/o in the US in the 70s was **66.7 in.**

$$H_0 : \mu = 66.7 \text{ vs. } H_1 : \mu > 66.7 \quad n = 10$$

$$\alpha = 0.05$$

$$\bar{x} = 68.442$$

$$s = 3.113$$

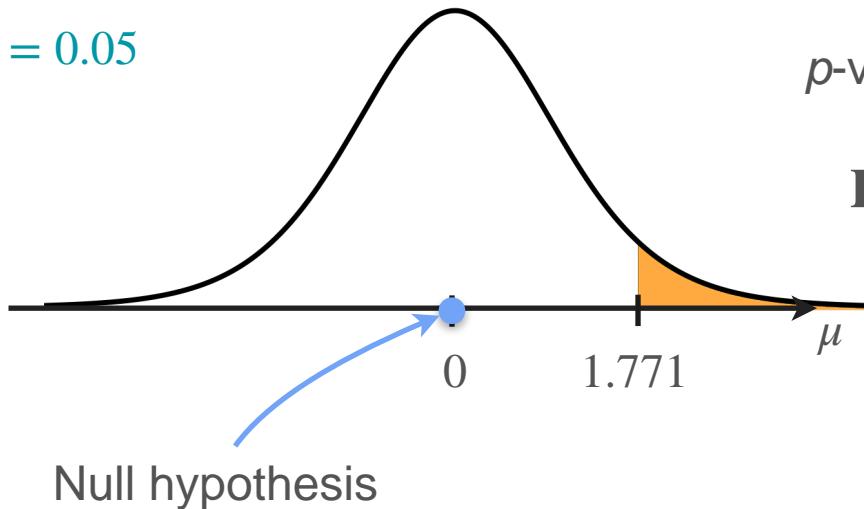
$$t = \frac{68.442 - 66.7}{3.113/\sqrt{10}} = 1.771$$

*p*-value:

$$P\left(\frac{\bar{X} - 66.7}{S/\sqrt{10}} > 1.771 \mid \mu = 66.7\right)$$

$$= 0.0552 > \alpha$$

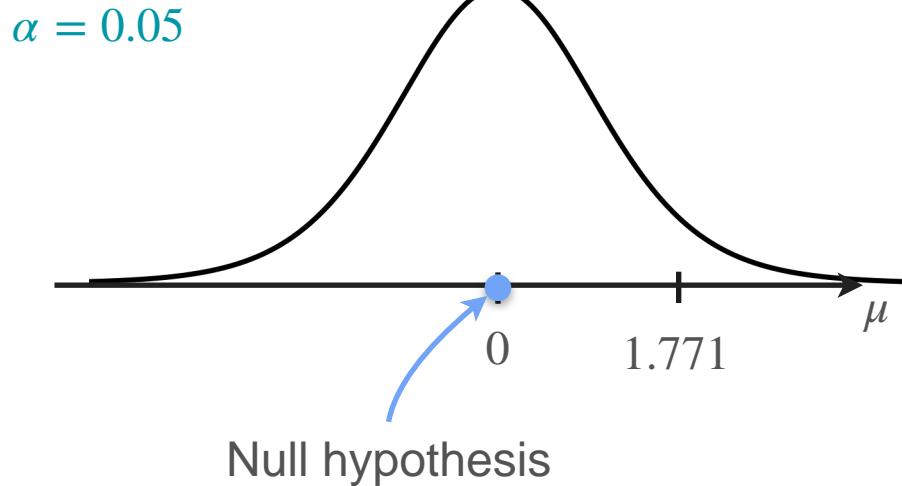
Conclusion: do not reject  $H_0$   
(with a 5% significance level)



# Two-Tailed Test for Gaussian Data (Unknown $\sigma$ )

The mean height for 18 y/o in the US in the 70s was **66.7** in.

$$n = 10 \quad \bar{x} = 68.442 \quad t = \frac{68.442 - 66.7}{3.113/\sqrt{10}} = 1.771$$
$$s = 3.113$$



# Two-Tailed Test for Gaussian Data (Unknown $\sigma$ )

The mean height for 18 y/o in the US in the 70s was **66.7** in.

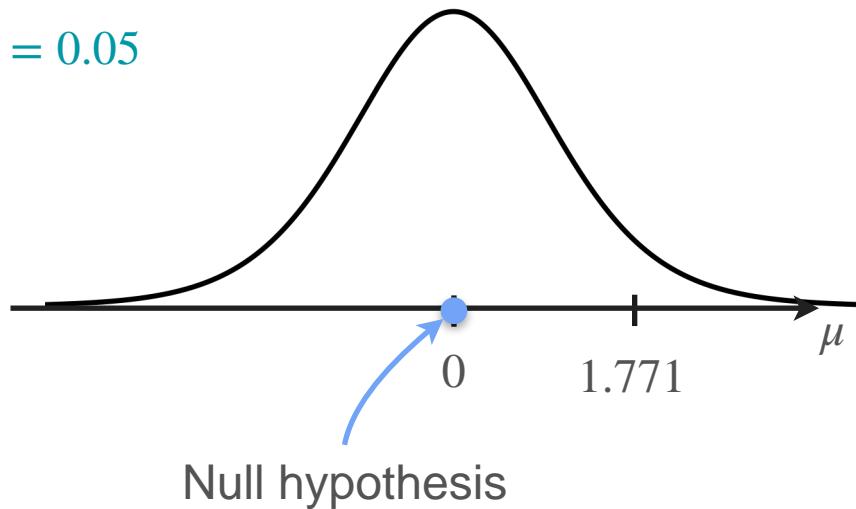
$$H_0 : \mu = 66.7 \text{ vs. } H_1 : \mu \neq 66.7 \quad n = 10$$

$$\bar{x} = 68.442$$

$$s = 3.113$$

$$t = \frac{68.442 - 66.7}{3.113/\sqrt{10}} = 1.771$$

$$\alpha = 0.05$$



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The mean height for 18 y/o in the US in the 70s was **66.7** in.

$$H_0 : \mu = 66.7 \text{ vs. } H_1 : \mu \neq 66.7$$

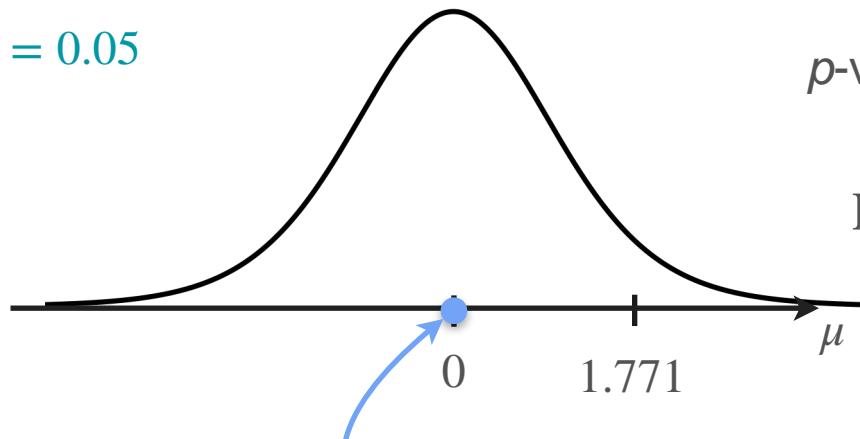
$$n = 10$$

$$\bar{x} = 68.442$$

$$s = 3.113$$

$$t = \frac{68.442 - 66.7}{3.113/\sqrt{10}} = 1.771$$

$$\alpha = 0.05$$



p-value:

$$P\left(\left|\frac{\bar{X} - 66.7}{S/\sqrt{10}}\right| > 1.771 \mid \mu = 66.7\right)?$$

# Two-Tailed Test for Gaussian Data (Unknown $\sigma$ )

The mean height for 18 y/o in the US in the 70s was **66.7** in.

$$H_0 : \mu = 66.7 \text{ vs. } H_1 : \mu \neq 66.7$$

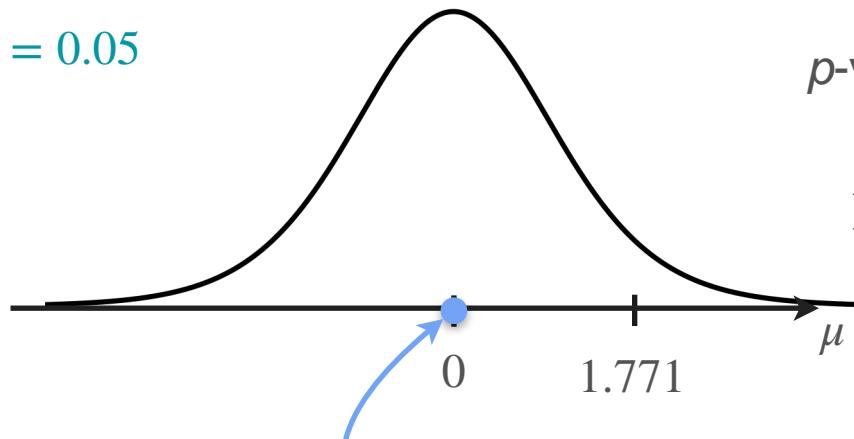
$$n = 10$$

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$$s = 3.113$$

$$t = \frac{68.442 - 66.7}{3.113/\sqrt{10}} = 1.771$$

$$\alpha = 0.05$$



p-value:

$$P\left(\left|\frac{\bar{X} - 66.7}{S/\sqrt{10}}\right| > |1.771| \middle| \mu = 66.7\right) ?$$

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The mean height for 18 y/o in the US in the 70s was **66.7** in.

$$H_0 : \mu = 66.7 \text{ vs. } H_1 : \mu \neq 66.7$$

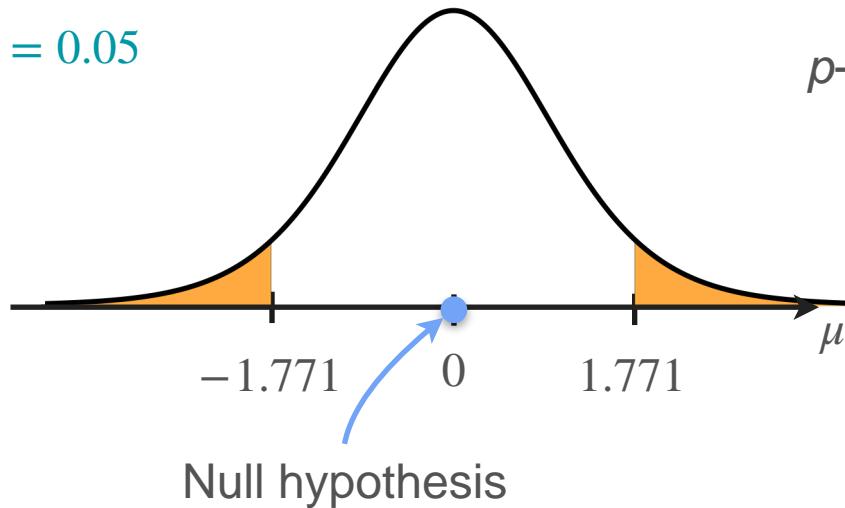
$$n = 10$$

$$\bar{x} = 68.442$$

$$s = 3.113$$

$$t = \frac{68.442 - 66.7}{3.113/\sqrt{10}} = 1.771$$

$$\alpha = 0.05$$



p-value:

$$P\left(\left|\frac{\bar{X} - 66.7}{S/\sqrt{10}}\right| > |1.771| \mid \mu = 66.7\right) = 0.1103$$

# Two-Tailed Test for Gaussian Data (Unknown $\sigma$ )

The mean height for 18 y/o in the US in the 70s was **66.7** in.

$$H_0 : \mu = 66.7 \text{ vs. } H_1 : \mu \neq 66.7$$

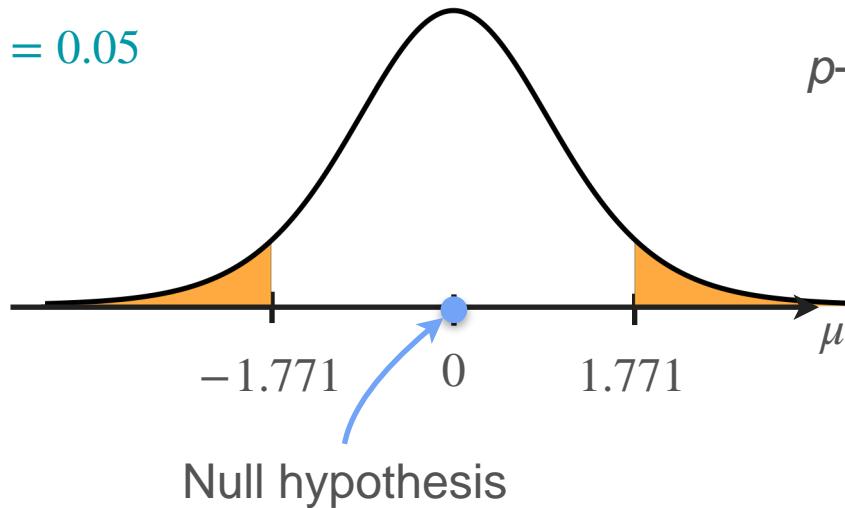
$$n = 10$$

$$\bar{x} = 68.442$$

$$s = 3.113$$

$$t = \frac{68.442 - 66.7}{3.113/\sqrt{10}} = 1.771$$

$$\alpha = 0.05$$



p-value:

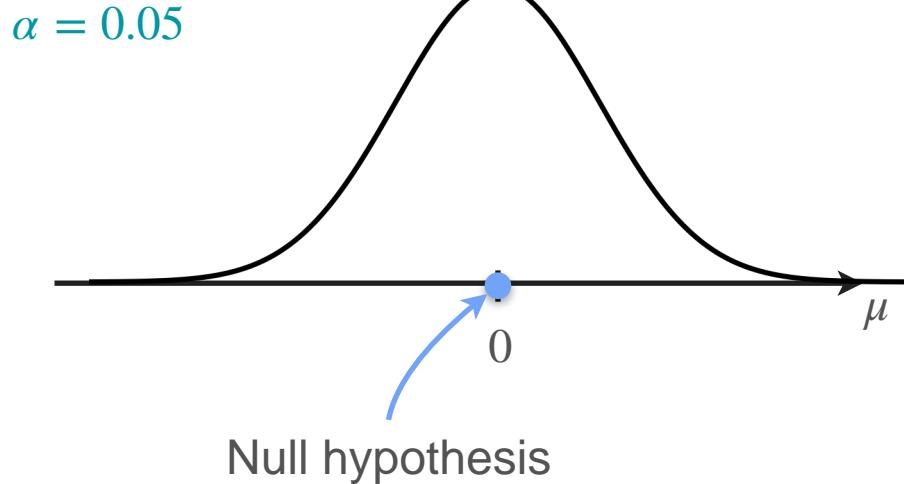
$$P\left(\left|\frac{\bar{X} - 66.7}{S/\sqrt{10}}\right| > |1.771| \mid \mu = 66.7\right) = 0.1103 > \alpha$$

Conclusion: do not reject  $H_0$   
(with a 5% significance level)

# Left-Tailed Test for Gaussian Data (Unknown $\sigma$ )

The mean height for 18 y/o in the US in the 70s was **66.7** in.

$$n = 10 \quad \bar{x} = 64.252 \quad t = \frac{64.252 - 66.7}{3.113/\sqrt{10}}$$



# Left-Tailed Test for Gaussian Data (Unknown $\sigma$ )

The mean height for 18 y/o in the US in the 70s was **66.7** in.

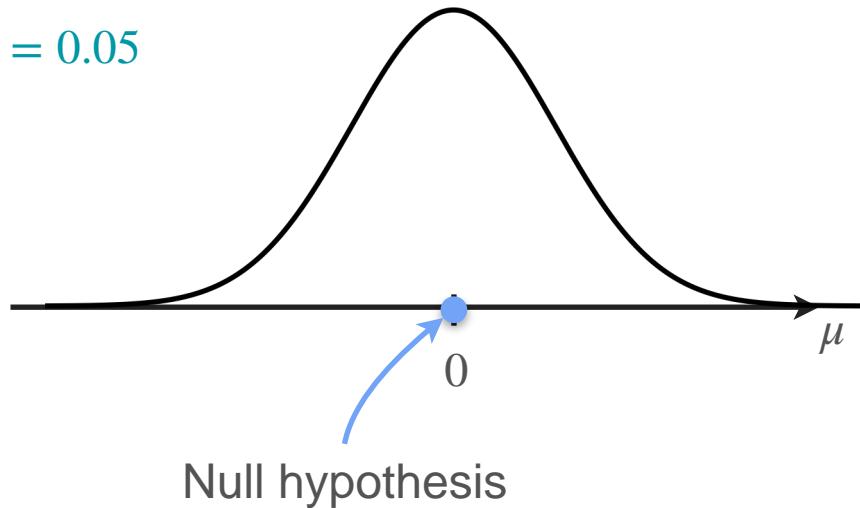
$$H_0 : \mu = 66.7 \text{ vs. } H_1 : \mu < 66.7 \quad n = 10$$

$$\bar{x} = 64.252$$

$$s = 3.113$$

$$t = \frac{64.252 - 66.7}{3.113/\sqrt{10}}$$

$$\alpha = 0.05$$



# Left-Tailed Test for Gaussian Data (Unknown $\sigma$ )

The mean height for 18 y/o in the US in the 70s was **66.7** in.

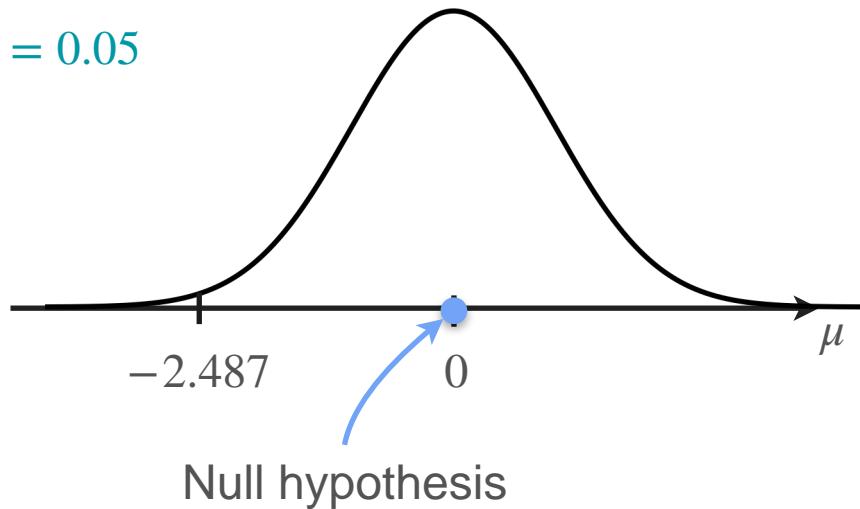
$$H_0 : \mu = 66.7 \text{ vs. } H_1 : \mu < 66.7 \quad n = 10$$

$$\bar{x} = 64.252$$

$$s = 3.113$$

$$t = \frac{64.252 - 66.7}{3.113/\sqrt{10}} = -2.487$$

$$\alpha = 0.05$$



# Left-Tailed Test for Gaussian Data (Unknown $\sigma$ )

The mean height for 18 y/o in the US in the 70s was 66.7 in.

$$H_0 : \mu = 66.7 \text{ vs. } H_1 : \mu < 66.7$$

$$n = 10$$

$$\bar{x} = 64.252$$

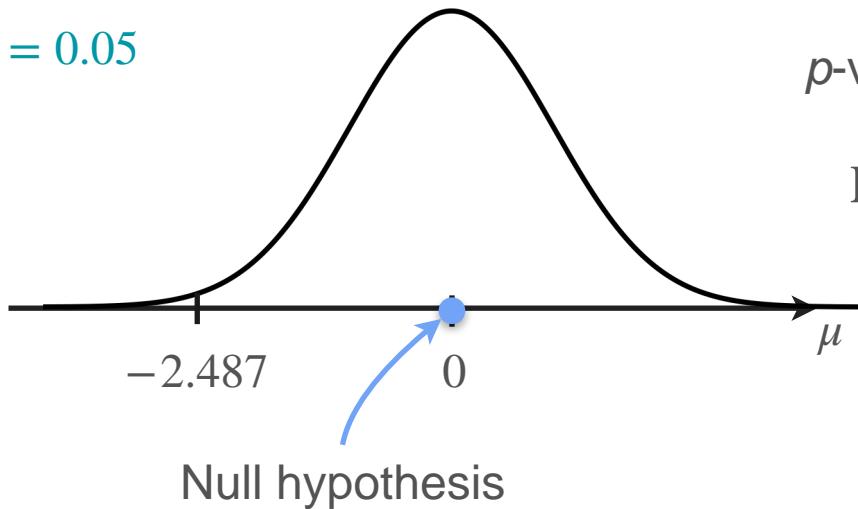
$$s = 3.113$$

$$t = \frac{64.252 - 66.7}{3.113/\sqrt{10}} = -2.487$$

$$\alpha = 0.05$$

p-value:

$$P\left(\frac{\bar{X} - 66.7}{S/\sqrt{10}} < -2.487 \mid \mu = 66.7\right) ?$$



# Left-Tailed Test for Gaussian Data (Unknown $\sigma$ )

The mean height for 18 y/o in the US in the 70s was 66.7 in.

$$H_0 : \mu = 66.7 \text{ vs. } H_1 : \mu < 66.7$$

$$n = 10$$

$$\bar{x} = 64.252$$

$$s = 3.113$$

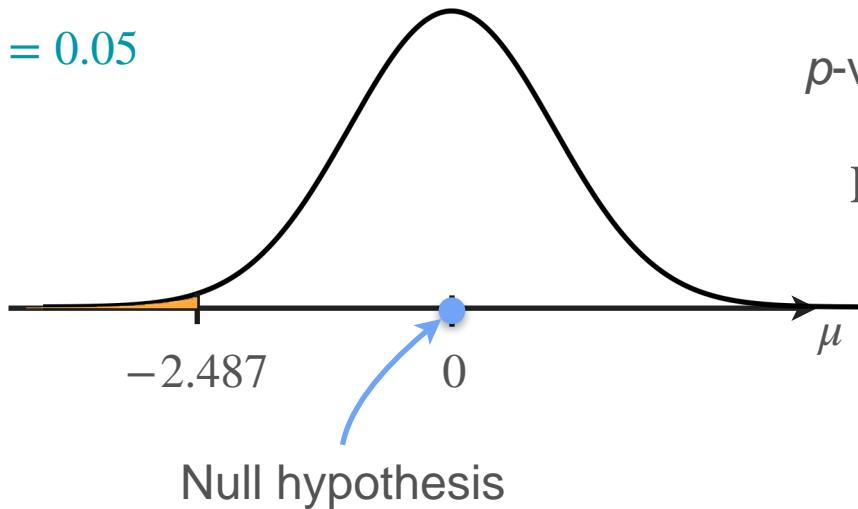
$$t = \frac{64.252 - 66.7}{3.113/\sqrt{10}} = -2.487$$

$$\alpha = 0.05$$

p-value:

$$P\left(\frac{\bar{X} - 66.7}{S/\sqrt{10}} < -2.487 \mid \mu = 66.7\right)$$

$$= 0.0173$$



# Left-Tailed Test for Gaussian Data (Unknown $\sigma$ )

The mean height for 18 y/o in the US in the 70s was **66.7** in.

$$H_0 : \mu = 66.7 \text{ vs. } H_1 : \mu < 66.7$$

$$n = 10$$

$$\bar{x} = 64.252$$

$$s = 3.113$$

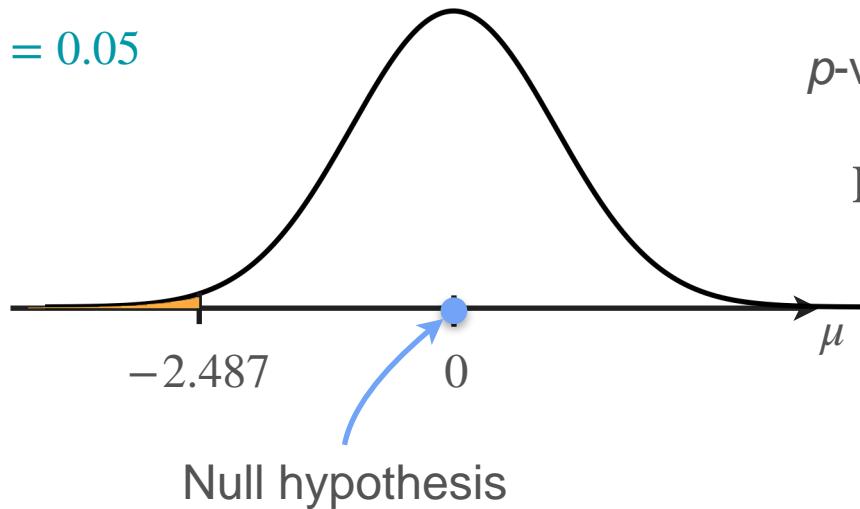
$$t = \frac{64.252 - 66.7}{3.113/\sqrt{10}} = -2.487$$

$$\alpha = 0.05$$

p-value:

$$P\left(\frac{\bar{X} - 66.7}{S/\sqrt{10}} < -2.487 \mid \mu = 66.7\right)$$

$$= 0.0173 < \alpha$$



Conclusion: reject  $H_0$   
(with a 5% significance level)



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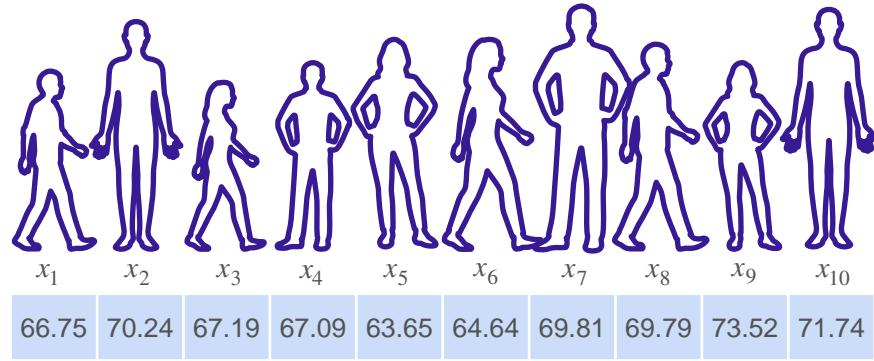
# Hypothesis Testing

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## Two sample t-test

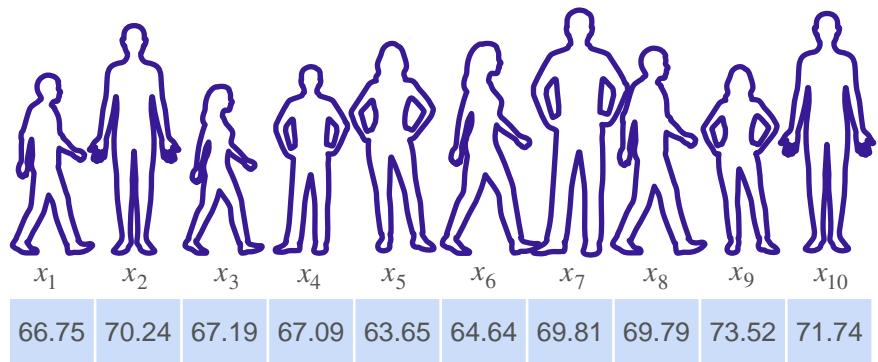
# Independent Two-Sample $t$ -Test

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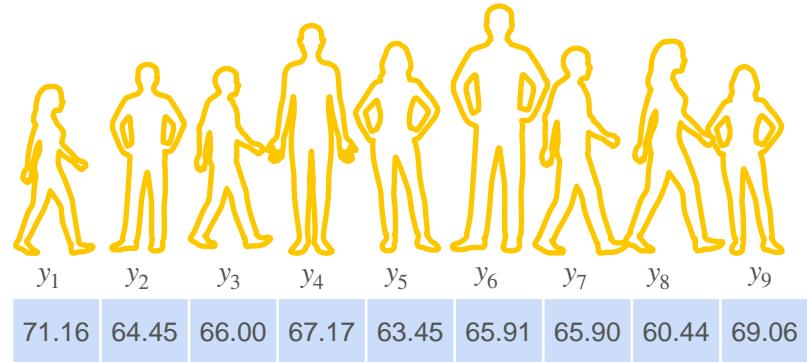


Height of 18 y/o in the US

# Independent Two-Sample $t$ -Test

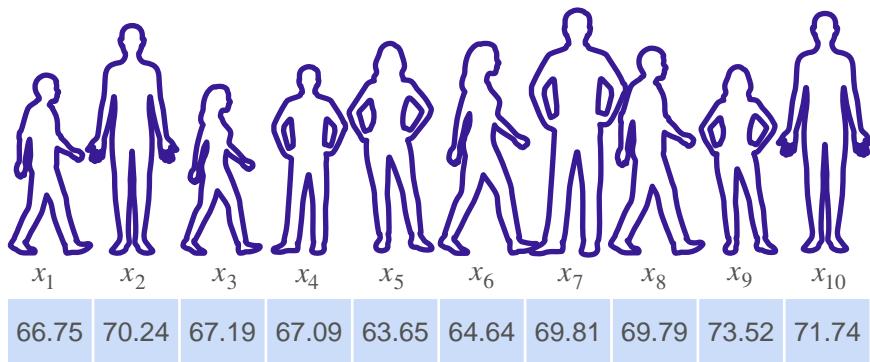


Height of 18 y/o in the US



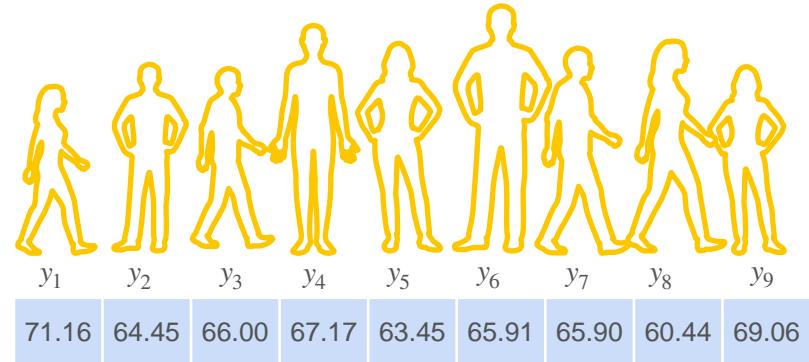
Height of 18 y/o in Argentina

# Independent Two-Sample $t$ -Test



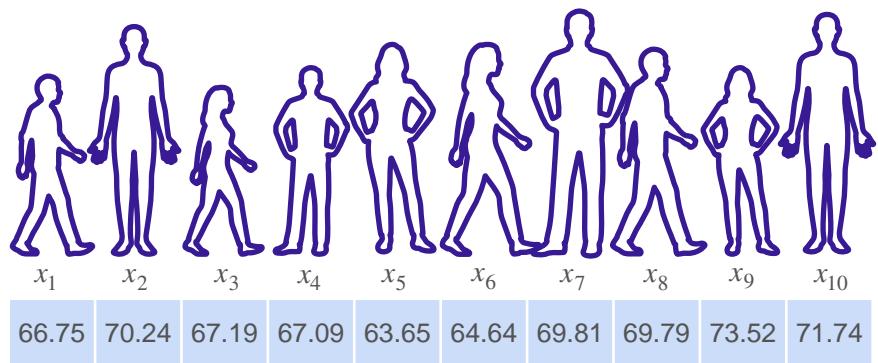
$$n_X = 10 \quad \bar{x} = 68.442$$
$$s_X = 3.113$$

Height of 18 y/o in the US



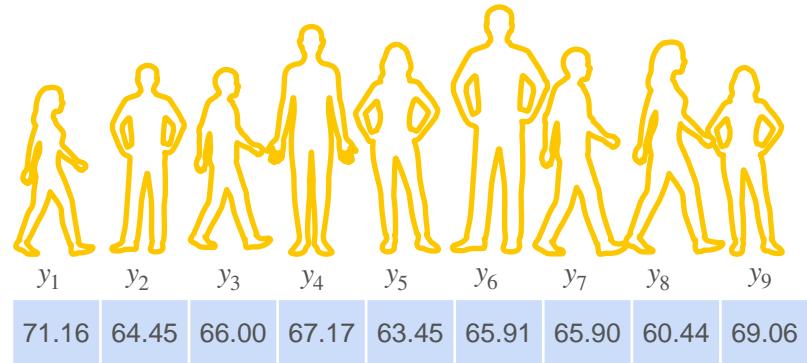
Height of 18 y/o in Argentina

# Independent Two-Sample $t$ -Test



$$n_X = 10 \quad \bar{x} = 68.442$$
$$s_X = 3.113$$

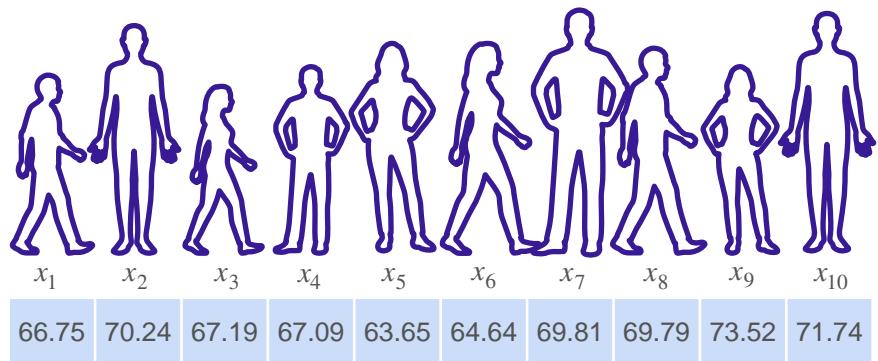
Height of 18 y/o in the US



$$n_Y = 9 \quad \bar{y} = 65.949$$
$$s_Y = 3.106$$

Height of 18 y/o in Argentina

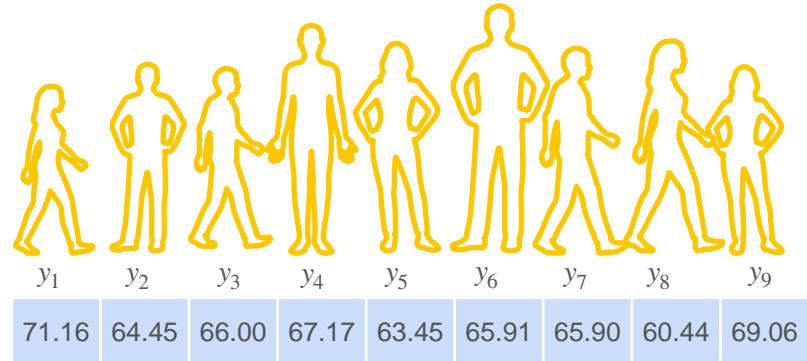
# Independent Two-Sample $t$ -Test



$$n_X = 10 \quad \bar{x} = 68.442$$
$$s_X = 3.113$$

Height of 18 y/o in the US

$$\mu_{US} \neq \mu_{Ar}$$

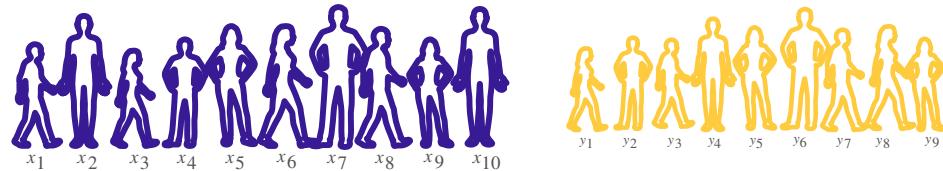


$$n_Y = 9 \quad \bar{y} = 65.949$$
$$s_Y = 3.106$$

Height of 18 y/o in Argentina

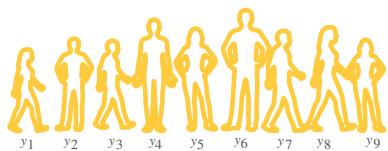
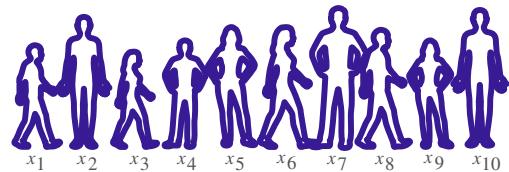
# Independent Two-Sample $t$ -Test: Hypothesis

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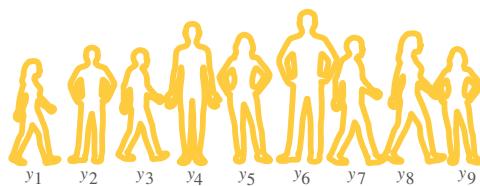
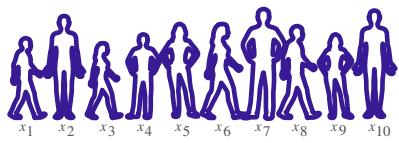


$$H_0 : \mu_{US} = \mu_{Ar} \text{ vs. } H_1 : \mu_{US} > \mu_{Ar}$$

# Independent Two-Sample $t$ -Test: Hypothesis

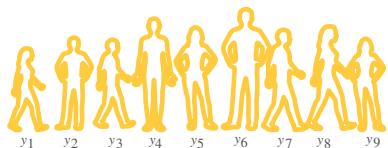
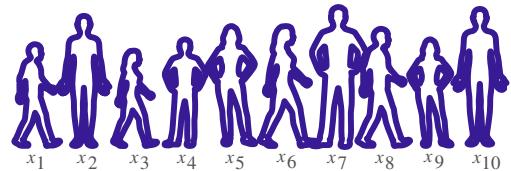


$$H_0 : \mu_{US} = \mu_{Ar} \text{ vs. } H_1 : \mu_{US} > \mu_{Ar}$$

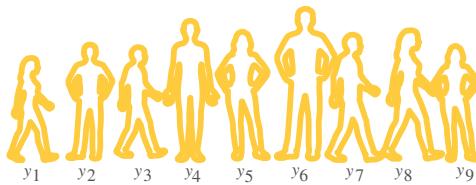
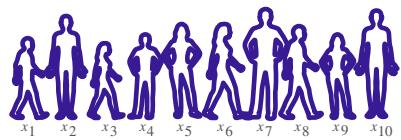


$$H_0 : \mu_{US} = \mu_{Ar} \text{ vs. } H_1 : \mu_{US} < \mu_{Ar}$$

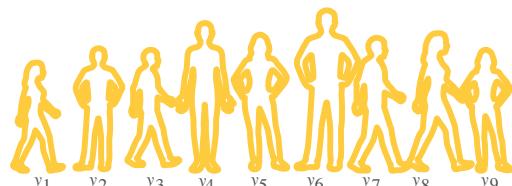
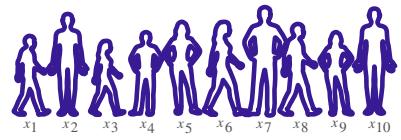
# Independent Two-Sample $t$ -Test: Hypothesis



$$H_0 : \mu_{US} = \mu_{Ar} \text{ vs. } H_1 : \mu_{US} > \mu_{Ar}$$

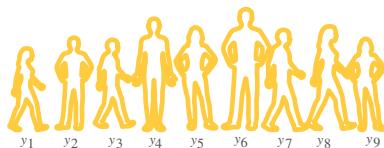
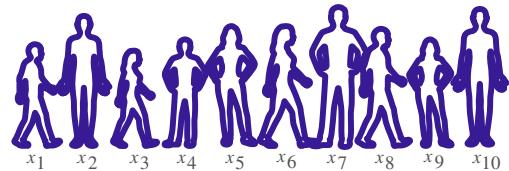


$$H_0 : \mu_{US} = \mu_{Ar} \text{ vs. } H_1 : \mu_{US} < \mu_{Ar}$$

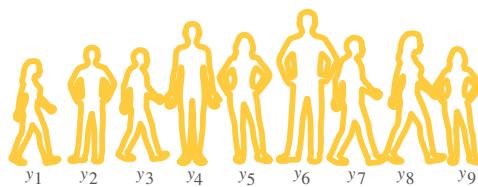
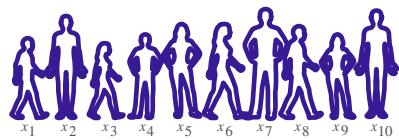


$$H_0 : \mu_{US} = \mu_{Ar} \text{ vs. } H_1 : \mu_{US} \neq \mu_{Ar}$$

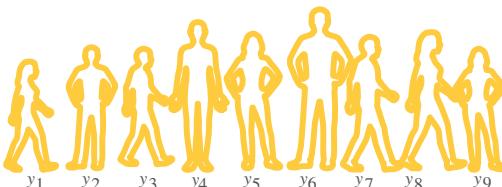
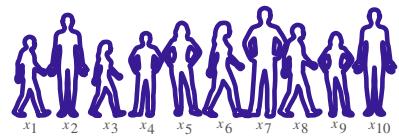
# Independent Two-Sample $t$ -Test: Hypothesis



$$H_0 : \mu_{US} - \mu_{Ar} = 0 \text{ vs. } H_1 : \mu_{US} - \mu_{Ar} > 0$$

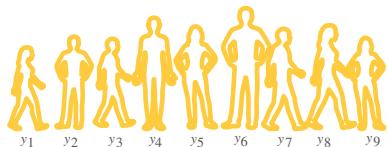
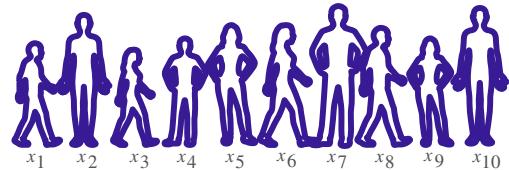


$$H_0 : \mu_{US} = \mu_{Ar} \text{ vs. } H_1 : \mu_{US} < \mu_{Ar}$$

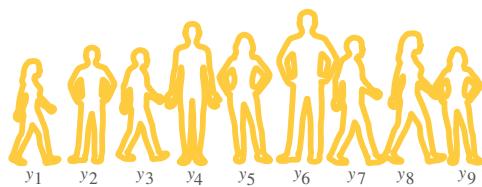
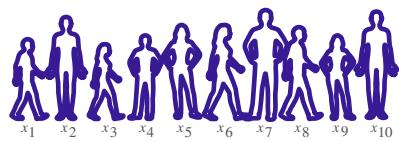


$$H_0 : \mu_{US} = \mu_{Ar} \text{ vs. } H_1 : \mu_{US} \neq \mu_{Ar}$$

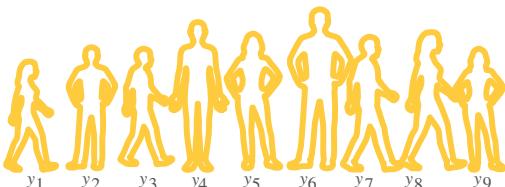
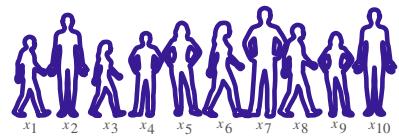
# Independent Two-Sample $t$ -Test: Hypothesis



$$H_0 : \mu_{US} - \mu_{Ar} = 0 \text{ vs. } H_1 : \mu_{US} - \mu_{Ar} > 0$$

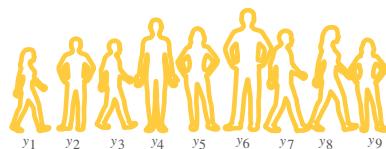
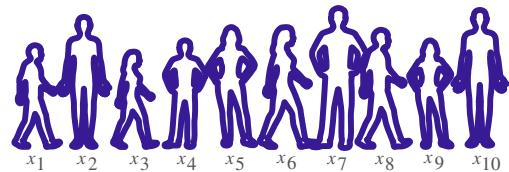


$$H_0 : \mu_{US} - \mu_{Ar} = 0 \text{ vs. } H_1 : \mu_{US} - \mu_{Ar} < 0$$

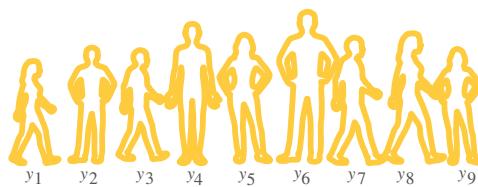
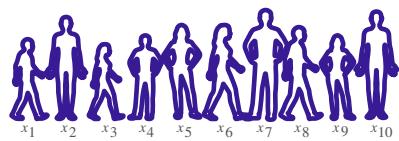


$$H_0 : \mu_{US} = \mu_{Ar} \text{ vs. } H_1 : \mu_{US} \neq \mu_{Ar}$$

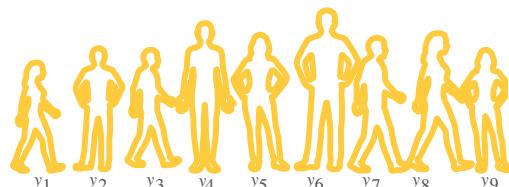
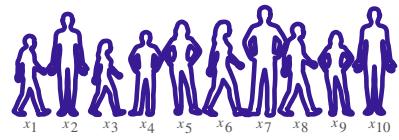
# Independent Two-Sample $t$ -Test: Hypothesis



$$H_0 : \mu_{US} - \mu_{Ar} = 0 \text{ vs. } H_1 : \mu_{US} - \mu_{Ar} > 0$$



$$H_0 : \mu_{US} - \mu_{Ar} = 0 \text{ vs. } H_1 : \mu_{US} - \mu_{Ar} < 0$$



$$H_0 : \mu_{US} - \mu_{Ar} = 0 \text{ vs. } H_1 : \mu_{US} - \mu_{Ar} \neq 0$$

# Independent Two-Sample $t$ -Test: Assumptions

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$$X \sim \mathcal{N}(\mu_{US}, \sigma_{US}^2)$$

$$Y \sim \mathcal{N}(\mu_{Arg}, \sigma_{Arg}^2)$$

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- All people in the sample from the two groups are different
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- Populations are normally distributed

$$X \sim \mathcal{N}(\mu_{US}, \sigma_{US}^2)$$

$$Y \sim \mathcal{N}(\mu_{Arg}, \sigma_{Arg}^2)$$

$$\bar{X} = \frac{1}{10} \sum_{i=1}^{10} X_i$$

$$\bar{Y} = \frac{1}{9} \sum_{i=1}^9 Y_i$$

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$$Y \sim \mathcal{N}(\mu_{Arg}, \sigma_{Arg}^2)$$

$$\bar{X} = \frac{1}{10} \sum_{i=1}^{10} X_i$$

$$\bar{X} - \bar{Y} \sim ?$$

$$\bar{Y} = \frac{1}{9} \sum_{i=1}^9 Y_i$$

# Independent Two-Sample $t$ -Test: Assumptions

- All people in the sample from the two groups are different
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$$Y \sim \mathcal{N}(\mu_{Arg}, \sigma_{Arg}^2)$$

$$\bar{X} = \frac{1}{10} \sum_{i=1}^{10} X_i$$

$$\bar{X} - \bar{Y} \sim \mathcal{N}\left( \quad, \quad \right)$$

$$\bar{Y} = \frac{1}{9} \sum_{i=1}^9 Y_i$$

# Independent Two-Sample $t$ -Test: Assumptions

- All people in the sample from the two groups are different
- Each person in both samples are independent
- Populations are normally distributed

$$X \sim \mathcal{N}(\mu_{US}, \sigma_{US}^2)$$

$$Y \sim \mathcal{N}(\mu_{Arg}, \sigma_{Arg}^2)$$

$$\bar{X} = \frac{1}{10} \sum_{i=1}^{10} X_i$$

$$\bar{X} - \bar{Y} \sim \mathcal{N}\left(\mu_{US} - \mu_{Arg}, \right)$$

$$\bar{Y} = \frac{1}{9} \sum_{i=1}^9 Y_i$$

# Independent Two-Sample $t$ -Test: Assumptions

- All people in the sample from the two groups are different
- Each person in both samples are independent
- Populations are normally distributed

$$X \sim \mathcal{N}(\mu_{US}, \sigma_{US}^2)$$

$$Y \sim \mathcal{N}(\mu_{Arg}, \sigma_{Arg}^2)$$

$$\bar{X} = \frac{1}{10} \sum_{i=1}^{10} X_i$$

$$\bar{X} - \bar{Y} \sim \mathcal{N}\left(\mu_{US} - \mu_{Arg}, \frac{\sigma_{US}^2}{10} + \frac{\sigma_{Arg}^2}{9}\right)$$

$$\bar{Y} = \frac{1}{9} \sum_{i=1}^9 Y_i$$

# Independent Two-Sample $t$ -Test: Assumptions

- All people in the sample from the two groups are different
- Each person in both samples are independent
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$$X \sim \mathcal{N}(\mu_{US}, \sigma_{US})$$

$$Y \sim \mathcal{N}(\mu_{Arg}, \sigma_{Arg})$$

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$$\bar{Y} = \frac{1}{9} \sum_{i=1}^9 Y_i$$

$$\bar{X} - \bar{Y} \sim \mathcal{N}\left(\mu_{US} - \mu_{Arg}, \sqrt{\frac{\sigma_{US}^2}{10} + \frac{\sigma_{Arg}^2}{9}}\right)$$

# Independent Two-Sample $t$ -Test: Statistic

$$X \sim \mathcal{N}(\mu_{US}, \sigma_{US})$$

$$Y \sim \mathcal{N}(\mu_{Arg}, \sigma_{Arg})$$

$$\bar{X} - \bar{Y} \sim \mathcal{N}\left(\mu_{US} - \mu_{Arg}, \frac{\sigma_{US}^2}{10} + \frac{\sigma_{Arg}^2}{9}\right)$$

# Independent Two-Sample $t$ -Test: Statistic

$$X \sim \mathcal{N}(\mu_{US}, \sigma_{US}) \quad Y \sim \mathcal{N}(\mu_{Arg}, \sigma_{Arg})$$
$$\bar{X} - \bar{Y} \sim \mathcal{N}\left(\mu_{US} - \mu_{Arg}, \frac{\sigma_{US}^2}{10} + \frac{\sigma_{Arg}^2}{9}\right) \xrightarrow{\hspace{1cm}} \frac{(\bar{X} - \bar{Y}) - (\mu_{US} - \mu_{Arg})}{\sqrt{\frac{\sigma_{US}^2}{10} + \frac{\sigma_{Arg}^2}{9}}} \sim \mathcal{N}(0, 1^2)$$

# Independent Two-Sample $t$ -Test: Statistic

$$X \sim \mathcal{N}(\mu_{US}, \sigma_{US}) \quad Y \sim \mathcal{N}(\mu_{Arg}, \sigma_{Arg})$$
$$\bar{X} - \bar{Y} \sim \mathcal{N}\left(\mu_{US} - \mu_{Arg}, \frac{\sigma_{US}^2}{10} + \frac{\sigma_{Arg}^2}{9}\right) \rightarrow \frac{(\bar{X} - \bar{Y}) - (\mu_{US} - \mu_{Arg})}{\sqrt{\frac{\sigma_{US}^2}{10} + \frac{\sigma_{Arg}^2}{9}}} \sim \mathcal{N}(0, 1^2)$$

You don't know  $\sigma_{US}$ ,  $\sigma_{Arg}$



# Independent Two-Sample $t$ -Test: Statistic

$$X \sim \mathcal{N}(\mu_{US}, \sigma_{US}) \quad Y \sim \mathcal{N}(\mu_{Arg}, \sigma_{Arg})$$
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You don't know  $\sigma_{US}$ ,  $\sigma_{Arg}$



Replace it with the sample standard deviation

# Independent Two-Sample $t$ -Test: Statistic

$$X \sim \mathcal{N}(\mu_{US}, \sigma_{US}) \quad Y \sim \mathcal{N}(\mu_{Arg}, \sigma_{Arg})$$
$$\bar{X} - \bar{Y} \sim \mathcal{N}\left(\mu_{US} - \mu_{Arg}, \frac{\sigma_{US}^2}{10} + \frac{\sigma_{Arg}^2}{9}\right) \rightarrow \frac{(\bar{X} - \bar{Y}) - (\mu_{US} - \mu_{Arg})}{\sqrt{\frac{\sigma_{US}^2}{10} + \frac{\sigma_{Arg}^2}{9}}} \sim \mathcal{N}(0, 1^2)$$

You don't know  $\sigma_{US}$ ,  $\sigma_{Arg}$



Replace it with the sample standard deviation

$$\frac{(\bar{X} - \bar{Y}) - (\mu_{US} - \mu_{Arg})}{\sqrt{\frac{S_X^2}{10} + \frac{S_Y^2}{9}}}$$

# Independent Two-Sample $t$ -Test: Statistic

$$X \sim \mathcal{N}(\mu_{US}, \sigma_{US}) \quad Y \sim \mathcal{N}(\mu_{Arg}, \sigma_{Arg})$$
$$\bar{X} - \bar{Y} \sim \mathcal{N}\left(\mu_{US} - \mu_{Arg}, \frac{\sigma_{US}^2}{10} + \frac{\sigma_{Arg}^2}{9}\right) \rightarrow \frac{(\bar{X} - \bar{Y}) - (\mu_{US} - \mu_{Arg})}{\sqrt{\frac{\sigma_{US}^2}{10} + \frac{\sigma_{Arg}^2}{9}}} \sim \mathcal{N}(0, 1^2)$$

You don't know  $\sigma_{US}$ ,  $\sigma_{Arg}$



Replace it with the sample standard deviation

$$T = \frac{(\bar{X} - \bar{Y}) - (\mu_{US} - \mu_{Arg})}{\sqrt{\frac{S_X^2}{10} + \frac{S_Y^2}{9}}} \sim t_{\nu}$$

# Independent Two-Sample $t$ -Test: Statistic

$$X \sim \mathcal{N}(\mu_{US}, \sigma_{US})$$

$$Y \sim \mathcal{N}(\mu_{Arg}, \sigma_{Arg})$$

$$\bar{X} - \bar{Y} \sim \mathcal{N}\left(\mu_{US} - \mu_{Arg}, \frac{\sigma_{US}^2}{10} + \frac{\sigma_{Arg}^2}{9}\right) \rightarrow \frac{(\bar{X} - \bar{Y}) - (\mu_{US} - \mu_{Arg})}{\sqrt{\frac{\sigma_{US}^2}{10} + \frac{\sigma_{Arg}^2}{9}}} \sim \mathcal{N}(0, 1^2)$$

You don't know  $\sigma_{US}$ ,  $\sigma_{Arg}$

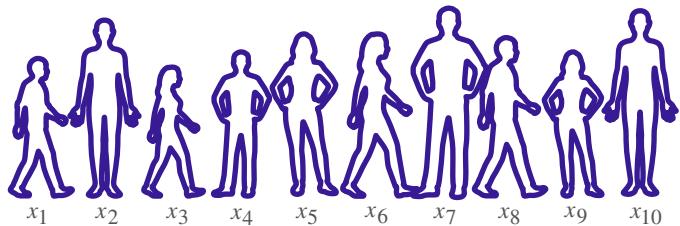


Replace it with the sample standard deviation

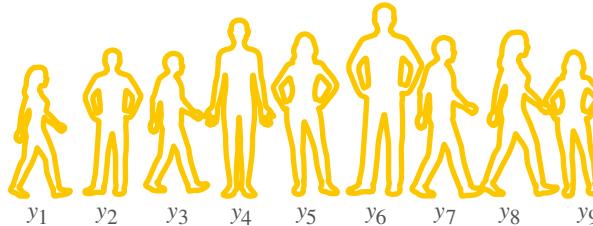
$$T = \frac{(\bar{X} - \bar{Y}) - (\mu_{US} - \mu_{Arg})}{\sqrt{\frac{s_X^2}{10} + \frac{s_Y^2}{9}}} \sim t_{\nu}$$

Degrees of freedom =  $\frac{\left(\frac{s_X^2}{n_X} + \frac{s_Y^2}{n_Y}\right)^2}{\frac{\left(\frac{s_X^2}{n_X}\right)^2}{n_X - 1} + \frac{\left(\frac{s_Y^2}{n_Y}\right)^2}{n_Y - 1}}$

# Independent Two-Sample $t$ -Test



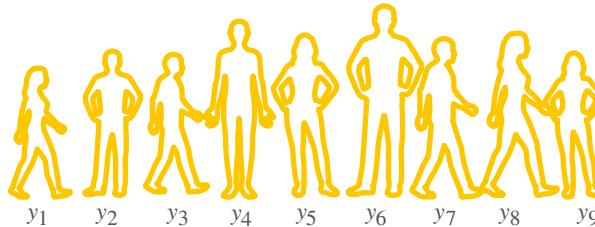
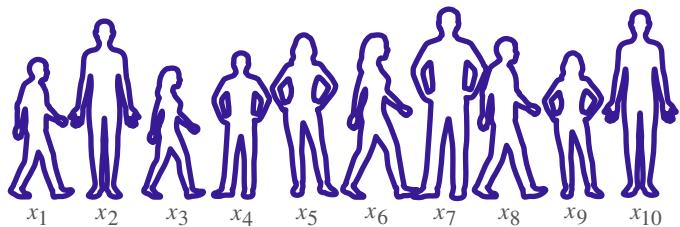
$$n_X = 10 \quad \bar{x} = 68.442$$
$$s_X = 3.113$$



$$n_Y = 9 \quad \bar{y} = 65.949$$
$$s_Y = 3.106$$

Degrees of freedom =  $\frac{\left( \frac{n_X - 1}{2} + \frac{n_Y - 1}{2} \right)^2}{\frac{\left( n_X - 1 \right)^2}{n_X - 1} + \frac{\left( n_Y - 1 \right)^2}{n_Y - 1}}$

# Independent Two-Sample $t$ -Test



$$n_X = 10 \quad \bar{x} = 68.442 \\ s_X = 3.113$$

$$n_Y = 9 \quad \bar{y} = 65.949 \\ s_Y = 3.106$$

Degrees of freedom =  $\frac{\overbrace{\left( \frac{2}{3.113} + \frac{2}{3.106} \right)^2}^{16.8}}{\left( \frac{2}{10} \right)^2 + \left( \frac{2}{9} \right)^2}$

# Independent Two-Sample $t$ -Test: Statistic

$$X \sim \mathcal{N}(\mu_{US}, \sigma_{US})$$

$$Y \sim \mathcal{N}(\mu_{Arg}, \sigma_{Arg})$$

$$\bar{X} - \bar{Y} \sim \mathcal{N}\left(\mu_{US} - \mu_{Arg}, \sqrt{\frac{\sigma_{US}^2}{10} + \frac{\sigma_{Arg}^2}{9}}\right) \rightarrow \frac{(\bar{X} - \bar{Y}) - (\mu_{US} - \mu_{Arg})}{\sqrt{\frac{\sigma_{US}^2}{10} + \frac{\sigma_{Arg}^2}{9}}} \sim \mathcal{N}(0,1)$$

You don't know  $\sigma_{US}$ ,  $\sigma_{Arg}$



Replace it with the sample standard deviation

$$T = \frac{(\bar{X} - \bar{Y}) - (\mu_{US} - \mu_{Arg})}{\sqrt{\frac{s_X^2}{10} + \frac{s_Y^2}{9}}} \sim t_{16.8}$$

Degrees of freedom =  $\frac{\left(\frac{s_X^2}{n_X} + \frac{s_Y^2}{n_Y}\right)^2}{\frac{\left(\frac{s_X^2}{n_X}\right)^2}{n_X - 1} + \frac{\left(\frac{s_Y^2}{n_Y}\right)^2}{n_Y - 1}}$

# Independent Two-Sample $t$ -Test: Statistic

$$X \sim \mathcal{N}(\mu_{US}, \sigma_{US})$$

$$Y \sim \mathcal{N}(\mu_{Arg}, \sigma_{Arg})$$

$$\bar{X} - \bar{Y} \sim \mathcal{N}\left(\mu_{US} - \mu_{Arg}, \sqrt{\frac{\sigma_{US}^2}{10} + \frac{\sigma_{Arg}^2}{9}}\right) \rightarrow \frac{(\bar{X} - \bar{Y}) - (\mu_{US} - \mu_{Arg})}{\sqrt{\frac{\sigma_{US}^2}{10} + \frac{\sigma_{Arg}^2}{9}}} \sim \mathcal{N}(0,1)$$

You don't know  $\sigma_{US}$ ,  $\sigma_{Arg}$



Replace it with the sample standard deviation

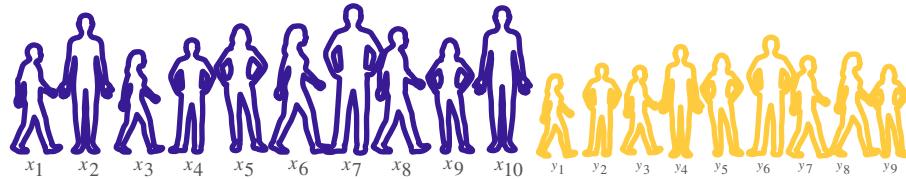
$$T = \frac{(\bar{X} - \bar{Y}) - (\mu_{US} - \mu_{Arg})}{\sqrt{\frac{s_X^2}{10} + \frac{s_Y^2}{9}}} \sim t_{16.8}$$

Degrees of freedom

$$= \frac{\left(\frac{s_X^2}{n_X} + \frac{s_Y^2}{n_Y}\right)^2}{\frac{\left(\frac{s_X^2}{n_X}\right)^2}{n_X - 1} + \frac{\left(\frac{s_Y^2}{n_Y}\right)^2}{n_Y - 1}}$$

# Independent Two-Sample $t$ -Test: Right Tailed Test

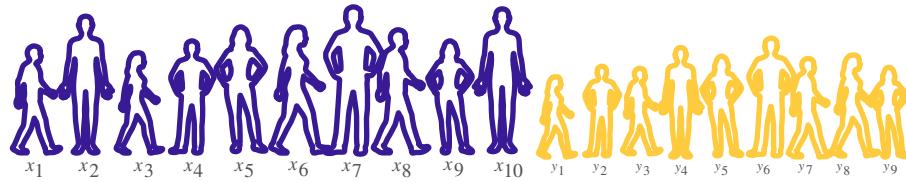
# Independent Two-Sample $t$ -Test: Right Tailed Test



$$n_X = 10$$

$$n_Y = 9$$

# Independent Two-Sample $t$ -Test: Right Tailed Test



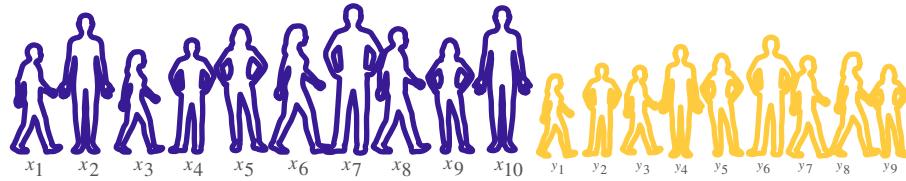
$$\bar{x} = 68.442$$

$$n_X = 10$$

$$s_X = 3.113$$

$$n_Y = 9$$

# Independent Two-Sample $t$ -Test: Right Tailed Test



$$\bar{x} = 68.442$$

$$n_X = 10$$

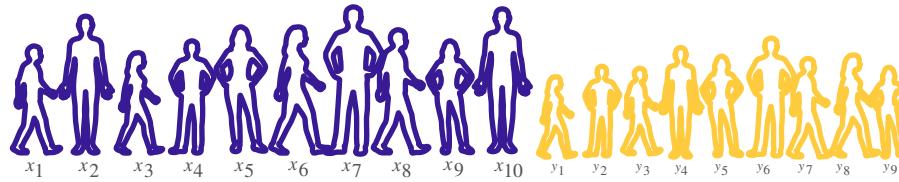
$$\bar{y} = 65.949$$

$$n_Y = 9$$

$$s_X = 3.113$$

$$s_Y = 3.106$$

# Independent Two-Sample $t$ -Test: Right Tailed Test



$$\bar{x} = 68.442$$

$$n_X = 10$$

$$\bar{y} = 65.949$$

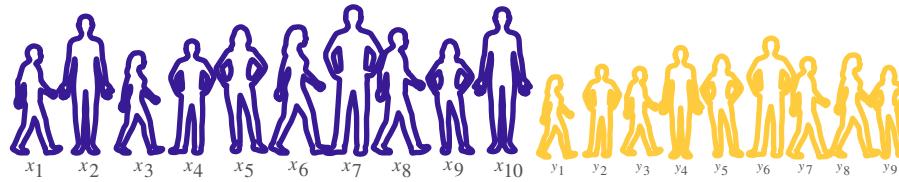
$$n_Y = 9$$

$$s_X = 3.113$$

$$s_Y = 3.106$$

$$H_0 : \mu_{US} - \mu_{Ar} = 0 \text{ vs. } H_1 : \mu_{US} - \mu_{Ar} > 0$$

# Independent Two-Sample $t$ -Test: Right Tailed Test



$$\bar{x} = 68.442$$

$$n_X = 10$$

$$\bar{y} = 65.949$$

$$n_Y = 9$$

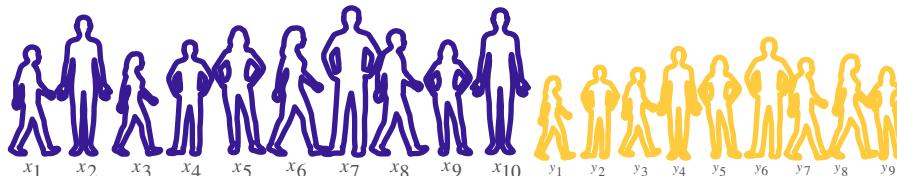
$$s_X = 3.113$$

$$s_Y = 3.106$$

$$H_0 : \mu_{US} - \mu_{Ar} = 0 \text{ vs. } H_1 : \mu_{US} - \mu_{Ar} > 0$$

$$\alpha = 0.05$$

# Independent Two-Sample $t$ -Test: Right Tailed Test



$$\bar{x} = 68.442$$

$$s_X = 3.113$$

$$n_X = 10$$

$$\bar{y} = 65.949$$

$$s_Y = 3.106$$

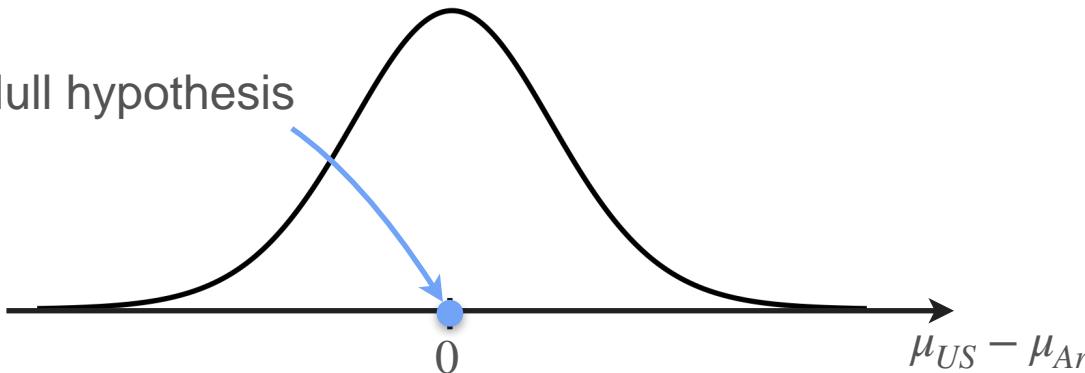
$$n_Y = 9$$

$$H_0 : \mu_{US} - \mu_{Ar} = 0 \text{ vs. } H_1 : \mu_{US} - \mu_{Ar} > 0$$

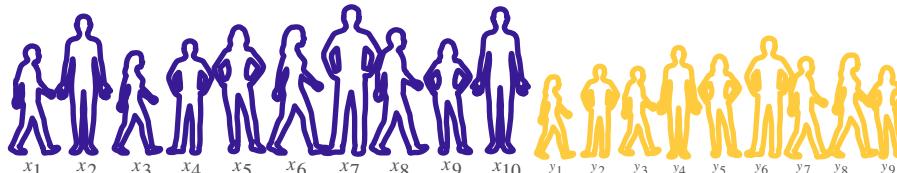
$$\alpha = 0.05$$

$$\text{If } H_0 \text{ is true: } T = \frac{(\bar{X} - \bar{Y}) - 0}{\sqrt{\frac{S_X^2}{10} + \frac{S_Y^2}{9}}} \sim t_{16.8}$$

Null hypothesis



# Independent Two-Sample $t$ -Test: Right Tailed Test



$$\bar{x} = 68.442$$

$$s_X = 3.113$$

$$n_X = 10$$

$$\bar{y} = 65.949$$

$$s_Y = 3.106$$

$$n_Y = 9$$

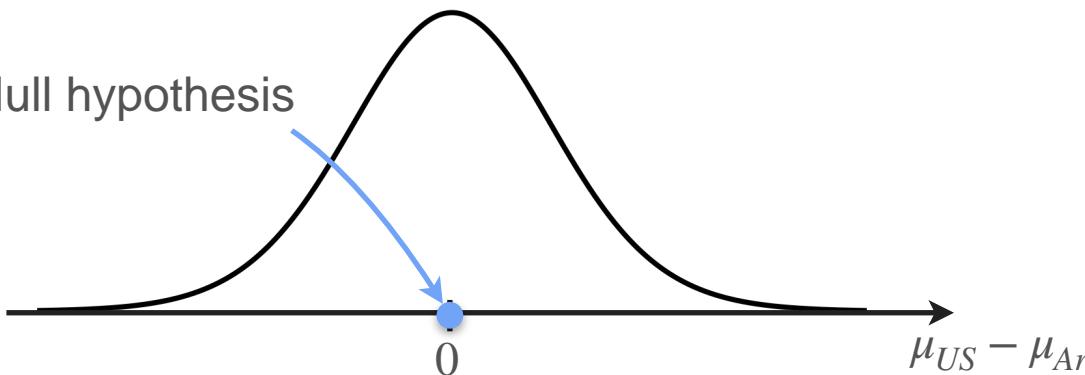
$$t = 1.7459$$

$$H_0 : \mu_{US} - \mu_{Ar} = 0 \text{ vs. } H_1 : \mu_{US} - \mu_{Ar} > 0$$

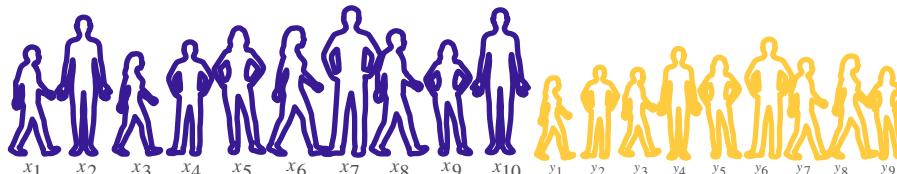
$$\alpha = 0.05$$

$$\text{If } H_0 \text{ is true: } T = \frac{(\bar{X} - \bar{Y}) - 0}{\sqrt{\frac{s_X^2}{10} + \frac{s_Y^2}{9}}} \sim t_{16.8}$$

Null hypothesis



# Independent Two-Sample $t$ -Test: Right Tailed Test



$$\bar{x} = 68.442$$

$$s_X = 3.113$$

$$n_X = 10$$

$$\bar{y} = 65.949$$

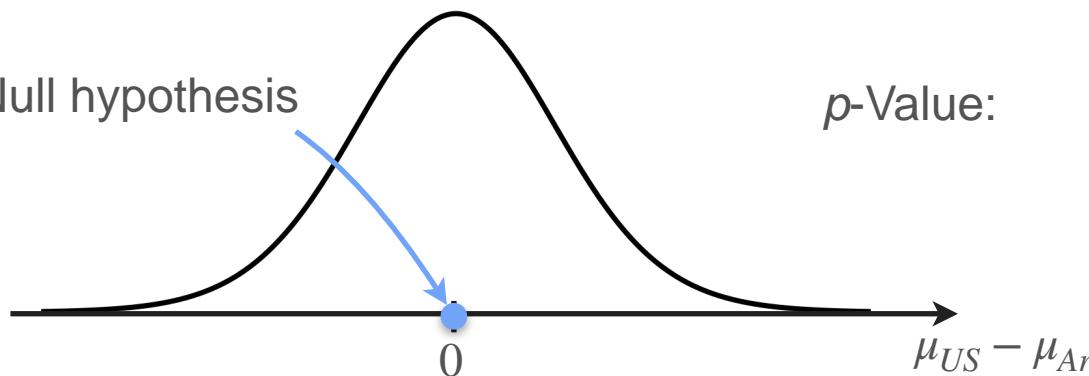
$$s_Y = 3.106$$

$$n_Y = 9$$

$$t = 1.7459$$

Null hypothesis

$p$ -Value:

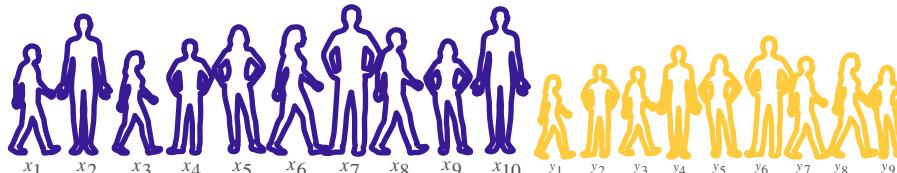


$$H_0 : \mu_{US} - \mu_{Ar} = 0 \text{ vs. } H_1 : \mu_{US} - \mu_{Ar} > 0$$

$$\alpha = 0.05$$

$$\text{If } H_0 \text{ is true: } T = \frac{(\bar{X} - \bar{Y}) - 0}{\sqrt{\frac{S_X^2}{10} + \frac{S_Y^2}{9}}} \sim t_{16.8}$$

# Independent Two-Sample $t$ -Test: Right Tailed Test



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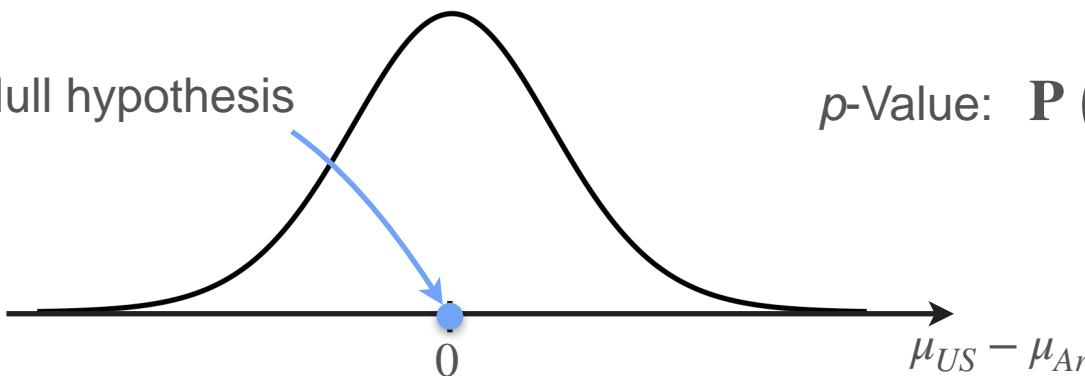
$$\bar{y} = 65.949$$

$$s_Y = 3.106$$

$$n_Y = 9$$

$$t = 1.7459$$

Null hypothesis



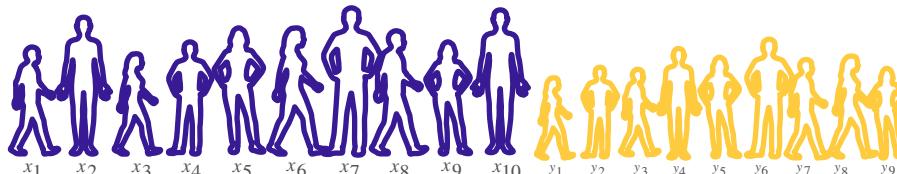
$$H_0 : \mu_{US} - \mu_{Ar} = 0 \text{ vs. } H_1 : \mu_{US} - \mu_{Ar} > 0$$

$$\alpha = 0.05$$

$$\text{If } H_0 \text{ is true: } T = \frac{(\bar{X} - \bar{Y}) - 0}{\sqrt{\frac{S_X^2}{10} + \frac{S_Y^2}{9}}} \sim t_{16.8}$$

$$p\text{-Value: } P(T > 1.7459 \mid \mu_{US} - \mu_{Ar} = 0)$$

# Independent Two-Sample $t$ -Test: Right Tailed Test



$$\bar{x} = 68.442$$

$$s_X = 3.113$$

$$n_X = 10$$

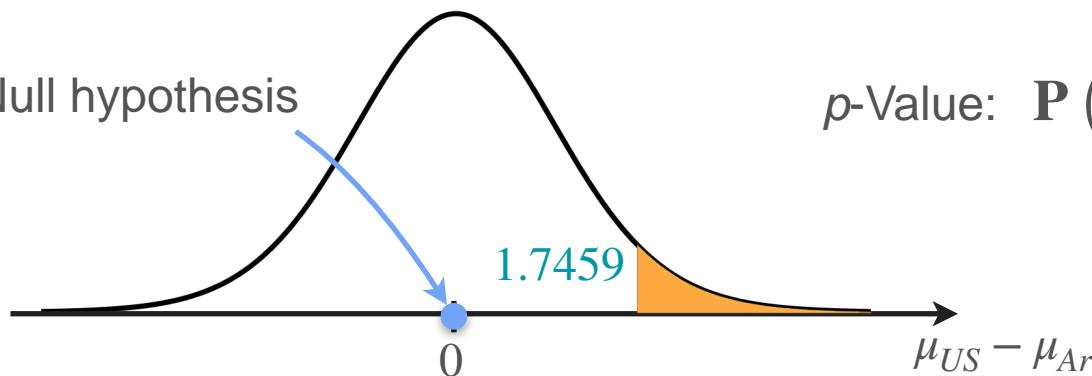
$$\bar{y} = 65.949$$

$$s_Y = 3.106$$

$$n_Y = 9$$

$$t = 1.7459$$

Null hypothesis



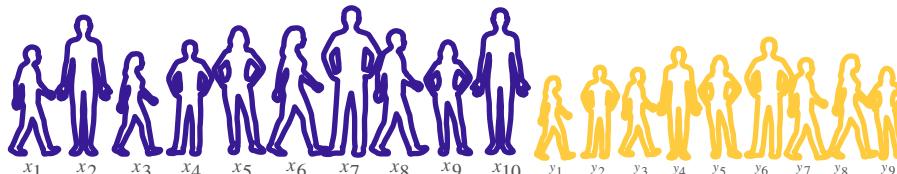
p-Value:  $P(T > 1.7459 \mid \mu_{US} - \mu_{Ar} = 0)$

$$H_0 : \mu_{US} - \mu_{Ar} = 0 \text{ vs. } H_1 : \mu_{US} - \mu_{Ar} > 0$$

$$\alpha = 0.05$$

$$\text{If } H_0 \text{ is true: } T = \frac{(\bar{X} - \bar{Y}) - 0}{\sqrt{\frac{s_X^2}{10} + \frac{s_Y^2}{9}}} \sim t_{16.8}$$

# Independent Two-Sample $t$ -Test: Right Tailed Test



$$\bar{x} = 68.442$$

$$s_X = 3.113$$

$$n_X = 10$$

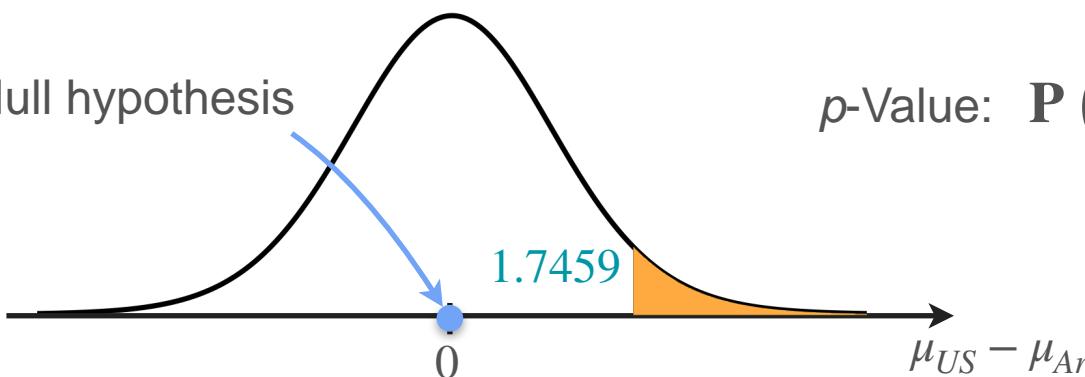
$$\bar{y} = 65.949$$

$$s_Y = 3.106$$

$$n_Y = 9$$

$$t = 1.7459$$

Null hypothesis



p-Value:  $P(T > 1.7459 \mid \mu_{US} - \mu_{Ar} = 0)$

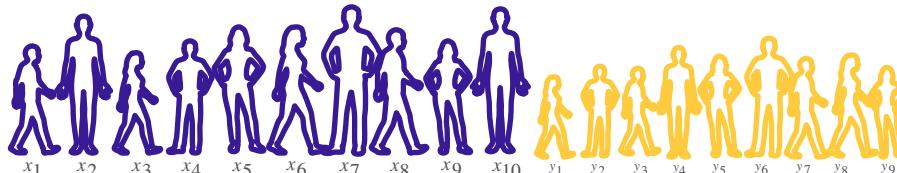
$$= 0.0495$$

$$H_0 : \mu_{US} - \mu_{Ar} = 0 \text{ vs. } H_1 : \mu_{US} - \mu_{Ar} > 0$$

$$\alpha = 0.05$$

$$\text{If } H_0 \text{ is true: } T = \frac{(\bar{X} - \bar{Y}) - 0}{\sqrt{\frac{s_X^2}{10} + \frac{s_Y^2}{9}}} \sim t_{16.8}$$

# Independent Two-Sample $t$ -Test: Right Tailed Test



$$\bar{x} = 68.442$$

$$s_X = 3.113$$

$$n_X = 10$$

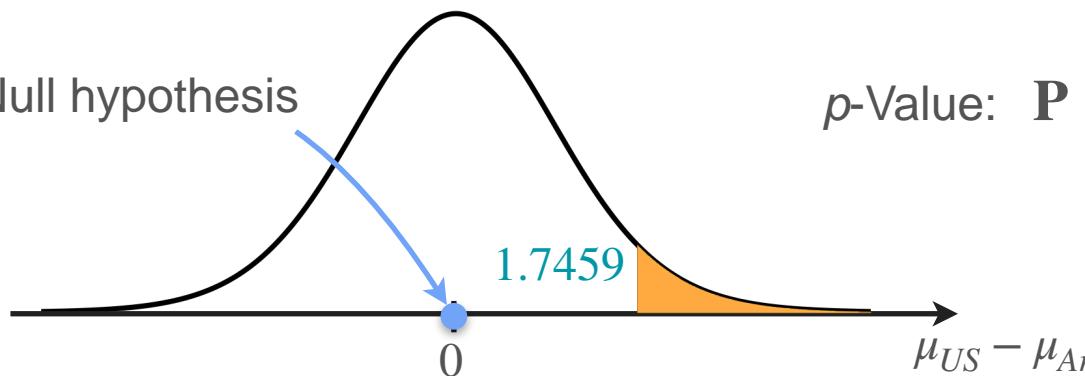
$$\bar{y} = 65.949$$

$$s_Y = 3.106$$

$$n_Y = 9$$

$$t = 1.7459$$

Null hypothesis



$$H_0 : \mu_{US} - \mu_{Ar} = 0 \text{ vs. } H_1 : \mu_{US} - \mu_{Ar} > 0$$

$$\alpha = 0.05$$

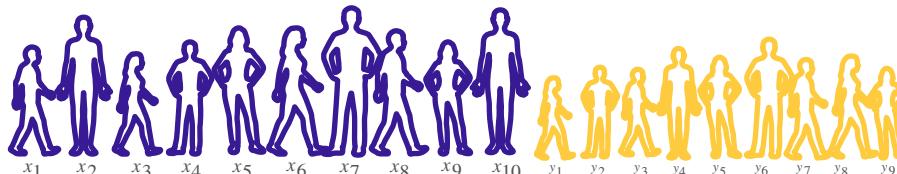
$$\text{If } H_0 \text{ is true: } T = \frac{(\bar{X} - \bar{Y}) - 0}{\sqrt{\frac{S_X^2}{10} + \frac{S_Y^2}{9}}} \sim t_{16.8}$$

$$p\text{-Value: } P(T > 1.7459 \mid \mu_{US} - \mu_{Ar} = 0)$$

$$= 0.0495 < 0.05$$

$\Rightarrow$  Reject  $H_0$  (and accept  $H_1$ )  
(with a 5% significance level)

# Independent Two-Sample $t$ -Test: Two Tailed Test



$$\bar{x} = 68.442$$

$$s_X = 3.113$$

$$n_X = 10$$

$$\bar{y} = 65.949$$

$$s_Y = 3.106$$

$$n_Y = 9$$

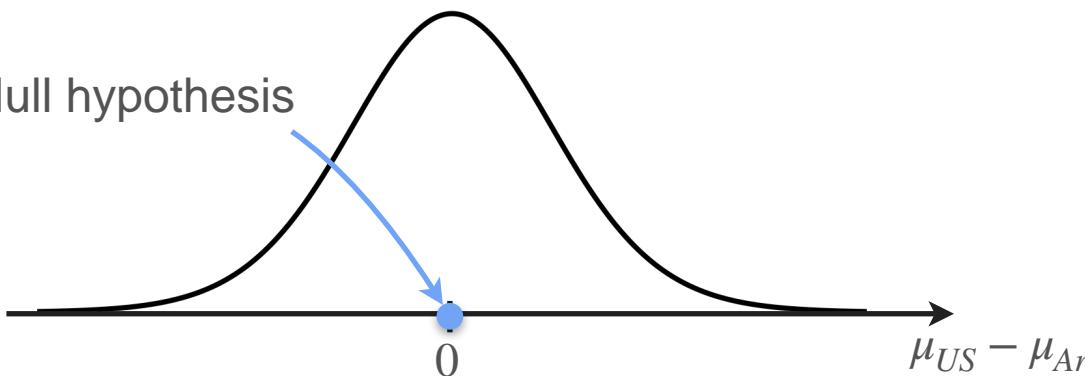
$$t = 1.7459$$

$$H_0 : \mu_{US} - \mu_{Ar} = 0 \text{ vs. } H_1 : \mu_{US} - \mu_{Ar} \neq 0$$

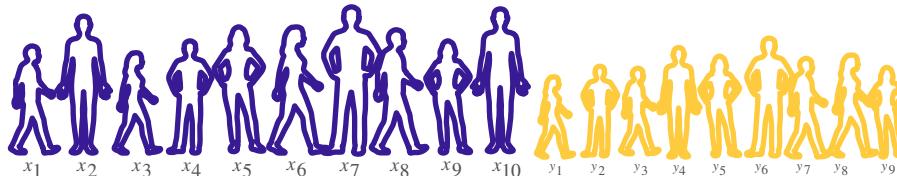
$$\alpha = 0.05$$

$$\text{If } H_0 \text{ is true: } T = \frac{(\bar{X} - \bar{Y}) - 0}{\sqrt{\frac{s_X^2}{10} + \frac{s_Y^2}{9}}} \sim t_{16.8}$$

Null hypothesis



# Independent Two-Sample $t$ -Test: Two Tailed Test



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$$s_X = 3.113$$

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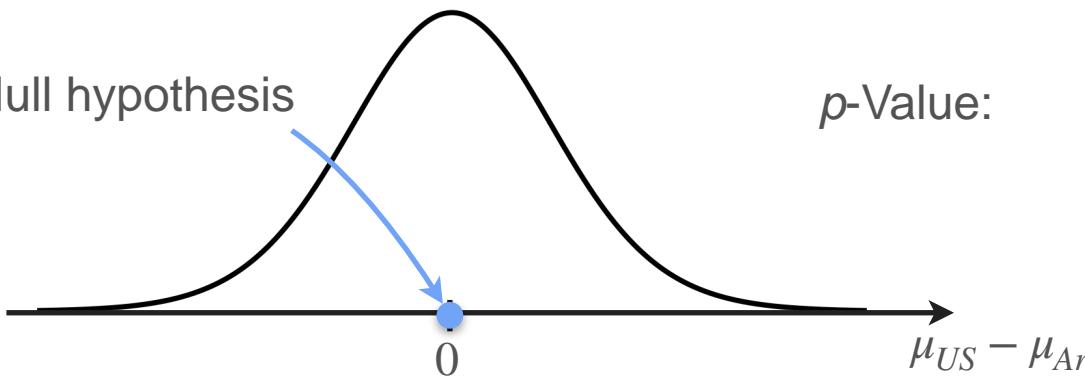
$$s_Y = 3.106$$

$$n_Y = 9$$

$$t = 1.7459$$

Null hypothesis

$p$ -Value:

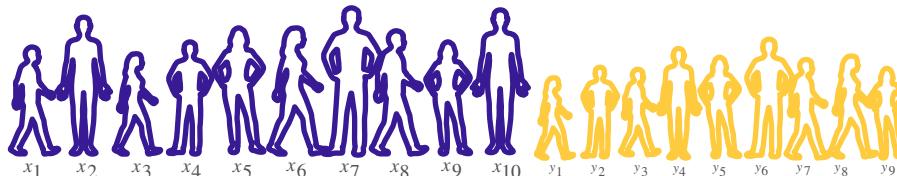


$$H_0 : \mu_{US} - \mu_{Ar} = 0 \text{ vs. } H_1 : \mu_{US} - \mu_{Ar} \neq 0$$

$$\alpha = 0.05$$

$$\text{If } H_0 \text{ is true: } T = \frac{(\bar{X} - \bar{Y}) - 0}{\sqrt{\frac{s_X^2}{10} + \frac{s_Y^2}{9}}} \sim t_{16.8}$$

# Independent Two-Sample $t$ -Test: Two Tailed Test



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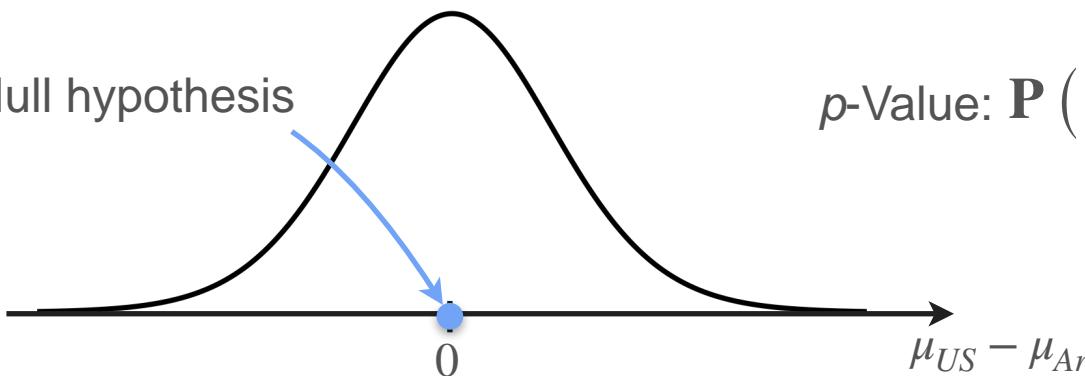
$$H_0 : \mu_{US} - \mu_{Ar} = 0 \text{ vs. } H_1 : \mu_{US} - \mu_{Ar} \neq 0$$

$$\alpha = 0.05$$

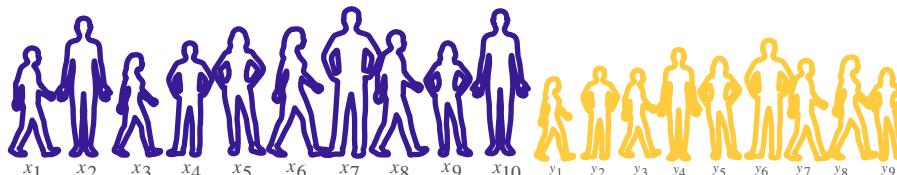
$$\text{If } H_0 \text{ is true: } T = \frac{(\bar{X} - \bar{Y}) - 0}{\sqrt{\frac{s_X^2}{10} + \frac{s_Y^2}{9}}} \sim t_{16.8}$$

Null hypothesis

$p$ -Value:  $\mathbf{P}(|T| > 1.7459 | \mu_{US} - \mu_{Ar} = 0)$



# Independent Two-Sample $t$ -Test: Two Tailed Test



$$\bar{x} = 68.442$$

$$s_X = 3.113$$

$$n_X = 10$$

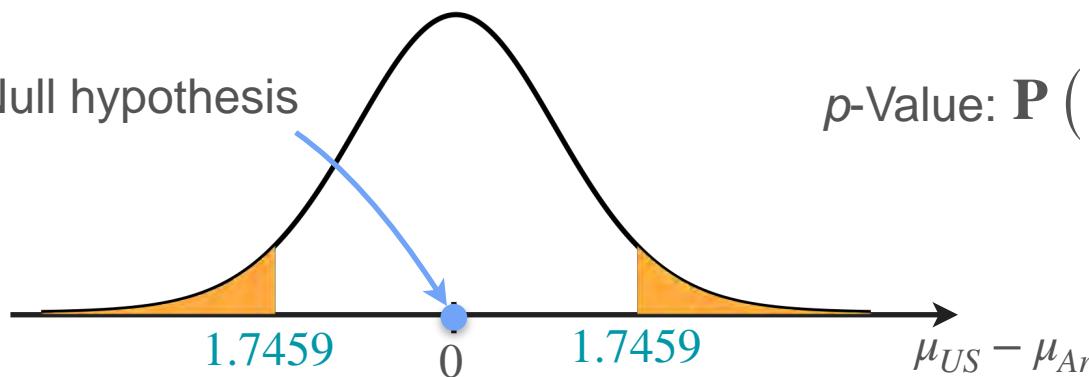
$$\bar{y} = 65.949$$

$$s_Y = 3.106$$

$$n_Y = 9$$

$$t = 1.7459$$

Null hypothesis



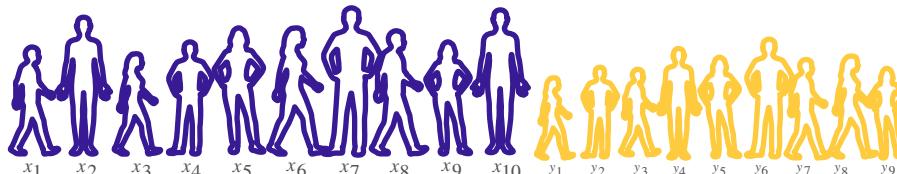
$p$ -Value:  $\mathbf{P}(|T| > 1.7459 | \mu_{US} - \mu_{Ar} = 0)$

$$H_0 : \mu_{US} - \mu_{Ar} = 0 \text{ vs. } H_1 : \mu_{US} - \mu_{Ar} \neq 0$$

$$\alpha = 0.05$$

$$\text{If } H_0 \text{ is true: } T = \frac{(\bar{X} - \bar{Y}) - 0}{\sqrt{\frac{s_X^2}{10} + \frac{s_Y^2}{9}}} \sim t_{16.8}$$

# Independent Two-Sample $t$ -Test: Two Tailed Test



$$\bar{x} = 68.442$$

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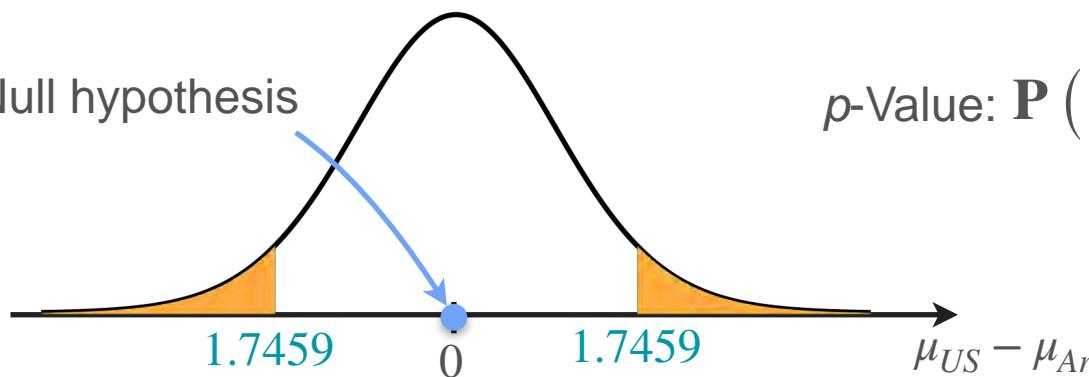
$$\bar{y} = 65.949$$

$$s_Y = 3.106$$

$$n_Y = 9$$

$$t = 1.7459$$

Null hypothesis



$$H_0 : \mu_{US} - \mu_{Ar} = 0 \text{ vs. } H_1 : \mu_{US} - \mu_{Ar} \neq 0$$

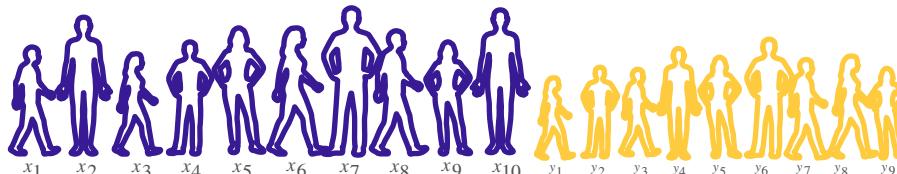
$$\alpha = 0.05$$

$$\text{If } H_0 \text{ is true: } T = \frac{(\bar{X} - \bar{Y}) - 0}{\sqrt{\frac{s_X^2}{10} + \frac{s_Y^2}{9}}} \sim t_{16.8}$$

$$p\text{-Value: } \mathbf{P}(|T| > 1.7459 | \mu_{US} - \mu_{Ar} = 0)$$

$$= 0.0991$$

# Independent Two-Sample $t$ -Test: Two Tailed Test



$$\bar{x} = 68.442$$

$$s_X = 3.113$$

$$n_X = 10$$

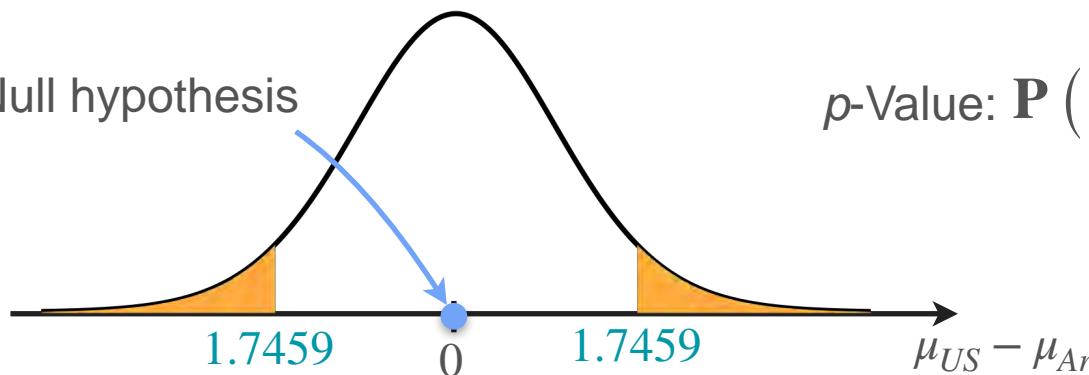
$$\bar{y} = 65.949$$

$$s_Y = 3.106$$

$$n_Y = 9$$

$$t = 1.7459$$

Null hypothesis



$p$ -Value:  $\mathbf{P}(|T| > 1.7459 | \mu_{US} - \mu_{Ar} = 0)$

$$= 0.0991 > 0.05$$

$\Rightarrow$  Do not reject  $H_0$   
(with a 5% significance level)



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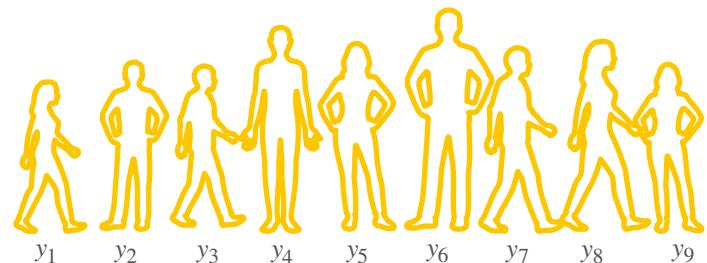
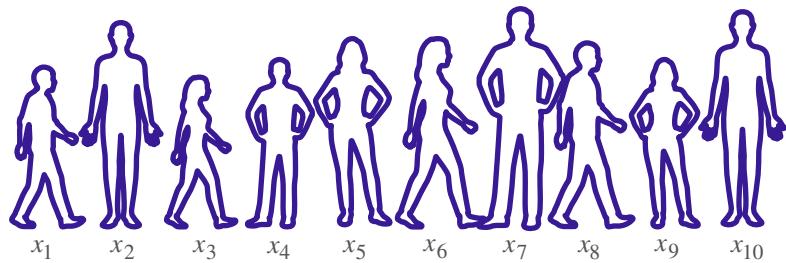
# Hypothesis Testing

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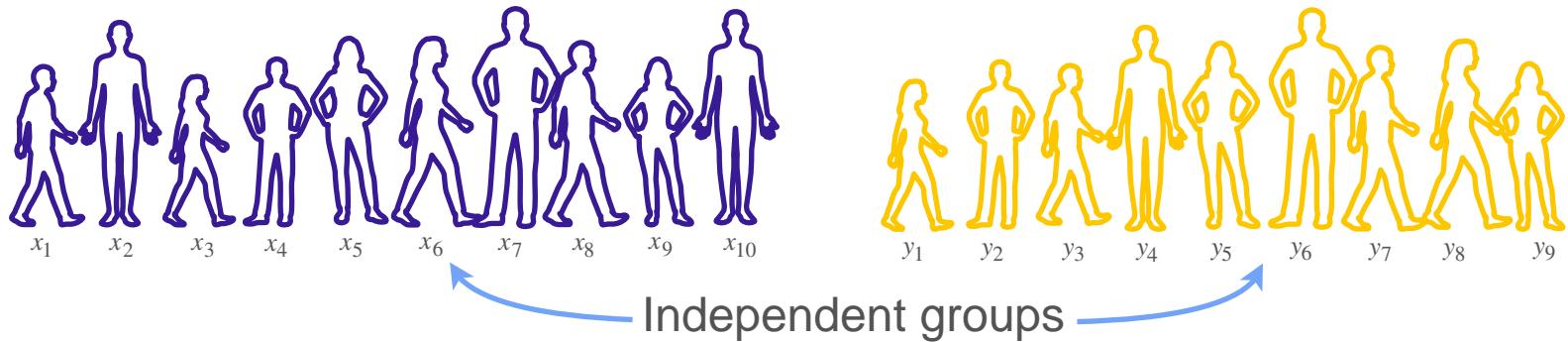
## Paired t-test

# Paired $t$ -Test and Two-Sample $t$ -Test

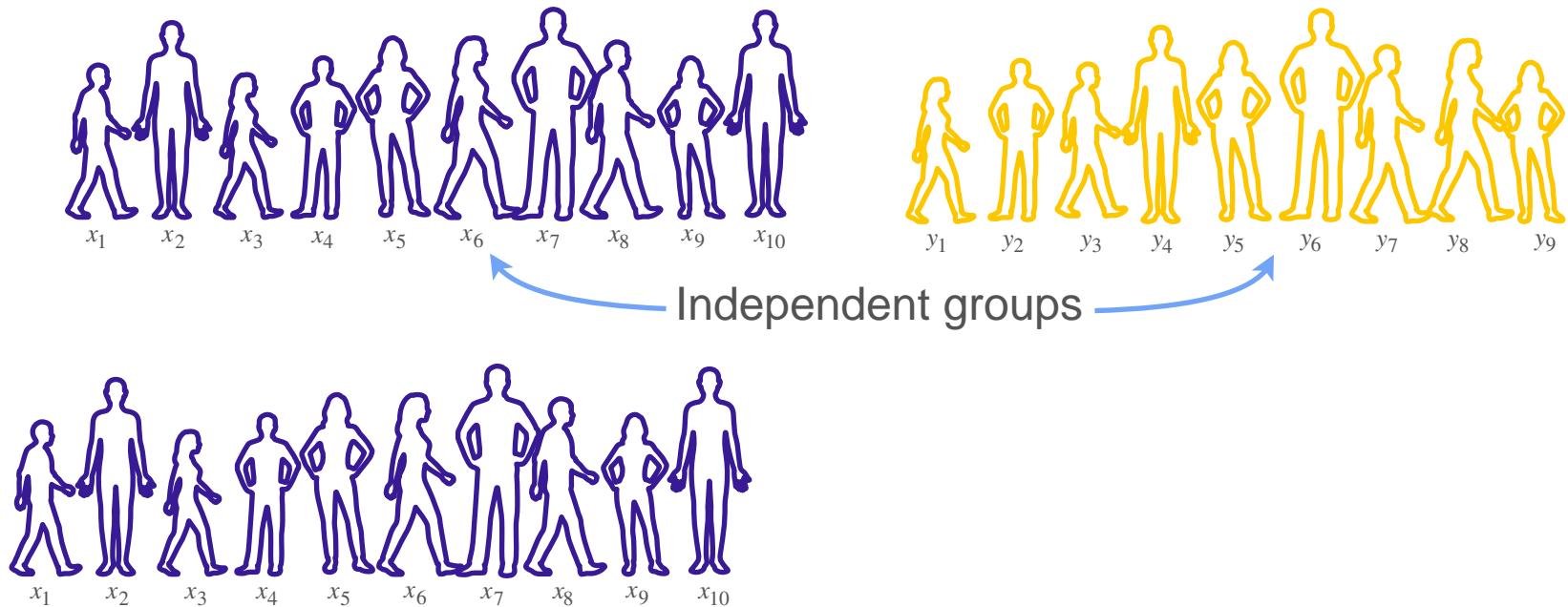
# Paired $t$ -Test and Two-Sample $t$ -Test



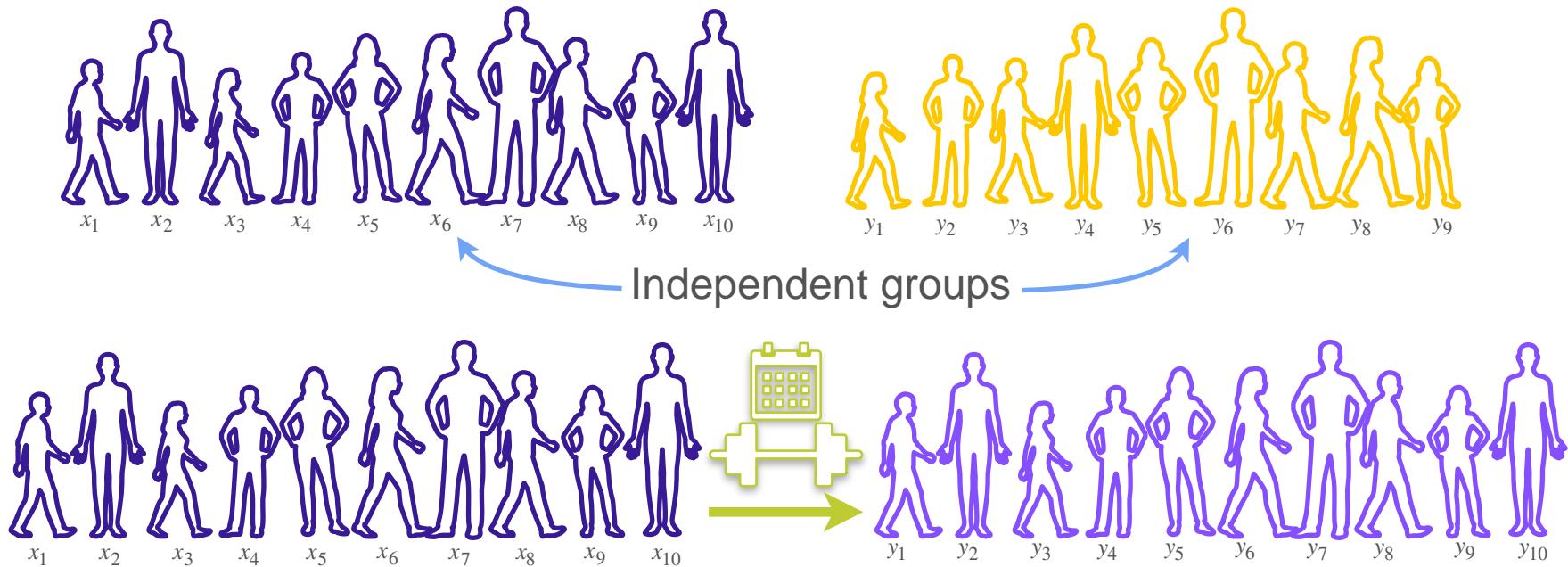
# Paired $t$ -Test and Two-Sample $t$ -Test



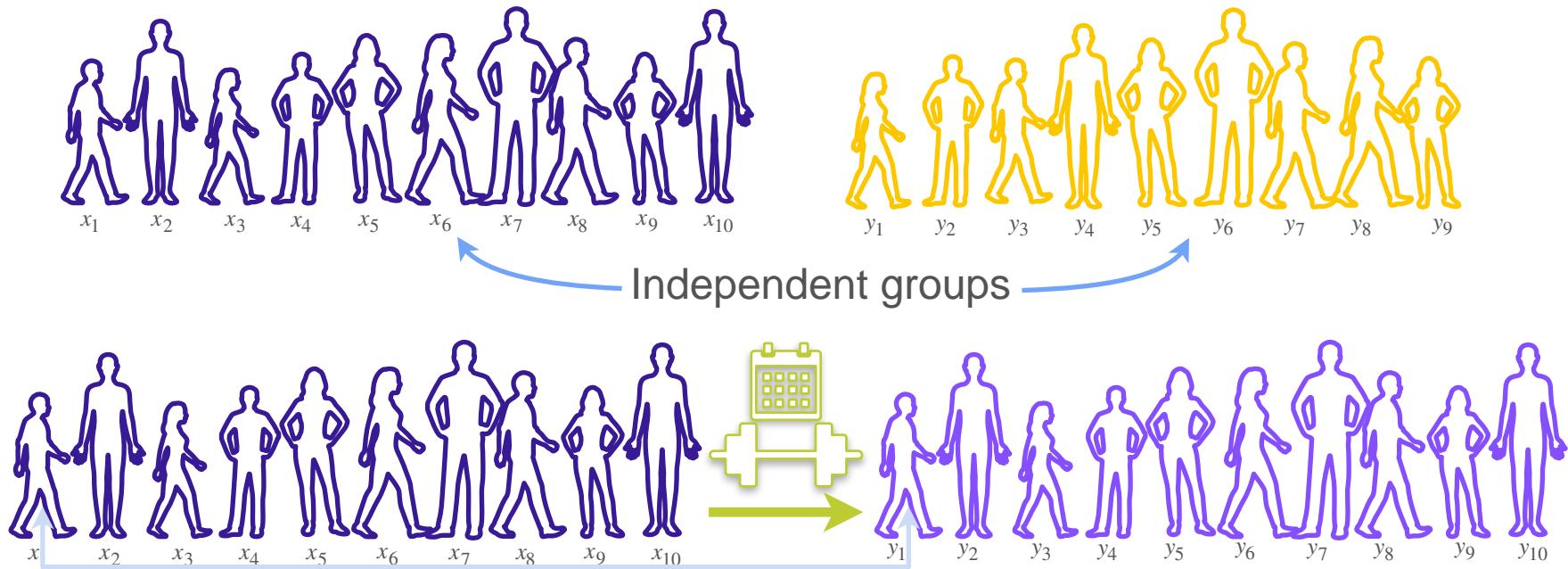
# Paired $t$ -Test and Two-Sample $t$ -Test



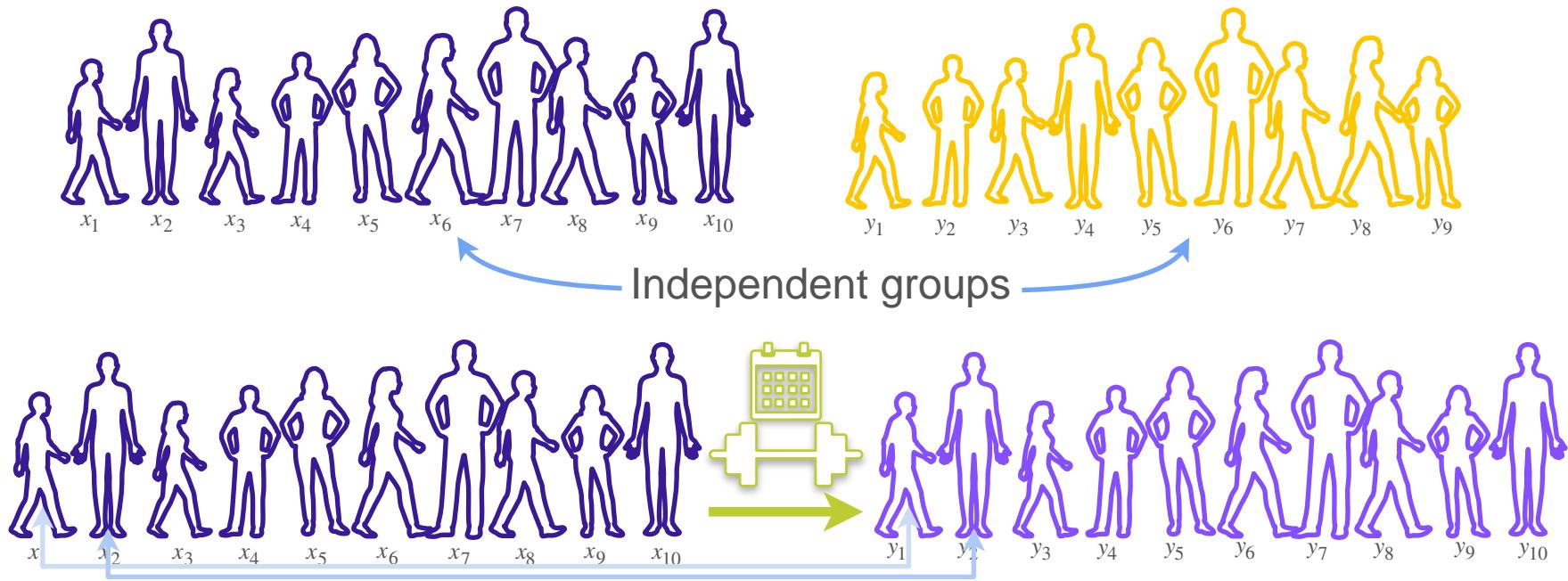
# Paired $t$ -Test and Two-Sample $t$ -Test



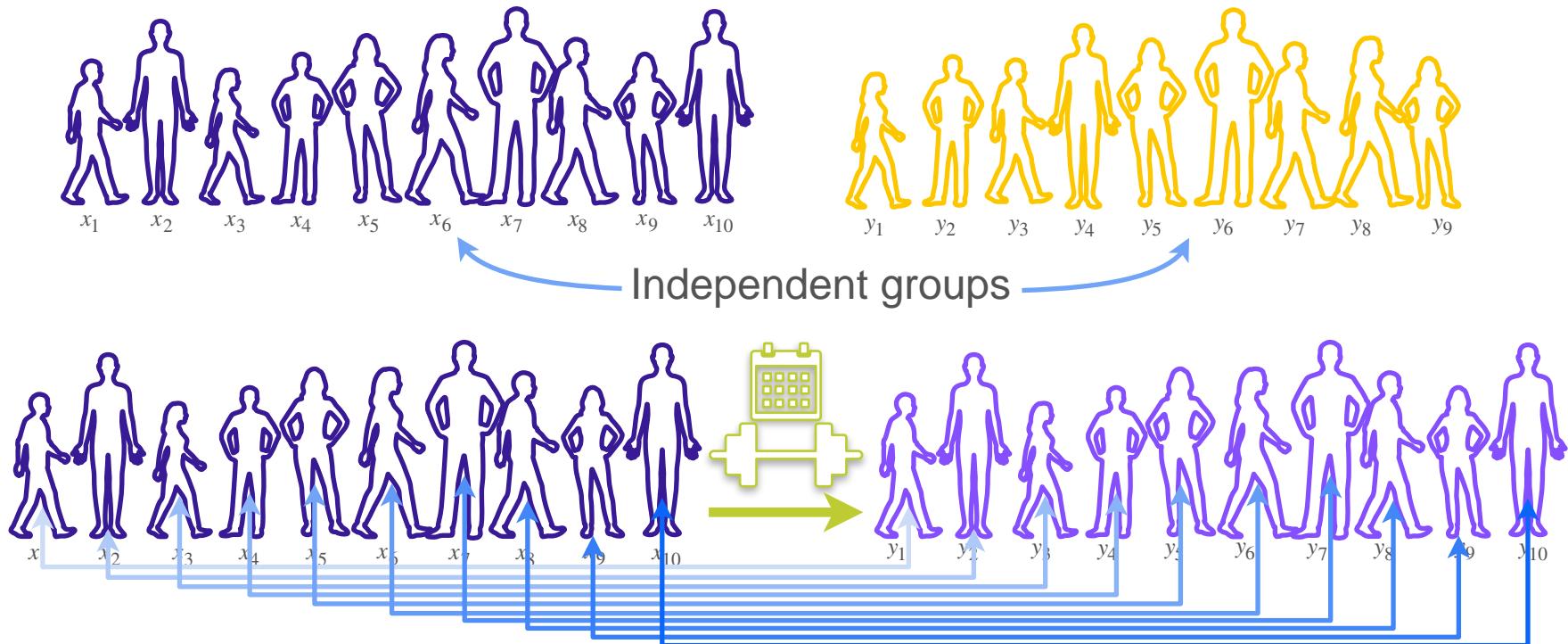
# Paired $t$ -Test and Two-Sample $t$ -Test



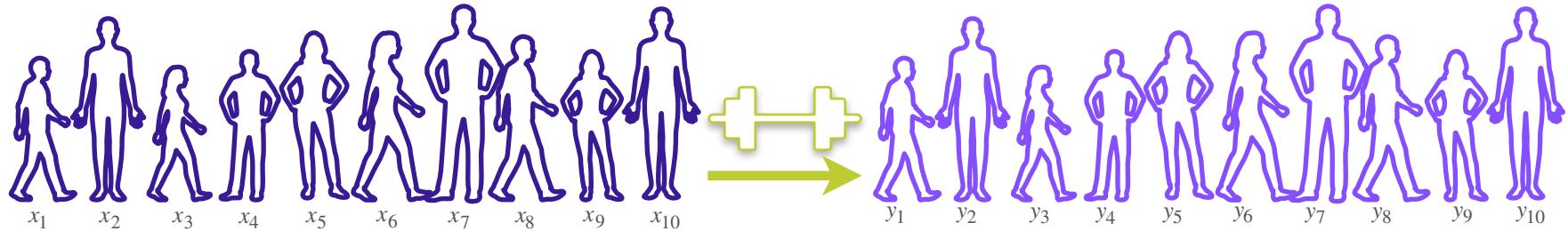
# Paired $t$ -Test and Two-Sample $t$ -Test



# Paired $t$ -Test and Two-Sample $t$ -Test

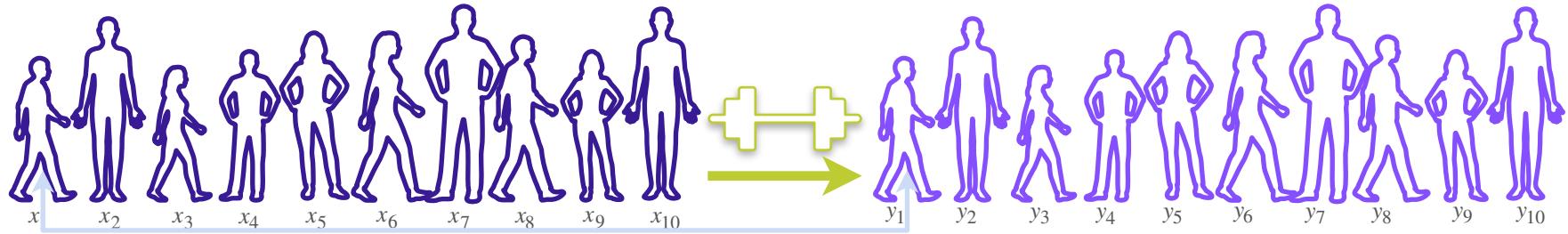


# Paired $t$ -Test: Statistic



Now you're interested in the difference between pair of samples

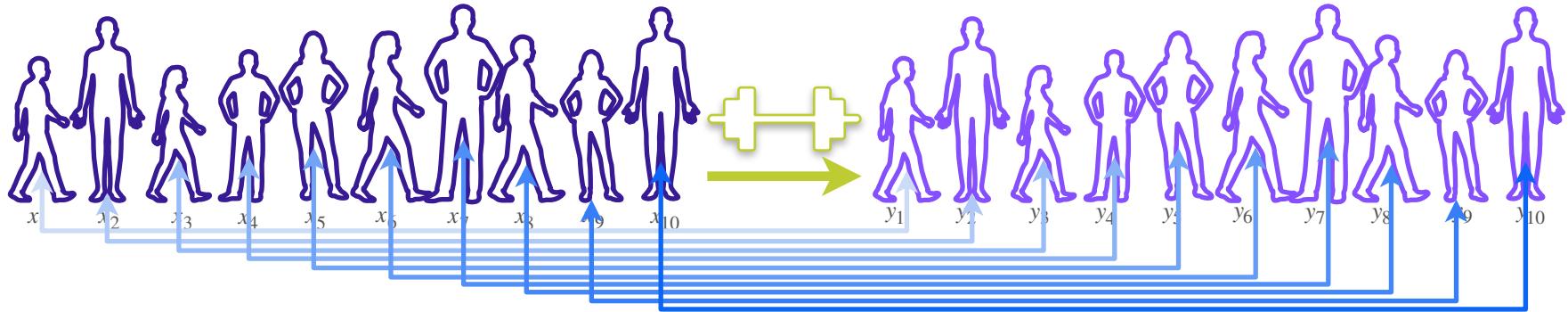
# Paired $t$ -Test: Statistic



Now you're interested in the difference between pair of samples

$$(X_1 - Y_1)$$

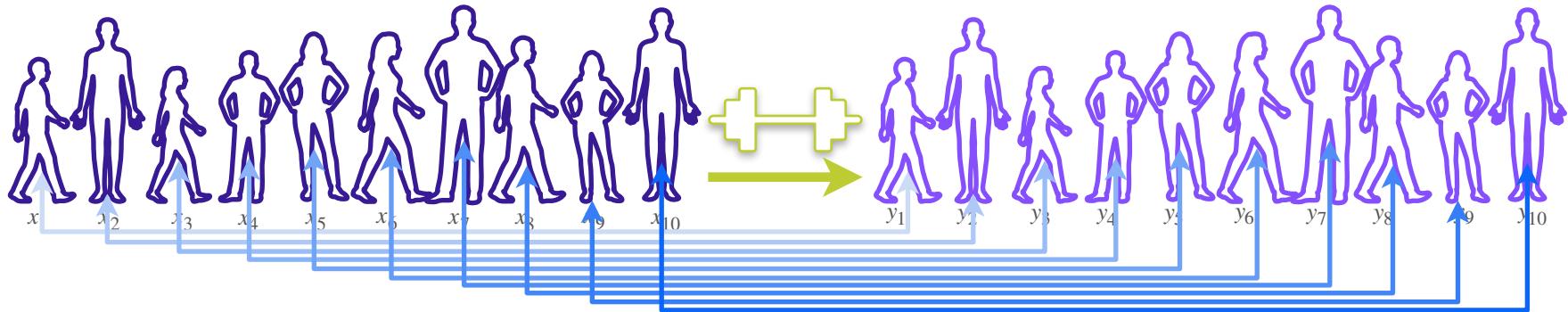
# Paired $t$ -Test: Statistic



Now you're interested in the difference between pair of samples

$$(X_1 - Y_1) \quad (X_2 - Y_2) \quad (X_3 - Y_3) \quad (X_4 - Y_4) \quad (X_5 - Y_5) \quad (X_6 - Y_6) \quad (X_7 - Y_7) \quad (X_8 - Y_8) \quad (X_9 - Y_9) \quad (X_{10} - Y_{10})$$

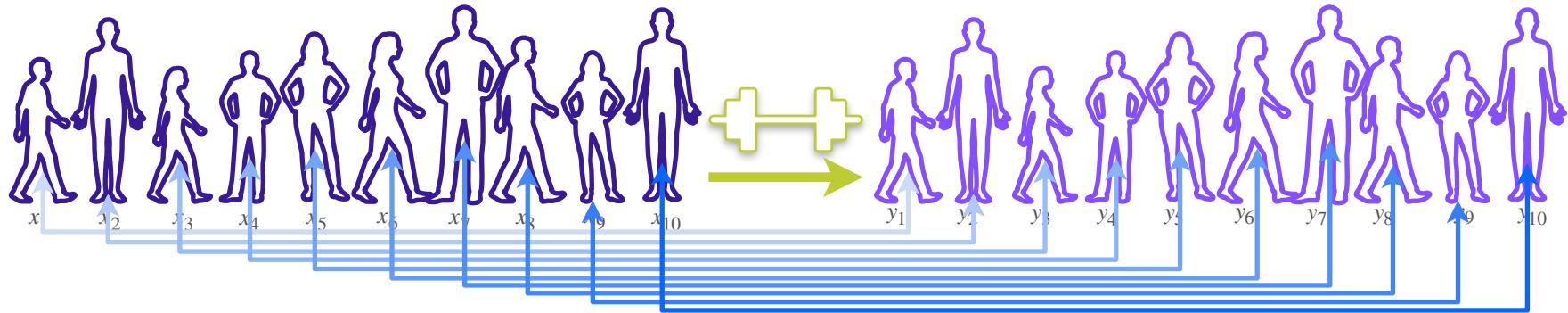
# Paired $t$ -Test: Statistic



Now you're interested in the difference between pair of samples

$$\frac{(X_1 - Y_1) + (X_2 - Y_2) + (X_3 - Y_3) + (X_4 - Y_4) + (X_5 - Y_5) + (X_6 - Y_6) + (X_7 - Y_7) + (X_8 - Y_8) + (X_9 - Y_9) + (X_{10} - Y_{10})}{10}$$

# Paired $t$ -Test: Statistic

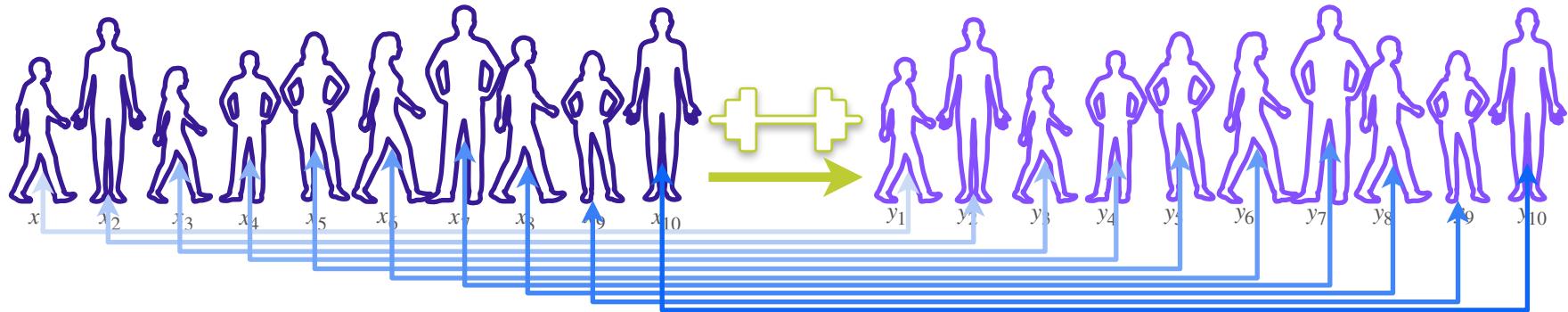


Now you're interested in the difference between pair of samples

$$\frac{(X_1 - Y_1) + (X_2 - Y_2) + (X_3 - Y_3) + (X_4 - Y_4) + (X_5 - Y_5) + (X_6 - Y_6) + (X_7 - Y_7) + (X_8 - Y_8) + (X_9 - Y_9) + (X_{10} - Y_{10})}{10}$$

$D_1$

# Paired $t$ -Test: Statistic



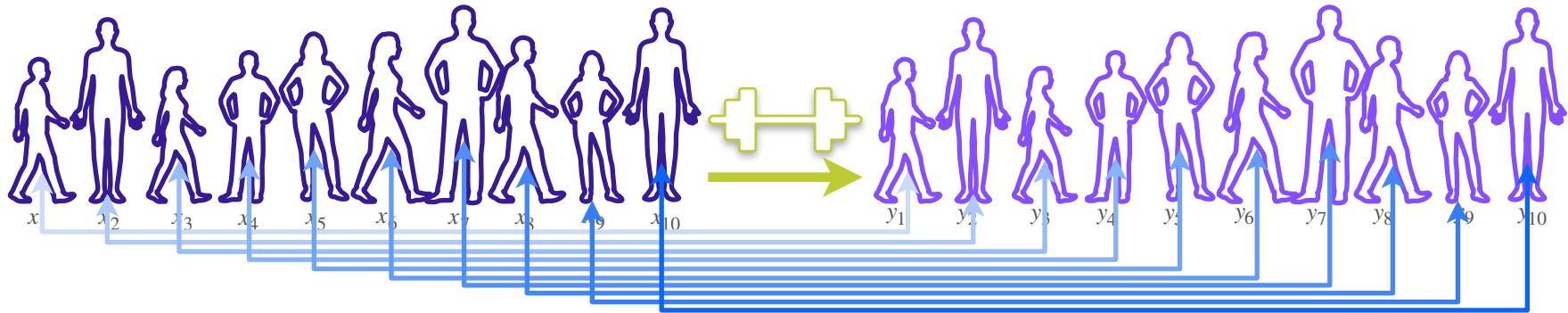
Now you're interested in the difference between pair of samples

$$\frac{(X_1 - Y_1) + (X_2 - Y_2) + (X_3 - Y_3) + (X_4 - Y_4) + (X_5 - Y_5) + (X_6 - Y_6) + (X_7 - Y_7) + (X_8 - Y_8) + (X_9 - Y_9) + (X_{10} - Y_{10})}{10}$$

$D_1$

$D_2$

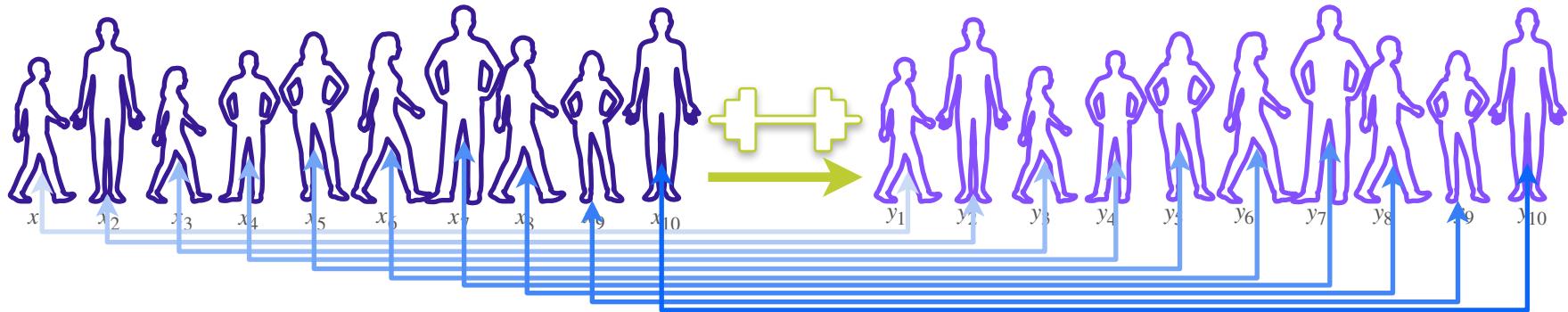
# Paired $t$ -Test: Statistic



Now you're interested in the difference between pair of samples

$$\frac{(X_1 - Y_1) + (X_2 - Y_2) + (X_3 - Y_3) + (X_4 - Y_4) + (X_5 - Y_5) + (X_6 - Y_6) + (X_7 - Y_7) + (X_8 - Y_8) + (X_9 - Y_9) + (X_{10} - Y_{10})}{10}$$
$$\frac{D_1 + D_2 + D_3 + D_4 + D_5 + D_6 + D_7 + D_8 + D_9 + D_{10}}{10}$$

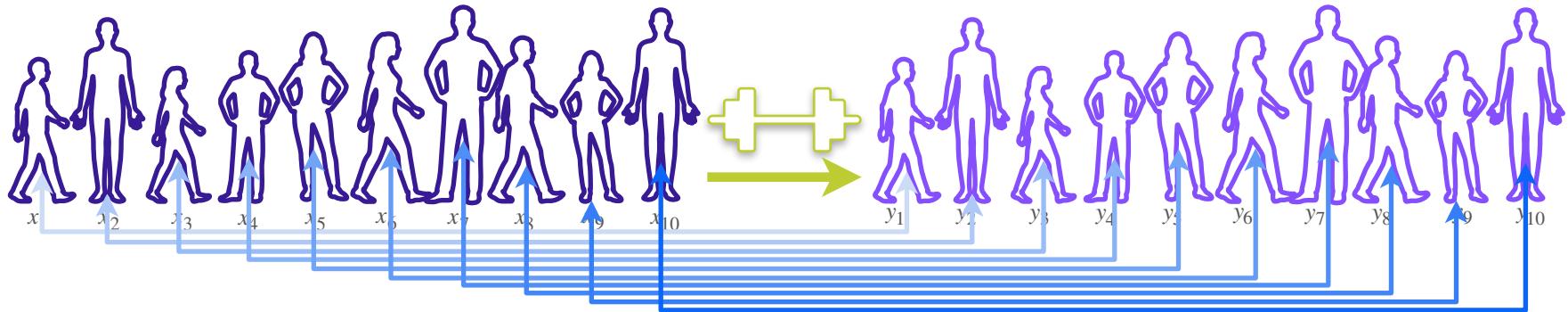
# Paired $t$ -Test: Statistic



Now you're interested in the difference between pair of samples

$$\frac{(X_1 - Y_1) + (X_2 - Y_2) + (X_3 - Y_3) + (X_4 - Y_4) + (X_5 - Y_5) + (X_6 - Y_6) + (X_7 - Y_7) + (X_8 - Y_8) + (X_9 - Y_9) + (X_{10} - Y_{10})}{10}$$
$$\frac{D_1 + D_2 + D_3 + D_4 + D_5 + D_6 + D_7 + D_8 + D_9 + D_{10}}{10}$$

# Paired $t$ -Test: Statistic

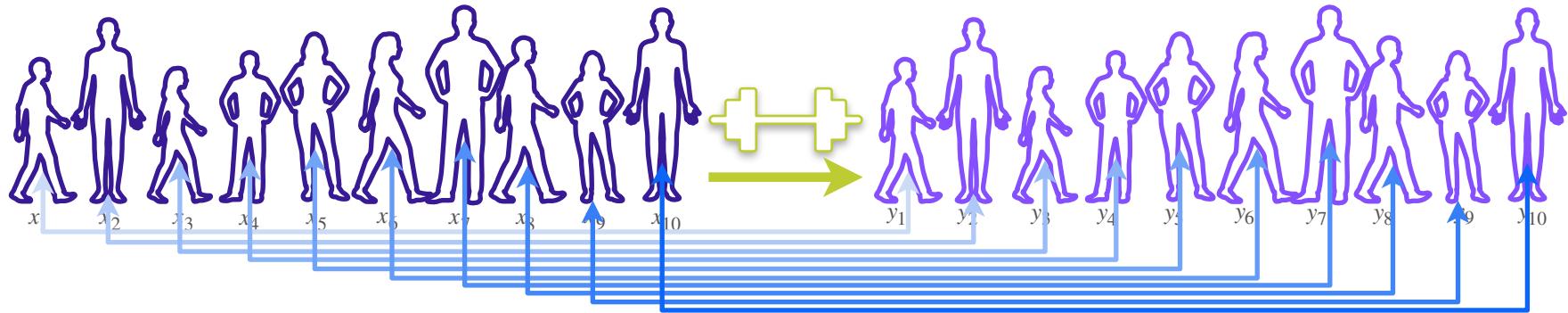


Now you're interested in the difference between pair of samples

$$\frac{(X_1 - Y_1) + (X_2 - Y_2) + (X_3 - Y_3) + (X_4 - Y_4) + (X_5 - Y_5) + (X_6 - Y_6) + (X_7 - Y_7) + (X_8 - Y_8) + (X_9 - Y_9) + (X_{10} - Y_{10})}{10}$$

$$\bar{D} = \frac{D_1 + D_2 + D_3 + D_4 + D_5 + D_6 + D_7 + D_8 + D_9 + D_{10}}{10}$$

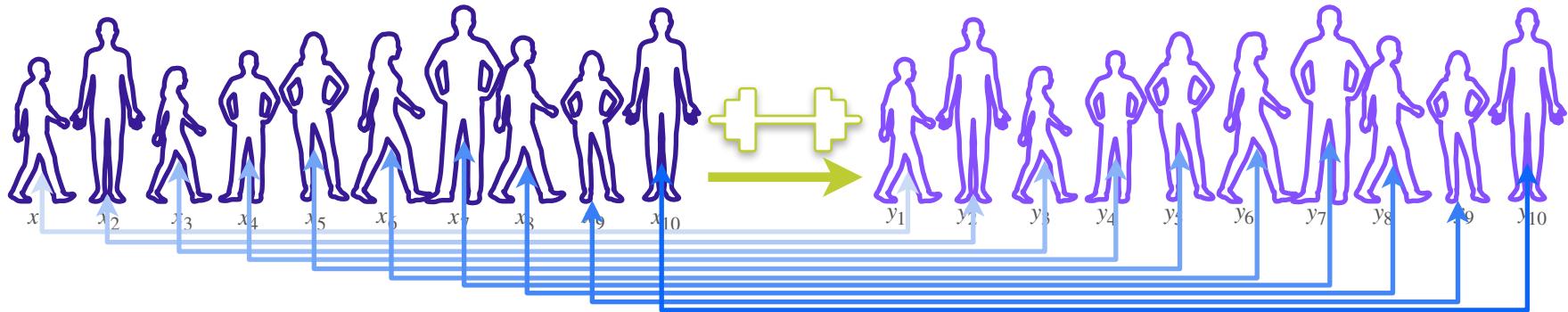
# Paired $t$ -Test: Statistic



Now you're interested in the difference between pair of samples

$$\bar{D} = \frac{D_1 + D_2 + D_3 + D_4 + D_5 + D_6 + D_7 + D_8 + D_9 + D_{10}}{10}$$

# Paired $t$ -Test: Statistic



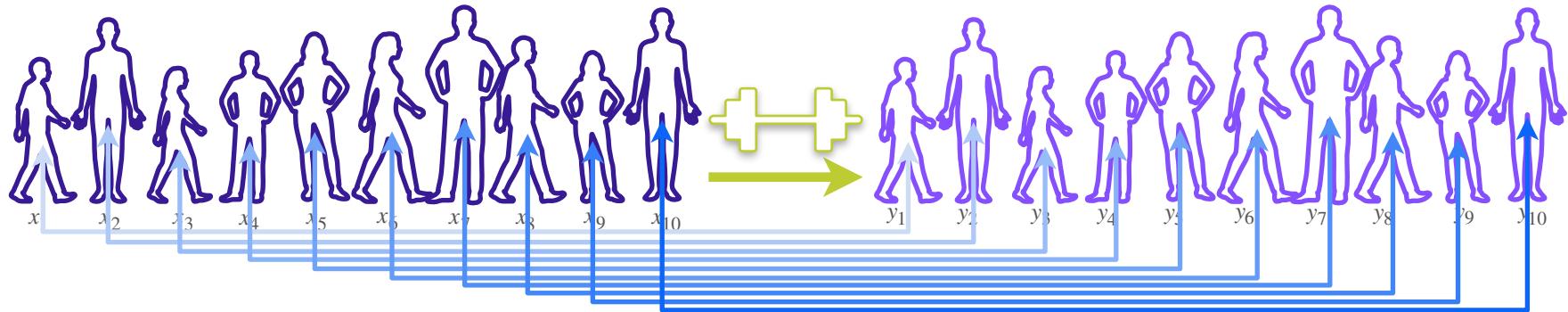
Now you're interested in the difference between pair of samples

$$\bar{D} = \frac{D_1 + D_2 + D_3 + D_4 + D_5 + D_6 + D_7 + D_8 + D_9 + D_{10}}{10}$$

If  $X_i, Y_i$  are gaussian  $\Rightarrow D_i = X_i - Y_i$  is gaussian.

$$D_i \stackrel{i.i.d}{\sim} \mathcal{N}(\mu_D, \sigma_D^2)$$

# Paired $t$ -Test: Statistic

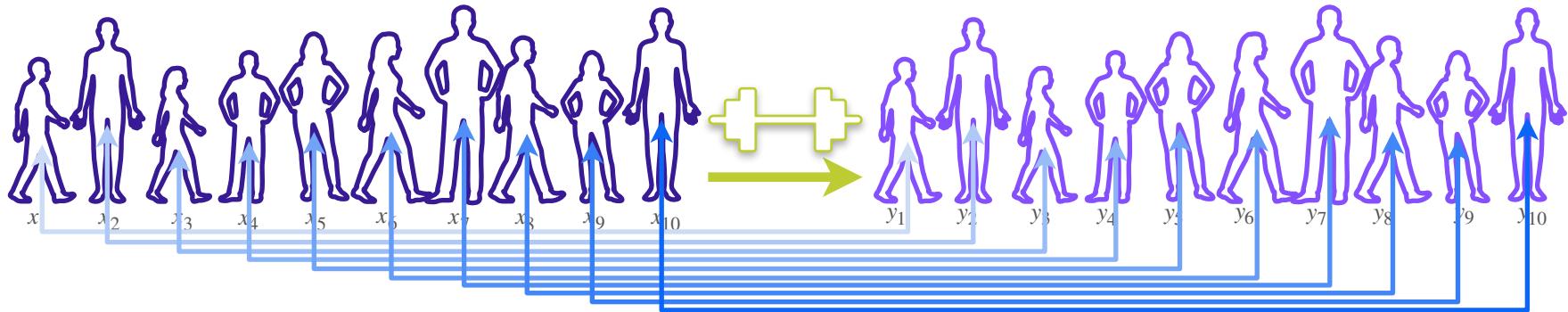


Now you're interested in the difference between pair of samples

$$\bar{D} = \frac{D_1 + D_2 + D_3 + D_4 + D_5 + D_6 + D_7 + D_8 + D_9 + D_{10}}{10}$$

If  $X_i, Y_i$  are gaussian  $\Rightarrow D_i = X_i - Y_i$  is gaussian.

# Paired $t$ -Test: Statistic



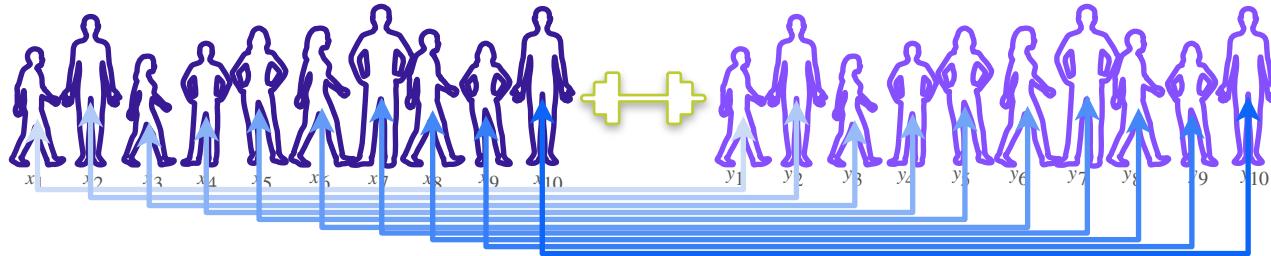
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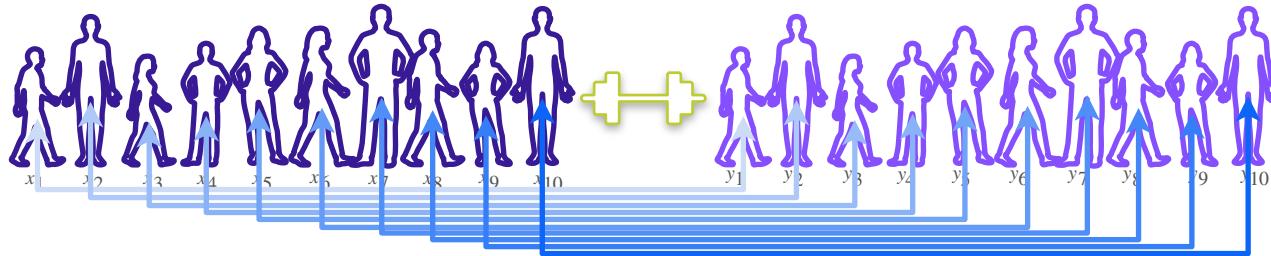
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Now you're interested in the difference between pair of samples

$$\bar{D} = \frac{D_1 + D_2 + D_3 + D_4 + D_5 + D_6 + D_7 + D_8 + D_9 + D_{10}}{10} \quad D_i = X_i - Y_i$$
$$D_i \stackrel{i.i.d}{\sim} \mathcal{N}(\mu_D, \sigma_D^2)$$

# Paired $t$ -Test: Statistic

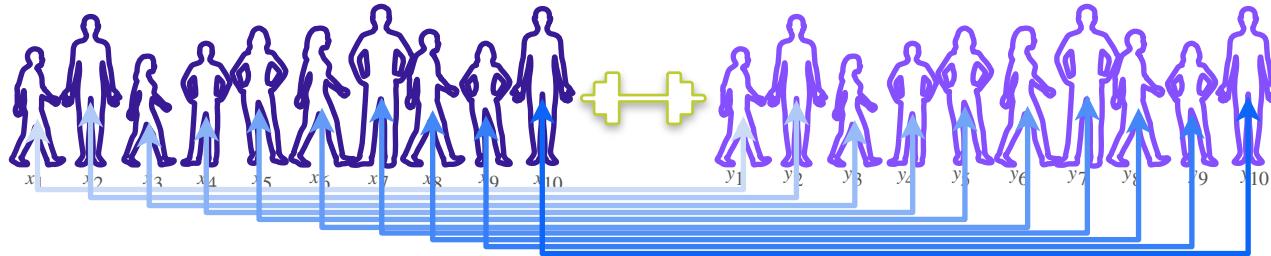


Now you're interested in the difference between pair of samples

$$\bar{D} = \frac{D_1 + D_2 + D_3 + D_4 + D_5 + D_6 + D_7 + D_8 + D_9 + D_{10}}{10} \quad D_i = X_i - Y_i$$
$$D_i \stackrel{i.i.d}{\sim} \mathcal{N}(\mu_D, \sigma_D^2)$$

$$\frac{\bar{D} - \mu_D}{\sigma_D / \sqrt{10}} \sim \mathcal{N}(0, 1^2)$$

# Paired $t$ -Test: Statistic



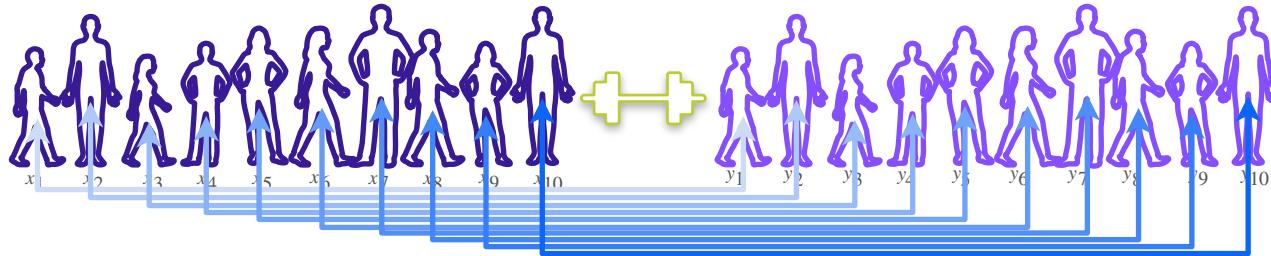
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$$\frac{\bar{D} - \mu_D}{\sigma_D / \sqrt{10}} \sim \mathcal{N}(0, 1^2)$$

But  $\sigma_D$  is unknown

# Paired $t$ -Test: Statistic



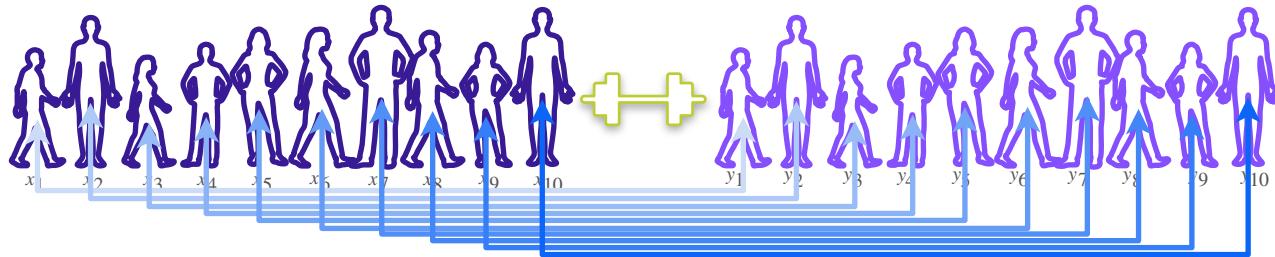
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But  $\sigma_D$  is unknown  $\Rightarrow \sigma_D \rightarrow S_D = \sqrt{\frac{\sum_{i=1}^{10} (D_i - \bar{D})^2}{10 - 1}}$

# Paired $t$ -Test: Statistic



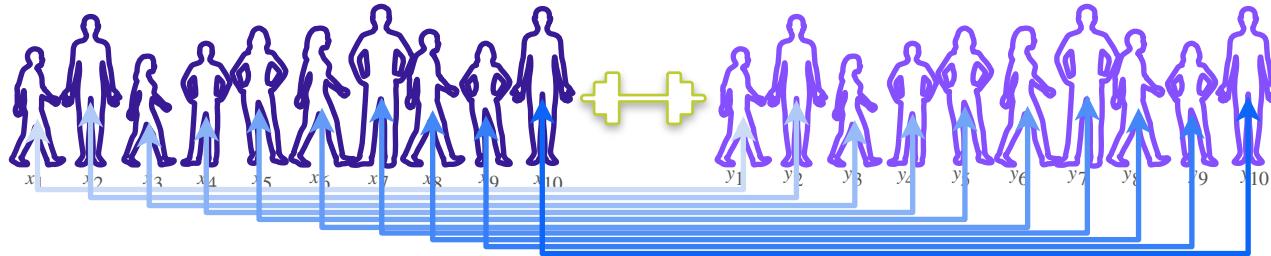
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# Paired $t$ -Test: Statistic



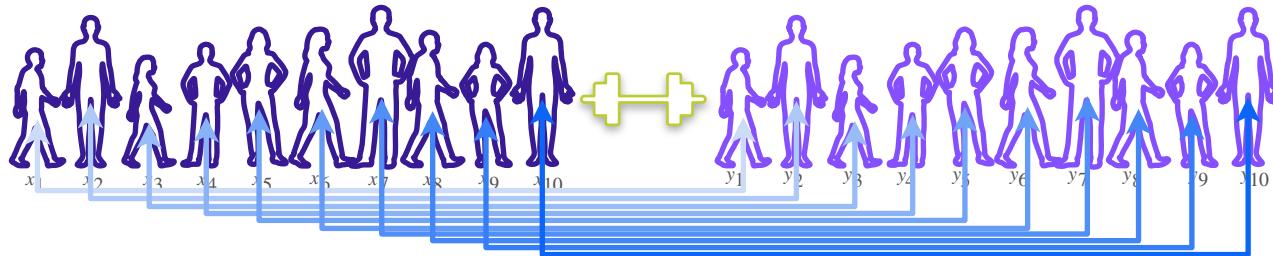
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# Paired $t$ -Test: Statistic



Now you're interested in the difference between pair of samples

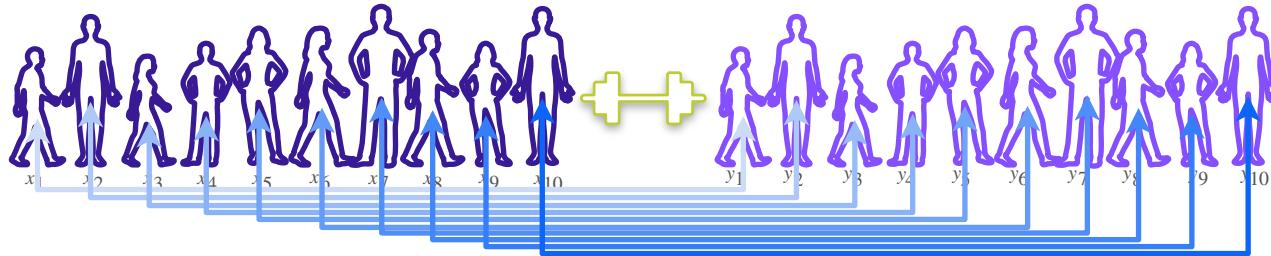
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$$\frac{\bar{D} - \mu_D}{\sigma_D / \sqrt{10}} \sim \mathcal{N}(0, 1^2)$$

But  $\sigma_D$  is unknown  $\Rightarrow \sigma_D \rightarrow S_D = \sqrt{\frac{\sum_{i=1}^{10} (D_i - \bar{D})^2}{10 - 1}}$   $\Rightarrow T = \frac{\bar{D} - \mu_D}{S_D / \sqrt{10}} \sim t_{10-1}$

$$H_0 : \mu_D = 0$$

# Paired $t$ -Test: Statistic



Now you're interested in the difference between pair of samples

$$\bar{D} = \frac{D_1 + D_2 + D_3 + D_4 + D_5 + D_6 + D_7 + D_8 + D_9 + D_{10}}{10} \quad D_i = X_i - Y_i$$
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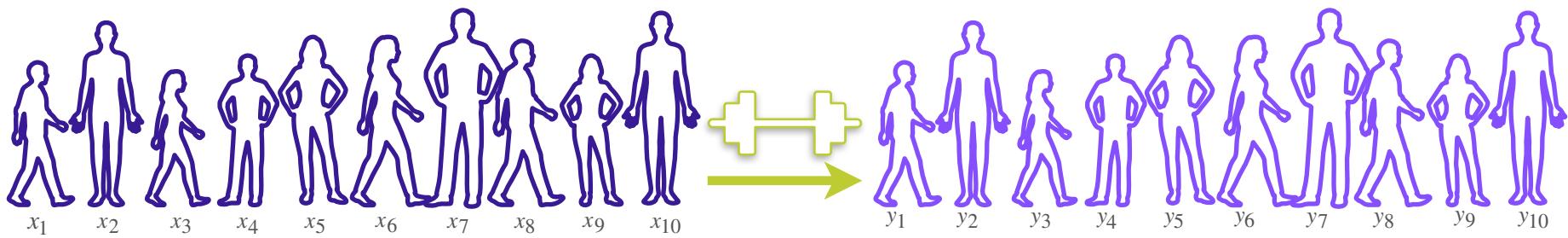
$H_0 : \mu_D = 0$  Test statistic

# Paired $t$ -Test: Observations

=

$d_i$

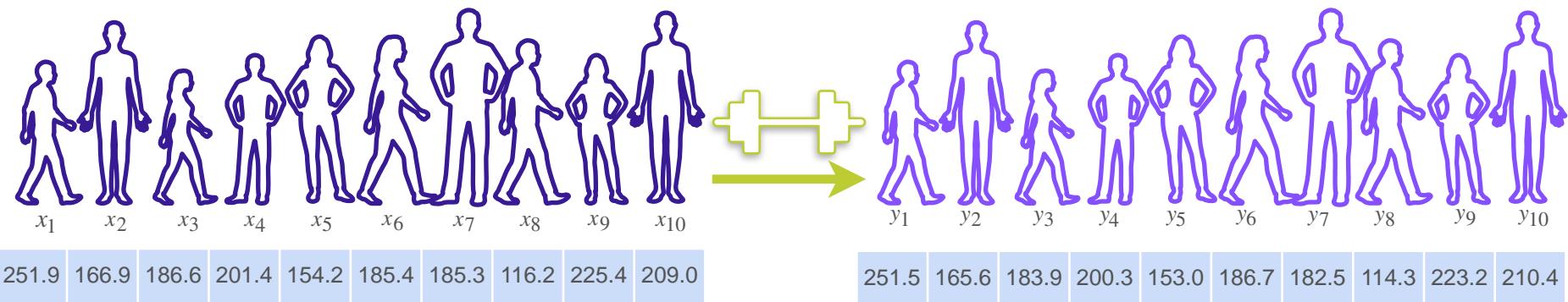
# Paired $t$ -Test: Observations



=

$$d_i$$

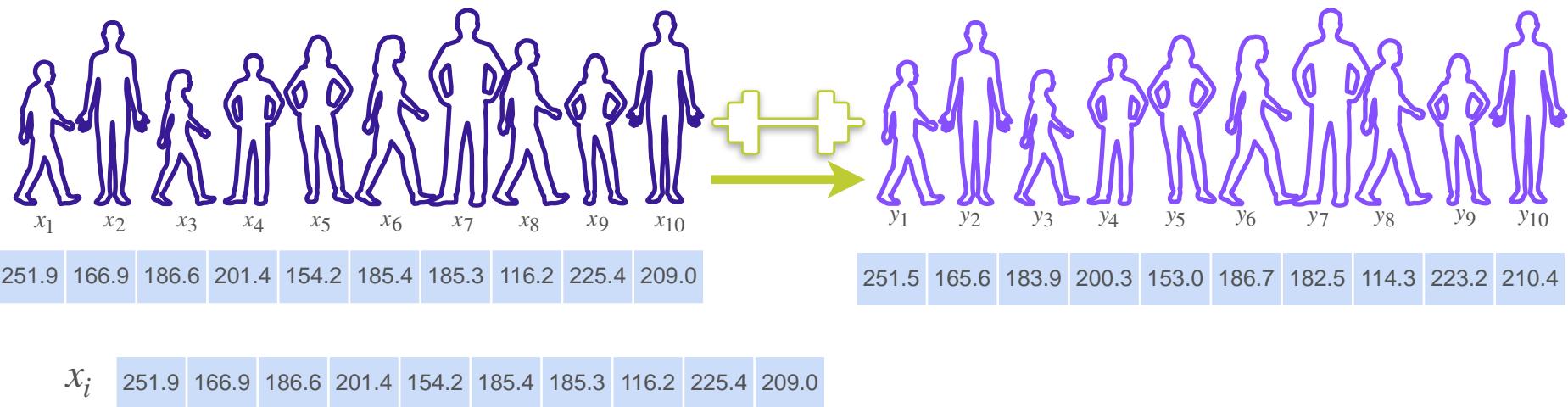
# Paired $t$ -Test: Observations



=

$d_i$

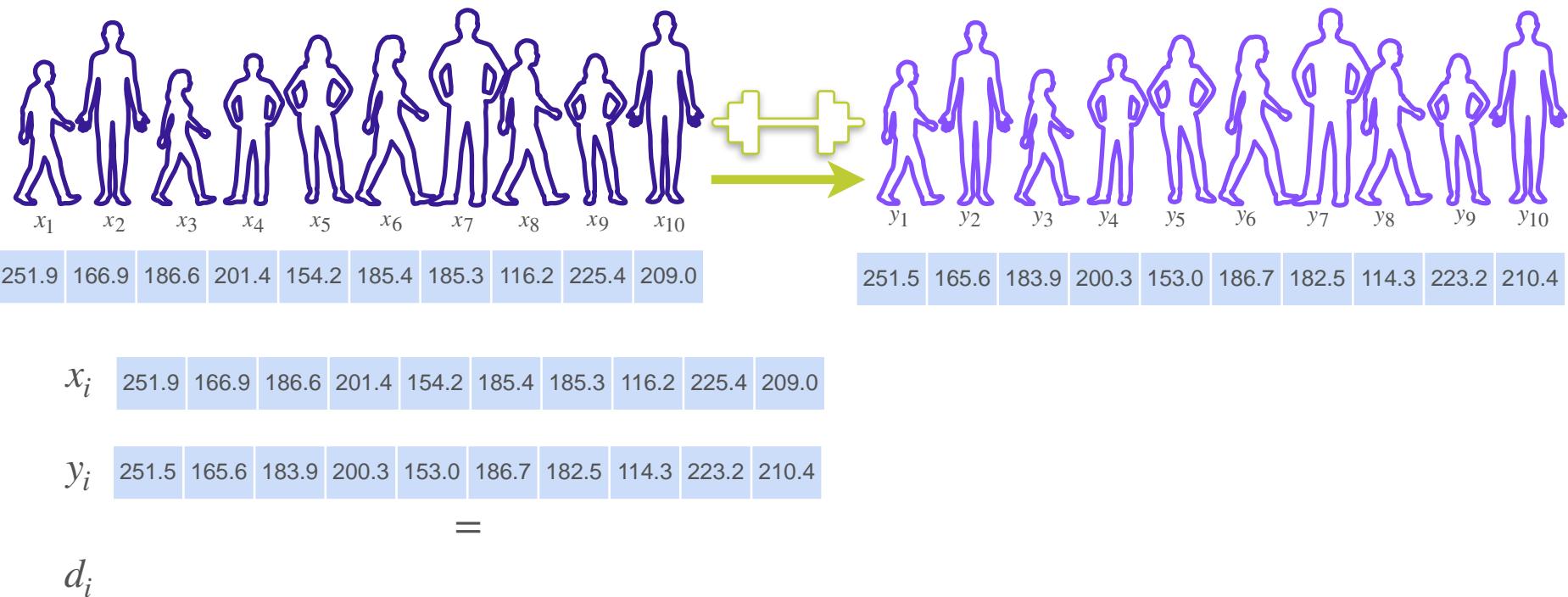
# Paired $t$ -Test: Observations



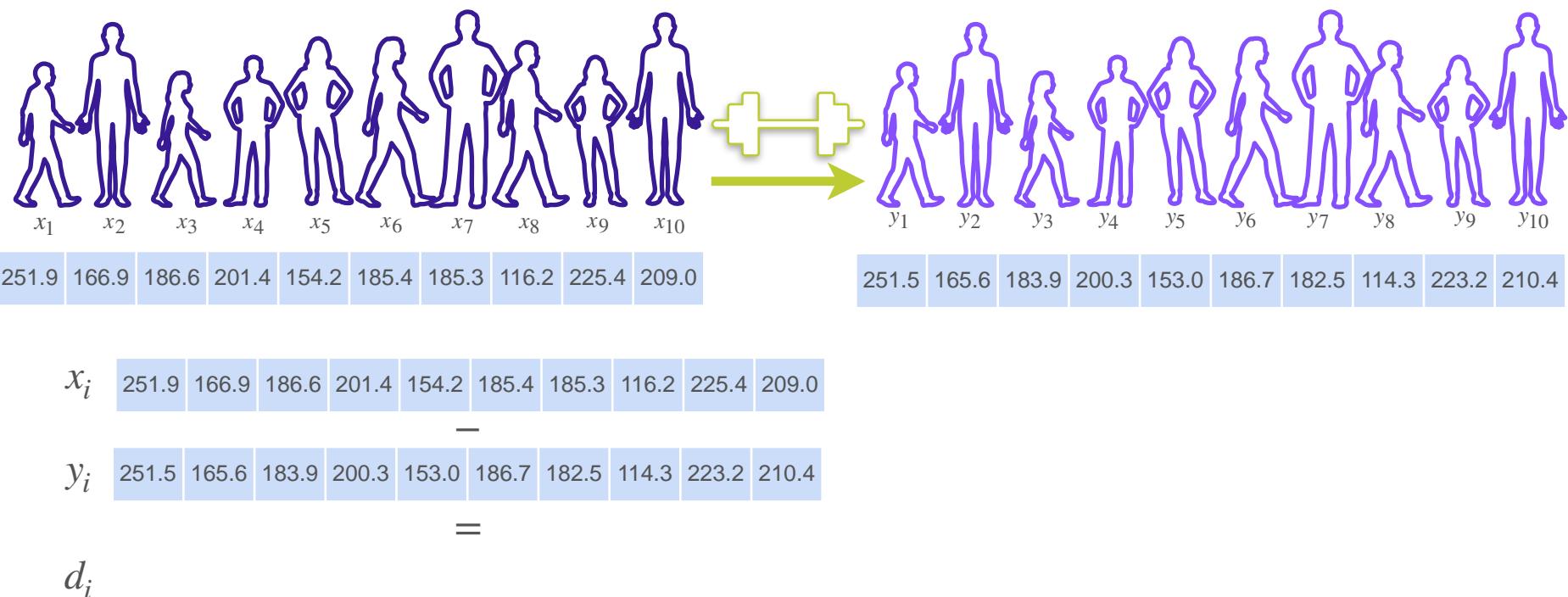
=

$d_i$

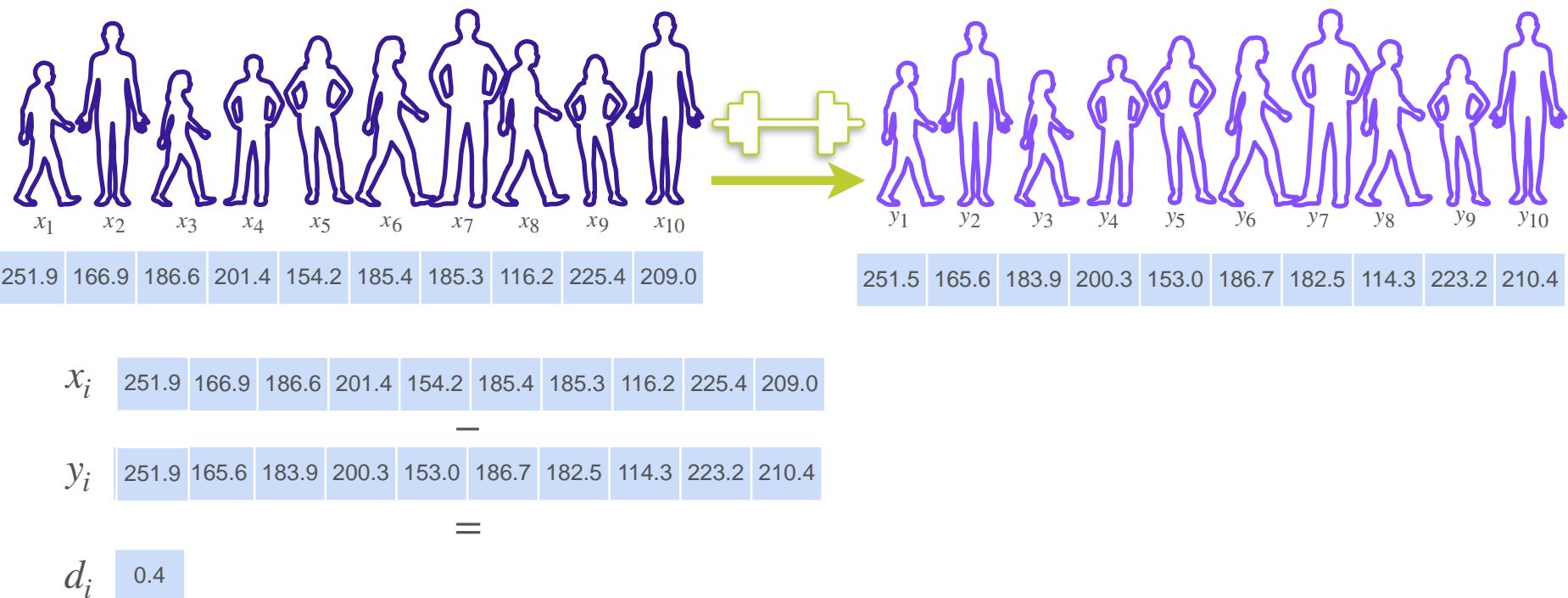
# Paired $t$ -Test: Observations



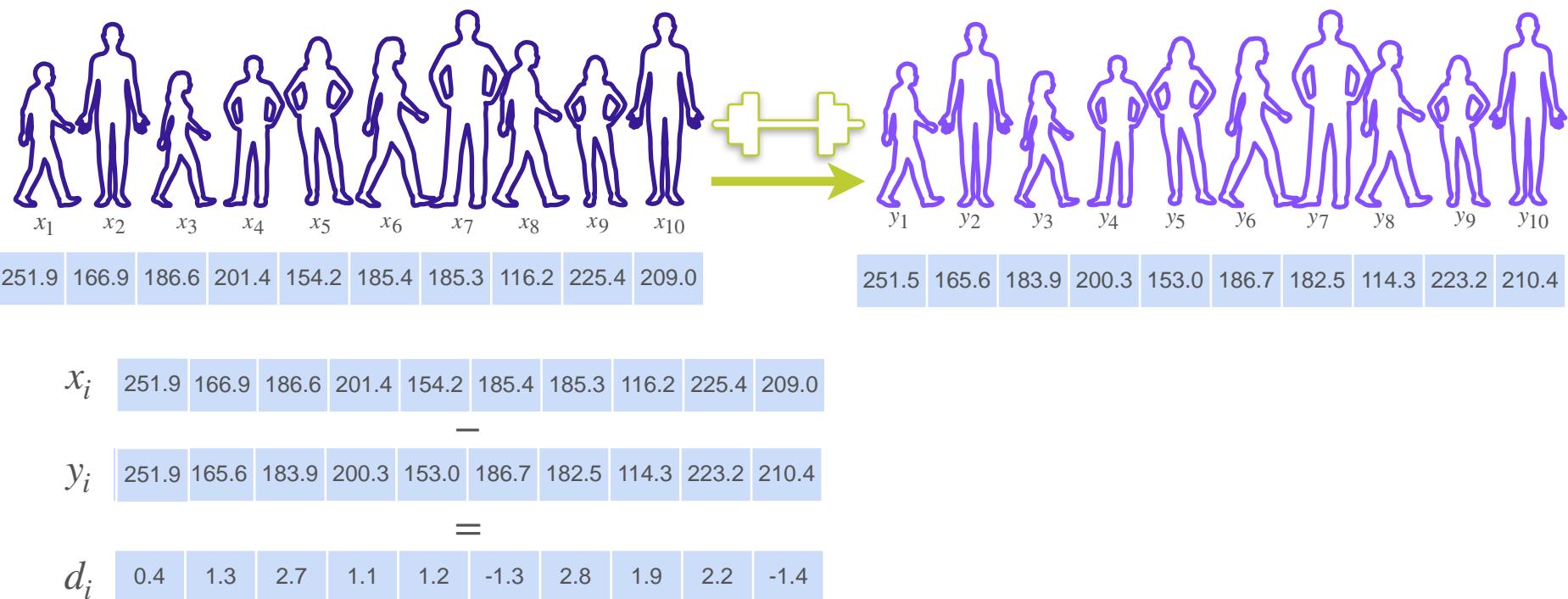
# Paired $t$ -Test: Observations



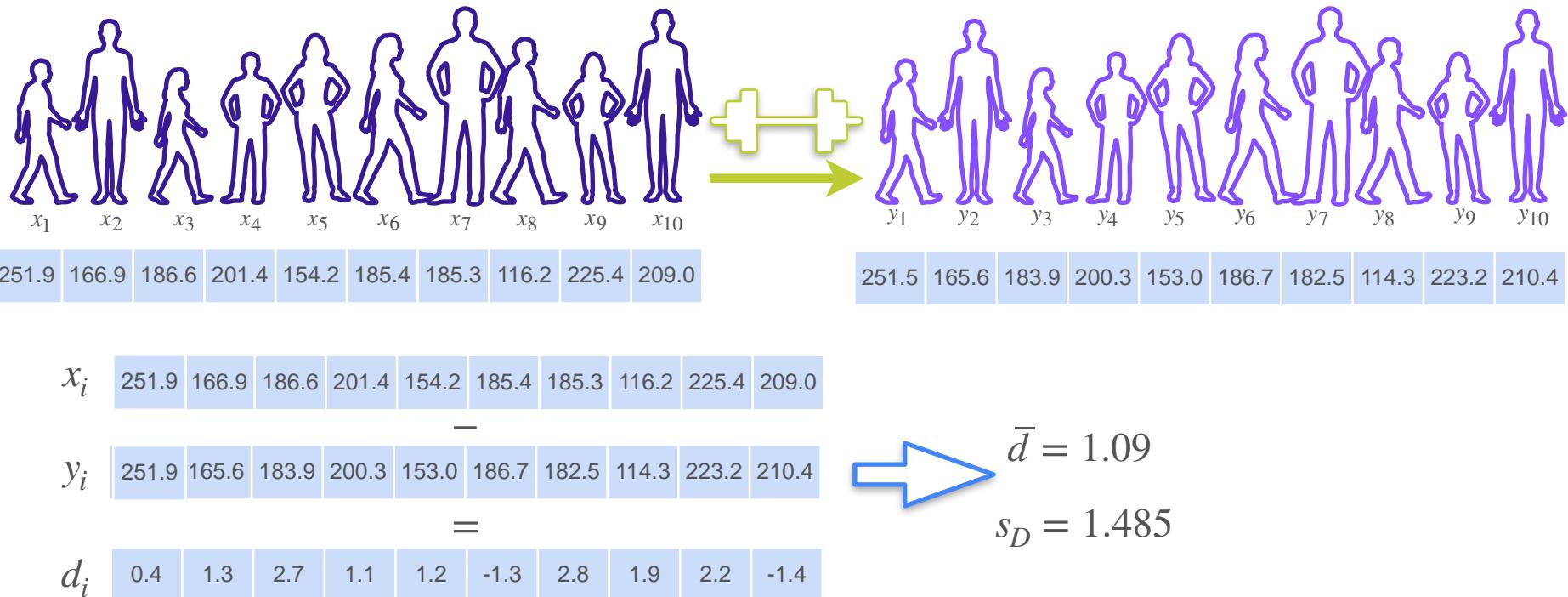
# Paired $t$ -Test: Observations



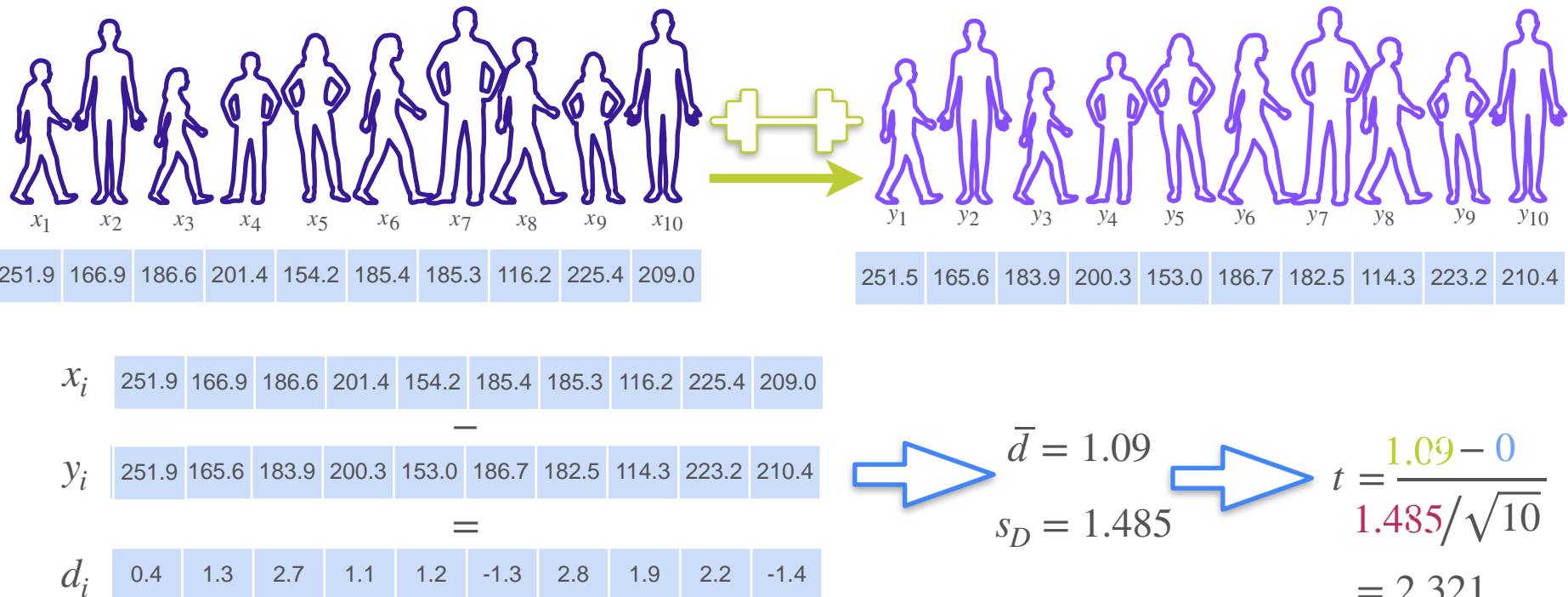
# Paired $t$ -Test: Observations



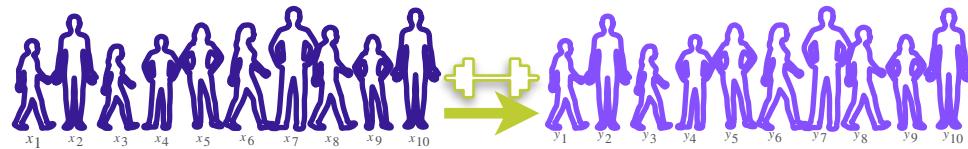
# Paired $t$ -Test: Observations



# Paired $t$ -Test: Observations



# Independent Two-Sample $t$ -Test: Right Tailed Test

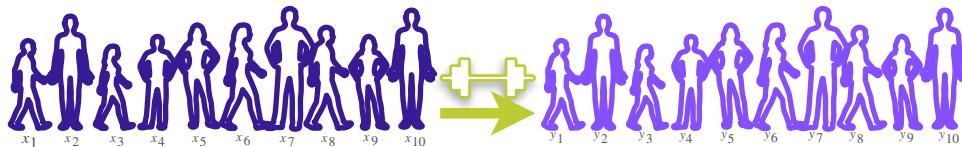


$$\bar{d} = 1.09$$

$$s_D = 1.485$$

$$n = 10$$

# Independent Two-Sample $t$ -Test: Right Tailed Test



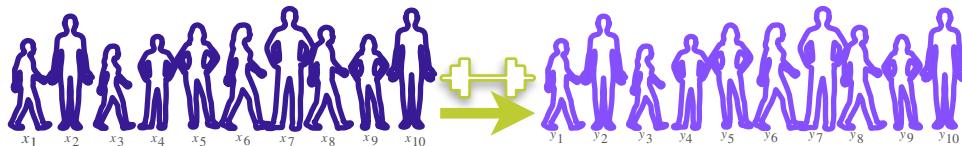
$$\bar{d} = 1.09$$

$$s_D = 1.485$$

$$n = 10$$

$$t = 2.321$$

# Independent Two-Sample $t$ -Test: Right Tailed Test



$$H_0 : \mu_D = 0 \text{ vs. } H_1 : \mu_D > 0$$

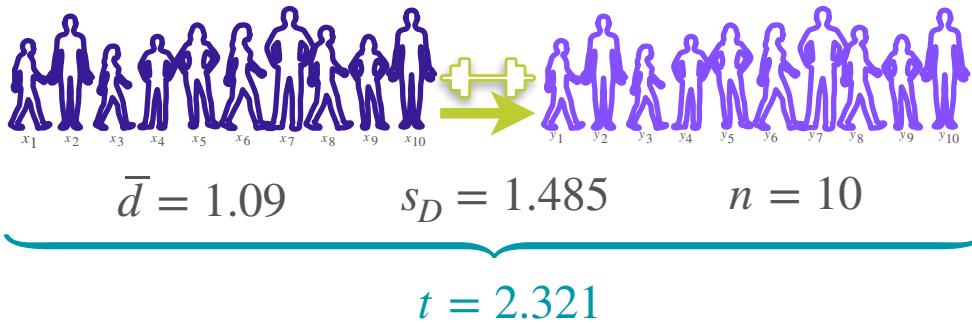
$$\bar{d} = 1.09$$

$$s_D = 1.485$$

$$n = 10$$

$$t = 2.321$$

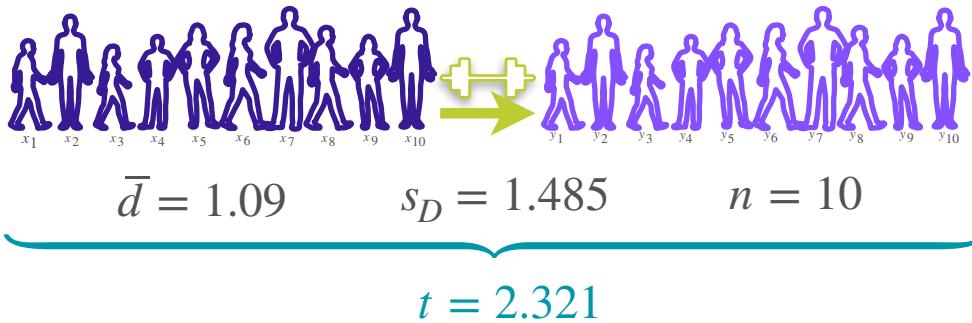
# Independent Two-Sample $t$ -Test: Right Tailed Test



$$H_0 : \mu_D = 0 \text{ vs. } H_1 : \mu_D > 0$$

$$\alpha = 0.05$$

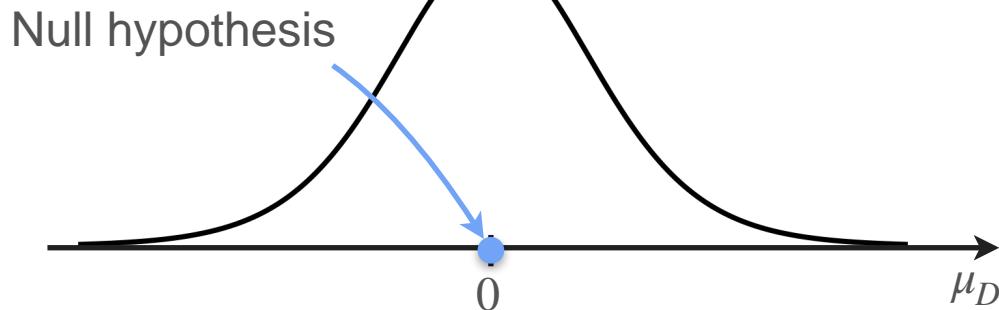
# Independent Two-Sample $t$ -Test: Right Tailed Test



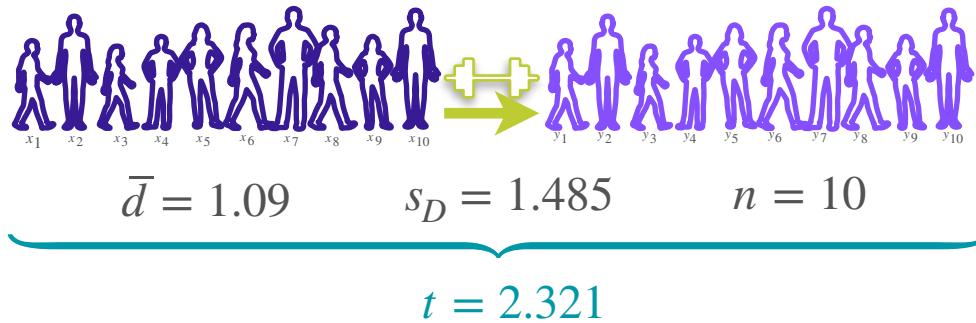
$$H_0 : \mu_D = 0 \text{ vs. } H_1 : \mu_D > 0$$

$$\alpha = 0.05$$

$$\text{If } H_0 \text{ is true: } T = \frac{\bar{D} - 0}{S_D / \sqrt{10}} \sim t_{10-1}$$



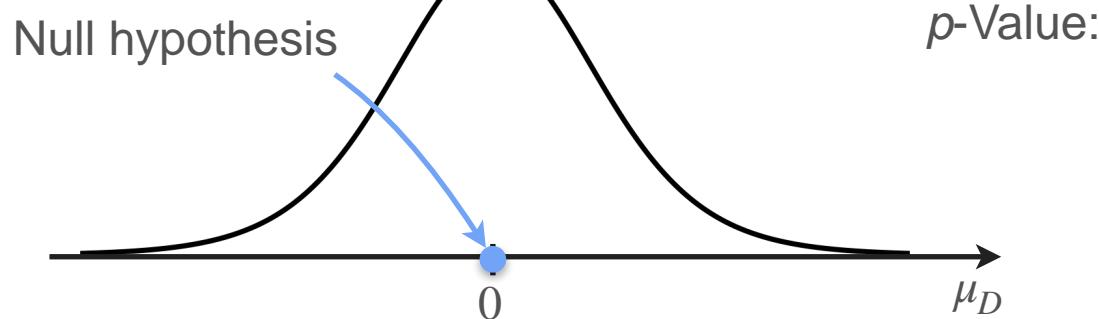
# Independent Two-Sample $t$ -Test: Right Tailed Test



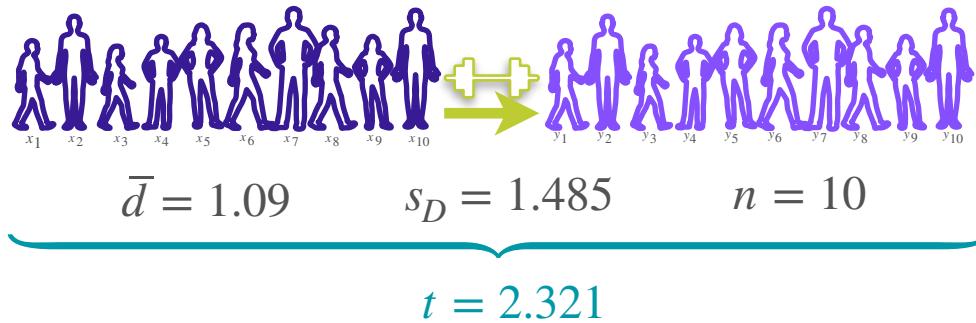
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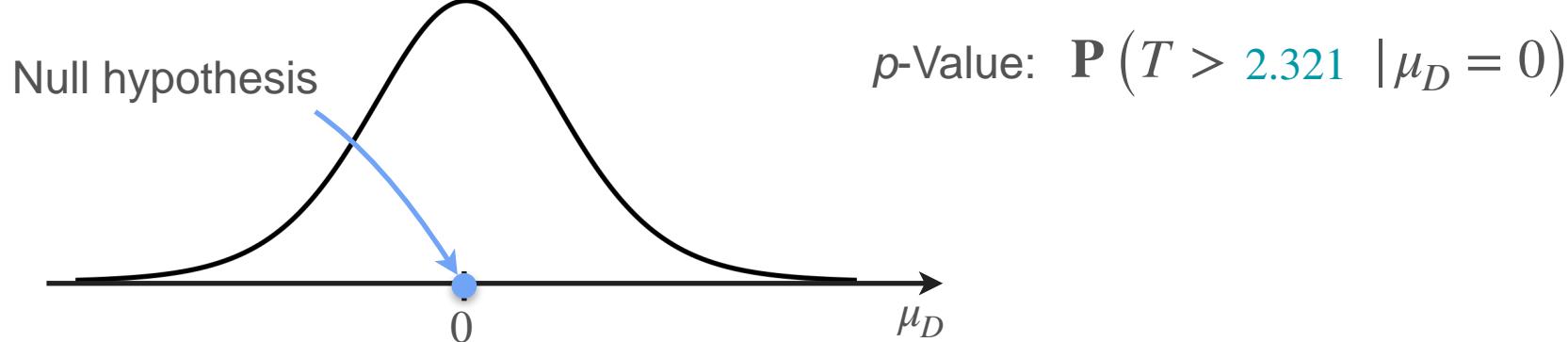
# Independent Two-Sample $t$ -Test: Right Tailed Test



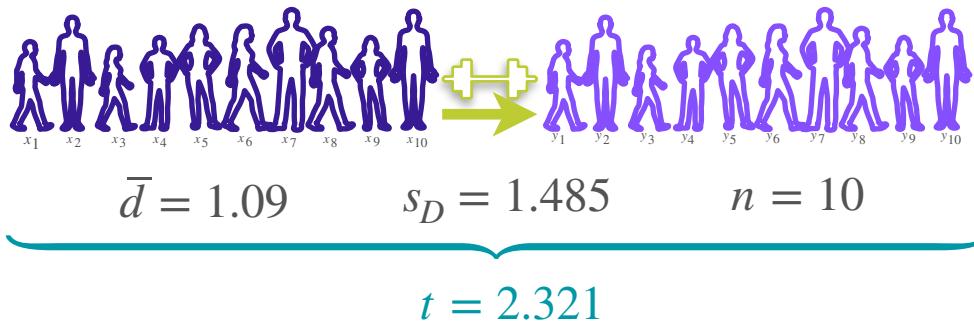
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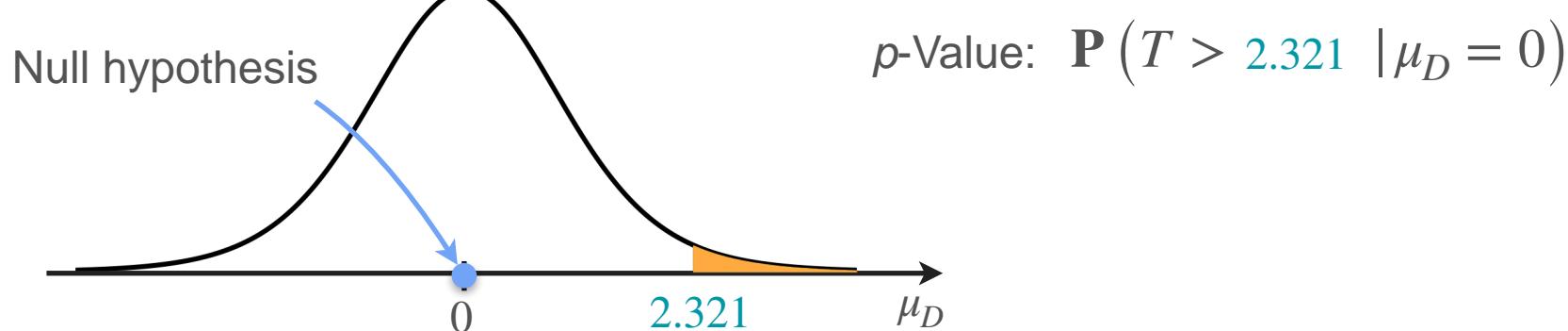
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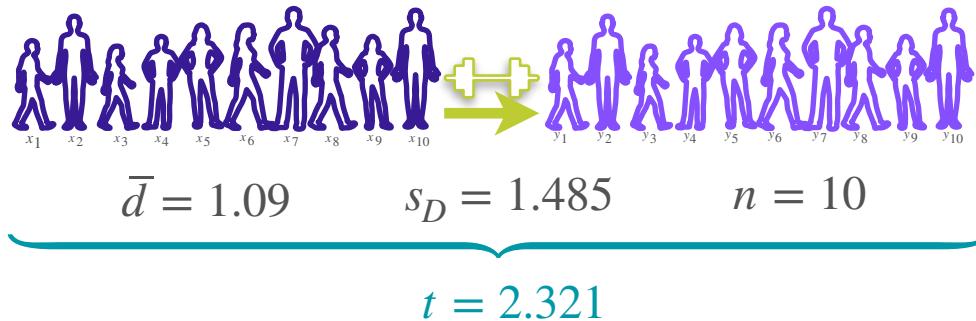
$$H_0 : \mu_D = 0 \text{ vs. } H_1 : \mu_D > 0$$

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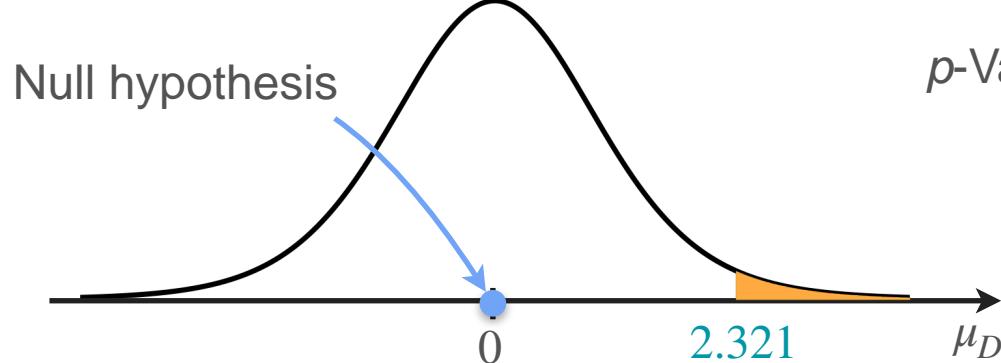
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$$H_0 : \mu_D = 0 \text{ vs. } H_1 : \mu_D > 0$$

$$\alpha = 0.05$$

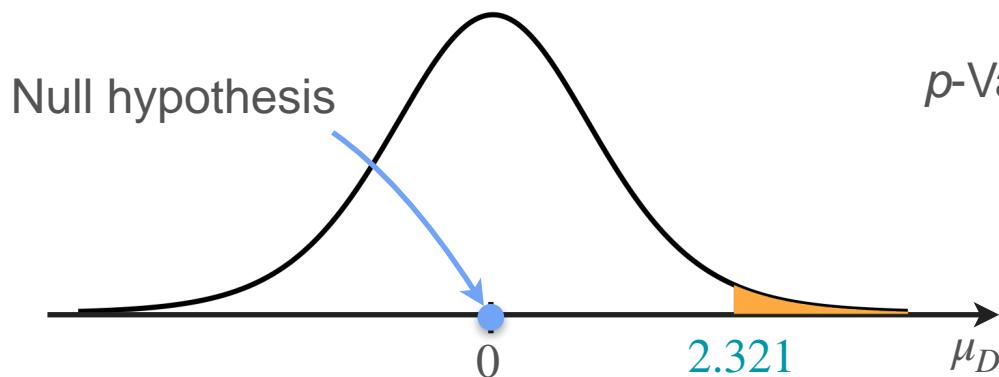
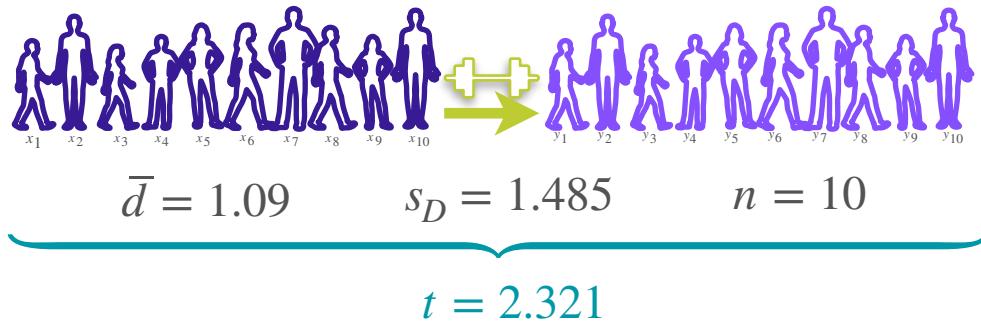
$$\text{If } H_0 \text{ is true: } T = \frac{\bar{D} - 0}{S_D / \sqrt{10}} \sim t_{10-1}$$



$$p\text{-Value: } P(T > 2.321 \mid \mu_D = 0)$$

$$= 0.0227$$

# Independent Two-Sample $t$ -Test: Right Tailed Test



$$H_0 : \mu_D = 0 \text{ vs. } H_1 : \mu_D > 0$$

$$\alpha = 0.05$$

$$\text{If } H_0 \text{ is true: } T = \frac{\bar{D} - 0}{S_D / \sqrt{10}} \sim t_{10-1}$$

$$p\text{-Value: } P(T > 2.321 \mid \mu_D = 0)$$

$$= 0.0227 < 0.05 \\ \Rightarrow \text{Reject } H_0 \text{ (and accept } H_1 \text{)} \\ (\text{with a 5\% significance level})$$



DeepLearning.AI

# Hypothesis Testing

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**ML Application: A/B testing**

# A/B Testing: Purchase Amount

# A/B Testing: Purchase Amount



Design A



Design B

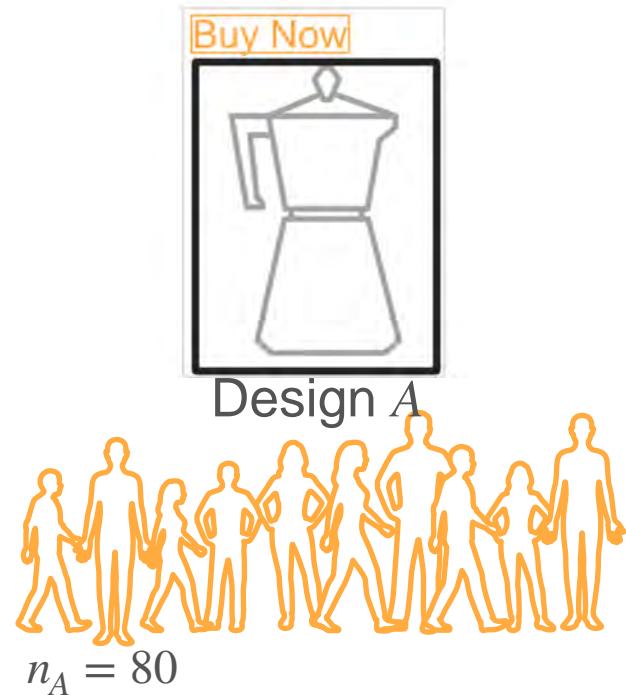
# A/B Testing: Purchase Amount



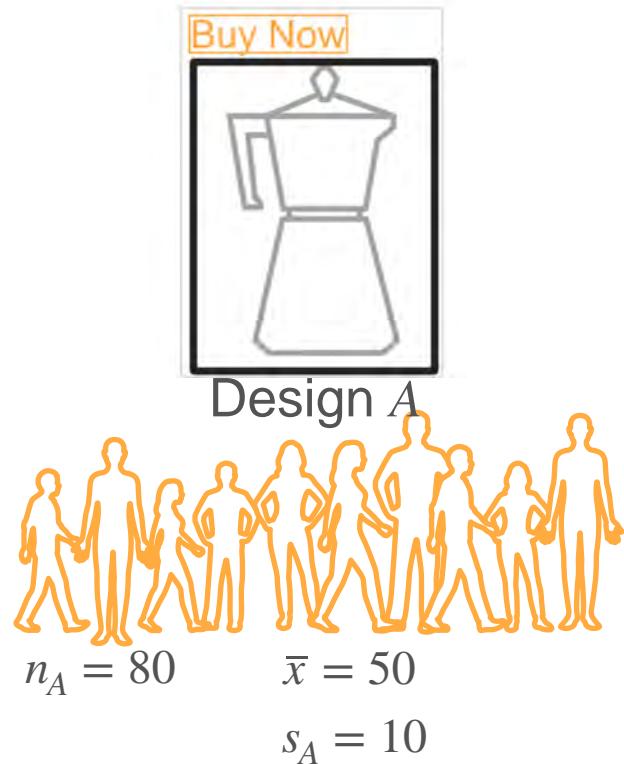
# A/B Testing: Purchase Amount



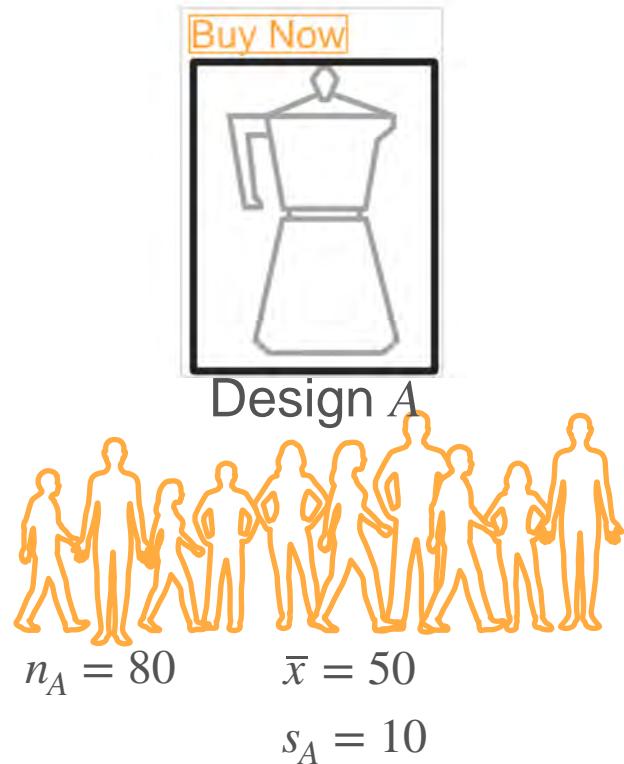
# A/B Testing: Purchase Amount



# A/B Testing: Purchase Amount



# A/B Testing: Purchase Amount



# A/B Testing: Purchase Amount



Design A



$$n_A = 80$$

$$\bar{x} = 50$$

$$n_B = 20 \quad \bar{y} = 55$$

$$s_A = 10$$



Design B



$$t = \frac{(\bar{x} - \bar{y}) - 0}{\sqrt{\frac{s_A^2}{n_A} + \frac{s_B^2}{n_B}}}$$

# A/B Testing: Purchase Amount



Design A



$$n_A = 80$$

$$\bar{x} = 50$$

$$n_B = 20$$

$$s_A = 10$$



Design B



$$H_0 : \mu_A = \mu_B \text{ vs. } H_1 : \mu_A < \mu_B$$

$$t = \frac{(\bar{x} - \bar{y}) - 0}{\sqrt{\frac{s_A^2}{n_A} + \frac{s_B^2}{n_B}}}$$

# A/B Testing: Purchase Amount



Design A



$$n_A = 80$$

$$\bar{x} = 50$$

$$n_B = 20$$

$$s_A = 10$$



Design B



$$H_0 : \mu_A - \mu_B = 0 \text{ vs. } H_1 : \mu_A - \mu_B < 0$$

$$t = \frac{(\bar{x} - \bar{y}) - 0}{\sqrt{\frac{s_A^2}{n_A} + \frac{s_B^2}{n_B}}}$$

# A/B Testing: Purchase Amount



Design A



$$n_A = 80$$

$$\bar{x} = 50$$

$$n_B = 20$$

$$s_A = 10$$



Design B



$$\bar{y} = 55$$

$$H_0 : \mu_A - \mu_B = 0 \text{ vs. } H_1 : \mu_A - \mu_B < 0$$

$$\alpha = 0.05$$

$$t = \frac{(\bar{x} - \bar{y}) - 0}{\sqrt{\frac{s_A^2}{n_A} + \frac{s_B^2}{n_B}}}$$

# A/B Testing: Purchase Amount



Design A



$$n_A = 80$$

$$\bar{x} = 50$$

$$n_B = 20$$

$$s_A = 10$$

$$\bar{y} = 55$$

$$s_B = 15$$



Design B



$$X \sim \mathcal{N}(\mu_A, \sigma_A^2)$$

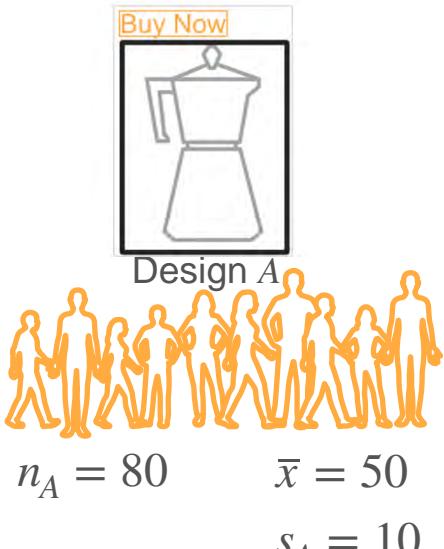
$$Y \sim \mathcal{N}(\mu_B, \sigma_B^2)$$

$$H_0 : \mu_A - \mu_B = 0 \text{ vs. } H_1 : \mu_A - \mu_B < 0$$

$$\alpha = 0.05$$

$$t = \frac{(\bar{x} - \bar{y}) - 0}{\sqrt{\frac{s_A^2}{n_A} + \frac{s_B^2}{n_B}}}$$

# A/B Testing: Purchase Amount



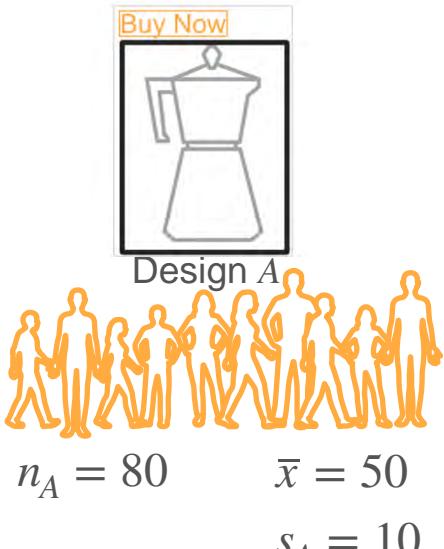
$$X \sim \mathcal{N}(\mu_A, \sigma_A^2) \quad Y \sim \mathcal{N}(\mu_B, \sigma_B^2)$$

$$H_0 : \mu_A - \mu_B = 0 \text{ vs. } H_1 : \mu_A - \mu_B < 0$$

$$\alpha = 0.05$$

$$\text{If } H_0 \text{ is true: } T = \frac{(\bar{X} - \bar{Y}) - 0}{\sqrt{\frac{S_A^2}{10} + \frac{S_B^2}{10}}} \sim t_{23.38}$$
$$t = \frac{(\bar{x} - \bar{y}) - 0}{\sqrt{\frac{s_A^2}{n_A} + \frac{s_B^2}{n_B}}}$$

# A/B Testing: Purchase Amount



$$X \sim \mathcal{N}(\mu_A, \sigma_A^2)$$

$$Y \sim \mathcal{N}(\mu_B, \sigma_B^2)$$

$$H_0 : \mu_A - \mu_B = 0 \text{ vs. } H_1 : \mu_A - \mu_B < 0$$

$$\alpha = 0.05$$

If  $H_0$  is true:  $T = \frac{(\bar{X} - \bar{Y}) - 0}{\sqrt{\frac{S_A^2}{10} + \frac{S_B^2}{10}}} \sim t_{23.38}$

$$t = \frac{(50 - 55) - 0}{\sqrt{\frac{10^2}{80} + \frac{15^2}{20}}} \\ -1.482$$

# A/B Testing: Purchase Amount



$$n_A = 80$$

$$\bar{x} = 50$$

$$s_A = 10$$

$$X \sim \mathcal{N}(\mu_A, \sigma_A^2)$$



$$n_B = 20$$

$$\bar{y} = 55$$

$$s_B = 15$$

$$Y \sim \mathcal{N}(\mu_B, \sigma_B^2)$$

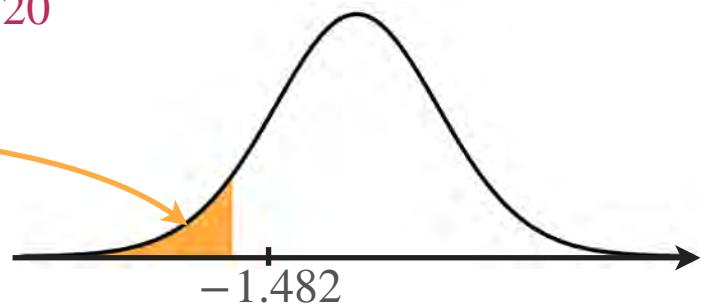
$$H_0 : \mu_A - \mu_B = 0 \text{ vs. } H_1 : \mu_A - \mu_B < 0$$

$$\alpha = 0.05$$

$$\text{If } H_0 \text{ is true: } T = \frac{(\bar{X} - \bar{Y}) - 0}{\sqrt{\frac{S_A^2}{10} + \frac{S_B^2}{10}}} \sim t_{23.38}$$

$$t = \frac{(50 - 55) - 0}{\sqrt{\frac{10^2}{80} + \frac{15^2}{20}}} \\ -1.482$$

p-Value:



# A/B Testing: Purchase Amount



$$n_A = 80$$

$$\bar{x} = 50$$

$$s_A = 10$$

$$X \sim \mathcal{N}(\mu_A, \sigma_A^2)$$



$$n_B = 20$$

$$\bar{y} = 55$$

$$s_B = 15$$

$$Y \sim \mathcal{N}(\mu_B, \sigma_B^2)$$

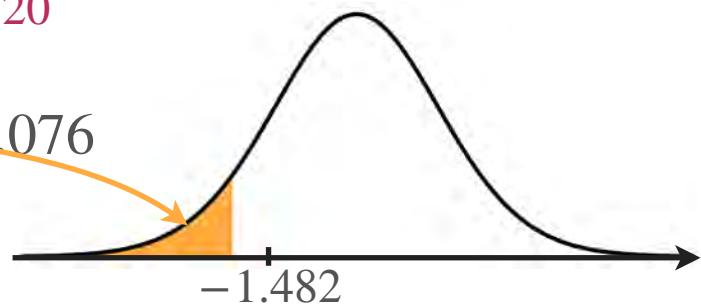
$$H_0 : \mu_A - \mu_B = 0 \text{ vs. } H_1 : \mu_A - \mu_B < 0$$

$$\alpha = 0.05$$

$$\text{If } H_0 \text{ is true: } T = \frac{(\bar{X} - \bar{Y}) - 0}{\sqrt{\frac{S_A^2}{10} + \frac{S_B^2}{10}}} \sim t_{23.38}$$

$$t = \frac{(50 - 55) - 0}{\sqrt{\frac{10^2}{80} + \frac{15^2}{20}}} \\ -1.482$$

$$p\text{-Value: } 0.076$$



# A/B Testing: Purchase Amount



$$n_A = 80$$

$$\bar{x} = 50$$

$$s_A = 10$$

$$X \sim \mathcal{N}(\mu_A, \sigma_A^2)$$



$$n_B = 20$$

$$\bar{y} = 55$$

$$s_B = 15$$

$$Y \sim \mathcal{N}(\mu_B, \sigma_B^2)$$

$$H_0 : \mu_A - \mu_B = 0 \text{ vs. } H_1 : \mu_A - \mu_B < 0$$

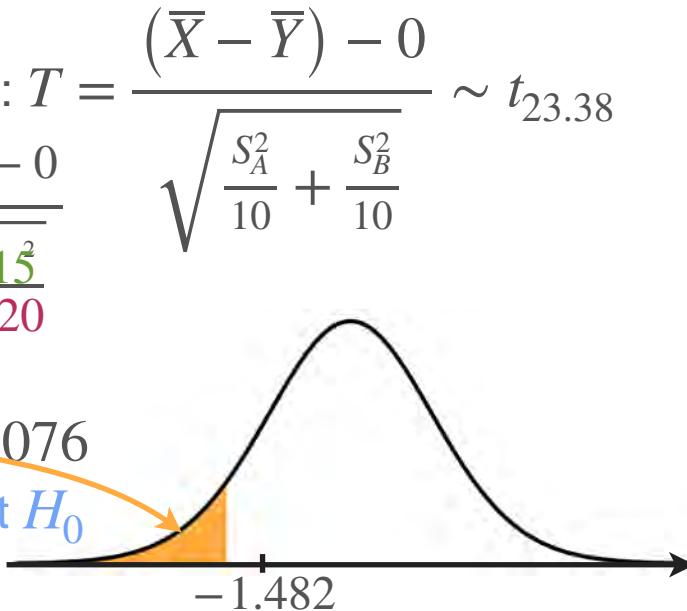
$$\alpha = 0.05$$

$$\text{If } H_0 \text{ is true: } T = \frac{(\bar{X} - \bar{Y}) - 0}{\sqrt{\frac{S_A^2}{10} + \frac{S_B^2}{10}}} \sim t_{23.38}$$

$$t = \frac{(50 - 55) - 0}{\sqrt{\frac{10^2}{80} + \frac{15^2}{20}}} = -1.482$$

$$p\text{-Value: } 0.076$$

Don't reject  $H_0$



# A/B Testing and $t$ -Tests

# A/B Testing and $t$ -Tests

A/B testing is a methodology for comparing two variations (A/B)

# A/B Testing and $t$ -Tests

A/B testing is a methodology for comparing two variations (A/B)

Propose variations  
(A/B)

# A/B Testing and $t$ -Tests

A/B testing is a methodology for comparing two variations (A/B)



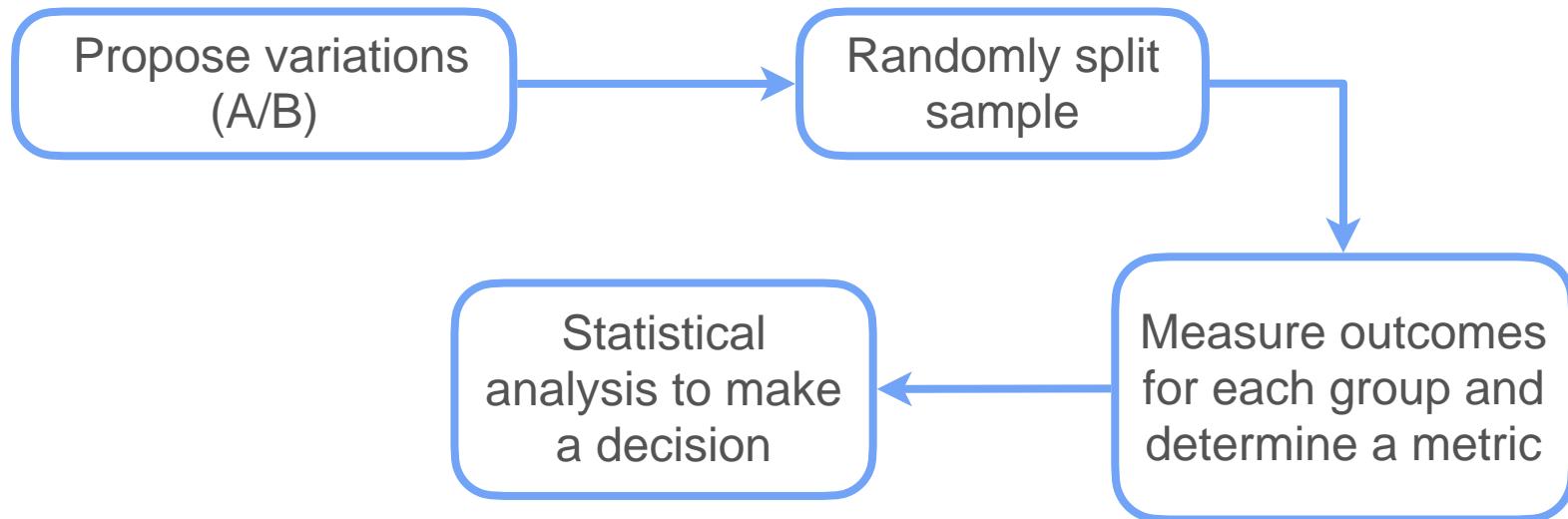
# A/B Testing and $t$ -Tests

A/B testing is a methodology for comparing two variations (A/B)



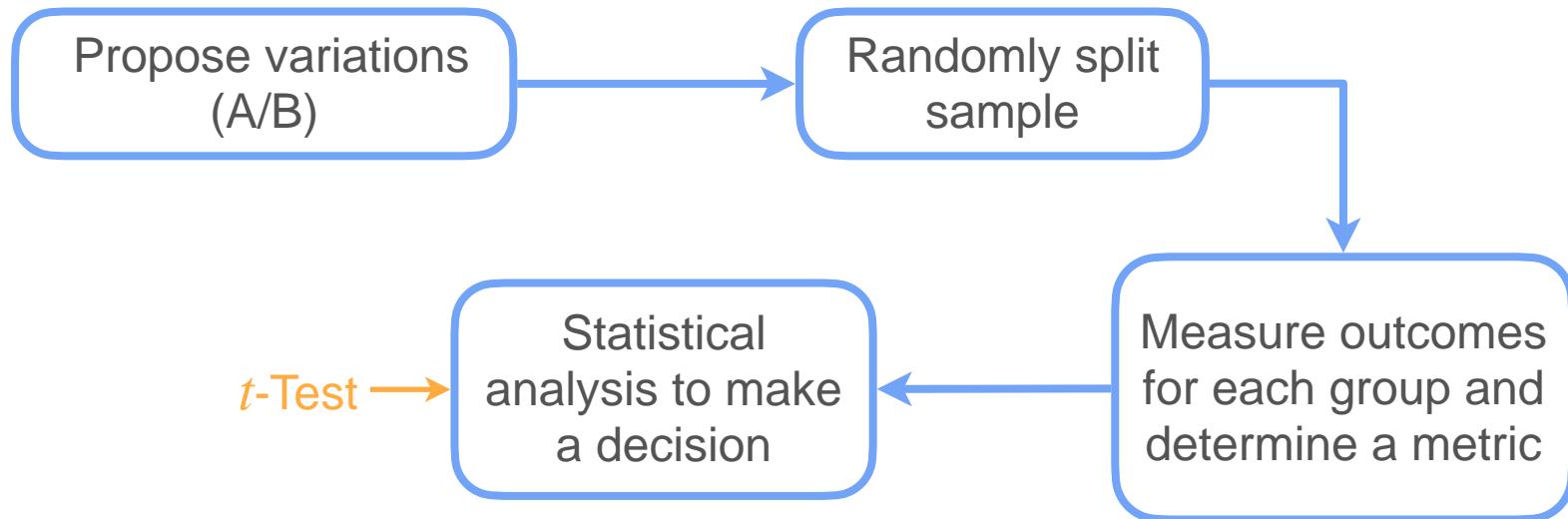
# A/B Testing and $t$ -Tests

A/B testing is a methodology for comparing two variations (A/B)



# A/B Testing and $t$ -Tests

A/B testing is a methodology for comparing two variations (A/B)



# A/B Testing: Conversion Rates

# A/B Testing: Conversion Rates



# A/B Testing: Conversion Rates



Design A  
Higher conversion rates?



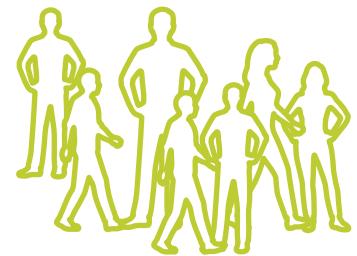
# A/B Testing: Conversion Rates



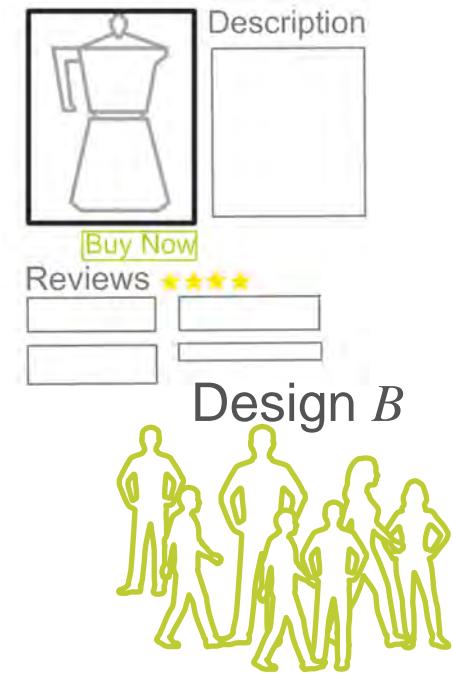
Design A  
Higher conversion rates?



Design B



# A/B Testing: Conversion Rates



# A/B Testing: Conversion Rates



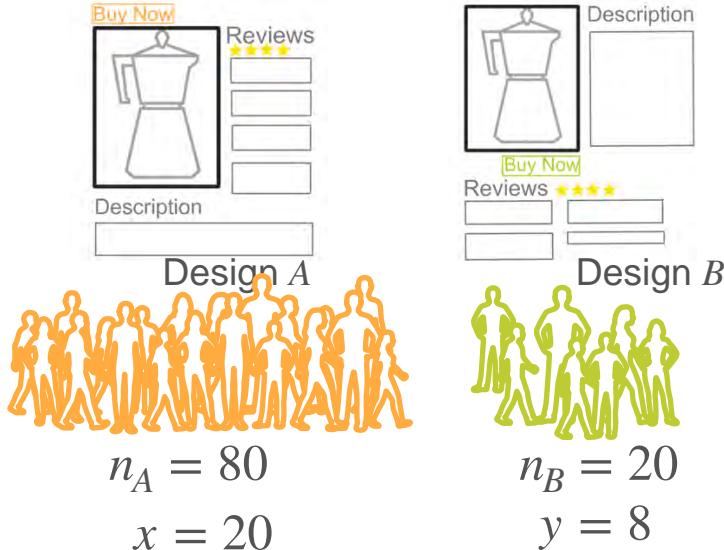
# A/B Testing: Conversion Rates



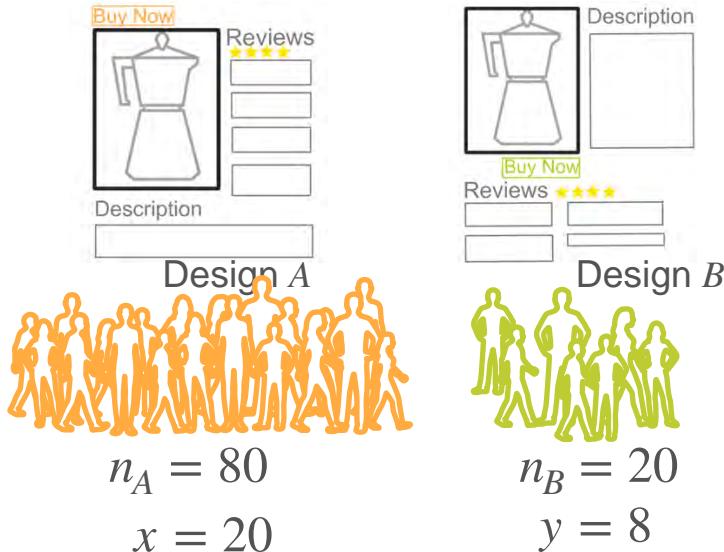
# A/B Testing: Conversion Rates



# A/B Testing: Conversion Rates



# A/B Testing: Conversion Rates

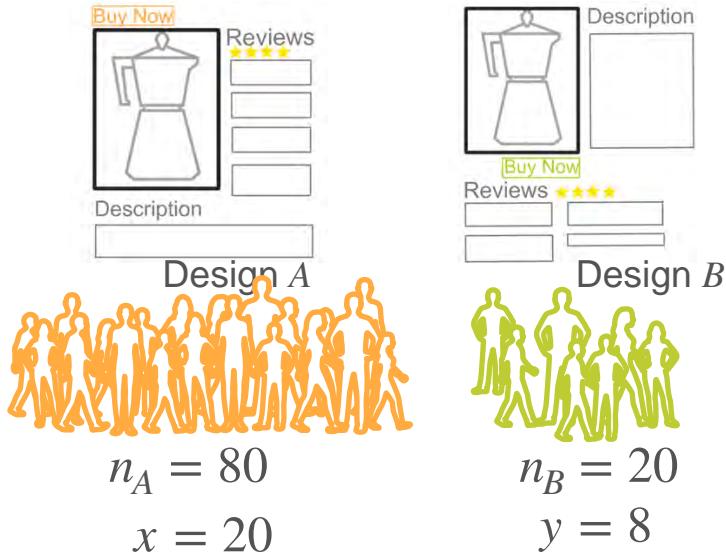


$$H_0 : p_A - p_B = 0 \text{ vs. } H_1 : p_A - p_B < 0$$

$p_A$  = Conversion rate from Design A

$p_B$  = Conversion rate from Design B

# A/B Testing: Conversion Rates



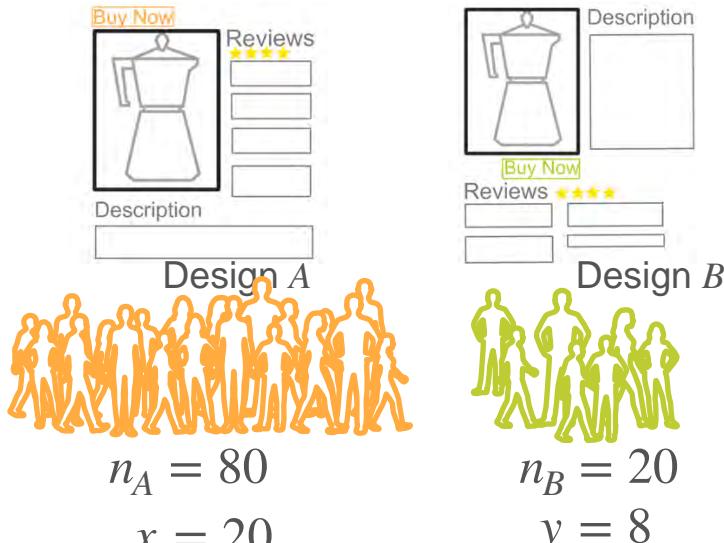
$$H_0 : p_A - p_B = 0 \text{ vs. } H_1 : p_A - p_B < 0$$

$p_A$  = Conversion rate from Design A

$p_B$  = Conversion rate from Design B

$$\alpha = 0.05$$

# A/B Testing: Conversion Rates



$$X \sim \text{Binomial}(n_A, p_A) \quad Y \sim \text{Binomial}(n_B, p_B)$$

$$H_0 : p_A - p_B = 0 \text{ vs. } H_1 : p_A - p_B < 0$$

$p_A$  = Conversion rate from Design A

$p_B$  = Conversion rate from Design B

$$\alpha = 0.05$$

# A/B Testing: Conversion Rates

# A/B Testing: Conversion Rates

Statistic?

# A/B Testing: Conversion Rates

Statistic?

Law of large numbers

$$\frac{X}{n_A} \rightarrow p_A$$

$$\frac{Y}{n_B} \rightarrow p_B$$

# A/B Testing: Conversion Rates

Statistic?

Law of large numbers

$$\frac{X}{n_A} \rightarrow p_A$$



$$\frac{X}{n_A} \sim \mathcal{N}\left(p_A, \frac{p_A(1-p_A)}{n_A}\right)$$

$$\frac{Y}{n_B} \rightarrow p_B$$

C.L.T.



$$\frac{Y}{n_B} \sim \mathcal{N}\left(p_B, \frac{p_B(1-p_B)}{n_B}\right)$$

# A/B Testing: Conversion Rates

Statistic?

$$\frac{X}{n_A} \stackrel{a}{\sim} \mathcal{N} \left( p_A, \frac{p_A(1 - p_A)}{n_A} \right)$$

$$\frac{Y}{n_B} \stackrel{a}{\sim} \mathcal{N} \left( p_B, \frac{p_B(1 - p_B)}{n_B} \right)$$

# A/B Testing: Conversion Rates

Statistic?

$$\left. \begin{aligned} \frac{X}{n_A} &\stackrel{a}{\sim} \mathcal{N} \left( p_A, \frac{p_A(1-p_A)}{n_A} \right) \\ \frac{Y}{n_B} &\stackrel{a}{\sim} \mathcal{N} \left( p_B, \frac{p_B(1-p_B)}{n_B} \right) \end{aligned} \right\}$$

# A/B Testing: Conversion Rates

Statistic?

$$\left. \begin{array}{l} \frac{X}{n_A} \stackrel{a}{\sim} \mathcal{N} \left( p_A, \frac{p_A(1-p_A)}{n_A} \right) \\ \frac{Y}{n_B} \stackrel{a}{\sim} \mathcal{N} \left( p_B, \frac{p_B(1-p_B)}{n_B} \right) \end{array} \right\} \frac{X}{n_A} - \frac{Y}{n_B} \rightarrow p_A - p_B$$

# A/B Testing: Conversion Rates

Statistic?

$$\left. \begin{array}{l} \frac{X}{n_A} \stackrel{a}{\sim} \mathcal{N} \left( p_A, \frac{p_A(1-p_A)}{n_A} \right) \\ \frac{Y}{n_B} \stackrel{a}{\sim} \mathcal{N} \left( p_B, \frac{p_B(1-p_B)}{n_B} \right) \end{array} \right\} \quad \begin{array}{l} \frac{X}{n_A} - \frac{Y}{n_B} \stackrel{a}{\sim} \mathcal{N} \left( p_A - p_B, \frac{p_A(1-p_A)}{n_A} + \frac{p_B(1-p_B)}{n_B} \right) \\ \frac{X}{n_A} - \frac{Y}{n_B} \rightarrow p_A - p_B \end{array}$$

# A/B Testing: Conversion Rates

Statistic?

$$\left. \begin{array}{l} \frac{X}{n_A} \stackrel{a}{\sim} \mathcal{N} \left( p_A, \frac{p_A(1-p_A)}{n_A} \right) \\ \frac{Y}{n_B} \stackrel{a}{\sim} \mathcal{N} \left( p_B, \frac{p_B(1-p_B)}{n_B} \right) \end{array} \right\} \frac{X}{n_A} - \frac{Y}{n_B} \rightarrow p_A - p_B \quad \begin{aligned} \frac{X}{n_A} - \frac{Y}{n_B} &\stackrel{a}{\sim} \mathcal{N} \left( p_A - p_B, \frac{p_A(1-p_A)}{n_A} + \frac{p_B(1-p_B)}{n_B} \right) \\ \frac{\left( \frac{X}{n_A} - \frac{Y}{n_B} \right) - (p_A - p_B)}{\sqrt{\frac{p_A(1-p_A)}{n_A} + \frac{p_B(1-p_B)}{n_B}}} &\stackrel{a}{\sim} \mathcal{N} (0, 1^2) \end{aligned}$$


# A/B Testing: Conversion Rates

$$\frac{\left( \frac{X}{n_A} - \frac{Y}{n_B} \right) - (p_A - p_B)}{\sqrt{\frac{p_A(1-p_A)}{n_A} + \frac{p_B(1-p_B)}{n_B}}} \stackrel{a}{\sim} \mathcal{N}(0,1)$$

# A/B Testing: Conversion Rates

If  $H_0$  is true  $\Rightarrow p_A = p_B = p$

$$\frac{\left(\frac{X}{n_A} - \frac{Y}{n_B}\right) - (p_A - p_B)}{\sqrt{\frac{p_A(1-p_A)}{n_A} + \frac{p_B(1-p_B)}{n_B}}} \stackrel{a}{\sim} \mathcal{N}(0,1)$$

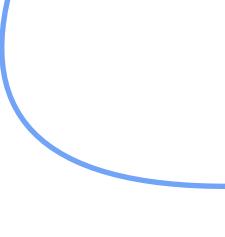
# A/B Testing: Conversion Rates

If  $H_0$  is true  $\Rightarrow p_A = p_B = p$

$$\frac{\left(\frac{X}{n_A} - \frac{Y}{n_B}\right) - (p - p)}{\sqrt{\frac{p(1-p)}{n_A} + \frac{p(1-p)}{n_B}}} \stackrel{a}{\sim} \mathcal{N}(0, 1^2)$$

# A/B Testing: Conversion Rates

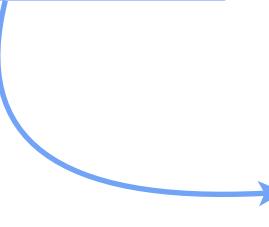
If  $H_0$  is true  $\Rightarrow p_A = p_B = p$

$$\frac{\left(\frac{X}{n_A} - \frac{Y}{n_B}\right) - (p - p)}{\sqrt{\frac{p(1-p)}{n_A} + \frac{p(1-p)}{n_B}}} \stackrel{a}{\sim} \mathcal{N}(0, 1^2)$$

$$= p(1-p) \left( \frac{1}{n_A} + \frac{1}{n_B} \right)$$

# A/B Testing: Conversion Rates

If  $H_0$  is true  $\Rightarrow p_A = p_B = p$

$$\frac{\left(\frac{X}{n_A} - \frac{Y}{n_B}\right) - (p - p)}{\sqrt{\frac{p(1-p)}{n_A} + \frac{p(1-p)}{n_B}}} \stackrel{a}{\sim} \mathcal{N}(0, 1^2)$$



$$= p(1-p) \left( \frac{1}{n_A} + \frac{1}{n_B} \right) = p(1-p)(n_A + n_B) \frac{1}{n_A n_B}$$

# A/B Testing: Conversion Rates

If  $H_0$  is true  $\Rightarrow p_A = p_B = p$

$$\frac{\left(\frac{X}{n_A} - \frac{Y}{n_B}\right) - (p - p)}{\sqrt{\frac{p(1-p)}{n_A} + \frac{p(1-p)}{n_B}}} \stackrel{a}{\sim} \mathcal{N}(0, 1^2)$$

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If  $H_0$  is true  $\Rightarrow p_A = p_B = p$

$$\frac{\left(\frac{X}{n_A} - \frac{Y}{n_B}\right) - (p - p)}{\sqrt{\frac{p(1-p)}{n_A} + \frac{p(1-p)}{n_B}}} \stackrel{a}{\sim} \mathcal{N}(0, 1^2)$$


# A/B Testing: Conversion Rates

If  $H_0$  is true  $\Rightarrow p_A = p_B = p$

$$\frac{\left(\frac{X}{n_A} - \frac{Y}{n_B}\right) - (p - p)}{\sqrt{\frac{p(1-p)}{n_A} + \frac{p(1-p)}{n_B}}} \stackrel{a}{\sim} \mathcal{N}(0, 1^2) \longrightarrow \frac{\left(\frac{X}{n_A} - \frac{Y}{n_B}\right) - 0}{\sqrt{(n_A + n_B)p(1-p)}} \sqrt{n_A n_B} \stackrel{a}{\sim} \mathcal{N}(0, 1^2)$$

# A/B Testing: Conversion Rates

If  $H_0$  is true  $\Rightarrow p_A = p_B = p$

$$\frac{\left(\frac{X}{n_A} - \frac{Y}{n_B}\right) - (p - p)}{\sqrt{\frac{p(1-p)}{n_A} + \frac{p(1-p)}{n_B}}} \stackrel{a}{\sim} \mathcal{N}(0, 1^2) \longrightarrow \frac{\left(\frac{X}{n_A} - \frac{Y}{n_B}\right) - 0}{\sqrt{(n_A + n_B)p(1-p)}} \sqrt{n_A n_B} \stackrel{a}{\sim} \mathcal{N}(0, 1^2)$$

But you don't know  $p$



Replace it by estimation!  $\hat{p} = \frac{X + Y}{n_A + n_B}$

# A/B Testing: Conversion Rates

If  $H_0$  is true  $\Rightarrow p_A = p_B = p$

$$\frac{\left(\frac{X}{n_A} - \frac{Y}{n_B}\right) - (p - p)}{\sqrt{\frac{p(1-p)}{n_A} + \frac{p(1-p)}{n_B}}} \stackrel{a}{\sim} \mathcal{N}(0, 1^2) \longrightarrow \frac{\left(\frac{X}{n_A} - \frac{Y}{n_B}\right) - 0}{\sqrt{(n_A + n_B)p(1-p)}} \sqrt{n_A n_B} \stackrel{a}{\sim} \mathcal{N}(0, 1^2)$$

But you don't know  $p$

↓  
Replace it by estimation!  $\hat{p} = \frac{X + Y}{n_A + n_B} \longrightarrow$

Test statistic

$$\frac{\left(\frac{X}{n_A} - \frac{Y}{n_B}\right) - 0}{\sqrt{(X + Y)\left(1 - \frac{X + Y}{n_A + n_B}\right)}} \sqrt{n_A n_B} \stackrel{a}{\sim} \mathcal{N}(0, 1^2)$$

# A/B Testing: Conversion Rates

# A/B Testing: Conversion Rates



# A/B Testing: Conversion Rates



Design A



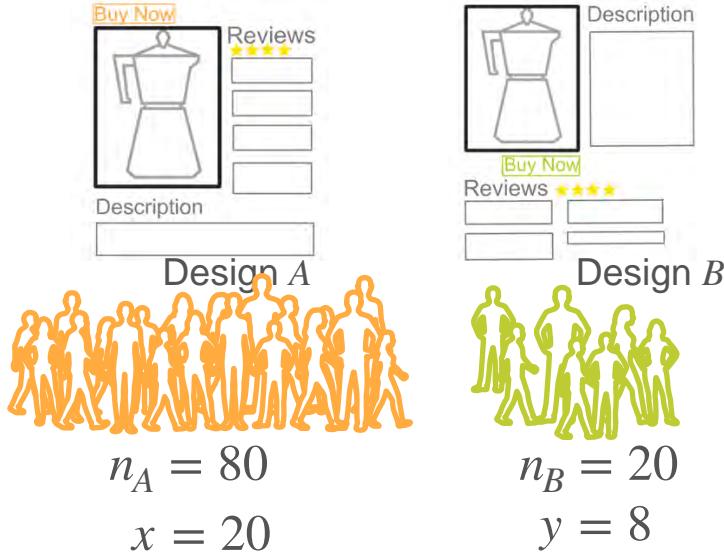
Design B



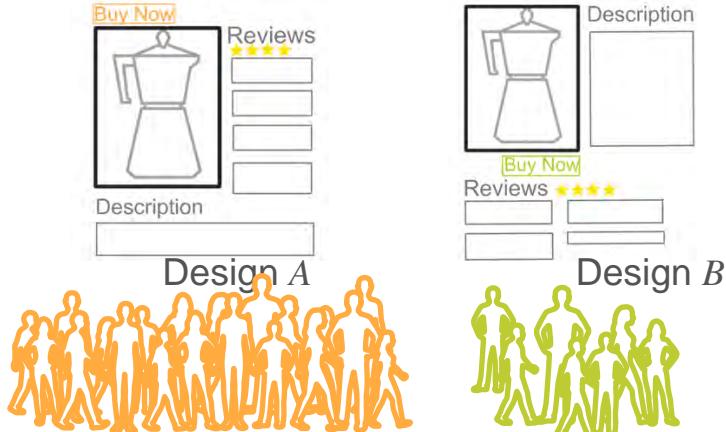
$$n_A = 80$$

$$x = 20$$

# A/B Testing: Conversion Rates



# A/B Testing: Conversion Rates



$$n_A = 80$$

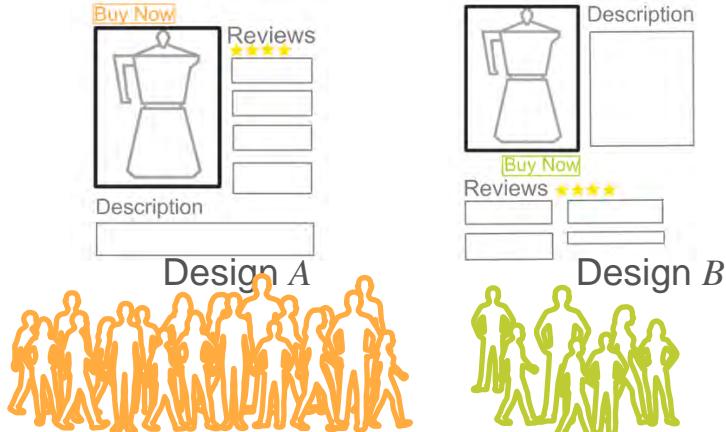
$$x = 20$$

$$n_B = 20$$

$$y = 8$$

$$X \sim \text{Binomial}(n_A, p_A) \quad Y \sim \text{Binomial}(n_B, p_B)$$

# A/B Testing: Conversion Rates



$$n_A = 80$$

$$x = 20$$

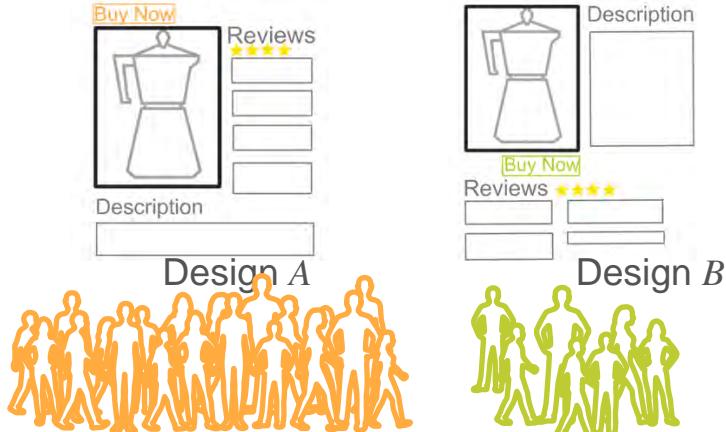
$$n_B = 20$$

$$y = 8$$

$$H_0 : p_A - p_B = 0 \text{ vs. } H_1 : p_A - p_B < 0$$

$$X \sim \text{Binomial}(n_A, p_A) \quad Y \sim \text{Binomial}(n_B, p_B)$$

# A/B Testing: Conversion Rates



$$n_A = 80$$

$$x = 20$$

$$X \sim \text{Binomial}(n_A, p_A) \quad Y \sim \text{Binomial}(n_B, p_B)$$

$$H_0 : p_A - p_B = 0 \text{ vs. } H_1 : p_A - p_B < 0$$

$$\alpha = 0.05$$

$$n_B = 20$$

$$y = 8$$

# A/B Testing: Conversion Rates



$$n_A = 80$$

$$x = 20$$



$$n_B = 20$$

$$y = 8$$

$$X \sim \text{Binomial}(n_A, p_A) \quad Y \sim \text{Binomial}(n_B, p_B)$$

$$H_0 : p_A - p_B = 0 \text{ vs. } H_1 : p_A - p_B < 0$$

$$\alpha = 0.05 \quad \text{If } H_0 \text{ is true } \Rightarrow p_A = p_B = p$$

$$Z = \frac{\left( \frac{X}{n_A} - \frac{Y}{n_B} \right) - 0}{\sqrt{(X+Y)\left(1 - \frac{X+Y}{n_A+n_B}\right)}} \sqrt{n_A n_B} \sim \mathcal{N}(0, 1^2)$$

# A/B Testing: Conversion Rates



$$H_0 : p_A - p_B = 0 \text{ vs. } H_1 : p_A - p_B < 0$$

$$\alpha = 0.05 \quad \text{If } H_0 \text{ is true } \Rightarrow p_A = p_B = p$$

$$Z = \frac{\left( \frac{X}{n_A} - \frac{Y}{n_B} \right) - 0}{\sqrt{(X+Y)\left(1 - \frac{X+Y}{n_A+n_B}\right)}} \sqrt{n_A n_B} \sim \mathcal{N}(0, 1^2)$$

$$z = \frac{\left( \frac{20}{80} - \frac{8}{20} \right) - 0}{\sqrt{(20+8)\left(1 - \frac{20+8}{80+20}\right)}} \sqrt{8020}$$

$$z = -1.336$$

# A/B Testing: Conversion Rates



$$n_A = 80$$

$$x = 20$$



$$n_B = 20$$

$$y = 8$$

$$X \sim \text{Binomial}(n_A, p_A) \quad Y \sim \text{Binomial}(n_B, p_B)$$

$$H_0 : p_A - p_B = 0 \text{ vs. } H_1 : p_A - p_B < 0$$

$$\alpha = 0.05 \quad \text{If } H_0 \text{ is true } \Rightarrow p_A = p_B = p$$

$$Z = \frac{\left( \frac{X}{n_A} - \frac{Y}{n_B} \right) - 0}{\sqrt{(X+Y)\left(1 - \frac{X+Y}{n_A+n_B}\right)}} \sqrt{n_A n_B} \sim \mathcal{N}(0, 1^2)$$

-2.07

# A/B Testing: Conversion Rates



$$n_A = 80$$

$$x = 20$$



$$n_B = 20$$
$$y = 8$$

$$X \sim \text{Binomial}(n_A, p_A) \quad Y \sim \text{Binomial}(n_B, p_B)$$

$$H_0 : p_A - p_B = 0 \text{ vs. } H_1 : p_A - p_B < 0$$

$$\alpha = 0.05 \quad \text{If } H_0 \text{ is true } \Rightarrow p_A = p_B = p$$

$$Z = \frac{\left( \frac{X}{n_A} - \frac{Y}{n_B} \right) - 0}{\sqrt{(X+Y)\left(1 - \frac{X+Y}{n_A+n_B}\right)}} \sqrt{n_A n_B} \sim \mathcal{N}(0, 1^2)$$

$$z = -1.336$$



# A/B Testing: Conversion Rates



$$n_A = 80$$

$$x = 20$$



$$n_B = 20$$

$$y = 8$$

$$X \sim \text{Binomial}(n_A, p_A) \quad Y \sim \text{Binomial}(n_B, p_B)$$

$$H_0 : p_A - p_B = 0 \text{ vs. } H_1 : p_A - p_B < 0$$

$$\alpha = 0.05 \quad \text{If } H_0 \text{ is true } \Rightarrow p_A = p_B = p$$

$$Z = \frac{\left( \frac{X}{n_A} - \frac{Y}{n_B} \right) - 0}{\sqrt{(X+Y)\left(1 - \frac{X+Y}{n_A+n_B}\right)}} \sqrt{n_A n_B} \sim \mathcal{N}(0, 1^2)$$

$$z = -1.336$$

*p*-value



# A/B Testing: Conversion Rates



$$n_A = 80$$

$$x = 20$$

$$X \sim \text{Binomial}(n_A, p_A) \quad Y \sim \text{Binomial}(n_B, p_B)$$



$$n_B = 20$$

$$y = 8$$

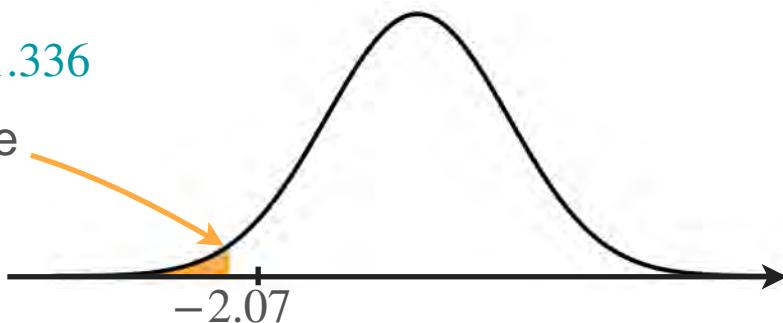
$$H_0 : p_A - p_B = 0 \text{ vs. } H_1 : p_A - p_B < 0$$

$$\alpha = 0.05 \quad \text{If } H_0 \text{ is true } \Rightarrow p_A = p_B = p$$

$$Z = \frac{\left( \frac{X}{n_A} - \frac{Y}{n_B} \right) - 0}{\sqrt{(X+Y)\left(1 - \frac{X+Y}{n_A+n_B}\right)}} \sqrt{n_A n_B} \sim \mathcal{N}(0, 1^2)$$

$$z = -1.336$$

p-value



# A/B Testing: Conversion Rates



$$n_A = 80$$

$$x = 20$$

$$X \sim \text{Binomial}(n_A, p_A) \quad Y \sim \text{Binomial}(n_B, p_B)$$



$$n_B = 20$$

$$y = 8$$

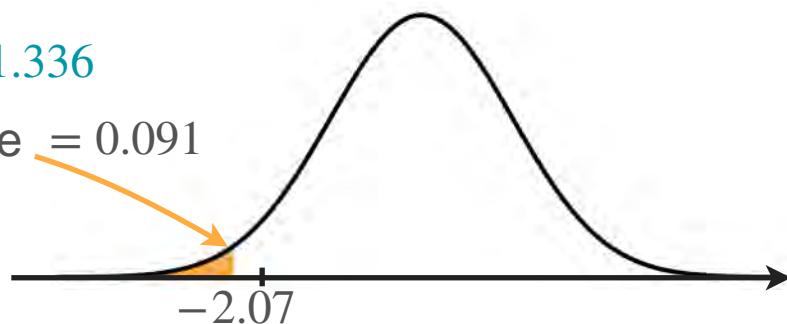
$$H_0 : p_A - p_B = 0 \text{ vs. } H_1 : p_A - p_B < 0$$

$$\alpha = 0.05 \quad \text{If } H_0 \text{ is true } \Rightarrow p_A = p_B = p$$

$$Z = \frac{\left( \frac{X}{n_A} - \frac{Y}{n_B} \right) - 0}{\sqrt{(X+Y)\left(1 - \frac{X+Y}{n_A+n_B}\right)}} \sqrt{n_A n_B} \sim \mathcal{N}(0, 1^2)$$

$$z = -1.336$$

$$p\text{-value} = 0.091$$



# A/B Testing: Conversion Rates



$$n_A = 80$$

$$x = 20$$

$$X \sim \text{Binomial}(n_A, p_A) \quad Y \sim \text{Binomial}(n_B, p_B)$$



$$n_B = 20$$

$$y = 8$$

$$H_0 : p_A - p_B = 0 \text{ vs. } H_1 : p_A - p_B < 0$$

$$\alpha = 0.05 \quad \text{If } H_0 \text{ is true} \Rightarrow p_A = p_B = p$$

$$Z = \frac{\left( \frac{X}{n_A} - \frac{Y}{n_B} \right) - 0}{\sqrt{(X+Y)\left(1 - \frac{X+Y}{n_A+n_B}\right)}} \sqrt{n_A n_B} \sim \mathcal{N}(0, 1^2)$$

$$z = -1.336$$

$$p\text{-value} = 0.091$$

Do not reject  
 $H_0$

$$-2.07$$