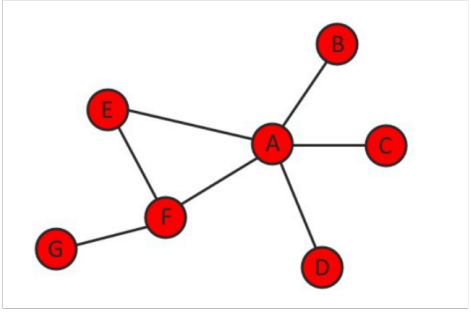
Graph

What is a Graph?

- A graph is a pictorial representation of a set of objects where some pairs of objects are connected by links. The interconnected objects are represented by points termed as vertices, and the links that connect the vertices are called edges.
- **Nodes / Vertices**
- **Edges**



Loop

In a graph, if an edge is drawn from vertex to itself, it is called a loop.

Pendent Vertex

By using degree of a vertex, we have a two special types of vertices. A vertex with degree one is called a pendent vertex.

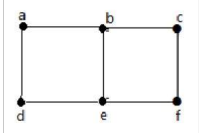
Isolated Vertex

A vertex with degree zero is called an isolated vertex.

Adjacency

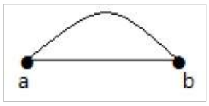
In a graph, two vertices are said to be adjacent, if there is an edge between the two vertices. Here, the adjacency of vertices is maintained by the single edge that is connecting those two vertices.

In a graph, two edges are said to be adjacent, if there is a common vertex between the two edges. Here, the adjacency of edges is maintained by the single vertex that is connecting two edges.



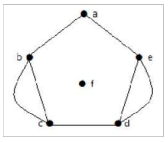
Parallel Edges

In a graph, if a pair of vertices is connected by more than one edge, then those edges are called parallel edges.



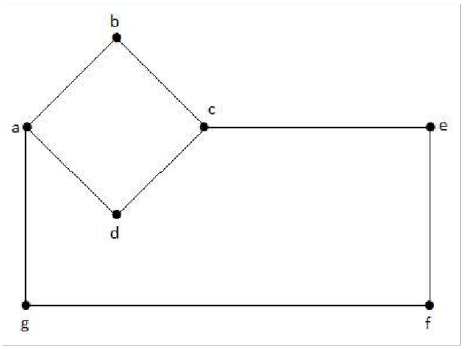
Multi Graph

A graph having parallel edges is known as a Multigraph.



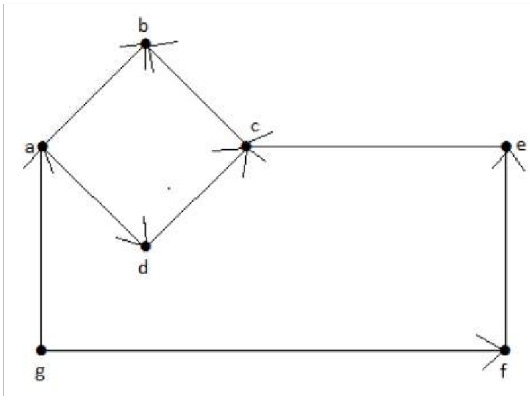
Non-Directed Graph

A non-directed graph contains edges but the edges are not directed ones.



Directed Graph

In a directed graph, each edge has a direction.

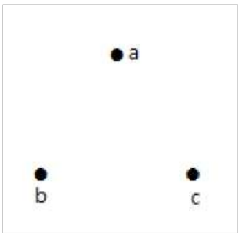


Trivial Graph

A graph with only one vertex is called a Trivial Graph.

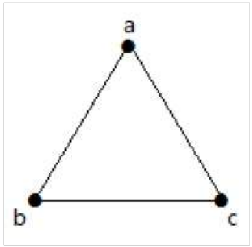
Null Graph

A graph having no edges is called a Null Graph.



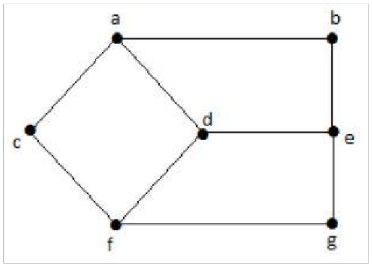
Simple Graph

A graph with no loops and no parallel edges is called a simple graph.



Distance between Two Vertices

It is number of edges in a shortest path between Vertex U and Vertex V.



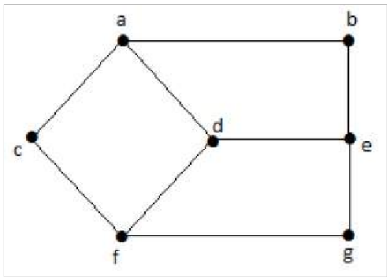
Distance from vertex ‘d’ to vertex ‘e’ or simply ‘de’ is 1 as there is one edge between them. There are many paths from vertex ‘d’ to vertex ‘e’ −

- da, ab, be
- df, fg, ge
- de (It is considered for distance between the vertices)
- df, fc, ca, ab, be
- da, ac, cf, fg, ge

Eccentricity of a Vertex

The maximum distance between a vertex to all other vertices is considered as the eccentricity of vertex.

The distance from a particular vertex to all other vertices in the graph is taken and among those distances, the eccentricity is the highest of distances.

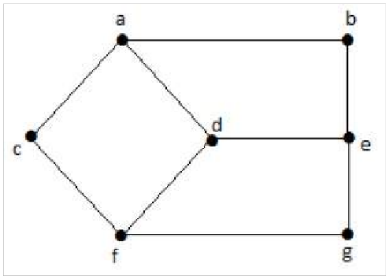


eccentricity of ‘a’ is 3.

- The distance from ‘a’ to ‘b’ is 1 (‘ab’),
- from ‘a’ to ‘c’ is 1 (‘ac’),
- from ‘a’ to ‘d’ is 1 (‘ad’),
- from ‘a’ to ‘e’ is 2 (‘ab’-‘be’) or (‘ad’-‘de’),
- from ‘a’ to ‘f’ is 2 (‘ac’-‘cf’) or (‘ad’-‘df’),
- from ‘a’ to ‘g’ is 3 (‘ac’-‘cf’-‘fg’) or (‘ad’-‘df’-‘fg’).

Radius of a Connected Graph

The minimum eccentricity from all the vertices is considered as the radius of the Graph G. The minimum among all the maximum distances between a vertex to all other vertices is considered as the radius of the Graph G



eccentricity

e(b) = 3

e(c) = 3

e(d) = 2

e(e) = 3

e(f) = 3

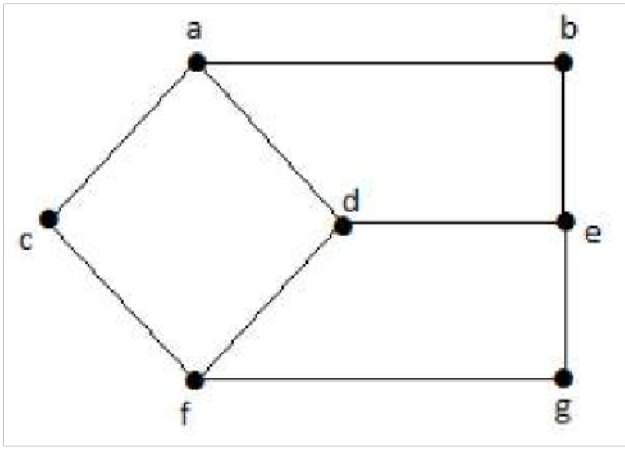
e(g) = 3

In the graph r(G) = 2, which is the minimum eccentricity for ‘d’.

Diameter of a Graph

The maximum eccentricity from all the vertices is considered as the diameter of the Graph G. The maximum among all the distances between a vertex to all other vertices is considered as the diameter of the Graph G.

In the graph, d(G) = 3; which is the maximum eccentricity.



eccentricity

e(b) = 3

e(c) = 3

e(d) = 2

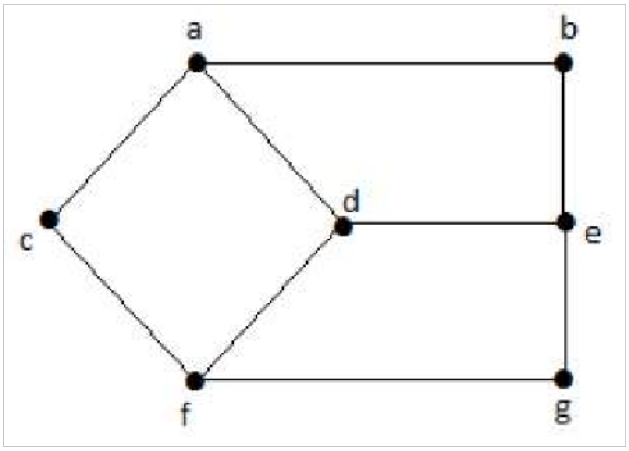
e(e) = 3

e(f) = 3

e(g) = 3

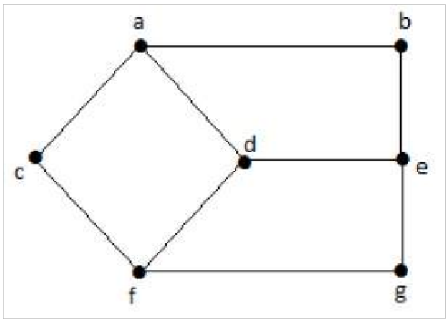
Circumference

The number of edges in the longest cycle of ‘G’ is called as the circumference of ‘G’.



In the example graph, the circumference is 6, which we derived from the longest cycle a-c-f-g-e-b-a or a-c-f-d-e-b-a.

Girth

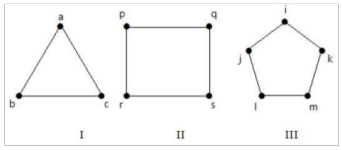


The number of edges in the shortest cycle of ‘G’ is called its Girth.

In the example graph, the Girth of the graph is 4, which we derived from the shortest cycle a-c-f-d-a or d-f-g-e-d or a-b-e-d-a.

Cycle Graph

A simple graph with ‘n’ vertices (n >= 3) and ‘n’ edges is called a cycle graph if all its edges form a cycle of length ‘n’.



Graph I has 3 vertices with 3 edges which is forming a cycle ‘ab-bc-ca’.

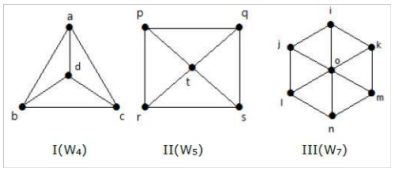
Graph II has 4 vertices with 4 edges which is forming a cycle ‘pq-qs-sr-rp’.

Graph III has 5 vertices with 5 edges which is forming a cycle ‘ik-km-ml-lj-ji’.

Hence all the given graphs are cycle graphs.

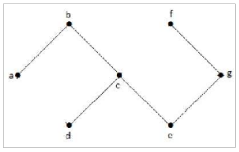
Wheel Graph

A wheel graph is obtained from a cycle graph Cn-1 by adding a new vertex. That new vertex is called a Hub which is connected to all the vertices of Cn.



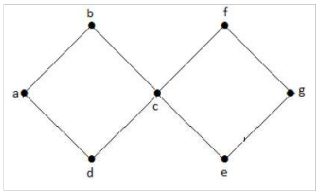
Acyclic Graph

A graph with no cycles is called an acyclic graph.



Cyclic Graph

A graph with at least one cycle is called a cyclic graph.



In the above example graph, we have two cycles a-b-c-d-a and c-f-g-e-c. Hence it is called a cyclic graph.