

A = LU

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Topics:

- Inverse of AB and A^T
- Product of elimination matrices
- $A = LU$
- How Expensive is elimination operation for $n \times n$ matrix

Inverse of AB and A^T

$$A A^{-1} = I = A^{-1} A$$

$$(AB) \underbrace{B^{-1} A^{-1}} = I$$

$$B^{-1} A^{-1} A B = I$$

Inverse of AB

$$A A^{-1} = I$$

$$\underbrace{(A^{-1})^T}_{\text{this is } (A^T)^{-1}} A^T = I$$

INVERSE OF A^T

Inverse of A^T

For a matrix, transposing and inverting can be done either way

Note:

U = Upper Triangular Matrix
L = Lower Triangular Matrix

$A = LU$ (No row exchanges)

Assuming that the matrix A doesn't require any row exchanges (non-zero pivots) in the initial step.

$$E_{21} A = U$$

$$E_{21} = \begin{bmatrix} 1 & 0 \\ -4 & 1 \end{bmatrix}$$

$$A = LU$$

$$(E_{21})^{-1} * E_{21} * A = (E_{21})^{-1} * U = L * U \Rightarrow L = E_{21}^{-1}$$

3 x 3 Matrix

$$E_{32} E_{31} E_{21} A = U \quad (\text{no row exchanges})$$

$$A = E_{21}^{-1} E_{31}^{-1} E_{32}^{-1} U$$

$$= L U$$

The preferred method of obtaining L is by product of individual E matrices rather than inverse of single E matrix because: L can be obtained by just keeping a record of all the multipliers of the individual elimination matrices.

$$A = LU$$

If no row exchanges, multipliers go directly into L.

Consider a matrix A such that it needs no row exchanges and E_{31} is identity matrix. Then,

Elimination matrices

$$E_{32} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -5 & 1 \end{bmatrix}, E_{21} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow E = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 10 & -5 & 1 \end{bmatrix} \Rightarrow EA = U$$

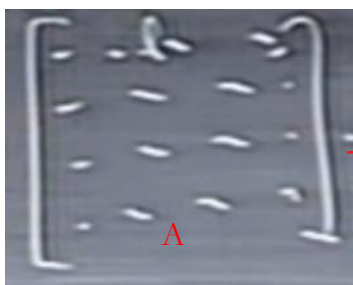
Inverses:

$$E_{21}^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, E_{32}^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 5 & 1 \end{bmatrix} \Rightarrow L = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 5 & 1 \end{bmatrix} \Rightarrow A = LU$$

The matrix E has elements that really do not help in obtaining L. Also, the matrix "A" can be forgotten as we get the first row of U in the first elimination step and also the multipliers and so on..

How many operation on n x n matrix A elimination?

The operations are: Multiplication + Subtraction



A size : 100 x 100



About 100^2 operations



About 99^2 operations

Approximately, number of operations is roughly equal to $= n^2 + (n-1)^2 + \dots + 3^2 + 2^2 + 1^2$

Count
 $n^2 + \dots + 1^2$
 $\approx \frac{1}{3} n^3$
 ON A

Cost of b n^2

Note:

- $1/3$ accounts for the reducing size of the elimination matrix
- Since elimination of A is more expensive than b, the RHS can be processed at a low cost separately.

Permutations: (when pivot elements are zero - row exchanges)

Permutations 3x3 6 P's $P^{-1} = P^T$

$\begin{bmatrix} 1 & & \\ & 1 & \\ & & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & & \\ & 0 & 1 \\ & 1 & 0 \end{bmatrix}, \begin{bmatrix} 1 & & \\ & 0 & 0 \\ & 1 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$