

# Matrix operations and inverses

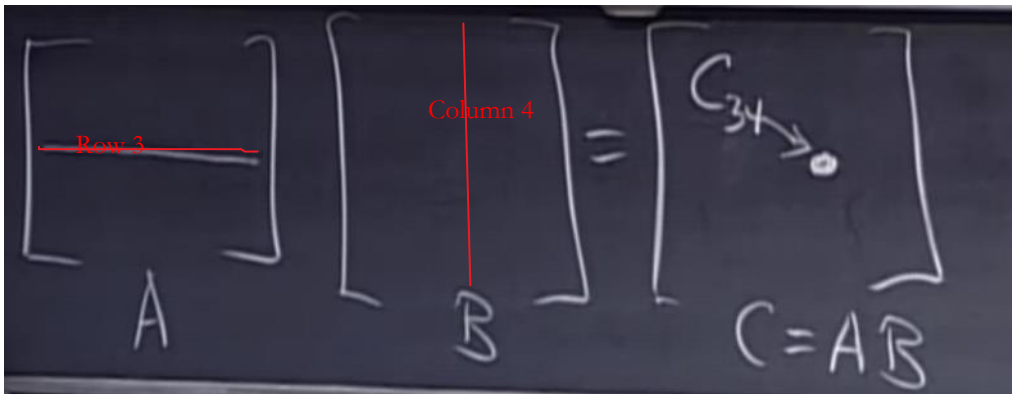
Sunday, October 22, 2017 9:28 PM

Topics:

- Matrix Multiplication
- Inverse of  $A$   $AB$   $A^T$
- Gauss-Jordan or find  $A^{-1}$

## 1. Matrix Multiplication : Method 1

Revising the rules for matrix multiplication, the value of an element in the matrix  $C_{ij}$ , in specific, let's take  $C_{34}$

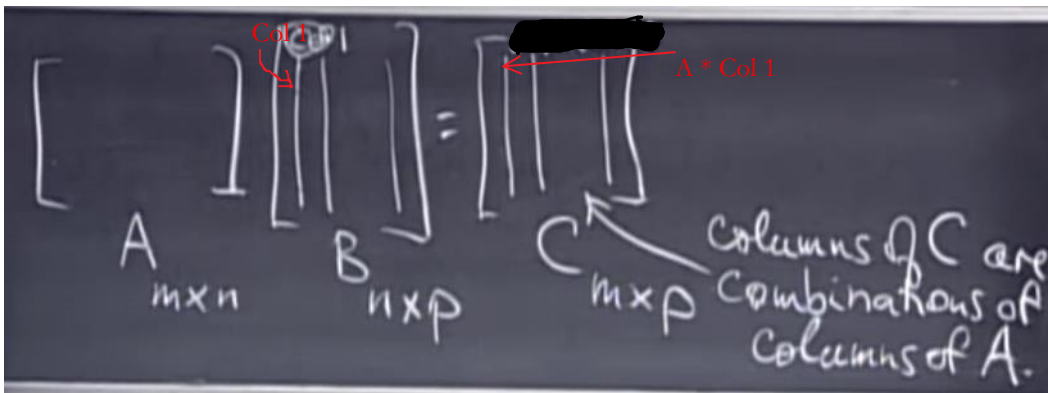


$$C_{34} = (\text{row 3 of } A) \cdot (\text{column 4 of } B)$$
$$= a_{31}b_{14} + a_{32}b_{24} + \dots = \sum_{k=1}^n a_{3k}b_{k4}$$

So the general rule is:

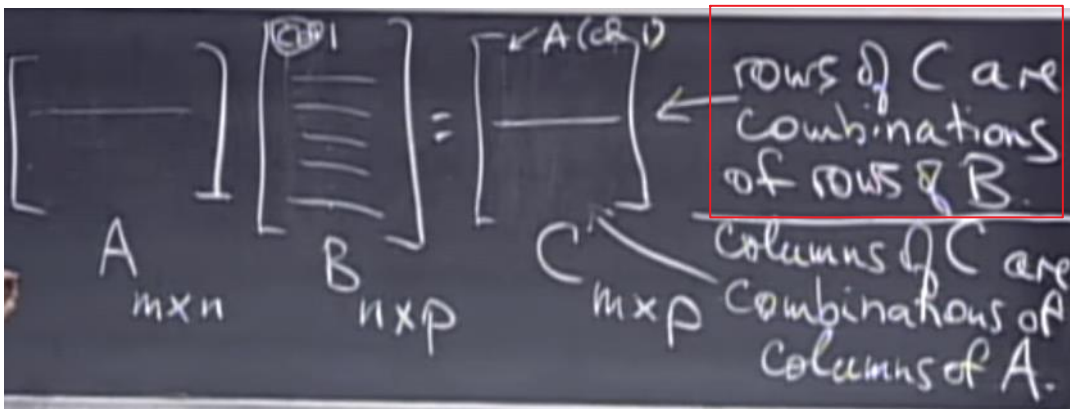
**If  $A$  is size  $(m,n)$  then  $B$  should be of size  $(n,p)$  and  $C$  would be of size  $(m,p)$**

## 1. Matrix Multiplication: Method 2 - Column Wise



Matrix multiplication can be seen as combinations of columns of "A" to get product "C"

#### 1. Matrix Multiplication: Method 3 - Row Wise



Matrix multiplication can be seen as combinations of rows of "B" to get product "C"

#### 4. Matrix Multiplication: Method 4 - $A \cdot B = \text{Sum of (columns of A)} \times (\text{rows of B})$

4<sup>th</sup> way  $AB = \text{Sum of (cols of A)} \times (\text{rows of B})$

$$\begin{bmatrix} 2 & 7 \\ 3 & 8 \\ 4 & 9 \end{bmatrix} \begin{bmatrix} 1 & 6 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix} \begin{bmatrix} 1 & 6 \end{bmatrix} + \begin{bmatrix} 7 \\ 8 \\ 9 \end{bmatrix} \begin{bmatrix} 0 & 0 \end{bmatrix}$$

Block Multiplication:

Block

$$\begin{bmatrix} A_1 & A_2 \\ A_3 & A_4 \end{bmatrix} \begin{bmatrix} B_1 & B_2 \\ B_3 & B_4 \end{bmatrix} = \begin{bmatrix} A_1 B_1 + A_2 B_3 & A_1 B_2 + A_2 B_4 \\ A_3 B_1 + A_4 B_3 & A_3 B_2 + A_4 B_4 \end{bmatrix}$$

$A \quad B$

Inverses:

### 1. Square Matrix

$$A^{-1} A = I = A A^{-1}$$

↑ if this exists  
invertible, nonsingular

**Note:** For a square matrix, left  $A^{-1}$  and right  $A^{-1}$  are the same. But not so for rectangular matrices (the shape doesn't allow)

- Singular Case / No inverse:

Consider matrix A:

1	3
2	6

The matrix A doesn't have an inverse because:

- The determinant is zero
- Since both the columns of A lie on the same line, it is not possible to get a combination of  $[1;0]$  columns which is required for identity matrix i.e.,
  - $A \cdot P \neq I$  where P is inverse
- A non-zero vector X exists such that  $A \cdot X = 0$

$$A \cdot X = \begin{bmatrix} 1 & 3 \\ 2 & 6 \end{bmatrix} \begin{bmatrix} 3 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Reason:

Consider,  $AX = 0$

on multiplying both sides by  $A^{-1}$ , we get LHS =  $A^{-1}AX = X$  and RHS =  $A^{-1}0 = 0$ . Implies  $X = 0$  which is not true.

**Clearly,  $RHS \neq LHS$ . Hence if there exists a non-zero vector X such that  $AX = 0$  then the A is non invertible**

- Positive Case i.e., Non-Singular Case:

$$\begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix} \begin{bmatrix} a & c \\ b & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$A \quad A^{-1} \quad I$

$A \times \text{column } j \text{ of } A^{-1} = \text{column } j \text{ of } I //$

Solving inverse is like solving a system of linear equations.

Gauss-Jordan Method (Solving two equations at once):

$$\begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad \begin{bmatrix} 1 & 3 & | & 1 & 0 \\ 2 & 7 & | & 0 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 3 & | & 1 & 0 \\ 0 & 1 & | & -2 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & | & 7 & -3 \\ 0 & 1 & | & -2 & 1 \end{bmatrix}$$

$A \quad I \quad I \quad A^{-1}$

Steps:

1. Augment the matrices A and I
2. Perform Eliminations on A till you get I, then
3. Gauss-Jordan method tells that, the augmented matrix is  $A^{-1}$

Proof:

Let "E" be the elimination matrix, then according to Gauss-Jordan method

$$> E(A \cdot I) = I \cdot A^{-1}$$

$$> EA = I \text{ which means } E \text{ is } A^{-1}$$

$$EA = I \text{ tells us } E = A^{-1}$$

$$E[A \quad I] = [I \quad A^{-1}]$$

$E = A^{-1}$