Elimination with matrices

Friday, October 20, 2017 10:50 PM

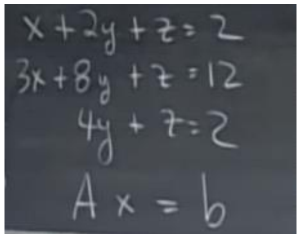
Topics:

- Elimination
  - Success
  - Failure
- Back Substitution
- Elimination Matrices
- Matrix Multiplication

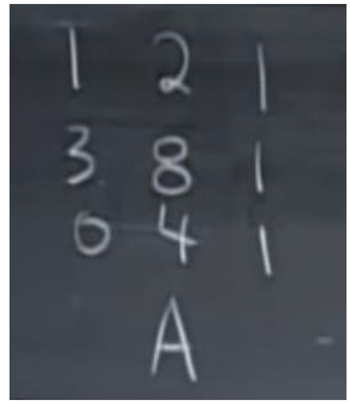
Method of Elimination:

Solving a system of equation using elimination the way all the software packages solves.

Eg equations: 3 equations and 3 unkowns

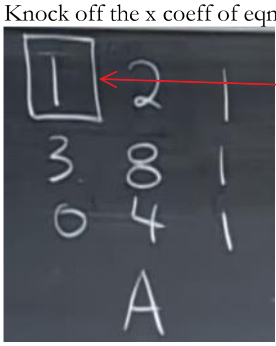


Matrix A is used elimination



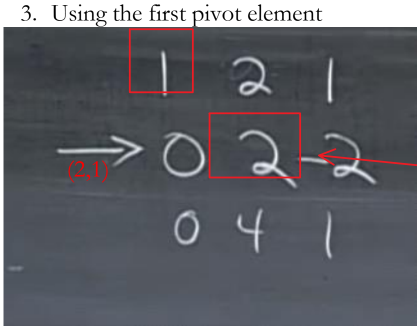
Method of elimination:

1. Identify if the first equation is acceptable
2. Knock off the x coeff of eqn2 by multiplying and subtract using the first row pivot element



First Pivot

3. Using the first pivot element



Second Pivot

4. Using the second pivot, eliminate the third row elements both x and y coeffs

Third pivot

Let's call this upper triangle as U

Note:
- Determinant of U is 10
- None of the pivots can be zero
- This is the operation done by most of the software programs

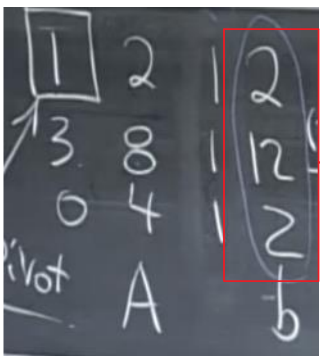
Failure Cases:

Here failure refers to failure to come up with 3 pivots .

In case of a zero in the first pivot position, the then that row is exchanged with lower rows, same applies to the second row (exchanged with 3rd row) BUT if there is zero in the 3rd pivot position leads to a **failure. Implies the matrix is not invertible**

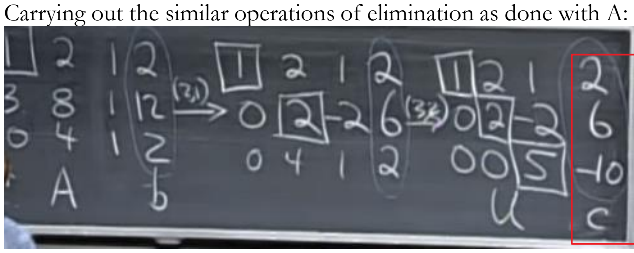
Note:
- The above operation is not complete as the elimination operation is not yet done for "b".
- This method of performing elimination operations first for "A" and then for "b" is followed by MATLAB

This involves bringing back the RHS (b) as an extra column.



This RHS is called the augmented matrix

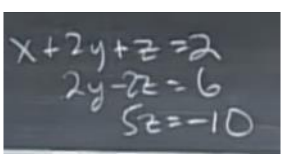
Carrying out the similar operations of elimination as done with A:



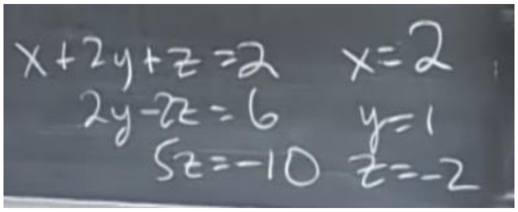
This is c

Back Substitution:

The final solution of elimination is of the form: Ux = c

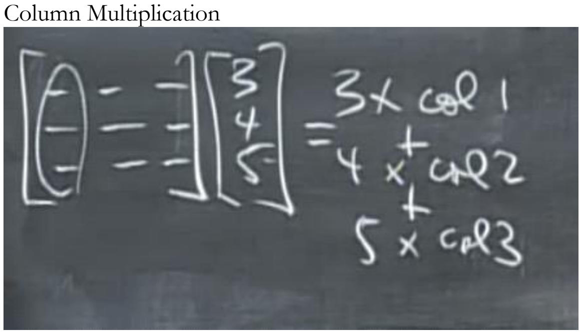


Back substitution is a method of solving a system of linear equation in the reverse order because the system is triangular matrix



Matrix Multiplication

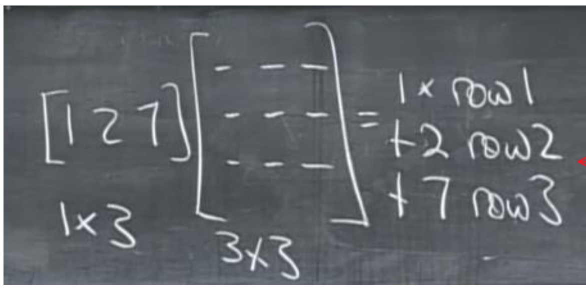
Column Multiplication



Matrix * column = column

Linear Combination of columns

Row Multiplication



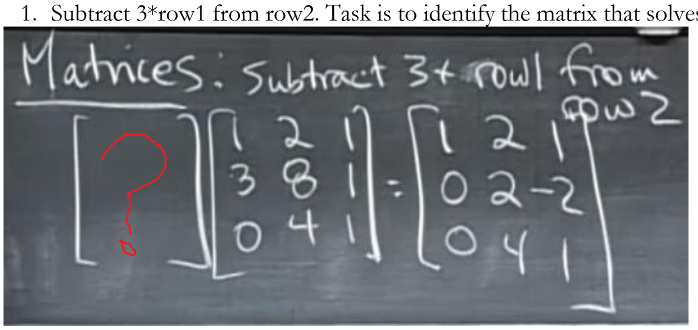
Row * matrix = row

Linear Combination of Rows

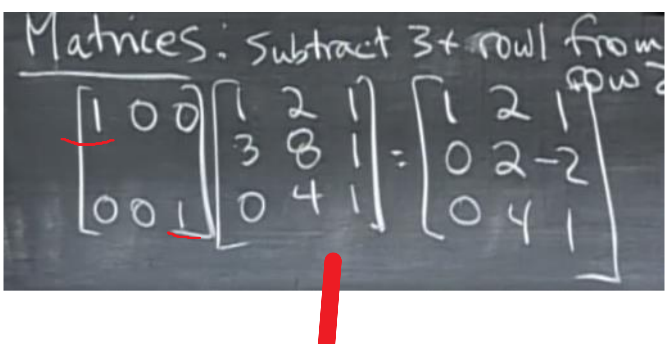
Matrix Operations:

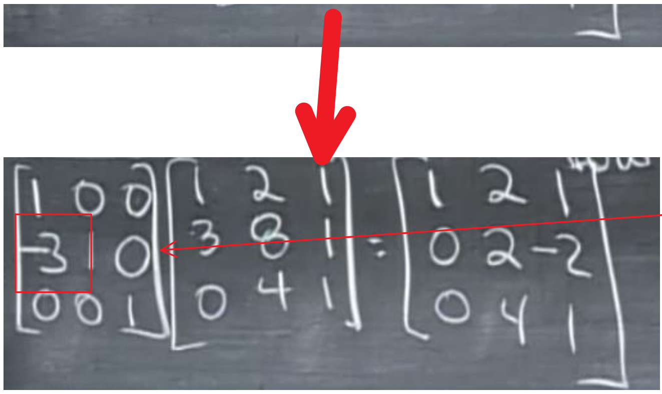
In the above elimination method, the first pivot element is used to eliminate x coeff and get 2 row pivot. The operation can be explained as follows:

1. Subtract 3*row1 from row2. Task is to identify the matrix that solves the equation below:



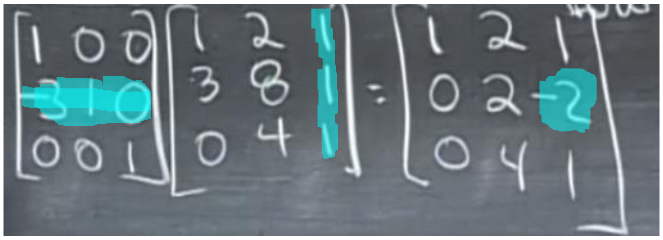
2. Approach: Since I and III row remain unchanged:





The matrix is called E_{21} as it is used to eliminate element at (2,1)

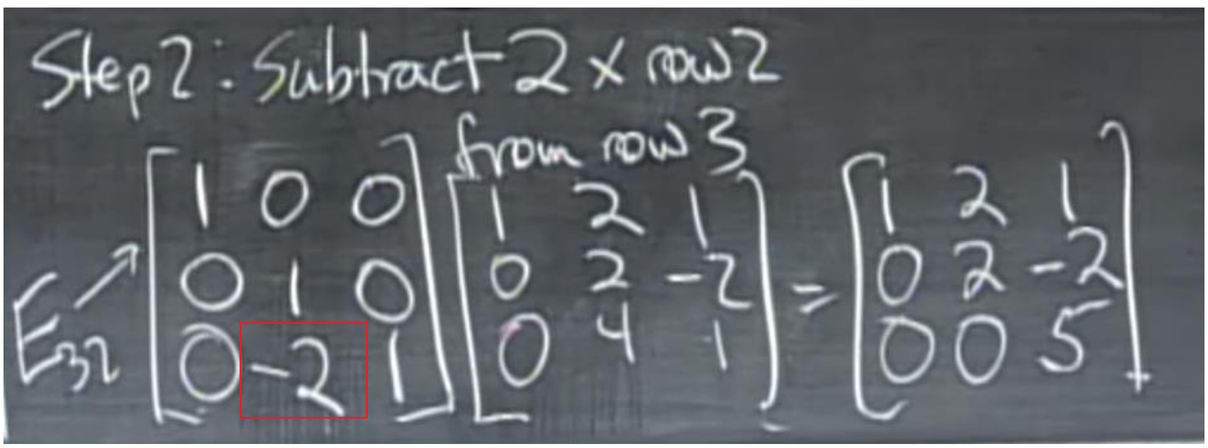
A validation check: Let's verify the element at (2,2)



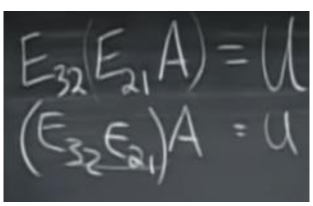
dot_product([-3 1 0], [1 ;1; 1]) = -2.

Hence, correct.

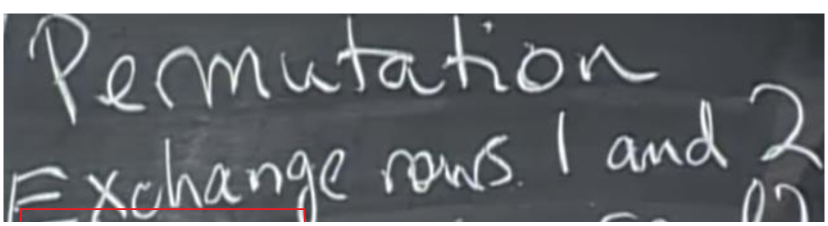
Continuing the Elimination with third row we get:

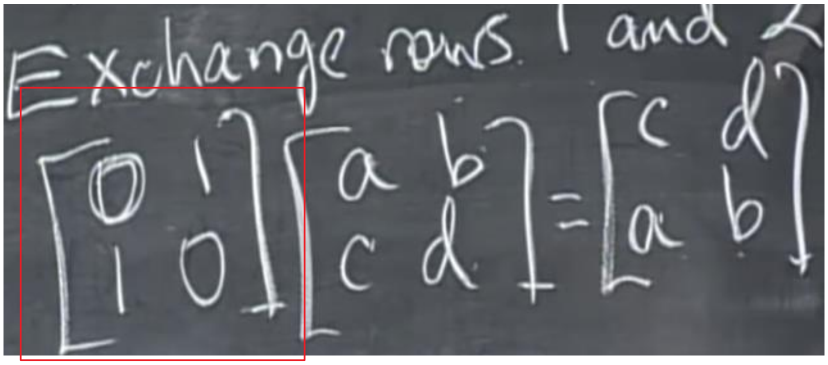


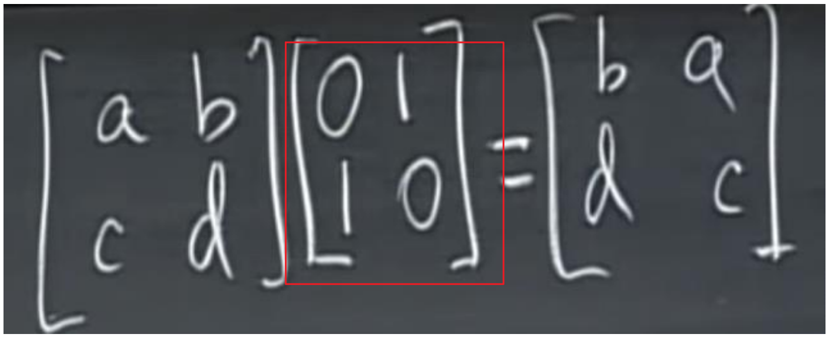
On summarizing, we get:



Permutation Matrix: Exchange Rows and Columns:







NOTE:

- **For columns operations, matrix multiplications comes on the right.**
- **For row operations, matrix multiplications comes on the left.**

Inverses: