

# The geometry of linear equations

Friday, October 20, 2017 8:51 AM

## Topics

- n-equations and n-unknowns
  - row-picture
  - column-picture (important)
  - matrix form

eqn1:  $2x - y = 0$

eqn2:  $-x + 2y = 3$

## Matrix Form

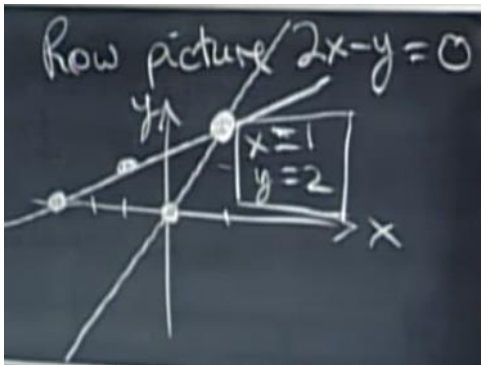
Handwritten matrix form of the system of equations:

$$\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \end{bmatrix}$$

The matrix is labeled **A**, the vector  $\begin{bmatrix} x \\ y \end{bmatrix}$  is labeled **x**, and the vector  $\begin{bmatrix} 0 \\ 3 \end{bmatrix}$  is labeled **b**.

## Row Picture

represent the equations on x-y plane and determine the point of intersection



## Column Picture

Handwritten column picture equation:

$$x \begin{bmatrix} 2 \\ -1 \end{bmatrix} + y \begin{bmatrix} -1 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \end{bmatrix}$$

Eqn1

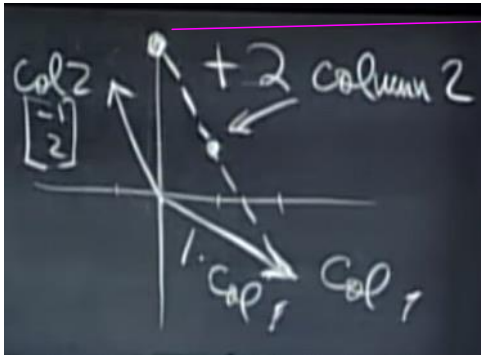
The above equation demands a linear combination of columns to get the  $\begin{bmatrix} 0 \\ 3 \end{bmatrix}$  vector

Representing Eqn1 as vector form:

Handwritten vector form of the column picture equation:

$$x \begin{bmatrix} 2 \\ -1 \end{bmatrix} + y \begin{bmatrix} -1 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \end{bmatrix}$$

From row picture we know the solution for the above set is  $x=1; y=2$ . These values are used in the column picture.



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Col2 vector is translated to the head of col1. The col2 meets  $[0;3]$  i.e., "b"

### 3D - Plane

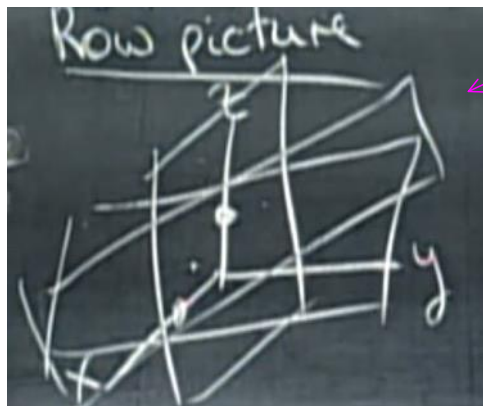
$$\begin{aligned} 2x - y &= 0 \\ -x + 2y - z &= -1 \\ -3y + 4z &= 4 \end{aligned}$$

$$A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -3 & 4 \end{bmatrix} \quad b = \begin{bmatrix} 0 \\ -1 \\ 4 \end{bmatrix}$$

### Row Picture

**Each row** in the coefficient matrix represents a plane

2	-1	0	Plane1
-1	2	-1	Plane2
0	-3	4	Plane3



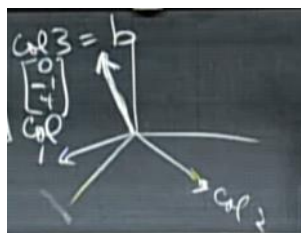
Represents 3 planes in 3D-Planes

They all meet at a point and that is the solution. The drawback is that, the visualization is getting difficult with the increase in the number of dimensions

### Column Picture:

$$x \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix} + y \begin{bmatrix} -1 \\ 2 \\ -3 \end{bmatrix} + z \begin{bmatrix} 0 \\ -1 \\ 4 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \\ 4 \end{bmatrix}$$

### 3D Space representation:



In this particular example, col3 is equal to "b".  
Hence, the linear combination is,  $X=0; Y=0; Z=1$ .

The above representations lead us to discuss this:

- Can I solve every  $Ax=b$ ? In other words,
- Do the linear combinations of the columns fill the 3D space?

When the 3 Columns(vectors) lie in the same plane. For example, when the Col3 is sum of Col2 and Col1 because all the combinations do not yield any new values as the all the values are within the plane. In such a situation, the **b** can only be the ones in that plane which is very limited as most of the values would be outside the plane that is a **singular case**.

### Multiplication of matrix and vector:

$Ax = b$  is the multiplication of a matrix, A and scalar, x. This can be done in two ways:

- Row-wise
- Column-wise

Row Wise

The diagram shows the equation  $Ax = b$  on a chalkboard. Matrix  $A$  is  $\begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$  and vector  $x$  is  $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$ . A red arrow points from the first row of  $A$  to the first element of the resulting vector  $b$ , which is 12. A purple arrow points from the second row of  $A$  to the second element of  $b$ , which is 7. A red arrow also points from the vector  $x$  to the same result vector  $b$ . The resulting vector  $b$  is shown as  $\begin{bmatrix} 12 \\ 7 \end{bmatrix}$ .

Column Wise

The diagram shows the equation  $Ax = b$  on a chalkboard. Matrix  $A$  is  $\begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$  and vector  $x$  is  $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$ . The calculation is shown as  $\begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = 1 \begin{bmatrix} 2 \\ 1 \end{bmatrix} + 2 \begin{bmatrix} 5 \\ 3 \end{bmatrix} = \begin{bmatrix} 12 \\ 7 \end{bmatrix}$ . A red bracket is drawn under the two column vectors  $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$  and  $\begin{bmatrix} 5 \\ 3 \end{bmatrix}$  to indicate they are being combined linearly.

The multiplication of matrix and vector can be seen as a linear combination of columns.

$Ax$  is a linear combination of columns of "A"

