Permutations and Transposes

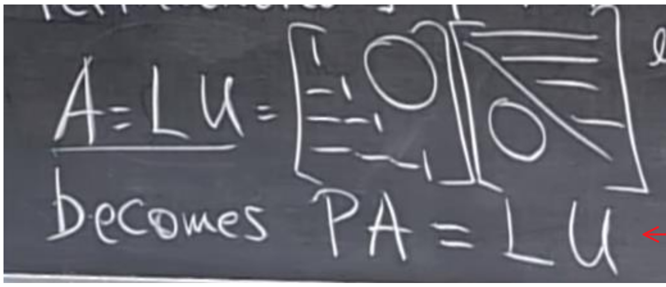
Monday, November 06, 2017 8:48 PM

Topics:
- PA=LU
- Transposes
- Vector Spaces and Subspaces

Permutations (P)

P are those matrices that execute row exchanges.

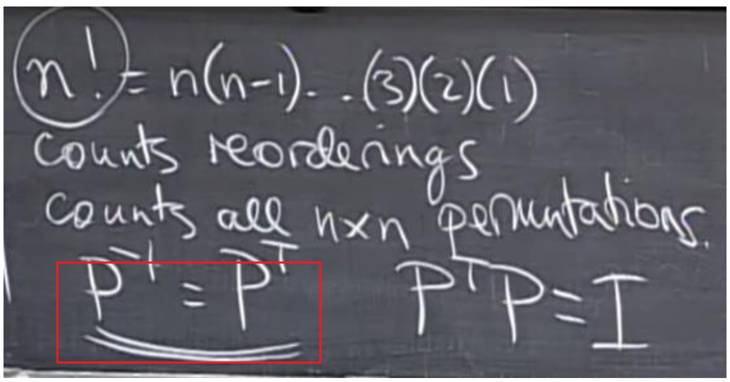
We know that:



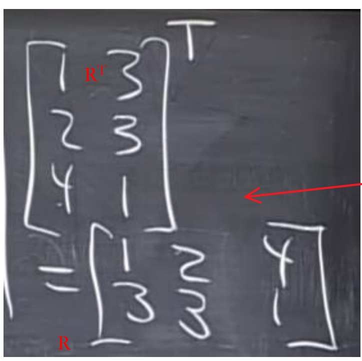
The operation A = LU is only possible if there exists no row exchanges i.e., P is identity matrix.
Note: Software like MATLAB performs first scans for zero pivot elements and performs permutations. Also, for computation accuracy, permutations are performed if the pivot elements are very numerically small. Here accuracy dominates over linear algebra in MATLAB

For any invertible A

P (Permutation Matrix) are those identity matrices with reordered rows and number of possibilities of P are n! where is the order of P. In addition, all the matrices are invertible.



Transpose:



$(A_{ij})^T = A_{ji}$

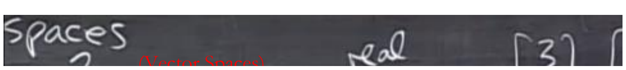
Symmetric Matrix : $A = A^T$

For any Rectangular Matrix R, $R^T * R$ is always an symmetric matrix.

Reason: Take transpose of $R^T * R$ i.e., $(R^T * R)^T = (R^T * R^{TT}) = (R^T * R)$

It is evident that both LHS and RHS are same, thus it is symmetric always.

Vector Spaces and Subspaces: AIM- To be able to add and multiply scalars and remain in the same space (including some rules).



SPACE - space (bunch) of vectors

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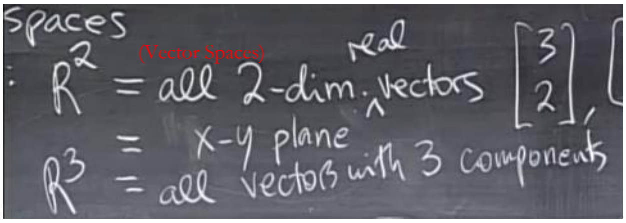
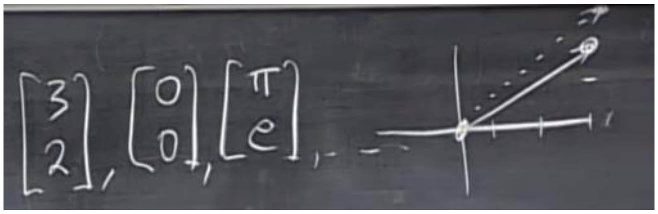
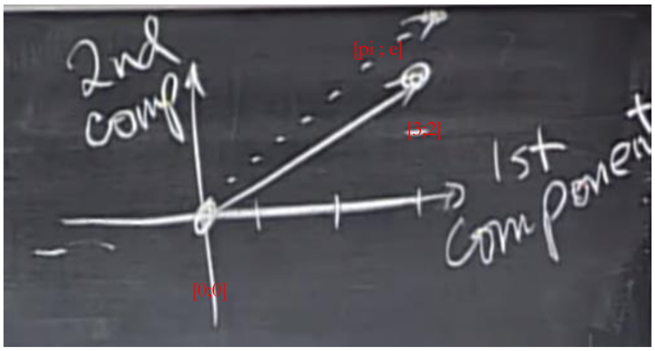


SPACE - space (bunch) of vectors
R^2 - X-Y plane
Similarly, R^n represents all (column)vector with n-real components

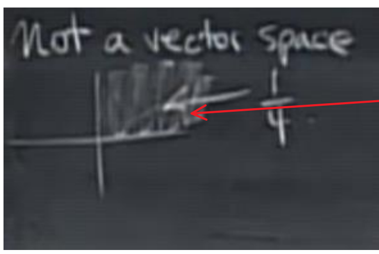
Example:





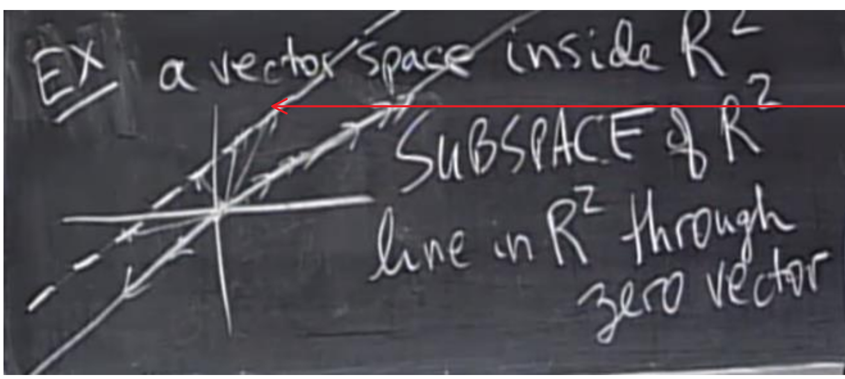
The whole plane is R^2 i.e., "X-Y" plane

Vector space has to be closed under vector multiplication and addition i.e., any linear combinations.



Consider first quarter (x and y are always positive)
Not a vector space for all real numbers especially multiplication with negative scalars, although the addition of vectors in first quarter always remain in first quarter. In other words it is not a closed vector space.

Subspace:



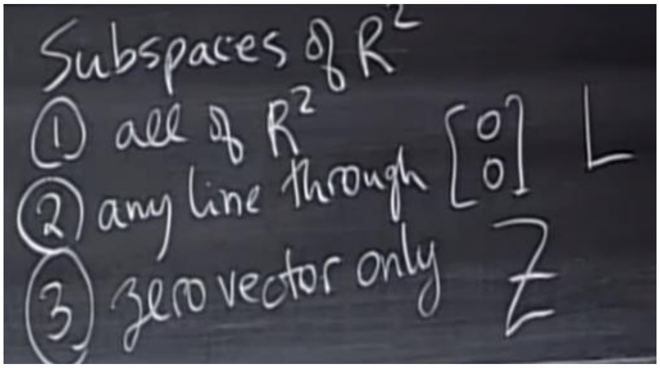
The dotted line is not a true Subspace in R^2 since it doesn't contain zero vector.

The line through the origin is a true subspace since any linear combination (vector addition and scalar mul remains on the line)

This is not a valid subspace since, any multiplication with say, zero is not a vector on that line.

This means every subspace should contain zero since multiplication with zero is zero vector.

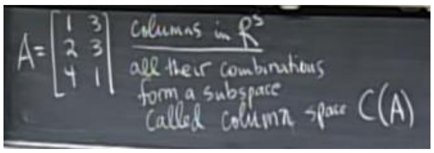
Examples of Subspaces of R^2



R^3 subspaces

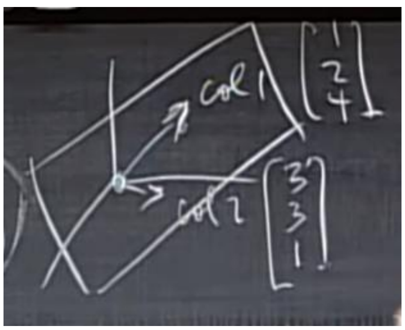
- All R^3
- Any plane through $[0\ 0]^T$
- Only $[0\ 0]^T$

Vector Subspace for matrix a is all possible linear combinations (addition and multiplication) of its vectors. For example take A:



To create a subspace from a matrix, take the linear combination of all the column vectors, this subspace is called column space.

The column space C(A)for two vectors in A for R^3 is the plane containing Col1 and Col2 and passing via origin.



Columns in $\mathbb{R}^5$

all their combinations
form a subspace
called column space $C(A)$