Matrix operations and inverses

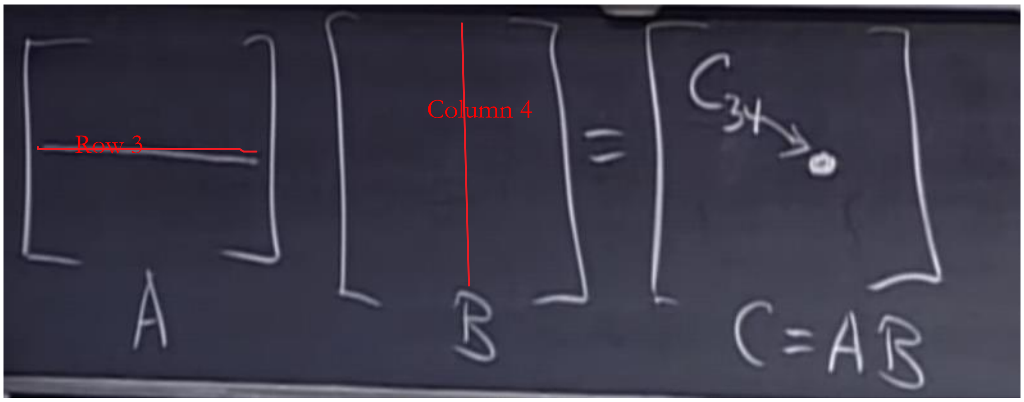
Sunday, October 22, 2017 9:28 PM

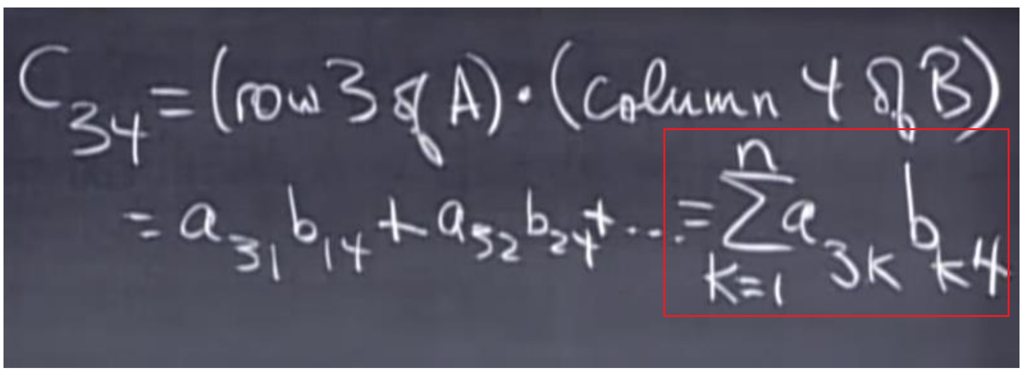
Topics:

- Matrix Multiplication
- Inverse of A AB A^T
- Gauss-Jordon or find A^{-1}

1. Matrix Multiplication : Method 1

Revising the rules for matrix multiplication, the value of an element in the matrix C_{ij} , in specific, let's take C_{34}

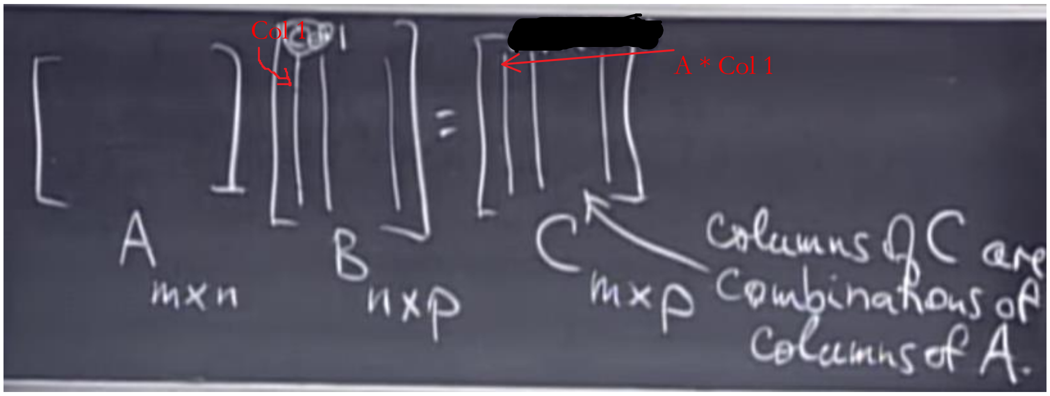




So the general rule is:

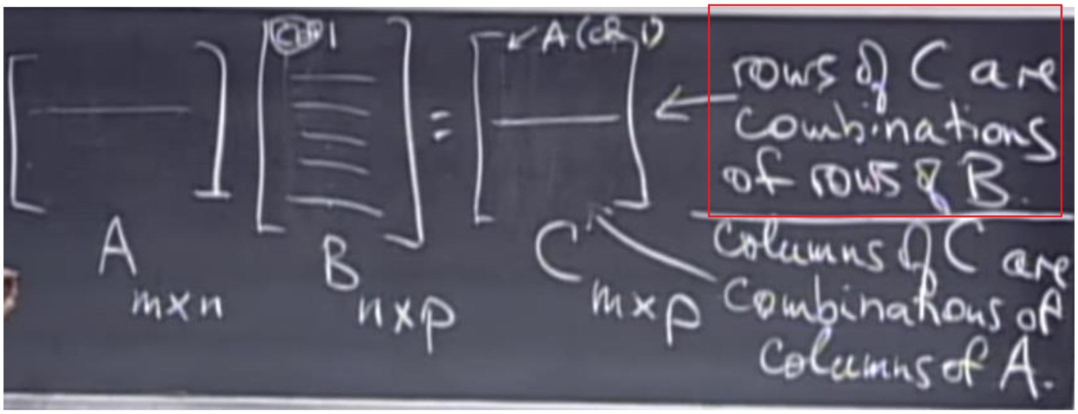
If A is size (m,n) then B should be of size (n,p) and C would be of size (m,p)

1. Matrix Multiplication: Method 2 - Column Wise



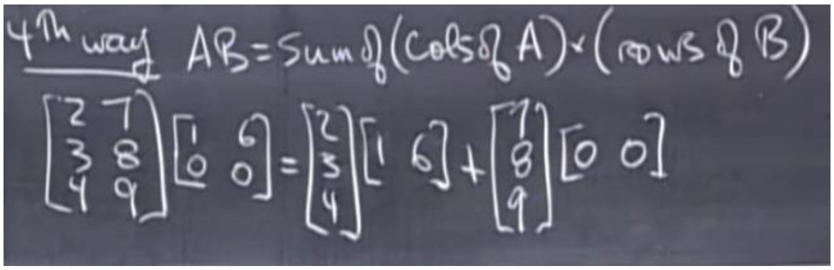
Matrix multiplication can be seen as combinations of columns of "A" to get product "C"

1. Matrix Multiplication: Method 3 - Row Wise

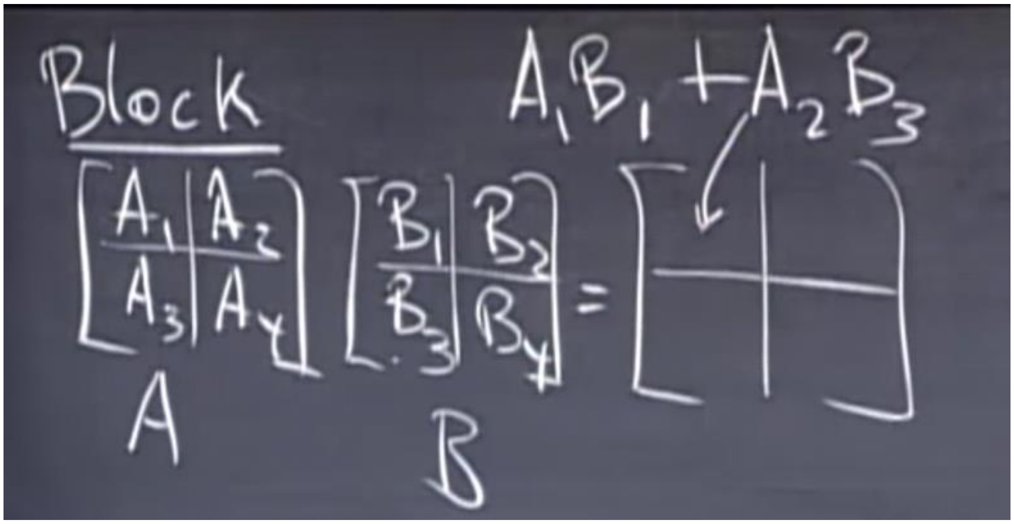


Matrix multiplication can be seen as combinations of rows of "B" to get product "C"

4. Matrix Multiplication: Method 4 - A*B = Sum of (columns of A) * (rows of B)

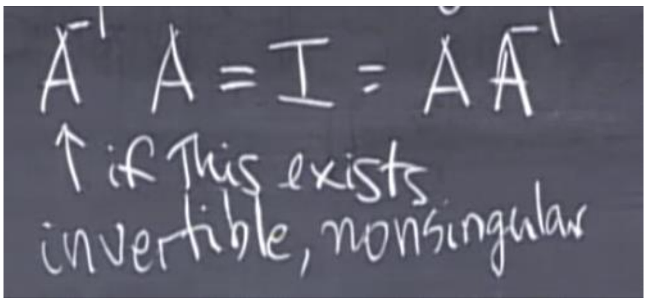


Block Multiplication:



Inverses:

1. Square Matrix



Note: For a square matrix, left A^{-1} and right A^{-1} are the same. But not so for rectangular matrices (the shape doesn't allow)

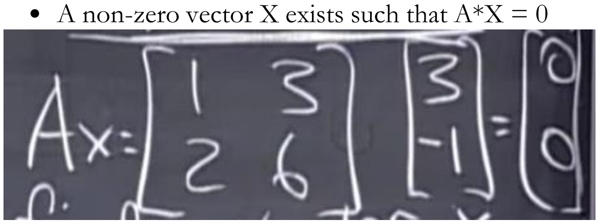
- Singular Case / No inverse:

Consider matrix A:

1	3
2	6

The matrix A doesn't have an inverse because:

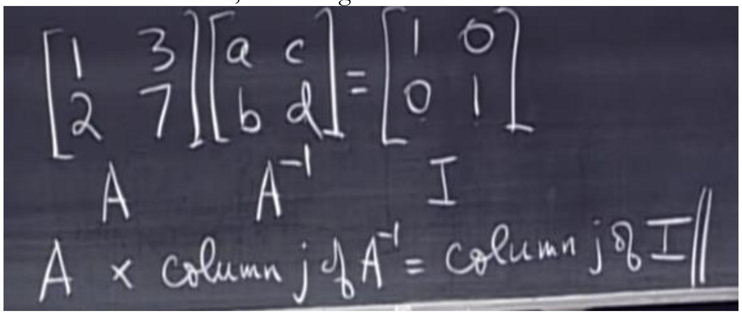
- The determinant is zero
- Since both the columns of A lie on the same line, it is not possible to get a combination of [1;0] columns which is required for identity matrix i.e,
  - A*P != I where P is inverse
- A non-zero vector X exists such that A*X = 0



Reason:
Consider, AX = 0
on multiplying both sides by A^{-1}, we get LHS = $A^{-1}AX$ =X and RHS = $A^{-1}0$ = 0. Implies X = 0 which is not true.

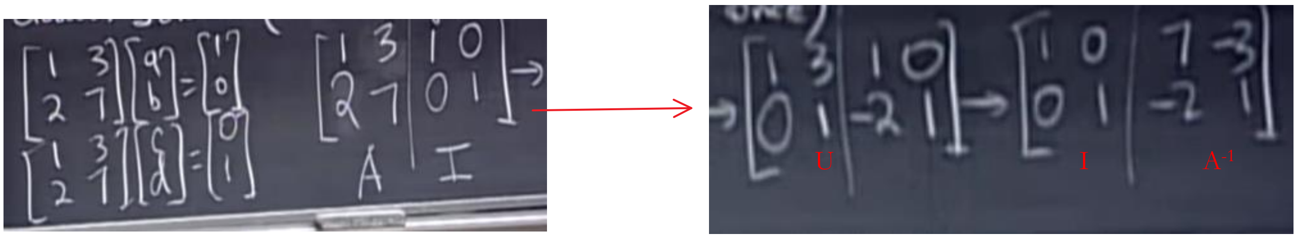
Clearly, RHS != LHS. Hence if there exists a non-zero vector X such that AX = 0 then the A is non invertible

- Positive Case i.e., Non-Singular Case:



Solving inverse is like solving a system of linear equations.

Gauss-Jordan Method (Solving two equations at once):



Steps:

1. Augment the matrices A and I
2. Perform Eliminations on A till you get I, then
3. Gauss-Jordan method tells that, the augmented matrix is A^{-1}

Proof:
Let "E" be the elimination matrix, then according to Gauss-Jordan method

>E(A*I) = I*A^{-1}
>EA = I which means E is A^{-1}