# Solving Ax = 0: pivot variables, special solutions

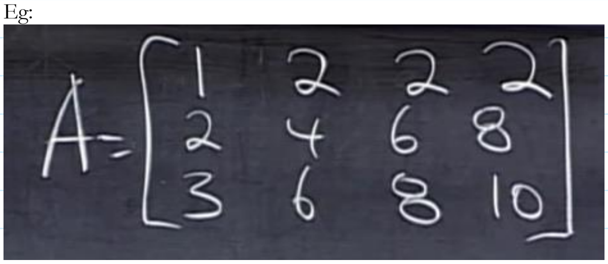
Friday, April 06, 2018 7:45 AM

Topics
- Computing Null Space AX = 0
- Pivot Variables - free variables
- Special Solutions - rref(A)=R

Computing Null Space:

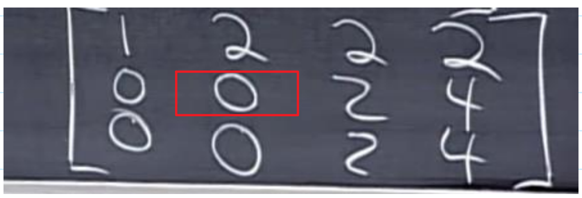
Method to solve a system of linear equations: AX=0

Eg:



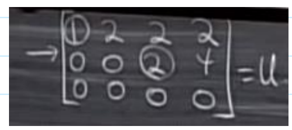

The method of eliminations leaves Null Space unchanged but changed the columns space.

Step 1: R3 = R3 - R1*3; R2 = R2 - R1*2




Continuing elimination with A(2,3)

Step 2: R3 = R3 - R2*2

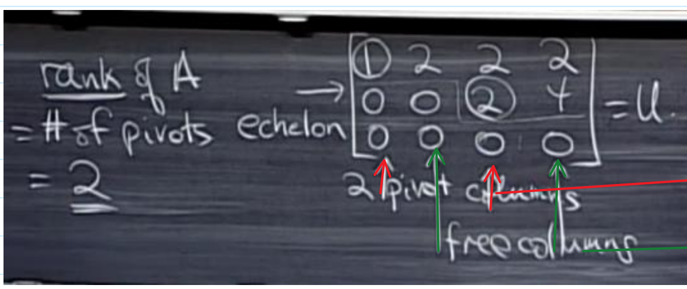



The second row pivot element is zero, also row exchange to swap the rows is also futile since A(3,2) = 0 as well. This implies **Col2 is not independent.**

The U is in echelon form. i.e., zeros form a stair case form.

NOTE: U has only two pivot elements.

- **RANK of A = Number of pivots (for the eg Rank =2)**
- The Solution U is the Null space of A



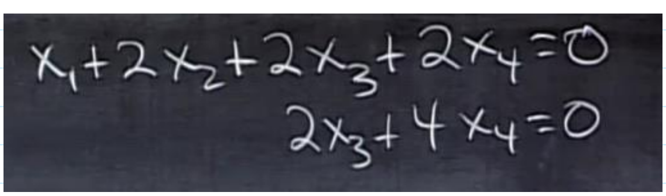

Pivot Columns

Free Columns

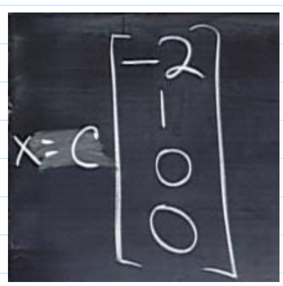
Free columns mean, X(2,1) and X(2,4) can take any values for solution of A.

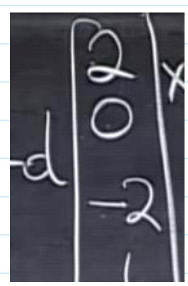
For example X = [x1, 1, x3, 0]$^T$

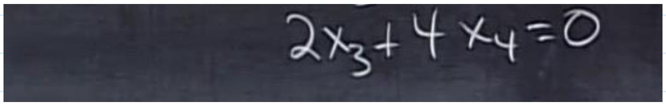
Hence solving the equations by substitution (x2 = 1 and x4 = 0), we get:

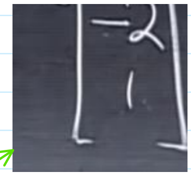



Solving






The variables associated with free columns are called free variables (x2 and x4)

NULL Space contains all the combinations of special solution (free variables). One for each free variable.

Similarly, by using another combination of free variables (x2 = 0. x4 =1) we get X = d[2 0 -2 1]$^T$

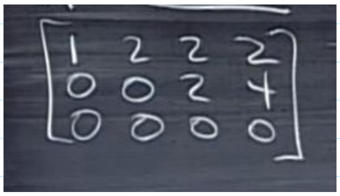
Free Variables:

For a "m" row and "n" column matrix with rank "r", number of free variables are "n-r"

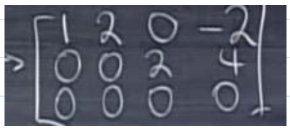
R - Reduced Row Echelon Form (Reduction of U):

In Reduced Row Echelon form, the elements above and below pivots are zero and pivots = 1

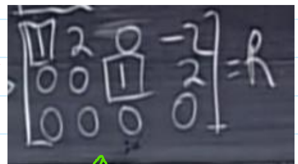
In the above example, the 3rd row is zero because,
R3 is a linear combination of R1 and R2 and elimination discovered it and eliminated it. Reduced form is in which the upper r ow is also reduced.



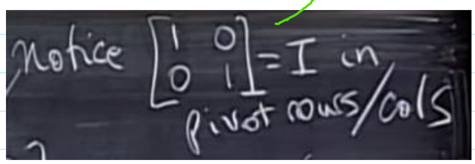
R1 = R1-R2



R2 = R2/2




In MATLAB R =rref(A)

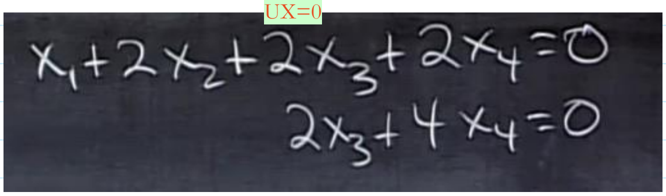



Inference from R:

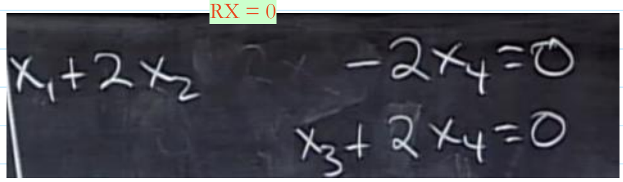
- Has an Identity matrix in pivot rows/ columns
- R3 is zero, indicating original row are combinations of other rows.
- Free columns
- Pivot Rows and Columns

Now UX can be reduced to RX

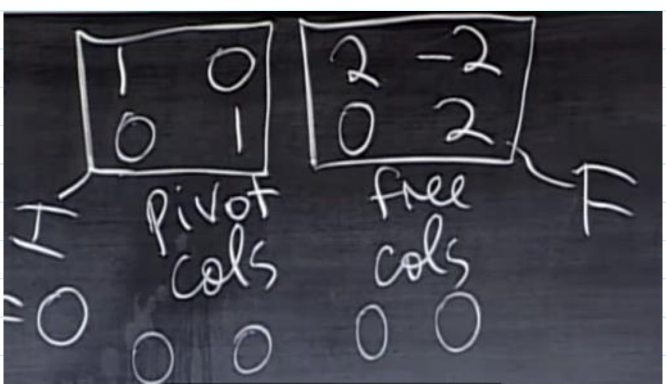
UX=0

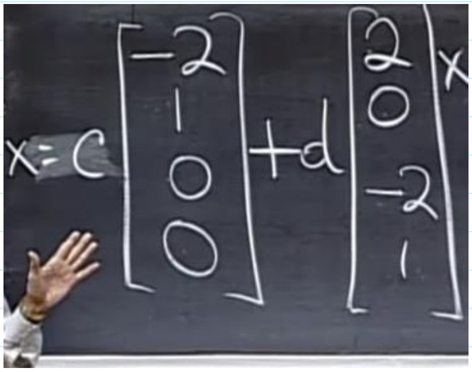



RX = 0



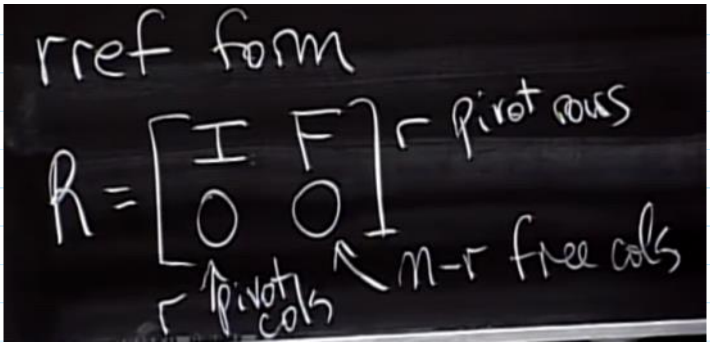

Here UX RX and AX are all equal. UX can be seen as a combination of Pivot and Free columns






The free column elements are in the NULL SPACE

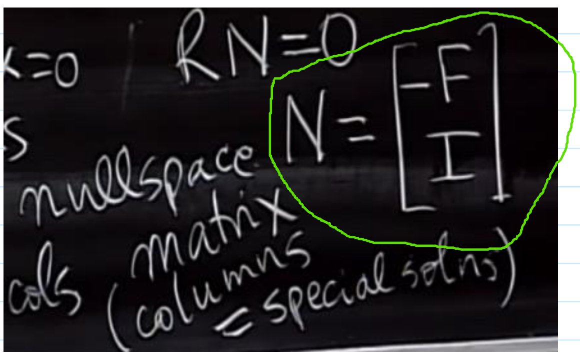
General "rref" form:




For RX = 0,

The Nullspace N matrix, the columns =  special solutions.

Also, RN = 0, where N is given by




0

The final solution can be shown as below: