Permutations and Transposes

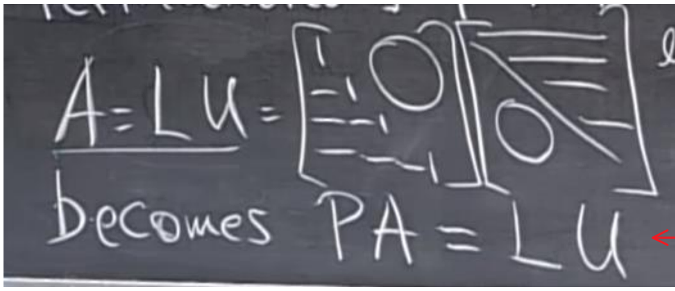
Monday, November 06, 2017 8:48 PM

Topics:
- PA=LU
- Transposes
- Vector Spaces and Subspaces

Permutations (P)

P are those matrices that execute row exchanges.

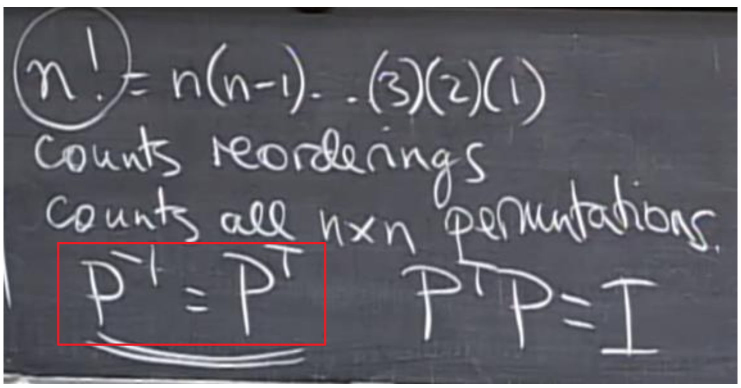
We know that:



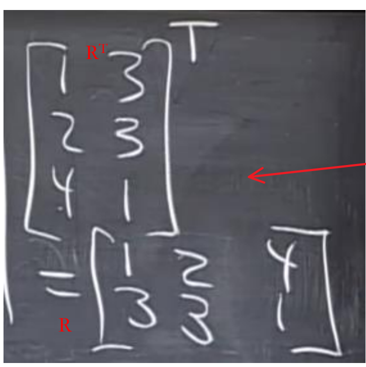
The operation A = LU is only possible if there exists no row exchanges i.e., P is identity matrix.
Note: Software like MATLAB performs first scans for zero pivot elements and performs permutations. Also, for computation accuracy, permutations are performed if the pivot elements are very small.

For any invertible A

P are those identity matrices with reordered rows and number of possibilities of P are n! where is the order of P. In additio n, all the matrices are invertible.



Transpose:



$(A_{ij})^T = A_{ji}$

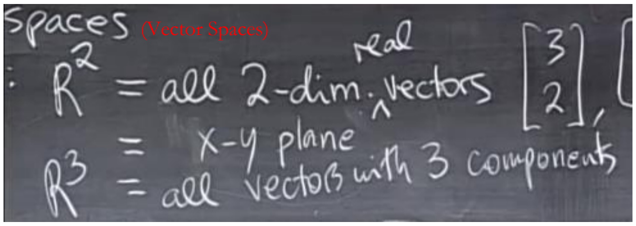
Symmetric Matrix : $A = A^T$

$R^T * R$ is always an symmetric matrix.

Reason: Take transpose of $R^T * R$ i.e., $(R^T * R)^T = (R^T * R^{TT}) = (R^T * R)$

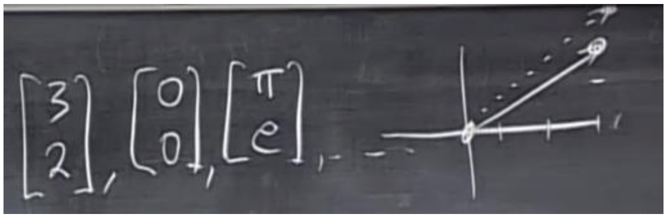
It is evident that both LHS and RHS are same, thus it is symmetric always.

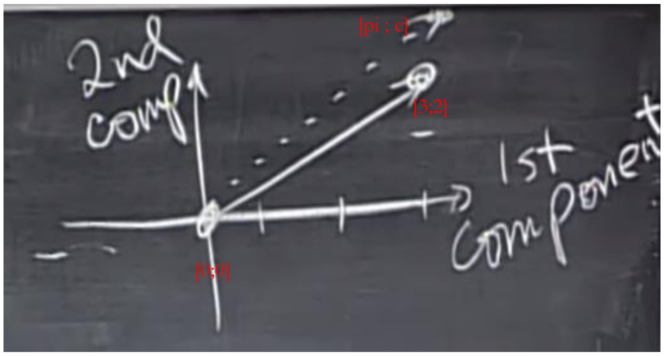
Vector Spaces and Subspaces:



Similarly, R^n represents all column vector with n-real components

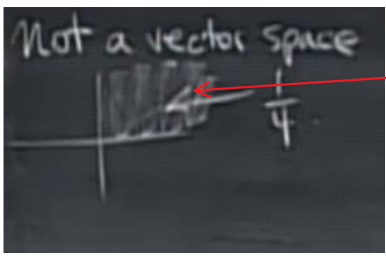
Example:





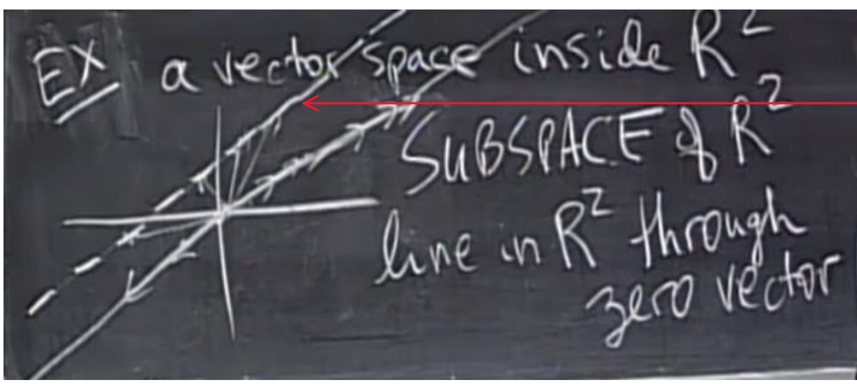
The whole plane is R^2 i.e., "X-Y" plane

Vector space has to be closed under vector multiplication and addition i.e., any linear combinations.



Not a vector space for all real numbers especially multiplication with negative numbers. In other words it is not a closed vector space.

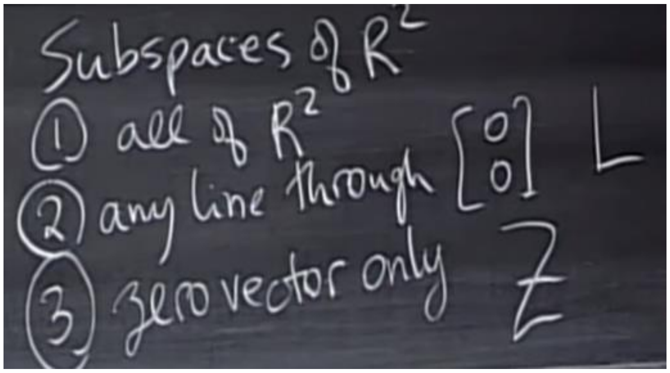
Subspace:



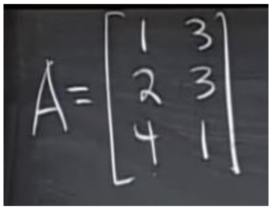
This is not a valid subspace since, any multiplication with say, zero is not a vector on that line.

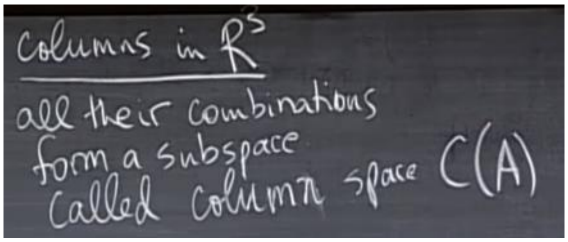
This means every subspace should contain zero since multiplication with zero is zero vector.

Examples of Subspaces of R^2



Vector Subspace for matrix a is all possible linear combinations (addition and multiplication) of its vectors. For example ta ke A:





The column space CA for two vectors in A for R^3 is the plane between Col1 and Col2 and passing via origin