The geometry of linear equations

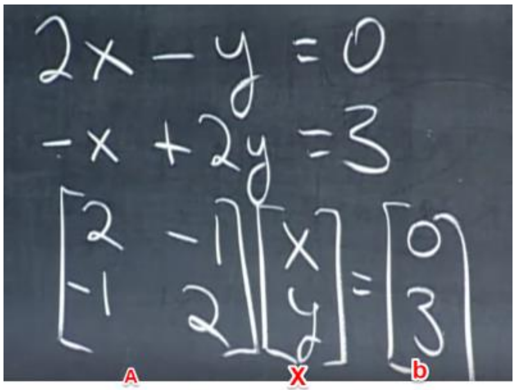
Friday, October 20, 2017 8:51 AM

Topics

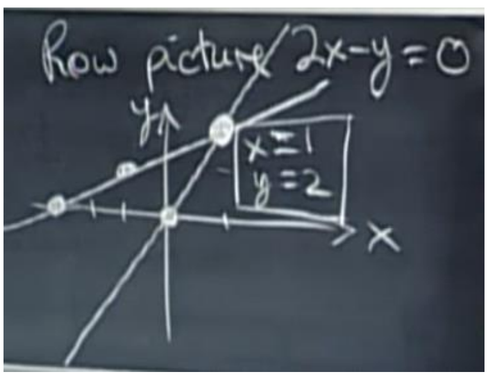
- n-equations and n-unknowns
    - row-picture
    - column-picture (important)
    - matrix form

eqn1: 2x - y = 0
eqn2: -x + 2y = 3

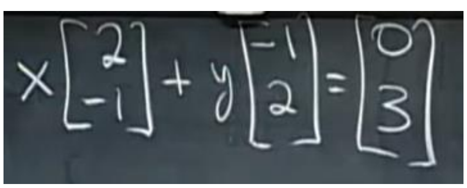
Matrix Form



Row Picture
represent the equations on x-y plane and determine the point of intersection



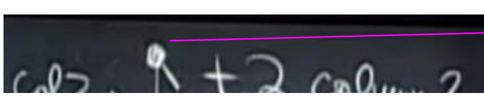
Column Picture



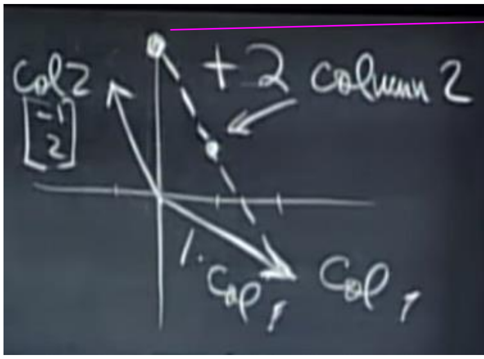
Eqn1

The above equation demands a linear combination of columns to get the [0;3] vector

Representing Eqn1 as vector form:



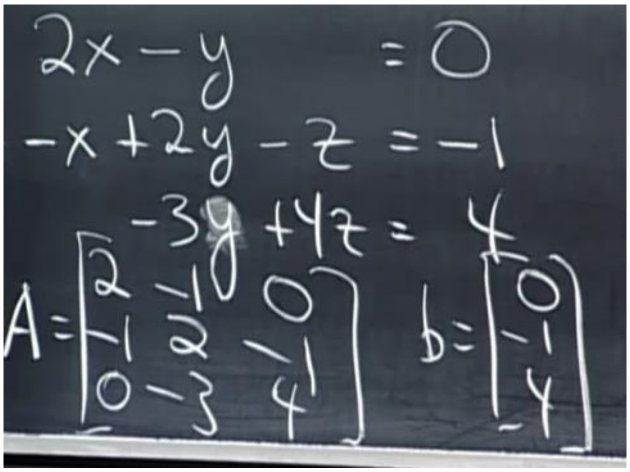
From row picture we know the solution for the above set is x=1;y=2. These values are used in the column picture.



From row picture we know the solution for the above set is x=1;y=2. These values are used in the column picture.

Col2 vector is translated to the head of col1. The col2 meets [0;3] i.e., "b"

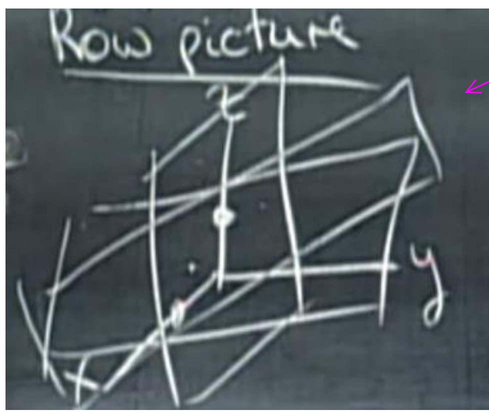
3D - Plane



Row Picture

Each row in the coefficient matrix represents a plane

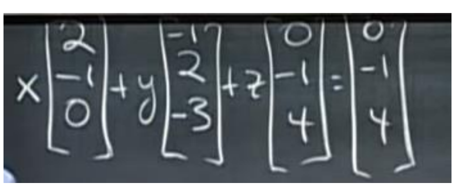
[2	-1	0	Plane1
-1	2	-1	Plane2
0	-3	4]	Plane3



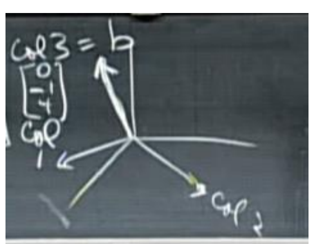
Represents 3 planes in 3D-Planes

They all meet at a point and that is the solution. The drawback is that, the visualization is getting difficult with the increase in the number of dimensions

Column Picture:



3D Space representation:



In this particular example, col3 is equal to "b". Hence, the linear combination is, X=0; Y=0; Z=1.

The above representations lead us to discuss this:

- Can I solve every Ax=b? In other words,
- Do the linear combinations of the columns fill the 3D space?

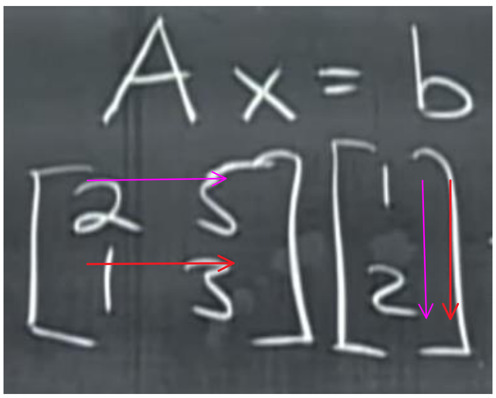
When the 3 Columns(vectors) lie in the same plane. For example, when the Col3 is sum of Col2 and Col1 because all the combinations do not yield any new values as the all the values are within the plane. In such a situation, the **b** can only the ones in that plane which is very limited as most of the values would be outside the plane that is a **singular case.**

Multiplication of matrix and vector:

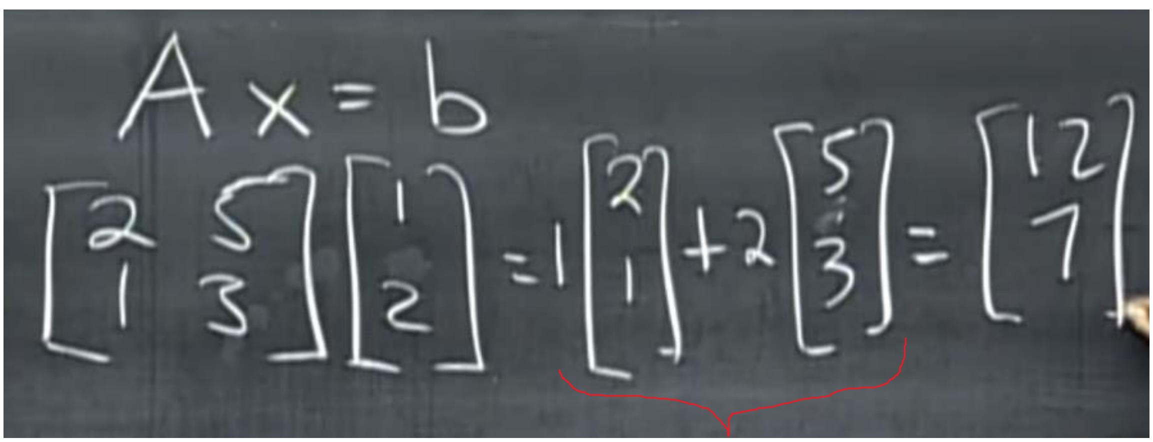
A x = b is the multiplication of a matrix, A and scalar, x. This can be done in two ways:

- Row-wise
- Column-wise

Row Wise



Column Wise



The multiplication of matrix and vector can be seen as a linear combination of columns.

Ax is a linear combination of columns of "**A**"