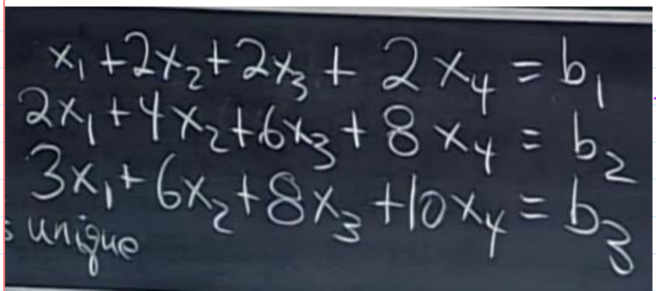
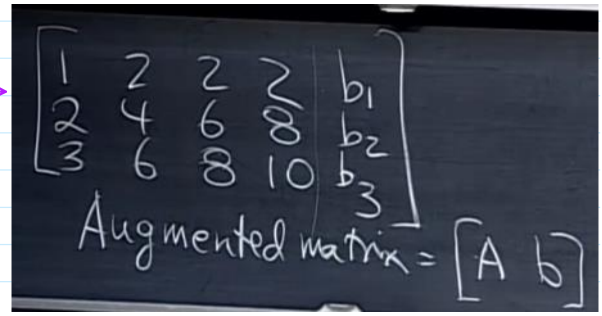
Complete Solution of AX=b

Monday, April 09, 2018 8:49 AM

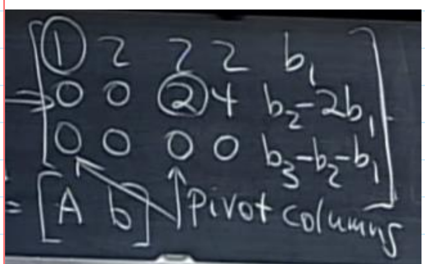
Consider a set of linear equations as below, it can be represented as RHS





On elimination of [A b],

1. R2 = R2 - 2R1;
2. R3 = R3 - 3R1;
3. R3 = R3 - R2



We can see that for R3,

0 = b3-b2-b1 -> b1 + b2 = b3. This is the **condition for solvability**,

If b = [1 5 6], this b is acceptable.

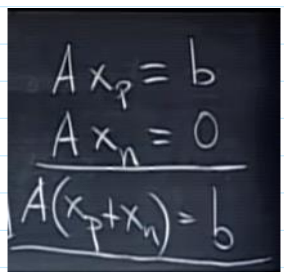
Condition for Solvability:

- AX = b is solvable, if b is in C(A)
- For a combination of rows of A gives a zero row,
  then the same combination of entities of b must give 0.

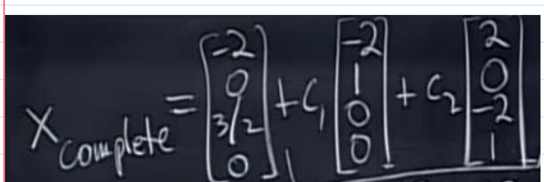
To find Complete Solution of AX = b

1. $X_{particular}$: Set free variables to zero and Solve AX = b for Pivot Variables.
   For the above equation, x2 and x4 are zero, on solving, we get : b = $[2\ 0\ 1.5\ 0]^T$
2. $X_{nullspace}$: Find the null space

Complete Solution is $X_{particular} + X_{nullspace}$

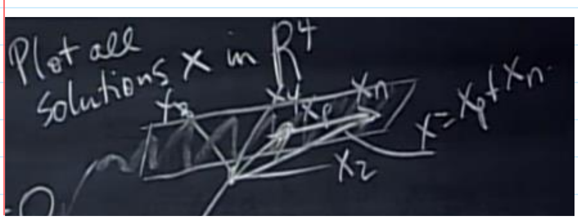


Complete Solution for above example

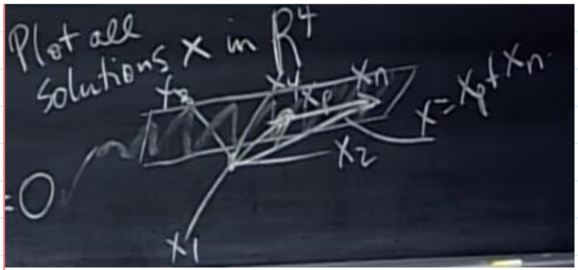


Note: X_p is a single solution while X_n is subspace hence c1 and c2.

The solution in R^4 can be shown as follows:



The solution is 2D figure **not a subspace** that passes through X_p.

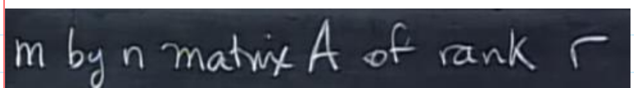


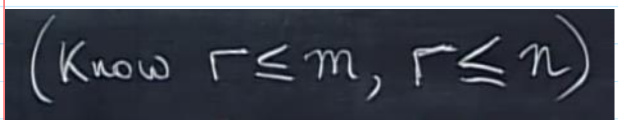
The solution is 2D figure **not a subspace** that passes through X_p.

Algorithm to find Complete Solution
1. Find the particular solution'
2. Find the null space.

The Bigger Picture:

For any,

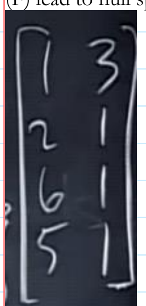




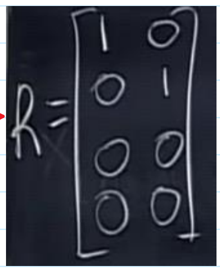
Since each column can have one pivot and **rank** cannot be more than number of rows

Maximum Ranks:
- Full column Rank: r =n

This implies all the columns have pivot, thus no free variables. Thus Null Space is only zero vector. N(A) = 0 (pivots because I and free variables (F) lead to null space)



R =rref(A)

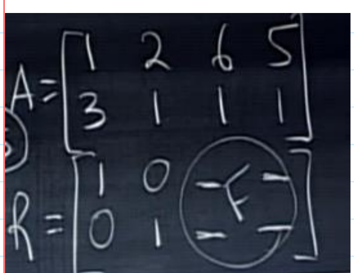


There are 4 equations but only two in X, hence there is not always a solution except X = [0 0 0 0]^{T}. A particular combination of b would have a solution (a linear combination of A) say b = [4 3 7 6]^T then X = [1 1 1 1]^T, that is X_P

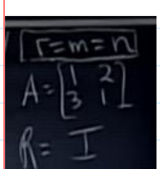
Complete Solution:
X = X_P i.e., only unique solution if it exists. (0 or 1 solutions)

- Full row Rank: r = m

Can solve AX =b for every b. Solutions exists.
And we have n-r free variables i.e., n-m

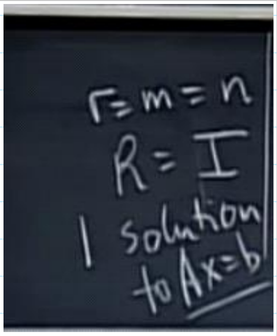
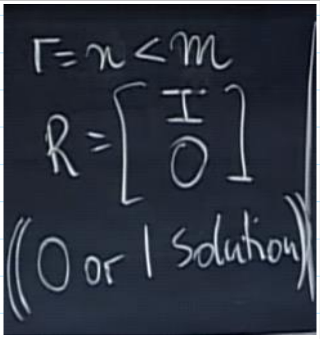
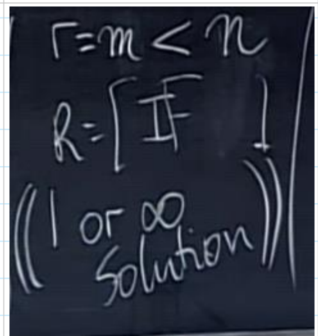
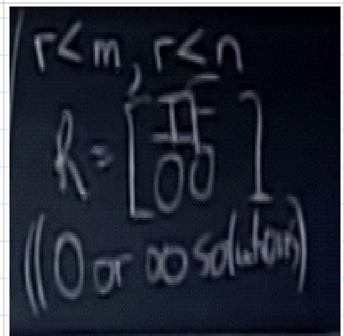


- For r = m =n



Null space is zero vector only.

AX=b is possible for every b, and has unique solution.

Solutions				
				

THE RANK represents all the information about the solution