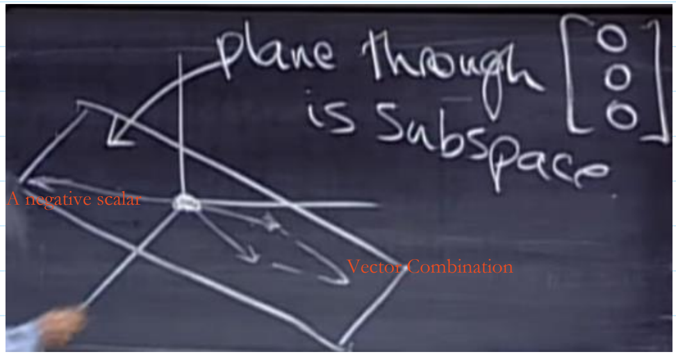
Column Space and Null Space

Wednesday, April 04, 2018 7:49 AM

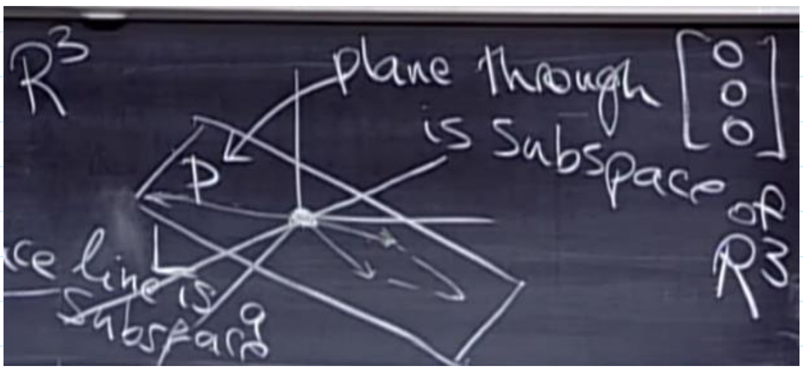
Topics
- Vector Spaces and Subspaces
- Column Space of A : Solving Ax = b
- Nullspace of A

Example:
In R^3, a Plane through $[0,0,0]^T$ is a subspace



Vector Space Requirements:
Bunch of vectors V, W and any scalars c, d such that,

- V + W and cV are **in the space**
- All combinations of cV + dW are in the space



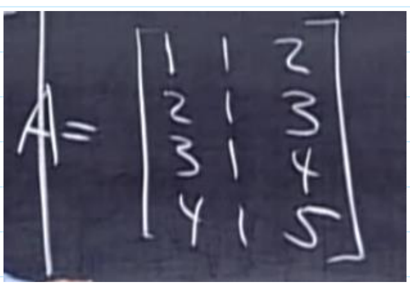
There are two subspaces P and L

P **U** L = all vectors in P or L (union). P **U** L is not a subspace
P ^ L = intersection of P and L, and the intersection is a subspace.

In general, for subspaces S and T,

S ^ T is a subspace because a vector is in both S and T

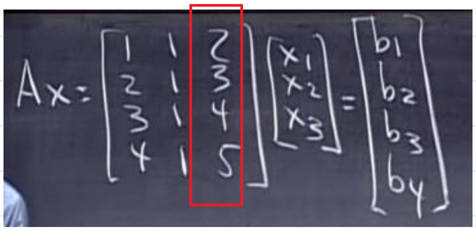
Column Space of a matrix A



A is vector in R^4,

C(A) is the column space of A in R^4. It is the linear combination of all the column vectors.

Ax = b , not all combinations of Ax doesn't solve b.



The col 3 can be neglected since it is the sum of col1 and col2, nothing new is generated from it. i.e., col3 is a dependent vector.

Col1 and Col2 are independent.

Hence, C(A) can be defined as a 2D subspace of R^4

Which b's allow the system of equations can be solve solved?

Ax is solved when b is in the C(A) column space i.e., b is the linear combinations of columns in A.

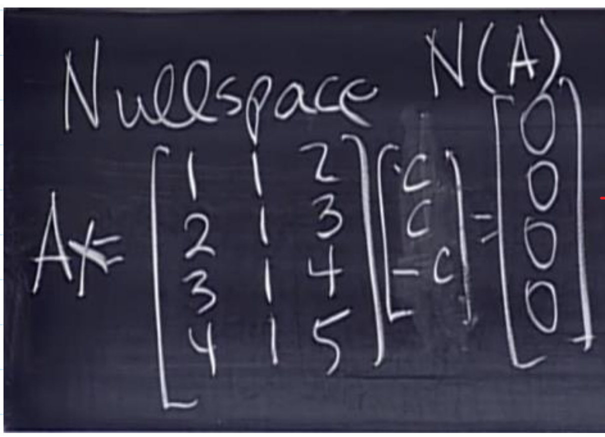
Note:
When b= 0, X =0, solves the equation. Thus has origin hence a subspace.

NULL SPACE

Nullspace of A = All solutions of X= $[x1,x2,x3]^T$ such that
AX = 0

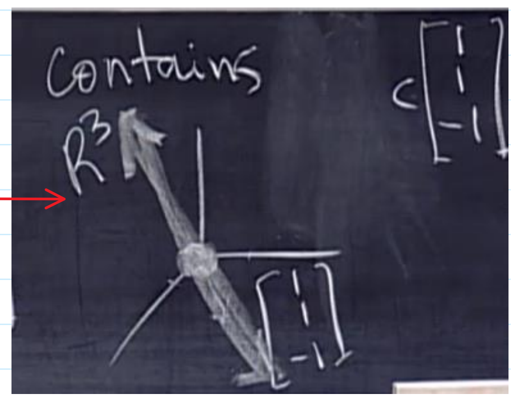
For a mxn matrix, C(A) is subspace of R^m while, N(A) is subspace of R^n

For the above example, N(A) is a line in R^3 passing via origin.

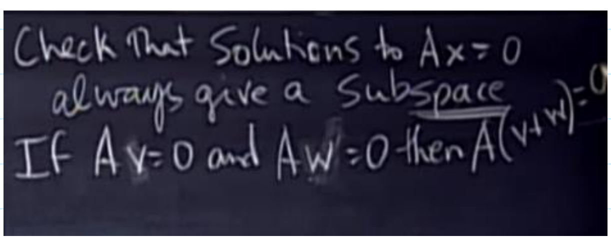


X = $[0,0,0]^T$;$[1,1,-1]^T$

So on ..



Check that the solutions of AX= 0 always gives a "Sub**<u>space"</u>**



<u>A(v+w) = A(v) + A(w) = 0 + 0 = 0</u>

<u>A(kv) = k*A(v) = 0</u> where k is a scalar.

Thus AX = 0 solution is a subspace

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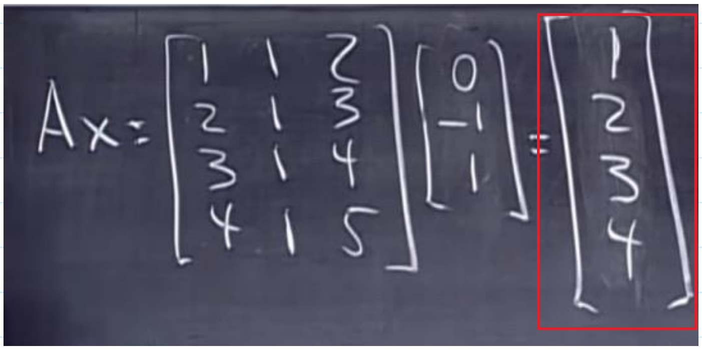
For any non-zero b, the solution X is not a subspace ($X = [0,0,0]^T$ is not a solution. In fact, X represents a line not passing through origin.



Methods to build a Subspace

1. From Columns Space - Using combinations of Columns of A
2. From Null Space - Satisfy the equation AX = 0 i.e., a system of equations that X has to satisfy.