

Permutations and Transposes

Monday, November 06, 2017 8:48 PM

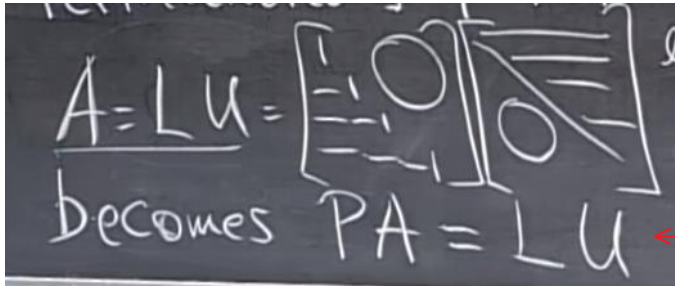
Topics:

- $PA=LU$
- Transposes
- Vector Spaces and Subspaces

Permutations (P)

P are those matrices that execute row exchanges.

We know that:



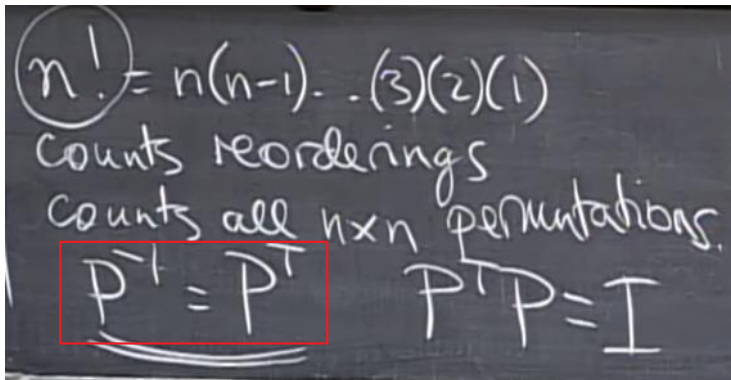
Handwritten chalkboard showing the equation $A=LU$ and its modification to $PA=LU$ with a permutation matrix P .

The operation $A = LU$ is only possible if there exists no row exchanges i.e., P is identity matrix.

Note: Software like MATLAB performs first scans for zero pivot elements and performs permutations. Also, for computation accuracy, permutations are performed if the pivot elements are very small.

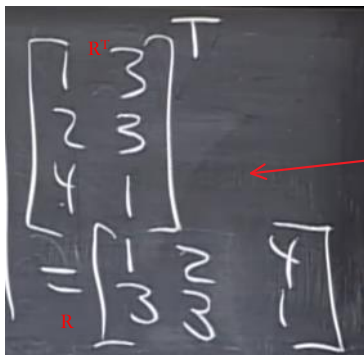
For any invertible A

P are those identity matrices with reordered rows and number of possibilities of P are $n!$ where n is the order of P. In addition, all the matrices are invertible.



Handwritten chalkboard showing the formula for $n!$ and the properties of permutation matrices P .

Transpose:



Handwritten chalkboard showing the transpose of a matrix R .

$$(A_{ij})^T = A_{ji}$$

Symmetric Matrix : $A = A^T$

$R^T * R$ is always an symmetric matrix.

Reason: Take transpose of $R^T * R$ i.e., $(R^T * R)^T = (R^T * R^{TT}) = (R^T * R)$

It is evident that both LHS and RHS are same, thus it is symmetric always.

Vector Spaces and Subspaces:

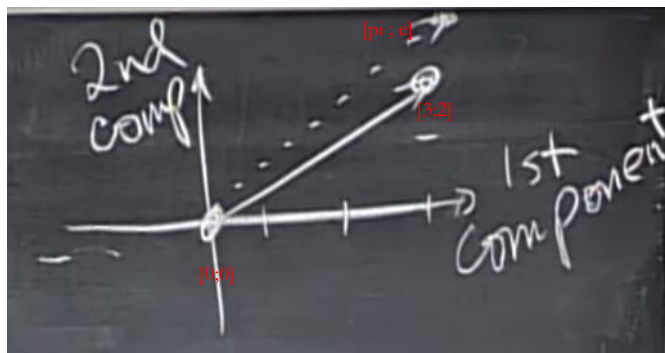
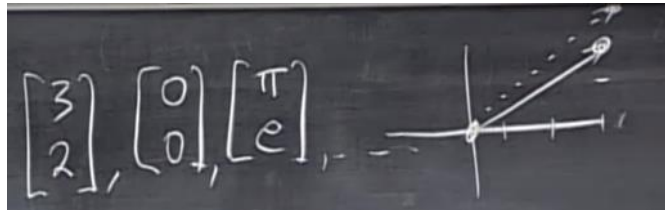
Spaces (Vector Spaces)

\mathbb{R}^2 = all 2-dim. ^{real} vectors $\begin{bmatrix} 3 \\ 2 \end{bmatrix}$, $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$, $\begin{bmatrix} \pi \\ e \end{bmatrix}$

\mathbb{R}^3 = all vectors with 3 components

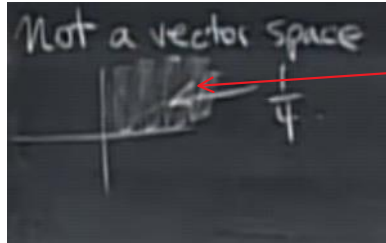
Similarly, \mathbb{R}^n represents all column vector with n-real components

Example:



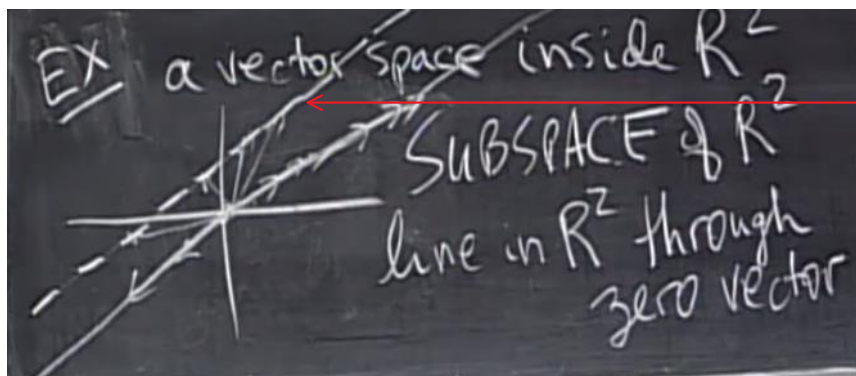
The whole plane is \mathbb{R}^2 i.e., "X-Y" plane

Vector space has to be closed under vector multiplication and addition i.e., any linear combinations.



Not a vector space for all real numbers especially multiplication with negative numbers. In other words it is not a closed vector space.

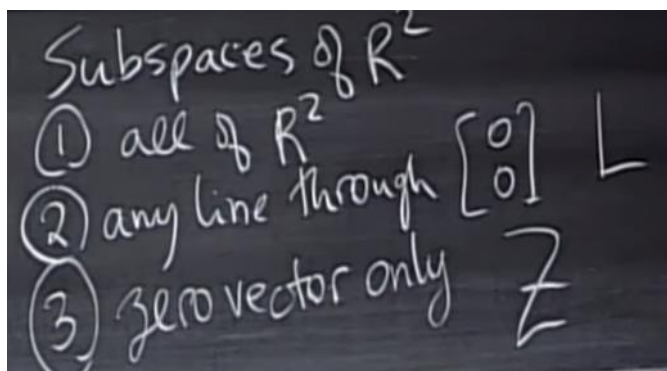
Subspace:



This is not a valid subspace since, any multiplication with say, zero is not a vector on that line.

This means every subspace should contain zero since multiplication with zero is zero vector.

Examples of Subspaces of \mathbb{R}^2



Vector Subspace for matrix A is all possible linear combinations (addition and multiplication) of its vectors. For example take A :

$$A = \begin{bmatrix} 1 & 3 \\ 2 & 3 \\ 4 & 1 \end{bmatrix}$$

columns in \mathbb{R}^3
 all their combinations
 form a subspace
 called column space $C(A)$

The column space CA for two vectors in A for \mathbb{R}^3 is the plane between Col1 and Col2 and passing via origin

