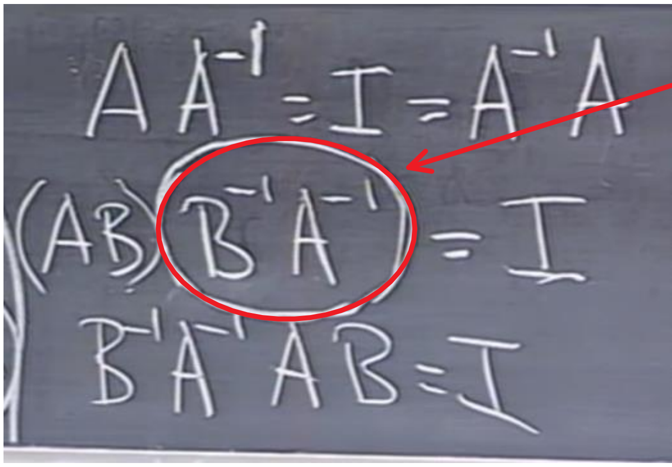
A = LU

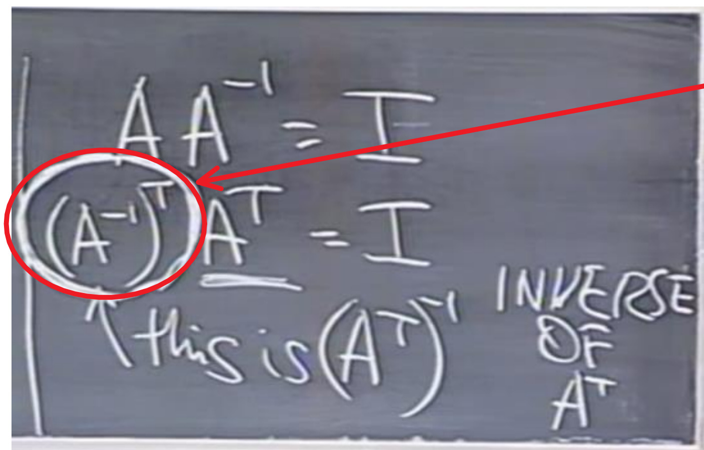
Thursday, November 02, 2017 8:24 PM

Topics:
- Inverse of AB and A^T
- Product of elimination matrices
- A = LU
- How Expensive is elimination operation for n x n matrix

Inverse of AB and A^T



Inverse of AB



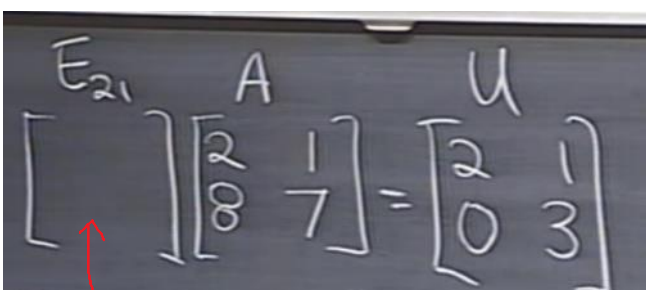
Inverse of A^T

For a matrix, transposing and inversing can be done either way

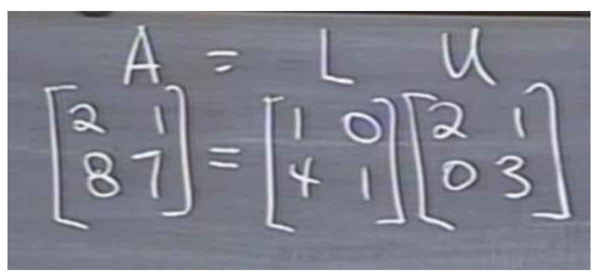
Note:
U = Upper Triangular Matrix
L = Lower Triangular Matrix

A = LU (No row exchanges)

Assuming that the matrix A doesn't require any row exchanges (non-zero pivots) in the initial step.

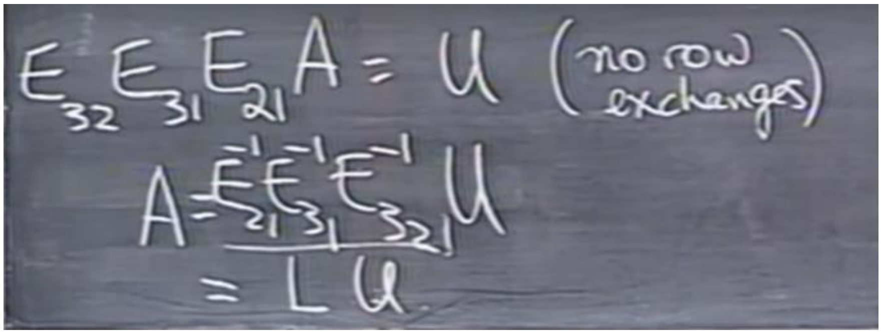


E_{21} =	1	0
	-4	1

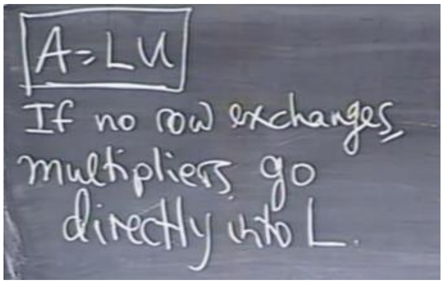


$(E_{21})^{-1} * E_{21} * A = (E_{21})^{-1} * U = L * U \Rightarrow L = E_{21}^{-1}$

3 x 3 Matrix



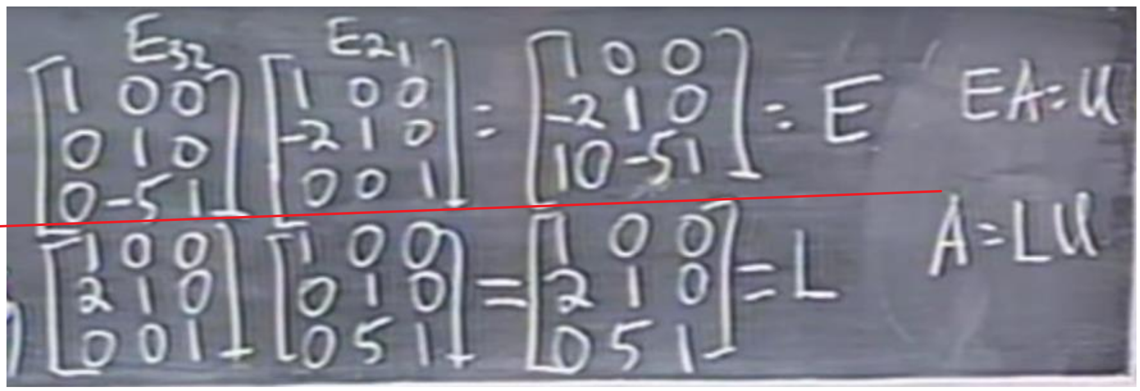
The preferred method of obtaining L is by product of individual E matrices rather than inverse of single E matrix because: L can be obtained by just keeping a record of all the multipliers of the individual elimination matrices.



Consider a matrix A such that it needs no row exchanges and E_{31} is identity matrix. Then,

Elimination matrices

Inverses:



The matrix E has elements that really do not help in obtaining L. Also, the matrix "A" can be forgotten as we get the first row of U in the first elimination step and also the multipliers and so on..

How many operation on n x n matrix A elimination?

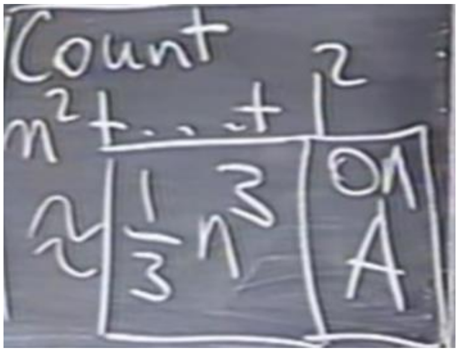
The operations are: Multiplication + Subtraction

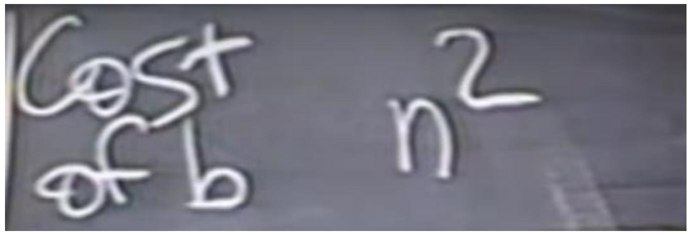
A size : 100 x 100

About 100^2 operations

About 99^2 operations

Approximately, number of operations is roughly equal to = $\mathbf{n^2 + (n-1)^2 + \ldots + 3^2 + 2^2 + 1^2}$





Note:

- 1/3 accounts for the reducing size of the elimination matrix
- Since elimination of A is more expensive than b, the RHS can be processed at a low cost separately.

Permutations: (when pivot elements are zero - row exchanges)