

Semester-IV Generic Elective (GE) Course -Mathematics

Any *one* of the following:

GE-4: Numerical Methods (with Practicals)

GE-4: Elements of Analysis

GE-4: Numerical Methods (with Practicals)

Total Marks: 150 (Theory: 75, Internal Assessment: 25, and Practical: 50)

Workload: 4 Lectures, 4 Practicals (per week) **Credits:** 6 (4+2)

Duration: 14 Weeks (56 Hrs. Theory + 56 Hrs. Practical) **Examination:** 3 Hrs.

Course Objectives: The goal of this paper is to acquaint students' various topics in Numerical Analysis such as solutions of nonlinear equations in one variable, interpolation and approximation, numerical differentiation and integration, direct methods for solving linear systems, numerical solution of ordinary differential equations using Computer Algebra System (CAS).

Course Learning Outcomes: After completion of this course, students will be able to:

- i) Find the consequences of finite precision and the inherent limits of numerical methods.
- ii) Appropriate numerical methods to solve algebraic and transcendental equations.
- iii) Solve first order initial value problems of ODE's numerically using Euler methods.

Unit 1: Errors and Roots of Transcendental and Polynomial Equations

Floating point representation and computer arithmetic, Significant digits; Errors: Roundoff error, Local truncation error, Global truncation error; Order of a method, Convergence and terminal conditions; Bisection method, Secant method, Regula-Falsi method, Newton-Raphson method.

Unit 2: Algebraic Linear Systems and Interpolation

Gaussian elimination method (with row pivoting), Gauss-Jordan method; Iterative methods: Jacobi method, Gauss-Seidel method; Interpolation: Lagrange form, Newton form, Finite difference operators, Gregory-Newton forward and backward difference interpolations, Piecewise polynomial interpolation (linear and quadratic).

Unit 3: Numerical Differentiation, Integration and ODE

Numerical differentiation: First and second order derivatives, Richardson extrapolation method; Numerical integration: Trapezoidal rule, Simpson's rule; Ordinary differential equation: Euler's method, Modified Euler's methods (Heun's and midpoint).

References:

1. Chapra, Steven C. (2018). *Applied Numerical Methods with MATLAB for Engineers and Scientists* (4th ed.). McGraw-Hill Education.
2. Fausett, Laurene V. (2009). *Applied Numerical Analysis Using MATLAB*. Pearson. India.
3. Jain, M. K., Iyengar, S. R. K., & Jain R. K. (2012). *Numerical Methods for Scientific and Engineering Computation* (6th ed.). New Age International Publishers. Delhi.

Additional Reading:

- i. Bradie, Brian (2006). *A Friendly Introduction to Numerical Analysis*. Pearson Education India. Dorling Kindersley (India) Pvt. Ltd. Third Impression, 2011.

Practical /Lab work to be performed in the Computer Lab:

Use of Computer Algebra System (CAS), for example MATLAB/Mathematica/Maple/Maxima/Scilab etc., for developing the following Numerical Programs:

1. Bisection method
2. Secant method and Regula–Falsi method
3. Newton–Raphson method
4. Gaussian elimination method and Gauss–Jordan method
5. Jacobi method and Gauss–Seidel method
6. Lagrange interpolation and Newton interpolation
7. Trapezoidal and Simpson’s rule.
8. Euler methods for solving first order initial value problems of ODE’s.

Teaching Plan (Theory of GE-4: Numerical Methods):

Weeks 1 and 2: Floating point representation and computer arithmetic, Significant digits; Errors: Roundoff error, Local truncation error, Global truncation error; Order of a method, Convergence and terminal conditions.

[2] Chapter 1 (Sections 1.2.3, 1.3.1, and 1.3.2).

[3] Chapter 1 (Sections 1.2, and 1.3).

Week 3 and 4: Bisection method, Secant method, Regula–Falsi method, Newton–Raphson method.

[2] Chapter 2 (Sections 2.1 to 2.3).

[3] Chapter 2 (Sections 2.2 and 2.3).

Week 5: Gaussian elimination method (with row pivoting), Gauss–Jordan method;

Iterative methods: Jacobi method, Gauss–Seidel method.

[2] Chapter 3 (Sections 3.1, and 3.2), Chapter 6 (Sections 6.1, and 6.2).

[3] Chapter 3 (Sections 3.2, and 3.4).

Week 6: Interpolation: Lagrange form, and Newton form.

[2] Chapter 8 (Section 8.1). [3] (Section 4.2).

Weeks 7 and 8: Finite difference operators, Gregory–Newton forward and backward difference interpolations.

[3] Chapter 4 (Sections 4.3, and 4.4).

Week 9: Piecewise polynomial interpolation: Linear, and quadratic.

[2] Chapter 8 [Section 8.3 (8.3.1, and 8.3.2)].

[1] Chapter 18 (Sections 18.1 to 18.3).

Weeks 10 and 11: Numerical differentiation: First and second order derivatives, Richardson extrapolation method.

[2] Chapter 11 [Sections 11.1 (11.1.1, 11.1.2 and 11.1.4)]

Weeks 12 and 13: Numerical integration: Trapezoidal rule, Simpson’s rule; Ordinary differential equations: Euler’s method.

[2] Chapter 11 [Section 11.2 (11.2.1, 11.2.2)].

[1] Chapter 22 (Sections 22.1, and 22.2 (up to Page 583)

Weeks 14: Modified Euler’s methods: Heun’s method, The midpoint method.

[1] Chapter 22 (Section 22.3)

Facilitating the Achievement of Course Learning Outcomes

Unit No.	Course Learning Outcomes	Teaching and Learning Activity	Assessment Tasks
1.	Find the consequences of finite precision and the inherent limits of numerical methods.	(i) Each topic to be explained with illustrations.	<ul style="list-style-type: none">• Presentations and class discussions.• Assignments and class tests.• Student presentations.• Mid-term examinations.• Practical and viva-voce examinations.• End-term examinations.
2.	Appropriate numerical methods to solve algebraic and transcendental equations.	(ii) Students to be encouraged to discover the relevant concepts.	
3.	Solve first order initial value problems of ordinary differential equations numerically using Euler methods.	(iii) Students to be given homework/assignments. (iv) Discuss and solve the theoretical and practical problems in the class. (v) Students to be encouraged to apply concepts to real world problems.	

Keywords: Bisection method, Secant method, Regula–Falsi method, Newton–Raphson method, Gauss–Seidel method, Piecewise polynomial interpolation, Richardson extrapolation method, Simpson’s rule.