Computational Physics – PHYS 241, Assignment 2

Prashant Singh, A53222484

Problem 1 (30 points):

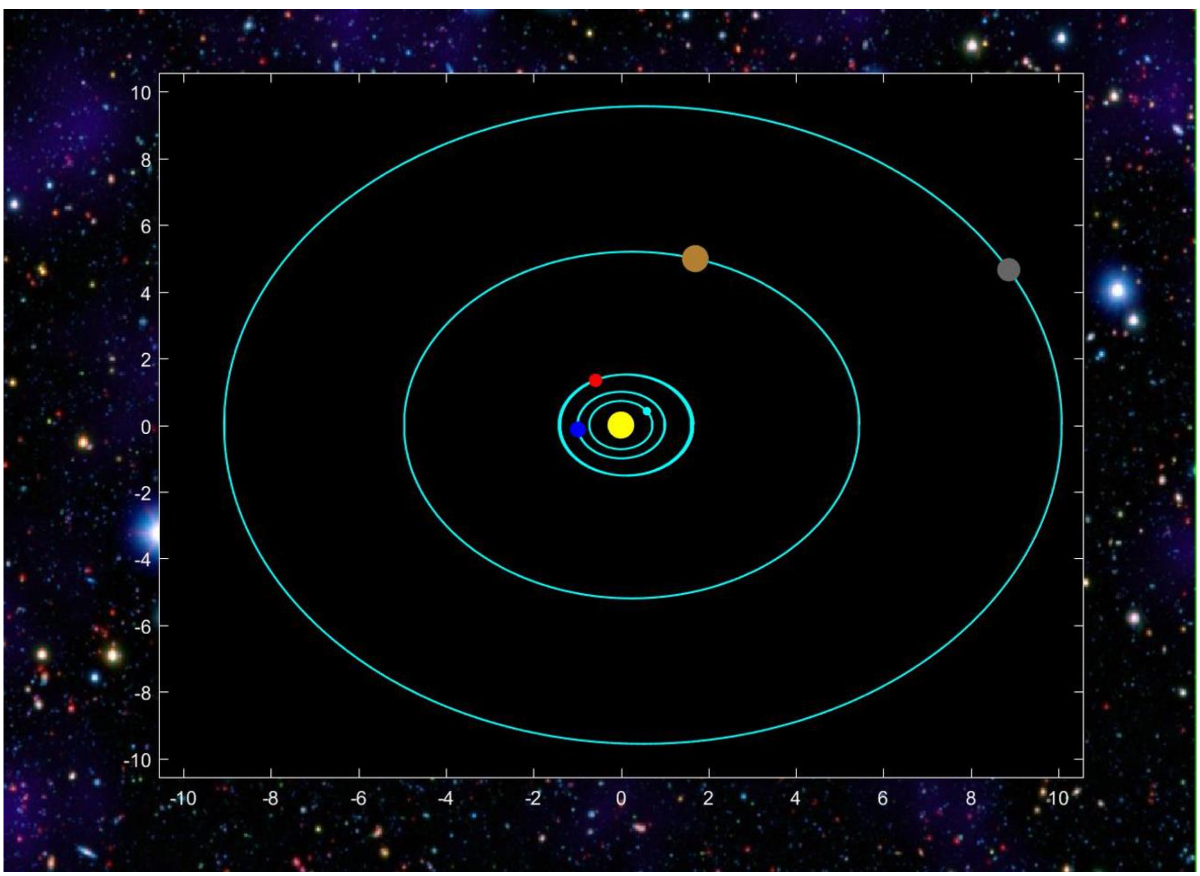
(a) Run your leapfrog code for Venus, Earth, Mars, Jupiter, Saturn, orbiting around the Sun over 50 Earth years. Put the Sun fixed at the origin of your coordinate system. Animate the revolving planets. You can use any platform for animation, like MATLAB based procedures.

(b) Make a stand-alone MPEG movie of the animation, or use some other equivalent media, like AVI.

Solution:

For this problem, I extended our C language leapfrog code from assignment 1 to calculate the orbits for five planets viz. Venus, Earth, Mars, Jupiter & Saturn in range of over 50 earth years. We retained the corresponding eccentricities & semi-major axis values of the planets to achieve the elliptical orbits.

For animation, I compiled all the simulation data of the orbits of 5 planets and compiled them into one animation program in MATLAB, which drew all the five orbits simultaneously. I also tried to scale planet sizes to some level (however, not to scale!) with Sun in the middle. Please find below the screenshot from the simulation video (***Revolving_Planets.mp4***):



Problem 2 (30 bonus points for PHY 141 and PHY 241):

- Based on reading material of the two links calculate the transfer orbit of the NASA Odyssey spacecraft to Mars.
- Sketch and outline the procedure and explain the transfer orbit.
- Provide a full simulation of the voyage and show that it requires the minimal amount of fuel.

Solution:

For this problem, I went through the reading material and comprehend the concept & physics behind the interplanetary trajectories & orbital transfer. This problem turned out to be quite challenging as well as highly interesting. It's fascinating how basic concepts of Physics can be applied on such large scale as to accomplish a challenging task of sending a space probe to Mars. Moreover, the fact that Orbital energy of Earth is used to give the required push to the probe and then just thrust it with additional velocity of 3.0Km/s is really intriguing as well as amazing.

To outline the procedure, we have planet Earth with it's almost circular orbit and planet Mars with its elliptic orbit. The physical facts which needs to be considered here are as follows:

1. Earth orbital radius = 1 AU
2. Mars orbital radius = 1.524 AU
3. Earth's orbital period = 1 Year
4. Mars orbital period = 1.88 Years

Now, we understand from the orbital transfer concepts, for space probe to reach Mars, we need to launch it at *Earth's perihelion* i.e. Earth's orbital radius of 1AU. Also, we need to make sure that it's the *probe's aphelion* where the probe will *meet* Mars and get into its gravitational pull. To make sure this happens we need to calculate few important factors. First, let's calculate the probe's elliptical orbit parameters:

a) $r_{perihelion}$= Earth orbital radius = 1 AU
b) $r_{aphelion}$= Mars orbital radius = 1.524 AU
c) a = **semimajor axis** of probe orbit = $(r_{perihelion}+ r_{aphelion})/2$ = (1 + 1.524)/2 = 1.262 AU
d) The eccentricity of the transfer orbit = e = 1 - $r_{perihelion}/a$ = 1 - 1/1.262 = 0.208
e) Kepler's third law is used to determine the probe's orbital period, P => $P^2 = a^3$
Probe's orbital period = $(1.262)^{3/2}$ = 1.418 years = **518 days**
f) Time of flight = Half of its orbital period = 518/2 = 259 days = **0.709 years**.
g) The degrees by which Mars would have moved during the probe's orbit = (360/1.88) * 0.709 = 136 degrees.
h) Therefore, the angular separation between Earth & Mars is = (180-136) = **44 degrees**. Therefore, we need to make sure that Mars angular separation between them is 44 degrees while calculating the orbital velocities of Mars. So Mars elliptical orbital position is calculated as, for x-component Px = Mars Orbital radius * cos(44*(pi/180)), Py = Mars Orbital radius * sin(44*(pi/180)). For orbital velocity also , we take into consideration the same factor to accommodate this 44 degrees separation.
i) For Probe's orbital parameters, we keep its position co-ordinates equal to Earth's as we launch it from Earth's perihelion co-ordinates.

j) For probe's orbital velocity, we need to give the probe an additional velocity of 3.0Km/s to compensate for the orbital energies difference.

k) Probe's circular velocity = $2\pi a/P$ = 2*(3.1416) *(1.262AU)/ (1.418 years) = **5.59 AU/yr**

l) Probe's perihelion velocity is = $v_{perihelion} = v_{circular}[(1+e)/(1-e)]^{1/2}$ = 5.59*4.73*[(1+0.208)/ (1-0.208)]$^{1/2}$ = **32.7 km/s** (conversion constant is 4.73 km/s per AU/yr)

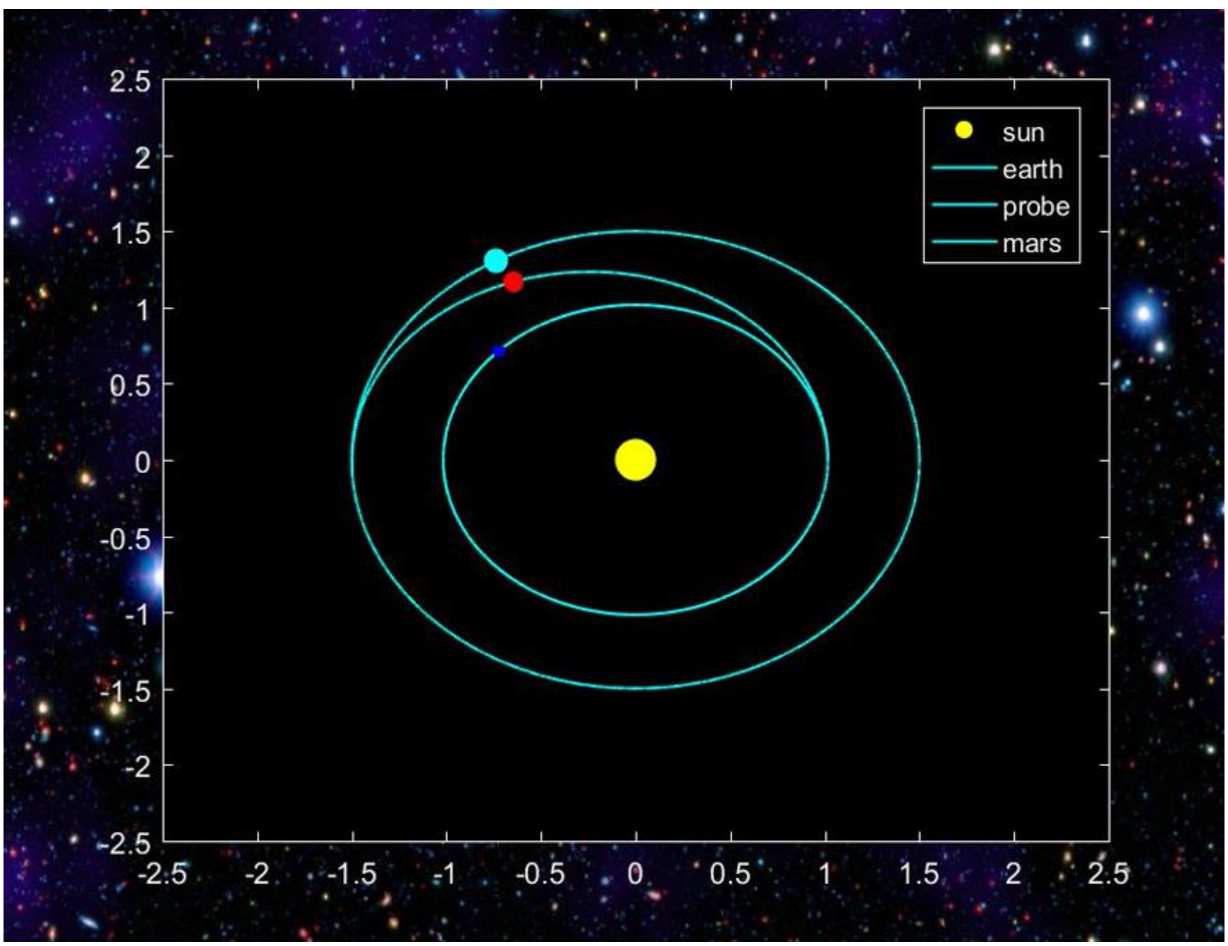
m) Earth's orbital velocity is = 29.7 km/s. Therefore, the probe's orbital velocity must be increased by 3Km/s, to give it the required push apart from the earth's orbital speed.

n) Therefore, in Code we need to set the probe's orbital velocity (y-component) with an increment of this value to the Earth's orbital velocity(y-component). This increment is calculated as: 3.0/4.73 = **0.63424 AU/Yr**.

Now, we can calculate the probe's orbit by restricting the time step integration to 0.709 years. For Earth & Mars, we will keep the time step integration as same as we did for calculating its orbit, i.e. 1 for Earth and 1.88 for Mars.

This was we calculated the probe's trajectory & thus simulated its orbital transfer from Earth to Mars in time-period equivalent to half of Mars orbital period.

For more details, on the calculations & how the same was coded, the C program file is attached in the zip. To illustrate the animation for the simulation, please find below the snapshot from the video:



Problem 3:

In this problem, we will study a simple model for the stability of the rings of Saturn. You can investigate mass ratios that lead to stable ring systems vs. unstable ones. If the mass of each ring body is no more than 2.3 times the mass of Saturn divided by the cube of the number of ring particles, then the system can be expected to be stable; otherwise not.

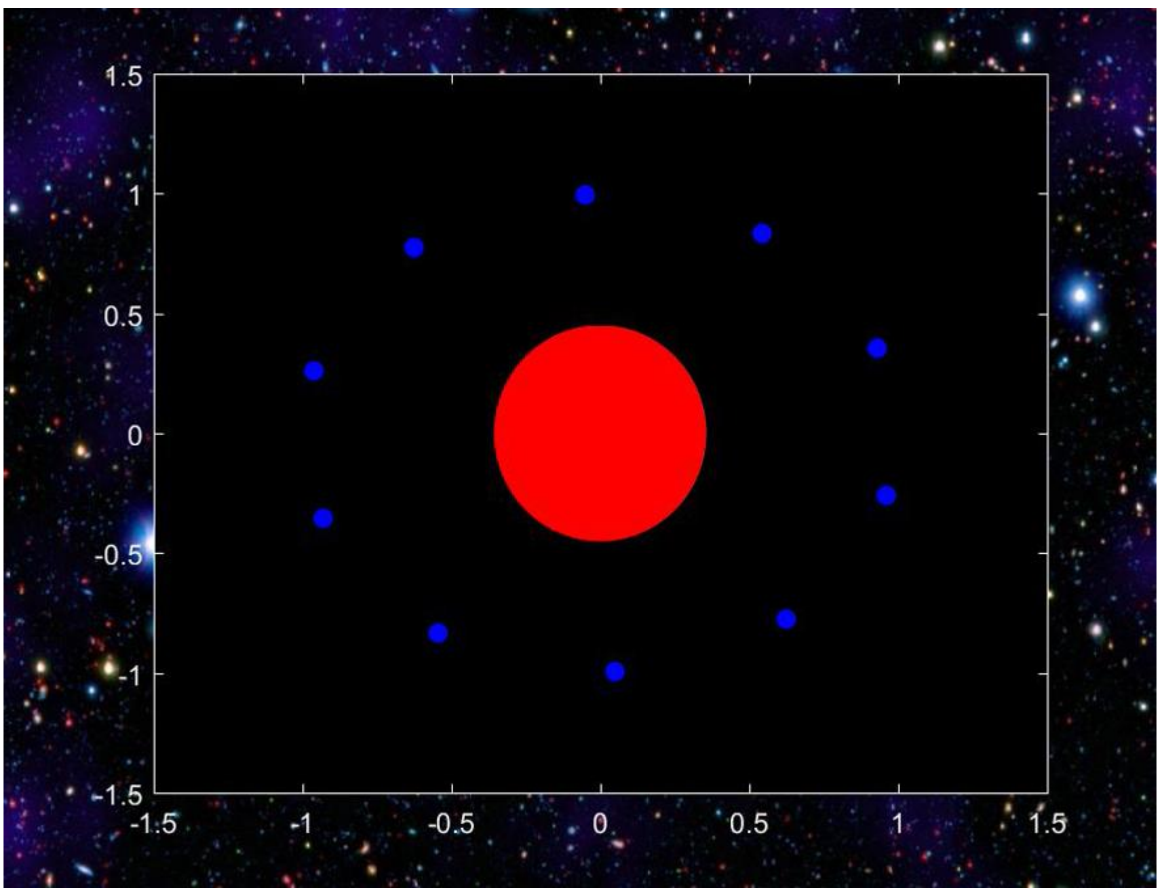
Solution:

For this problem, I worked on my own code where-in I solved it as n-body problem. I take 10 bodies revolving around the Saturn for two cases: first, being in circular orbit & the second case being the elliptical orbits.

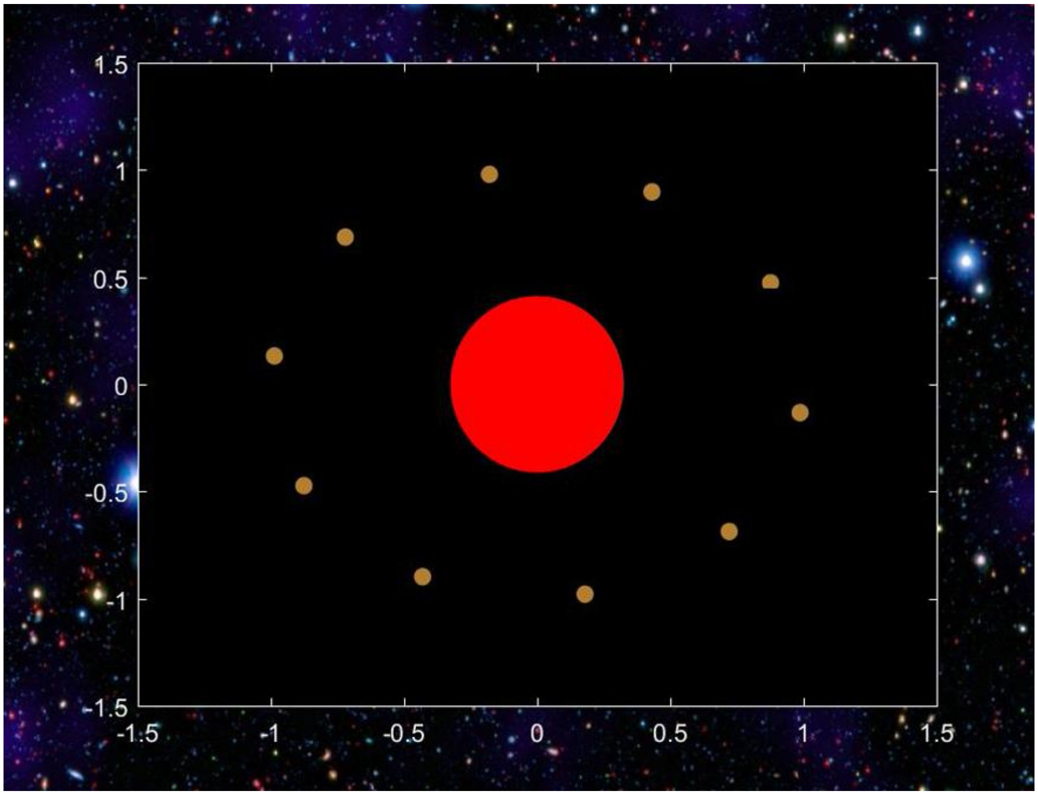
a) For stable rings, we have considered the fact that gamma value remains below 2.3. For our case, we have used the value of gamma as 2.0. This results in rings around Saturn as stable. Where, Gamma = (Mass_body * n^3)/Mass_Saturn, where n =number of bodies forming the ring.
b) For unstable orbit, we have set gamma as 100. This is due to as described in the problem statement, that this shows the “instability” very quickly.

Please find below the snapshot from videos for the stable & unstable ring around Saturn:

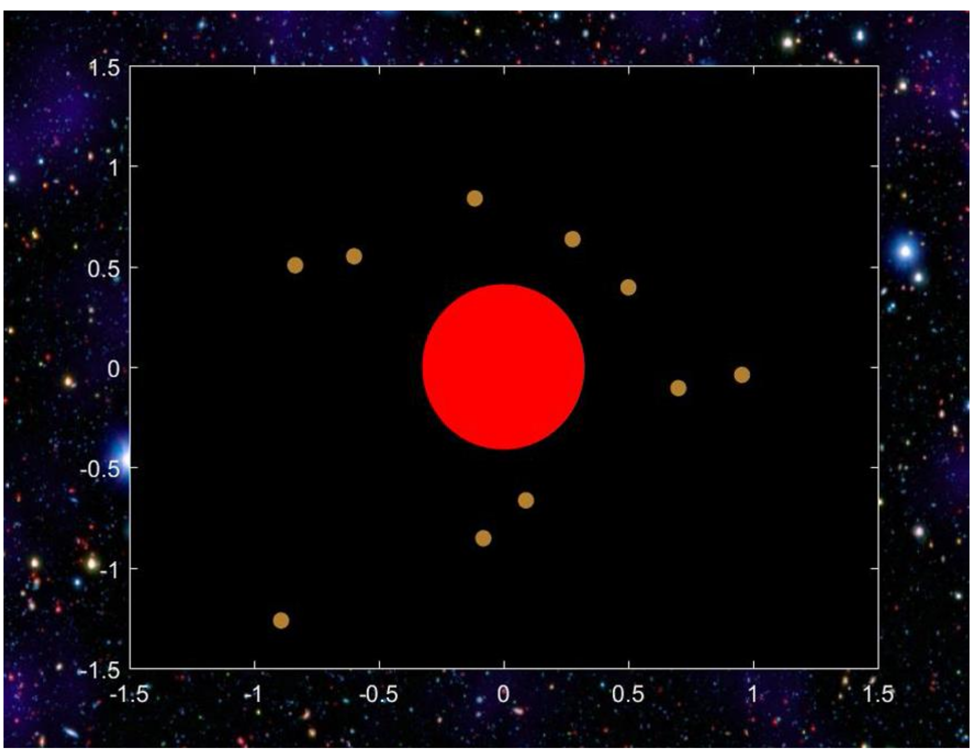
1. **<u>Saturn Stable Ring (Circular Orbit):</u>**



2. Saturn Stable Ring (Elliptical Orbit):



3. Saturn Unstable Ring:



Online links to the uploaded animation videos for the problems (These videos are in the zip file too):

- Revolving Planets - https://www.dropbox.com/s/55nitx24yzjzpqs/Revolving_Planets.avi?dl=0
- Space Probe Orbital Transfer – https://www.dropbox.com/s/xny5w6mp8dag082/Space_Probe.avi?dl=0
- Saturn Stable Rings (Circular) - https://www.dropbox.com/s/fj5mfofyp3mznui/saturn_stable_circular.avi?dl=0
- Saturn Stable Rings (Elliptical) - https://www.dropbox.com/s/ug6albhj0hhzswj/Saturn_StableRings_Elliptic.avi?dl=0
- Saturn Unstable Rings - https://www.dropbox.com/s/e29e07wjn8iwhwu/Saturn_UnStableRings.avi?dl=0