INDUSTRIAL ENGINEERING
FALL 2021

DECISION MODELS
IE 2086
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FINANCIAL HEDGING USING STOCHASTIC SIMULATION

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Background

When a U.S. company does business around the world, it has to deal with foreign exchange risk, that is, the risk that a foreign currency loses value against the dollar. For example, a company whose operations are in the U.S. (and thus pays for R&D in USD) would be hurt if it held German marks (DM) and the German mark loses value.

Suppose the current exchange rate is $0.6513/DM and the company has 100 million DM. This translates to $65.13 million. If the DM loses value (i.e., the USD strengthens), then the exchange rate may be $0.6413, and the company has lost $1 million

How do we fix this?

A put option is a special contract designed to mitigate (hedge) such risk.

This is how it works: You pay a cost of c for a "put option" with strike k. The put option gives you the option, hence the name, to convert your DM next year at an exchange rate of k (this is called exercising the option).

The put option mechanism

If next year's exchange rate D falls below k, then your option is valuable because it allows you to "bring up" the exchange rate to k. We have to subtract the cost, so the option gives you the net value of k - D - c. If D ≥ k, then our option is worthless and it has a net value of -c. Putting this together, we have the following:

$$\boxed{\textbf{Net Payoff} = \max(k-D\,,\,0) - c}$$

Mechanism in action

D is a random variable (next year's rate), but k and c are known (characteristics of the contract known this year). Suppose k = 0.62 and c = 0.0137.

Let us assume D = 0.5 (i.e., rate falls a lot). Then the Net Payoff = 0.12 – 0.0137 = 0.1063.
If D = 0.61 (i.e., rate falls a little). Then Net Payoff = 0.01 – 0.0137 = -0.0037.

In both cases, we exercise the option, but in the second case, the value of the option was not enough to offset c, the cost of the contract.

Let EG be the revenue generated next year in Germany. Here are two formulas:

$$\boxed{\textbf{Unhedged USD Revenue} = E_G * D}$$

$$\boxed{\begin{array}{c}\textbf{Hedged USD Revenue} = \textbf{Unhedged USD Revenue} +\\ \textbf{Number of Options} * \textbf{Net Payoff}\end{array}}$$

About the Project

In this project we'll consider a company that generates revenue E_G = 643 million DM next year in Germany and E_B = 272 million pounds next year in Great Britain. Using the current exchange rates of 0.6531 for DM and 1.234 for the British pound, the total revenue is around 756 million USD. We will denote next year's exchange rates by D for the DM and B for the British pound.

Below are the determination formulas where $R_D \sim N(0,9^2)$ and $R_B \sim N(0,11^2)$ and correlated with $\rho = 0.675$. Notice that R_D AND R_B are percentage change from 0.6531 and 1.234 (this year's rates) .

$$\mathbf{D = 0.6531 * (1 + R_D/100)}$$
$$\mathbf{B = 1.234 * (1 + R_B/100)}$$

We consider 9 possible put options on the German mark (k_1) and British pound (k_2) with respective costs (c_1, c_2) as shown below

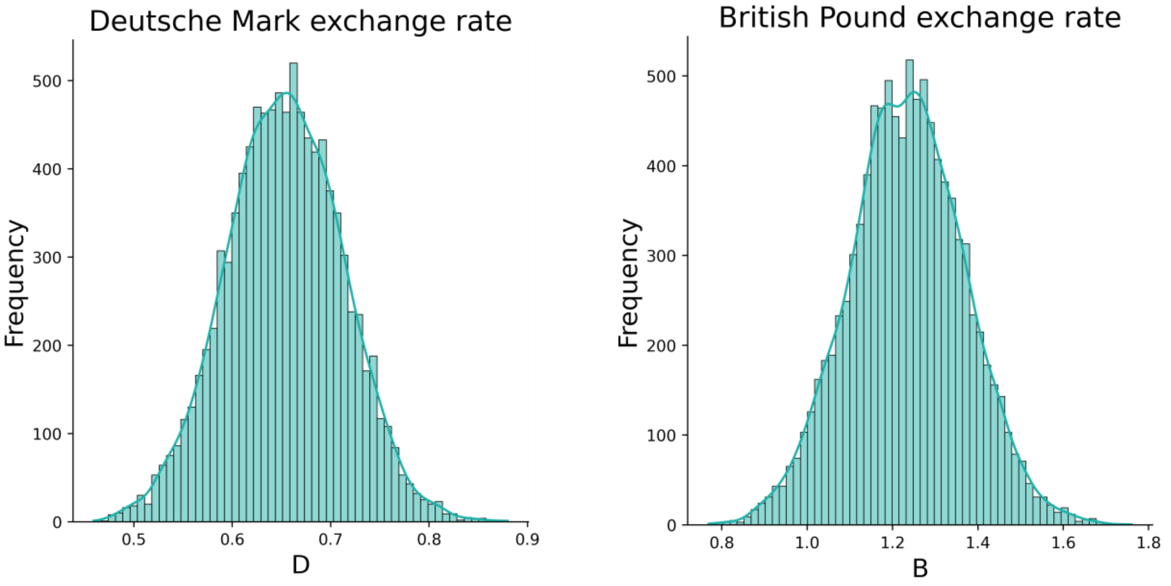
K_1	C_1	K_2	C_2
0.66	0.0858	1.3	0.1372
0.65	0.0322	1.25	0.0826
0.64	0.0208	1.2	0.0451
0.63	0.0170	1.15	0.0283
0.62	0.0137	1.1	0.0161
0.61	0.0108	1.05	0.0079
0.60	0.0084	1.00	0.0033
0.59	0.0063	0.95	0.0011
0.55	0.0014	0.90	0.0002

Problem statement

We assume that the company will buy 500 million put options on DM and 500 million put options on the British pound. We are interested in which pair of k's and c's should the company buy. We approach this by determining the combination that maximized the estimated probability that the hedge revenue will be greater than 706 million USD.

Generating 10000 samples of D and B from R_D AND R_B

	RD	RB	D	B
0	-15.7479	-13.702	0.550251	1.064917
1	3.084124	7.765856	0.673242	1.329831
2	10.37732	-0.03601	0.720874	1.233556
.	.	.	.	.
9997	-4.04101	-15.0907	0.626708	1.04778
9998	6.594543	9.709754	0.696169	1.353818
9999	-1.89163	-3.55306	0.640746	1.190155

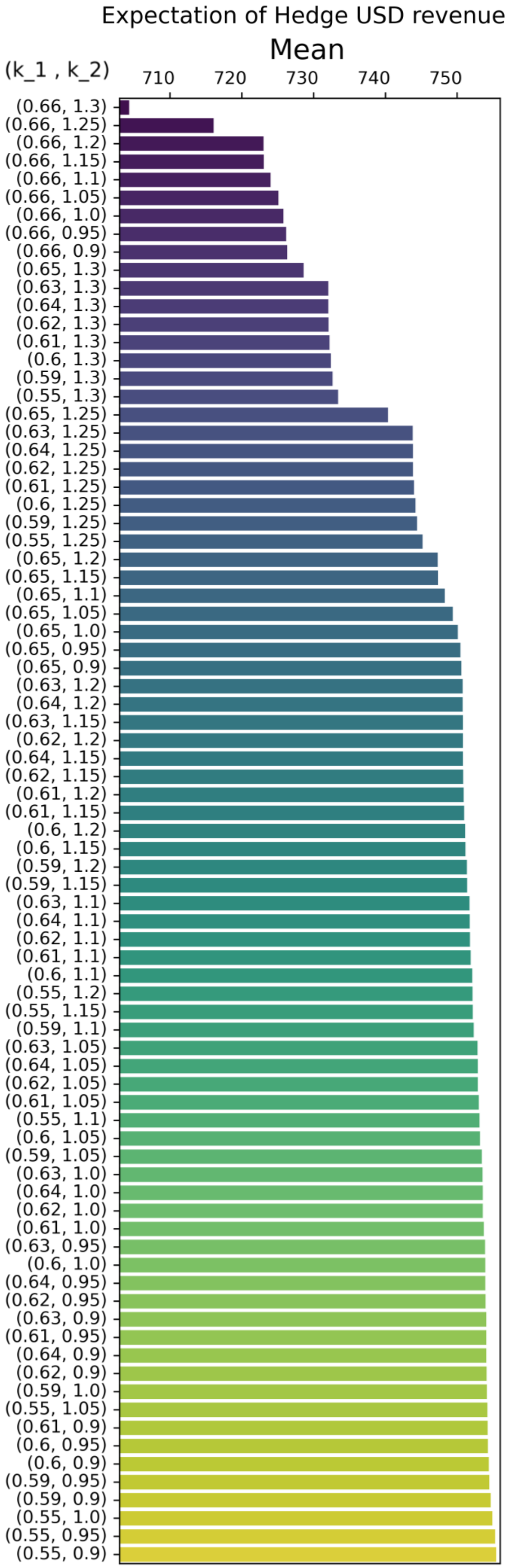


Hedge USD revenue for different combinations of (k_1,k_2) pairs

	(0.66, 1.3)	(0.66, 1.25)	.	(0.55, 0.95)	(0.55, 0.9)
0	704.3508	706.6348	.	642.2009	642.6454
1	683.0748	710.3588	.	793.3413	793.7858
2	720.7375	723.0215	.	797.7818	798.2263
3	668.7791	684.1297	.	756.2432	756.6877
.	.	.	.	.	.
9996	691.1375	693.4215	.	746.5813	747.0258
9997	719.1914	721.4754	.	686.7021	687.1466
9998	704.3412	731.6252	.	814.6077	815.0522
9999	688.7372	691.0212	.	734.4543	734.8988

- We have assumed 9 different cost and strike pairs of put options for each of the currencies. This boils down to 81 different experiments of calculating the hedge USD revenue.
- From the plot shown the lowest expectation of Hedge USD revenue is when the strikes for Deutsche Mark (k_1) and the British Pound (k_2) are k_1 = 0.66 and k_2 = 1.3 while the highest are k_1 = 0.55 and k_2 = 0.9 respectively.
- With that k_1 and k_2 values the costs corresponding to them are c_1 = 0.0858 and c_2 = 0.1372 for the lowest and c_1 = 0.0014 and c_2 = 0.0002 for the highest.

	High	Low
K_1	0.55	0.66
C_1	0.0014	0.0858
K_2	0.9	1.3
C_2	0.0002	0.1372
Expectation	**755.423**	**704.326**



Strikes (k_1,k_2) with the probability that Hedged USD Revenue will be greater than 706 million USD

(k_1,k_2)	Probabilities
(0.66, 1.3)	0.4
(0.66, 1.25)	0.5319
.	.
(0.55, 0.95)	0.7628
(0.55, 0.9)	0.7649

- The pairs of k_1 and k_2 for the highest and lowest probability of the hedge USD revenue greater than 706 million are the same as the pairs that produced the highest mean values.

	High	Low
K_1	0.55	0.66
C_1	0.0014	0.0858
K_2	0.9	1.3
C_2	0.0002	0.1372
Probability	**755.423**	**704.326**