

INDUSTRIAL ENGINEERING
FALL 2021

DECISION MODELS
IE 2086
PROF. DANIEL JIANG

FINANCIAL HEDGING USING STOCHASTIC SIMULATION

SUBMITTED BY
SABELLA PRASANNA
4450873
prs98@pitt.edu

Background

When a U.S. company does business around the world, it has to deal with foreign exchange risk, that is, the risk that a foreign currency loses value against the dollar. For example, a company whose operations are in the U.S. (and thus pays for R&D in USD) would be hurt if it held German marks (DM) and the German mark loses value.

Suppose the current exchange rate is \$0.6513/DM and the company has 100 million DM. This translates to \$65.13 million. If the DM loses value (i.e., the USD strengthens), then the exchange rate may be \$0.6413, and the company has lost \$1 million.

How do we fix this?

A put option is a special contract designed to mitigate (hedge) such risk.

This is how it works: You pay a cost of c for a “put option” with strike k . The put option gives you the option, hence the name, to convert your DM next year at an exchange rate of k (this is called exercising the option).

The put option mechanism

If next year's exchange rate D falls below k , then your option is valuable because it allows you to “bring up” the exchange rate to k . We have to subtract the cost, so the option gives you the net value of $k - D - c$. If $D \geq k$, then our option is worthless and it has a net value of $-c$. Putting this together, we have the following:

$$\text{Net Payoff} = \max(k - D, 0) - c$$

Mechanism in action

D is a random variable (next year's rate), but k and c are known (characteristics of the contract known this year). Suppose $k = 0.62$ and $c = 0.0137$.

Let us assume $D = 0.5$ (i.e., rate falls a lot). Then the Net Payoff = $0.12 - 0.0137 = 0.1063$.

If $D = 0.61$ (i.e., rate falls a little). Then Net Payoff = $0.01 - 0.0137 = -0.0037$.

In both cases, we exercise the option, but in the second case, the value of the option was not enough to offset c , the cost of the contract.

Let E_G be the revenue generated next year in Germany. Here are two formulas:

$$\text{Unhedged USD Revenue} = E_G * D$$

$$\text{Hedged USD Revenue} = \text{Unhedged USD Revenue} + \text{Number of Options} * \text{Net Payoff}$$

About the Project

In this project we'll consider a company that generates revenue $E_G = 643$ million DM next year in Germany and $E_B = 272$ million pounds next year in Great Britain. Using the current exchange rates of 0.6531 for DM and 1.234 for the British pound, the total revenue is around 756 million USD. We will denote next year's exchange rates by D for the DM and B for the British pound.

Below are the determination formulas where $R_D \sim N(0,9^2)$ and $R_B \sim N(0,11^2)$ and correlated with $\rho = 0.675$. Notice that R_D AND R_B are percentage change from 0.6531 and 1.234 (this year's rates) .

$$\begin{aligned} D &= 0.6531 * (1 + R_D/100) \\ B &= 1.234 * (1 + R_B/100) \end{aligned}$$

We consider 9 possible put options on the German mark (k_1) and British pound (k_2) with respective costs (c_1, c_2) as shown below

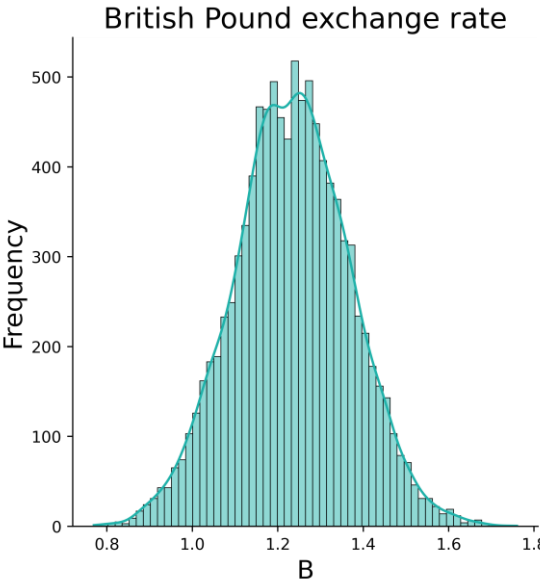
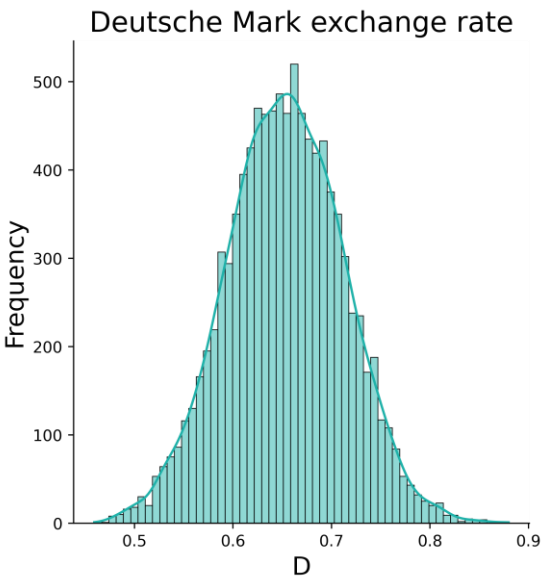
| K_1 | C_1 | K_2 | C_2 |
|-------|--------|-------|--------|
| 0.66 | 0.0858 | 1.3 | 0.1372 |
| 0.65 | 0.0322 | 1.25 | 0.0826 |
| 0.64 | 0.0208 | 1.2 | 0.0451 |
| 0.63 | 0.0170 | 1.15 | 0.0283 |
| 0.62 | 0.0137 | 1.1 | 0.0161 |
| 0.61 | 0.0108 | 1.05 | 0.0079 |
| 0.60 | 0.0084 | 1.00 | 0.0033 |
| 0.59 | 0.0063 | 0.95 | 0.0011 |
| 0.55 | 0.0014 | 0.90 | 0.0002 |

Problem statement

We assume that the company will buy 500 million put options on DM and 500 million put options on the British pound. We are interested in which pair of k 's and c 's should the company buy. We approach this by determining the combination that maximized the estimated probability that the hedge revenue will be greater than 706 million USD.

Generating 10000 samples of D and B from R_D AND R_B

| | RD | RB | D | B |
|------|----------|----------|----------|----------|
| 0 | -15.7479 | -13.702 | 0.550251 | 1.064917 |
| 1 | 3.084124 | 7.765856 | 0.673242 | 1.329831 |
| 2 | 10.37732 | -0.03601 | 0.720874 | 1.233556 |
| . | . | . | . | . |
| 9997 | -4.04101 | -15.0907 | 0.626708 | 1.04778 |
| 9998 | 6.594543 | 9.709754 | 0.696169 | 1.353818 |
| 9999 | -1.89163 | -3.55306 | 0.640746 | 1.190155 |



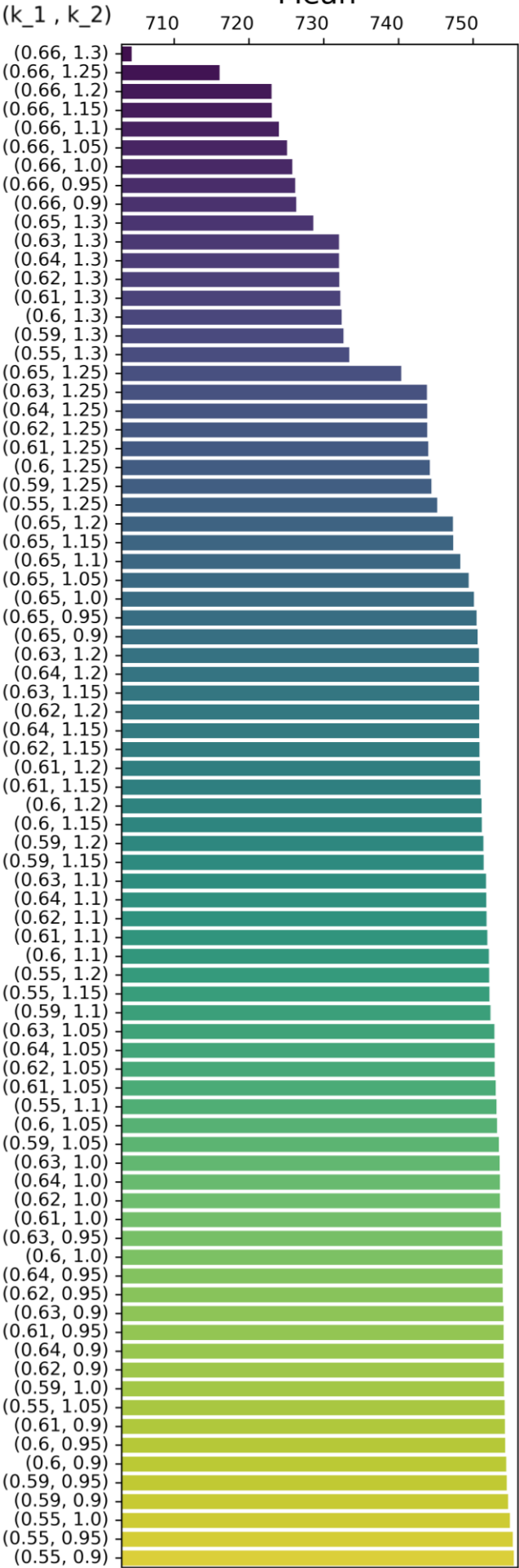
Hedge USD revenue for different combinations of (k₁,k₂) pairs

| | (0.66, 1.3) | (0.66, 1.25) | . | (0.55, 0.95) | (0.55, 0.9) |
|------|-------------|--------------|---|--------------|-------------|
| 0 | 704.3508 | 706.6348 | . | 642.2009 | 642.6454 |
| 1 | 683.0748 | 710.3588 | . | 793.3413 | 793.7858 |
| 2 | 720.7375 | 723.0215 | . | 797.7818 | 798.2263 |
| 3 | 668.7791 | 684.1297 | . | 756.2432 | 756.6877 |
| . | . | . | . | . | . |
| 9996 | 691.1375 | 693.4215 | . | 746.5813 | 747.0258 |
| 9997 | 719.1914 | 721.4754 | . | 686.7021 | 687.1466 |
| 9998 | 704.3412 | 731.6252 | . | 814.6077 | 815.0522 |
| 9999 | 688.7372 | 691.0212 | . | 734.4543 | 734.8988 |

Expectation of Hedge USD revenue
Mean

- We have assumed 9 different cost and strike pairs of put options for each of the currencies. This boils down to 81 different experiments of calculating the hedge USD revenue.
- From the plot shown the lowest expectation of Hedge USD revenue is when the strikes for Deutsche Mark (k_1) and the British Pound (k_2) are $k_1 = 0.66$ and $k_2 = 1.3$ while the highest are $k_1 = 0.55$ and $k_2 = 0.9$ respectively.
- With that k_1 and k_2 values the costs corresponding to them are $c_1 = 0.0858$ and $c_2 = 0.1372$ for the lowest and $c_1 = 0.0014$ and $c_2 = 0.0002$ for the highest.

| | High | Low |
|-------------|---------|---------|
| K_1 | 0.55 | 0.66 |
| C_1 | 0.0014 | 0.0858 |
| K_2 | 0.9 | 1.3 |
| C_2 | 0.0002 | 0.1372 |
| Expectation | 755.423 | 704.326 |



Strikes (k_1,k_2) with the probability that Hedged USD Revenue will be greater than 706 million USD

| (k_1,k_2) | Probabilities |
|--------------|---------------|
| (0.66, 1.3) | 0.4 |
| (0.66, 1.25) | 0.5319 |
| . | . |
| (0.55, 0.95) | 0.7628 |
| (0.55, 0.9) | 0.7649 |

- The pairs of k_1 and k_2 for the highest and lowest probability of the hedge USD revenue greater than 706 million are the same as the pairs that produced the highest mean values.

| | High | Low |
|-------------|--------|--------|
| K_1 | 0.55 | 0.66 |
| C_1 | 0.0014 | 0.0858 |
| K_2 | 0.9 | 1.3 |
| C_2 | 0.0002 | 0.1372 |
| Probability | 0.95 | 0.4 |

