OMMC 2025 Main Round

OMMC Staff

May 2025

HOW TO SUBMIT

1. During the testing period, go to our official test portal: https://ommc-test-portal-2025.vercel.app/. **One person submits per team.** Teams can have 1 to 4 people, inclusive.

2. Register an account using Google or Discord.

3. Fill out team registration information and answers.

4. Submit your answers and all answers and information will be saved to your account. **The submission and your registration information are to be sent in TOGETHER.**

5. You can access this information, and **send in an updated submission if you choose,** if you sign back in to your account later within the testing period. **We will take the last set of answers submitted.**

NO, you do not need to "sign up" beforehand in the conventional sense. Just sign into the test portal **ANYTIME DURING THE TESTING PERIOD and SEND IN A RESPONSE.**

RULES

1. This is a **25-question, untimed examination** that can be worked on by a team of up to 4 people. Only one person from each team has to submit answers. If we receive multiple submissions from a team, the most recent submission will be graded.

2. All answers are positive rational numbers inputted as "a/b" for relatively prime positive integers a, b where $b > 1$, or nonnegative integers. Your score will be the number of correct answers; there is neither partial credit nor a penalty for wrong answers. Top scoring teams will move onto the final round.

3. No aids other than writing utensils, scratch paper, rulers, compasses, erasers, and a **four-function/scientific calculator** are allowed. The use of **graphing calculators, smartphones, smartwatches, and/or outside websites is NOT ALLOWED**. In particular, **Desmos, Geogebra, Wolfram Alpha, and other similar websites are forbidden**. Failure to follow this rule will result in your test score being voided.

4. Discussion of any aspect of the test outside your team is **not allowed** until submissions are closed and discussion is opened. Failure to follow this rule will result in your test score being voided.

5. Submissions open on **May 17th, 2025, 8:00 AM EDT** and close at **May 25th, 2025, 8:00 AM EDT**.

6. Diagrams are not necessarily to scale.

Any questions on the above should be emailed to ommcofficial@gmail.com. With that, good luck! The OMMC team has spent a lot of time on this contest and we hope that you enjoy your experience.

PROBLEMS

1. There are 2025 balls in a row. Their colors from left to right are

   Red, Green, Blue, Red, Red, Green, Green, Blue, Blue, Red, Red, Red, Green, ...

   and so on, where one more Red, Green, and Blue ball is used each cycle. Each Red ball is worth 3 points. Each Green ball is worth 2 points. Each Blue ball is worth 1 point. Find the sum of the points of all 2025 balls.

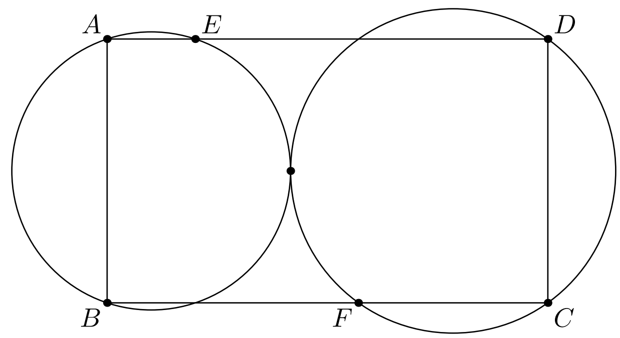
2. Three consecutive even numbers have a six-digit product whose first and last digits are 4 and 8, respectively. Find the smallest among these three numbers.

3. In rectangle $ABCD$, semicircles with diameters $\overline{AB}$ and $\overline{CD}$ meet at P and Q. If $PQ = 16$ and $AB = 20$, find the area of $ABCD$.

4. Due to a clerical error when trying to order some Crispi cookies for an office party, we, the OMMC staff, ended up with a massive excess of cookies that nobody is able to finish. We have decided to reward our contestants by splitting up our excess cookies into gift baskets and mailing them out for free. After compiling a list of our top contestants, Evan has realized that if we distribute our cookies equally among all of the people on our list, everyone will receive 20 cookies, with 25 remaining.

   Given that there are at least a million people on the list right now, Evan has determined that he must strike at least P people from the list so that all of the remaining cookies can be split evenly among everyone on the list. Find the minimum possible value of P.

5. Rectangle $ABCD$ has $AB = 12$ and $BC = 30$. Points E and F lie on sides AD and BC, respectively, such that $AE = 9$ and the circumcircles of ABE and CDF are tangent to each other. Find the length of CF.



6. Let x, y, and z be real numbers. Given that there exists some real number c such that

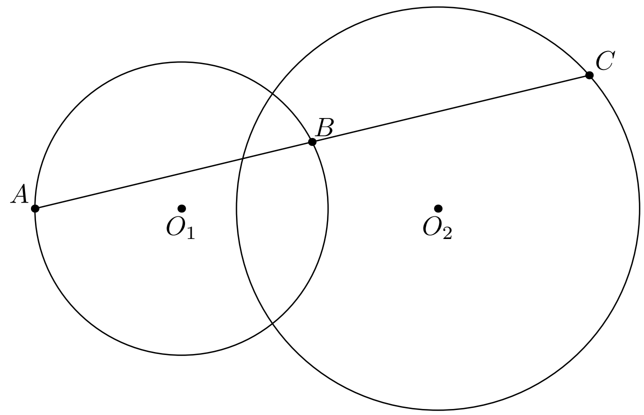
$$xy + yz + zx \;=\; x^2 + 2yz - 1 \;=\; y^2 + 2zx - 2 \;=\; z^2 + 2xy - c,$$

find c^2.

7. Phrygia chooses points A, B, C, and D at random on the surface of a sphere. She then draws the shortest paths on the sphere's surface from A to B and from C to D. Find the probability that these two paths intersect.

8. Find the number of ways to shade 40 cells in a 9×9 square grid such that no two shaded cells share an edge.

9. Two circles have radii 8 and 11 and their centers are 14 units apart. The line passing through their centers intersects the smaller circle at A, where A is outside the larger circle. If points B on the smaller circle and C on the larger satisfy that B is the midpoint of AC, then $AB = BC = \sqrt{m}$ for a positive integer m. Find m.



10. The number of positive divisors of n^2 is three times the number of positive divisors of n. Find the smallest possible number of positive divisors of n^3.

11. Let x, y, z be positive real numbers satisfying

$$\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = \frac{1}{2025}.$$

Find the distance from the origin to the plane formed by the points $(x, 0, 0)$, $(0, y, 0)$, and $(0, 0, z)$.

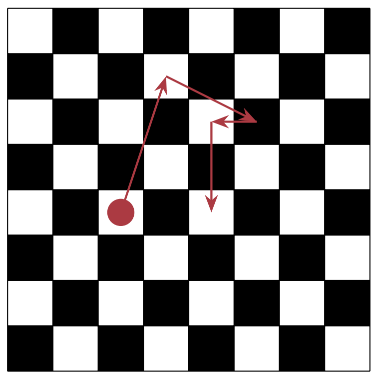
12. Cyclic quadrilateral $ABCD$ has area 3 such that $AB + AD = 13$ and $CB - CD = 5$. Find $\tan(\angle BAD)$.

13. Let a and b be positive integers such that $a^2 + 2b$ and $b^2 + 9a$ are both perfect squares. Find the sum of all possible values of a.

14. The exradii of $\triangle ABC$ are the roots of the polynomial $x^3 - 30x^2 + 50x - 20$. If s is the semiperimeter and a, b, c are the sidelengths of $\triangle ABC$, find $ab + bc + ca - s^2$.

The three exradii of a triangle are the radii of circles tangent to one side of the triangle and the extensions of the other two sides in the direction opposite their common vertex.

15. Ritwin moves a checker on an 8×8 square checkerboard. First, he puts the checker on one of the cells. Next, in each move, he moves the checker to any cell which he has not visited before, such that the distance traveled in each move is different, where the distance between two cells is the Euclidean distance between their centers. Find the maximum number of times Ritwin can move the checker.



16. Define $f(x)$ for positive integers x as the smallest positive integer $k < x$ such that $\binom{x}{k}$ is divisible by 7 if one exists, otherwise $f(x) = 0$. Find the maximum value that $n \cdot f(n)$ takes over all $n \in [7, 100]$.

17. In $\triangle ABC$, points D, E, and F are the intersections of the angle bisector of $\angle A$ with BC and the perpendicular bisectors of AB and AC, respectively. Let O, O_B, and O_C be the circumcenters of $\triangle ABC$, $\triangle BDE$, and $\triangle CDF$, respectively. Find the maximum value of the ratio of the areas of $\triangle OO_BO_C$ and $\triangle OBC$.

18. Let $p(x)$ be a degree 6 monic polynomial (with not necessarily real coefficients) satisfying that $p(x) \mid p(x^2)$. Find the sum of all distinct values of $p(1)$.

19. 100 cows are standing in a line. Farmer John wants to separate the cows into herds (but some may not end up in a herd) and designate a leader for each herd with the following rules:

    - Each herd is a contiguous group of cows.
    - Between any two consecutive herds is exactly one cow that is not in a herd.
    - Each herd must have exactly one leader, and this leader is in the herd.
    - The first and last cows are in a herd.

    If N is the number of ways Farmer John can do so, find the remainder when N is divided by 1000.

20. Let $ABCD$ be a quadrilateral inscribed in circle Ω, with diagonals AC and BD intersecting at P. Let ω be a circle passing through D and tangent to line AC at P, and suppose ω and Ω intersect at D and Q. If $BP = 1, AP = 2, CP = 3$, and $\angle APB = \angle BCD$, and $DQ = \frac{a\sqrt{b}}{c}$ for positive integers a, b, c with $\gcd(a, c) = 1$ and b squarefree, find $a + b + c$.

21. Let $z_1, \ldots z_{2187}$ be nonnegative real numbers summing to 1. Let S be the minimum of $\sqrt{z_i z_{3i}}$ over all indices i, where indices are taken mod 2187. Then, the maximum value of S is equal to $\sqrt{\frac{a}{b}} - \sqrt{\frac{c}{d}}$ where a, b, c, and d are positive integers satisfying $\gcd(a, b) = \gcd(c, d) = 1$. Find $a + c$.

22. A group of ten people play a strange game with two teams: 7 people are town and 3 people are spies, with one of the spies being the Chief. The game ends with a vote, which works as follows:

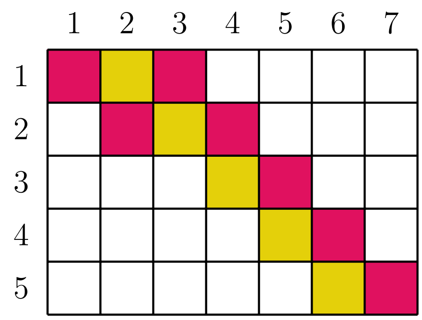
    - The spies close their eyes, and the town each simultaneously casts a vote for who to eject.
    - Then, the spies *collectively* get one vote.
    - Everyone who is tied for the most votes is ejected. The town wins if no town is ejected and the Chief is ejected; otherwise, the spies win.

    The game has devolved to the point where team information is public, but no one knows who the Chief is, including the Chief themselves. The town may publicly discuss a strategy before voting, with the spies also knowing the strategy. Each of the town has access to *independent* random number generators. With optimal strategy from both sides, find the probability that the town wins.

23. A 5×7 grid has rows numbered 1 through 5 and columns numbered 1 through 7. Let N be the number of ways to choose and color 12 cells in the grid with the following properties:

- Each chosen cell is colored either ruby or citrine.
- The sum of the rows of the ruby cells is a multiple of 5.
- The sum of the columns of the citrine cells is a multiple of 7.

Find the remainder when N is divided by 65537.

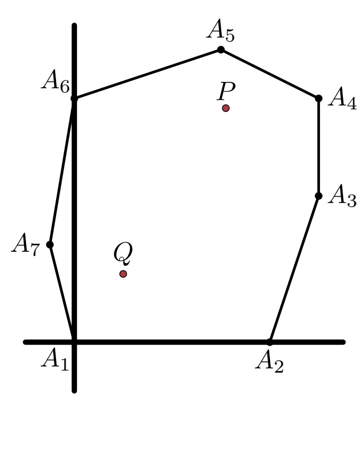


24. Let $A_1A_2A_3A_4A_5A_6A_7$ be a heptagon with a pair of isogonal conjugates P and Q inside the heptagon. Suppose $A_1 = (0,0)$, $A_2 = (4,0)$, $A_3 = (5,3)$, $A_4 = (5,5)$, $A_5 = (3,6)$, and $A_6 = (0,5)$. If $A_7 = \left(-\frac{a}{n}, \frac{b}{n}\right)$ and

$$P, Q = \left(\frac{c}{m}, \frac{d}{m}\right) \pm \left(\frac{\sqrt{-e+\sqrt{f}}}{m}, \frac{\sqrt{e+\sqrt{f}}}{m}\right)$$

for positive integers a, b, c, d, e, f, m, and n satisfying $\gcd(a,b,n) = 1$ and m is as small as possible, find $a+b+c+d+e+f+m+n$.

Two points P and Q are said to be isogonal conjugates in $A_1A_2A_3A_4A_5A_6A_7$ if $\angle PA_iA_{i-1} = \angle QA_iA_{i+1}$ for all positive integers $1 \le i \le 7$, where $A_0 = A_7$ and $A_1 = A_8$.



25. Let N be the number of ways to orient the 10122 edges of the pictured 3×2025 grid such that there are no directed cycles. Find the remainder when N is divided by 2495.