

* HEAPS

- Why? we say find the smallest no. \rightarrow

3	8	4	19	20	2	36
---	---	---	----	----	---	----

$O(N)$ gives you smallest no.

- you can reduce it to $O(1)$ by inserting by sorted \rightarrow

2	3	4	8	19	20
---	---	---	---	----	----

$O(1)$

\therefore min. at 0th index

- But it takes $O(N \log N)$ to insert item



How to reduce it?

\downarrow
using
Heaps

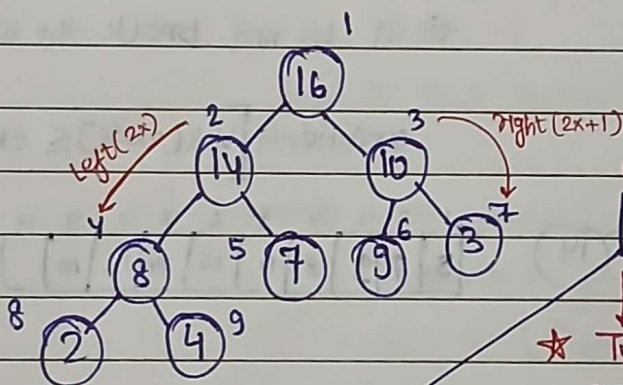
- Heaps are generally stored in arrays but represented as Trees.

- How Heaps work?

level order Traversal

1	2	3	4	5	6	7	8	9
16	14	10	8	7	9	3	2	4

Heaps stored in array



How it is represented?

① Complete Tree

② Every node value \geq All of its children

★ Two imp. conditions in minHeap Data structures

root $\rightarrow i = 1$

- for complete binary tree
- parent of $i \rightarrow i/2$
 - left of $i \rightarrow 2 \times i$
 - right of $i \rightarrow 2 \times i + 1$

eg. ⑭ \rightarrow index(2) \rightarrow parent of 4 $\therefore 4/2 = 2$
5 \times 5/2 = [2.5] = 2

eg. parent of 7 $\rightarrow 7/2 = 3.5 = 3 \rightarrow$ (10)

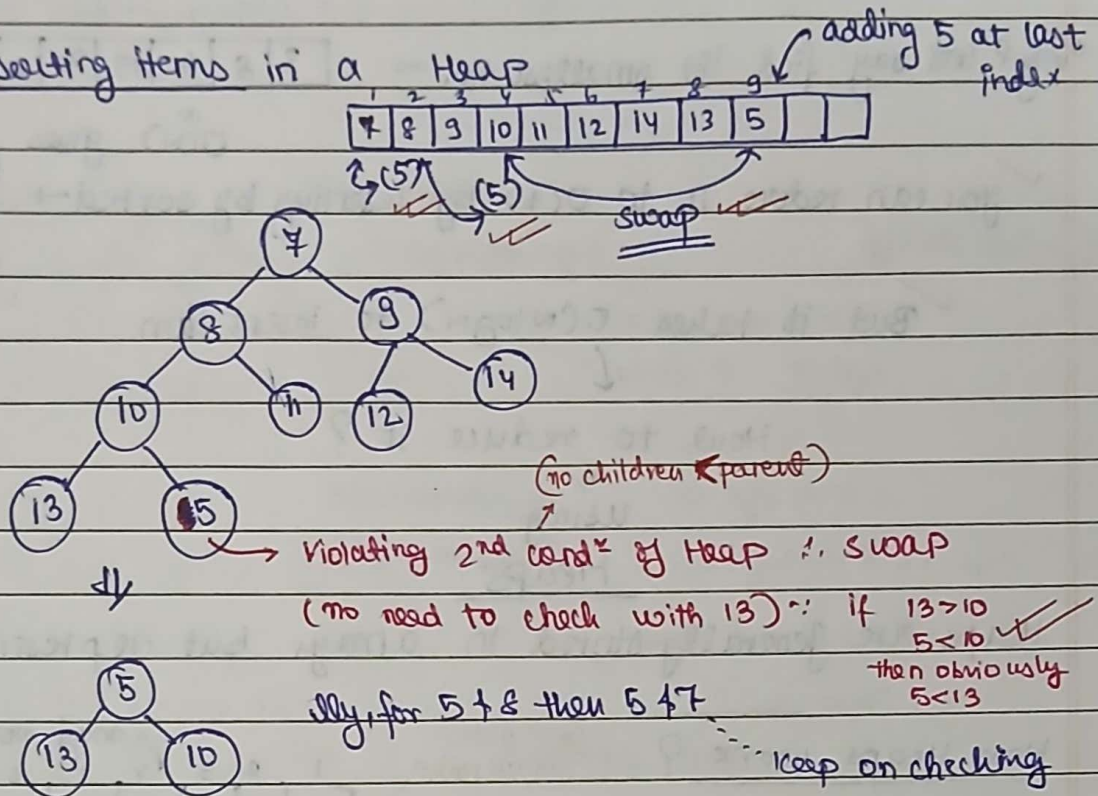
No pointers required

because of this formula.

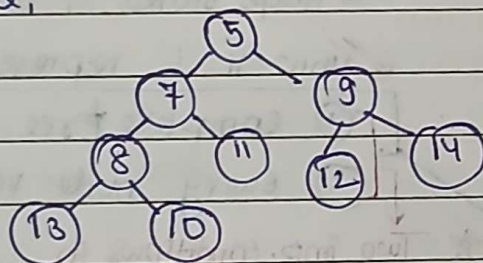
[height = $\log(N)$]

- How to solve problem of delete & insert?

→ Inserting Items in a Heap



Hence,



condition → $Val(node) \leq childrens$

1	2	3	4	5	6	7	8	9	10
5	7	9	8	11	12	14	13	10	

✓ This is known as UpHeap method

Since, we're checking one at every level & levels are $\log n$
∴ complexity → $O(\log n)$

→ Removing items

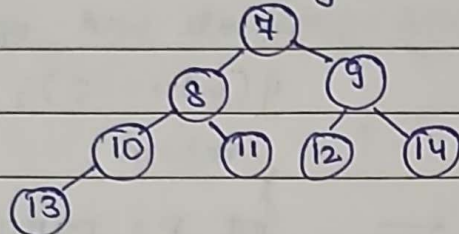
→ from here, if we remove 5 (remove from 1st index) *
∴ put the element which is last of index (here it is 10) → put on 1st index in place of 5.

arr[1] = 10;

(updated)

- Now check if left is smaller or right is smaller
- left $\therefore 4 \therefore$ swap with 7 (automatically adjusted with 9 \rightarrow obvious)
- Now for 10 again check left or right which is smaller
- left $\therefore 8 \therefore$ swap with 8
- ~~Agg~~ Again now 13 (larger) \therefore stop.

Hence \rightarrow



\therefore checking each level \therefore complexity $\Rightarrow O(\log N)$

[revise Generics in OOPs]

\Rightarrow Code:-

\downarrow type T is extending comparable so we compare 2 objects

```
class Heap<T> extends Comparable<T> {
```

```
    private ArrayList<T> list;
```

```
    public Heap() {
```

\rightarrow Heap constructor

```
        list = new ArrayList<>();
```

```
    }
```

```
    private void swap(int first, int second) {
```

```
        T temp = list.get(first);
```

```
        list.set(first, list.get(second));
```

```
        list.set(second, temp);
```

```
    }
```

} basic Swapping

Since in the theory we discussed index starting from 1 but in code we'll start from 0 & adjust others similarly.

```
    private int parent(int index) {
```

```
        return (index - 1) / 2;
```

\therefore parent was $i/2$

\therefore left

```
        return index * 2 + 1;
```

\therefore left was $i * 2$
(Soch $\#$ dekho $\#$ index tree se)

\therefore right

```
        return index * 2 + 2;
```

$\therefore i * 2 + 1$

Inserting → public void insert(T value)

this will add at last ← list.add(value);
Index

now swapping bottom to up (upheap) { upheap(list.size()-1);

private void upheap(int index) {

if (index == 0) {

return;

start checking ← int p = parent(index);

from parent of last Index

if this index < parent

if (list.get(index).compareTo(list.get(p)) < 0) {

swap if yes ← swap(index, p);

repeat till above upheap(p);

}

Removing →

public T remove() throws Exception {

if (list.isEmpty()) { ←

throw new Exception("removing from an empty heap!");

first remove item → T temp = list.get(0);
at 0th Index

taking last & removing from there
for placing last index ← T last = list.remove(list.size()-1);
item to its place

if (!list.isEmpty()) {

list.set(0, last);

starting from 1st index to bottom ← downheap(0);

return temp;

}

```
private void downheap(int index){
```

```
    int min = index;
```

```
    int left = left(index);
```

```
    int right = right(index);
```

if min > left then swap both

(if left exists) →

```
    if (left < list.size && list.get(min).compareTo(list.get(left)) > 0){
```

```
        min = left;
```

```
    }
```

do for right

Now calling for next →

```
    if (min != index){
```

```
        swap(min, index);
```

```
        downheap(min);
```

```
    }
```

• Heap Sort

→ You have a heap, till the heap is empty keep removing items & put it in a list.

∴ removing n-items till last level of $\log n$

∴ complexity → $O(n \log n)$.

∴ ⇒

```
public ArrayList<T> heapSort() throws Exception{
```

```
    ArrayList<T> data = new ArrayList<>();
```

```
    while (!list.isEmpty()) {
```

```
        data.add(list.remove());
```

```
    }
```

```
    return data;
```

```
}
```

```
}
```

time
already
sorted
hai

Main func. →
(Copying into ArrayList)

import java.util.ArrayList;

PSVM () throws Exception {

Heap Integer > heap = new Heap > ();

heap.insert (34);

⋮

ArrayList list = heap.heapSort();
cout (list);

}

~~cc~~ This is also called Priority Queue.

eg. getting min. item is the priority. ✓

eg. " max " " " " " " ✓

~~cc~~ Priority Queue as a Linked List

(3) → (4) → (8) → (18)

To insert 10 (while checking) at correct position

as in linked list we cannot jump in b/w ∴ start from head & end till tail.

∴ checks 3 ✓ checks 4 ✓ checks 8 ✓ checks 18 ✗ ∴ place 10 before 18.

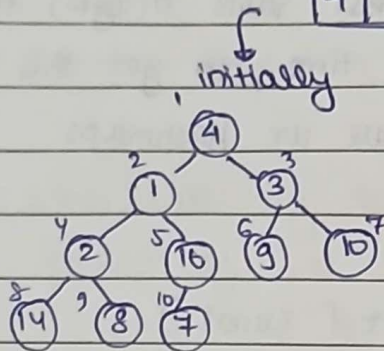
(3) → (4) → (8) → (10) → (18)

Time complexity → $O(N)$ ∴ Not good for LL

∴ use Heaps.

Q. Create a max Heap from an unsorted Array.

1	2	3	4	5	6	7	8	9	10
4	1	3	2	16	9	10	14	8	7



→ Property condition of max heap $Val(node) \geq childrens$

Bottom starting from $N/2 \rightarrow$ because all after $\frac{N}{2}$ i.e. $\frac{N}{2}+1, \dots, n$ are leaf nodes (they don't have any child to check).
(So start from 5th)

code \rightarrow for $(i = N/2 \text{ till } 1)$
 \downarrow downheap(i) ✓

write on own \leftarrow

[Time complexity = $O(N)$] not $O(N/2 * \log(N))$

$\left. \begin{matrix} \frac{N}{2} \\ \frac{N}{4} \\ \frac{N}{8} \\ \frac{N}{16} \\ \vdots \end{matrix} \right\}$ Harmonic Progression sum *