

Master Project

Water Sterilizer Optimization

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The beginning

- **Where?** Cosmology and subatomic physics laboratory in Grenoble

¹Le journal la CNRS, <http://www2.cnrs.fr/presse/journal/2853.htm>

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I was, then, convinced that I could improve UV lamps technology. It was enough to adapt particle accelerators technologies to lower energies¹.

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I was, then, convinced that I could improve UV lamps technology. It was enough to adapt particle accelerators technologies to lower energies¹.

- **When?** 25 January 2006, RC Lux is created

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The steriliser



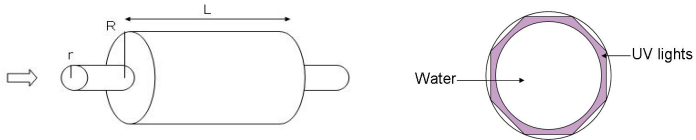
Award winner of most innovative
technologies for environment

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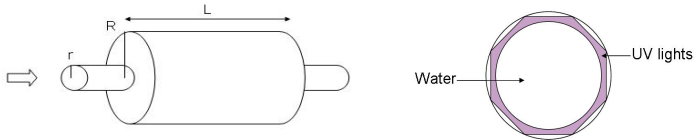
Definition

- **Context:** To model water sterilization by UV radiation with this device :



Definition

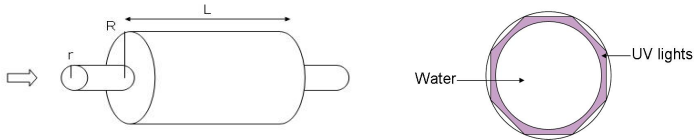
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- **Goal:** To find the optimal radius r and R for which :

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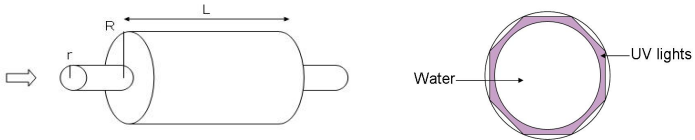
- **Context:** To model water sterilization by UV radiation with this device :



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 - * At the end of the pipe, the water is sterilized.

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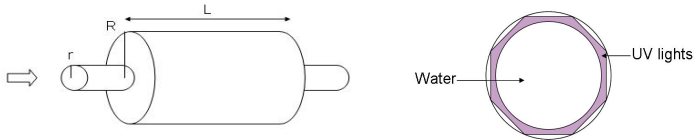
- **Context:** To model water sterilization by UV radiation with this device :



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 - * At the end of the pipe, the water is sterilized.
 - * The flow must be 2 or 4 $l.min^{-1}$.

Definition

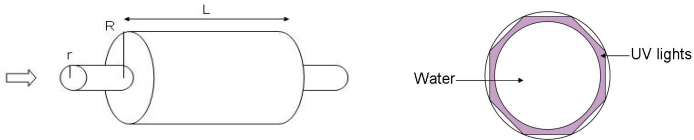
- **Context:** To model water sterilization by UV radiation with this device :



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 - * At the end of the pipe, the water is sterilized.
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 - * $R \in [7mm - 20mm]$ and $r \in [2mm - 6mm]$

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- **Goal:** To find the optimal radius r and R for which :
 - * At the end of the pipe, the water is sterilized.
 - * The flow must be 2 or 4 $l.min^{-1}$.
 - * $R \in [7mm - 20mm]$ and $r \in [2mm - 6mm]$
- **Expected Result:** To create a computer program to search these radius.

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Methodology

Different concerned domains:

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Our research approach:

Simplified case \Rightarrow real case:

- 0D mean velocity

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- 2D axisymmetric with a Poiseuille's Profile

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Our research approach:

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- 0D mean velocity
- 1D mean velocity
- 2D axisymmetric with a Poiseuille's Profile
- 2D axisymmetric with simplified Navier-Stokes' equations

Methodology

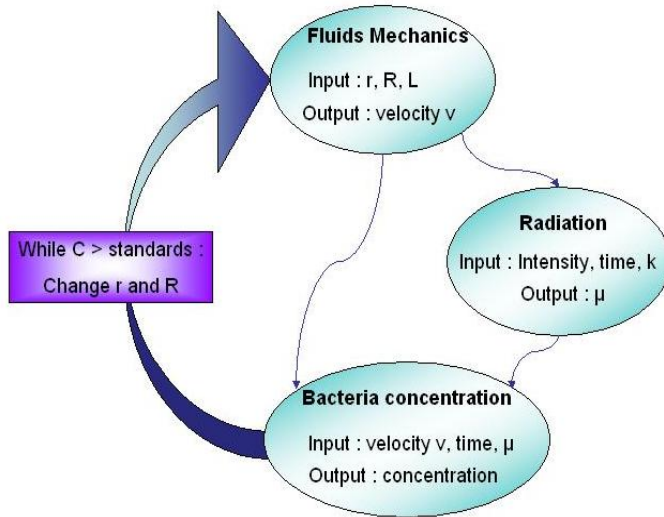


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Objectives

First:

⇒ To complete the 0D model before mid-February.

⇒ To finish the 2D model and give representative results for r and R to obtain sterilised water.

If miracle:

⇒ To do the same with the 3D model.

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General Case

Incompressible fluids velocity are governed by Navier-Stokes equations.
We look for the velocity \vec{u}

The Navier-Stokes Equations

$$\begin{cases} \frac{\partial \vec{u}}{\partial t} + \vec{u} \cdot \nabla \vec{u} - \mu \Delta \vec{u} + \nabla p = f \\ \operatorname{div}(\vec{u}) = 0 \end{cases} \quad (1)$$

where : \vec{u} : the fluid velocity
 p : the pressure

0D Case

- RC-Lux gives us: the pressure loss Δp
- We look for: the velocity v

The Darcy-Weisbach Equation (version 1)

$$h_l = f \cdot \frac{L}{D} \cdot \frac{v^2}{2g} \quad (2)$$

h_l : the head loss due to friction

f : a Darcy friction factor

where :

L : the length of the pipe and D : the diameter of the pipe

g : the gravitational constant

The Darcy-Weisbach Equation (version 2)

Since $\Delta p = \rho g h_l$, where ρ is the flow's density, we have :

$$\Delta p = f \cdot \frac{L}{D} \cdot \frac{\rho v^2}{2} \quad (3)$$

0D Case

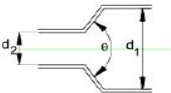
$$\Delta p = \left(\sum_{p=1}^3 f_p \frac{L_p}{D_p} + K_e + K_c \right) \cdot \frac{\rho v^2}{2} \quad (4)$$

Swamee-Jain Equation

$$f_p = \frac{0.25}{\left[\log \left(\frac{\epsilon}{3.7 D_p} + \frac{5.74 \cdot \nu^{0.9}}{(v \cdot D_p)^{0.9}} \right) \right]^2} \quad (5)$$

And :

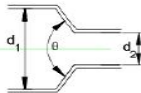
$\beta = \frac{d_2}{d_1}$



Sudden Expansion

$$K_e = \frac{(1 - \beta^2)^2}{\beta^4}$$

$\theta \geq 45^\circ$



Sudden Contraction

$$K_c = \frac{0.5 (1 - \beta^2) \sqrt{\sin \frac{\theta}{2}}}{\beta^4}$$

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General Case

Single Stage Exponential Decay Equation

$$S(t) = e^{-klt} \quad (6)$$

Where S : the surviving ratio of the initial population $\frac{c}{c_0}$
 t : the exposure time
 I : the UV radiation intensity
 k : the sensitivity coefficient of the microorganisms to UV exposure.

Beer-Lambert Law

$$I(x) = I_0 \cdot e^{-\alpha x \rho} \quad (7)$$

Where I_0 is the intensity of the incident light
 α , the absorption coefficient
 ρ the density of water.

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General Case

We are interested in solving this equation in order to know concentration at the end of the sterilizer.

Bacteria's Concentration Equation

$$\frac{\partial c}{\partial t} + \underbrace{\vec{u} \cdot \nabla c}_{\text{advection}} + \underbrace{\mu \cdot c}_{\text{reaction}} = f \quad (8)$$

Where, c : the bacteria concentration
 u : the fluid velocity
 μ : a constant which represents the bacteria's destruction

0D Case

In a first approximation:

Simplified Bacteria's Concentration Equation

$$\begin{cases} \frac{\partial c}{\partial t} = -\mu \cdot c \\ c(t=0) = c_0 \end{cases} \quad (9)$$

The solution of this problem is so:

$$c(t) = c_0 \cdot e^{-\mu t} \quad (10)$$

Using the formula about radiation explained earlier, we obtain:

The Concentration at time t:

$$c(t) = c_0 \cdot S(t) = c_0 \cdot e^{-kIt} \quad (11)$$

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Problem Formulation

Constrained optimisation problem

$$\begin{cases} \text{Min}_{(r,R) \in \mathcal{D}} \alpha C(r, R) + \beta \text{Vol}(r, R) \\ C(r, R) \leq C_s \\ 2 \leq Q(r, R) \leq 4 \end{cases} \quad (12)$$

- $\mathcal{D} = [2, 6] \times [7, 20]$
- Vol represents the volume of the device
- α and β weights on concentration and volume
- The dimension of the radius is the millimetre.
- We use $\Delta p = kQ^2$ to have the constraint on the flow.

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First Approximation : $\beta = 0$

Our optimisation problem

$$\begin{cases} \text{Min}_{(r,R)} c(r, R) &= c_0 \exp(-kl \frac{z}{U(r,R)}) \text{ under the constraints} \\ Q_1(r, R) &= 2 - \pi r_{\text{exit}}^2 U(r, R) \leq 0 \\ Q_2(r, R) &= \pi r_{\text{exit}}^2 U(r, R) - 4 \leq 0 \end{cases}$$

$$\frac{2}{\pi r^2} \leq U(r, R) \leq \frac{4}{\pi r^2}$$

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Second Approximation : $\beta = 0$ and $r = 2mm$

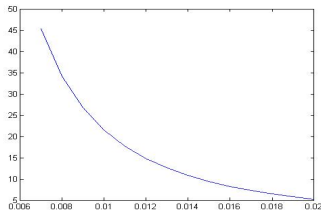
$$\Rightarrow \min_{R \in [7, 20]} C(R)$$

Formula

$$\left(f_1(v) \frac{L_1 + L_3}{2r} + f_2(v, R) \frac{L_2}{2R} + K_e + K_c \right) \cdot \frac{\rho v^2}{2} - \Delta p = 0 \quad (13)$$

$$c(R) = c_0 \cdot e^{-kl \frac{L_2}{v(R)}} \quad (14)$$

1st Step : $v(R)$



2nd Step : $C(R)$

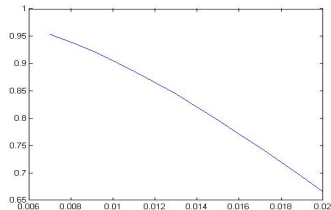


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In the Future

- 0 Space Dimension :
 - * To find the minimum bacteria concentration with r and R .
 - * To optimize with the volume
- 1 Space Dimension
- 2 Space Dimension

Conclusion

Difficulties:

- Delay in our schedule.
- More complex than we expected.
- Almost no communication with the RC-Lux company.

Positive Points:

- Very interesting and concrete subject with different scientific domains.
- Learning of many things.
- Participation to an industrial project.

Thank you for your attention