Names: Hai Huynh
Prudhviraj Sheela
Aman Masipeddi

Assignment-1 (Phase-2) Report

Problem-2

From Problem 1, for a queue with size of K, we have the probability P(i), where i = 0, 1, … K, meaning the probability that there are n transactions inside the block. The formula for such probability is as follows:

$$P_{(i)} = \frac{1-\rho}{1-\rho^{K+1}}\rho^{i}, \text{ where } \rho = \frac{\lambda}{\mu}$$

If we want to find the the average of all possible sizes of the blocks, the formula must be

$$P_{average}(i) = \frac{P'(i) + P''(i) + P''(i) + \ldots P^{\infty}(i)}{1+1+1+\ldots+1}$$

(1)

such that $\Sigma_{i=0}^{\infty} \quad P_{average}(i) = 1$

(2)

With P', P'', P''', … are the probabilities that there are i transactions in the blocks of sizes K', K'', K''', …

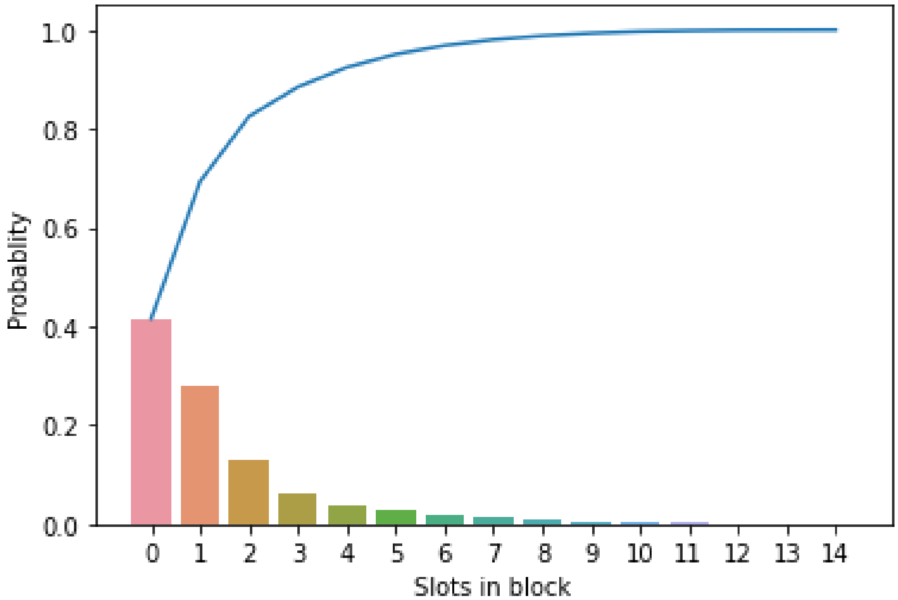
After much thought and study, we believed that there are no feasible closed form formulas for this problem due to the variant sizes of K', K'', K''' … in the formula and we were not able to extract a pattern out of it. However, we believe that this problem is only feasible in the case of analyzing a snapshot of the system with the recorded numbers of those block sizes. The simulations below indeed show that they do follow the above formula (1) and the condition (2)

In our simulation, we randomly generated N blocks of random sizes between 1-25 and we obtained the following results,

Case-I:

λ= 2, μ= 3, 5 blocks of size 1, 12, 2, 15, 10

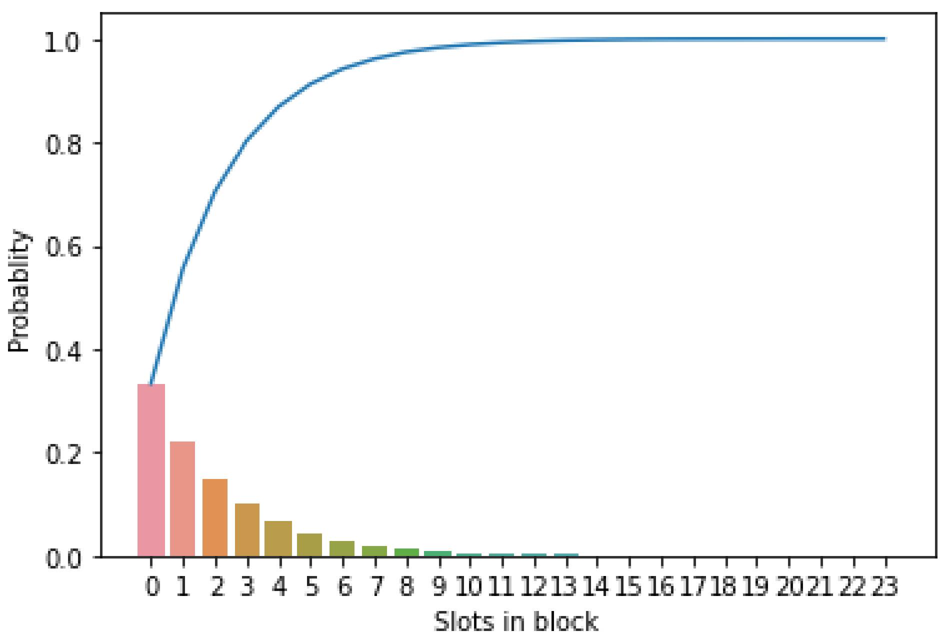
Generated Simulation Model:



Case-II:

λ = 2, μ = 3, 3 blocks of size 16, 24, 20

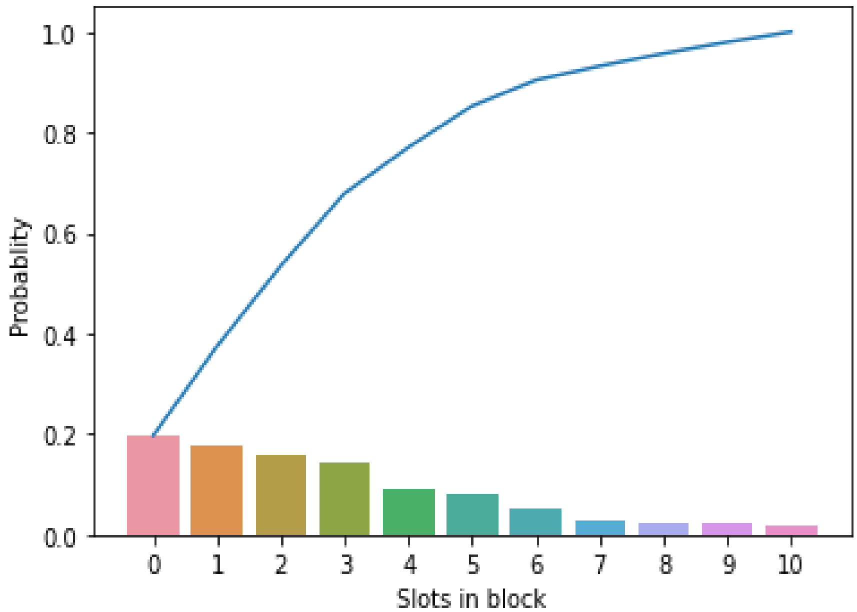
Generated Simulation Model:



Case-III:

λ = 9, μ = 10, 5 blocks of size 10, 5, 10, 3, 6

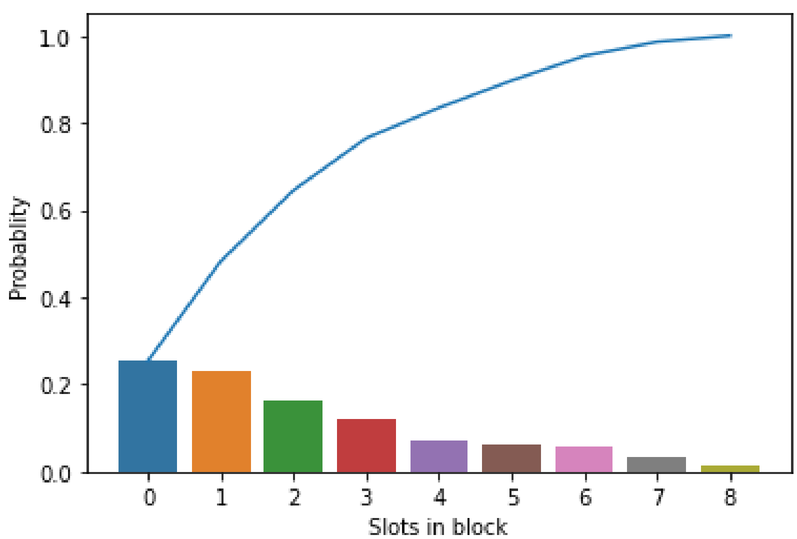
Generated Simulation Model:



Case-IV:

λ = 9, μ = 10, 10 blocks of size 8, 8, 7, 1, 7, 6, 6, 3, 2, 3

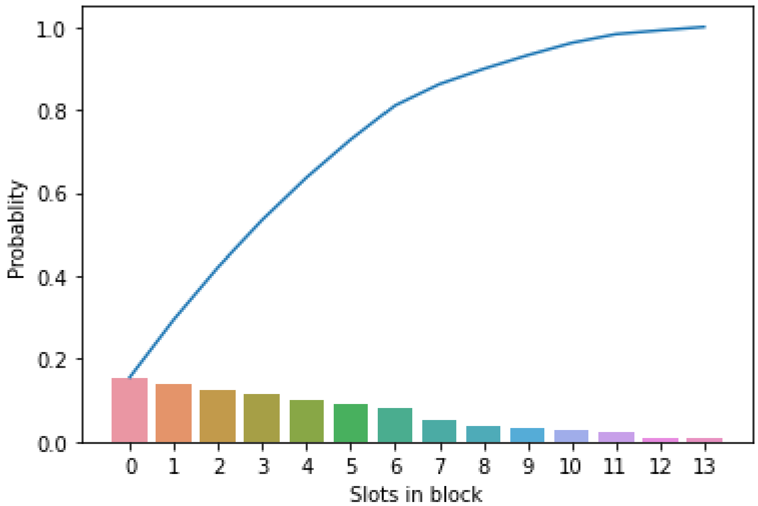
Generated Simulation Model:



Case-V:

λ = 4, μ = 10, 10 blocks of size 7, 13, 6, 11, 6, 13, 10, 11

Generated Simulation Model:



We could observe that the values for $P(i)$ are in the range of [0, 1] and the sum of those accumulated probabilities of those $P(i)$'s approaches 1.0. This is consistent with the conditions assumed in this problem.

Problem-3

In the previous problems we have observed that all the transactions are being served only when the block gets full. But as the problem-3, states that it must follow a pseudo asynchronous block processing where each transaction needs to be served immediately when it is arrived into the block. So, in order to generate a model as stated in the assignment we have gone through some of the existing queuing models to find out a start point. It turns out there exists models that seem to fit the description of the system well.

We believe that the (M/M/1: ∞/FIFO) queuing model described in [2] fits the assignment requirements the best. In this (M/M/1 : ∞/FIFO) queuing model the arrival which is followed is based on the Marcovian principle and the service distribution followed is also Marcovian, where number of server is one and the size of queue is also Marcovian (infinite) and the service discipline followed is based on the "First Come First Serve" (FCFS) principle and the calling source is also finite. So, this basically varies for every unit of time where a transaction is serviced immediately as it arrives into the block.

Considering the above principle following are the assumptions and derivation that can be obtained accordingly,

n = Number of transactions in the system

λ = Arrival Rate

μ = Service Rate

$r = \lambda/\mu$ = Utilization Rate

$Pn(t)$ = Probability of 'n' customers in system at time 't'

Probability of one arrival in the system during $\rightarrow \Delta t = \lambda\Delta t + O(h)$

Probability of more than one arrival in the system during $\rightarrow \Delta t = O(h)$

Probability of no arrival in the system during during $\rightarrow \Delta t = (1 - \lambda\Delta t) + O(h)$

Probability of one customer being serviced in time $\rightarrow \Delta t = \mu\Delta t + O(h)$

Probability of more than one customer being serviced in time $\rightarrow \Delta t = O(h)$

Probability of not even a single service customer being serviced in time $\rightarrow \Delta t = (1 - \mu\Delta t) + O(h)$

$\rightarrow$ Let us assume that $Pn(t+\Delta t)$ be the probability of 'n' transactions in the block at the time $(t+\Delta t)$.

Stage - I: For $(n > 0)$

$\rightarrow Pn(t + \Delta t) = Pn(t)(1 - \lambda\Delta t)(1 - \mu\Delta t) + Pn{+}1(t)(1 - \lambda\Delta t)(\mu\Delta t) + Pn{-}1(t)(\lambda\Delta t)$

$\rightarrow Pn(t + \Delta t) - Pn(t) = -Pn(t)(\lambda\Delta t + \mu\Delta t) + Pn{+}1(t)(\mu\Delta t) + Pn{-}1(t)(\lambda\Delta t) + o(\Delta t)$

$\rightarrow [(Pn(t + \Delta t) - Pn(t)) / \Delta t] = -(\lambda + \mu)Pn + Pn{-}1(t)\lambda + Pn{+}1(t)\mu + [o(\Delta t)/\Delta t]$

Now, applying a limit for the above equation [$\Delta t \rightarrow 0$], we obtain the following derived equation,

$\therefore (\Delta/\Delta t)\,[Pn(t)] = -(\lambda + \mu)Pn + Pn\text{-}1(t)\lambda + Pn\text{+}1(t)\mu \rightarrow$ (Eq 1)

Stage - II: For (n = 0)

$\rightarrow P_0(t + \Delta t) = P_0(t)(1 - \lambda\Delta t) - P_1(t)(\mu\Delta t)(1 - \lambda\Delta t) + o(\Delta t)$

$\rightarrow P_0(t + \Delta t) - P_0(t) = -P_0(t)(\lambda\Delta t) + P_1(t)(\mu\Delta t) + o(\Delta t)$

$\rightarrow [P_0(t + \Delta t) - P_0(t)] / [\Delta t] = -\lambda P_0 + \mu P_1(t) + [o(\Delta t)/\Delta t]$

Now, applying a limit for the above equation [$\Delta t \rightarrow 0$], we obtain the following derived equation,

$\therefore (\Delta/\Delta t)\,[P_0(t)] = -\lambda P_0 + \mu P_1(t)$, where (n=0) $\rightarrow$ (Eq 2)

Stage - III: Now let us consider a steady state condition based on the above two stages considered.

$\rightarrow Pn(t) = (\Delta/\Delta t)\,[Pn(t)] = 0$

Now the above derived equation is reduced as follows,

$\rightarrow -(\lambda + \mu)Pn + (\lambda)Pn\text{-}1 + (\mu)Pn\text{+}1 = 0$, where (n>0) considering (Eq-1)

$\rightarrow -\lambda P_0 + \mu P_1 = 0$, where (n=0) considering (Eq-2)

Now, from this we can obtain each values of ‘P’ in the following fashion,

$\rightarrow P_1 = (\lambda/\mu)P_0$

Now, on substituting different values of ‘n’ we obtain the following probabilities,

$\rightarrow P_2 = (\lambda/\mu)^2 P_0$

$\rightarrow P_3 = (\lambda/\mu)^3 P_0$

On calculating each probabilities in a continuous fashion we obtain the final result,

$\rightarrow P_n = (\lambda/\mu)^n P_0$, where the value of (n > 0)

Finally the generalized probabilities obtained are,

$\rightarrow P_0 = [1 - (\lambda/\mu)] = (1 - r)$ and $Pn = (\lambda/\mu)^n[1 - (\lambda/\mu)] = (r^n)\,(1-r)$

So, considering the above formula that we have obtained each transaction (i) follows the FCFS principle that whenever it enters the block it gets served immediately and in the same sequence the next incoming transactions are served as follows.

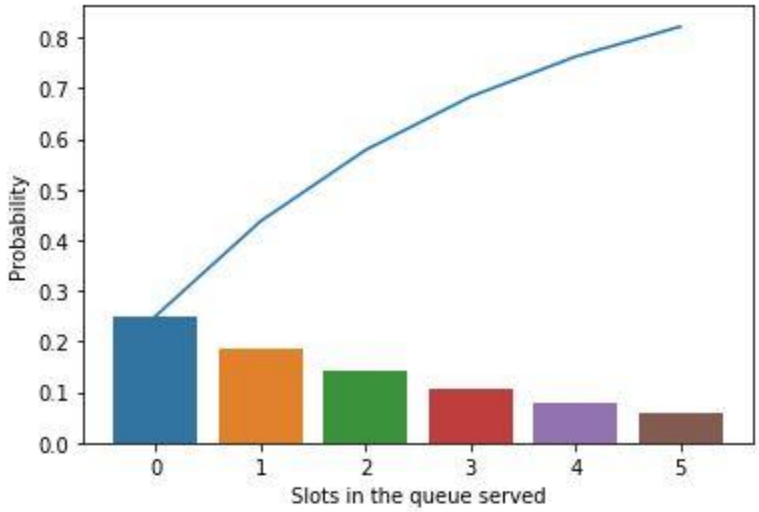
Therefore, on a whole $\Sigma P(i) = 1$ for ‘i’ in the range of $(0,\infty)$. This proves the concept of pseudo asynchronous block processing.

Applying the above formulas into a Python simulation with the following values, we obtained the following results that could show the pseudo asynchronous block processing. As the block size can be until a range of infinite, we have randomly tried generating some queue size and then applied the algorithm for generating its corresponding results accordingly.

Case - I:

The configurations for this are Queue size (K) = 5, λ = 3, μ = 4

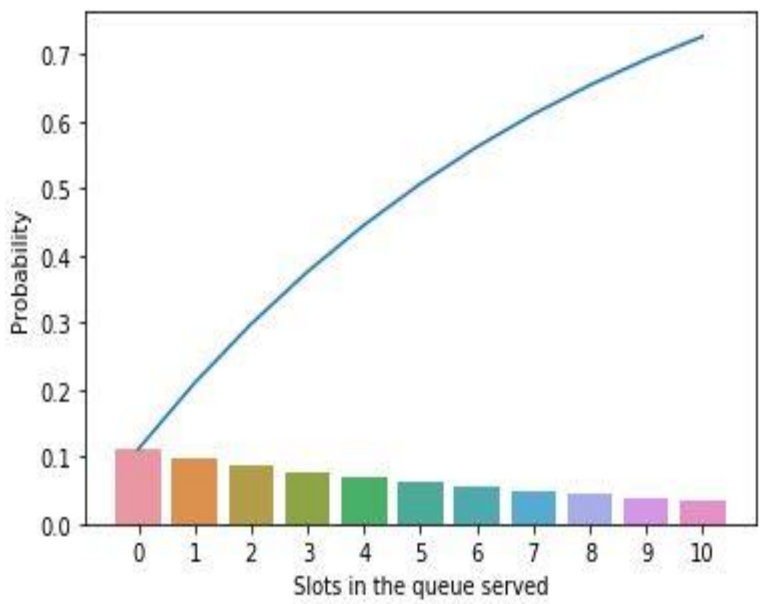
Generated Simulation Model:



Case - II:

The configurations for this are Queue size (K) = 10, λ = 8, μ = 9

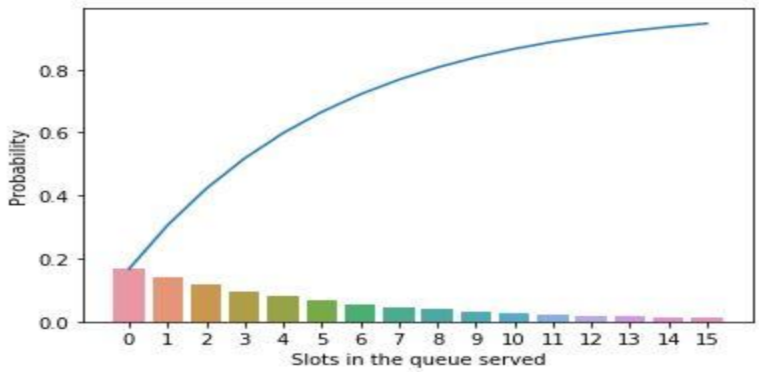
Generated Simulation Model:



Case - III:

The configurations for this are Queue size (K) = 15, λ = 5, μ = 6

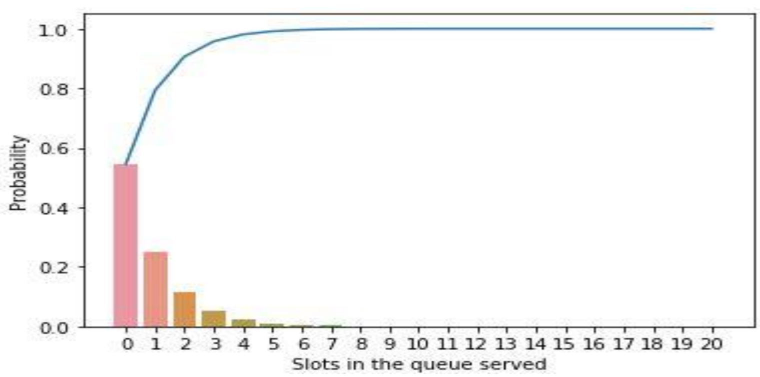
Generated Simulation Model:



Case - IV:

The configurations for this are Queue size (K) = 20, λ = 5, μ = 11

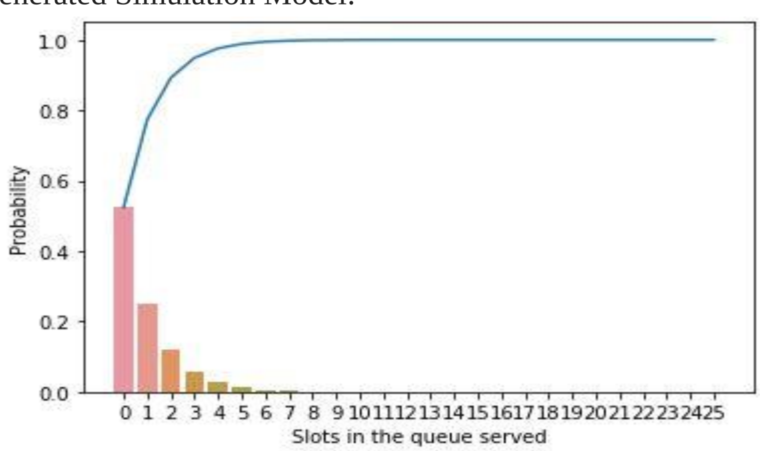
Generated Simulation Model:



Case - V:

The configurations for this are Queue size (K) = 25, λ = 10, μ = 21

Generated Simulation Model:



From the above all 5 cases of different configurations that have been considered for the "Problem-3" we can see that the cumulative probabilities add up to 1.0 and the individual P(n) values stay within the range of (0.0,1.0) and this model follows the principle of Pseudo Asynchronous Block Processing.

Contribution:

Hai Huynh: 37.5%
Prudhviraj Sheela: 37.5%
Aman Masipeddi: 25%

References

[1] Hillier, F. S., & Lieberman, G. J. (2010). Introduction to operations research.

[2] https://hithaldia.in/faculty/sas_faculty/Dr_M_B_Bera/Lecture%20note_5_CE605A&CHE705B.pdf