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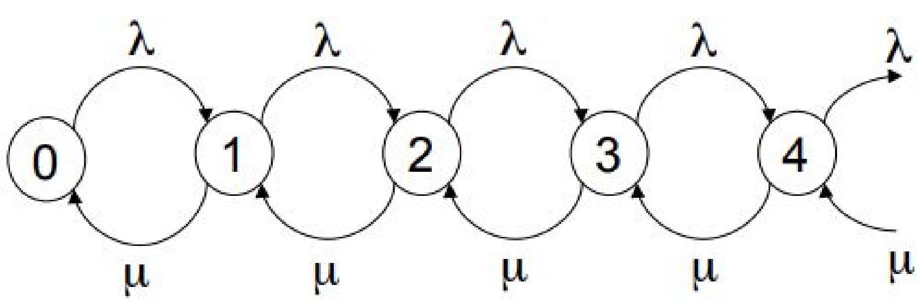
Assignment 1 Report

Based on the description of the system in the assignment, we studied the existing queuing models to find a starting point. It turns out that there exist models that seem to fit the description of the system well. We believe that the M/M/1/K queuing model described in [1] fits the assignment requirements the best. The M/M/1/K queuing model consists of the Poisson arrival rate, exponential service rate, one server, and a limited queue size.

In this model, we assume that when the new batch of transactions arrives, the individual transaction enters the queue one by one. When they are serviced, one transaction will be serviced at a time, as described in the following diagram



With P(n) is the probability of n transactions are in the system and ρ is the ratio of λ and μ. At equilibrium, the rate in = rate out, so

State 0:

$$\mu P_1 = \lambda P_0 \quad \Rightarrow \quad P_1 = \frac{\lambda}{\mu} P_0$$

State1:

$$\lambda P_0 + \mu P_2 = (\lambda + \mu) P_1 \quad \Rightarrow \quad P_2 = \frac{1}{\mu}\left((\lambda + \mu)\frac{\lambda}{\mu} P_0 - \frac{\mu}{\mu}\lambda P_0 \right) = \frac{\lambda^2}{\mu^2} P_0$$

State n:

$$P_n = \frac{\lambda\lambda\cdots\lambda}{\mu\mu\cdots\mu} P_0 = \left(\frac{\lambda}{\mu}\right)^n P_0 = \rho^n P_0$$

And we need $\sum_{n=0}^{K} \rho^n P_0 = 1$, with K, the size of the queue, which leads to $P_0 = \frac{1}{\sum_{n=0}^{K} \rho^n}$

For a finite geometry series like this, the formula is

$$\sum_{n=0}^{N} x^n = \frac{1-x^{N+1}}{1-x}$$

Therefore, the probability that the system is idle (or have 0 transactions in the system) is

$$\begin{aligned} P_0 &= \frac{1}{\sum_{n=0}^{K} (\lambda/\mu)^n} \\ &= 1 \Big/ \left[\frac{1-(\lambda/\mu)^{K+1}}{1-\lambda/\mu} \right] \\ &= \frac{1-\rho}{1-\rho^{K+1}}, \end{aligned}$$

And the probability that the system has n transactions is

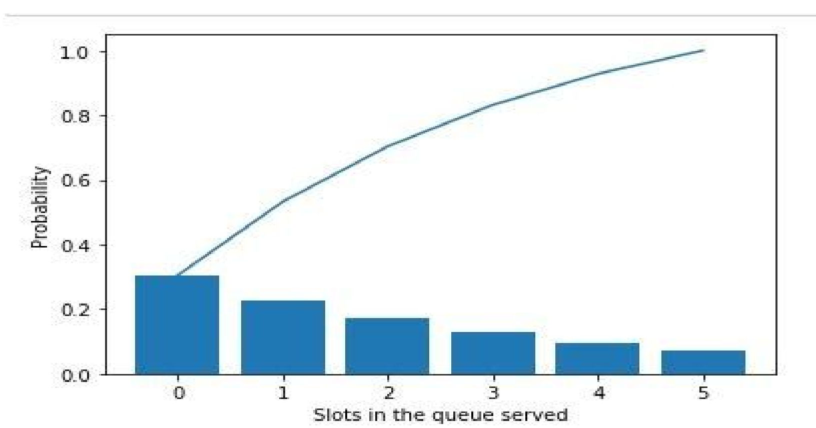
$P_n = P_0 \rho^n$, or $P_n = \frac{1-\rho}{1-\rho^{K+1}} \rho^n,$

Applying the above formulas into a Python simulation with the following values, we obtained the following results

Case-I:

The configurations for this are Queue Size (K) = 5 , λ = 3, μ=4

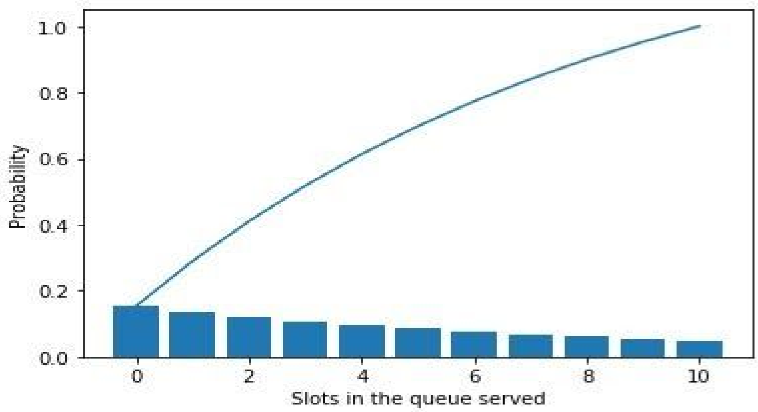
Generated Simulation Model:



Case-II:

The configurations for this are Queue Size (K) = 10 , $\lambda = 8$, $\mu=9$

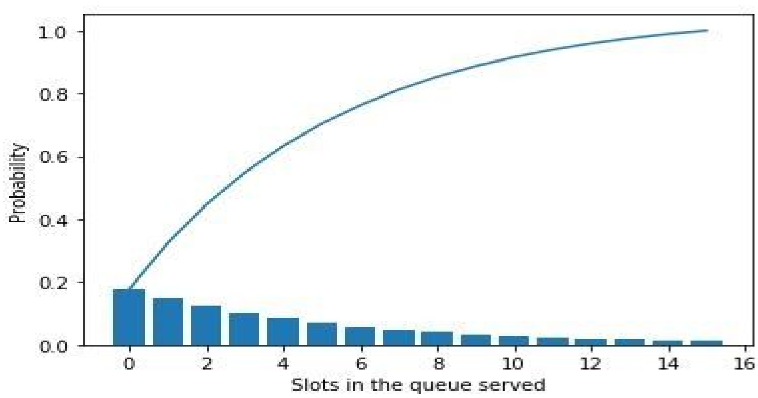
Generated Simulation Model:



Case-III:

The configurations for this are Queue Size (K) = 15 , $\lambda = 5$, $\mu=6$

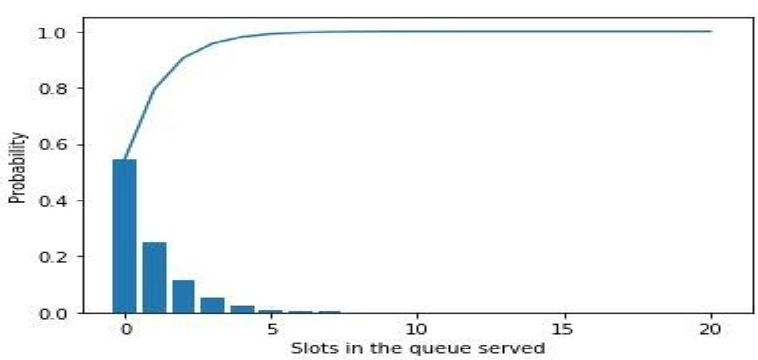
Generated Simulation Model:



Case-IV:

The configurations for this are Queue Size (K) = 20 , $\lambda = 5$, $\mu = 11$

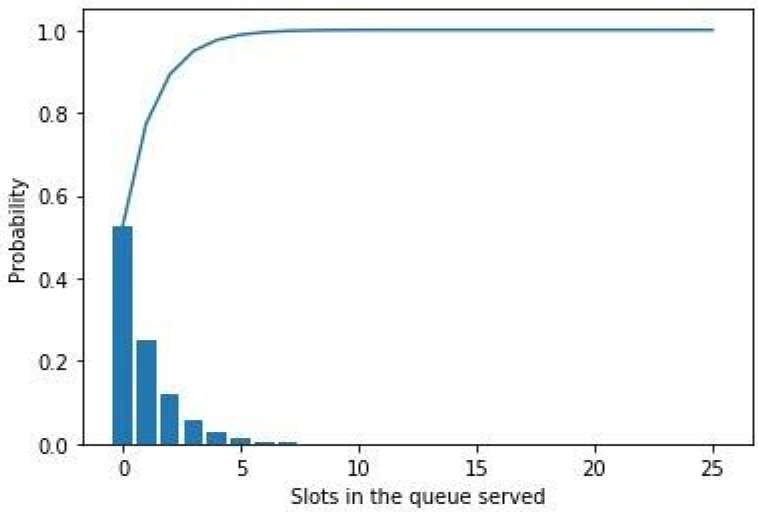
Generated Simulation Model:



Case-V:

The configurations for this are Queue Size (K) = 25 , λ = 10, μ = 21

Generated Simulation Model:



From the above all the 5 cases of different configurations we can see that the cumulative probabilities add up to 1.0 and the individual P(n) values stay within the range of 0.0 to 1.0.

Contribution:

Hai Huynh: 40%

Prudhviraj Sheela: 35%

Aman Masipeddi: 25%

References

[1] Hillier, F. S., & Lieberman, G. J. (2010). Introduction to operations research.