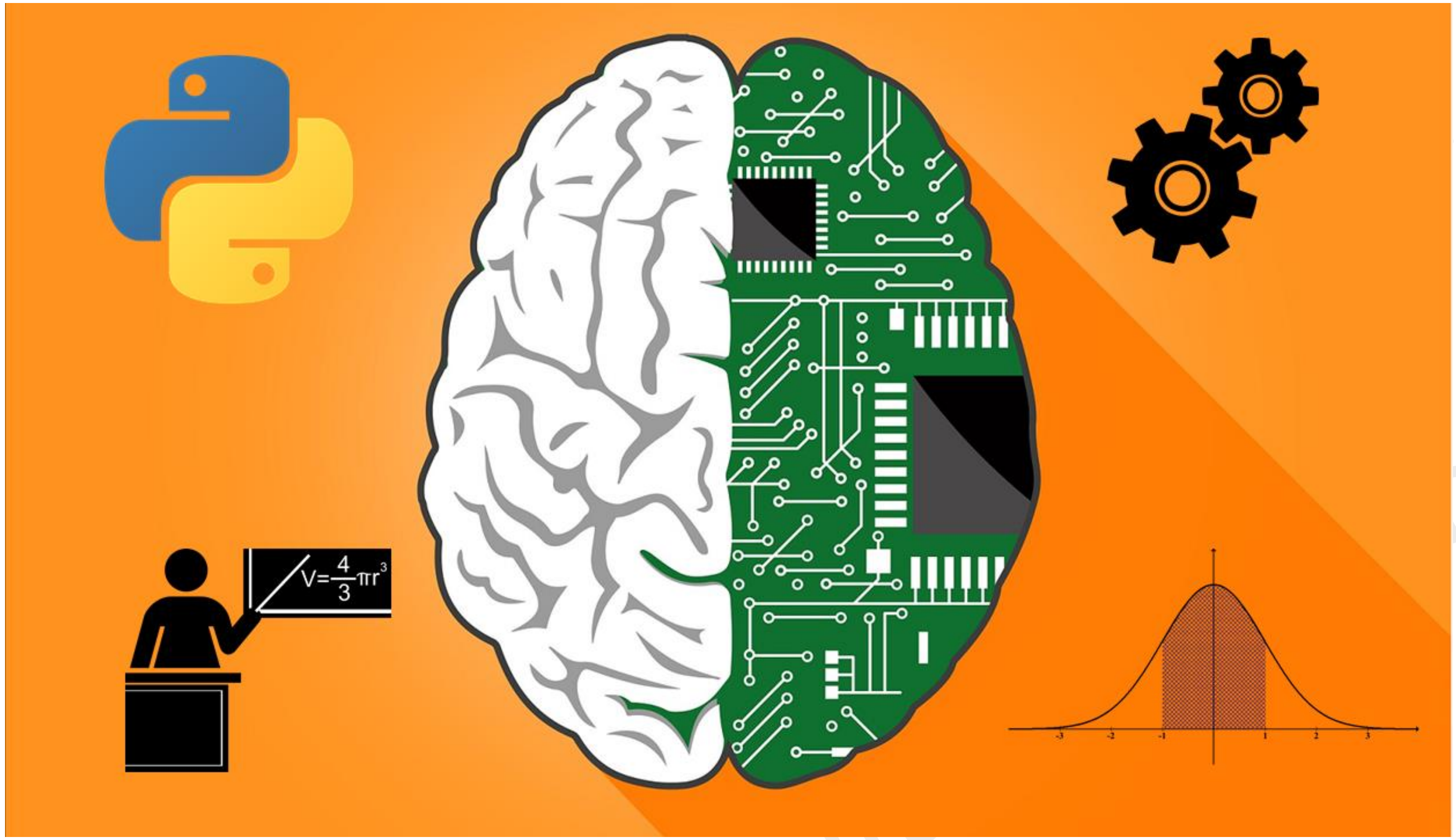


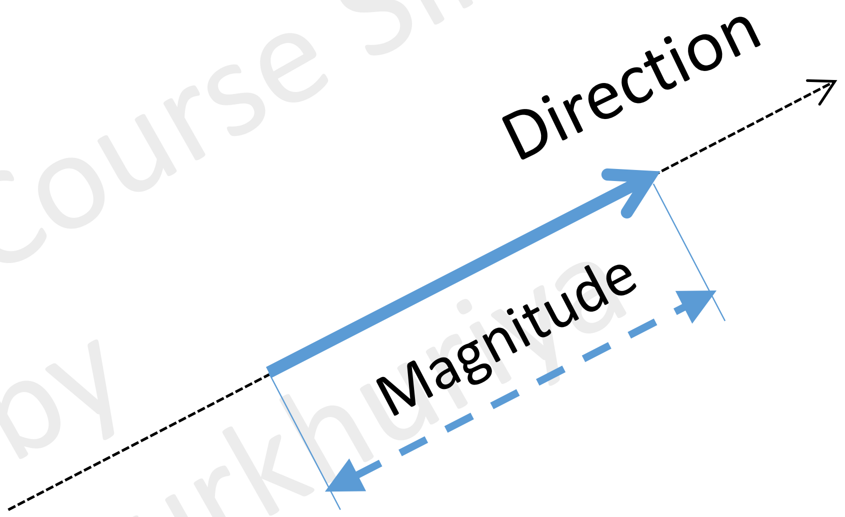
Complete Data Science and Machine Learning Using Python

By
Jitesh Khurkhuriya

Vectors

What is a vector?

$\vec{V}$



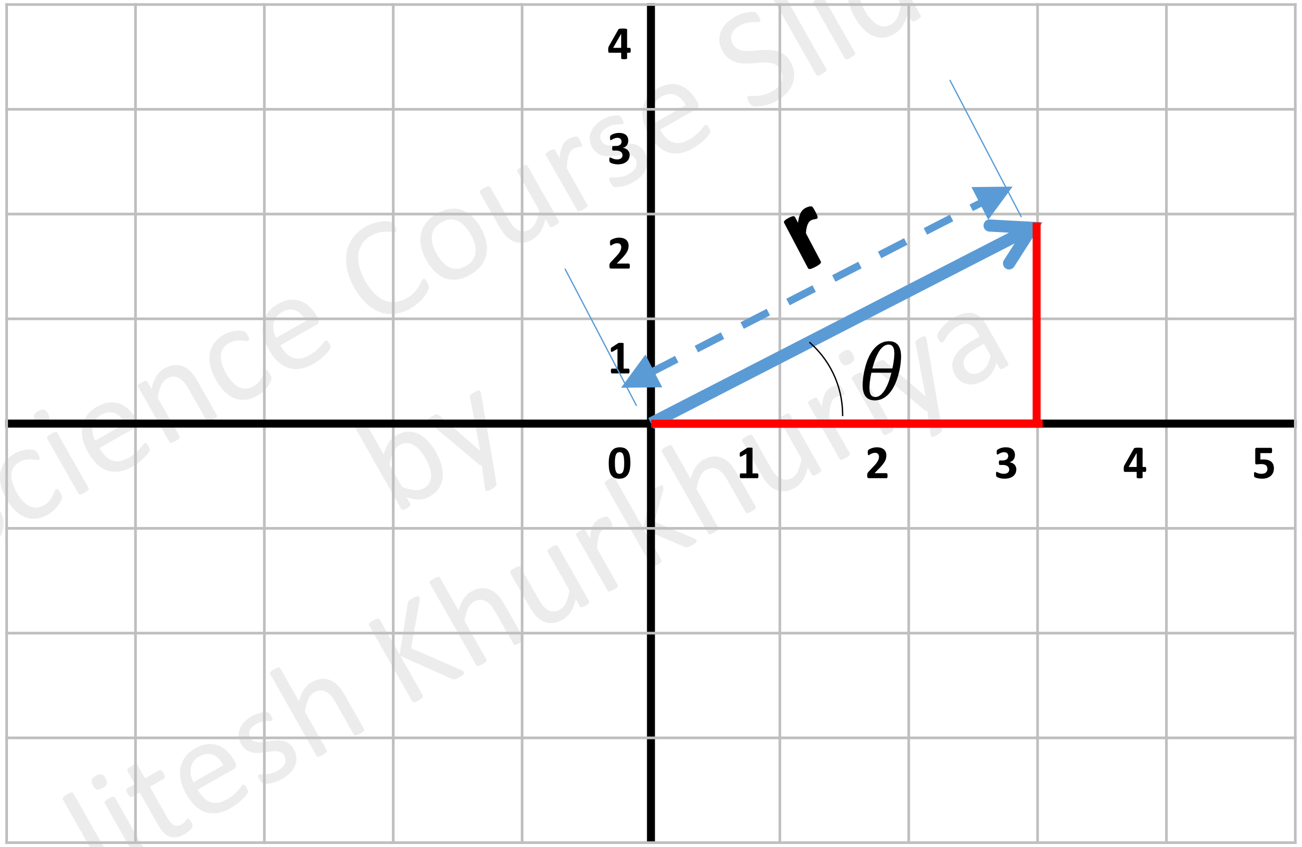
What is a vector?

Cartesian:

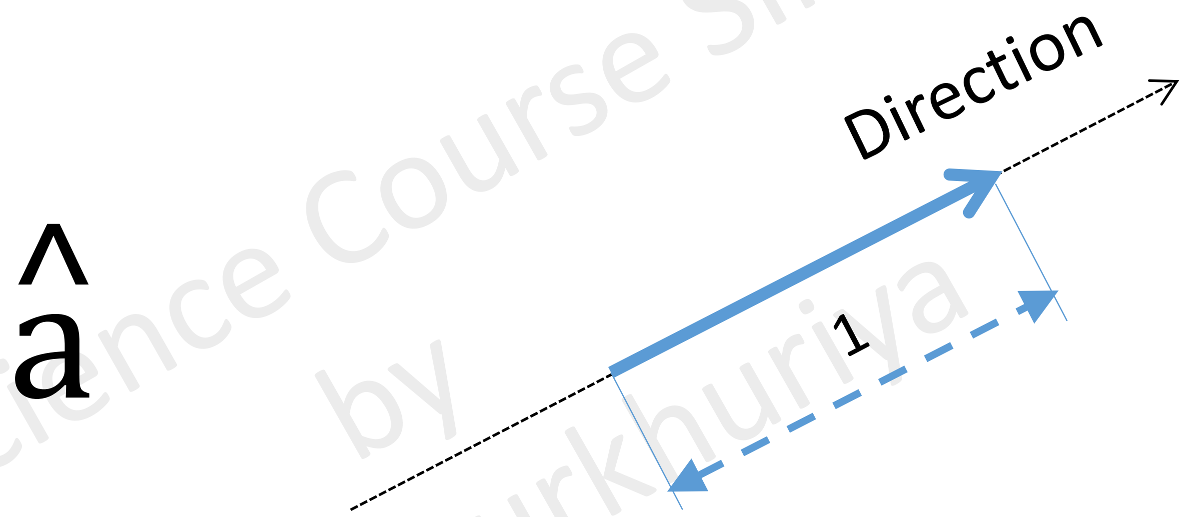
$$\vec{V} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

Polar:

$$\vec{V} = (r, \theta)$$

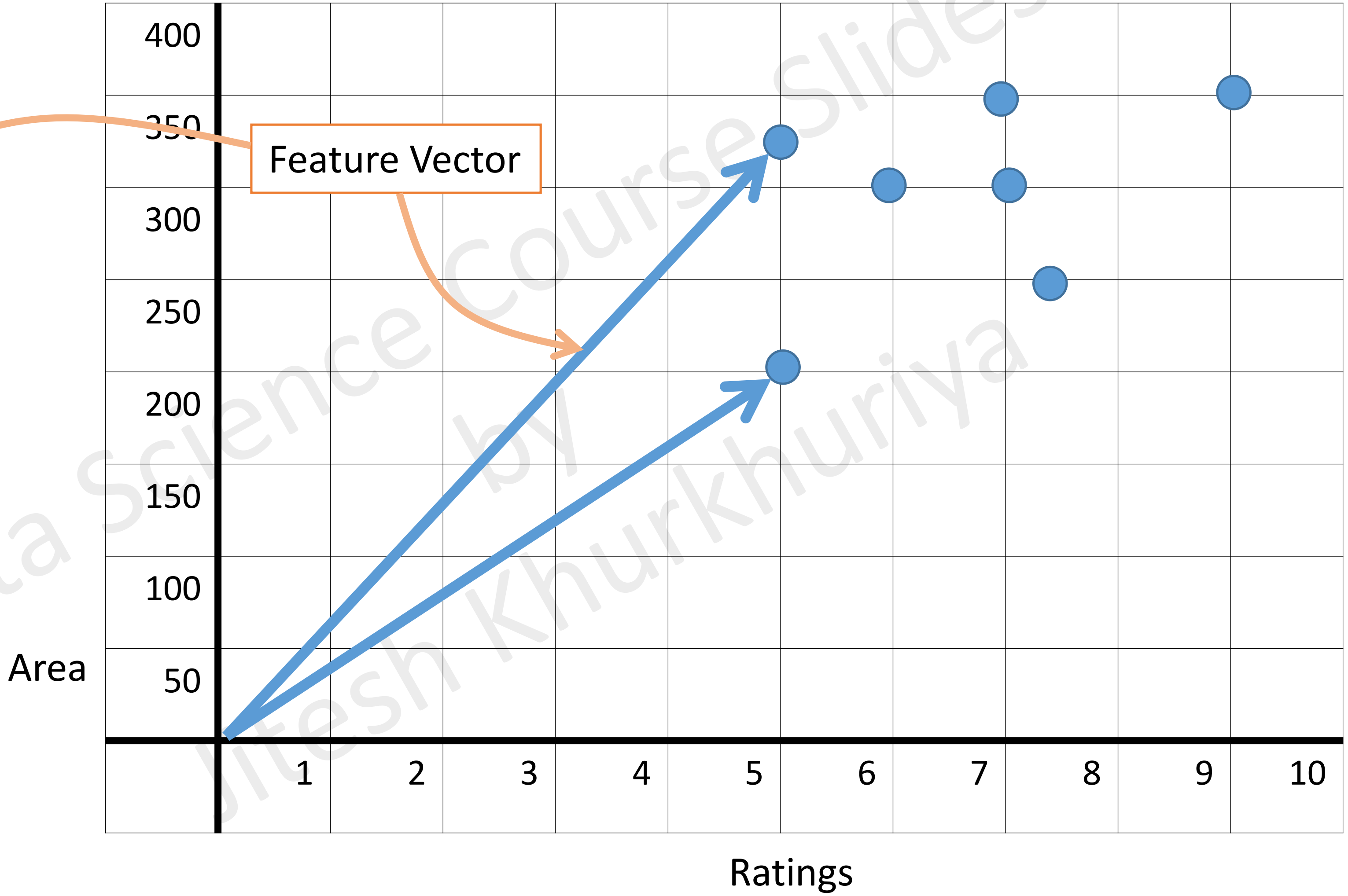


Unit Vector



Vectors in Machine Learning

Rating	Area sq. mtr
5	200
7	300
5	325
8	250
6	300
7	350
7.5	250
9	350



Vector Arithmetic

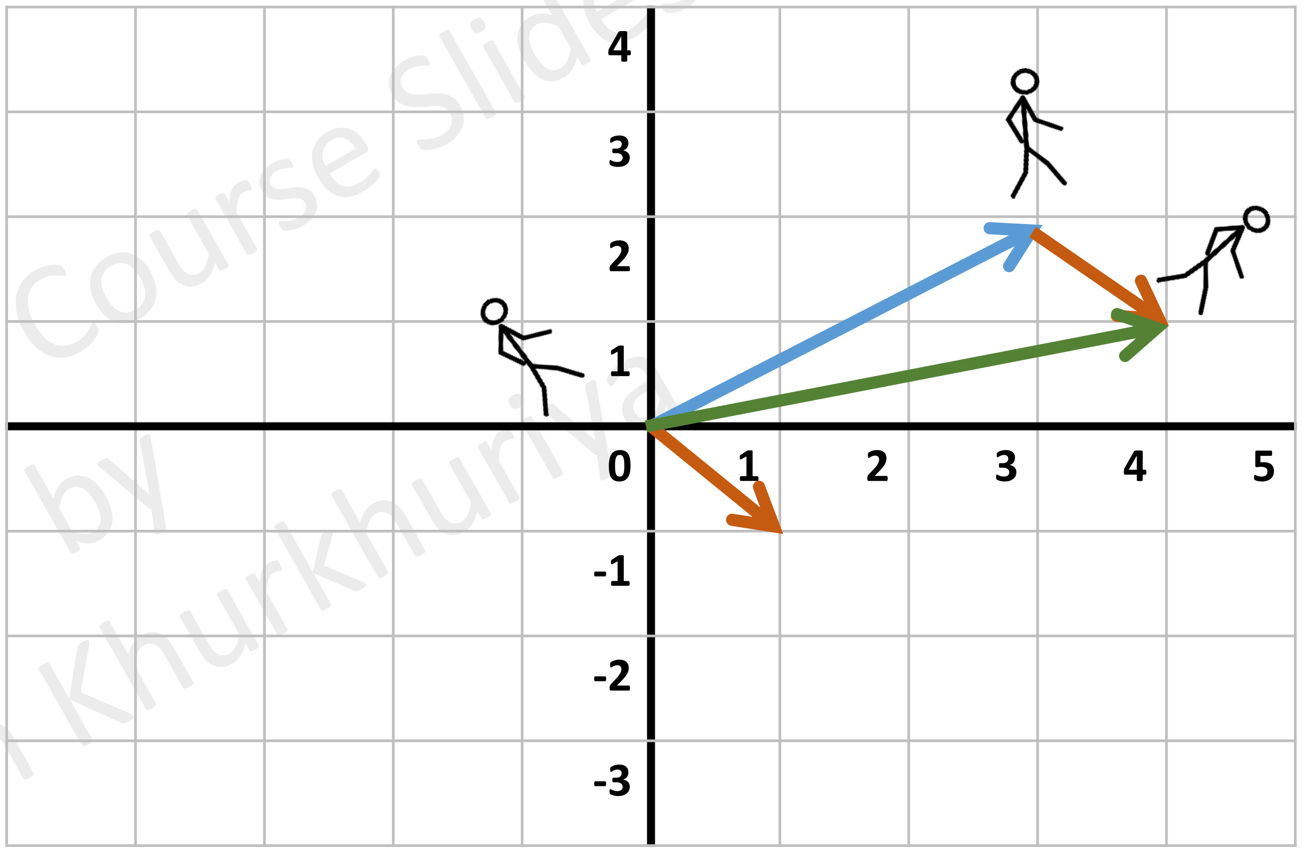
- Addition
- Subtraction
- Multiplication

Vector Addition

$$\vec{V}_1 = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

$$\vec{V}_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$\vec{V}_1 + \vec{V}_2 = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$$

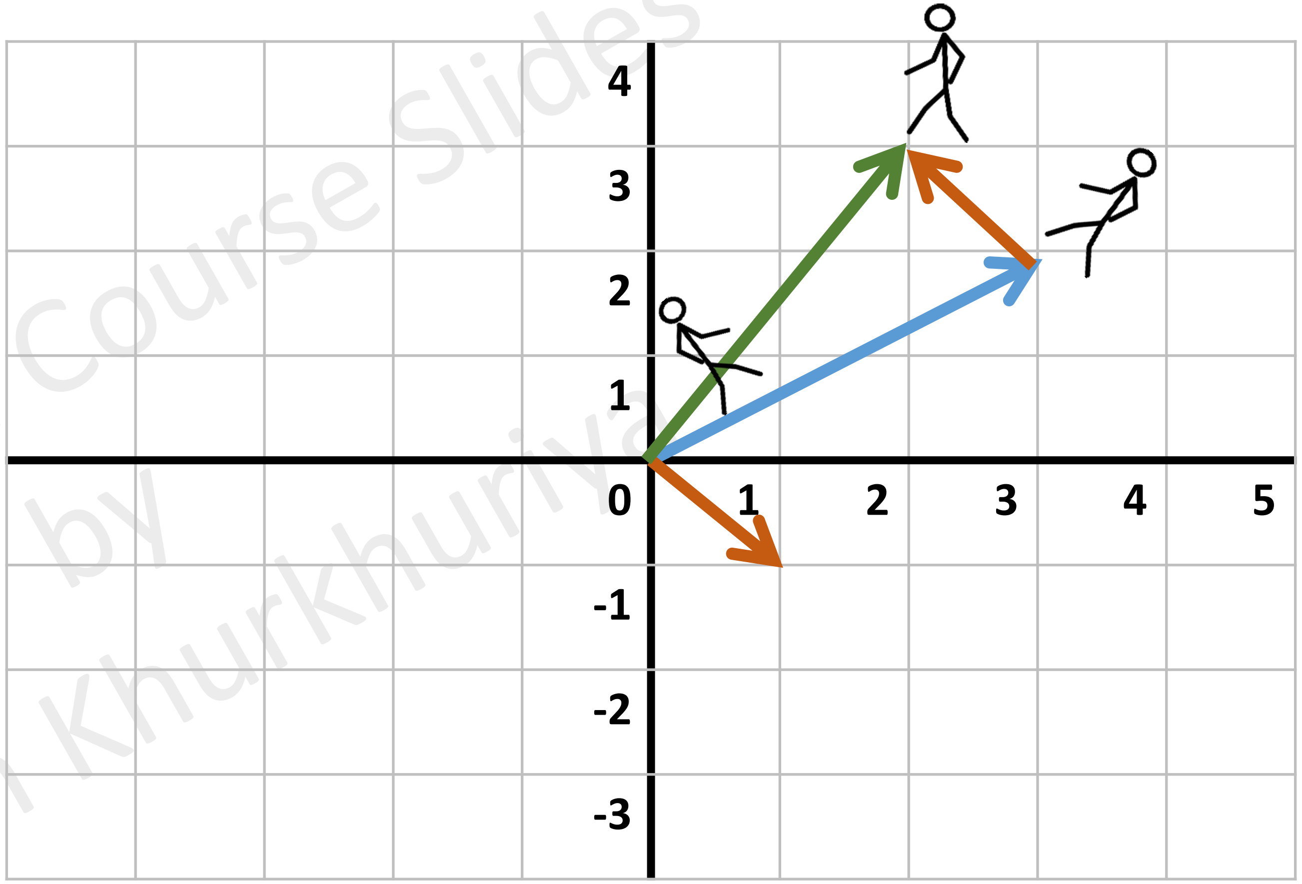


Vector Subtraction

$$\vec{V}_1 = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

$$\vec{V}_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$\vec{V}_1 - \vec{V}_2 = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

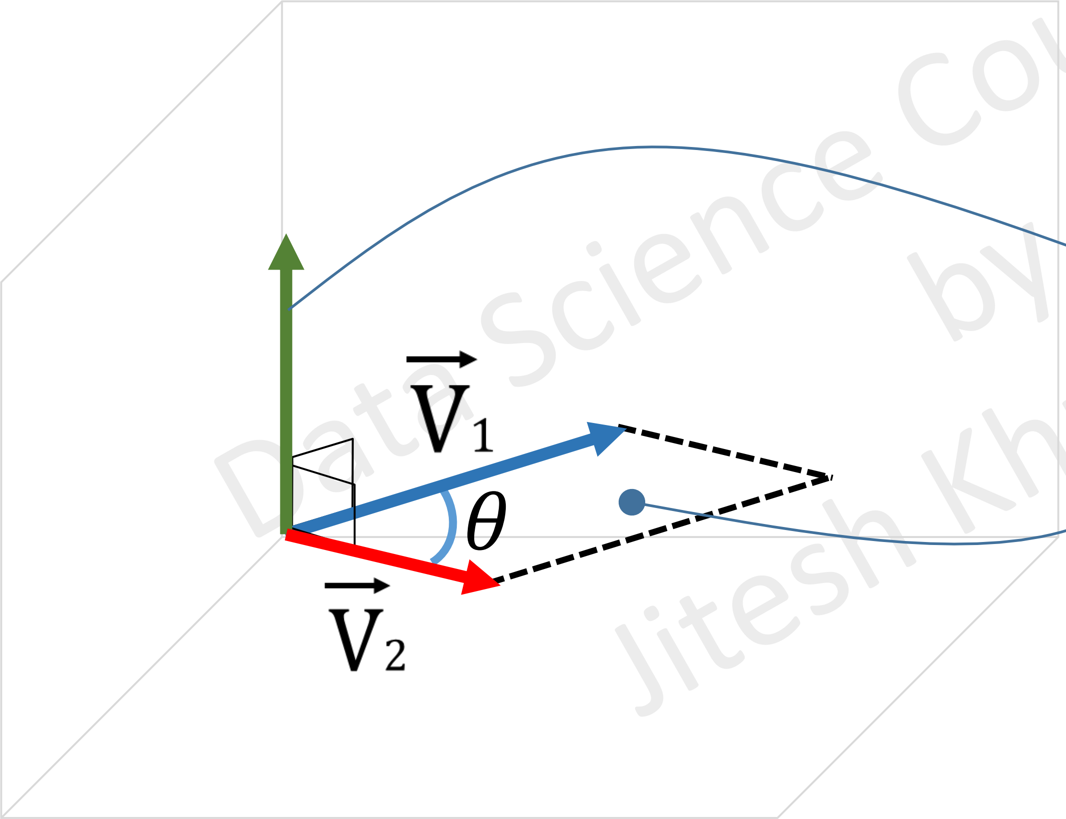


Vector Multiplication

$$\vec{V}_1 * \vec{V}_2 = \boxed{|V_1| * |V_2| \sin(\theta)} * \hat{n}$$

Unit vector

Magnitude



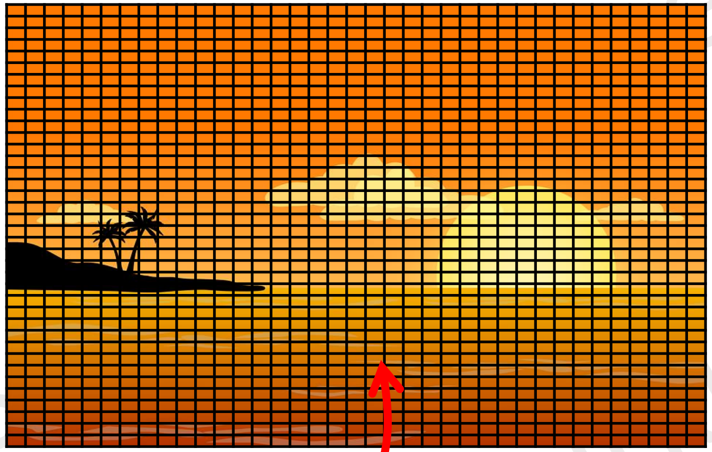
Matrices

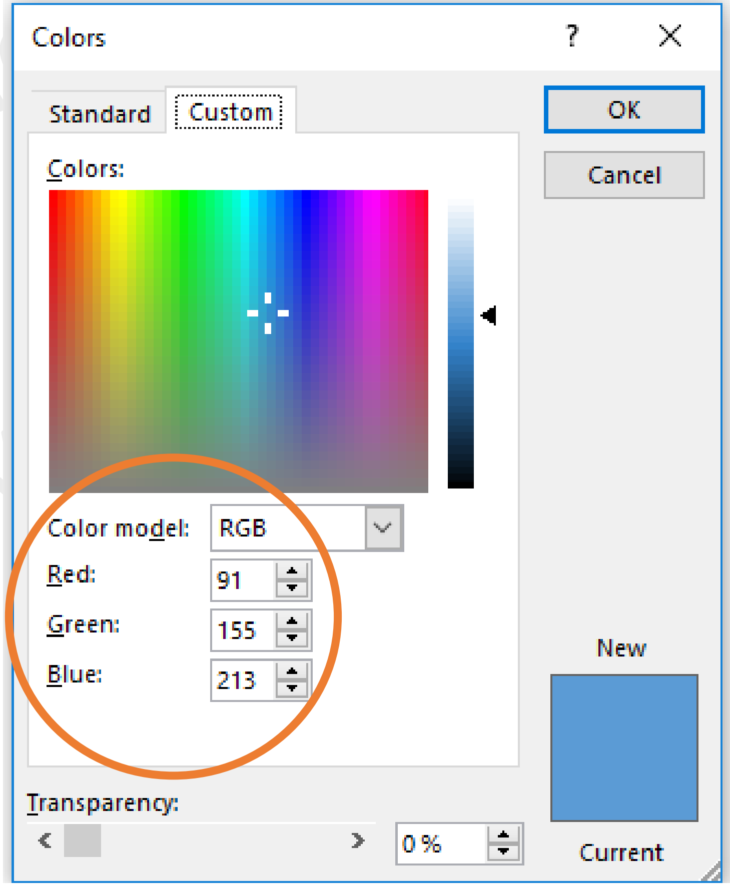
What is a Matrix?

$$X = \underbrace{\begin{bmatrix} 2 & 3 & 4 \\ 1 & 6 & 7 \end{bmatrix}}_{\text{Columns}} \left.\vphantom{\begin{bmatrix} 2 \\ 1 \end{bmatrix}}\right\} \text{Rows}$$

Why should we learn Matrices?

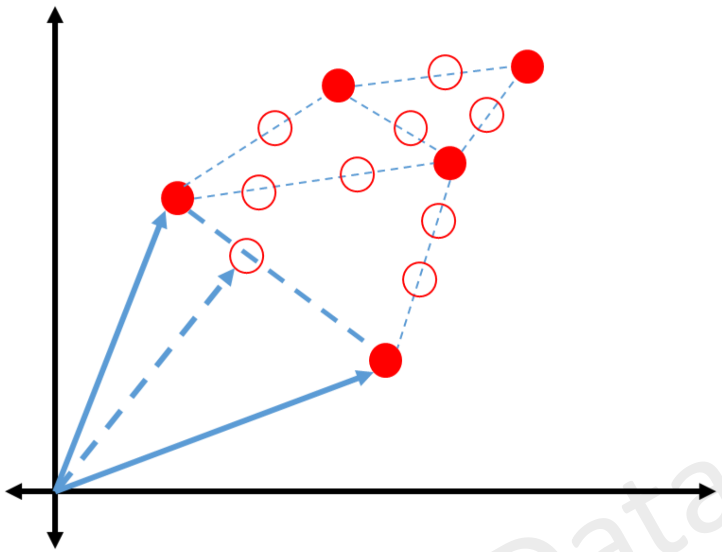
Matrix of Pixels



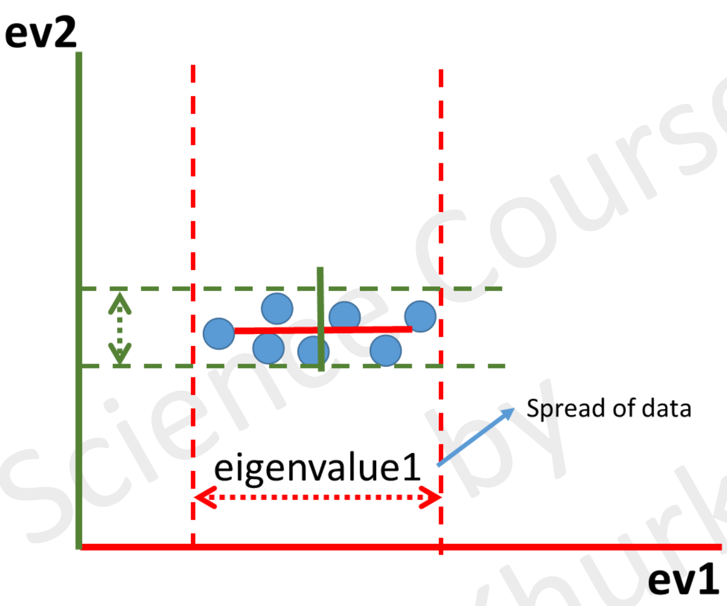


$$\begin{array}{c c} R & \\ G & \\ B & \end{array} \begin{bmatrix} 230 \\ 169 \\ 43 \end{bmatrix}$$

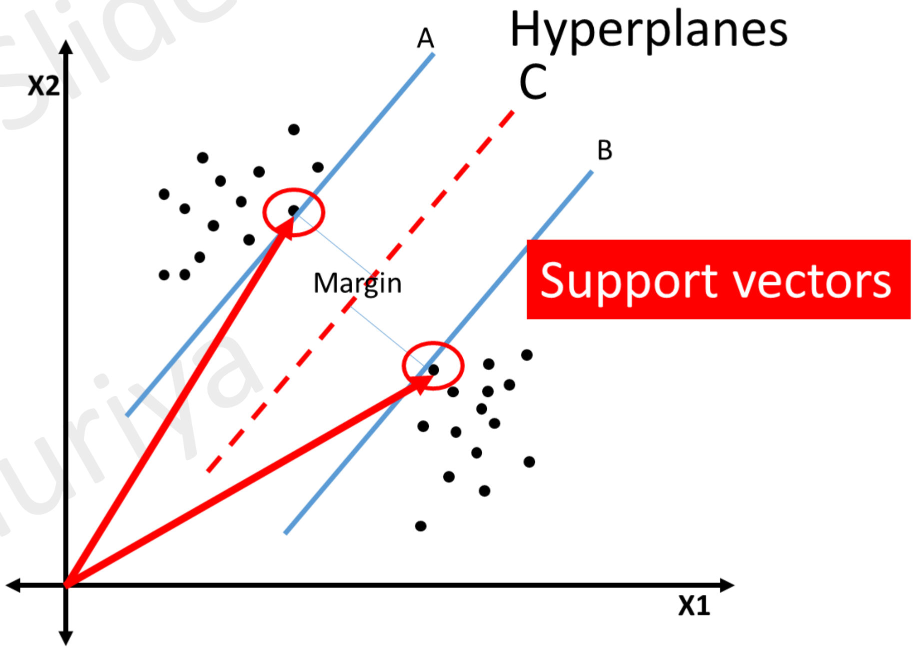
Some more examples of Vectors and Matrices



SMOTE



PCA



SVM

Matrix Addition

$$X = \begin{bmatrix} 2 & 3 & 4 \\ 1 & 6 & 7 \end{bmatrix} \qquad Y = \begin{bmatrix} 1 & 8 & -1 \\ 5 & -2 & -3 \end{bmatrix}$$

$$X + Y = \begin{bmatrix} 2+1 & 3+8 & 4+(-1) \\ 1+5 & 6+(-2) & 7+(-3) \end{bmatrix} = \begin{bmatrix} 3 & 11 & 3 \\ 6 & 4 & 4 \end{bmatrix}$$

Matrix Subtraction

$$X = \begin{bmatrix} 2 & 3 & 4 \\ 1 & 6 & 7 \end{bmatrix} \qquad Y = \begin{bmatrix} 1 & 8 & -1 \\ 5 & -2 & -3 \end{bmatrix}$$

$$X - Y = \begin{bmatrix} 2-1 & 3-8 & 4-(-1) \\ 1-5 & 6-(-2) & 7-(-3) \end{bmatrix} = \begin{bmatrix} 1 & -5 & 5 \\ -4 & 8 & 10 \end{bmatrix}$$

Matrix Multiplication – Scalar

$$X = \begin{bmatrix} 2 & 3 & 4 \\ 1 & 6 & 7 \end{bmatrix}$$

$$2 * X = 2 * \begin{bmatrix} 2 & 3 & 4 \\ 1 & 6 & 7 \end{bmatrix} \Rightarrow \begin{bmatrix} 4 & 6 & 8 \\ 2 & 12 & 14 \end{bmatrix}$$

Matrix Multiplication – Dot product

$$X = \begin{bmatrix} 2 & 3 & 4 \\ 1 & 6 & 7 \end{bmatrix} \qquad A = \begin{bmatrix} 2 & 3 \\ 1 & 6 \\ 2 & 3 \end{bmatrix}$$

$$X\,.\,A = \begin{bmatrix} 2 & 3 & 4 \\ 1 & 6 & 7 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 1 & 6 \\ 2 & 3 \end{bmatrix}$$

Matrix Multiplication – Dot product

$$X = \begin{bmatrix} 2 & 3 & 4 \\ 1 & 6 & 7 \end{bmatrix} \qquad A = \begin{bmatrix} 2 & 3 \\ 1 & 6 \\ 2 & 3 \end{bmatrix}$$

$$X \,.\, A = \begin{bmatrix} 2 & 3 & 4 \\ 1 & 6 & 7 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 1 & 6 \\ 2 & 3 \end{bmatrix}$$

Matrix Multiplication – Dot product

$$X = \begin{bmatrix} 2 & 3 & 4 \\ 1 & 6 & 7 \end{bmatrix} \qquad A = \begin{bmatrix} 2 & 3 \\ 1 & 6 \\ 2 & 3 \end{bmatrix}$$

$$X \,.\, A = \begin{bmatrix} 2 & 3 & 4 \\ 1 & 6 & 7 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 1 & 6 \\ 2 & 3 \end{bmatrix} \Rightarrow \begin{bmatrix} 15 & 36 \\ 22 & 60 \end{bmatrix}$$

$$(1*3) + (6*6) + (7*3) = 60$$

Matrix Multiplication – Dot product

$$X = \begin{bmatrix} 2 & 3 & 4 \\ 1 & 6 & 7 \end{bmatrix} \qquad A = \begin{bmatrix} 2 & 3 \\ 1 & 6 \\ 2 & 3 \end{bmatrix} \qquad X\,.\,A = \begin{bmatrix} 15 & 36 \\ 22 & 60 \end{bmatrix}$$

$$2 \; X \; \boxed{3 \qquad\qquad 3} \; X \; 2 \qquad\qquad 2 \; X \; 2$$

Matrix Division

$$A = \begin{bmatrix} 3 & 4 & -1 \\ 7 & -3 & 2 \end{bmatrix}$$

$$X = \begin{bmatrix} 2 & 3 & 4 \\ 1 & 6 & 7 \end{bmatrix}$$

$$\frac{A}{X} = A \,.\, X^{-1}$$

Important Matrix Terms

Identity Matrix

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Identity Matrix

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad A = \begin{bmatrix} 2 & 3 & 4 \\ 1 & 6 & 1 \\ 2 & 3 & 8 \end{bmatrix}$$

$$A * I = A$$

Determinant of a Matrix

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$\text{Determinant} = ad - bc$$

Inverse of a Matrix

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$1/A = \text{Inverse of A} = A^{-1}$$

$$A^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

Transpose of a matrix

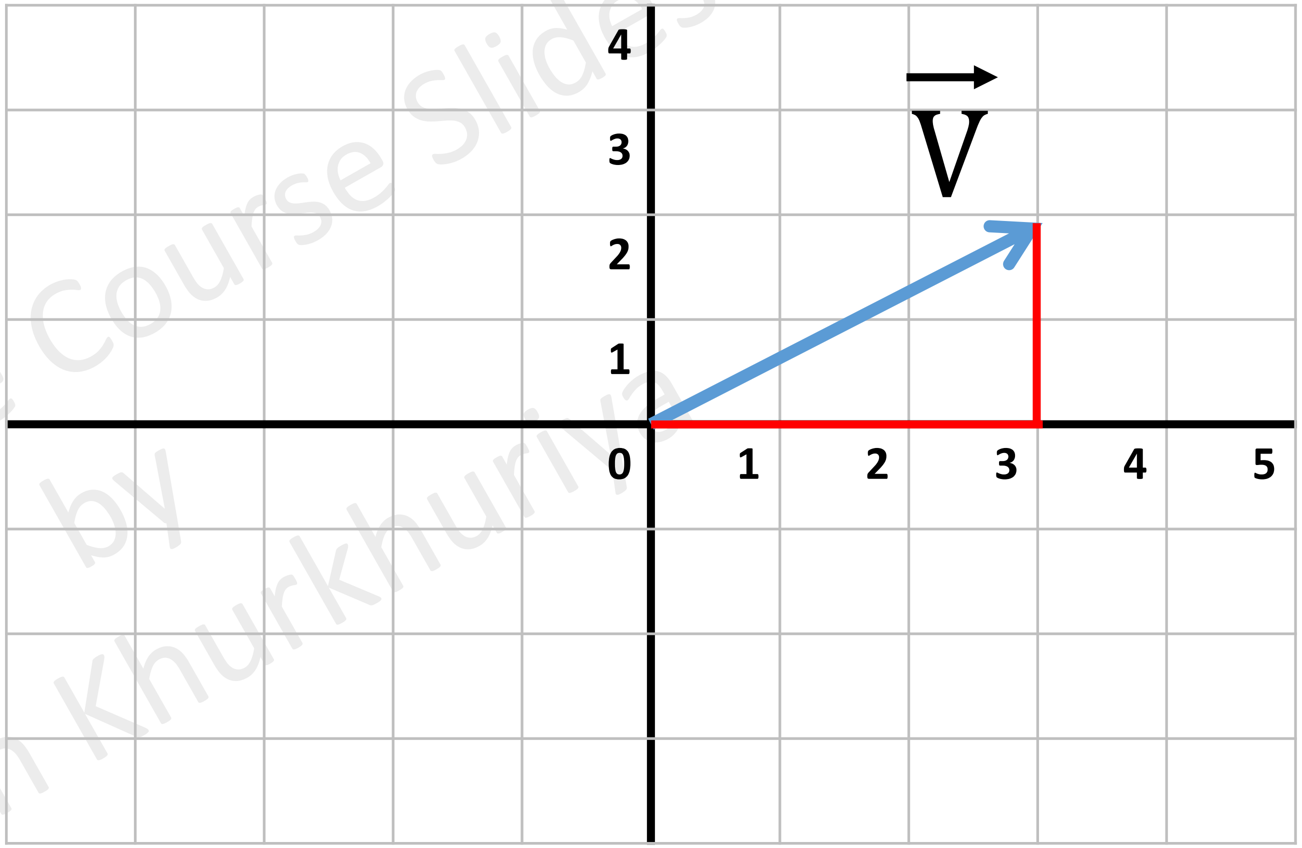
$$X = \begin{bmatrix} 2 & 3 & 4 \\ 1 & 6 & 7 \end{bmatrix} \Rightarrow X^T = \begin{bmatrix} 2 & 1 \\ 3 & 6 \\ 4 & 7 \end{bmatrix}$$

Vector Transformation using Matrix

Vector Transformation

$$T = \begin{bmatrix} \boxed{1 \quad -1} \\ \boxed{1 \quad\ \ 2} \end{bmatrix} \qquad \vec{V} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

2 x 2   2 x 1



$$T \, . \, \vec{V} = \begin{bmatrix} (1*3) + (-1*2) \\ (1*3) + (2*2) \end{bmatrix}$$

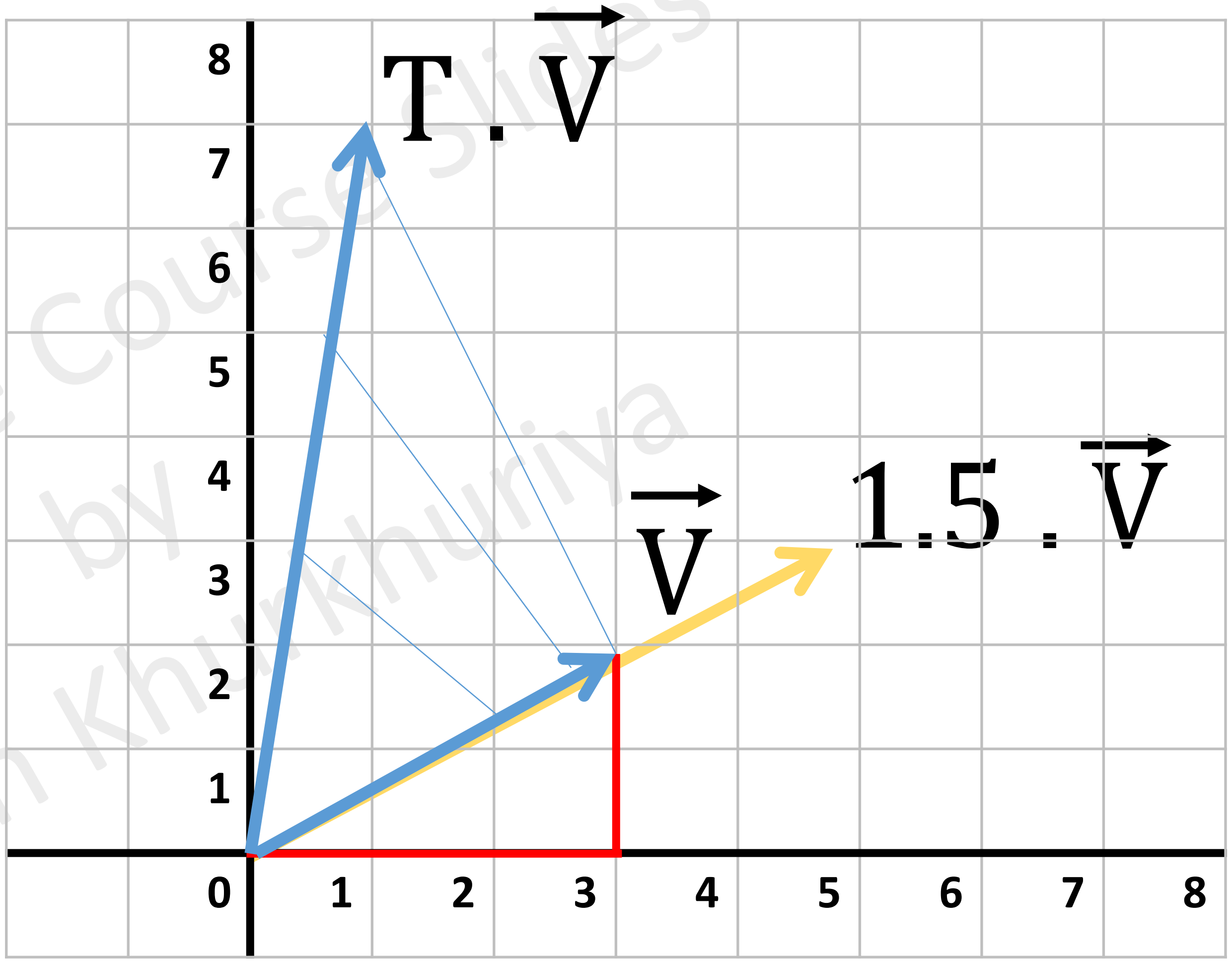
$$= \begin{bmatrix} 3 - 2 \\ 3 + 4 \end{bmatrix}$$

Vector Transformation

$$T = \begin{bmatrix} 1 & -1 \\ 1 & 2 \end{bmatrix} \qquad \vec{V} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

$$T . \vec{V} = \begin{bmatrix} 1 \\ 7 \end{bmatrix}$$

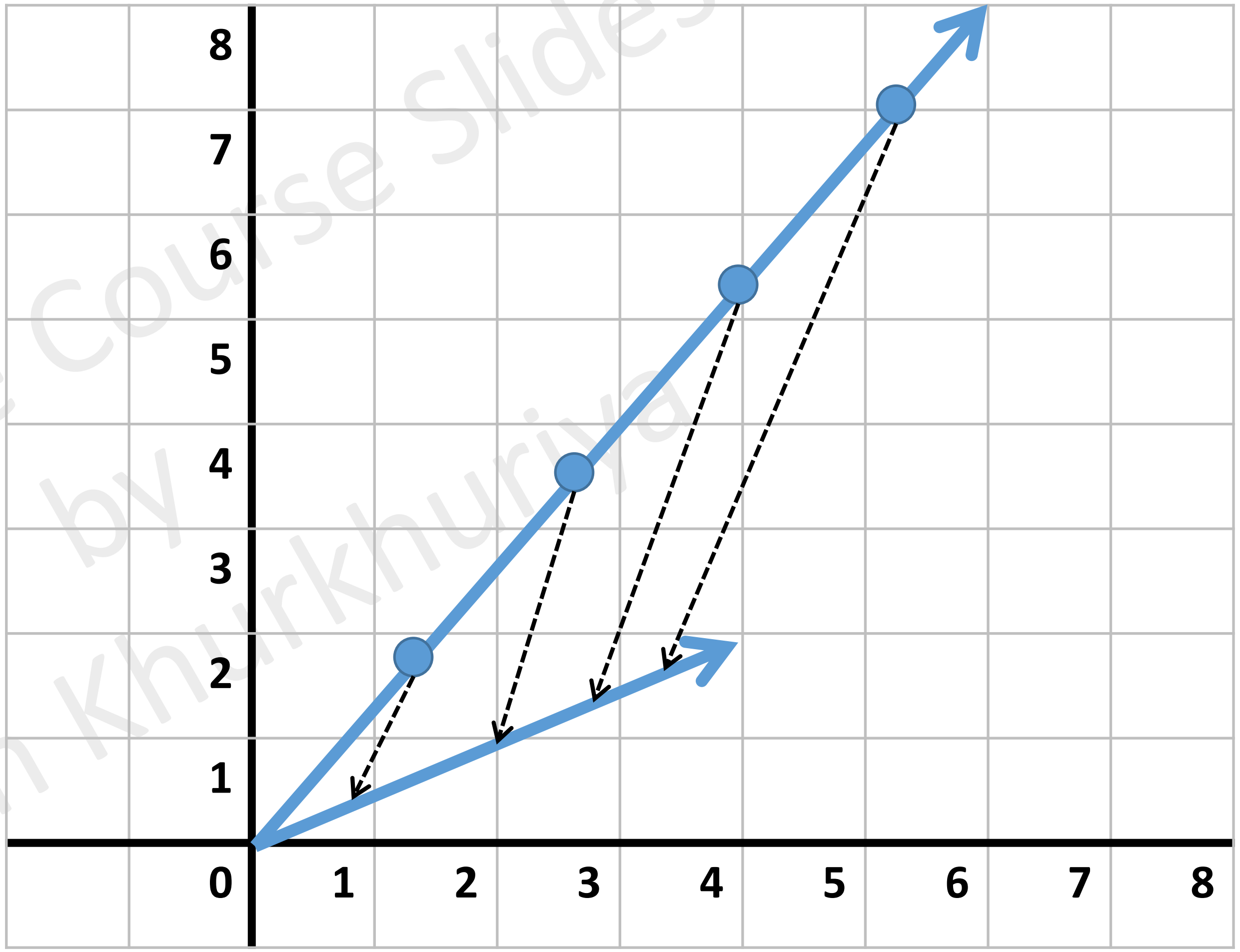
$$1.5 . \vec{V} = \begin{bmatrix} 4.5 \\ 3 \end{bmatrix}$$



Vector Transformation

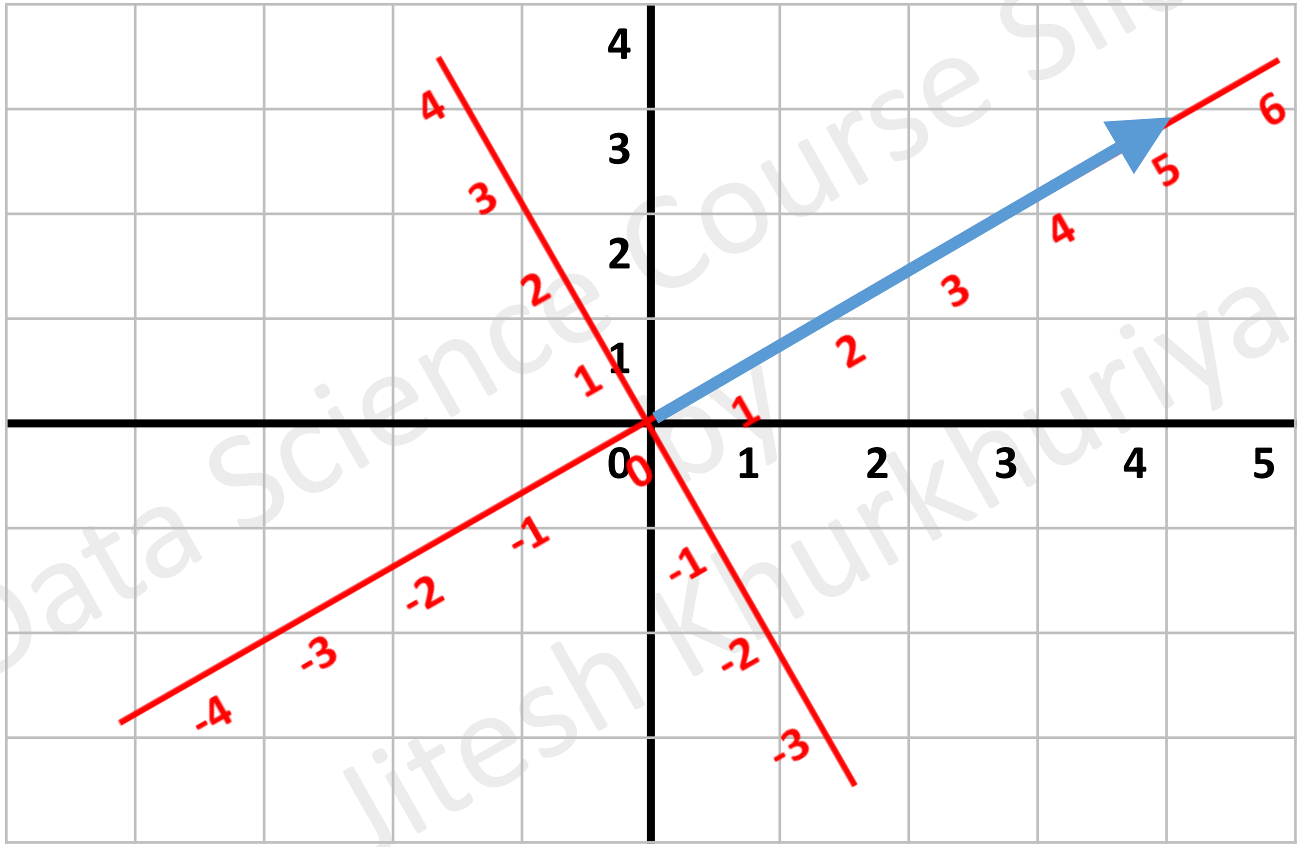
$$T = \begin{bmatrix} 2 & -1 \\ 1 & -0.5 \end{bmatrix} \qquad \vec{V} = \begin{bmatrix} 6 \\ 8 \end{bmatrix}$$

$$T\,.\,\vec{V} = \begin{bmatrix} 4 \\ 2 \end{bmatrix}$$



Change of Basis

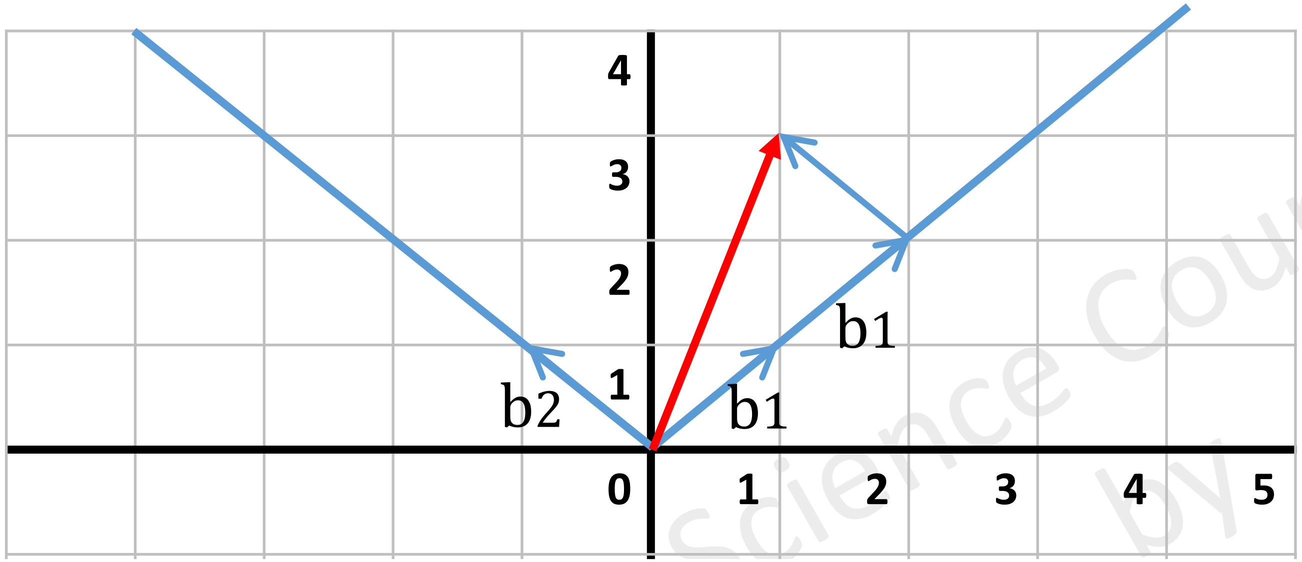
Change of Basis – Alternate Coordinates



$$\vec{V} = \begin{bmatrix} 4 \\ 3 \end{bmatrix}$$

$$\vec{V} = \begin{bmatrix} 5 \\ 0 \end{bmatrix}$$

Change of Basis – Alternate Coordinates

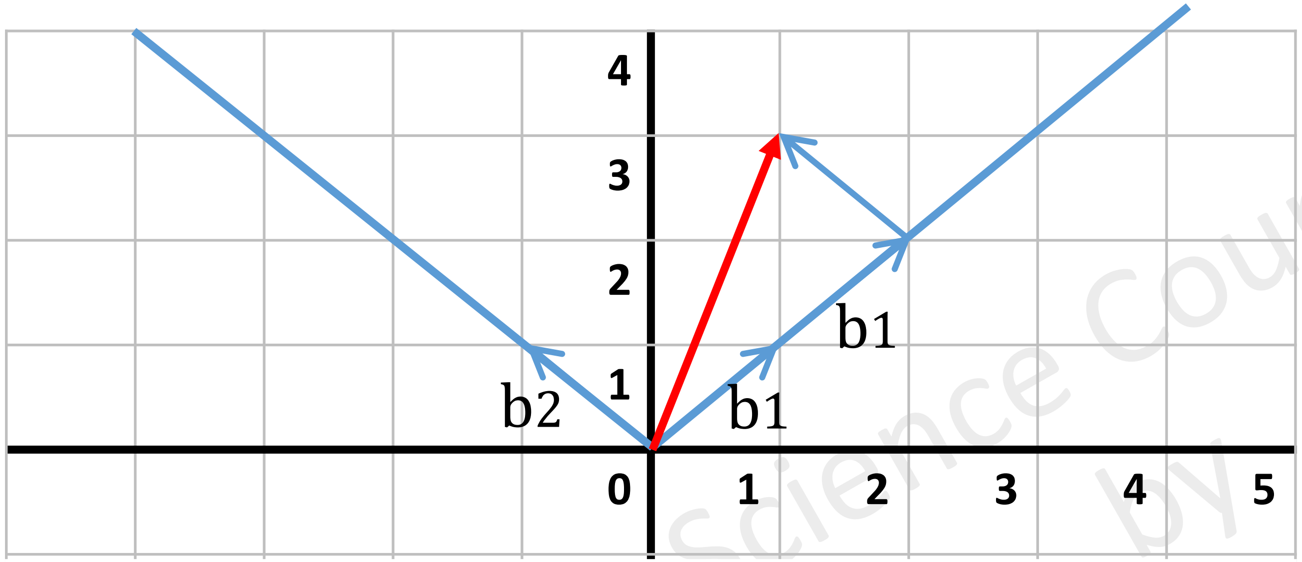


$$b_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \qquad b_2 = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

$$\vec{W} = \begin{pmatrix} 2 \\ 1 \end{pmatrix} = 2b_1 + b_2$$

$$\vec{V} = 2\begin{pmatrix} 1 \\ 1 \end{pmatrix} + 1\begin{pmatrix} -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 2-1 \\ 2+1 \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$$

Change of Basis – Alternate Coordinates

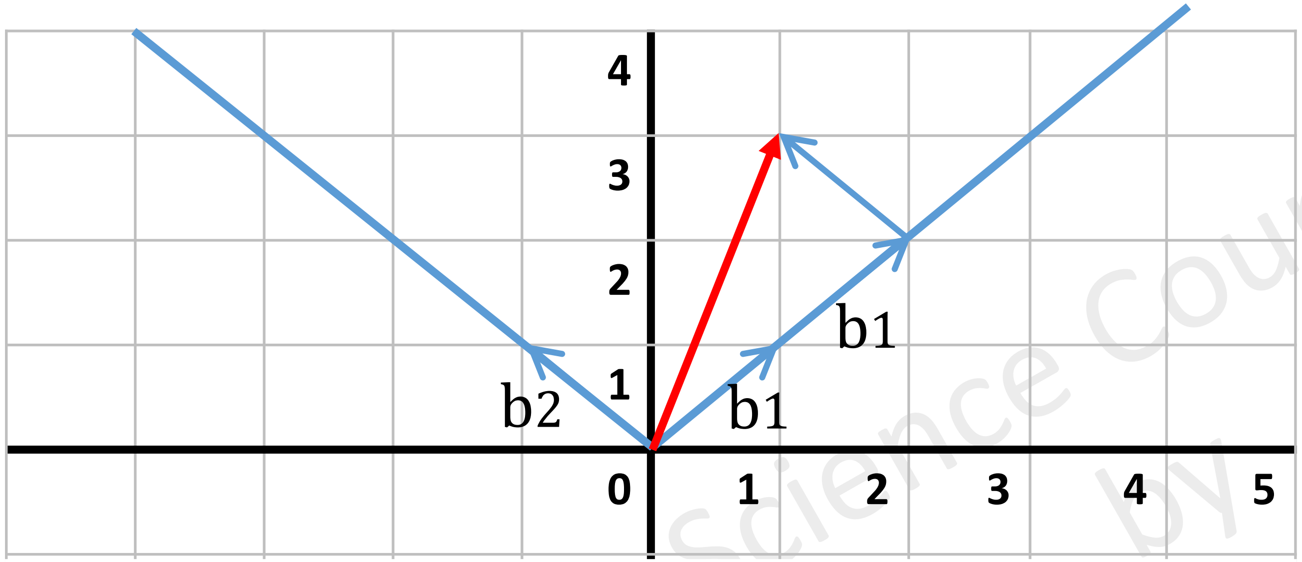


$$b_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \qquad b_2 = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

$$\vec{W} = \begin{pmatrix} 2 \\ 1 \end{pmatrix} = 2b_1 + b_2$$

$$\vec{V} = 2\begin{pmatrix} 1 \\ 1 \end{pmatrix} + 1\begin{pmatrix} -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 2-1 \\ 2+1 \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \end{pmatrix} = \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

Change of Basis – Alternate Coordinates

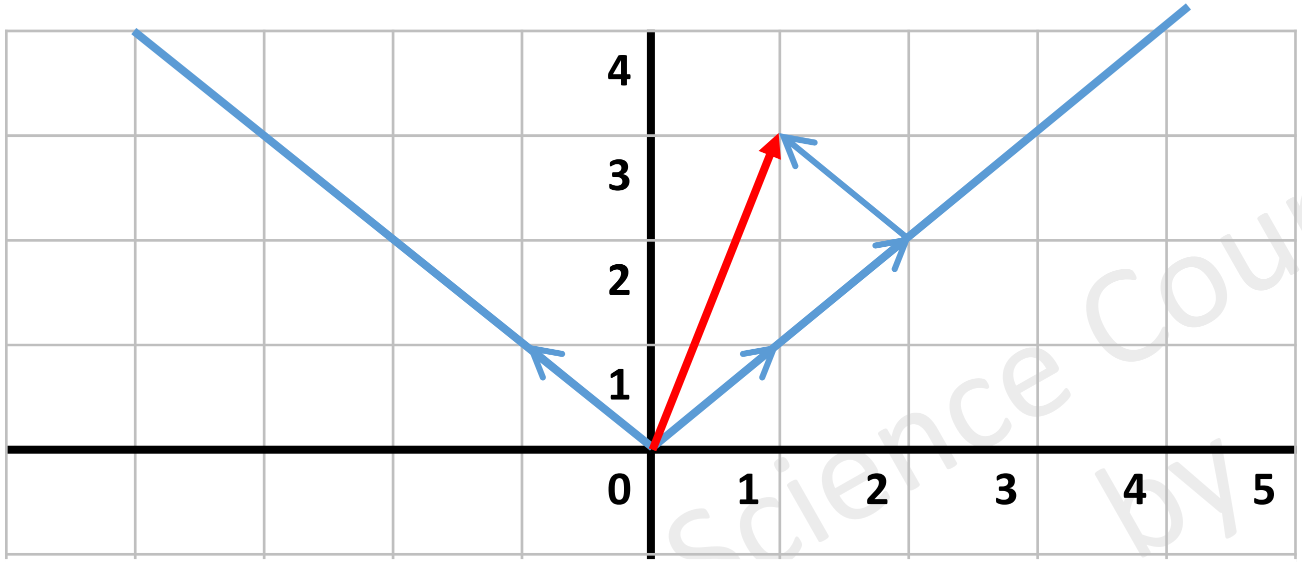


$$b_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \qquad b_2 = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

$$\vec{W} = \begin{pmatrix} 2 \\ 1 \end{pmatrix} = 2b_1 + b_2$$

$$\vec{V} = \begin{pmatrix} 1 \\ 3 \end{pmatrix} = \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

Change of Basis – Alternate Coordinates



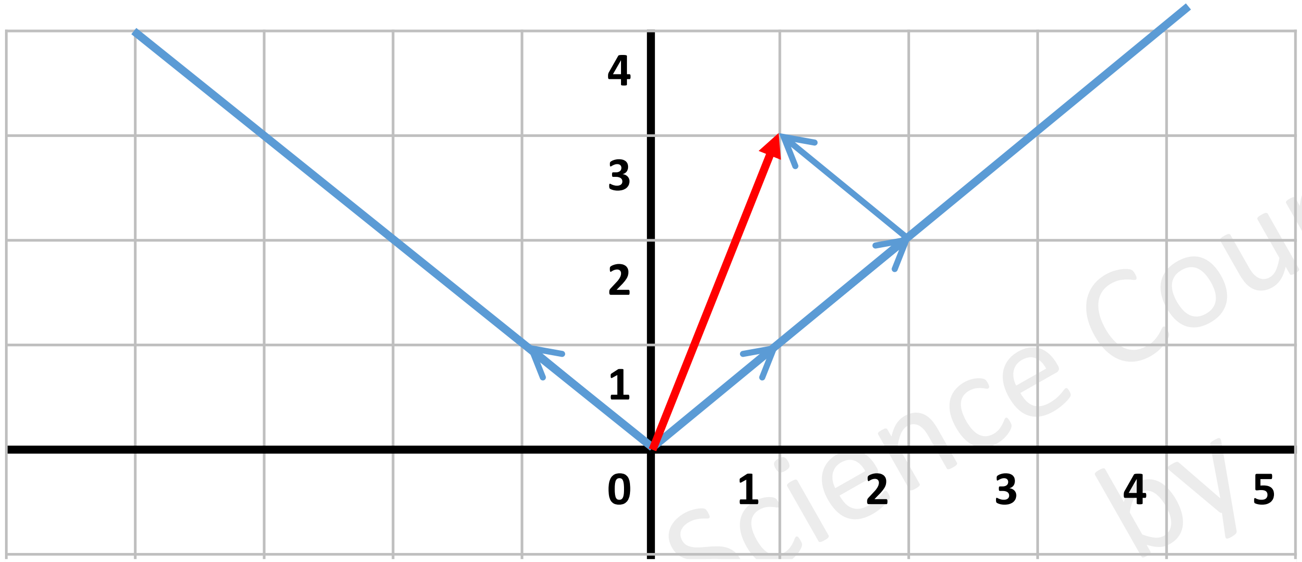
$$b_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \quad b_2 = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

$$\vec{W} = \begin{pmatrix} 2 \\ 1 \end{pmatrix} = 2b_1 + b_2$$

$$\vec{V} = \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} . \vec{W}$$

Matrix Transformation of $\vec{W}$

Change of Basis – Alternate Coordinates

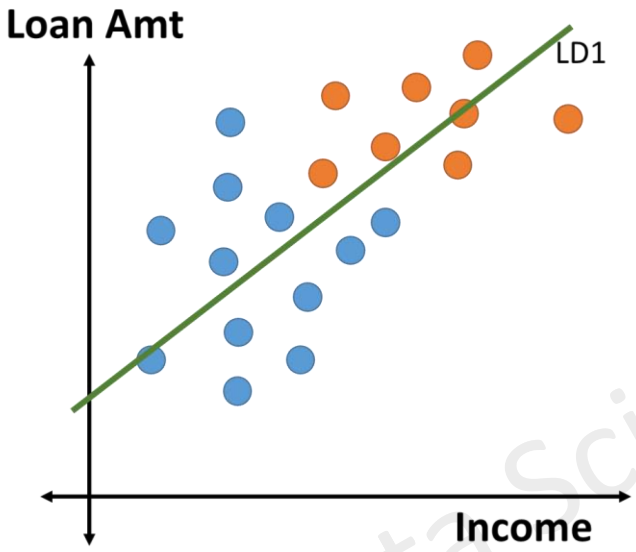


$$b_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \quad b_2 = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

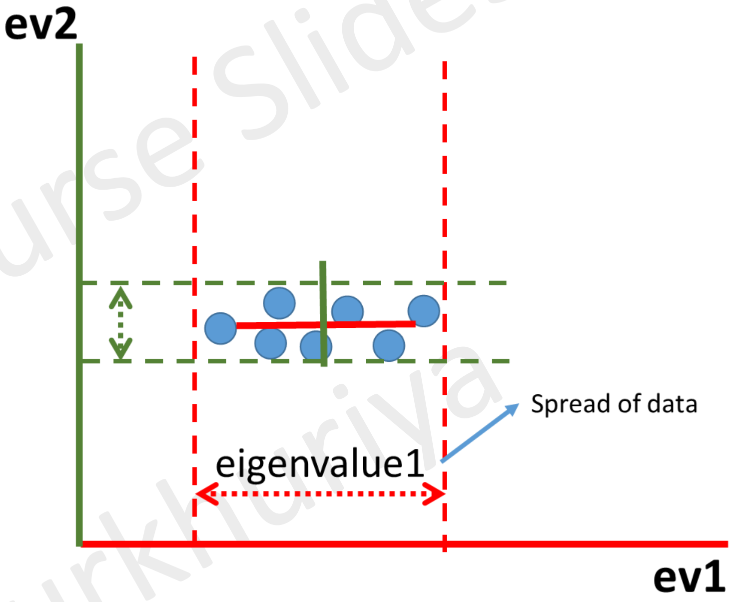
$$\vec{W} = \begin{pmatrix} 2 \\ 1 \end{pmatrix} = 2b_1 + b_2$$

$$\vec{W} = \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}^{-1} . \vec{V}$$

Why we are learning this?



Linear Discriminant Analysis



Principal Component Analysis

Eigenvectors and Eigenvalues

Eigenvector and Eigenvalues?

- A non-zero vector that changes by a scalar during linear transformation.

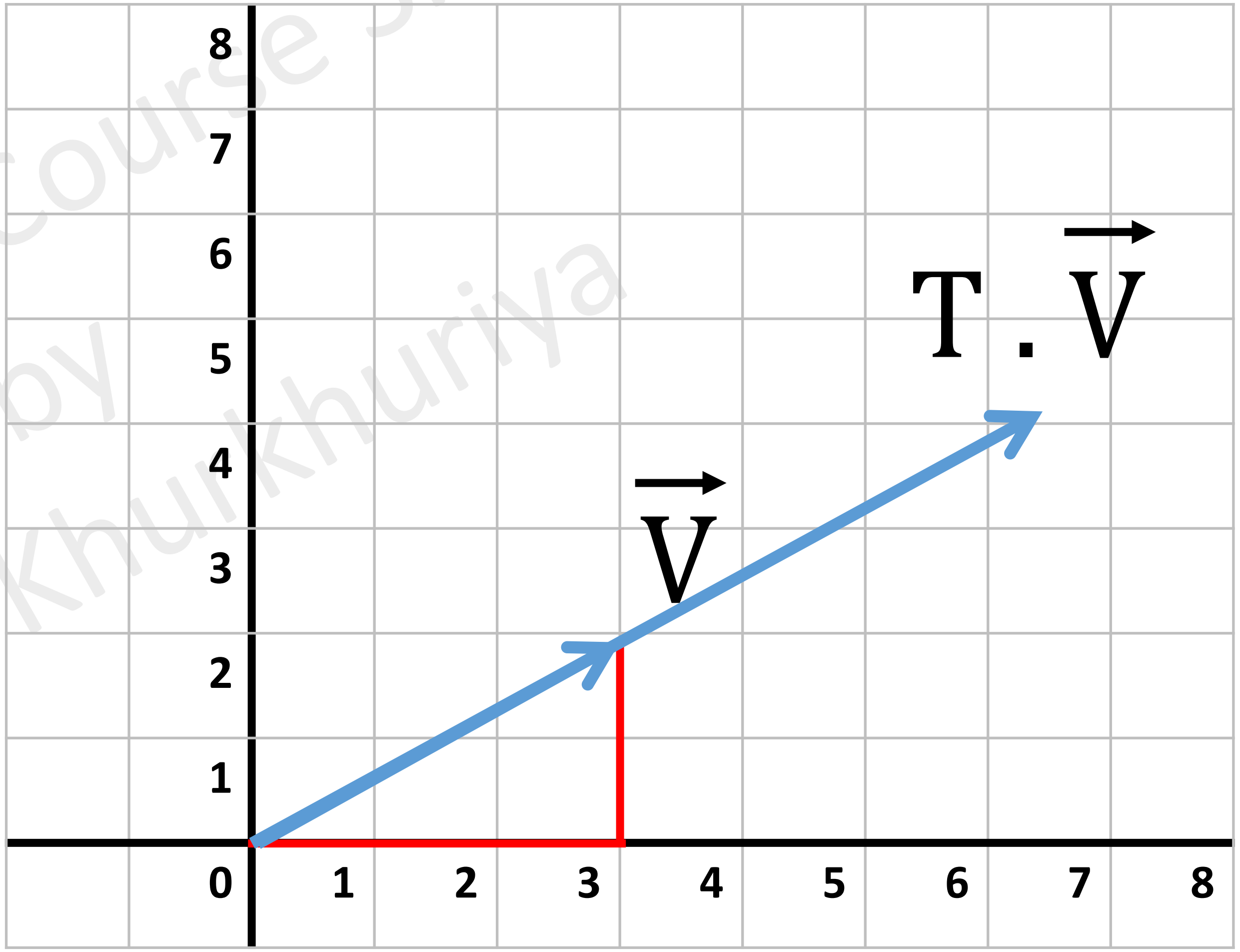
- Scalar value by which it changes its magnitude is eigenvalue

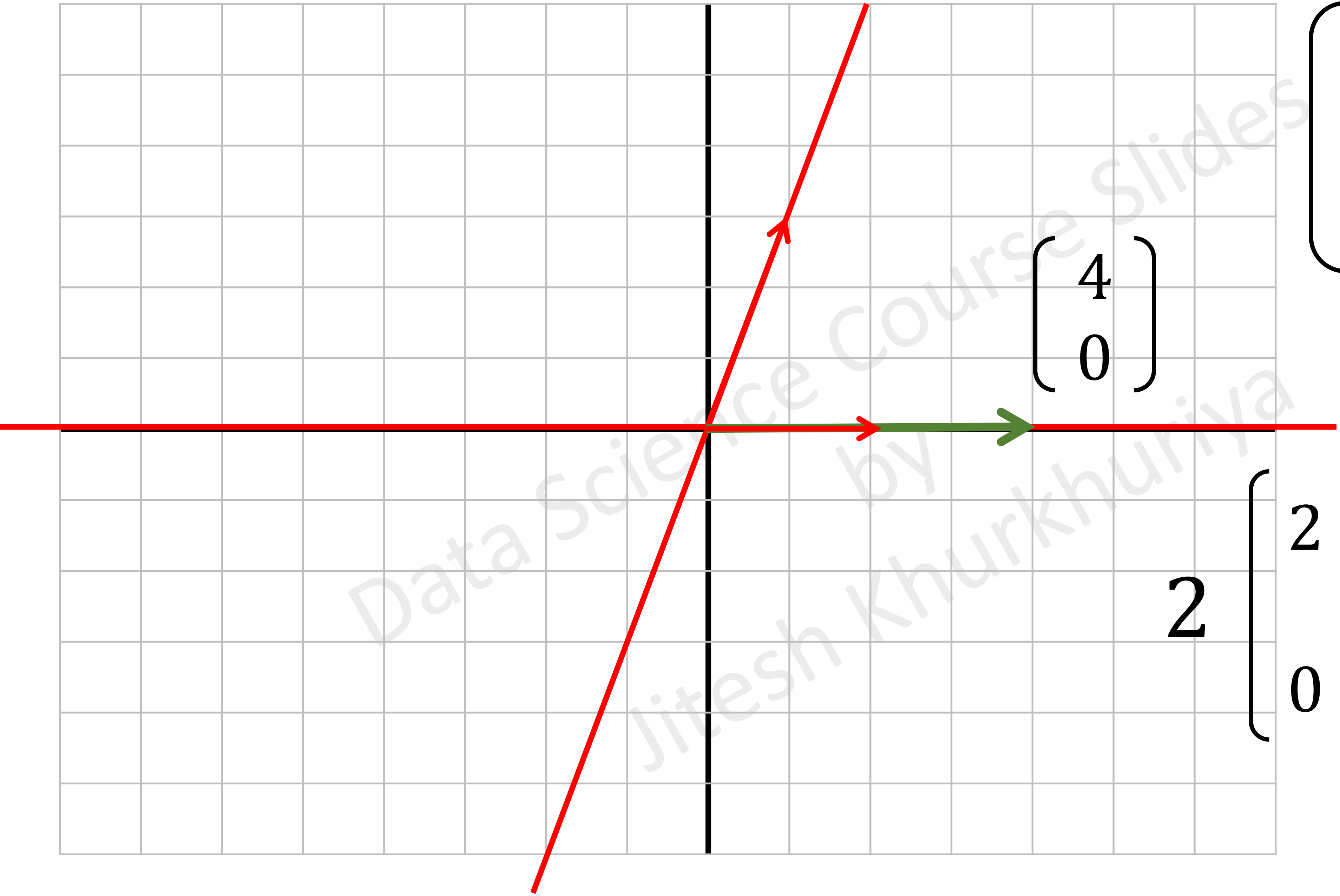
Note: Only thing that's changing is our perception of the coordinates.

What is an Eigenvector and Eigenvalues?

- A non-zero vector that changes by a scalar during linear transformation

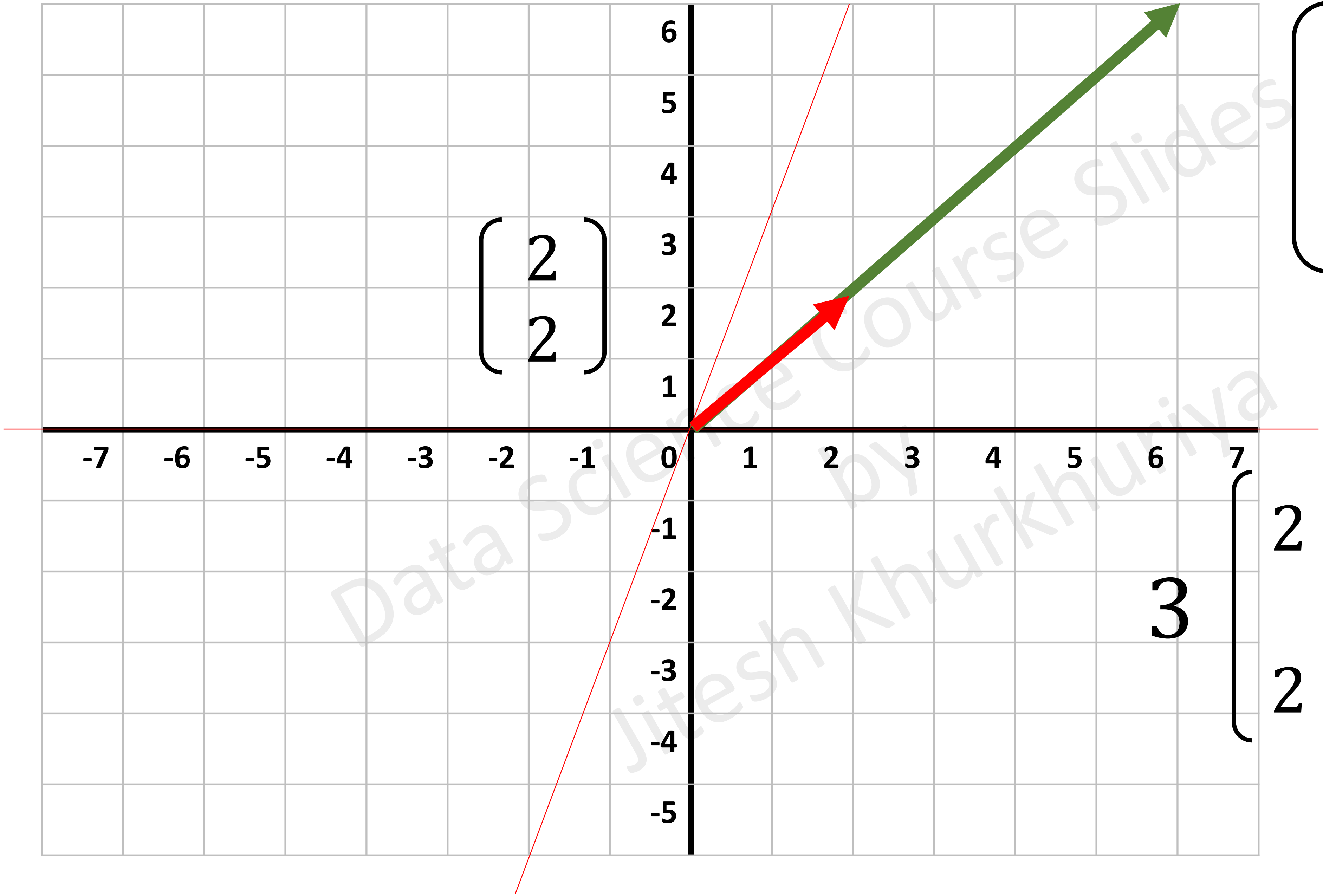
$$T.\vec{V} = \lambda.\vec{V}$$





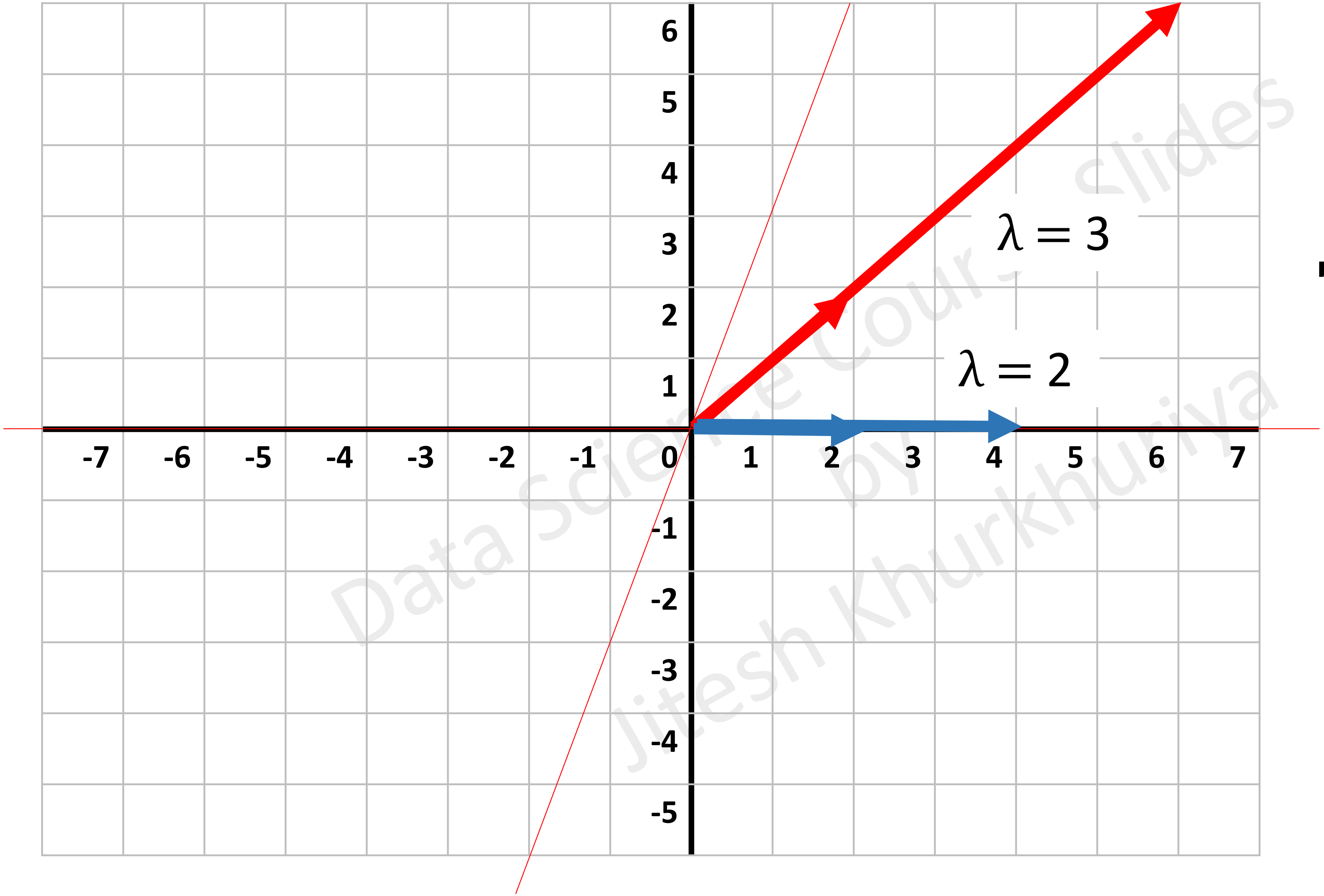
$$\begin{pmatrix} 2 & 1 \\ 0 & 3 \end{pmatrix} \begin{pmatrix} 2 \\ 0 \end{pmatrix}$$

$$2 \begin{pmatrix} 2 \\ 0 \end{pmatrix} = \begin{pmatrix} 4 \\ 0 \end{pmatrix}$$



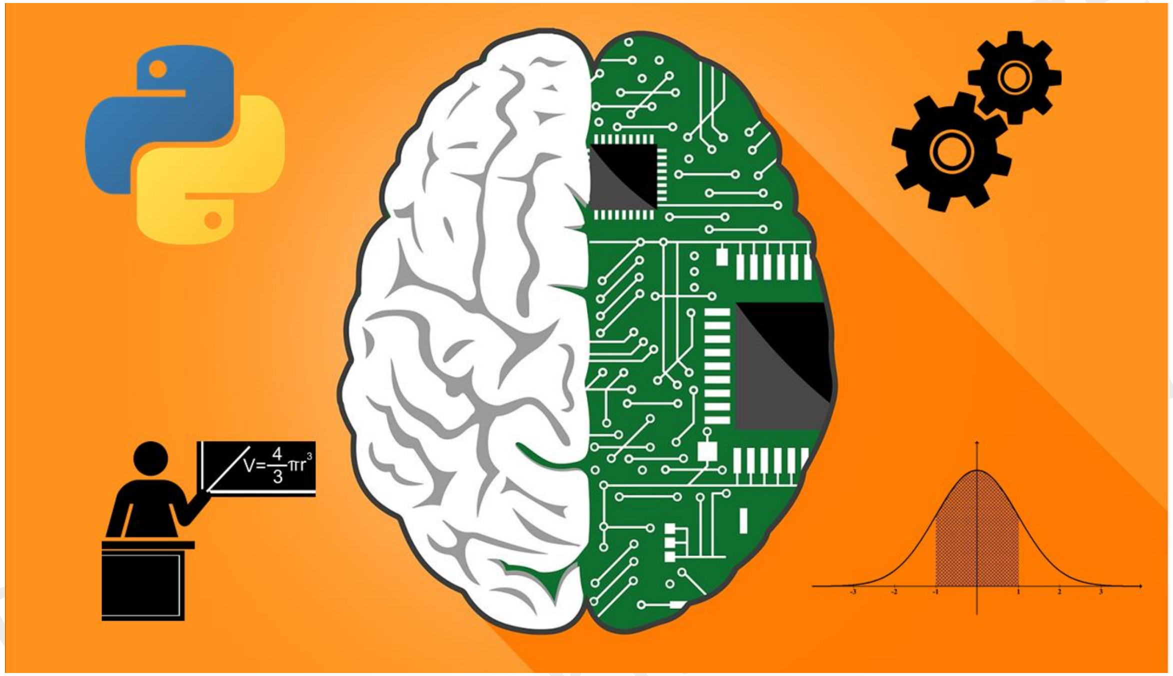
$$\begin{pmatrix} 2 & 1 \\ 0 & 3 \end{pmatrix} \begin{pmatrix} 2 \\ 2 \end{pmatrix}$$

$$3 \begin{pmatrix} 2 \\ 2 \end{pmatrix} = \begin{pmatrix} 6 \\ 6 \end{pmatrix}$$



$$T.\vec{V} = \lambda.\vec{V}$$

Complete Data Science and Machine Learning Using Python



Thank You!