FUNCTIONS

OBJECTIVE PROBLEMS

1. Let A be a set of n distinct elements. Then the total number of distinct functions from A to A is

a) n^2 b) n^n c) 2n d) n!

2. Set A has 3 elements and set B has 4 elements. The number of injections that can be defined from A into B is

a) 144 b) 12 c) 24 c) 64

3. Let n(A) = 4 and n(B) = K. The number of all possible injections from A to B is 120 then k =

a) 9 b) 24 c) 5 d) 6

4. Let n(A) = 4 and n(b) = 5. The number of all possible many-one functions from A to B is

a) 625 b) 20 c) 120 d) 505

5. Set A contains 3 elements and set B contains 2 elements. The number of onto functions from A onto B is

a) 3 b) 6 c) 8 d) 9

6. Let A = {1, 2, 3, ….., n} and B = {a, b, c}, the number of functions from A to B that are onto is

a) $3^n - 2^n$ b) $3^n - 2^n = 1$ c) $3(2^n - 1)$ d) $3^n - 3(2^n - 1)$

7. Set A has 3 elements, Set B has 4 elements. The number of surjections that can be defined from A to B is

a) 144 b) 12 c) 0 d) 64

8. **Let A, B are two sets each with 10 elements, then the number of all possible bijections from A to B is**

a) 20 b) 10! C) 100 d) 1000

9. **The number of one-one onto functions that can be defined from (1, 2, 3, 4) onto set B is 24 then n(B) =**

a) 4 b) 2 c) 3 d) 6

10. **A = {1, 2, 3, 4), B = {a, b, c, d, e}, then the number of all possible constant functions from A to B is**

a) 9 b) 4 c) 5 d) 16

11. **If** $f(x)=\cos(\log x)$**, then** $f(x)f(y)-\frac{1}{2}[f(x/y)+f(xy)]=$

(a) -1 (b) $\frac{1}{2}$ (c) -2 (d)None of these

12. **If** $f(x)=\sin\log x$**, then the value of** $f(xy)+f\left(\frac{x}{y}\right)-2f(x).\cos\log y$ **is equal to**

(a)1 (b) 0 (c)–1 (d) $\sin\log x.\cos\log y$

13. **If** $f(x)=\cos(\log x)$**, then** $f(x^2)f(y^2)-\frac{1}{2}\left[f\left(\frac{x^2}{2}\right)+f\left(\frac{x^2}{y^2}\right)\right]$ **has the value**

(a)–2 (b)–1 (c)1/2 (d)None of these

14. **If** $f(x)=\cos(\log x)$**, then the value of** $f(x).f(4)-\frac{1}{2}\left[f\left(\frac{x}{4}\right)+f(4x)\right]$

(a)1 (b)–1 (c)0 (d) ± 1

15. **Given the function** $f(x)=\frac{a^x+a^{-x}}{2},(a>2)$**. Then** $f(x+y)+f(x-y)=$

(a) $2f(x).f(y)$ (b) $f(x).f(y)$ (c) $\frac{f(x)}{f(y)}$ (d)None of these

16. **Let** $f:R\to R$ **be defined by** $f(x)=2x+|x|$**, then** $f(2x)+f(-x)-f(x)=$

(a) $2x$ (b) $2|x|$ (c) $-2x$ (d) $-2|x|$

17. **If** $e^{f(x)}=\frac{10+x}{10-x}, x\in(-10,10)$ **and** $f(x)=kf\left(\frac{200x}{100+x^2}\right)$**, then** $k=$

(a) 0.5 (b) 0.6 (c) 0.7 (d)0.8

18. **If** $f(x)=\log\left[\frac{1+x}{1-x}\right]$**, then** $f\left[\frac{2x}{1+x^2}\right]$ **is equal to**

(a) $[f(x)]^2$ (b) $[f(x)]^3$ (c) $2f(x)$ (d) $3f(x)$

19. **If** $f(x)=\cos[\pi^2]x+\cos[-\pi^2]x$**, then**

(a) $f\left(\frac{\pi}{4}\right)=2$ (b) $f(-\pi)=2$ (c) $f(\pi)=1$ (d) $f\left(\frac{\pi}{2}\right)=-1$

20. **If** $y=f(x)=\frac{ax+b}{cx-a}$**, then** x **is equal to**

(a) $1/f(x)$ (b) $1/f(y)$ (c) $yf(x)$ (d) $f(y)$

21. **The function equivalent to** $\log x^2$ **is**

(a) $2\log x$ (b) $2\log|x|$ (c) $|\log x^2|$ (d) $(\log x)^2$

22. **The graph of the function** $y=f(x)$ **is symmetrical about the line** $x=2$**, then**

(a) $f(x)=-f(-x)$ (b) $f(2+x)=f(2-x)$ (c) $f(x)=f(-x)$ (d) $f(x+2)=f(x-2)$

23. **If** $f(x)=\frac{1}{\sqrt{x+2\sqrt{2x-4}}}+\frac{1}{\sqrt{x-2\sqrt{2x-4}}}$ **for** $x>2$**, then** $f(11)=$

(a) 7/6 (b) 5/6 (c) 6/7 (d)5/7

24. **If** $f:IR\to IR$ **is defined by** $f(x)=3x-4$**, then** $f^{-1}:IR\to IR$ **is**

(a) $4-3x$ (b) $\frac{x+4}{3}$ (c) $\frac{1}{3x-4}$ (d) $\frac{3}{x+4}$

25. **Which of the following function is invertible?**

(a) $f(x)=2^x$ (b) $f(x)=x^3-x$ (c) $f(x)=x^2$ (d)None of these

26. **The inverse of the function** $\frac{10^x-10^{-x}}{10^x+10^{-x}}$ **is**

(a) $\frac{1}{2}\log_{10}\left(\frac{1+x}{1-x}\right)$ (b) $\frac{1}{2}\log_{10}\left(\frac{1-x}{1+x}\right)$ (c) $\frac{1}{4}\log_{10}\left(\frac{2x}{2-x}\right)$ (d)None of these

27. **If** $f(x)=\frac{x}{1+x}$**, then** $f^{-1}(x)$ **is equal to**

(a) $\frac{(1+x)}{x}$ (b) $\frac{1}{(1+x)}$ (c) $\frac{(1+x)}{(1-x)}$ (d) $\frac{x}{(1-x)}$

28. **If** $f(x)=\frac{x}{\sqrt{1+x^2}}$**, then** $(fofof)(x)=$

(a) $\frac{3x}{\sqrt{1+x^2}}$ (b) $\frac{x}{\sqrt{1+3x^2}}$ (c) $\frac{3x}{\sqrt{1+x^2}}$ (d)None of these

29. **If** $f(x)=\log_a x$ **and** $F(x)=a^x$ **, then** $F[f(x)]$ **is**

(a) $f[F(x)]$ (b) $f[F(2x)]$ (c) $F|\ f(2x)|$ (d) $F[(x)]$

30. **Let *f* and *g* be functions defined by** $f(x)=\frac{x}{x+1}$, $g(x)=\frac{x}{1-x}$ **, then** $(fog)(x)$ **is**

(a) $\frac{1}{x}$ (b) $\frac{1}{x-1}$ (c) $x-1$ (d) x

31. **If** $f(x)=\frac{\alpha x}{x+1}, x\neq -1$**. Then, for what value of** α **is** $f(f(x))=x$

(a) $\sqrt{2}$ (b) $-\sqrt{2}$ (c) 1 (d) –1

32. **If** $f(x)=(a-x^n)^{1/n}$**, where** $a>0$ **and *n* is a positive integer, then** $f[f(x)]=$

(a) x^3 (b) x^2 (c) x (d) None of these

33. **If** $f(x_1)-f(x_2)=f\left(\frac{x_1-x_2}{1-x_1x_2}\right)$ **for** $x_1,x_2\in[-1,1]$**, then** $f(x)$ **is**

(a) $\log\frac{(1-x)}{(1+x)}$ (b) $\tan^{-1}\frac{(1-x)}{(1+x)}$ (c) $\log\frac{(1+x)}{(1-x)}$ (d) all the above

34. **If** $f(x)=\begin{cases} x, & \text{when } x \text{ is rational} \\ 0, & \text{when } x \text{ is irrational}\end{cases}$ **;** $g(x)=\begin{cases} 0, & \text{when } x \text{ is rational} \\ x, & \text{when } x \text{ is irrational}\end{cases}$ **then** $(f-g)$ **is**

(a) One-one onto (b) One-one not onto

(c) Not one-one but onto (d) Not one-one not onto

35. **If** $e^x=y+\sqrt{1+y^2}$ **, then *y* =**

(a) $\frac{e^x+e^{-x}}{2}$ (b) $\frac{e^x-e^{-x}}{2}$ (c) e^x+e^{-x} (d) e^x-e^{-x}

36. **If** $f(x)=\frac{x-|x|}{|x|}$**, then** $f(-1)=$

(a) 1 (b) –2 (c) 0 (d) +2

37. **If** $f(x+ay, x-ay)=axy$**, then** $f(x,y)$ **is equal to**

(a) xy (b) $x^2-a^2y^2$ (c) $\frac{x^2-y^2}{4}$ (d) $\frac{x^2-y^2}{a^2}$

38. **Let** $f(x)=\begin{cases}\frac{1}{2}, if\ 0\le x\le\frac{1}{2}\\ \frac{1}{3}, if\ \frac{1}{2}<x\le 1\end{cases}$**, then *f* is**

(a) A rational function (b)A trigonometric function

(c) A step function (d)An exponential function

39. **Let** $f:(2,3)\to(0,1)$ **be defined by** $f(x)=x-[x]$ **then** $f^{-1}(x)$ **equals**

(a) $x-2$ (b) $x+1$ (c) $x-1$ (d) $x+2$

40. **The domain of** $\sin^{-1}\left[\log_3\left(\frac{x}{3}\right)\right]$ **is**

(a) [1, 9] (b) [–1, 9] (c) [–9, 1] (d) [–9, –1]

41. **The domain of the function** $f(x)=\sin^{-1}[\log_2(x/2)]$ **is**

(a) [1, 4] (b) [–4, 1] (c) [–1, 4] (d) None of these

42. **The domain of the function** $f(x)=\log(\sqrt{x-4}+\sqrt{6-x})$ **is**

(a) $[4,\infty)$ (b) $(-\infty, 6]$ (c) $[4, 6]$ (d) None of these

43. **Domain of the function** $f(x)=\left[\log_{10}\left(\frac{5x-x^2}{4}\right)\right]^{1/2}$ **is**

(a) $-\infty<x<\infty$ (b) $1\le x\le 4$ (c) $4\le x\le 16$ (d)None

44. **Domain of the function** $f(x)=\sqrt{2-2x-x^2}$ **is**

(a) $-\sqrt{3}\le x\le\sqrt{3}$ (b) $-1-\sqrt{3}\le x\le -1+\sqrt{3}$

(c) $-2\le x\le 2$ (d) $-2+\sqrt{3}\le x\le -2-\sqrt{3}$

45. **Domain of the function** $\frac{\sqrt{1+x}-\sqrt{1-x}}{x}$ **is**

(a) (–1, 1) (b) (–1, 1)–{0} (c)[–1, 1] (d)[–1, 1]–{0}

46. **The largest possible set of real numbers which can be the domain of** $f(x)=\sqrt{1-\frac{1}{x}}$ **is**

(a) $(0,1)\cup(0,\infty)$ (b) $(-1,0)\cup(1,\infty)$ (c) $(-\infty,-1)\cup(0,\infty)$ (d) $(-\infty,0)\cup(1,\infty)$

47. **Domain of the function** $f(x)=\dfrac{x^2-3x+2}{x^2+x-6}$ **is**

(a) $\{x : x \in R, \ \ x \neq 3\}$

(b) $\{x : x \in R, \ \ x \neq 2\}$

(c) $\{x : x \in R\}$

(d) $\{x : x \in R, \ \ x \neq 2, x \neq -3\}$

48. **The domain of the function** $y=\dfrac{1}{\sqrt{|x|-x}}$ **is**

(a) $(-\infty, 0)$ (b) $(-\infty, 0]$ (c) $(-\infty, -1)$ (d) $(-\infty, \infty)$

49. **Function** $\sin^{-1}\sqrt{x}$ **is defined in the interval**

(a) (–1, 1) (b) [0, 1] (c) [–1, 0] (d) (–1, 2)

50. **The domain of the function** $f(x)=\dfrac{\sin^{-1}(3-x)}{\ln(|x|-2)}$ **is**

(a) [2, 4] (b) (2, 3) $\cup$ (3, 4] (c) [2, ∞) (d) $(-\infty, -3)\cup[2, \infty)$

51. **The domain of** $f(x)=\dfrac{\log_2(x+3)}{x^2+3x+2}$ **is**

(a) $R-\{-1, -2\}$ (b) $(-2, +\infty)$ (c) $R-\{-1, -2, -3\}$ (d) $(-3, +\infty)-\{-1, -2\}$

52. **The domain of the function** $f(x)=\log_{3+x}(x^2-1)$ **is**

(a) $(-3, -1)\cup(1, \infty)$ (b) $[-3, -1)\cup[1, \infty)$

(c) $(-3, -2)\cup(-2, -1)\cup(1, \infty)$ (d) $[-3, -2)\cup(-2, -1)\cup[1, \infty)$

53. **Domain of definition of the function** $f(x)=\dfrac{3}{4-x^2}+\log_{10}(x^3-x)$**, is**

(a) (1, 2) (b) $(-1, 0)\cup(1, 2)$ (c) $(1, 2)\cup(2, \infty)$ (d) $(-1, 0)\cup(1, 2)\cup(2, \infty)$

54. **Domain of the function** $\sqrt{\log\left\{(5x-x^2)/6\right\}}$ **is**

(a) (2, 3) (b) [2, 3] (c) [1, 2] (d) [1, 3]

55. **The domain of the function** $\sqrt{\log(x^2-6x+6)}$ **is**

(a) $(-\infty, \infty)$

(b) $(-\infty, 3-\sqrt{3})\cup(3+\sqrt{3}, \infty)$

(c) $(-\infty, 1]\cup[5, \infty)$

(d) $[0, \infty)$

56. The domain of the function $f(x)=\exp(\sqrt{5x-3-2x^2})$ **is**

(a) $\left[1, -\frac{3}{2}\right]$ (b) $\left[\frac{3}{2}, \infty\right]$ (c) $[-\infty, 1]$ (d) $\left[1, \frac{3}{2}\right]$

57. The domain of the function $f(x)=\frac{\sin^{-1}(x-3)}{\sqrt{9-x^2}}$ **is**

(a) [1, 2) (b) [2, 3) (c) [1, 2] (d) [2, 3]

58. The domain of the function $f(x)=\sin^{-1}\{(1+e^x)^{-1}\}$ **is**

(a) $\left(\frac{1}{4}, \frac{1}{3}\right)$ (b) [–1, 0] (c) [0, 1] (d) [–1, 1]

59. Domain of the function $\sqrt{2-x}-\frac{1}{\sqrt{9-x^2}}$ **is**

(a) (–3, 1) (b) [–3, 1] (c) (–3, 2] (d) [–3, 1)

60. Domain of the function $f(x)=\frac{x-3}{(x-1)\sqrt{x^2-4}}$ **is**

(a) (1, 2) (b) $(-\infty, -2)\cup(2, \infty)$

(c) $(-\infty, -2)\cup(1, \infty)$ (d) $(-\infty, \infty)-\{1, \pm 2\}$

61. The domain of the function $f(x)=\sqrt{\log\frac{1}{|\sin x|}}$ **is**

(a) $R-\{2n\pi, n\in I\}$ (b) $R-\{n\pi, n\in I\}$

(c) $R-\{-\pi, \pi\}$ (d) $(-\infty, \infty)$

62. The function $f(x)=\frac{\sec^{-1}x}{\sqrt{x-[x]}}$, **where [.] denotes the greatest integer less than or equal to** ***x*** **is defined for all** ***x*** **belonging to**

(a) R (b) $R-\{(-1, 1)\cup(n|\ n\in Z)\}$

(c) $R^+-(0, 1)$ (d) $R^+-\{n|\ n\in N\}$

63. Domain of $f(x)=\log|\log x|$ **is**

(a) $(0, \infty)$ (b) $(1, \infty)$ (c) $(0, 1)\cup(1, \infty)$ (d) $(-\infty, 1)$

64. Domain of function $f(x)=\sin^{-1}5x$ **is**

(a) $\left(-\frac{1}{5}, \frac{1}{5}\right)$ (b) $\left[-\frac{1}{5}, \frac{1}{5}\right]$ (c) R (d) $\left(0, \frac{1}{5}\right)$

65. The domain of the function $f(x)=\log(\sqrt{x-4}+\sqrt{6-x})$ **is**

(a) $[4, \infty)$ (b) $(-\infty, 6]$ (c) $[4, 6]$ (d) None of these

66. **Domain of the function** $f(x)=\sin^{-1}(1+3x+2x^2)$ **is**

(a) $(-\infty,\infty)$ (b) $(-1,1)$ (c) $\left[-\frac{3}{2},0\right]$ (d) $\left(-\infty,\frac{-1}{2}\right)\cup(2,\infty)$

67. **The range of the function** $f(x)=\frac{x+2}{|x+2|}$ **is**

(a) {0, 1} (b) {–1, 1} (c) R (d) $R-\{-2\}$

68. **Range of the function** $\frac{1}{2-\sin 3x}$ **is**

(a) [1, 3] (b) $\left[\frac{1}{3},1\right]$ (c) (1, 3) (d) $\left(\frac{1}{3},1\right)$

69. **If** x **is real, then value of the expression** $\frac{x^2+14x+9}{x^2+2x+3}$ **lies between**

(a) 5 and 4 (b) 5 and –4 (c) – 5 and 4 (d) None of these

70. **If the function** $f:[1,\infty)\to[1,\infty)$ **is defined by** $f(x)=2^{x(x-1)}$, **then** $f^{-1}(x)$ **is**

(a) $\left(\frac{1}{2}\right)^{x(x-1)}$ (b) $\frac{1}{2}(1+\sqrt{1+4\log_2 x})$

(c) $\frac{1}{2}(1-\sqrt{1+4\log_2 x})$ (d) Not defined

71. **The range of function The range of function** $f(x)=x^2-6x+7$ **is**

(a) $(-\infty,\infty)$ (b) $[-2,\infty)$ (c) $(-2,3)$ (d) $(-\infty,-2)$

72. **Range of the function** $f(x)=\frac{x^2+x+2}{x^2+x+1}; x\in R$ **is**

(a) $(1,\infty)$ (b) $(1,11/7]$ (c) $(1,7/3]$ (d) $(1,7/5]$

73. **The function** $f:R\to R$ **is defined by** $f(x)=\cos^2 x+\sin^4 x$ **for** $x\in R$, **then** $f(R)=$

(a) $\left(\frac{3}{4},1\right]$ (b) $\left[\frac{3}{4},1\right)$ (c) $\left[\frac{3}{4},1\right]$ (d) $\left(\frac{3}{4},1\right)$

74. **If** $f:R\to R$, **then the range of the function** $f(x)=\frac{x^2}{x^2+1}$ **is**

(a) R^- (b) R^+ (c) R (d) $R\times R$

75. **Range of the function** $f(x)=9-7\sin x$ **is**

(a) (2, 16) (b) [2, 16] (c) [–1, 1] (d) (2, 16]

76. **The range of** $f(x)=\sec\left(\frac{\pi}{4}\cos^2 x\right), -\infty<x<\infty$ **is**

(a) $[1,\sqrt{2}]$ (b) $[1,\infty)$ (c) $[-\sqrt{2},-1]\cup[1,\sqrt{2}]$ (d) $(-\infty,-1]\cup[1,\infty)$

77. If $f(x) = a\cos(bx+c)+d$, then range of $f(x)$ is

(a) $[d+a, d+2a]$ (b) $[a-d, a+d]$ (c) $[d+a, a-d]$ (d) $[d-a, d+a]$

78. Range of the function $f(x) = \frac{x^2}{x^2+1}$ is

(a) (–1, 0) (b) (–1, 1)

(c) [0, 1) (d) (1, 1)

79. The domain of $\sin^{-1}(\log_3 x)$ is

(a) [–1, 1] (b) [0, 1] (c) [0, ∞] (d) R

80. For $\theta > \frac{\pi}{3}$, the value of $f(\theta) = \sec^2\theta + \cos^2\theta$ always lies in the interval

(a) (0, 2) (b) [0, 1] (c) (1, 2) (d) $[2, \infty)$

81. The Domain of function $f(x) = \log_e(x-[x])$ is

(a) R (b) R-Z (c) $(0, +\infty)$ (d) Z

82. If $f(x) = x^2+1$, then $f^{-1}(17)$ and $f^{-1}(-3)$ will be

(a) 4, 1 (b) 4, 0 (c) 3, 2 (d) None of these

83. If $f(x) = |\cos x|$ and $g(x) = [x]$, then $gof(x)$ is equal to

(a) $|\cos[x]|$ (b) $|\cos x|$ (c) $[|\cos x|]$ (d) $|[\cos x]|$

84. Let $g(x) = 1+x-[x]$ and $f(x) = \begin{cases} -1, & x < 0 \\ 0, & x = 0, \\ 1, & x > 0 \end{cases}$ then for all x, $f(g(x))$ is equal to

(a) x (b) 1 (c) $f(x)$ (d) $g(x)$

85. A real valued function $f(x)$ satisfies the function equation $f(x-y) = f(x)f(y) - f(a-x)f(a+y)$ where a is a given constant and $f(0) = 1$, $f(2a-x)$ is equal to

(a) $f(a)+f(a-x)$ (b) $f(-x)$ (c) $-f(x)$ (d) $f(x)$

86. Let $g(x) = 1+x-[x]$ and

$f(x) = \begin{cases} -1, & If\ x < 0 \\ 0, & If\ x = 0, \\ 1, & if\ x > 0 \end{cases}$ then for all values of x the value of $fog(x)$

(a) x (b) 1 (c) $f(x)$ (d) $g(x)$

87. The function $f: R \to R,\ f(x) = x^2, \forall x \in R$ is

(a) Injection but not surjection (b) Surjection but not injection

(c) Injection as well as surjection (d) Neither injection nor surjection

88. If $f : R \to R$, then $f(x) = |x|$ is

(a) One-one but not onto (b) Onto but not one-one

(c) One-one and onto (d) None of these

89. Which one of the following is a bijective function on the set of real numbers

(a) $2x - 5$ (b) $|x|$ (c) x^2 (d) $x^2 + 1$

90. If $(x, y) \in R$ and $x, y \neq 0$; $f(x, y) \to \frac{x}{y}$, then this function is a/an

(a) Surjection (b) Bijection (c) One-one (d) None of these

91. The function $f(x) = \sin\left(\log(x + \sqrt{x^2 + 1})\right)$ is

(a) Even function (b) Odd function

(c) Neither even nor odd (d) Periodic function

92. If $f(x) = \sin^2 x$ and the composite function $g\{f(x)\} = |\sin x|$, then the function $g(x)$ is equal to

(a) $\sqrt{x - 1}$ (b) $\sqrt{x}$ (c) $\sqrt{x + 1}$ (d) $-\sqrt{x}$

93. If $f(x) = 2x^6 + 3x^4 + 4x^2$ then $f'(x)$ is

(a) Even function (b) An odd function

(c) Neither even nor odd (d) None of these

94. The function $f(x) = \log(x + \sqrt{x^2 + 1})$, is

(a) An even function (b) An odd function

(c) A Periodic function (d) Neither an even nor odd function

95. The period of $f(x) = x - [x]$, if it is periodic, is

(a) $f(x)$ is not periodic (b) $\frac{1}{2}$ (c) 1 (d)2

96. A function *f* from the set of natural numbers to integers defined by

$$f(n) = \begin{cases} \frac{n-1}{2}, & \text{when } n \text{ is odd} \\ -\frac{n}{2}, & \text{when } n \text{ is even} \end{cases}$$ **, is**

(a) One-one but not onto (b) Onto but not one-one

(c) One-one and onto both (d) Neither one-one nor onto

97. Let $f : N \to N$ defined by $f(x) = x^2 + x + 1$, $x \in N$, then *f* is

(a) One-one onto
(b) Many one onto
(c) One-one but not onto
(d) None of these

98. The function $f : R \to R$ defined by $f(x) = (x-1)\ (x-2)(x-3)$ is

(a) One-one but not onto
(b) Onto but not one-one
(c) Both one-one and onto
(d) Neither one-one nor onto

99. If $f : [0, \infty) \to [0, \infty)$ and $f(x) = \dfrac{x}{1+x}$, then *f* is

(a) One-one and onto
(b) One-one but not onto
(c) Onto but not one-one
(d) Neither one-one nor onto

100. Let the function $f : R \to R$ be defined by $f(x) = 2x + \sin x, x \in R$. Then *f* is

(a) One-to-one and onto
(b)One-to-one but not onto
(c) Onto but not one-to-one
(d) Neither one-to-one nor onto

101. $f(x) = x + \sqrt{x^2}$ is a function from $R \to R$, then $f(x)$ is

(a) Injective (b) Surjective (c) Bijective (d) None of these

102. Range of $f(x) = \dfrac{x^2 + 34x - 71}{x^2 + 2x - 7}$ is

(a) [5, 9] (b) $(-\infty, 5] \cup [9, \infty)$ (c) (5, 9) (d) None of these

103. Which of the following functions is inverse of itself

(a) $f(x) = \dfrac{1-x}{1+x}$ (b) $f(x) = 5^{\log x}$ (c) $f(x) = 2^{x(x-1)}$ (d) None of these

104. If $f(x) = 3x - 5$, then $f^{-1}(x)$

(a) Is given by $\dfrac{1}{3x-5}$
(b) Is given by $\dfrac{x+5}{3}$
(c) Does not exist because *f* is not one-one
(d) Does not exist because *f* is not onto

105. If $f : R \to S$ defined by $f(x) = \sin x - \sqrt{3}\cos x + 1$ is onto, then the interval of *S* is

(a) [–1, 3] (b) [1, 1] (c) [0, 1] (d) [0, –1]

106. Let $f : R \to R$ be a function defined by $f(x) = \dfrac{x-m}{x-n}$, where $m \neq n$. Then

(a) *f* is one-one onto
(b) *f* is one-one into
(c) *f* is many one onto
(d) *f* is many one into

107. Let $f(x) = \frac{x^2 - 4}{x^2 + 4}$ **for** $|x| > 2$**, then the function** $f : (-\infty, -2] \cup [2, \infty) \rightarrow (-1, 1)$ **is**

(a) One-one into (b) One-one onto

(c) Many one into (d) Many one onto

108. Let *X* and *Y* be subsets of *R*, the set of all real numbers. The function $f : X \rightarrow Y$ **defined by** $f(x) = x^2$ **for** $x \in X$ **is one-one but not onto if (Here** R^+ **is the set of all positive real numbers)**

(a) $X = Y = R^+$ (b) $X = R, Y = R^+$

(c) $X = R^+, Y = R$ (d) $X = Y = R$

109. Function $f : R \rightarrow R, f(x) = x^2 + x$ **is**

(a) One-one onto (b) One-one into

(c) Many-one onto (d) Many-one into

FUNCTIONS

HINTS AND SOLUTIONS

1. (b).

$n(A) = n(B) = n$

and no. functions $= n(B)^{n(A)}$

2. (c) Synopsis 3. (c) Synopsis 4. (d) Synopsis 5. (b) Synopsis

6. (d) 7. (c) 8. (b) 9. (a) 10. (c) Synopsis

11. (d) $f(x) = \cos(\log x) \Rightarrow f(y) = \cos(\log y)$

$$f(x).f(y) - \frac{1}{2}\left[f\left(\frac{x}{y}\right) + f(xy)\right]$$

$$= \cos(\log x)\cos(\log y) - \frac{1}{2}\left[\cos\left(\log\frac{x}{y}\right) + \cos(\log xy)\right]$$

$$= \cos(\log x)\cos(\log y) - \frac{1}{2}[2\cos(\log x)\cos(\log y)] = 0.$$

12. (b) $f(xy) = \sin\log xy = \sin(\log x + \log y)$ .....(i)

$f(x/y) = \sin\log(x/y) = \sin(\log x - \log y)$ .....(ii)

$\therefore f(xy) + f(x/y) = 2\sin\log x \cos\log y$

$2\sin\log x\cos\log y - 2\sin\log x\cos\log y = 0$.

13. (d)

14. (c)

15. (a) $f(x+y) + f(x-y)$

$$= \frac{1}{2}\left[a^{x+y} + a^{-x-y} + a^{x-y} + a^{-x+y}\right]$$

$$= \frac{1}{2}\left[a^{x}(a^{y} + a^{-y}) + a^{-x}(a^{y} + a^{-y})\right]$$

$$= \frac{1}{2}(a^{x} + a^{-x})\,(a^{y} + a^{-y}) = 2f(x)\,f(y).$$

16. (b) $f(2x) = 2(2x) + |2x| = 4x + 2|x|$,

$y = x^2 + 1$,

$f(x) = 2x + |x| \implies f(2x) + f(-x) - f(x)$

$= 4x + 2|x| + |x| - 2x - 2x - |x| = 2|x|$.

17. (a) $e^{f(x)} = \frac{10+x}{10-x} \Rightarrow f(x) = \log\left(\frac{10+x}{10-x}\right)$

$$\Rightarrow f\left(\frac{200x}{100+x^2}\right) = \log\left[\frac{10+\frac{200x}{100+x^2}}{10-\frac{200x}{100+x^2}}\right] = \log\left[\frac{10(10+x)}{10(10-x)}\right]^2$$

$$= 2\log\left(\frac{10+x}{10-x}\right) = 2f(x)$$

$$\therefore f(x) = \frac{1}{2} f\left(\frac{200x}{100+x^2}\right) \Rightarrow k = \frac{1}{2} = 0.5.$$

18. (c) $f(x) = \log(x+\sqrt{x^2+1})$

$$\therefore f\left(\frac{2x}{1+x^2}\right) = \log\left[\frac{1+\frac{2x}{1+x^2}}{1-\frac{2x}{1+x^2}}\right] = \log\left[\frac{x^2+1+2x}{x^2+1-2x}\right]$$

$$= \log\left[\frac{1+x}{1-x}\right]^2 = 2\log\left[\frac{1+x}{1-x}\right] = 2f(x).$$

19. (d) $f(x) = \cos[\pi^2]x + \cos[-\pi^2]x$

$$f(x) = \cos(9x) + \cos(-10x) = \cos(9x) + \cos(10x)$$

$$= 2\cos\left(\frac{19x}{2}\right)\cos\left(\frac{x}{2}\right)$$

$$f\left(\frac{\pi}{2}\right) = 2\cos\left(\frac{19\pi}{4}\right)\cos\left(\frac{\pi}{4}\right);\ f\left(\frac{\pi}{2}\right) = 2\times\frac{-1}{\sqrt{2}}\times\frac{1}{\sqrt{2}} = -1.$$

20. (d) $y = \frac{ax+b}{cx-a} \Rightarrow x(cy-a) = b+ay \Rightarrow x = \frac{ay+b}{cy-a} = f(y)$.

21. (b) domain of $\log x^2$ is R-{0} and the domain of $\log|x|$ is also R-{0}.

$\log x^2$ and $2\log|x|$ are identical functions.

22. (b) $f(x) = f(-x) \Rightarrow f(0+x) = f(0-x)$ is symmetrical about $x=0$.

$\therefore f(2+x) = f(2-x)$ is symmetrical about $x=2$.

23. (c) $f(x) = \frac{1}{\sqrt{x+2\sqrt{2x-4}}} + \frac{1}{\sqrt{x-2\sqrt{2x-4}}}$

$$f(11) = \frac{1}{\sqrt{11+2\sqrt{18}}} + \frac{1}{\sqrt{11-2\sqrt{18}}}$$

$$= \frac{1}{3+\sqrt{2}} + \frac{1}{3-\sqrt{2}} = \frac{3-\sqrt{2}}{7} + \frac{3+\sqrt{2}}{7} = \frac{6}{7}.$$

24. (b) $f(x) = 3x-4$. let $y = f^{-1}(x) \Rightarrow f(y) = x$

$$\Rightarrow 3y-4 = x \Rightarrow 3y = x+4$$

$\Rightarrow y = \frac{x+4}{3} \Rightarrow f^{-1}(x) = \frac{x+4}{3}$.

25. (a) A function is invertible if it is bijection.

26. (a) $y = \frac{10^x - 10^{-x}}{10^x + 10^{-x}} \Rightarrow x = \frac{1}{2}\log_{10}\left(\frac{1+y}{1-y}\right)$

Let $y = f(x) \Rightarrow x = f^{-1}(y)$

$\Rightarrow f^{-1}(y) = \frac{1}{2}\log_{10}\left(\frac{1+y}{1-y}\right) \Rightarrow f^{-1}(x) = \frac{1}{2}\log_{10}\left(\frac{1+x}{1-x}\right)$

27. (d) $f(x) = \frac{x}{1+x}$. Let $y = f(x) \Rightarrow x = f^{-1}(y)$

$\therefore y = \frac{x}{1+x} \Rightarrow y + yx = x \Rightarrow x = \frac{y}{1-y}$

$\Rightarrow f^{-1}(y) = \frac{y}{1-y} \Rightarrow f^{-1}(x) = \frac{x}{1-x}$.

28. (b) $(fofof)(x) = (fof)(f(x)) = (fof)\left(\frac{x}{\sqrt{1+x^2}}\right)$

$$= f\left[\frac{\left(\frac{x}{\sqrt{1+x^2}}\right)}{\sqrt{1+\frac{x^2}{1+x^2}}}\right] = f\left(\frac{x\sqrt{1+x^2}}{\sqrt{1+x^2}\sqrt{1+2x^2}}\right)$$

$$= f\left(\frac{x}{\sqrt{1+2x^2}}\right) = \frac{\frac{x}{\sqrt{1+2x^2}}}{\sqrt{\left[1+\frac{x^2}{1+2x^2}\right]}} = \frac{x}{\sqrt{1+3x^2}}.$$

29. (a) $F[f(x)] = F(\log_a x) = a^{\log_a x} = x$

$f[F(x)] = f(a^x) = \log_a a^x = x \log_a a = x$.

30. (d) $(fog)(x) = f(g(x)) = f\left(\frac{x}{1-x}\right) = \frac{\frac{x}{1-x}}{\frac{x}{1-x}+1} = \frac{x}{x+1-x} = x$.

31. (d) $f(f(x)) = \frac{\alpha f(x)}{f(x)+1} = \frac{\alpha\left(\frac{\alpha x}{x+1}\right)}{\left(\frac{\alpha x}{x+1}+1\right)} = \frac{\alpha^2.x}{\alpha x + x + 1}$

$\therefore x = \frac{\alpha^2.x}{(\alpha+1)x+1}$ or $x((\alpha+1)x + 1 - \alpha^2) = 0$

Or $(\alpha+1)x^2 + (1-\alpha^2)x = 0$.

$\Rightarrow \alpha + 1 = 0, 1 - \alpha^2 = 0$, $\therefore \alpha = -1$.

32. (c) $f[f(x)] = [a - \{f(x)\}^n]^{1/n} = [a - (a - x^n)]^{1/n} = x$.

33. (d) When $x_1=-1$ and $x_2=1$, then

$$f(-1)-f(1)=f\left[\frac{-1-1}{1+1(1)}\right]=f(-1)\Rightarrow f(1)=0$$

Which is satisfied when $f(x)=\tan^{-1}\left(\frac{1-x}{1+x}\right)$

When $x_1=x_2=0$, then

$$f(0)-f(0)=f\left[\frac{0-0}{1-0}\right]=f(0)\Rightarrow f(0)=0$$

When $x_1=-1$ and $x_2=0$ then

$$f(-1)-f(0)=f\left(\frac{-1-0}{1-0}\right)=f(-1)\Rightarrow f(0)=0$$

Which is satisfied when $f(x)=\log\left(\frac{1-x}{1+x}\right)$ and $f(x)=\log\left(\frac{1+x}{1-x}\right)$.

34. (a) $(f-g)(x)=\begin{cases} x,\ x\in Q \\ -x,\ x\notin Q\end{cases}$

35. (b) $\because e^x=y+\sqrt{1+y^2}$

$\therefore e^x-y=\sqrt{1+y^2}$

$(e^x-y)^2=(1+y^2)$

$e^{2x}+y^2-2ye^x=1+y^2\Rightarrow e^{2x}-1=2ye^x$

$\Rightarrow 2y=\frac{e^{2x}-1}{e^x}\Rightarrow 2y=e^x-e^{-x}$

$y=\frac{e^x-e^{-x}}{2}$.

36. (b) $f(-1)=\frac{-1-|-1|}{|-1|}=\frac{-1-1}{1}=-2$.

37. (c) $f(x+ay,x-ay)=axy$ .....(i)

Let $x+ay=u$ and $x-ay=v$

Then $x=\frac{u+v}{2}$ and $y=\frac{u-v}{2a}$

Substituting the value of x and y in (i), we obtain

$$f(u,v)=\frac{u^2-v^2}{4}\Rightarrow f(x,y)=\frac{x^2-y^2}{4}.$$

38. (c)

39. (d) $f(x)=x-[x]$, here [x]=2

$\therefore f(x)=y=x-2 \Rightarrow x=y+2=f^{-1}(y) \Rightarrow f^{-1}(x)=x+2$.

40. (a) $y=\sin^{-1}\left[\log_3\left(\frac{x}{3}\right)\right] \Rightarrow -1 \le \log_3\left(\frac{x}{3}\right) \le 1$

$\Rightarrow \frac{1}{3} \le \frac{x}{3} \le 3 \Rightarrow 1 \le x \le 9 \Rightarrow x \in [1,9]$.

41. (a) $f(x)=\sin^{-1}[\log_2(x/2)]$, Domain of $\sin^{-1}x$ is $x \in [-1,1]$

$\Rightarrow -1 \le \log_2(x/2) \le 1 \Rightarrow \frac{1}{2} \le \frac{x}{2} \le 2 \Rightarrow 1 \le x \le 4$

$\therefore x \in [1,4]$.

42. (c) $f(x)=\log(\sqrt{x-4}+\sqrt{6-x})$

$\Rightarrow x-4 \ge 0$ and $6-x \ge 0 \Rightarrow x \ge 4$ and $x \le 6$

$\therefore$ Domain of $f(x)$ = $[4, 6]$.

43. (b) We have $f(x)=\left[\log_{10}\left(\frac{5x-x^2}{4}\right)\right]^{1/2}$ …..(i)

From (i), clearly $f(x)$ is defined for those values of x for which $\log_{10}\left[\frac{5x-x^2}{4}\right] \ge 0$

$\Rightarrow \left(\frac{5x-x^2}{4}\right) \ge 10^0 \Rightarrow \left(\frac{5x-x^2}{4}\right) \ge 1$

$\Rightarrow x^2-5x+4 \le 0 \Rightarrow (x-1)(x-4) \le 0$

44. (b) $2-2x-x^2 \ge 0 \Rightarrow -1-\sqrt{3} \le x \le -1+\sqrt{3}$.

45. (d) $1+x \ge 0 \Rightarrow x \ge -1$; $1-x \ge 0 \Rightarrow x \le 1,\ x \ne 0$

Domain is $[-1,1]-\{0\}$.

46. (d) $1-\frac{1}{x} > 0 \Rightarrow x > 1$. Also, $x \ne 0$.

$\therefore$ Required interval $=(-\infty,0)\cup(1,\infty)$

47. (d) $(x-2)(x+3) \ne 0$

Domain is $\{x : x \in R,\ x \ne 2,\ x \ne -3\}$.

48. (a) $|x|-x > 0$

$|x| > x$ but $|x| = x$ for x positive and $|x| > x$ for x negative. Domain is $(-\infty, 0)$.

49. (b) Let $y = \sin^{-1}\sqrt{x} \Rightarrow \sqrt{x} = \sin y$

$\Rightarrow x = \sin^2 y, \ \therefore 0 \le x \le 1$

50. (b) $f(x) = \dfrac{\sin^{-1}(3-x)}{\log[\,|x|-2]}$

Let $g(x) = \sin^{-1}(3-x) \Rightarrow -1 \le 3 - x \le 1$

Domain of $g(x)$ is [2, 4]

And let $h(x) = \log[\,|x|-2] \Rightarrow |x| - 2 > 0$

$\Rightarrow |x| > 2 \ \Rightarrow x < -2$ Or $x > 2 \ \Rightarrow (-\infty, -2) \cup (2, \infty)$

We know that

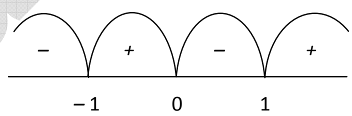
$(f/g)(x) = \dfrac{f(x)}{g(x)} \forall x \in D_1 \cap D_2 - \{x \in R : g(x) = 0\}$

$\therefore$ Domain of $f(x) = (2,4] - \{3\} = (2,3) \cup (3,4]$.

51. (d) Here $x + 3 > 0$ and $x^2 + 3x + 2 \ne 0$

$\therefore x > -3$ and $(x+1)(x+2) \ne 0$, *i.e.* $x \ne -1, -2$

$\therefore$ Domain $= (-3, \infty) - \{-1, -2\}$



52. (c) $x^2 - 1 > 0$

$\Rightarrow x^2 > 1, \ \Rightarrow x < -1$ or $x > 1$ and $3 + x > 0$

$\therefore x > -3$ and $x \ne -2$

$\therefore D_f = (-3, -2) \cup (-2, -1) \cup (1, \infty)$.

53. (d) $f(x) = \dfrac{3}{4 - x^2} + \log_{10}(x^3 - x)$. $4 - x^2 \ne 0 \Rightarrow x \ne \pm\sqrt{4}$

$x^3 - x > 0 \Rightarrow x(x^2 - 1) > 0 \Rightarrow x > 0, x > 1$

54. (b) $\log\left\{\dfrac{5x - x^2}{6}\right\} \ge 0 \Rightarrow \dfrac{5x - x^2}{6} \ge 1$ or $x^2 - 5x + 6 \le 0$ or $(x-2)(x-3) \le 0$. Hence $2 \le x \le 3$.

55. (c) $\log(x^2 - 6x + 6) \ge 0$

$\Rightarrow x^2 - 6x + 6 \ge 1 \ \Rightarrow (x-5)(x-1) \ge 0$

$\Rightarrow x \le 1$ or $x \ge 5$. $\Rightarrow$ Domain is $(-\infty, 1] \cup [5, \infty)$.

56. (d) $5x - 3 - 2x^2 \ge 0 \ \Rightarrow (x-1)\left(x - \dfrac{3}{2}\right) \ge 0$

$\therefore$ Domain is $[1, 3/2]$.

57. (b) $9-x^2>3 \Rightarrow -3<x<3$ .....(i)

$-1 \le (x-3) \le 1 \Rightarrow 2 \le x \le 4$ .....(ii)

From (i) and (ii), $2 \le x < 3$ *i.e.*, [2, 3)

58. (a) $-1 \le \frac{1}{1+e^x} \le 1$

$2<e^x<3 \Rightarrow 3<(e^x+1)<4 \Rightarrow \frac{1}{4}<\frac{1}{1+e^x}<\frac{1}{3}$

$\therefore$ Domain of $f(x)=\left(\frac{1}{4},\frac{1}{3}\right)$.

59. (c) (i) $x \le 2$ (ii) $\sqrt{9-x^2}>0 \Rightarrow |x|<3$ or $-3<x<3$.

Domain is $(-3, 2]$.

60. (b) $|x|>2$ and $x \ne 1$

61. (b) $f(x)=\sqrt{\log\frac{1}{|\sin x|}} \Rightarrow \sin x \ne 0 \Rightarrow x \ne n\pi+(-1)^n 0$

$\Rightarrow x \ne n\pi$. Domain of $f(x)$ $=R-\{n\pi, n \in I\}$.

62. (b) The function $\sec^{-1} x$ is defined for all $x \in R-(-1, 1)$ and the function $\frac{1}{\sqrt{x-[x]}}$ is defined for all $x \in R-Z$. So the given function is defined for all $x \in R-\{(-1, 1) \cup (n \mid n \in Z)\}$.

63. (c) $f(x)=\log|\log x|$, $f(x)$ is defined if $|\log x|>0$ and $x>0$ *i.e.*, if $x>0$ and $x \ne 1$ $(\because |\log x|>0 \text{ if } x \ne 1)$

$\Rightarrow x \in (0,1)\cup(1,\infty)$.

64. (b) $-1 \le 5x \le 1 \Rightarrow \frac{-1}{5} \le x \le \frac{1}{5}$. Domain is $\left[\frac{-1}{5},\frac{1}{5}\right]$.

65. (c) $f(x)=\log(\sqrt{x-4}+\sqrt{6-x})$

$\Rightarrow x-4 \ge 0$ and $6-x \ge 0 \Rightarrow x \ge 4$ and $x \le 6$

$\therefore$ Domain = $[4, 6]$.

66. (c) $-1 \le 1+3x+2x^2 \le 1$

$2x^2+3x+1 \ge -1$; $2x^2+3x+2 \ge 0$

$x=\frac{-3 \pm \sqrt{9-16}}{6}$ $=\frac{-3 \pm i\sqrt{7}}{6}$ Which is imaginary , not allowed

$2x^2+3x+1 \le 1$

$\Rightarrow 2x^2+3x \le 0 \Rightarrow 2x\left(x+\frac{3}{2}\right) \le 0$

$\Rightarrow \frac{-3}{2} \le x \le 0 \Rightarrow x \in \left[-\frac{3}{2}, 0\right]$

Domain of function = $\left[\frac{-3}{2}, 0\right]$.

67. (b) $f(x) = \frac{x+2}{|x+2|}$

$$f(x) = \begin{cases} -1, & x < -2 \\ 1, & x > -2 \end{cases}$$

$\therefore$ Range of $f(x)$ is $\{-1, 1\}$.

68. (b) $f(x) = \frac{1}{2 - \sin 3x}$, $\sin 3x \in [-1, 1]$

Hence $f(x)$ lies in $\left[\frac{1}{3}, 1\right]$.

69. (c) $\frac{x^2 + 14x + 9}{x^2 + 2x + 3} = y \Rightarrow x^2 + 14x + 9 = x^2 y + 2xy + 3y$

$\Rightarrow x^2(y-1) + 2x(y-7) + (3y-9) = 0$

Since x is real, $\therefore 4(y-7)^2 - 4(3y-9)(y-1) > 0$

$\Rightarrow 4(y^2 + 49 - 14y) - 4(3y^2 + 9 - 12y) > 0$

$\Rightarrow 4y^2 + 196 - 56y - 12y^2 - 36 + 48y > 0$

$\Rightarrow 8y^2 + 8y - 160 < 0 \Rightarrow y^2 + y - 20 < 0$

$\Rightarrow (y+5)(y-4) < 0$;

70. (b) Given $f(x) = 2^{x(x-1)} \Rightarrow x(x-1) = \log_2 f(x)$

$\Rightarrow x^2 - x - \log_2 f(x) = 0 \Rightarrow x = \frac{1 \pm \sqrt{1 + 4\log_2 f(x)}}{2}$

Only $x = \frac{1 + \sqrt{1 + 4\log_2 f(x)}}{2}$ lies in the domain

$\therefore f^{-1}(x) = \frac{1}{2}[1 + \sqrt{1 + 4\log_2 x}]$.

71. (b) $x^2 - 6x + 7 = (x-3)^2 - 2$

Minimum value is – 2 and maximum ∞.

Hence range of function is $[-2, \infty]$.

72. (c) $f(x)=1+\frac{1}{\left(x+\frac{1}{2}\right)^2+\frac{3}{4}}$ $\Rightarrow$ Range $=(1,7/3]$.

73. (c) $y=f(x)=\cos^2 x+\sin^4 x$

$\Rightarrow y=f(x)=\cos^2 x+\sin^2 x(1-\cos^2 x)$

$\Rightarrow y=\cos^2 x+\sin^2 x-\sin^2 x\cos^2 x$

$\Rightarrow y=1-\sin^2 x\cos^2 x \Rightarrow y=1-\frac{1}{4}.\sin^2 2x$

$\therefore \frac{3}{4}\le f(x)\le 1$, $(\because 0\le \sin^2 2x\le 1)$

$\Rightarrow f(R)\in[3/4,1]$.

74. (b) R^+ {as y is always positive $\forall x\in R$}.

75. (b) $y=f(x)=9-7\sin x$. Range $=[2,16]$.

76. (a) $f(x)=\sec\left(\frac{\pi}{4}\cos^2 x\right)$

$0\le\cos^2 x\le 1$ at $\cos x=0,\ f(x)=1$ and at $\cos x=1,\ f(x)=\sqrt{2}$; $\therefore 1\le x\le\sqrt{2}\Rightarrow x\in[1,\sqrt{2}]$.

77. (d) $f(x)=a\cos(bx+c)+d$ .....(i)

For minimum $\cos(bx+c)=-1$

$\Rightarrow f(x)=-a+d=(d-a)$

For maximum $\cos(bx+c)=1$

$\Rightarrow f(x)=a+d=(d+a)$

$\therefore$Range of $f(x)=[d-a,d+a]$.

78. (c) Let $y=\frac{x^2}{x^2+1}$

$\Rightarrow (y-1)x^2+0x+y=1, y\ne 1$ For real values of x,

We have $D\ge 0\Rightarrow -4y(y-1)\ge 0\Rightarrow y(y-1)\le 0\Rightarrow y\in[0,1)$

$0\le\frac{x^2}{x^2+1}<1$.

79. (e) $-1\le\log_3 x\le 1$; $3^{-1}\le x\le 3 \Rightarrow \frac{1}{3}\le x\le 3$

$\therefore$ Domain of function $=\left[\frac{1}{3},3\right]$

80. (d) $-1 \le \cos\theta \le 1 \Rightarrow \cos^2\theta \le 1$

And $\sec^2\theta \ge 1$ for $\theta > \frac{\pi}{3}$, $\sec\theta \ge 2$

$\Rightarrow \sec^2\theta \ge 4$. $\therefore$ Required interval $= [2, \infty)$

81. (a) The domain of $\log_e\{x-[x]\}$ is R, because $[x] \le 0$

82. (d) Let $y = x^2 + 1 \Rightarrow x = \pm\sqrt{y-1}$

$\Rightarrow f^{-1}(y) = \pm\sqrt{y-1} \Rightarrow f^{-1}(x) = \pm\sqrt{x-1}$

$\Rightarrow f^{-1}(17) = \pm\sqrt{17-1} = \pm 4$ and $f^{-1}(-3) = \pm\sqrt{-3-1} = \pm\sqrt{-4}$, which is not possible

83. (c) $gof(x) = g\{f(x)\} = [|\cos x|]$.

84. (b) Here $g(x) = 1 + n - n = 1, x = n \in Z$

$1 + n + k - n = 1 + k$, $x = n + k$ (where $n \in Z, 0 < k < 1$)

Now $f(g(x)) = \begin{cases} -1, & g(x) < 0 \\ 0, & g(x) = 0 \\ 1, & g(x) > 0 \end{cases}$

Clearly, $g(x) > 0$ for all *x*. So, $f(g(x)) = 1$ for all *x*.

85. (c) $f(a-(x-a)) = f(a)f(x-a) - f(0)f(x)$ .....(i)

Put $x = 0, y = 0$; $f(0) = (f(0))^2 - [f(a)]^2 \Rightarrow f(a) = 0$

$[\because f(0) = 1]$. From (i), $f(2a-x) = -f(x)$.

86. (b) $g(x) = 1 + \{x\}$; $f\{g(x)\} = f\{1 + \{x\}\} = f(k) = 1$

Where, $k = 1 + \{x\}, 1 \le k < 2$

87. (d)

88. (d) $f(-1) = f(1) = 1 \Rightarrow$ f is not one

Range of the function is $R^+ \cup \{0\}$ which is not equal to co domain. Therefore f is not onto.

so *f* is neither one-one nor onto.

89. (a) $|x|$ is not one-one; x^2 is not one-one;

$x^2 + 1$ is not one-one. But $2x - 5$ is one-one because $f(x) = f(y) \Rightarrow 2x - 5 = 2y - 5 \Rightarrow x = y$

Now $f(x) = 2x - 5$ is onto. $\therefore f(x) = 2x - 5$ is bijective.

90. (a)

91. (b) $f(x) = \sin\left(\log(x + \sqrt{1 + x^2})\right)$

$\Rightarrow f(-x)+f(x)=0$

$\therefore f(x)$ is odd function.

92. (b) $(gof)(x)=|\sin x|$ and $f(x)=\sin^2 x$

$\Rightarrow g(\sin^2 x)=|\sin x|;\ \therefore g(x)=\sqrt{x}$.

93. (b) $f(x)=2x^6+3x^4+4x^2$

$f(-x)=2(-x)^6+3(-x)^4+4(-x)^2=f(x)$

$\Rightarrow f(x)$ is an even function and derivative of an even function is always odd.

94. (b) $f(x)=\log(x+\sqrt{x^2+1})$

and $f(-x)=-\log(x+\sqrt{x^2+1})=-f(x)$

$f(x)$ is odd function.

95. (c)

96. (c) $f(1)=0,\ f(2)=-1,\ f(3)=1,\ f(4)=-2,\ f(5)=2$

And $f(6)=-3$ so on.

No two elements of domain have same image. And range of the function is set of all integers. Hence *f* is one-one and onto function.

97. (a) Let $x,y\in N$ such that $f(x)=f(y)$

Then $f(x)=f(y)\Rightarrow x^2+x+1=y^2+y+1$

$\Rightarrow (x-y)(x+y+1)=0\Rightarrow x=y$ Or $x=(-y-1)\notin N$

$\therefore f$ is one-one.

Again, since for each $y\in N$, there exist $x\in N$

$\therefore f$ is onto.

98. (b) $f(x)=(x-1)(x-2)(x-3)$

$\Rightarrow f(1)=f(2)=f(3)=0 \Rightarrow f(x)$ is not one-one.

For each $y\in R$, there exists $x\in R$ such that $f(x)=y$. Therefore *f* is onto. Hence $f:R\to R$ is onto but not one-one.

99. (b) $f'(x)=\dfrac{1}{(1+x)^2}>0,\forall x\in[0,\infty)$ and range $\in[0,1)$

$\Rightarrow$ Function is one-one but not onto

100. (a) $f'(x)=2+\cos x>0$. So, $f(x)$ is strictly monotonic increasing so, $f(x)$ is one-to-one and onto.

101. (d) $f(x) = x + \sqrt{x^2} = x + |x|$

Clearly f is not one-one as $f(-1) = f(-2) = 0 \Rightarrow f$ is not one-one

$f(x) \ge 0, \forall x \in R, \Rightarrow$ Range of $f = (0, \infty) \subset R$. $\Rightarrow$ f is not onto.

102. (b) Let $\frac{x^2 + 34x - 71}{x^2 + 2x - 7} = y$

$\Rightarrow x^2(1-y) + 2(17-y)x + (7y - 71) = 0$

$\Rightarrow \Delta \ge 0$

$\Rightarrow y^2 - 14y + 45 \ge 0 \Rightarrow y \ge 9, y \le 5$.

103. (a) $fof(x) = f(f(x)) = f\left(\frac{1-x}{1+x}\right) = \frac{1 - \frac{1-x}{1+x}}{1 + \frac{1-x}{1+x}} = x, \forall x$

$\therefore fof = I \Rightarrow f$ is the inverse of itself.

104. (b) f is one-one and onto, so f^{-1} exists and is $f^{-1}(x) = \frac{x+5}{3}$.

105. (a) $-\sqrt{1 + (-\sqrt{3})^2} \le (\sin x - \sqrt{3}\cos x) \le \sqrt{1 + (-\sqrt{3})^2}$

$-2 \le (\sin x - \sqrt{3}\cos x) \le 2$

$-2 + 1 \le (\sin x - \sqrt{3}\cos x + 1) \le 2 + 1$

$-1 \le (\sin x - \sqrt{3}\cos x + 1) \le 3$ *i.e.*, range $= [-1, 3]$

$\therefore$ For f to be onto $S = [-1, 3]$

106. (b) For any $x, y \in R$, we have

$f(x) = f(y) \Rightarrow \frac{x-m}{x-n} = \frac{y-m}{y-n} \Rightarrow x = y$

$\therefore$ f is one-one.

Let $\alpha \in R$ such that $f(x) = \alpha \Rightarrow \frac{x-m}{x-n} = \alpha \Rightarrow x = \frac{m - n\alpha}{1-\alpha}$

Clearly $x \notin R$ for $\alpha = 1$. So, f is not onto.

107. (c) Let $f(x) = f(y) \Rightarrow \frac{x^2-4}{x^2+4} = \frac{y^2-4}{y^2+4}$

$\Rightarrow \frac{x^2-4}{x^2+4} - 1 = \frac{y^2-4}{y^2+4} - 1 \Rightarrow x^2 + 4 = y^2 + 4$

$\Rightarrow x = \pm y$, $\therefore f(x)$ is many-one.

Now for each $y \in (-1,1)$, there does not exist $x \in X$ such that $f(x) = y$. Hence f is into.

108. (c) $f(x_1) = f(x_2) \Rightarrow x_1^2 = x_2^2 \Rightarrow x_1 = x_2$, [if $X = R^+$]

$\Rightarrow f$ is one-one. Since $R_f = R^+ \subseteq R = Y$; $\therefore f$ is not onto.

109. (d) $\because f(0) = f(-1) = 0$ hence $f(x)$ is many one. But there is no pre-image of -1. Hence $f(x)$ is into function. So function is many-one into.