



$$H_1 = x_1 * w_1 + x_2 * w_2 + b_1$$

Activation function is sigmoid = $\frac{1}{1+e^{-x}}$

$$\text{out } H_1 = \frac{1}{1+e^{-H_1}}$$

Forward Pass

$$H_1 = w_1 x_1 + w_2 x_2 + b_1 = 0.3775$$

$$\text{out } H_1 = \frac{1}{1+e^{-0.3775}} = 0.593269992$$

$$\text{out } H_2 = 0.596884378$$

$$y_1 = \text{out } H_1 * w_5 + \text{out } H_2 * w_6 + b_2 = 1.105905967$$

$$\text{out } y_1 = 0.75136507$$

$$\text{out } y_2 = 0.772928465$$

I/P

$$x_1 = 0.05$$

$$x_2 = 0.10$$

Bias

$$b_1 = 0.35$$

$$b_2 = 0.60$$

Initial weights

$$w_1 = 0.15$$

$$w_2 = 0.20$$

$$w_3 = 0.25$$

$$w_4 = 0.30$$

$$w_5 = 0.40$$

$$w_6 = 0.45$$

$$w_7 = 0.50$$

$$w_8 = 0.55$$

$$T_1 = 0.01$$

$$T_2 = 0.99$$

Calculation Total Error

$$E_{\text{Total}} = \sum \frac{1}{2} (\text{target} - \text{output})^2$$

$$= \frac{1}{2} (0.01 - 0.75136)^2 + \frac{1}{2} (0.99 - 0.772)^2$$

$$E_{\text{Total}} = 0.299371109$$

Backward Pass

To calculate error at $w_5 = \frac{\partial E_{total}}{\partial w_5}$

If ∂E_{total} there is no w_5 , hence we cannot differentiate we will split it now

$$\frac{\partial E_{total}}{\partial w_5} = \frac{\partial E_{total}}{\partial out_{y1}} + \frac{\partial out_{y1}}{\partial y_1} + \frac{\partial y_1}{\partial w_5}$$

(1) (2) (3)

$$\frac{\partial E_{total}}{\partial w_5} = \frac{1}{2} (T_1 - out_{y1})^2 + \frac{1}{2} (T_2 - out_{y2})^2$$

$$\frac{\partial E_{total}}{\partial out_{y1}} = 2 \cdot \frac{1}{2} (T_1 - out_{y1})^{2-1} \cdot -1 + 0 = (out_{y1} - T_1) = 0.74136507$$

For term (1)

$$\frac{\partial out_{y1}}{\partial y_1} = \frac{1}{1 + e^{-y_1}}$$

$$\frac{\partial out_{y1}}{\partial y_1} = out_{y1} (1 - out_{y1}) = 0.75136 (1 - 0.75136) = 0.186815602$$

For term (2)

$$\begin{aligned} \sigma(x) &= \frac{1}{1 + e^{-x}} \\ \frac{\partial}{\partial x} \sigma(x) &= \frac{e^{-x}}{(1 + e^{-x})^2} \\ &= \frac{(1 - e^{-x}) - 1}{(1 + e^{-x})^2} \\ &= \frac{1}{1 + e^{-x}} \left(1 - \frac{1}{1 + e^{-x}} \right) \end{aligned}$$

$$y_1 = out_{H1} + w_5 + out_{H2} + w_6 + b_2$$

$$\frac{\partial y_1}{\partial w_5} = out_{H1} = 0.593269992$$

For term (3)

$$\frac{\partial E_{total}}{\partial w_5} = (1) * (2) * (3) = 0.082167041$$

$$w_5 = w_5 - \eta \frac{\partial E_{total}}{\partial w_5}$$

$$w_5 = 0.35891648$$

Now we are operating in the hidden layer

$$\frac{\partial E_{total}}{\partial w_1} = \frac{\partial E_{total}}{\partial a_{outH1}} * \frac{\partial a_{outH1}}{\partial H_1} * \frac{\partial H_1}{\partial w_1}$$

$$\downarrow$$

$$\frac{\partial E_{total}}{\partial a_{outH1}} = \frac{\partial E_1}{\partial a_{outH1}} + \frac{\partial E_2}{\partial a_{outH1}}$$

$$\downarrow$$

$$\frac{\partial E_1}{\partial a_{outH1}} = \frac{\partial E_1}{\partial y_1} * \frac{\partial y_1}{\partial a_{outH1}}$$

$$\downarrow$$

$$\frac{\partial E_1}{\partial y_1} = \frac{\partial E_1}{\partial a_{out y_1}} * \frac{\partial a_{out y_1}}{\partial y_1}$$

$$= (0.01 - 0.75136) * (0.186815602)$$

$$= 0.71436507 * 0.1868$$

$$= 0.138$$

$$\frac{\partial a_{outH1}}{\partial H_1} = a_{outH1} (1 - a_{outH1}) = 0.5932 (1 - 0.5932) = 0.241360$$

$$\frac{\partial H_1}{\partial w_1} = x_1 = 0.05$$

$$\frac{\partial E_{total}}{\partial w_1} = (0.03635) * (0.24300) * (0.05) = 0.000438568$$

updating w_1

$$w_1^* = w_1 - \eta \left(\frac{\partial E_{total}}{\partial w_1} \right)$$

$$w_1^* = 0.15 - \eta (0.000438)$$

$$w_1^* = 0.149780716$$

$$E_1 = \frac{1}{2} (T - a_{out y_1})^2$$

$$E_2 = \frac{1}{2} (T_2 - a_{out y_2})^2$$

$$\frac{\partial g_1}{\partial a_{outH1}} = w_5 = 0.40$$

$$\frac{\partial E_1}{\partial a_{outH1}} = 0.138 * 0.40 = 0.055$$

$$\frac{\partial E_2}{\partial a_{outH1}} = -0.019049117$$

$$\frac{\partial E_{total}}{\partial a_{outH1}} = 0.055 - 0.019$$

$$= 0.03635$$