Linear Algebra

Dimension of matrix : no. of rows x no. of columns.

$$A = \begin{bmatrix} 1402 & 191 \\ 1371 & 821 \\ 949 & 1437 \\ 147 & 1448 \end{bmatrix}$$

Rows x colum.

4x3 matric

$\mathbb{R}^{4\times2}$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

2x3 matr.

$\mathbb{R}^{2\times3}$

A_{ij} = "i, j entry" in the i^{th} row, j^{th} colum.

$A_{11} = 1402$ $\quad$ $A_{32} = 1437$ $\quad$ A_{43} = Row 4 & Colu 3 = Undefined.

$A_{12} = 191$ $\quad$ $A_{41} = 147$

Vector = An n x 1 matrix $y = \begin{bmatrix} 460 \\ 232 \\ 315 \\ 174 \end{bmatrix}$ $n = 4$ $\quad \mathbb{R}^4$

← 4 dimensional Vector

y_i = i^{th} element

$y_1 = 460$, $y_2 = 232$, $y_3 = 315$, $y_4 = 174$

1-indexed Vs 0-indexed.

$$y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} \qquad y = \begin{bmatrix} y_0 \\ y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

Addition

$$\begin{bmatrix} 1 & 0 \\ 2 & 5 \\ 3 & 1 \end{bmatrix} + \begin{bmatrix} 4 & 0.5 \\ 2 & 5 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 5 & 0.5 \\ 4 & 10 \\ 3 & 2 \end{bmatrix}$$

3x2 $\qquad$ 3x2 $\qquad$ 3x2

$$\begin{bmatrix} 1 & 0 \\ 2 & 5 \\ 3 & 1 \end{bmatrix} + \begin{bmatrix} 4 & 0.5 \\ 2 & 2 \end{bmatrix} = \boxed{\text{Error}}$$

Scalar multiplication

$$3 \times \underset{3\times 2}{\begin{bmatrix} 1 & 0 \\ 2 & 5 \\ 3 & 1 \end{bmatrix}} = \underset{3\times 2}{\begin{bmatrix} 3 & 0 \\ 6 & 15 \\ 9 & 3 \end{bmatrix}} = \begin{bmatrix} 1 & 0 \\ 2 & 5 \\ 3 & 1 \end{bmatrix} \times 3$$

$$\begin{bmatrix} 4 & 0 \\ 6 & 3 \end{bmatrix} / 4 = \frac{1}{4} \begin{bmatrix} 4 & 0 \\ 6 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 3/2 & 3/4 \end{bmatrix}$$

Combination of operand

$$\underbrace{3 \times}_{\text{Scalar mult}} \begin{bmatrix} 1 \\ 4 \\ 2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 5 \end{bmatrix} - \underbrace{\begin{bmatrix} 3 \\ 0 \\ 2 \end{bmatrix} / 3}_{\text{Scalar division}}$$

$$\begin{bmatrix} 3 \\ 12 \\ 6 \end{bmatrix} \underset{\substack{\text{matrix add} \\ \text{vector add}}}{+} \begin{bmatrix} 0 \\ 0 \\ 5 \end{bmatrix} \underset{\substack{\text{vector subtract} \\ \text{matric subt}}}{-} \begin{bmatrix} 1 \\ 0 \\ 2/3 \end{bmatrix}$$

$$= \begin{bmatrix} 3 \\ 12 \\ 11 \end{bmatrix} - \begin{bmatrix} 1 \\ 0 \\ 2/3 \end{bmatrix} = \begin{bmatrix} 2 \\ 12 \\ 11-2/3 \end{bmatrix}$$

matrix multiplication

$$\begin{bmatrix} 1 & 3 \\ 4 & 0 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 5 \end{bmatrix} = \begin{bmatrix} 1\times1 + 3\times5 \\ 4\times1 + 0\times5 \\ 2\times1 + 1\times5 \end{bmatrix} = \begin{bmatrix} 16 \\ 4 \\ 7 \end{bmatrix} \mathbb{R}^3$$

3×2 $m\times1$

$$\overset{A}{\begin{bmatrix} \; \end{bmatrix}} \times \overset{B}{\begin{bmatrix} \; \end{bmatrix}} = \overset{y}{\begin{bmatrix} \; \end{bmatrix}}$$

$m\times n$ $n\times1$

col

This two should be equa

m dimensional vector.

$$\begin{bmatrix} 1 & 2 & 1 & 5 \\ 0 & 3 & 0 & 4 \\ -1 & -2 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 14 \\ 13 \\ -7 \end{bmatrix}$$

3×4 4×1 3×1

House sizes

$h_\theta(x) = -40 + 0.25x$

$$\left.\begin{matrix} 2104 \\ 1416 \\ 1534 \\ 852 \end{matrix}\right\} \quad \begin{bmatrix} 1 & 2104 \\ 1 & 1416 \\ 1 & 1534 \\ 1 & 852 \end{bmatrix} \times \begin{bmatrix} -40 \\ 0.25 \end{bmatrix} = \begin{bmatrix} 1\times-40 + 2104\times0.25 \\ 1\times-40 + 1416\times0.25 \end{bmatrix}$$

4×2 2×1

Prediction = Data matrix x parameter

Inverse matrix

1 = "Identity"

$3(3^{-1}) = 1$ $\quad 12 \times (12^{-1}) = 1$

$12 \times 1/12 = 1$

not all number have an inverse $0(0^{-1})$ undefine.

Matrix Inverse

If A is an $m \times m$ matrix, and if it has an inverse

$A(A^{-1}) = A^{-1}A = I$

Eg:- $\underbrace{\begin{bmatrix} 3 & 4 \\ 2 & 16 \end{bmatrix}}_{\substack{2\times2 \\ A}} \underbrace{\begin{bmatrix} 0.4 & -0.1 \\ -0.05 & 0.075 \end{bmatrix}}_{A^{-1}} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I_{2\times2}$

matrices that don't have an inverse is called "singular"

$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

Matrix Transpose

$A = \begin{bmatrix} 1 & 2 & 0 \\ 3 & 5 & 9 \end{bmatrix}$ $\quad A^T = \begin{bmatrix} 1 & 3 \\ 2 & 5 \\ 0 & 9 \end{bmatrix}$

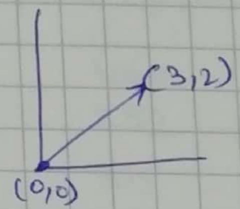
2×3 $\quad$ 3×2

Vectors

→ Eigen Vector

when a matrix is multiplied with a vector and results in the same vector

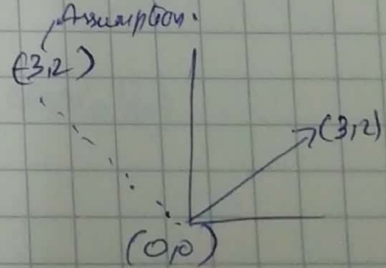
$$\begin{bmatrix} 2 & 3 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 11 \\ 5 \end{bmatrix}$$



$$\begin{bmatrix} 2 & 3 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 2 \end{bmatrix} = \cancel{13} \begin{bmatrix} 12 \\ 8 \end{bmatrix} = 3 \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

Eigen vector.

- → Eigen vectors are only possible in square matrix (2x2 & 4x4)
- → If square matrix has "n" dimension (n×n) we will get "n" Eigen vectors
- → All Eigen vector are ~~orthogonal~~ perpendicular in nature



Eigen Values

$$\underbrace{M}_{\text{Square matrix}} * \underbrace{V}_{\text{Eigen vect}} = \underbrace{\lambda}_{\text{Eigen value}} * \underbrace{V}_{\text{Eigen vector}}$$

$$\underbrace{\begin{bmatrix} 2 & 3 \\ 2 & 1 \end{bmatrix}}_{m} \underbrace{\begin{bmatrix} 3 \\ 2 \end{bmatrix}}_{V} = \underbrace{4}_{\lambda} \underbrace{\begin{bmatrix} 3 \\ 2 \end{bmatrix}}_{V}$$