Optimizing Public Bus Network Scheduling

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Abstract

This report explores the optimization of urban bus scheduling systems using operations analytics techniques, based on the model proposed by Berhan et al. (2014). The original linear programming (LP) model aimed to minimize the total number of bus trips while meeting passenger demand, constrained by fleet size, route-specific trip limits, and minimum service levels.

To validate and extend this model, multiple implementations were developed using Python and Google OR-Tools. A first version strictly replicated the original formulation on a small subset of routes, confirming mathematical correctness and structure. A second, extended prototype introduced a third bus type and a proportional demand model across shifts, successfully achieving optimal results at a smaller scale.

However, when scaled up to all 93 routes, neither the original model nor the extended version could be solved optimally. Despite attempts to increase flexibility with an additional vehicle type and imputed demand data, both versions remained infeasible, primarily due to overly rigid constraints and limitations in real-world data approximation.

The findings highlight the importance of iterative modeling and sensitivity analysis, emphasizing the challenges of achieving feasible and scalable solutions in large-scale public transit scheduling under strict operational constraints.

Table of Contents

Chapter 1: Introduction

1.1 Problem Description

Bus scheduling in public transportation is the process of assigning a fleet of buses to routes and time slots to meet passenger demand efficiently. This operational planning task must balance service availability against cost constraints. Transit agencies must deploy enough buses to avoid crowding and long waits, but not so many that buses run mostly empty and waste resources. The scheduling decision is thus a trade-off: using too many buses increases operating costs, while too few harms service quality and reliability (Berhan et al., 2014).

In the context of Addis Ababa's Anbessa City Bus Service Enterprise (ACBSE)—the city's sole public bus provider—this problem is especially pronounced. ACBSE operates over 110 urban routes using a fixed daily timetable, meaning the same number of buses is assigned to each route regardless of time-of-day demand. This fixed bus assignment approach has led to operational and financial inefficiencies (Berhan et al., 2014).

1.2 Objectives of the Study

The primary objective of this study is to analyze and optimize urban bus scheduling through the application of Operations Analytics techniques. By examining the current scheduling practices used by Anbessa City Bus Service Enterprise (ACBSE) in Addis Ababa, this project aims to enhance decision-making in public transportation by minimizing operational inefficiencies and improving service reliability.

Building upon the linear programming model proposed by Berhan et al. (2014), this project seeks not only to replicate the original model but also to extend it through a Python-based implementation using OR-Tools. This computational approach enables the integration of route-level passenger demand, heterogeneous bus types, and real-time shift constraints. Furthermore, the extended model allows for future inclusion of additional business considerations, such as minimizing fuel costs, reducing idle time, or optimizing service quality during peak congestion periods.

1.3 Scope of the Study

The scope of this study is to analyze and optimize urban bus scheduling using Operations Analytics techniques through progressive model development. Using Anbessa City Bus Service Enterprise (ACBSE) in Addis Ababa as a case study, the research began by replicating the original linear programming model proposed by Berhan et al. (2014) on a reduced dataset of selected routes to verify mathematical validity and constraint logic.

The study then extended the model by introducing a third bus type and proportional shift-based demand, tested through a small-scale prototype to explore feasibility improvements. Following this, the full original model was implemented across 93 routes using imputed demand data. However, under rigid constraints, the model failed to find a feasible solution.

To address this limitation, a final extended version was developed. It incorporated flexible trip allocations through a buffer mechanism and maintained theoretical consistency while adapting to real-world operational variability. The model was successfully solved for full demand, offering practical recommendations for scalable and resilient transit planning.

Chapter 2: Literature Review and Model Implementation

2.1 Review of the Selected Article

The article by Berhan, Mengistu, Beshah, and Kitaw (2014) addresses inefficiencies in public bus scheduling within Addis Ababa’s Anbessa City Bus Service Enterprise (ACBSE). The fixed schedule system caused overcrowded and underutilized buses at different times. The authors proposed a linear programming (LP) model to optimize bus deployment across 93 routes and four shifts. The LP model aimed to minimize the total number of trips while satisfying passenger demand, fleet capacity, and service requirements.

2.2 Mathematical Model

The LP model includes two types of buses, each with different capacities. The objective is to minimize total trips:
Minimize

$$Z = \sum_{93}^{93} \sum_{j=1}^{4} (x_{ij} + y_{ij}) (1)$$

Subject to constraints on demand satisfaction, fleet size, trip limits per route, and non-negativity. Decision variables x_{ij} and y_{ij}represent trips by each bus type. Parameters include demand D_{ij}, trip factors T_{ij}, proportions P_i, and shift requirements w_j.

$$60^* x_{ij} + 90^* y_{ij} \geq D_{ij} (2)$$

$$\sum_{93}^{93} \sum_{j=1}^{4} x_{ij} \leq 600^* \sum_{93}^{93} \sum_{j=1}^{4} T_{ij}\ (3)$$

$$\sum_{93}^{93} \sum_{j=1}^{4} y_{ij} \leq 90^* \sum_{93}^{93} \sum_{j=1}^{4} T_{ij}\ (4)$$

$$\sum_{93}^{93} \sum_{j=1}^{4} (x_{ij} + y_{ij}) \leq 93^* \sum_{j=1}^{4} w_j (5)$$

$$x_{ij} \leq 600^* P_i T_{ij} (6)$$

$$y_{ij} \leq 90^* P_i T_{ij} (7)$$

$$\sum_{i=1}^{93} P_i = 1 (8)$$

$$x_{ij}, y_{ij} \geq 0 (9)$$

$$\forall_i,\ i = 1,2,3, \ldots 93, \quad \forall_j,\ i = 1,2,3,4$$

2.3 Implementation of the Model Using Google OR-Tools

To explore the practical viability of the LP model proposed by Berhan et al. (2014), the team implemented it using Google OR-Tools in Python. A phased approach was adopted to ensure validation and scalability.

Phase 1: Small-Scale Prototype (Original Model)

An initial prototype was created using a subset of 8 routes and four daily shifts. This version remained 100% faithful to the original mathematical formulation, including all constraints related to demand fulfillment, fleet size, trip factors, and minimum trips per shift. The solver returned an optimal solution, confirming that the original model structure is valid when applied to a manageable dataset.

Phase 1.5: Small Extended Prototype (with Enhancements)

To explore enhancements prior to scaling, a second small-scale prototype was developed. This version added a third bus type with higher capacity and replaced fixed proportions with proportional shift-based demand. It successfully validated the benefits of heterogeneous fleet design and dynamic demand modeling, informing the structure of the full extended model.

2.4 Discussion of Results

The original LP model demonstrated feasibility in the small-scale prototype but failed when applied to the full dataset of 93 routes and 640,000 daily passengers. The solver returned an infeasible status, even though all constraints and equations matched the original formulation by Berhan et al. (2014).

This infeasibility points to practical limitations rather than theoretical flaws.

The key challenges included:

- Overly rigid allocation of trips based strictly on fixed proportions (P_i).
- Strict shift-based trip limits that did not account for variations in route-specific demand.
- Limited empirical data that required imputation which may have introduced distributional inconsistencies.

These results suggested the need for a more flexible and adaptable version of the model—one that could preserve the spirit of the original formulation while relaxing selected constraints to accommodate real-world operational dynamics. However, even after introducing a third bus type and proportional demand handling in the extended model, the full-scale implementation across 93 routes remained infeasible. This outcome further underscores the difficulty of rigid LP formulations in large-scale applications without sufficient slack or dynamic data inputs.

Chapter 3: Extension of the Mathematical Model

3.1 Extended Model Proposal and Justification

The original linear programming (LP) model proposed by Berhan et al. (2014) aimed to optimize urban bus scheduling in Addis Ababa using fixed constraints, fixed route demand proportions (P_1 to P_{x3}), and rigid fleet allocations. However, when re-implementing the model with a total daily demand of 640,000 passengers, the model proved infeasible. This outcome revealed structural limitations in the formulation and inspired the development of an extended version of the model to regain feasibility while preserving its theoretical integrity.

Part 1: Interpretation of the Original Model's Infeasibility

The infeasibility of the original model under current input parameters is not a flaw in formulation, but a diagnostic insight into the operational limitations of the transit system. Specifically:

- Rigid Demand Fulfillment: The model required 100% of the estimated demand (640k passengers) to be satisfied.
- Strict Fleet Allocation: Each route was capped to a fixed percentage of the total bus trips.
- Inflexible Frequency Constraints: A minimum number of trips per shift was enforced for all routes.

Part 2: Minimal Adjustment Strategy – Extended Model

To restore feasibility without discarding the core structure of the original model, we introduced a minimal and justified modification: the addition of a third bus type, with lower capacity but more flexible usage. This extension maintains the model's integrity while providing operational realism.

Before implementing the full 93-route version of the extended model, we tested this strategy on a small-scale prototype (6 routes) that included the third bus type and proportional demand across shifts. This version demonstrated feasibility and optimality, validating the use of additional bus types and time-of-day demand decomposition. Based on these results, the full extended model was constructed.

$$Z = \sum_{93}^{93} \sum_{j=1}^{4} (x_{ij} + y_{ij} + z_{ij}) (1)$$

$$60^* x_{ij} + 90^* y_{ij} + 10^* z_{ij} \geq D_{ij} (2)$$

$$\sum_{93}^{93} \sum_{j=1}^{4} x_{ij} \leq 600^* \sum_{93}^{93} \sum_{j=1}^{4} T_{ij} \ (3)$$

$$\sum_{93}^{93} \sum_{j=1}^{4} y_{ij} \leq 90^* \sum_{93}^{93} \sum_{j=1}^{4} T_{ij} \ (4)$$

$$\sum_{93}^{93} \sum_{j=1}^{4} (x_{ij} + y_{ij} + z_{ij}) \leq 93^* \sum_{j=1}^{4} w_j (5)$$

$$x_{ij} \leq 600^* P_i T_{ij} (6)$$

$$y_{ij} \leq 90^* P_i T_{ij} (7)$$

$$\sum_{i=1}^{93} P_i = 1 (8)$$

$$x_{ij}, y_{ij}, z_{ij} \geq 0 (9)$$

$$\sum_{93}^{93} \sum_{j=1}^{4} z_{ij} \leq 10^* \sum_{93}^{93} \sum_{j=1}^{4} T_{ij} \ (10)$$

$$z_{ij} \leq 10^* P_i T_{ij} (11)$$

$$\forall_i, \ i = 1,2,3, \ldots 93, \quad \forall_j, \ i = 1,2,3,4$$

Part 3: Academic Justification

The extended version of the model demonstrated feasibility and optimality only at the small-scale level. The introduction of a third bus type improved flexibility and suggested potential for better demand coverage. However, when applied to the full dataset of 93 routes, the model still failed to reach feasibility.

Key observations included:

- Persistent infeasibility despite introducing additional flexibility.
- Improved structure and realism at a conceptual level, but limited impact when scaled under rigid constraints.
- The infeasibility suggests that either further relaxation of constraints or more accurate demand/fleet data would be necessary for real-world implementation.

While the extended model did not solve the full-scale problem optimally, it provided valuable insights into constraint sensitivity and the limits of linear programming for large-scale transit operations.

3.2 Implementation of the Extended Model

To overcome the infeasibility of the original LP model proposed by Berhan et al. (2014), we implemented an extended version using Google OR-Tools in Python, applying integer programming techniques. This implementation introduced a third bus type to increase flexibility under high-demand conditions given:

- Passenger demand proportions P_i
- Demand per shift D_i^j
- Trip factors T_i^j

This approach allowed us to simulate realistic operational data aligned with the original study's assumptions. While the extended model achieved feasibility and optimality in small-scale tests, the full-scale implementation across 93 routes remained infeasible. The solver could not satisfy all constraints under current input conditions, indicating the need for further adjustments or data enhancements.

3.3 Discussion of Extended Model Results

The extended version of the model provided conceptual improvements in feasibility and flexibility. The introduction of a third bus type offered additional capacity and adaptability in the small-scale prototype. This allowed the solver to satisfy demand under more realistic, shift-based assumptions and demonstrated the potential value of heterogeneous fleet modeling.

However, when the extended model was applied to the full dataset of 93 routes and 640,000 passengers, it remained infeasible. Despite relaxing certain assumptions and using a more flexible structure, the solver was unable to satisfy all constraints simultaneously. This result highlights the persistence of over-constraining factors in the original design, such as strict trip caps, rigid demand satisfaction, and uniform frequency requirements.

Key outcomes included:

- Feasibility and optimality achieved only in the small-scale version.
- Improved conceptual adaptability for large-scale use, but still infeasible when fully scaled.
- Identification of bottlenecks and constraint sensitivity that inform future modeling directions.

While the extended model did not solve the full-scale problem, it provided useful diagnostics and emphasized the importance of soft constraints, scalable data generation, and iterative refinement in large urban scheduling problems.

Chapter 4: Conclusion and Recommendations

4.1 Summary of Findings

This study explored the modeling and optimization of urban bus scheduling using both the original linear programming model proposed by Berhan et al. (2014) and an extended version developed to address infeasibility and practical limitations.

Key Findings from the Original Model:

- The original model aimed to minimize the total number of bus trips while satisfying strict constraints on demand coverage, trip limits based on fleet availability, and minimum service frequency per shift.
- When re-implemented with a total demand of 640,000 passengers and complete data for 93 routes, the model proved infeasible under its original constraints.
- This infeasibility revealed that the combination of fixed route allocations (based on historical proportions), rigid fleet caps, and uniform frequency standards made the model too restrictive to handle peak demand scenarios.
- The result served as an analytical signal that the system, as modeled, could not realistically support the projected demand without some operational flexibility.

Key Findings from the Extended Model:

- To address infeasibility, an extended version of the model was developed by introducing a small buffer in the trip assignment constraints. This allowed the model to allocate slightly more trips to high-demand routes when needed.
- A looped buffer strategy was considered to explore the minimum constraint relaxation that might restore feasibility. Preliminary analysis suggested that increasing trip caps by approximately 28% could potentially lead to a feasible solution. However, under the current formulation and input data, the extended

model remained infeasible at full scale. This highlights the need for further model adjustments and more flexible constraint handling in future iterations.
- The extended model preserved all original objectives and most constraints, minimizing the total number of trips while ensuring demand coverage and maintaining shift-level service frequency.
- This model also proved more adaptable to real-world operational challenges, such as demand variability and route-specific peaks, without requiring a complete overhaul of the mathematical formulation.

Overall Improvements in Bus Scheduling Optimization:

- At the small scale, the extended model successfully provided an optimal solution that the original model could not, demonstrating greater flexibility and robustness.
- It maintained theoretical rigor while introducing practical adaptability, making it a more effective decision-support tool for transit agencies.
- The buffer-based approach helped highlight how small policy adjustments (e.g., relaxing strict proportional trip limits) can significantly improve system performance and model feasibility.

4.2 Recommendations

Based on the analysis and results of both the original and extended models, several recommendations can be made for transit agencies aiming to optimize urban bus scheduling:

Recommendations for Transit Agencies:

- Introduce Controlled Flexibility in Resource Allocation:
  Instead of assigning bus trips strictly according to historical demand proportions, agencies should consider allowing control adjustments based on real-time or forecasted passenger load. This flexibility can improve service quality on high-demand routes without compromising overall efficiency.
- Use Optimization Tools with Adaptive Parameters:
  Tools like Google OR-Tools can be integrated into planning systems to run simulations under varying constraints. By using adjustable parameters such as buffers or slack variables, planners can evaluate multiple scenarios and choose the one that balances cost and coverage best.
- Monitor System Stress Points:
  The use of a buffer in our extended model highlighted the minimum level of flexibility required to restore feasibility. Transit planners can use similar techniques to identify which constraints (e.g., route caps, fleet size, or service frequency) are most limiting during peak periods.

- Adopt Data-Driven Scheduling Policies:
  Instead of static schedules, consider demand-driven planning where trip assignments are updated regularly based on passenger trends. This requires access to reliable data but can lead to better fleet utilization and passenger satisfaction.

Suggestions for Future Research:

- Incorporating Dynamic Passenger Demand:
  Future models could benefit from integrating time-of-day fluctuations, seasonal variations, or special events to reflect more realistic demand patterns.

- Integration of Real-Time Data:
  Real-time GPS and passenger count data can be used to update schedules dynamically. This would allow for responsive adjustments, especially during service disruptions or sudden demand spikes.

- Modeling Environmental Objectives:
  Expanding the model to include fuel consumption, emissions reduction, or electric fleet constraints can align transit optimization with broader sustainability goals.

- Considering Multiple Depots and Bus Types:
  Adding complexities such as different garage locations and more diverse bus categories can provide a more granular and practical scheduling approach.

By leveraging these recommendations and continuing to refine optimization models, transit agencies can move toward more efficient, equitable, and sustainable public transport systems.

References

Berhan, E., Mengistu, D., Beshah, B., & Kitaw, D. (2014). *Modeling and analysis of bus scheduling systems of urban public bus transport*. *International Journal of Computer Information Systems and Industrial Management Applications*, *6*, 403–414. https://www.mirlabs.org/ijcisim/regular_papers_2014/IJCISIM_62.pdf

Churchill, E. (2020). *Operations analytics*. Wharton School Press.

Google OR-Tools. (n.d.). *Operations research tools for optimization*. Google Developers. https://developers.google.com/optimization

Mitrović-Minić, S. (2013). Vehicle routing problems. In S. I. Gass & M. C. Fu (Eds.), *Encyclopedia of Operations Research and Management Science* (pp. 1588–1594). Springer. https://doi.org/10.1007/978-1-4419-1153-7_1158

Van Rossum, G., & Drake, F. L. (2009). *Python 3 reference manual*. CreateSpace.

Appendices

Appendix A: Python Scripts – Original Model

A.1 – Prototype Script for Original Model (Small-Scale)

Berhan_bus_original_small.py
This script contains a small-scale version of the original linear programming (LP) model proposed by Berhan et al. (2014), implemented using Google OR-Tools. It uses only a few routes and shifts to validate the correctness of the mathematical formulation and constraint logic.

A.2 – Prototype Script for Extended Model (Small-Scale):
Berhan_bus_extended_small.py
This version extends the small-scale prototype by introducing a third bus type to enhance operational flexibility. It tests whether additional vehicle types improve feasibility while maintaining the structure of the original model.

A.3 – Full Model Script for Original Formulation (Infeasible):
Berhan_bus_original_full.py
This script applies the original LP model to all 93 routes and four shifts using imputed data to simulate full-scale operations. Despite being a faithful replication of Berhan et al.'s model, it results in infeasibility due to the rigidity of constraints when scaled up.

A.4 – Full Model Script for Extended Formulation (Still Infeasible):
Berhan_bus_extended_full.py
This version applies the extended model—with a third bus type—to the full dataset of 93 routes. While it improves realism by introducing heterogeneity in the fleet, the model remains infeasible due to strict demand satisfaction and fleet allocation constraints.

The python scripts will be uploaded via cascade, but they will also be available in https://github.com/prumucena1979/DAMO-610-6

Appendix B: Individual Contribution & AI Usage Declaration (uploaded on cascade)

This section documents the individual contributions of each team member to the project and describes how AI tools were used responsibly throughout the development process. AI support was limited to permissible tasks, such as idea generation, grammar refinement, code review assistance, and formatting suggestions, in alignment with the institution's academic integrity guidelines.