Data Streams

Internet Analytics (COM-308)

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Overview

- Performance criteria:
  - Data volume
  - Computational cost
- We have paid some attention to this:
  - Examples: SGD vs "full" gradient descent; power method for PageRank
- But: assumption that we always have access to all the data!
  - Allows iterative algorithms
  - Many algorithms require "random access"
- Data streams:
  - Relax these assumptions in different ways
  - Tradeoff: compute simple quantities very efficiently

Motivating example (1)

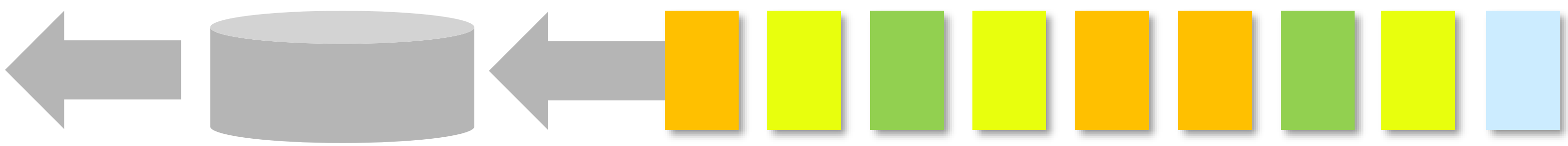
- Internet backbone router
- Order of magnitude:
  - 100s of interfaces at 10s of Gbps
  - = several billion pkts/sec!
  - …at expensive SRAM speeds!
- Traffic analysis app to detect DDoS attack:
  - What are the **dominant** flows?
  - How many **different** (unique) source IP addresses in a minute?

Motivating example (2)

- Implantable medical devices:
  - Resource-limited: memory, computation, energy
  - Rare/unpredictable read-outs
- Sensor reads:
  - Storing full trace may not be feasible
  - Extract key statistics & maintain over time

Data stream model

- Computing statistics with sub-linear memory
- Example:
  - n numbers: how many unique values (colors) k?

$k = 4$ 

- How to solve with $\theta(n \log n)$ memory?
  - Keep every value in some efficient data structure; compare & count
- How to solve with $o(n)$ memory?
  - Cannot solve exactly
- Streaming algorithms: ∞ data, finite memory
  - Approximation
  - (Pseudo-)randomization

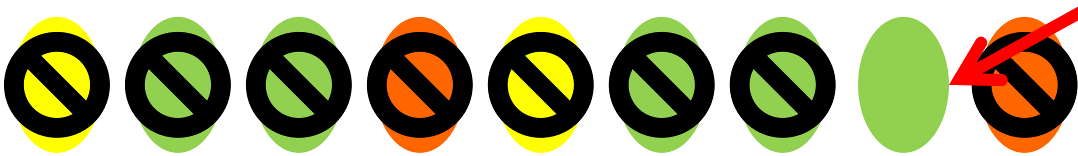
Estimates with sublinear memory

- Counting # of elements:
  - $O(\log n)$ space
- Maximum value:
  - $O(1)$ space
- Average value:
  - Sum / counter: $O(\log n)$ space
- Heavy hitters:
  - Most frequent values
  - Table of values so far has size $O(n \log n)$
- Number of distinct elements?
  - I.e., do not double-count multiple occurrences
  - Table of values so far: $O(n \log n)$

(1): Heavy hitters

- Find most frequent values in a stream
- Trivial solution: keep counters for every value, sort at the end
  - Cost: space $\Omega(m)$ (m: alphabet size)
- Threshold criterion θ:
  - Report every value a that has occurred $> \theta n$ times
- Case $\theta = 1/2$: majority (at most one value)
- Trick: pairwise annihilation of different observed values

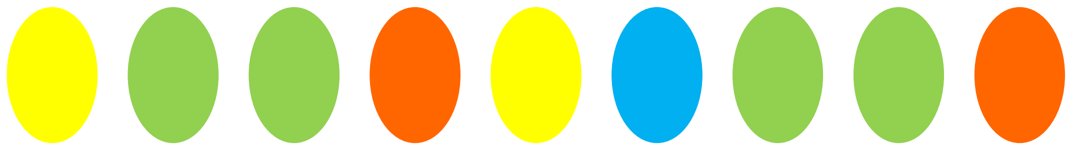
May or may not be majority color → 2nd pass to re-tally



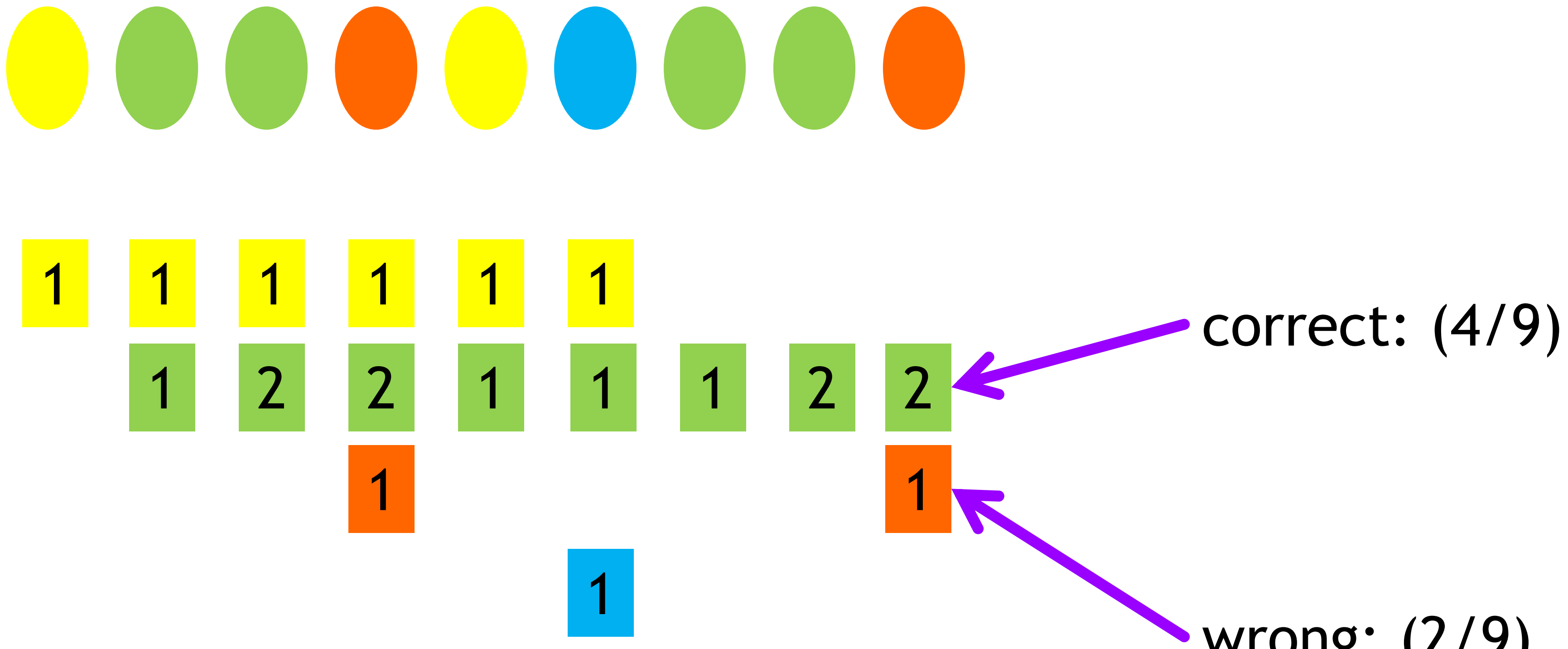
Heavy hitters: algorithm

- For $\theta < 1/2$, need larger "annihilation set" K
- Algorithm:
  - Annihilation set K contains (value, count) tuples
  - For every new sample x, update (or create) tuple in K
  - When $|K| \geq 1/\theta$ → decrease all counts by 1, and delete those hitting 0
- Property:
  - At the end, K is a superset of values with frequency θn
  - Second pass needed to get actual counts
- Cost: $O(\log n / \theta)$ space, $O(n)$ complexity

Heavy hitters: example ($\theta = 1/3$)



Heavy hitters: example ($\theta = 1/3$)

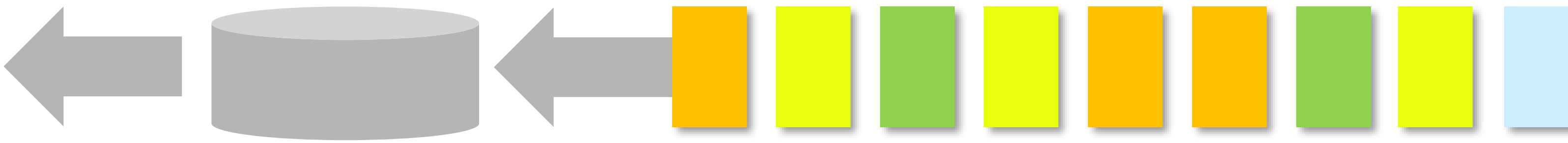


Heavy hitters: proof

- Lemma:
  - A value x that occurs at least θn times in the stream is in K at time n (with count ≥ 1)
- Proof:
  - Def: decrement action = dec all nonzero counters by one
  - Each decrement action decreases the total count (sum of all counters) by at least $1/\theta$
  - Therefore, at most θn decrement actions
  - Each individual value x gets decremented by at most one for each decrement action → at most θn decrements per x
  - If x occurs more than θn times, its counter cannot be 0 at time n

(2) Counting distinct elements

$k = 4$

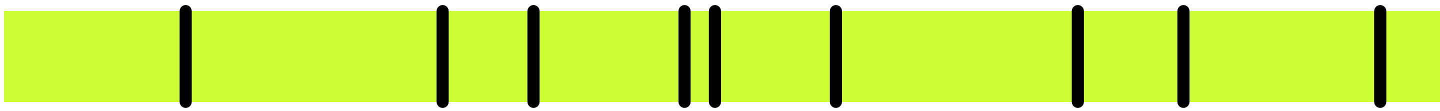


- Number of distinct elements?
  - Table of values seen: $O(n \log n)$
  - Hash table
  - Can we do better?
    - No – if we need exact answer
- Approximation:
  - Many streaming algorithms trade off small loss in precision (approximation) with large gain in memory requirement

Approximate count-distinct

- Flajolet-Martin algorithm
- Intuition:
  - Hash function $h(x)$:
    - Pseudorandom:
      - Random: $x \neq y$ → $h(x)$ and $h(y)$ can be regarded as independent uniform random variables
      - Pseudo: $x = y$ → $h(x) = h(y)$
  - k distinct values $x_1, \dots, x_k$ → hashed values $h(x_1), \dots, h(x_k)$

Count-distinct: single hash function



$\min_i h(x_i)$

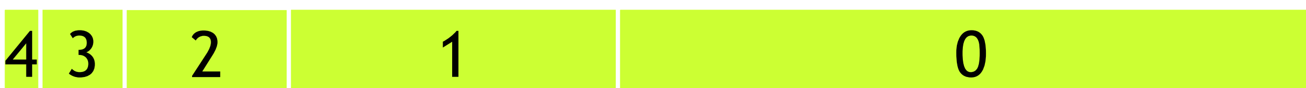
- Normalize $H = 1$
- What is $y = E[\min_i h(x_i)]$?
  - $k + 1$ intervals → $y = (k+1)^{-1}$
- Now suppose we have n values $x_1, \ldots, x_n$, but not necessarily distinct
- What is $y = E[\min_i h(x_i)]$ now?
  - Still $y = (k+1)^{-1}$, where k is # of distinct elements
- Therefore $\hat{k} = \frac{1}{\min_i h(x_i)} - 1$ is an estimator for # of distinct elements in $x_1, \ldots, x_n$

Count-distinct: multiple hash functions

- Improving the estimate:
  - Obtain multiple independent estimates & average, by using multiple hash functions:
  - Assumption: $h_1(x)$ and $h_2(x)$ can be regarded as independent RVs

Flajolet-Martin algorithm

- Discretization of minimum estimator → compression
- Binary representation:
- Number of leading 0s (or trailing)
  - Example: 0010101111000101011010 → $z = 2$, ie, $x \in \{001000000\ldots, 001111111\ldots\}$



- Maximum # of zeroes:

$$R = z(\min_i h(x_i)) = \max_i z(h(x_i))$$

- Estimate: $\hat{k} = 2^R / 0.77351$

FM algorithm: implementation

		$h_j(x_i)$		R_j
x_i	⇨	001010011101011100101	⇨	2
		010111100101101110011		1
		001000101100111011100		2
x_{i+1}	⇨	001111101101011100101	⇨	2
		000101101110011100100		3
		100010110011101110011		0

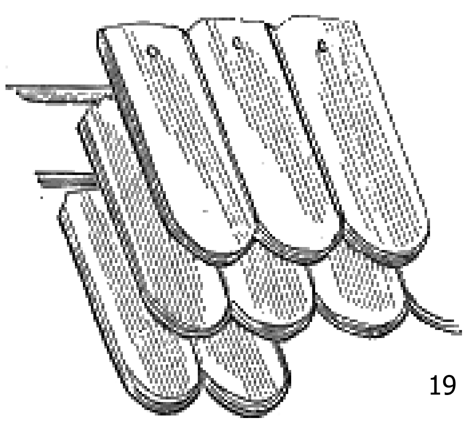
- Combining $\{R_j\}$ into a single estimate for $\hat{k}$:
  - 2^R has power-law distribution → $\text{median}\{2^{R_1}, \dots, 2^{R_J}\}$ is better than average
  - (or average of medians over subgroups)

(3) Document similarity

- Given: Corpus of documents
- Efficient way to find pairwise similarity
- Applications:
  - Web crawling: eliminate copies or similar version of documents
  - Filter search results: only show sufficiently different docs
- Solutions:
  - Cosine similarity or similar: too costly for some applications

Document similarity: sketch

- How to compare two docs very efficiently?
- Approach 1: vector space model
  - Two docs are similar if vectors of word frequencies are similar
  - Drawback: bag-of-words approach loses all structure
- Approach 2: use more local structure → shingles (or n-grams)
  - k-shingle: ordered sequence of k consecutive words
  - "bag of shingles" – large, sparse vocabulary
  - Example: "And now for something completely different" → 3-shingles: "and now for", "now for something", "for something completely",…



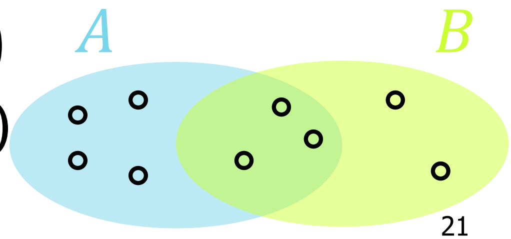
Document similarity: shingles

- Two documents characterized by their sets of shingles A and B
- Similarity: Jaccard index
  - $sim(A,B) = \frac{|A \cap B|}{|A \cup B|}$
- Problem:
  - Vector-space representation: $|V|^k$-sized vocabulary
  - Sparse representation: most shingles unique → $\theta(n)$ space
- Solution:
  - Sketch: compressed version of vector of shingles

Document sketch

- Goal: compact representation of A, B to estimate $sim(A, B)$
- Hash function $h(.) \in [1, H]$: maps shingle to integer
  - Assume H is large enough (e.g., 2^{64}) to avoid collisions
- Def: sketch $s(A) = \min_{a \in A} h(a)$
- Lemma: $P\big(s(A) = s(B)\big) = \dfrac{|A \cap B|}{|A \cup B|}$
- Proof:
  - Hash function → Each element x in $A \cup B$ has same prob. of being min
  - No collision assumption → $P\big(s(A) = s(B)\big)$

$$= P(\text{random } x \in A \cup B \text{ is in } A \cap B)$$
$$= |A \cap B| / |A \cup B|$$



Document sketch

- To reduce variance: multiple hash functions (e.g., 100-1000)
- $s(A) = (\min h_1(A), \min h_2(A), \dots, \min h_J(A))$
- $\widehat{sim}(A,B) = \frac{\left|\{s : h_j(A) = h_j(B)\}\right|}{J}$
  - Unbiased
- Doc sketch comparison:
  - A few hundred integer comparison operations → very efficient

$s(A)$	17	204	94	144	78	12	204	42	…
$s(B)$	87	204	185	91	78	12	84	42	…

(4) Distances and nearest-neighbor

- So far: counts, set similarity
- Another general problem: high-dimensional data $x_i \in \mathbb{R}^d$, with d large
  - Examples: images; time series (financial, sensors,…)
- Euclidean distance $\|x_i - x_j\|$
- Often, one needs nearest-neighbor queries:
  - For a given x, find x_i minimizing $\|x_i - x\|$
  - Examples: find most similar images; find user with similar pref vector in recommender system
- Large d and n: very costly computation
- How can we bring down dimensionality?

Randomized dim reduction

- Johnson-Lindenstrauss Lemma:
  - Given: n points $x_i \in \mathbb{R}^d$, error tolerance $0 < \epsilon < 1$
  - There is a (linear) function $f: \mathbb{R}^d \to \mathbb{R}^{d'}$, such that
  - $(1-\epsilon)\|x_i - x_j\|^2 \leq \|f(x_i) - f(x_j)\|^2 \leq (1+\epsilon)\|x_i - x_j\|^2$
  - for $d' > 8\ln(n)/\epsilon^2$

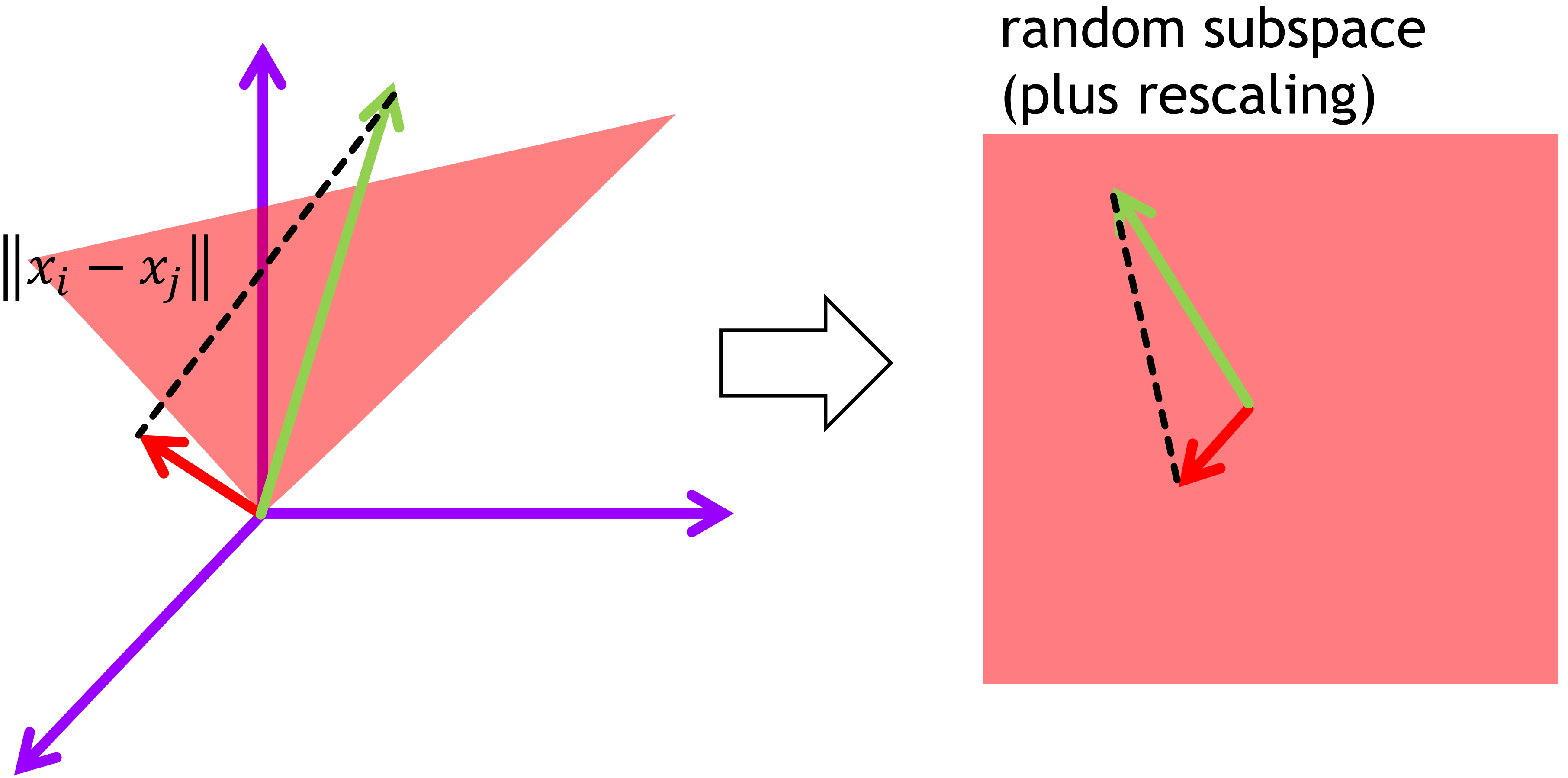
Does not depend on d

Very benign growth in n

- Numerical example:
  - n =1000 images of d =1m pixels each; suppose $\epsilon = 0.2$
  - $d' \cong 1400$

Random projection

- Techniques: project into a **random** low-dim subspace



Random projection

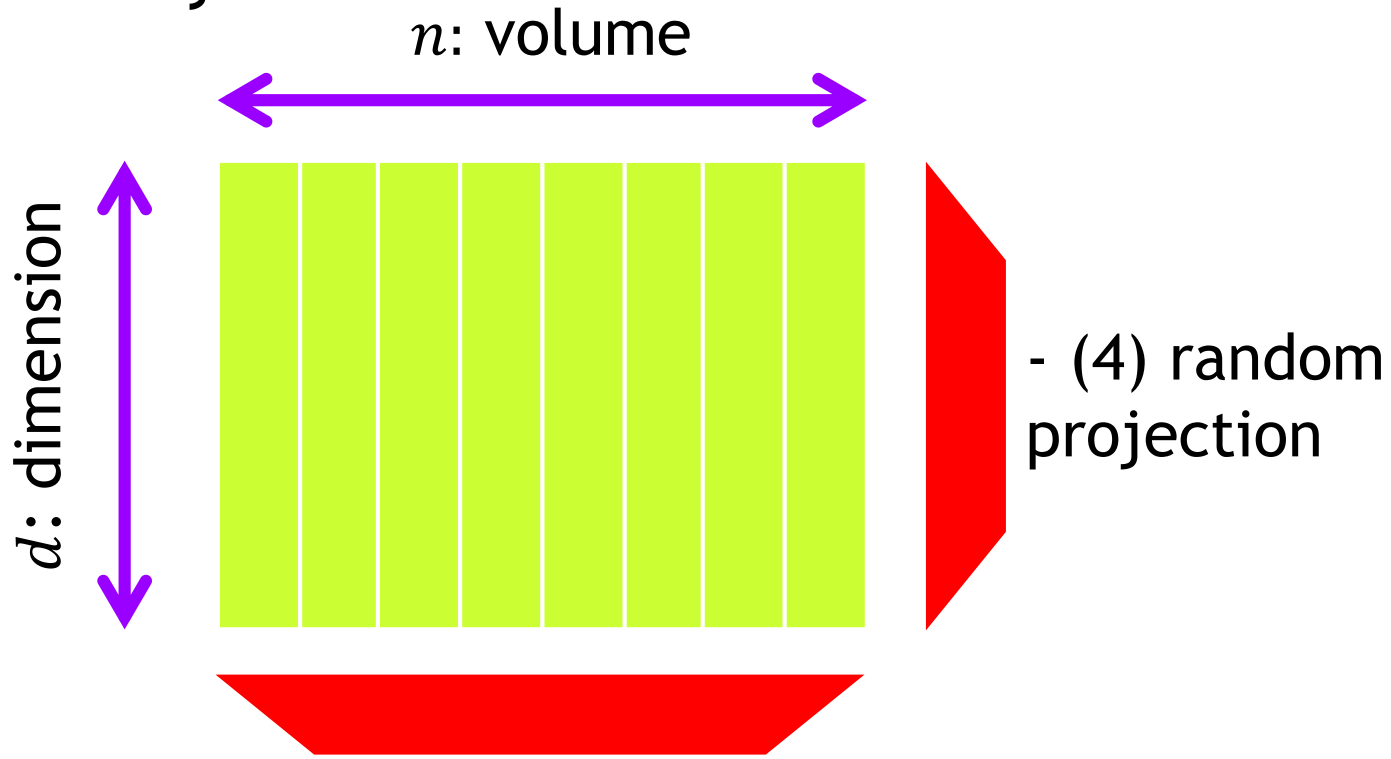
- Intuition: why can we project into a space of dim d' independently of original dim d?
  - High dimension: almost everything is far away
  - When projecting into d' -dim random subspace:
    - For $\|x_i - x_j\|$ small → it stays small
    - For $\|x_i - x_j\|$ large → for distance to collapse, d' different dimensions would have to "miss" the difference
    - Probability of this happening drops very quickly with d'
    - $\ln(n)$ factor: price to pay for this to hold for n^2 different pairs

Dim reduction with random projections

- Very efficient way of maintaining distances between large collection of objects
- Version:
  - Random vectors (components restricted to {-1,+1})
- Example application:
  - Autonomous security camera taking pictures
  - Cannot transmit all pictures, but want to answer queries of the type “when did things look different?”, or cluster similar scenes

Summary

- "Big Data" challenges: volume and curse of dimensionality



- (1) Heavy-hitter
- (2) Flajolet-Martin sketch
- (3) Doc shingle sketch

References

- [A. Rajaraman, J. D. Ullman: Mining of Massive Datasets, 2012 (chapter 4)]
- [S. Muthukrishnan: Data Streams – Algorithms and Applications, Foundations and Trends in Theoretical CS, 1:2, 2005]