Recommender Systems 1

Internet Analytics (COM-308)

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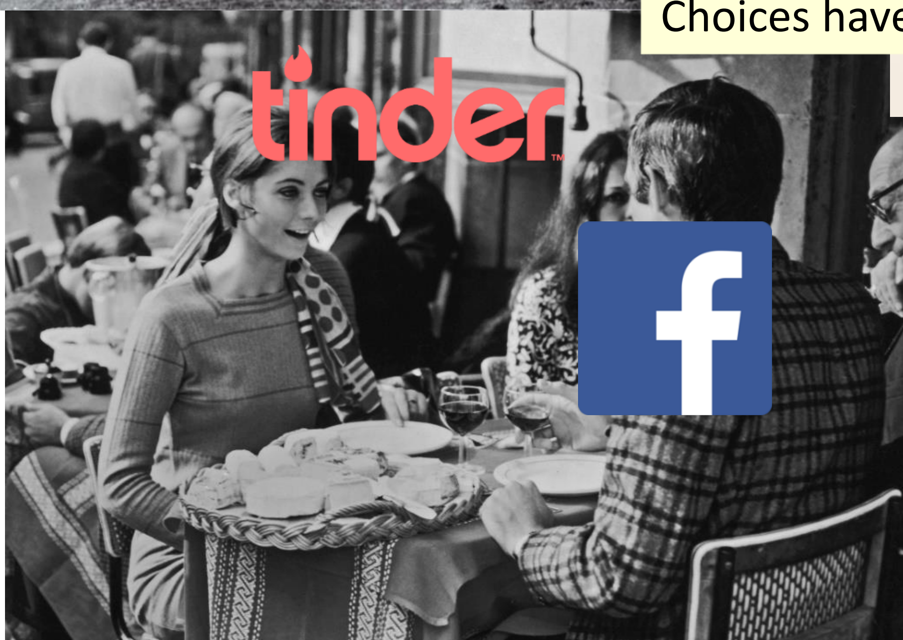
Overview

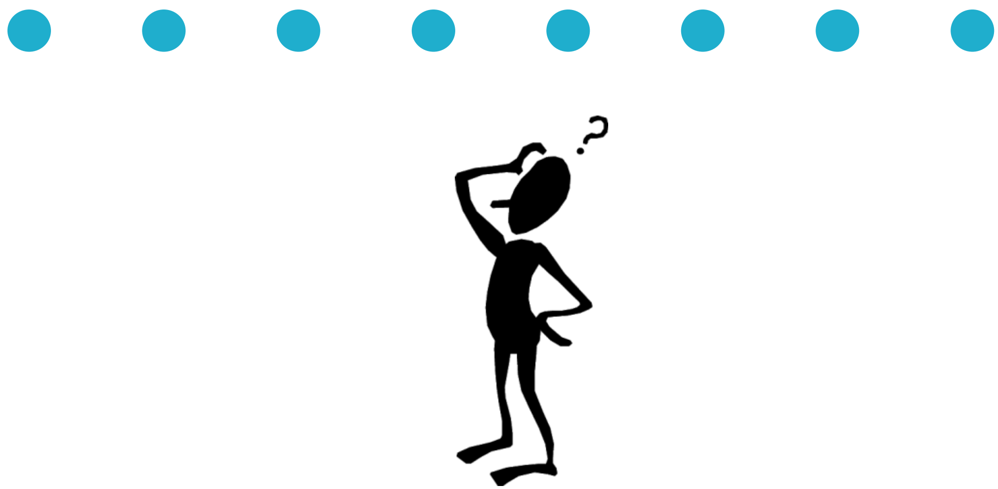
- Motivation: why are recommenders so prevalent today?
- Collaborative filtering vs content-based recommenders
- Example: Netflix Prize
- Neighborhood methods
- Latent factor methods
  - Overfitting, regularization, stochastic gradient descent

Choices: the good old days → the brave new world

Choices have exploded!

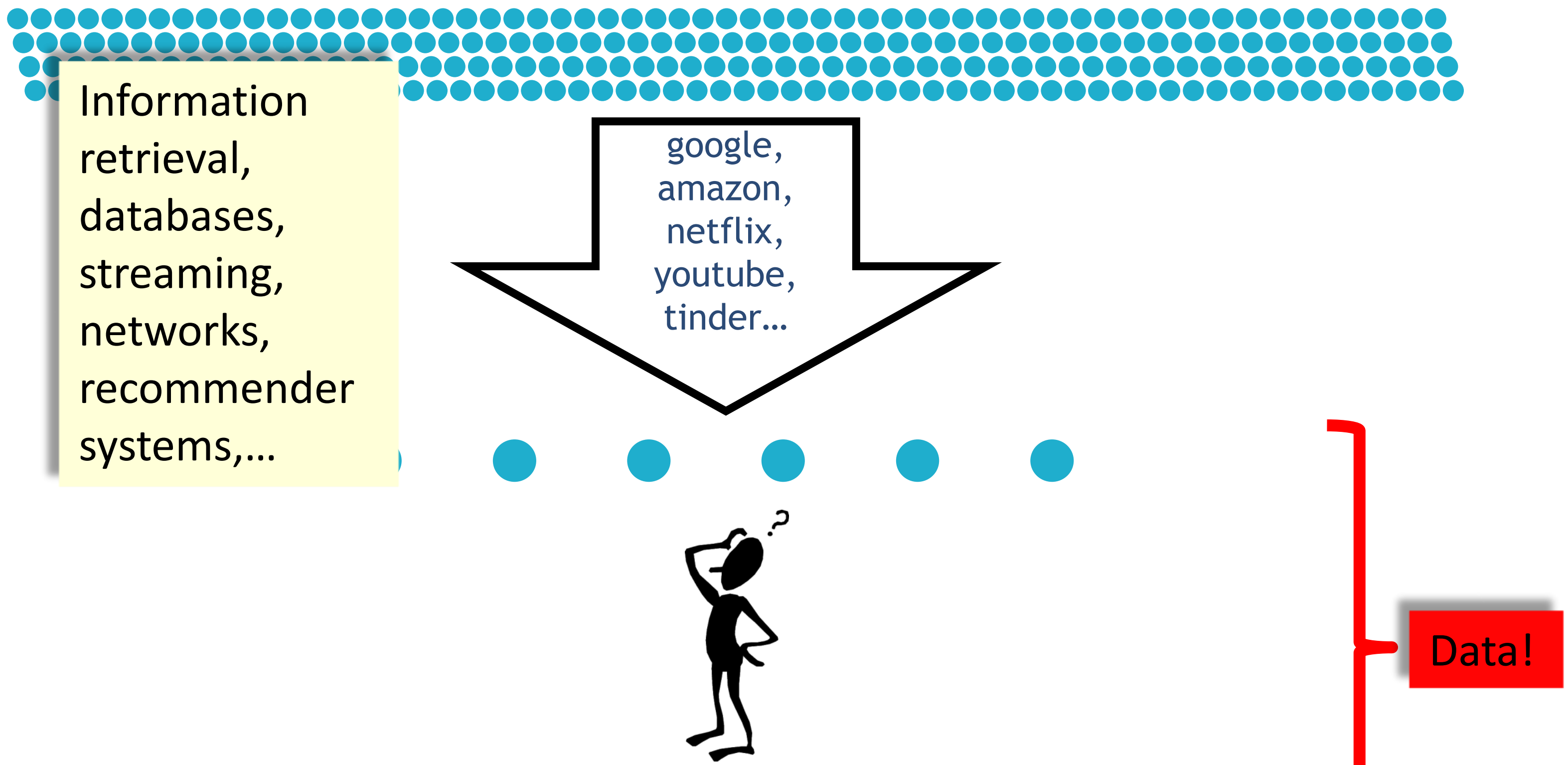


Choosing in the old days



- Small number of alternatives
  - Physical limits: shelf space, weight of the encyclopedia, surface area of the dance floor, cost of the record collection,…

Filtering + choosing



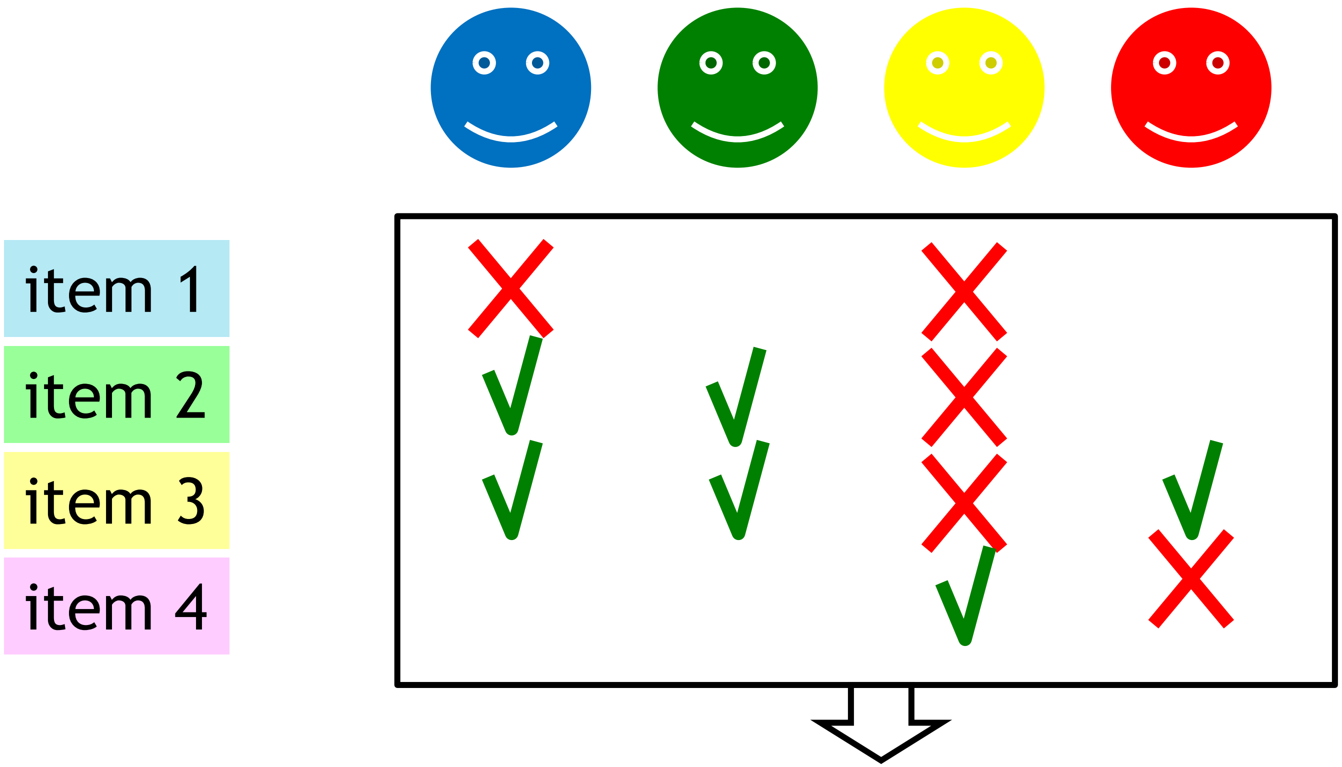
- Filtering: search, recommender systems
- Choosing: the “last cognitive mile”

Situation today

- Traditional retailers:
  - Shelf space & warehouse: expensive → carry only items with sufficient sales volume
- Online:
  - Potentially unlimited catalogue for digital goods (or physical goods – amazon, iTunes, …)
  - Needs better filters: search & recommendations
- Recommenders are integral part of most online services
  - Amazon, Youtube, Spotify, LinkedIn, Twitter, …
  - Even search: google → “filter bubble”
- Limited user interface (mobile!):
  - Recommendations even more important than search

Collaborative filtering

- Content-agnostic → learning from other users



Model for (user, item)→rating

Overview: recommender systems

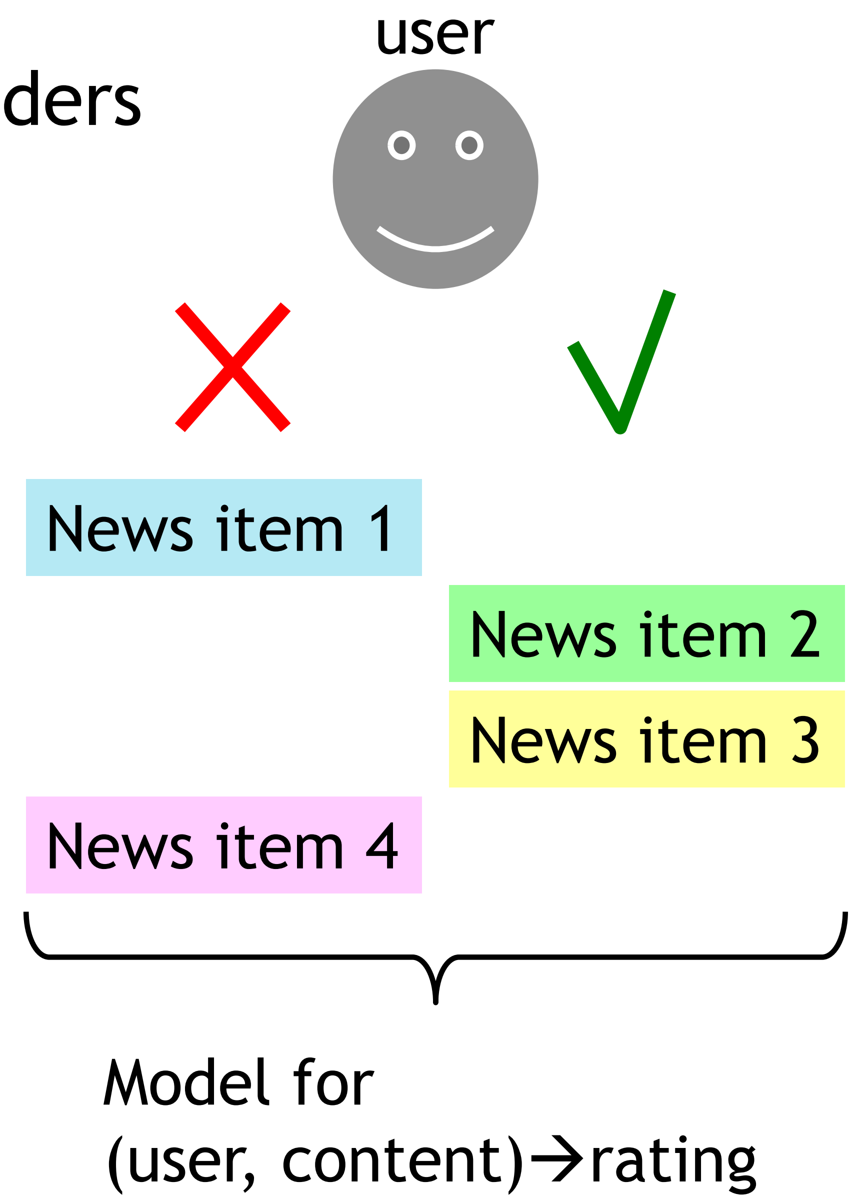
- Content-based recommenders

item 1:
"Plane hijacked…"

item 2:
"soccer game…"

item 3:
"swiss skiers win…"

item 4:
"50.3% vote yes…"



The Netflix competition

- Netflix: mail-based DVD rental company (today streaming)
- New form of research outsourcing: Netflix Prize
  - Goal: “increase performance of in-house system by 10%”
  - Prize: 1m USD + yearly progress prize
  - Anyone can participate
  - Careful setup to avoid reverse-engineering of dataset, overfitting, etc.
- Early example of data-driven competitions
  - Kaggle etc.

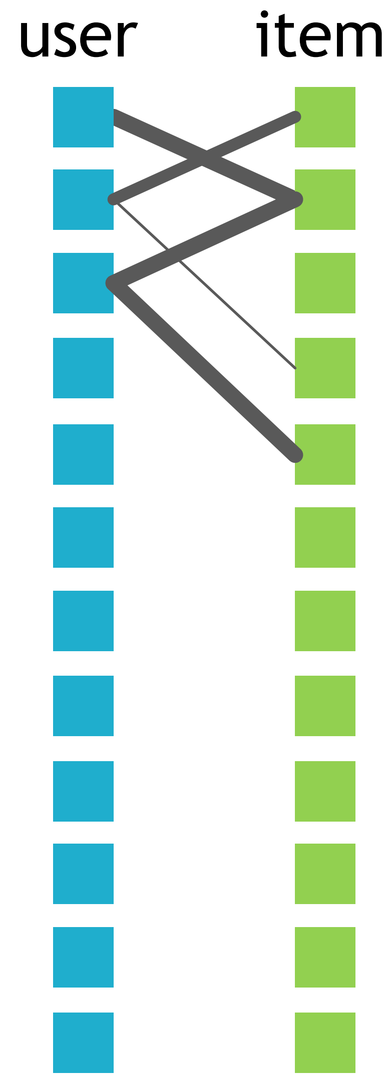
Model

- Set of users U (size n), set of items I (size m)
- Utility or rating function $r: U \times I \rightarrow R$
  - R: e.g. 0-5 stars; probability of liking an item; yes/no;...
- Collecting r_{ui} values:
  - Amazon: buying a product
  - Youtube: watching/liking a video
  - News reader: opening a news item from list
  - In general: depends on context and design
- Explicit vs implicit:
  - Explicit: Ask people to rate (stars, etc.)
    → effort, sparse, reliable
  - Implicit: derive from actions (delete, save, forward, etc.)
    → for free, dense, noisy

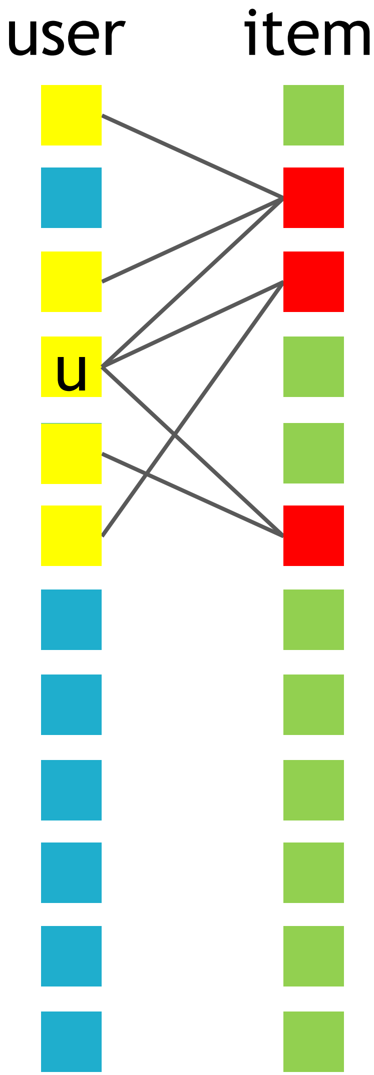
Model

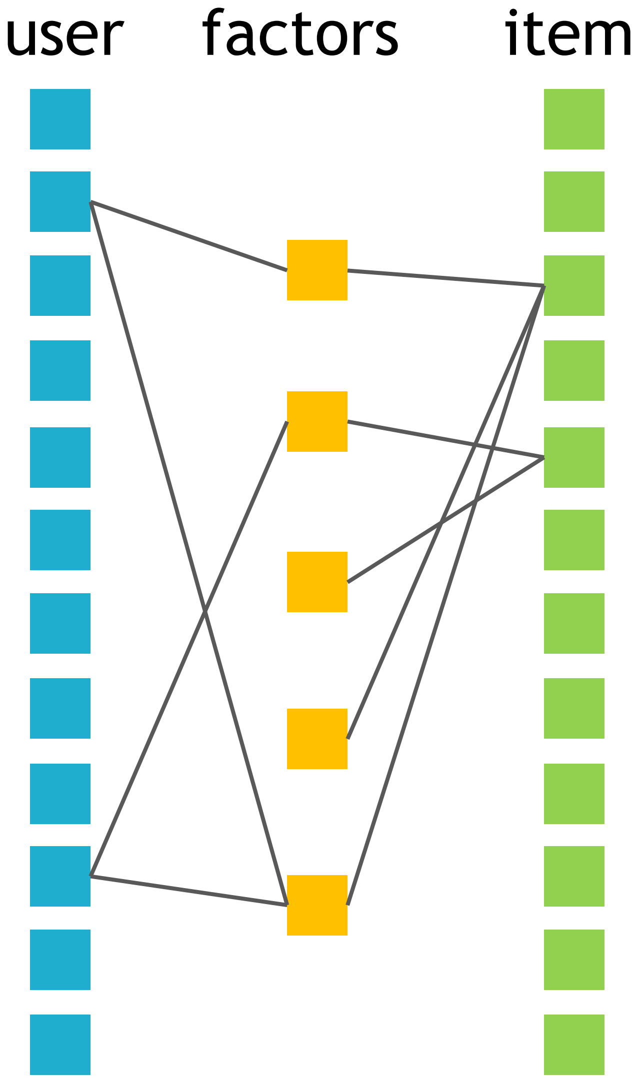
- Ratings matrix R:
  - Captures preference on some scale
  - Matrix representation:
    - Missing elements = unknown
  - Bipartite weighted graph representation

	1	2	3	4	5	6	7	8
1		5		2	4			
2	4		3	1			3	
3		5	4		5		4	
4						1	1	2
5	3					3		
6			2		4			



Neighborhood vs latent factor methods





Performance criterion for Netflix Prize

- RMSE: root mean squared error:
  - $RMSE = \sqrt{\sum_{(u,i)} \frac{(r_{ui} - \hat{r}_{ui})^2}{C}},$
    with C=# of rated pairs
- Why RMS?
  - Penalizes larger errors
- Why not RMS?
  - Often only interested in precision on top ratings
  - Often not interested in absolute value, only order

Baseline predictor

- Assumption:
  - Global average rating $\bar{r}$
  - Each user u has a bias (or average opinion) b_u
  - Each item i has a bias (or average quality) b_i
- First approximation:
  - $b_{ui} = \bar{r} + b_u + b_i$
  - No "interaction" between users and items
- Learning: data → model parameters?
  - $n + m$ parameters
  - Data: up to nm
  - In general overdetermined → find best solution

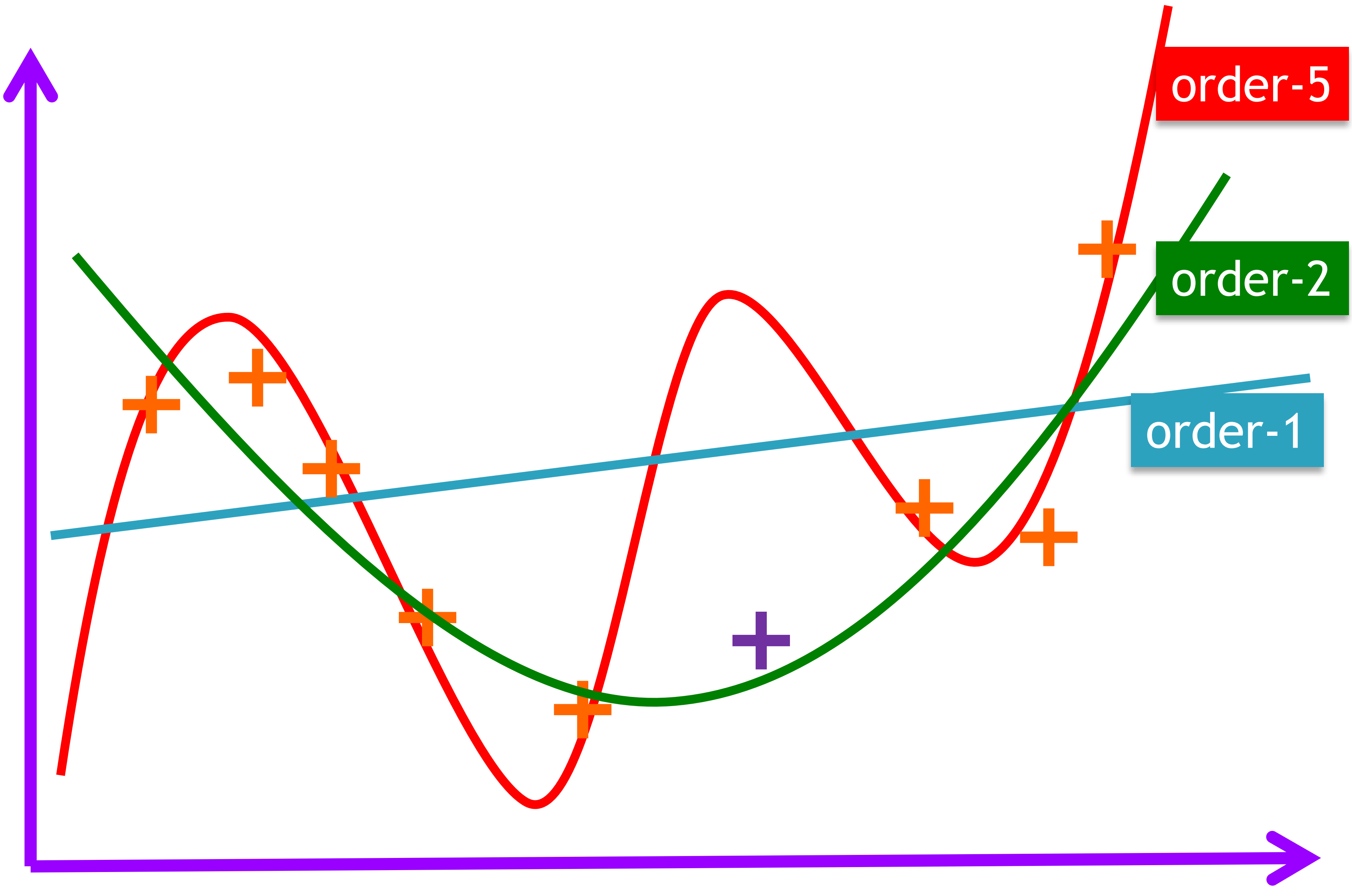
Learning baseline predictor

- Given: training set of ratings $R = \{(u, i, r_{ui})\}$
- Could just use averages per user/item
  - Not optimal
- Min RMS on training set:
  - $\min_{\{b_u, b_i\}} \sum_{(u,i)\in R} (r_{ui} - b_{ui})^2$
- General form of quadratic min problem:

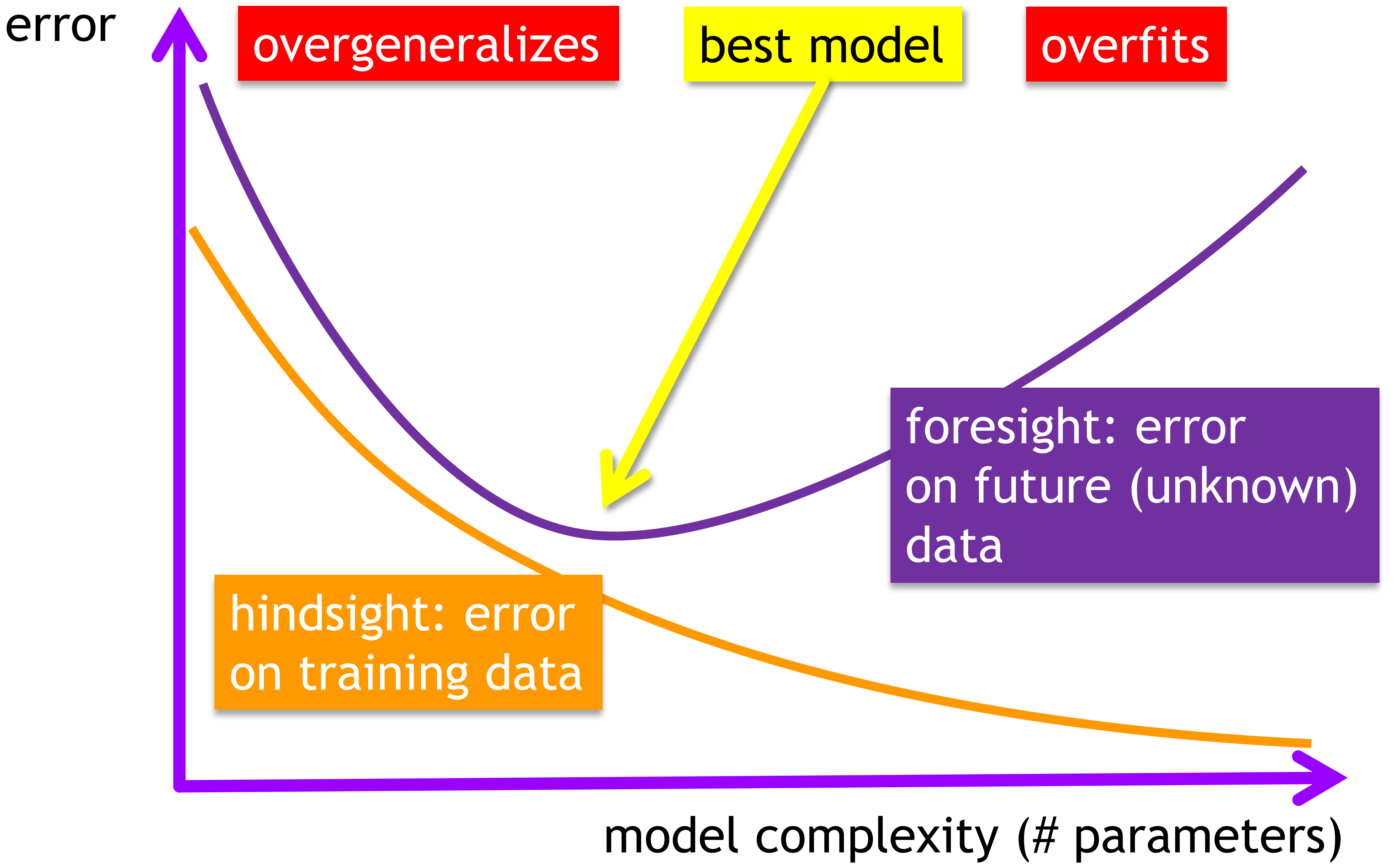
$$\|Ab - c\|_2^2 = (Ab - c)^T(Ab - c) =$$
$$= b^T A^T A b - 2b^T A^T c + c^T c$$

- Derivative w.r.t. b
  - $2(A^T A)b - 2A^T c = 0$ → find b
- This may lead to overfitting → regularization

Learning: overfitting



Hindsight vs Foresight

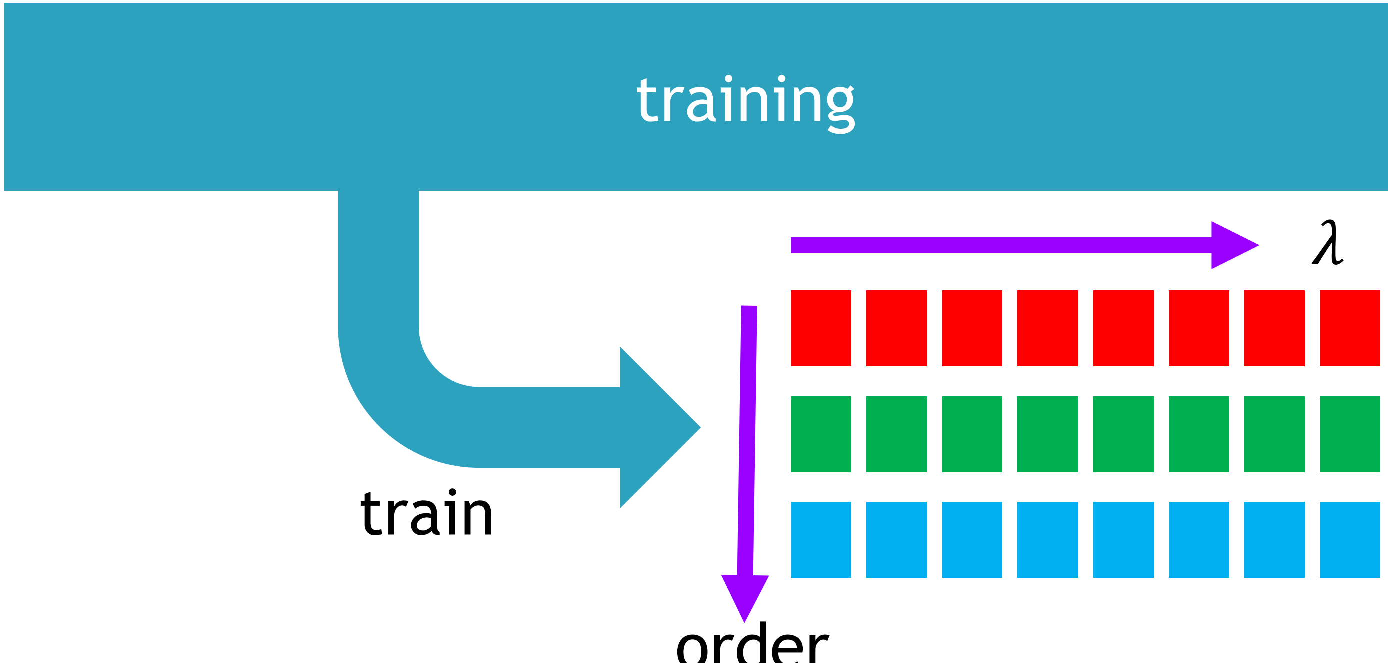


Regularization: penalizing complexity

- Cost function to minimize:
  - $\min_{\theta} f_{\theta}(X)$: X is the data (here $\{r_{ui}\}$), θ the model parameters ($\{b_u, b_i\}$)
- Penalize complexity:
  - Overfitting: complex model does well on training set, but poorly on future data (or test set)
  - We want to use complex models (e.g., higher order polynomial), but avoid overfitting
  - Solution: build in preference for "small" parameters
- Regularizer:
  - $\min_{\theta} f_{\theta}(X) + \lambda g(\theta)$
  - Choice of $g(\theta) \geq 0$ depends on context
  - Example: $g(\theta) = \|\theta\|_2^2$

Validation: simulating foresight

- Model selection:
  - Degree of polynomial
  - Regularization (hyper)parameter λ
- We don't have future data → set aside some training data and pretend it's future data
  - Validation set



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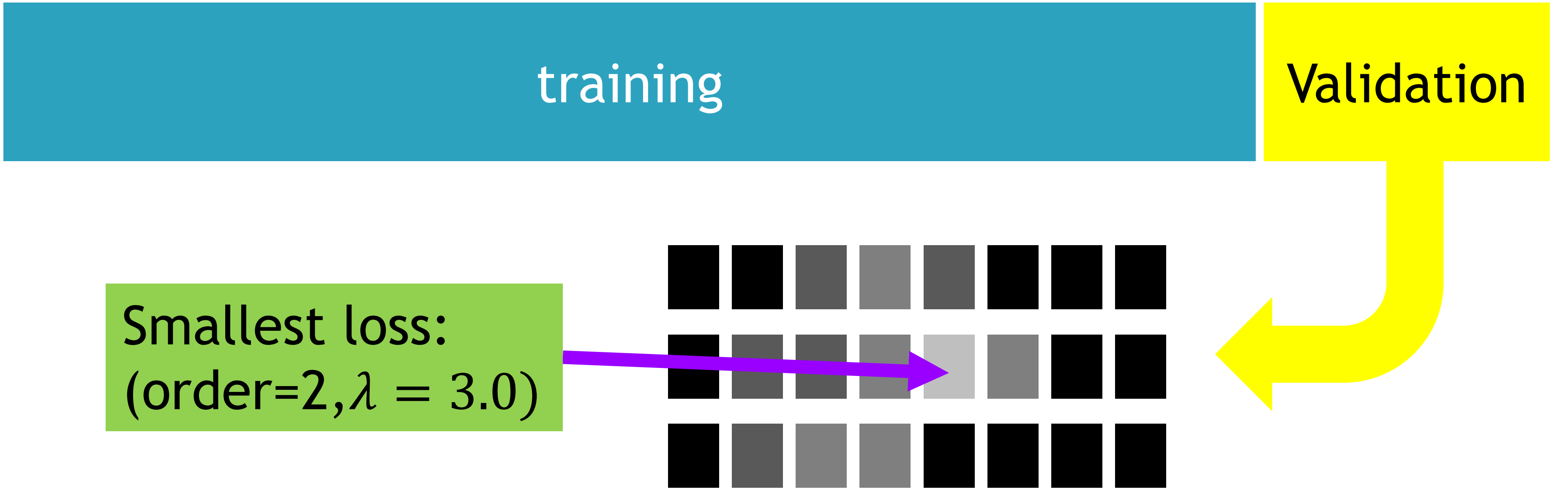
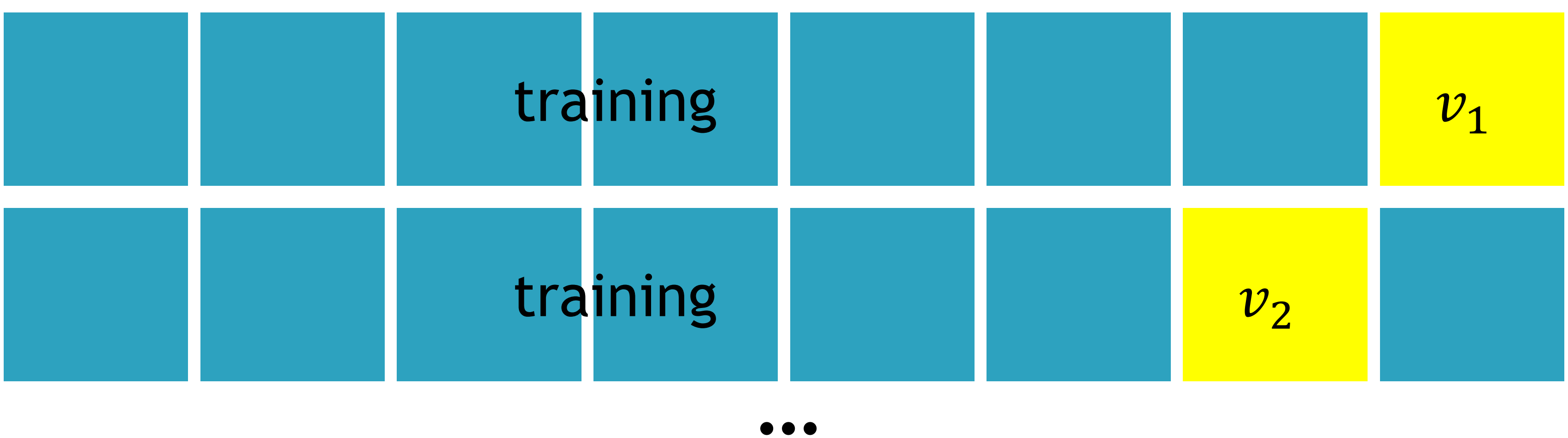


Cross-validation: averaged validation

- If data is not abundant: validation is costly
  - Tradeoff between training and validation data
- k-fold CV:
  - Chop data into equal sized blocks (e.g., $k = 10$)
- For each block $x = 1, \dots, k$:
  - Train model on all other blocks (training set)
  - Evaluate model on v_x (validation set)
- Performance = average error over all iterations



Regularized bias estimates

- Quadratic form to minimize to obtain the bias terms $\{b_u\}$ and $\{b_i\}$:

$$(b_u^*, b_i^*) =$$

$$= \underset{\{b_u, b_i\}}{\operatorname{argmin}} \sum_{(u,i)\in R} (r_{ui} - \bar{r} - b_u - b_i)^2 + \lambda_1 \left(\sum_u b_u^2 + \sum_i b_i^2 \right)$$

Residual error after baseline predictor

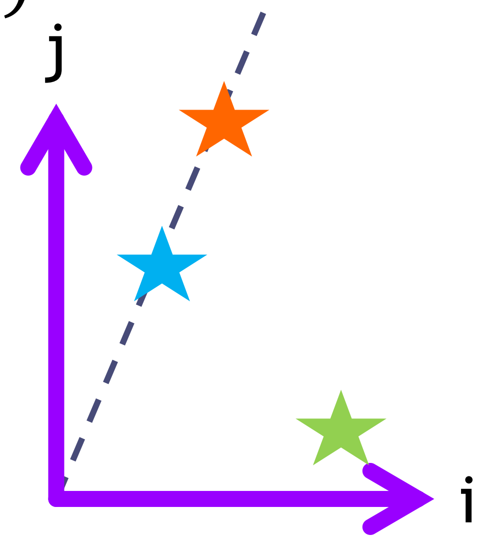
- Residual error:
  - Captures dependence between user and item
  - $\tilde{r}_{ui} = r_{ui} - b_{ui} = r_{ui} - (\bar{r} + b_u + b_i)$
  - $\tilde{R} = [\tilde{r}_{ui}]$
- How to capture residual error?
- Much higher-dimensional model (nm parameters) than baseline ($n + m$ parameters)
  - Overfitting even more of an issue than for b_u, b_i

Neighborhood models

- Goal: estimate rating r_{ui} for user u and item i
- Approach: pairwise user-user or item-item correlation
- User-user:
  - For user u, find other users $\{v\}$ that are similar to u
  - Combine $\{r_{vi}\}$ into an estimate for r_{ui}
- Item-item:
  - For item i, find other items $\{j\}$ that are similar to i
  - Combine $\{r_{uj}\}$ into an estimate for r_{ui}

Similarity metric (user-user variant)

- Similarity between users u and v?
- Degree of agreement in ratings for joint items
- Cosine similarity:
  - Def: x_u, x_v are vectors of ratings $\tilde{r}_u, \tilde{r}_v$ over items rated by both
  - $sim(u, v) = \frac{x_u^T x_v}{\|x_u\|_2 \|x_v\|_2} = \cos(< x_u, x_v)$
- For user u, evaluate $sim(u, v)$ over all other users v; retain L highest → set L_u



Neighborhood model

- Combine "opinions" of all the similar users L_u:
  - $$\hat{r}_{ui} = \bar{r} + b_u + b_i + \frac{\sum_{v \in L_u} sim(u,v)\tilde{r}_{vi}}{\sum_{v \in L_u} sim(u,v)}$$
- Pros:
  - Intuitively appealing and natural
- Cons:
  - Hard to tune: choice of similarity metric $sim(.,.)$, L (and other parameters depending on variant)
  - Sparsity: for many (u,i) pairs, no rating possible (if nobody in set L_u has rated i)

Neighborhood model: user-user vs item-item

- Two symmetric approaches:
  - User-user: for user u, find similar users v, and their ratings of i
  - Item-item: for item i, find similar items j, and their ratings by u
- In practice they are different:
  - Argument for item-item: in most applications, items tend to “cluster” better (e.g., movies belong to a single genre); users are more “mixed” (e.g., one user may like many genres)
  - Argument for user-user: often the application is not “estimate r_{ui}”, but “get highest rated items for u”

Recommending best item i to a user u

- With user-user:
  - First find users v having rated same movie(s)
  - Estimate all unrated items for u from these users
  - Equal to two-hop neighborhood in bipartite rating graph

	1	2	3	4	5	6	7	8
1		5		2	2			
2	4		3	1	1		4	
u	???	5	4	???	1	???	4	???
4						1	1	2
5	3		?		?	3		
6		?	2		4		?	

Recommending best item i to a user u

- With item-item:
  - For each item i not rated by u, find similar items L_i
  - Obtain $\hat{r}_{ui}$ from user u's ratings for $j \in L_i$
  - Much more costly: many unrated items $j \in L_i$

para puntuar n items de un usuario u hay que mirar
n items, por eso es tan costoso

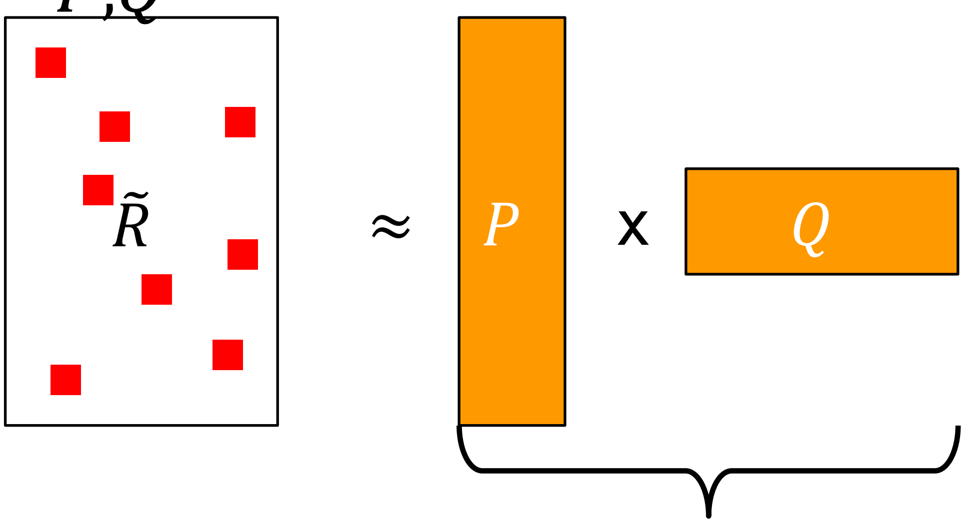
	1	2	3	4	5	6	7	8
1		5		2	2			
2	4		3	1	1		4	
u	???	5	4	???	1	???	4	???
4						1	1	2
5	3		5		?	3	4	
6			2		4			

Latent factor models

- Dimensionality-reduction technique
  - Hypothesis: simpler (lower-dim) space capturing user-item dependencies
- Assume K concepts/latent factors/taste dimensions
  - Movies: Comedy vs drama; historic vs sci-fi; intellectual vs entertainment; romantic vs action; specific cast, directors; etc.
- Each user u has a K-dim factor vector p_u:
  - $p_u[k]$: degree to which user u enjoys/hates factor k
- Each item i has a a K-dim factor vector q_i:
  - $q_i[k]$: degree to which item i possesses factor k

Latent factor models

- $\hat{r}_{ui} = \bar{r} + b_u + b_i + p_u^T q_i$
- Note:
  - Contrary to SVD, no requirement that P, Q be orthogonal
- Training the model on data:
  - $\min_{P,Q} \sum_{(u,i) \in R} \left(\tilde{r}_{ui} - p_u^T q_i \right)^2$

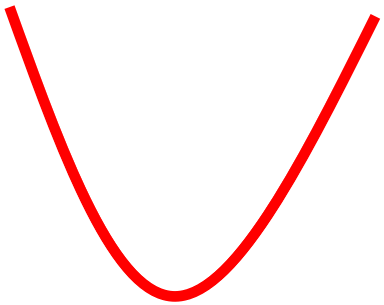


How to find P, Q?

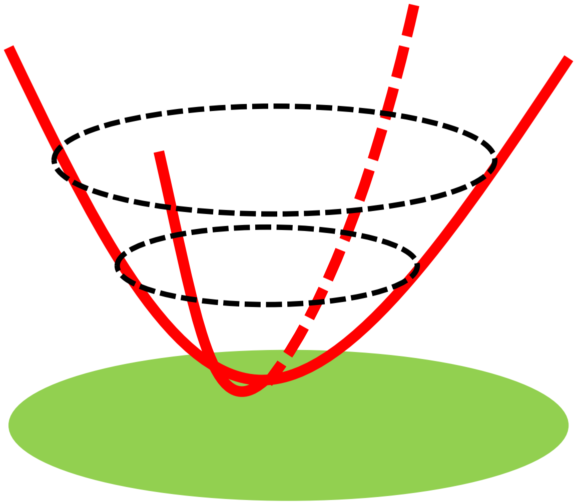
parameters $\theta = (P, Q)$: $d = K(n+m)$ degrees of freedom

Convex optimization

- Convex function $f(\theta)$, $\theta = (\theta_1, \dots, \theta_d)$
  - For every two points θ, ϕ, the line between $f(\theta)$ and $f(\phi)$ is «above» the function



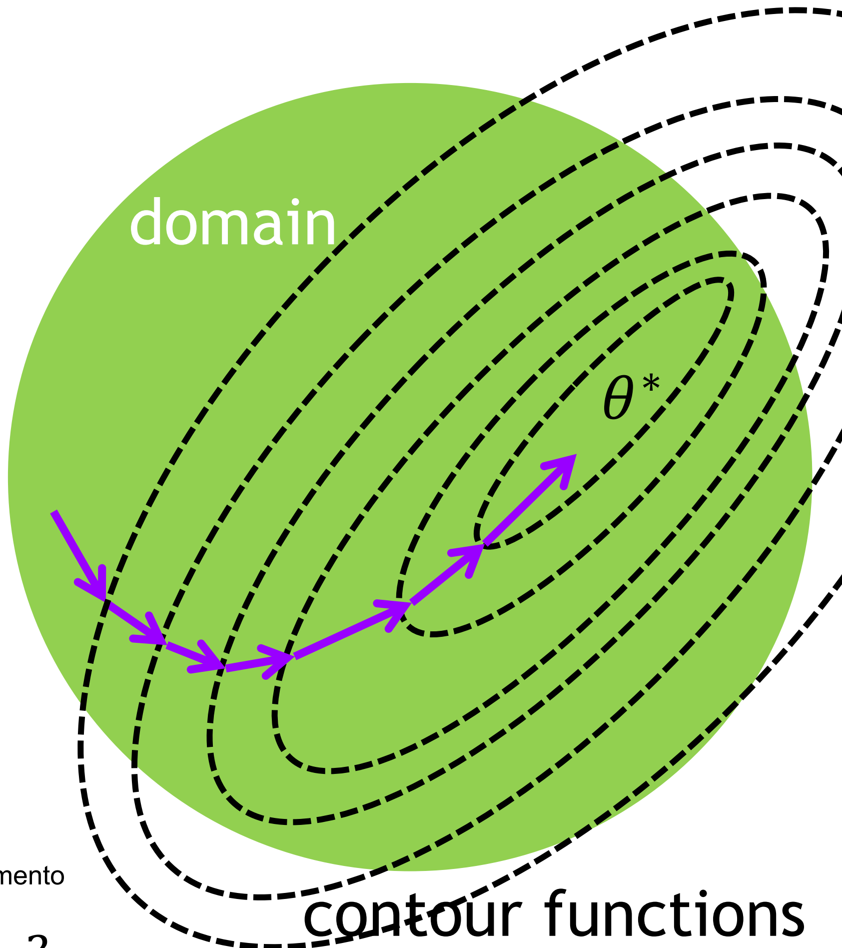
- Convex domain:
  - For every two points θ, ϕ in the domain, the line connecting them also in domain
- Test for convexity of differentiable function:
  - Hessian $(\nabla^2 f)_{ij} = \dfrac{\partial^2 f}{\partial \theta_i \partial \theta_j}$ must be PSD (positive semidefinite)



- Convex optimization:
  - Local min = global min → we can use methods for local min search

Minimization: stochastic gradient descent

- Gradient $(\nabla f)_i = \frac{\partial f}{\partial \theta_i}$
- Gradient descent:
  - $\theta^{(k+1)} = \theta^{(k)} - \alpha \nabla f(\theta^{(k)})$
  - α: learning rate
  - Intuition: move in direction of local decrease
- In ML, f is often a sum over the data:
  - $f = \sum f_n$ muy costoso pq habria que aplicar f a cada elemento
    - Here: $f = \sum_{(u,i)\in R} \left(\tilde{r}_{ui} - p_u^T q_i\right)^2$
    - Log-likelihood: $f = \log P(Y|\theta) = \sum \log P(Y_n|\theta)$
  - Gradient costly to compute! ($(O(n)$ per step)

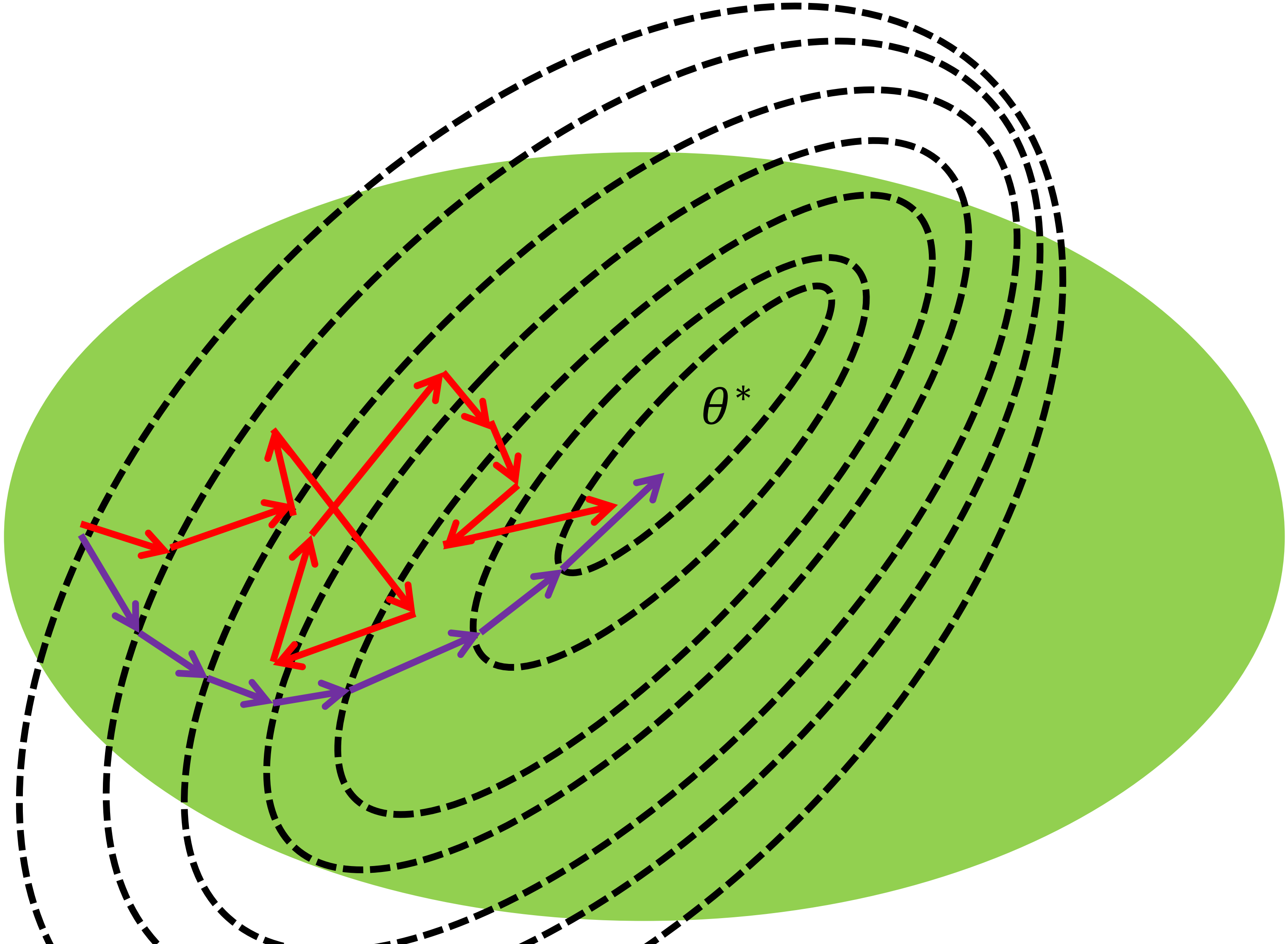


Stochastic gradient descent

- Gradient descent: expensive
  - Sum over all data points $(u, i) \in R$
- Stochastic gradient descent:
  - Idea: noisy but cheap gradient approximation
  - Pick a random data point (u, i) (or some other order)
  - Compute gradients w.r.t. this data point (or a small batch of data points)
  - Iterate until convergence
- Intuition:
  - Random walk that is biased towards minimum
  - Pro: gradient much cheaper to compute
  - Con: random walk may "veer" in the wrong direction
  - Worth it if "detours" do not outweigh reduction in computational cost

Stochastic gradient descent

- Gradient descent vs stochastic gradient descent



Regularized latent factor model

- Cost function with regularizer:
  - $f(P,Q) = \sum_{(u,i)\in R}\left(\tilde{r}_{ui} - p_u^T q_i\right)^2 +$
    $+\lambda_2\left(\|P\|_F^2 + \|Q\|_F^2\right)$
  - We try to find $(P^*, Q^*) = \operatorname*{argmin}_{P,Q} f(P,Q)$
  - Regularizer: favors smaller-magnitude factor vectors
- Not actually a convex problem!
  - But convex (quadratic) over P, Q individually (bi-convex)
  - Finding minimal (P^*, Q^*) is actually NP-hard

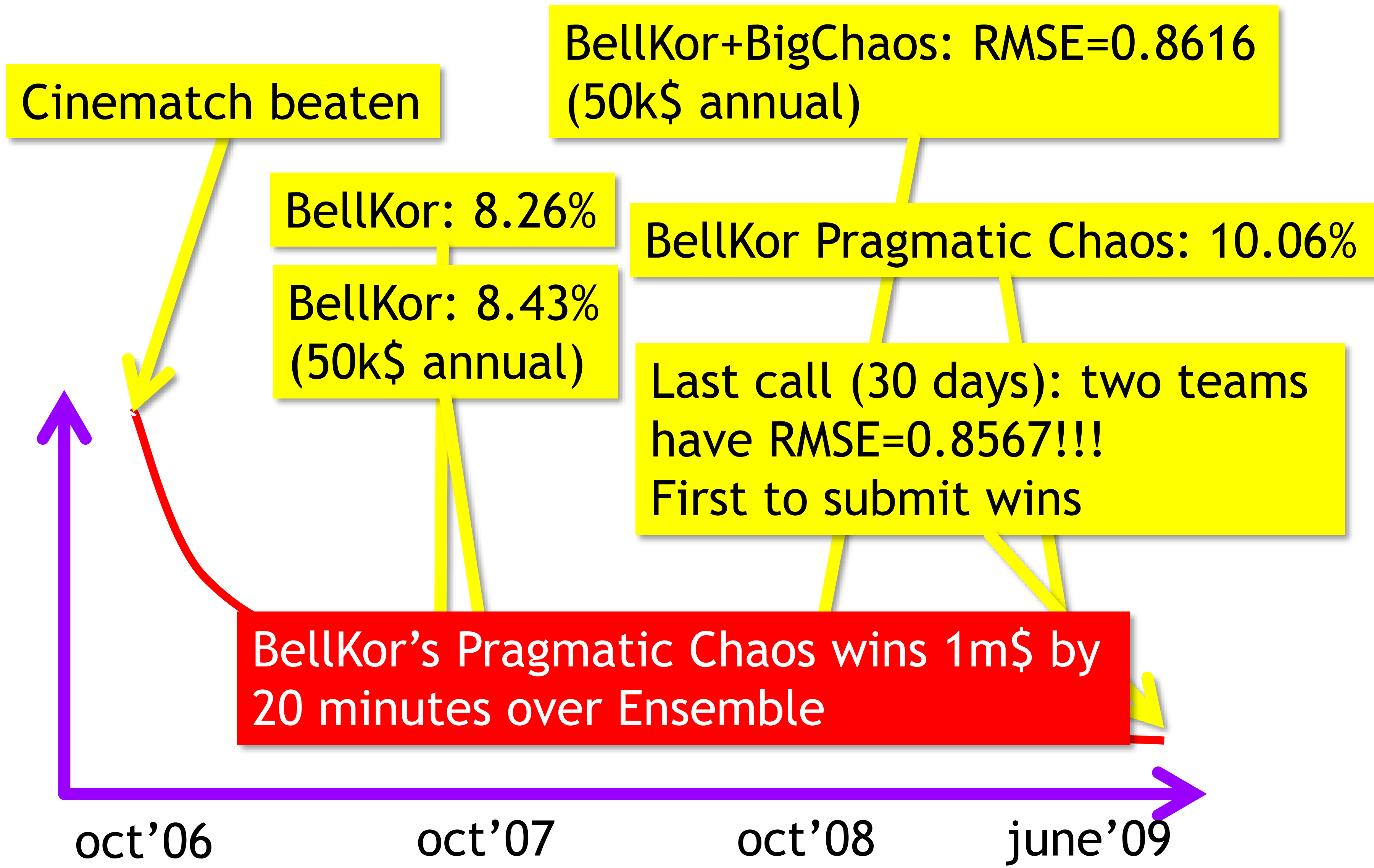
ALS vs SGD

- Two solutions to deal with non-convexity:
  - Optimize using SGD anyway → no guarantee of finding global minimum, but in practice it usually finds a good solution
  - Alternating Least Squares (ALS):
    - Fix P, minimize in Q: quadratic form
    - Fix Q, minimize in P: quadratic form
    - Repeat until convergence
- From studies with real data: SGD usually wins, except for very sparse datasets

Netflix Prize: outcomes and stats

- Data set:
  - ~500k users, ~18k movies
  - 100m ratings over 5 years
- Recommender system for movies: Cinematch
  - RMSE = 0.9514
  - One week until Cinematch got outperformed!
- Stats:
  - 5000 teams (200 USD/team)
  - 44000 submissions
- Netflix required for all results to be published

Netflix Prize



Summary & lessons

- Advantages of collaborative filtering (CF):
  - Content-independent: works for any type of item
  - Big data: exploits large user population
- CF drawbacks:
  - Cold start (new user and new item)
  - Sparsity: most user-item pairs never observed
- Extensions:
  - Context: location, time, mood, etc.
  - Temporal factors: e.g., age of a movie – critical in netflix challenge
- Next lecture:
  - Using content to recommend
- …and: “time is money”! ;-) (1m$/20 minutes)

References

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