Uplink User-Assisted Relaying in Cellular Networks

Presented by: Prudhvi Porandla (110070039)

Guide: Prof. S. N. Merchant

Dual Degree Phase 1 Presentation

27th October 2015

Overview

- Introduction
- Partial Decode-and-Forward Relaying
- PDF in Cellular Networks
- Cooperation Policies
- Simulations and Results
- Future Work

Introduction

- Relaying cooperative communications will play important roles in future generations wireless networks.
- Relay-aided cooperative communication techniques represent a promising technology that improves average rate
- We use Partial Decode-and-Forward relaying scheme
- We will explore two policies by which active User Equipments(UEs) pick their relays

Partial Decode-and-Forward Relaying

Two Phases

Total transmission period is divided into phases: 1. Broadcast phase and 2. Multicast phase as shown in the figure below.

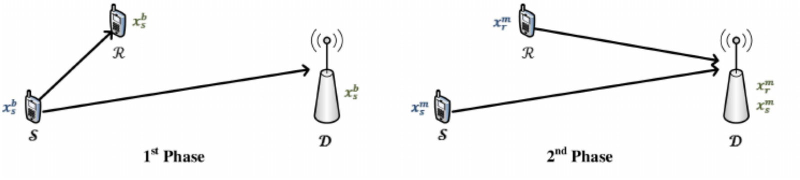

Figure: Two phases in PDF relaying.

Partial Decode-and-Forward Relaying

Transmit Signals

The signals transmitted by source and relay are as follows:

$$\begin{aligned} \text{Phase 1:} \quad x_s^b &= \sqrt{P_s^b} U_s^b, \\ \text{Phase 2:} \quad x_r^m &= \sqrt{P_r^m} U_s^{m_1}, \\ x_s^m &= \sqrt{P_s^{m_1}} U_s^{m_1} + \sqrt{P_s^{m_2}} V_s^{m_2} \end{aligned}$$

- All codewords above are picked from independent Gaussian codebooks with zero mean and unit variance.
- $\alpha_1 P_s^b + \alpha_2 P_s^m = P_s, P_s^{m_1} + P_s^{m_2} = P_s^m, \alpha_2 P_r^m = P_r$ where $\alpha_2 = 1 - \alpha_1$

Partial Decode-and-Forward Relaying

Received Signals

Signals received at relay, BS during broadcast(b) and multicast(m) phases:

$$Y_r^b = h_{sr} x_s^b + Z_r^b, \quad Y_d^b = h_{sd} x_s^b + Z_d^b$$

Z_r^b and Z_d^b are *i.i.d* $\mathcal{CN}(0, \sigma^2)$ that represent noises at $\mathcal{R}$ and $\mathcal{D}$.

Partial Decode-and-Forward Relaying

Received Signals

Signals received at relay, BS during broadcast(b) and multicast(m) phases:

$$Y_r^b = h_{sr} x_s^b + Z_r^b, \quad Y_d^b = h_{sd} x_s^b + Z_d^b$$

Z_r^b and Z_d^b are *i.i.d* $\mathcal{CN}(0, \sigma^2)$ that represent noises at $\mathcal{R}$ and $\mathcal{D}$.

$$Y_d^m = h_{sd} x_s^m + h_{rd} x_r^m + Z_d^m$$

The above expression is true only if $\mathcal{D}$ has knowledge about the phase offset between $\mathcal{S}$ and $\mathcal{R}$.

Partial Decode-and-Forward Relaying

Achievable Rate

With received signals as above and joint ML decoding rule at destination, the achievable rate for this relaying scheme is:

$$R_{PDF} \leq min(C_1 + C_2, C_3)$$

$$\begin{aligned}
\text{where } C_1 &= \alpha_1 \log\left(1 + |h_{sr}|^2 P_s^b\right), \\
C_2 &= \alpha_2 \log\left(1 + |h_{sd}|^2 P_s^{m_2}\right), \\
C_3 &= \alpha_1 \log\left(1 + |h_{sd}|^2 P_s^b\right) \\
&\quad + \alpha_2 \log\left(1 + |h_{sd}|^2 P_s^{m_2} + \left(|h_{sd}|\sqrt{P_s^{m_1}} + |h_{rd}|\sqrt{P_r^m}\right)^2\right)
\end{aligned}$$

PDF in Cellular Networks

Network Geometry

- Active users in different cells using same resource block are are distributed according to a homogeneous and stationary Poisson point process (PPP) Φ_1 with intensity λ_1
- Idle UEs that can participate in relaying are distributed according to another PPP Φ_2 with intensity λ_2.
- Φ_1 and Φ_2 are independent.
- BS is uniformly distributed in the Voronoi cell of its served UE

PDF in Cellular Networks

Network Geometry

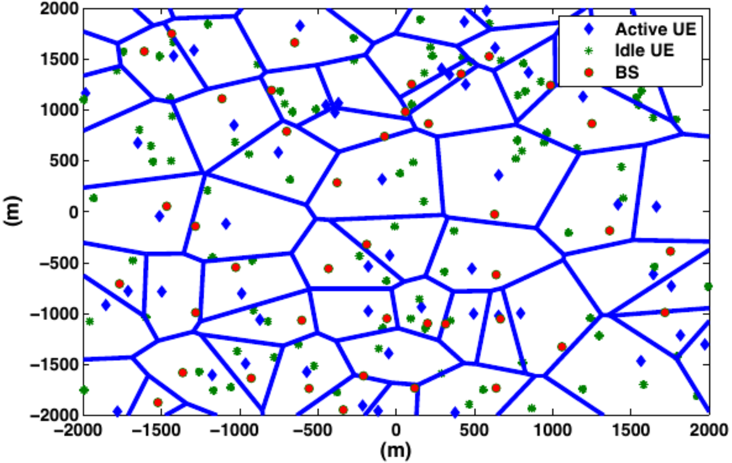

Figure: Sample Network Layout

Consider i^{th} active UE, we model the received signals at the relay and base station in this cell during 1st phase as

$$Y^b_{r,i} = h^{(i)}_{sr} x^b_{s,i} + I^b_{r,i} + Z^b_{r,i},$$

$$Y^b_{d,i} = h^{(i)}_{sd} x^b_{s,i} + I^b_{d,i} + Z^b_{d,i}$$

where $I^b_{r,i}$ and $I^b_{d,i}$ represent the interference received at the i^{th} relay and destination. In second phase of the transmission, the received signal at the BS can be modelled as

$$Y^m_{d,i} = h^{(i)}_{sd} x^m_{s,i} + h^{(i)}_{rd} x^m_{r,i} + I^m_{d,i} + Z^m_{d,i}$$

PDF in Cellular Networks

Interference

$$I_{r,i}^{b} = \sum_{k \neq i} B_k h_{sr}^{(k,i)} x_{s,k}^{b} + (1 - B_k) h_{sr}^{(k,i)} x_{s,k},$$

$$I_{d,i}^{b} = \sum_{k \neq i} B_k h_{sd}^{(k,i)} x_{s,k}^{b} + (1 - B_k) h_{sd}^{(k,i)} x_{s,k},$$

$$I_{d,i}^{m} = \sum_{k \neq i} B_k \left(h_{sd}^{(k,i)} x_{s,k}^{m} + h_{rd}^{(k,i)} x_{r,k}^{m} \right) + (1 - B_k) h_{sd}^{(k,i)} x_{s,k}$$

where $B_k = 1$ if k^{th} user has a relay. B_k can be seen as a Bernoulli RV with success probability equal to cooperation probability of the policy by which relays are picked

PDF in Cellular Networks

Interference

- Interference at either the relay or destination is the sum of many signals undergoing independent fading from nodes distributed in the infinite 2-D plane
- Approximate the interference as a complex Gaussian distribution by using law of large numbers
- Distributions: $I^b_{d,i} \sim \mathcal{CN}(0, \mathcal{Q}^b_{d,i})$, $I^m_{d,i} \sim \mathcal{CN}(0, \mathcal{Q}^m_{d,i})$, and $I^b_{r,i} \sim \mathcal{CN}(0, \mathcal{Q}_{r,i})$

PDF in Cellular Networks

Equivalent Standard Channel Model

$$\tilde{Y}^b_{r,i} = \tilde{h}^{(i)}_{sr} x^b_{s,i} + \tilde{Z}^b_{r,i},$$
$$\tilde{Y}^b_{d,i} = \tilde{h}^{(i)}_{sd} x^b_{s,i} + \tilde{Z}^b_{d,i},$$
$$\tilde{Y}^m_{d,i} = \tilde{h}^{(i)}_{sd} x^m_{s,i} + \tilde{h}^{(i)}_{rd} x^m_{r,i} + \tilde{Z}^m_{d,i}$$

where the new channel fading terms are defined as

$$\tilde{h}^{(i)}_{sr} = \frac{h^{(i)}_{sr}}{\sqrt{\mathcal{Q}_{r,i} + \sigma^2}}, \quad \tilde{h}^{(b,i)}_{sd} = \frac{h^{(i)}_{sd}}{\sqrt{\mathcal{Q}^b_{d,i} + \sigma^2}}$$

$$\tilde{h}^{(m,i)}_{sd} = \frac{h^{(i)}_{sd}}{\sqrt{\mathcal{Q}^m_{d,i} + \sigma^2}}, \quad \tilde{h}^{(i)}_{rd} = \frac{h^{(i)}_{rd}}{\sqrt{\mathcal{Q}^m_{d,i} + \sigma^2}}$$

and the noise terms are now all $\mathcal{CN}(0,1)$

Cooperation Policies

Ideal Policy E_1

$$E_1 = \left\{ |\tilde{h}_{(sr)}^{(k)}|^2 \geq |\tilde{h}_{(sd)}^{(k)}|^2 \right\}$$
$$\backsimeq \left\{ \frac{g_{sr} r_2^{-\alpha}}{\mathcal{Q}_{r,k}} \geq \frac{g_{sd} r_1^{-\alpha}}{\mathcal{Q}_{d,k}^{b}} \right\}$$

where r_1 and r_2 denote the direct distance between $\mathcal{S}$ and $\mathcal{D}$ and cooperation distance between $\mathcal{S}$ and its closest idle UE, respectively and α is pathloss exponent. Active UE nodes should know instantaneous SINRs of the relay link($\mathcal{S} - \mathcal{R}$) and the direct link($\mathcal{S} - \mathcal{D}$)

Cooperation Policies

Pure Geometric Policy E_2

$$E_2 = \{r_2 \leq r_1, D \leq r_1\}$$

where D is the distance between $\mathcal{R}$ and $\mathcal{D}$.

Cooperation Policies

Pure Geometric Policy E_2

$$E_2 = \{r_2 \leq r_1, D \leq r_1\}$$

where D is the distance between $\mathcal{R}$ and $\mathcal{D}$.

- More practical than E_1

Cooperation Policies

Pure Geometric Policy E_2

$$E_2 = \{r_2 \leq r_1, D \leq r_1\}$$

where D is the distance between $\mathcal{R}$ and $\mathcal{D}$.

- More practical than E_1
- Does not require full knowledge of interference at decision making node.

Cooperation Policies

Pure Geometric Policy E_2

$$E_2 = \{r_2 \leq r_1, D \leq r_1\}$$

where D is the distance between $\mathcal{R}$ and $\mathcal{D}$.

- More practical than E_1
- Does not require full knowledge of interference at decision making node.
- Only requires the decision making nodes to know the distances from the active user to the nearest idle user and to the base station.

Cooperation Policies

Hybrid Policy E_3

This policy is proposed for slow fading channels where small scale fading parameters estimation and their feedback to the decision making node is feasible.

$$E_3 = \{g_{sd} r_1^{-\alpha} \leq g_{sr} r_2^{-\alpha}, D \leq r_1\}$$

Note that this cooperation policy is still independent of the interference as in the pure geometric cooperation policy E_2.

Cooperation Probabilities

Distributions of r_1, r_2

The distribution of the distance r_1 between the i^{th} UE and its associated BS can be shown to be Rayleigh distributed

$$f_{r_1}(r_1) = 2\pi\lambda_1 r_1 e^{-\lambda_1 \pi r_1^2},$$

Cooperation Probabilities

Distributions of r_1, r_2

The distribution of the distance r_1 between the i^{th} UE and its associated BS can be shown to be Rayleigh distributed

$$f_{r_1}(r_1) = 2\pi\lambda_1 r_1 e^{-\lambda_1 \pi r_1^2},$$

Similarly the distribution of the source-to-relay distance r_2 between the i^{th} UE and its associated relaying UE can be also shown to be Rayleigh distributed

$$f_{r_2}(r_2) = 2\pi\lambda_2 r_2 e^{-\lambda_2 \pi r_2^2}$$

Cooperation Probabilities

Distributions of r_1, r_2

The distribution of the distance r_1 between the i^{th} UE and its associated BS can be shown to be Rayleigh distributed

$$f_{r_1}(r_1) = 2\pi\lambda_1 r_1 e^{-\lambda_1 \pi r_1^2},$$

Similarly the distribution of the source-to-relay distance r_2 between the i^{th} UE and its associated relaying UE can be also shown to be Rayleigh distributed

$$f_{r_2}(r_2) = 2\pi\lambda_2 r_2 e^{-\lambda_2 \pi r_2^2}$$

Can be proved from the null probability of a two dimensional PPP

Cooperation Probabilities

ρ_2, ρ_3

For policy E_2

$$\rho_2 = \int_{-\pi/2}^{-\pi/3} \frac{2\lambda_2 cos^2\psi_0}{\pi(\lambda_1 + 4\lambda_2 cos^2\psi_0)} d\psi_0 + \int_{\pi/3}^{\pi/2} \frac{2\lambda_2 cos^2\psi_0}{\pi(\lambda_1 + 4\lambda_2 cos^2\psi_0)} d\psi_0 + \frac{\lambda_2}{3(\lambda_1 + \lambda_2)}$$

Cooperation Probabilities

ρ_2, ρ_3

$$\rho_3 = \int_0^2 f_\beta(z) \int_{-\pi/2}^{-cos^{-1}(z/2)} \frac{2\lambda_2 cos^2\psi_0}{\pi(\lambda_1 + 4\lambda_2 cos^2\psi_0)} d\psi_0 dz$$
$$+ \int_0^2 f_\beta(z) \int_{cos^{-1}(z/2)}^{\pi/2} \frac{2\lambda_2 cos^2\psi_0}{\pi(\lambda_1 + 4\lambda_2 cos^2\psi_0)} d\psi_0 dz$$
$$+ \int_0^2 f_\beta(z) \frac{\lambda_2 z^2 cos^{-1}(z/2)}{\pi(\lambda_1 + \lambda_2 z^2)} dz$$
$$+ \int_2^\infty f_\beta(z) \int_{-\pi/2}^{\pi/2} \frac{2\lambda_2 cos^2\psi_0}{\pi(\lambda_1 + 4\lambda_2 cos^2\psi_0)} d\psi_0 dz$$

where $\beta = \left(\frac{g_{sr}}{g_{sd}}\right)^{1/\alpha}$ and $f_\beta(z)$ is pdf of β which can be shown to be

$$f_\beta(z) = \frac{\alpha z^{\alpha-1}}{(1+z^\alpha)^2}$$

Cooperation Probabilities

Proof

$$\begin{aligned}\rho_2 &= \mathbb{P}\{E_2\} \\ &= \mathbb{P}\{r_2 \leq r_1, r_1^2 + r_2^2 - 2r_1 r_2 cos\psi_0 \leq r_1^2\} \\ &= \mathbb{P}\{r_2 \leq r_1, r_2 \leq 2r_1 cos\psi_0\}\end{aligned}$$

when $|\psi_0| < \pi/3, r_1 < 2r_1 cos\psi_0$

$\Rightarrow$ if $r_2 < r_1$, r_2 satisfies both inequalities.

Accordingly, we define $\mathcal{E}_1$ and $\mathcal{E}_2$ as follows

$$\begin{aligned}\mathcal{E}_1 &= (2\pi)^2\lambda_1\lambda_2 \int_0^{\infty}\int_0^{2r_1 cos\psi_0} r_1 r_2 e^{-\pi(\lambda_1 r_1^2 + \lambda_2 r_2^2)} dr_2 dr_1 \\ &= \frac{2\lambda_2 cos^2\psi_0}{\pi(\lambda_1 + 4\lambda_2 cos^2\psi_0)}\end{aligned}$$

Cooperation Probabilities

Proof

$$\mathcal{E}_2 = (2\pi)^2 \lambda_1 \lambda_2 \int_0^\infty \int_0^{r_1} r_1 r_2 e^{-\pi(\lambda_1 r_1^2 + \lambda_2 r_2^2)} dr_2 dr_1$$
$$= \frac{\lambda_2}{2\pi(\lambda_1 + \lambda_2)}$$

$$\text{Now, } \rho_2 = \int_{-\pi/3}^{\pi/3} \mathcal{E}_2 d\psi_0 + 2 \int_{\pi/3}^{\pi/2} \mathcal{E}_1 d\psi_0$$
$$= \frac{\lambda_2}{3(\lambda_1 + \lambda_2)} + 2 \int_{\pi/3}^{\pi/2} \mathcal{E}_1 d\psi_0$$

Cooperation Probabilities

Proof

$$
\begin{aligned}
\rho_3 &= \mathbb{P}\{E_3\} \\
&= \mathbb{P}\{r_2 \leq \left(\frac{g_{sr}}{g_{sd}}\right)^{1/\alpha} r_1, r_1^2 + r_2^2 - 2r_1 r_2 cos\psi_0 \leq r_1^2\} \\
&= \mathbb{P}\{r_2 \leq \beta r_1, r_2 \leq 2r_1 cos\psi_0\} \\
&= \mathbb{P}\{r_2 \leq 2r_1 cos\psi_0\} \qquad \text{for } \beta > 2 \\
&= \mathbb{P}\{r_2 \leq \beta r_1\} \qquad \text{for } \beta < 2 \text{ and } |\psi_0| < cos^{-1}(\beta/2) \\
&= \mathbb{P}\{r_2 \leq 2r_1 cos\psi_0\} \qquad \text{for } \beta < 2 \text{ and } cos^{-1}(\beta/2) < |\psi_0| < \pi/2
\end{aligned}
$$

Cooperation Probabilities

Proof

$$\therefore \rho_3 = 2\int_0^2 f_\beta(z)\int_{cos^{-1}(z/2)}^{\pi/2} \mathcal{E}_1 d\psi_0 dz + \int_0^2 f_\beta(z)\int_{-cos^{-1}(z/2)}^{cos^{-1}(z/2)} \mathcal{E}_3 d\psi_0 dz$$
$$+ \int_2^\infty f_\beta(z)\int_{-\pi/2}^{\pi/2} \mathcal{E}_1 d\psi_0 dz$$

$\mathcal{E}_1$ is defined in part i. of the proof and $\mathcal{E}_3 = \frac{\lambda_2 z^2}{2\pi(\lambda_1+\lambda_2 z^2)}$ which is nothing but $\mathcal{E}_2$ with $\lambda_2 = \lambda_2 z^2$

Cooperation Probabilities
Proof

$f_\beta(z)$, the pdf of β, can be obtained as follows

$$
\begin{aligned}
F_\beta(z) &= \mathbb{P}\left\{\left(\frac{x_1}{x_2}\right)^{1/\alpha} \leq z\right\} = \mathbb{P}\{x_1 \leq z^\alpha x_2\} \\
&= \int_0^\infty \int_0^{z^\alpha x_2} e^{-(x_1+x_2)} dx_1 dx_2 \quad \text{since } g_{sr}, g_{sd} \sim Exp(1) \\
&= 1 - \frac{1}{1+z^\alpha}, \qquad z \in [0, \infty)
\end{aligned}
$$

The pdf $f_\beta(z)$ is then obtained by differentiating $F_\beta(z)$:

$$
f_\beta(z) = \frac{dF_\beta(z)}{dz} = \frac{\alpha z^{\alpha-1}}{(1+z^\alpha)^2} \quad z \in [0, \infty)
$$

Simulations

Generate UEs

Consider a square region of area A.

- Pick $N_1 \sim \text{poisson}(\lambda_1 A)$
- Pick $N_2 \sim \text{poisson}(\lambda_2 A)$
- Distribute N_1 UEs uniformly in the region and mark them as active UEs
- Distribute N_2 UEs uniformly in the region and mark them as idle UEs

Simulations

Generating Base Stations

- Ideally, BSs corresponding to a UE should be picked uniformly from the Voronoi region of UE.

Simulations

Generating Base Stations

- Ideally, BSs corresponding to a UE should be picked uniformly from the Voronoi region of UE.

- One way to do that is to triangulate the polygonal Voronoi region, choose a triangle weighted by area, choose a point in that triangle.

Simulations

Generating Base Stations

- Ideally, BSs corresponding to a UE should be picked uniformly from the Voronoi region of UE.
- One way to do that is to triangulate the polygonal Voronoi region, choose a triangle weighted by area, choose a point in that triangle.
- Difficult to implement

Simulations

Generating Base Stations

BSs are generated in the following manner:

- Pick a number N_3 greater than N_1
- Distribute N_3 BSs uniformly in the whole square region

Simulations

Generating Base Stations

BSs are generated in the following manner:

- Pick a number N_3 greater than N_1
- Distribute N_3 BSs uniformly in the whole square region
- Go to each active UE and check if there are any BSs in its Voronoi region

Simulations

Generating Base Stations

- If there is more than one BS in the region, pick any one of them randomly.

- Since BSs are uniform over the whole region, they are also uniform in each Voronoi region

- But some UEs might not have a BS unlike in the actual method

- Finally, discard all active UEs without a BS

Simulations

Transmission Powers

The average powers used by source and relay are as follows:

- Source and relays use equal power $\Rightarrow P_{s,i} = P_{r,i}$
- $\alpha_1 P_{s,i}^{b} + \alpha_2 P_{s,i}^{m} = P_{s,i}$
- Source uses equal power during broadcast and multicast phases $\Rightarrow P_{s,i}^{b} = P_{s,i}^{m}$
- $P_{s,i}^{m_1} = \beta_1 P_{s,i}^{m}$ and $P_{s,i}^{m_2} = (1 - \beta_1) P_{s,i}^{m}$
- β_1 is allocated optimally to maximize the transmission rate of the active user.

Simulations

Other Parameters

Path loss exponent $\alpha = 3$

$$\rho = \frac{\text{Number of users with relays}}{\text{Total number of active users}}$$

Fraction of transmission time used for 1st phase $\alpha_1 = 0.5$

Results

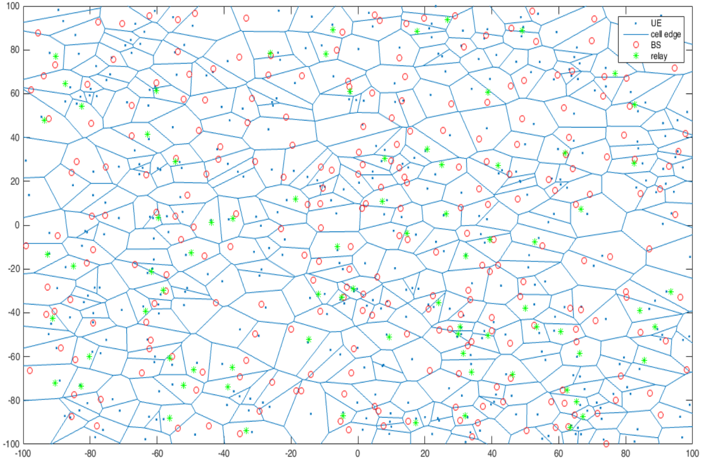

Figure: Sample Network Layout

Results

In the reference paper, there was a mistake in the derivation of expression for cooperation probability(ρ_3) of policy E_3.
$\mathcal{E}_2$ was used in place of $\mathcal{E}_3$

$$\mathcal{E}_3 = \frac{\lambda_2 z^2}{2\pi(\lambda_1 + \lambda_2 z^2)}$$

$$\mathcal{E}_2 = \frac{\lambda_2}{2\pi(\lambda_1 + \lambda_2)}$$

Results

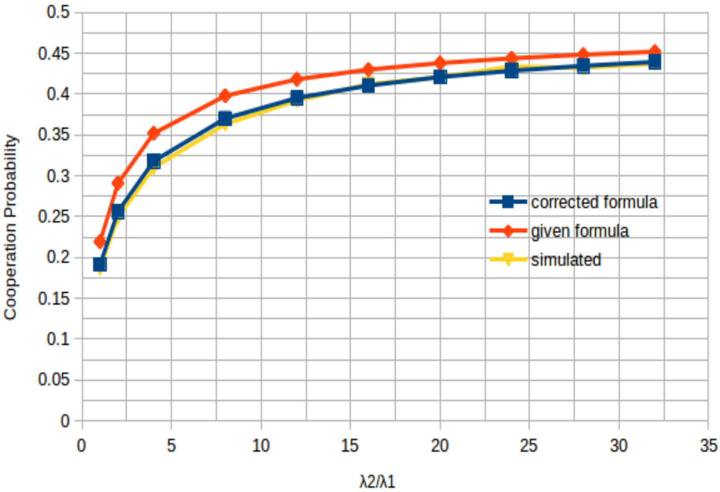

Figure: Cooperation probability of E_3.

Results

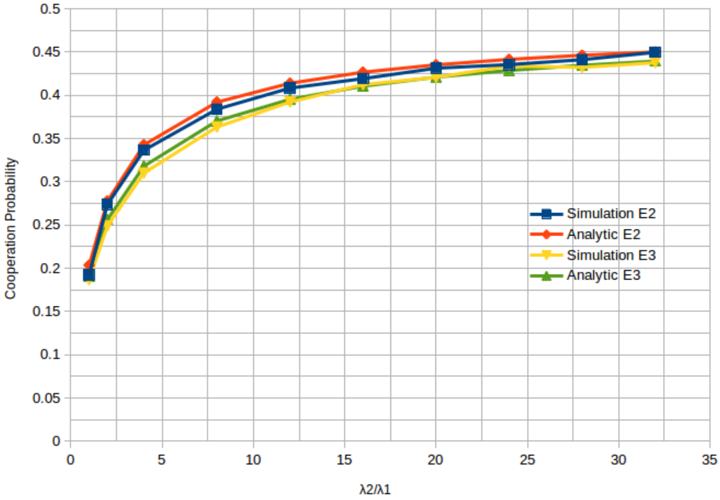

Figure: Cooperation probabilities v/s User ratio density.

Results

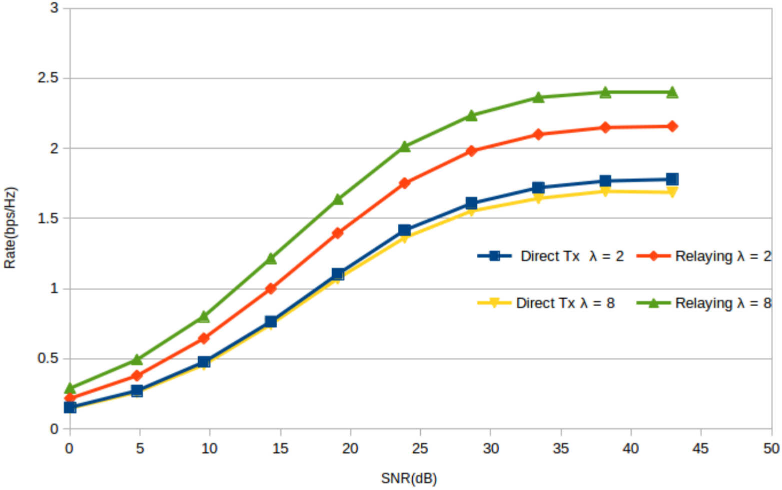

Figure: Per user average rate v/s SNR.

Future Work

New Policy

- In the policies E_2, E_3, only the first neighbour of the active UE is seen as a potential relay.
- Any relay which is at a distance r_2 from active UE and satisfying $r_2 < r_1, D < r_1$ can be treated as a potential relay

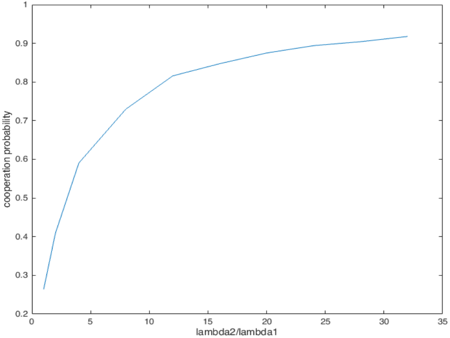

Figure: Cooperation Probability v/s User density ratio

Future Work

New Policy

- This also means more interference
- Simulate and see if the rate decreases due to increased interference
- If results are not negative, develop an analytical expression for cooperation probability

Future Work

β_1

Power used by UE to transmit the common codeword in second phase $P_s^{m_1} = \beta_1 P_s^m$

- Currently, β_1 is obtained by optimizing the final rate

Future Work

β_1

Power used by UE to transmit the common codeword in second phase $P_s^{m_1} = \beta_1 P_s^m$

- Currently, β_1 is obtained by optimizing the final rate

- This is not practical.

Future Work

β_1

Power used by UE to transmit the common codeword in second phase $P_s^{m_1} = \beta_1 P_s^m$

- Currently, β_1 is obtained by optimizing the final rate
- This is not practical.
- An algorithm can be developed to estimate β_1

Future Work

Power Control

- We assumed that all nodes transmit at maximum power

Future Work

Power Control

- We assumed that all nodes transmit at maximum power
- A distance based power control method can be applied

Thank You!!