Uplink User-Assisted Relaying in Cellular Networks

Dual Degree Project 1st Stage Report

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Abstract

Currently, there are 31,254 level crossings and around 40% of them are unmanned. The unmanned crossings are responsible for the maximum number of train accidents. The m ain objective of this project is to reduce the number of such accidents by building a reliable system th at can consistently detect a train moving towards the crossing and sets off an alarm at the crossing.

Contents

1   Introduction

The solution to this problem is to build a system that can turn on an alarm at the crossing at least 1 min before the train reaches the crossing and turn off the alarm when the train passes the crossing. To do this, we designed a sensor unit, using two inductive proximity sensors, that can detect a train and its direction, and an alarm unit. The sensor unit will be placed 1.5 km away from the crossing while the alarm unit will be placed at the crossing. When a train passes over the two sensors of the sensor unit, it detects the direction of the train, counts the number of axles[1](n) and sends this information to the alarm unit. If the train is moving towards the crossing, alarm unit turns on the alarm. When the train passes over the single sensor placed at the crossing, the alarm unit down counts the number of axles from n and turns off the alarm when the count reaches 0.

In the next sections we present the block and circuit diagrams of various units of the system, different algorithms used to detect the direction of train and also how false alarm cases are handled.

1.1   Motivation

1.2   Work Reported

1.3   Organization of this report

2   Partial Decode-and-Forward Relaying

In this section, we discuss the signal design, channel model and achievable rate of PDF relaying scheme.

2.1   Signal Design

Consider a source $\mathcal{S}$, its relay $\mathcal{R}$ and the destination $\mathcal{D}$. Each transmission block is divided into two phases: 1. broadcast transmission in which $\mathcal{S}$ broadcasts to both $\mathcal{R}$ and $\mathcal{D}$. 2. multiple access transmission in which both $\mathcal{S}$ and $\mathcal{R}$ transmit to $\mathcal{D}$. In each block of transmission, $\mathcal{S}$ splits its information into a common part and a private part. The common part is encoded via U_s^b in the 1st phase and $U_s^{m_1}$ in the 2nd phase; and the private part is encoded via $V_s^{m_2}$ in the 2nd phase. The relay $\mathcal{R}$ decodes the information sent by $\mathcal{S}$ in first phase and encodes the same information using $U_s^{m_1}$ in the 2nd phase.
The signals transmitted by $\mathcal{R}$ and $\mathcal{S}$ are as follows:

$$\text{Phase 1:} \quad x_s^b = \sqrt{P_s^b} U_s^b, \tag{1}$$

$$\text{Phase 2:} \quad x_r^m = \sqrt{P_r^m} U_s^{m_1}, \tag{2}$$

$$x_s^m = \sqrt{P_s^{m_1}} U_s^{m_1} + \sqrt{P_s^{m_2}} V_s^{m_2} \tag{3}$$

All codewords above are picked from independent Gaussian codebooks with zero mean and unit variance.

Power Constraints: Let P_s and P_r be the transmit powers of $\mathcal{S}$ and $\mathcal{R}$ respectively and α_1 be the fraction of transmission time allocated to first phase, then the following average power constraints should to be satisfied:

$$\alpha_1 P_s^b + \alpha_2 P_s^m = Ps, \quad \alpha_2 P_r^m = P_r \tag{4}$$

where $\alpha_2 = 1 - \alpha_1$

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[1]We actually count the number of wheels on one side, 4 wheels on each side $\implies$ 4 axles

2.2 Channel Model

Considering the transmit signals presented above and assuming flat fading over the two phases, the received signals at $\mathcal{R}$ and $\mathcal{D}$ during first phase are

$$Y_r^b = h_{sr}x_s^b + Z_r^b, \quad Y_d^b = h_{sd}x_s^b + Z_d^b \tag{5}$$

where b denotes broadcast mode, Z_r^b and Z_d^b are *i.i.d* circularly-symmetric complex gaussians with mean 0 and variance σ^2 - $\mathcal{CN}(0, \sigma^2)$ that represent noises at $\mathcal{R}$ and $\mathcal{D}$.
Similarly the received signal at $\mathcal{D}$ during second phase can be modelled as

$$Y_d^m = h_{sd}x_s^m + h_{rd}x_r^m + Z_d^m \tag{6}$$

here m denotes multicast transmission; all others have usual meaning. The above expression is true only if $\mathcal{D}$ has knowledge about the phase offset between $\mathcal{S}$ and $\mathcal{R}$. This assumption is justified by noting that the phase offset between the two nodes can be estimated at base station.

2.3 Achievable Rate

With transmit signals in equations 1- 3 and joint ML decoding rule at $\mathcal{D}$, the achievable rate for this relaying scheme is:
$$R_{PDF} \leq min(C_1 + C_2, C_3) \tag{7}$$

$$\text{where } C_1 = \alpha_1 \log\left(1 + |h_{sr}|^2 P_s^b\right), \tag{8}$$

$$C_2 = \alpha_2 \log\left(1 + |h_{sd}|^2 P_s^{m_2}\right), \tag{9}$$

$$C_3 = \alpha_1 \log\left(1 + |h_{sd}|^2 P_s^b\right) + \alpha_2 \log\left(1 + |h_{sd}|^2 P_s^{m_2} + \left(|h_{sd}|\sqrt{P_s^{m_1}} + |h_{rd}|\sqrt{P_r^m}\right)^2\right) \tag{10}$$

C_1 represents the rate of the common part that can be decoded at $\mathcal{R}$, C_2 the private part that can be decoded at $\mathcal{D}$ provided the common part has been decoded correctly, and C_3 both the common and private parts that can be jointly decoded at $\mathcal{D}$. These rates are achievable provided full CSIR at all receivers and the source-relay phase offset knowledge.

Now that we know what PDF relaying scheme is and the achievable rate, let us see how this scheme performs in cellular networks. To analyse system performance under PDF relaying, we need to know network geometry i.e., how the users and base stations are distributed, how many users can take advantage of relaying, how users identify a potential relay etc. In the next couple of sections we describe network geometry, received signals and interference model when relaying is deployed in the whole network, and cooperation policies.

3 Cellular Network Geometry and User-Assisted Relaying

3.1 Network geometry model

Consider a cellular system which consists of multiple cells, each cell has a single base station and each base station serves multiple users. Each of the users uses a distinct frequency block. Each user is served by the single base station that is closest to that user.
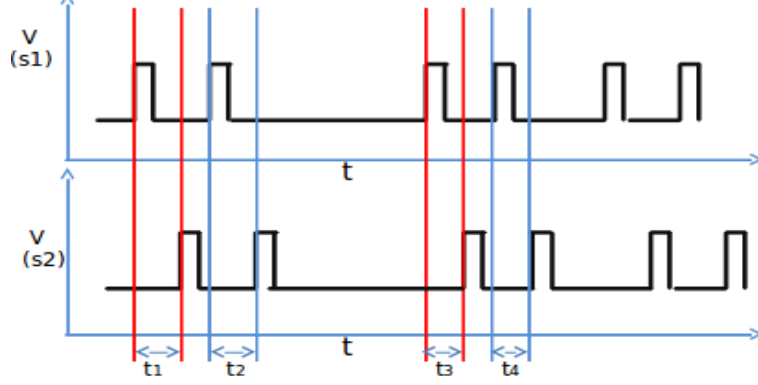
Figure 1: pulses on s1, s2 as train crosses the sensor unit

We use stochastic geometry to describe the uplink cellular network as shown in Fig. 1. We assume that the active users in different cells that use the same resource block and cause interference to each other are distributed on a two-dimensional plane according to a homogeneous and stationary Poisson point process (PPP) Φ_1 with intensity λ_1. The set of user equipments(UEs) that are in idle state and can participate in relaying are distributed according to another PPP Φ_2 with intensity λ_2. We assume Φ_1 and Φ_2 are independent. Furthermore, under the assumption that each BS serves a single mobile in a given resource block, the BS should be closer to its served UE than to any other UE. Therefore we assume each BS is uniformly distributed in the Voronoi cell of its served UE. Fig. 2 shows an example layout of the model.
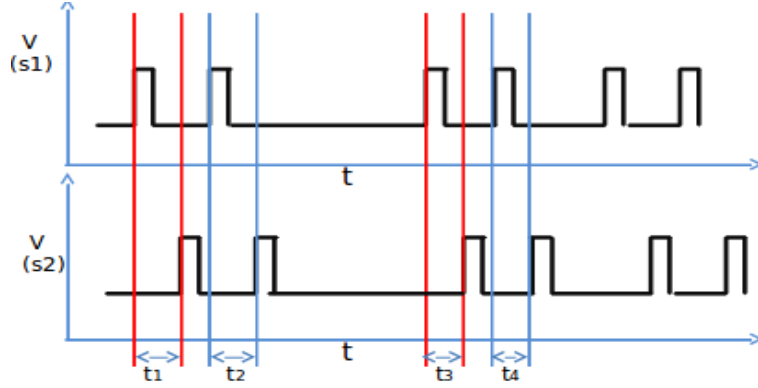
Figure 2: pulses on s1, s2 as train crosses the sensor unit

3.2 Channel Model

In this section, we describe the channel model when PDF relaying is deployed in cellular network. In this case, there will be out-of-cell inteference in addition to noise. The interference is due to frequency reuse in other cells.

Consider i^{th} cell, we model the received signals at the relay and base station in this cell during 1st phase as

$$Y_{r,i}^b = h_{sr}^{(i)} x_{s,i}^b + I_{r,i}^b + Z_{r,i}^b,$$

$$Y_{d,i}^b = h_{sd}^{(i)} x_{s,i}^b + I_{d,i}^b + Z_{d,i}^b \tag{11}$$

where $I_{r,i}^b$ and $I_{d,i}^b$ represent the interference received at the ith relay and destination.

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In second phase of the transmission, the received signal at the BS can be modelled as

$$Y_{d,i}^m = h_{sd}^{(i)} x_{s,i}^m + h_{rd}^{(i)} x_{r,i}^m + I_{d,i}^m + Z_{d,i}^m \tag{12}$$

3.3 Interference

To model interference, we assume prefect frame synchronization. LTE-Advanced imposes very strict requirements on synchronization anyway. Interference at the relay during first phase and at the destination(BS) during first and second phases can be expressed as

$$I_{r,i}^b = \sum_{k \neq i} B_k h_{sr}^{(k,i)} x_{s,k}^b + (1 - B_k) h_{sr}^{(k,i)} x_{s,k},$$

$$I_{d,i}^b = \sum_{k \neq i} B_k h_{sd}^{(k,i)} x_{s,k}^b + (1 - B_k) h_{sd}^{(k,i)} x_{s,k},$$

$$I_{d,i}^m = \sum_{k \neq i} B_k \left(h_{sd}^{(k,i)} x_{s,k}^m + h_{rd}^{(k,i)} x_{r,k}^m \right) + (1 - B_k) h_{sd}^{(k,i)} x_{s,k} \tag{13}$$

the summation is over all active users. Here, $h_{sd}^{(k,i)}$ and $h_{rd}^{(k,i)}$, respectively, are the channel fading from the kth active UE in Φ_1 and the associated relaying UE in Φ_2 to the BS associated with the ith active UE in Φ_1; and $h_{sr}^{(k,i)}$ is the channel fading from the kth active UE in Φ_1 to the relaying UE associated with the ith active UE in Φ_1.

B_k in above expressions is a Bernoulli random variable with success probability ρ. $B_k = 1$ is used to indicate the kth active UE's decision to exploit the help of another idle UE, a relay, and apply the relaying transmission strategy, and $B_k = 0$ indicates that the kth UE has no relay. In section 4, we derive the cooperation probability ρ for different cooperation policies.

For a given setting of nodes locations, based on the interference model in Eq. 13, we can use the fact that interference at either the relay or destination is the sum of an infinite number of signals undergoing independent fading from nodes distributed in the infinite 2-D plane and use the law of large numbers to approximate the interference as a complex Gaussian distribution. Also, since the transmitted codewords complex Gaussian with zero mean, mean of interference is zero. To fully characterize interference as a complex Gaussian distribution, we define their distributions as $I_{d,i}^b \sim \mathcal{CN}(0, \mathcal{Q}_{d,i}^b), I_{d,i}^m \sim \mathcal{CN}(0, \mathcal{Q}_{d,i}^m)$, and $I_{r,i}^b \sim \mathcal{CN}(0, \mathcal{Q}_{r,i})$ with the variances derived later in Section 5. The power of these interference terms which correspond to the variance of the Gaussian random variables are function of node locations and hence vary with different network realizations.

3.4 Equivalent Standard Channel Model

Using the interference model discussed above, we can convert the channel model in case of relaying into the standard form to capture the effects of interference into the channel fading as

$$\tilde{Y}_{r,i}^b = \tilde{h}_{sr}^{(i)} x_{s,i}^b + \tilde{Z}_{r,i}^b,$$
$$\tilde{Y}_{d,i}^b = \tilde{h}_{sd}^{(i)} x_{s,i}^b + \tilde{Z}_{d,i}^b,$$
$$\tilde{Y}_{d,i}^m = \tilde{h}_{sd}^{(i)} x_{s,i}^m + \tilde{h}_{rd}^{(i)} x_{r,i}^m + \tilde{Z}_{d,i}^m$$

where the new channel fading terms are defined as

$$\tilde{h}_{sr}^{(i)} = \frac{h_{sr}^{(i)}}{\sqrt{\mathcal{Q}_{r,i} + \sigma^2}}, \quad \tilde{h}_{sd}^{(b,i)} = \frac{h_{sd}^{(i)}}{\sqrt{\mathcal{Q}_{d,i}^b + \sigma^2}} \quad \tilde{h}_{sd}^{(m,i)} = \frac{h_{sd}^{(i)}}{\sqrt{\mathcal{Q}_{d,i}^m + \sigma^2}}, \quad \tilde{h}_{rd}^{(i)} = \frac{h_{rd}^{(i)}}{\sqrt{\mathcal{Q}_{d,i}^m + \sigma^2}}$$

and the noise terms are now all $\mathcal{CN}(0, 1)$. Using these equivalent standard channels, we can compute the transmission rate as in Eq. 7

4 Cooperation Policies and Probability

In this section, we look at three cooperation policies: an ideal policy E_1, a pure geometric policy E_2 and a hybrid policy E_3 that defines whether an active UE should select an inactive UE to use it in PDF relaying. Also, expressions for cooperation probabilities of E_2 and E_3 are derived.

4.1 Policies

4.1.1 Ideal Policy E_1

The ideal cooperation policy E_1 requires the active UE nodes to know instantaneous SINRs of the relay link($\mathcal{S} - \mathcal{R}$) and the direct link($\mathcal{S} - \mathcal{D}$). The policy is defined as

$$E_1 = \left\{ |\tilde{h}_{(sr)}^{(k)}|^2 \geq |\tilde{h}_{(sd)}^{(k)}|^2 \right\}$$

$$\simeq \left\{ \frac{g_{sr} r_2^{-\alpha}}{\mathcal{Q}_{r,k}} \geq \frac{g_{sd} r_1^{-\alpha}}{\mathcal{Q}_{d,k}^b} \right\}$$

where r_1 and r_2 denote the direct distance between $\mathcal{S}$ and $\mathcal{D}$ and cooperation distance between $\mathcal{S}$ and its closest idle UE, respectively and α is pathloss exponent. This event E_1 identifies whether an idle UE will be associated as a relay for the kth UE and participate in transmission. Noise variance σ^2 is ignored since interference dominates.

Since interference at relay and destination during first phase is more or less the same and g_{sr}, g_{sd} are identically distributed, we can safely ignore them and propose a policy that depends only on distances.

4.1.2 Pure Geometric Policy E_2

This policy is defined as

$$E_2 = \{r_2 \leq r_1, D \leq r_1\} \tag{14}$$

where D is the distance between $\mathcal{R}$ and $\mathcal{D}$. In words, if source's(active UE's) nearest idle neighbour is in the intersection region of two circles of radius r_1 centered at source and destination, then that idle UE will be chosen to act as a relay.

E_2 is more practical than policy E_1 in the sense that it does not require full knowledge of both the channel fading and the interference at the decision making node. Instead, it only requires the decision making nodes to know the distances from the active user to the nearest idle user and to the base station. It represents a practical decision making strategy for fast fading channels, requiring no knowledge of the channel fading.

4.1.3 Hybrid Policy E_3

This policy is proposed for slow fading channels where small scale fading parameters estimation and their feedback to the decision making node is feasible.

$$E_3 = \{g_{sd} r_1^{-\alpha} \leq g_{sr} r_2^{-\alpha}, D \leq r_1\} \tag{15}$$

Note that this cooperation policy is still independent of the interference as in the pure geometric cooperation policy E_2.

4.2 Cooperation Probabilities

In this part of the section we derive cooperation probabilities ρ_2 and ρ_3 for the policies E_2 and E_3 respectively. For the ideal policy E_1, analytic evaluation of the cooperation probability is rather complicated because of the inter-dependency between the cooperation decision and con- sequential interference among different cells, hence we use numerical simulations instead. Consider a random BS and its associated active UE. The distribution of the distance r_1 between the ith UE and its associated BS can be shown to be Rayleigh distributed directly from the null probability of a two dimensional PPP distribution.

we can assume due to the stationarity of the PPP, i.e., location of the origin doesn't change the distribution of points, and the independence of Φ_2 from BSs distribution that the location of the UE associated with the BS under study represents the origin point of Φ_2. Then, each UE in Φ_1 chooses the closest UE in Φ_2 to assist it in relaying its message to the serving BS. Hence, similar to source-to- destination distance, the distribution of the source-to-relay distance r_2 between the ith UE and its associated relaying UE can be also shown to be Rayleigh distributed from the null probability of a two dimensional PPP. Therefore,

$$f_{r_1}(r_1) = 2\pi\lambda_1 r_1 e^{-\lambda_1\pi r_1^2},$$

$$f_{r_2}(r_2) = 2\pi\lambda_2 r_2 e^{-\lambda_2\pi r_2^2} \tag{16}$$

Theorem 1. *Cooperation Probabilities. The probability of deploying user-assisted relaying for a randomly located active user within a cell can be evaluated as follows:*

    i. For policy E_2

$$\rho_2 = \int_{-\pi/2}^{-\pi/3} \frac{2\lambda_2 cos^2\psi_0}{\pi(\lambda_1 + 4\lambda_2 cos^2\psi_0)}d\psi_0 + \int_{\pi/3}^{\pi/2} \frac{2\lambda_2 cos^2\psi_0}{\pi(\lambda_1 + 4\lambda_2 cos^2\psi_0)}d\psi_0 + \frac{\lambda_2}{3(\lambda_1 + \lambda_2)} \tag{17}$$

    ii. For policy E_3

$$\rho_3 = \int_0^2 f_\beta(z)\int_{-\pi/2}^{-cos^{-1}(z/2)} \frac{2\lambda_2 cos^2\psi_0}{\pi(\lambda_1 + 4\lambda_2 cos^2\psi_0)}d\psi_0 dz$$

$$+ \int_0^2 f_\beta(z)\int_{cos^{-1}(z/2)}^{\pi/2} \frac{2\lambda_2 cos^2\psi_0}{\pi(\lambda_1 + 4\lambda_2 cos^2\psi_0)}d\psi_0 dz$$

$$+ \int_0^2 f_\beta(z)\frac{\lambda_2 z^2 cos^{-1}(z/2)}{\pi(\lambda_1 + \lambda_2 z^2)}dz$$

$$+ \int_2^\infty f_\beta(z)\int_{-\pi/2}^{\pi/2} \frac{2\lambda_2 cos^2\psi_0}{\pi(\lambda_1 + 4\lambda_2 cos^2\psi_0)}d\psi_0 dz$$

where $\beta = \left(\frac{g_{sr}}{g_{sd}}\right)^{1/\alpha}$ and $f_\beta(z)$ is pdf of β which can be shown to be

$$f_\beta(z) = \frac{\alpha z^{\alpha-1}}{(1 + z^\alpha)^2} \tag{18}$$

Proof.   i.

$$\rho_2 = \mathbb{P}\{E_2\}$$
$$= \mathbb{P}\{r_2 \le r_1, r_1^2 + r_2^2 - 2r_1 r_2 cos\psi_0 \le r_1^2\}$$
$$= \mathbb{P}\{r_2 \le r_1, r_2 \le 2r_1 cos\psi_0\}$$

when $|\psi_0| < \pi/3$, $r_1 < 2r_1 cos\psi_0 \Rightarrow$ if $r_2 < r_1$, r_2 satisfies both inequalities. Accordingly, we define $\mathcal{E}_1$ and $\mathcal{E}_2$ as follows

$$\mathcal{E}_1 = (2\pi)^2\lambda_1\lambda_2 \int_0^\infty \int_0^{2r_1 cos\psi_0} r_1 r_2 e^{-\pi(\lambda_1 r_1^2 + \lambda_2 r_2^2)}dr_2 dr_1$$

$$= \frac{2\lambda_2 cos^2\psi_0}{\pi(\lambda_1 + 4\lambda_2 cos^2\psi_0)}$$

$$\mathcal{E}_2 = (2\pi)^2\lambda_1\lambda_2 \int_0^\infty \int_0^{r_1} r_1 r_2 e^{-\pi(\lambda_1 r_1^2 + \lambda_2 r_2^2)}dr_2 dr_1$$

$$= \frac{\lambda_2}{2\pi(\lambda_1 + \lambda_2)}$$

$$\text{Now, } \rho_2 = \int_{-\pi/3}^{\pi/3} \mathcal{E}_2 d\psi_0 + 2 \int_{\pi/3}^{\pi/2} \mathcal{E}_1 d\psi_0$$

$$= \frac{\lambda_2}{3(\lambda_1 + \lambda_2)} + 2 \int_{\pi/3}^{\pi/2} \mathcal{E}_1 d\psi_0$$

ii.

$$\rho_3 = \mathbb{P}\{E_3\}$$

$$= \mathbb{P}\{r_2 \le \left(\frac{g_{sr}}{g_{sd}}\right)^{1/\alpha} r_1, r_1^2 + r_2^2 - 2r_1 r_2 cos\psi_0 \le r_1^2\}$$

$$= \mathbb{P}\{r_2 \le \beta r_1, r_2 \le 2r_1 cos\psi_0\}$$

$$= \mathbb{P}\{r_2 \le 2r_1 cos\psi_0\} \qquad \text{for } \beta > 2$$

$$= \mathbb{P}\{r_2 \le \beta r_1\} \qquad \text{for } \beta < 2 \text{ and } |\psi_0| < cos^{-1}(\beta/2)$$

$$= \mathbb{P}\{r_2 \le 2r_1 cos\psi_0\} \qquad \text{for } \beta < 2 \text{ and } cos^{-1}(\beta/2) < |\psi_0| < \pi/2$$

$$\therefore \rho_3 = 2 \int_0^2 f_\beta(z) \int_{cos^{-1}(z/2)}^{\pi/2} \mathcal{E}_1 d\psi_0 dz + \int_0^2 f_\beta(z) \int_{-cos^{-1}(z/2)}^{cos^{-1}(z/2)} \mathcal{E}_3 d\psi_0 dz$$

$$+ \int_2^\infty f_\beta(z) \int_{-\pi/2}^{\pi/2} \mathcal{E}_1 d\psi_0 dz$$

$\mathcal{E}_1$ is defined in part i. of the proof and $\mathcal{E}_3 = \frac{\lambda_2 z^2}{2\pi(\lambda_1 + \lambda_2 z^2)}$ which is nothing but $\mathcal{E}_2$ with $\lambda_2 = \lambda_2 z^2$. $f_\beta(z)$, the pdf of β, can be obtained as follows

$$F_\beta(z) = \mathbb{P}\left\{\left(\frac{x_1}{x_2}\right)^{1/\alpha} \le z\right\} = \mathbb{P}\{x_1 \le z^\alpha x_2\}$$

$$= \int_0^\infty \int_0^{z^\alpha x_2} e^{-(x_1 + x_2)} dx_1 dx_2 \quad \text{since } g_{sr}, g_{sd} \sim Exp(1)$$

$$= 1 - \frac{1}{1 + z^\alpha}, \qquad z \in [0, \infty)$$

The pdf $f_\beta(z)$ is then obtained by differentiating $F_\beta(z)$:

$$f_\beta(z) = \frac{dF_\beta(z)}{dz} = \frac{\alpha z^{\alpha-1}}{(1 + z^\alpha)^2} \quad z \in [0, \infty)$$

$\square$

5 Interference Analysis

User-assisted relaying actually increases the amount of out- of-cell interference in the network as some idle users are now transmitting when relaying information of active users. It is therefore necessary to understand this out-of-cell interference power, particularly its distribution, in order to assess the overall impact of user-assisted relaying on system performance.

5.1 First Two Moments of Interference Power

Since it is difficult to describe the exact distribution of out-of-cell interference power, here we choose to model the interference power to the cell under study as a Gamma distribution by fitting the first two moments of the interference power analytically developed using stochastic geometry of the field of interferers outside that cell. The expressions for interference power can be developed from Eqs. 13.

$$\mathcal{Q}_{d,i}^b = \sum_{k \neq i} B_k \left| h_{sd}^{(k,i)} \right|^2 P_{s,k}^b + (1 - B_k) \left| h_{sd}^{(k,i)} \right|^2 P_{s,k} \tag{19}$$

$$\mathcal{Q}_{d,i}^m = \sum_{k \neq i} \left[B_k \left(\left| h_{sd}^{(k,i)} \right|^2 P_{s,k}^m + \left| h_{rd}^{(k,i)} \right|^2 P_{r,k}^m \right) \right] + (1 - B_k) \left| h_{sd}^{(k,i)} \right|^2 P_{s,k} \tag{20}$$

$$\mathcal{Q}_{r,i} = \sum_{k \neq i} B_k \left| h_{sr}^{(k,i)} \right|^2 P_{s,k}^b + (1 - B_k) \left| h_{sr}^{(k,i)} \right|^2 P_{s,k} \tag{21}$$

Theorem 2. *Interference Power Statistics For network-wide deployment of user-assisted relaying, the out-of-cell interference generated at the destination BS and the relaying UE have the following statistics:*

i. *The first two moments, mean and variance, of interference power at the destination BS during the 1st and 2nd phase, respectively, are*

$$\mathbb{E}[\mathcal{Q}_{d,i}^b] = \frac{2\pi \lambda_1 \zeta_1}{\alpha - 2} R_c^{2-\alpha}, \qquad \mathbb{E}[\mathcal{Q}_{d,i}^m] = \frac{2\pi \lambda_1 \zeta_3}{\alpha - 2} R_c^{2-\alpha} \tag{22}$$

$$var[\mathcal{Q}_{d,i}^b] = \frac{\pi \lambda_1 \zeta_2}{\alpha - 1} R_c^{2(1-\alpha)}, \qquad var[\mathcal{Q}_{d,i}^m] = \frac{\pi \lambda_1 \zeta_4}{\alpha - 1} R_c^{2(1-\alpha)} \tag{23}$$

ii. *The first two moments, mean and variance, of interfer- ence power at the idle UE associated as a relay with the ith active UE are*

$$\mathbb{E}[\mathcal{Q}_{r,i}] = \lambda_1 \zeta_1 \int_0^{2\pi} \int_{R_c}^{\infty} (r^2 + D^2 - 2rD cos\theta)^{\alpha/2} r dr d\theta \tag{24}$$

$$var[\mathcal{Q}_{r,i}] = \lambda_1 \zeta_2 \int_0^{2\pi} \int_{R_c}^{\infty} (r^2 + D^2 - 2rD cos\theta)^{\alpha/2} r dr d\theta \tag{25}$$

$$where \quad \zeta_1 = \rho_1 P_{s,k}^b + (1 - \rho_1) P_{s,k} \tag{26}$$

$$\zeta_2 = 2[\rho_1 (P_{s,k}^b)^2 + (1 - \rho_1) P_{s,k}^2], \tag{27}$$

$$\zeta_3 = \rho_1 (P_{s,k}^m + P_{r,k}^m) + (1 - \rho_1) P_{s,k}, \tag{28}$$

$$\zeta_4 = 2[\rho_1 (P_{s,k}^m + P_{r,k}^m)^2 + (1 - \rho_1) P_{s,k}^2 - \rho_1 P_{s,k}^m P_{r,k}^m] \tag{29}$$

Proof.

$$\mathbb{E}[\mathcal{Q}_{d,i}^b] = - \frac{\partial \mathcal{L}_{\mathcal{Q}_{d,i}^b}(s)}{\partial s} \bigg|_{s=0},$$

$$var[\mathcal{Q}_{d,i}^b] = - \frac{\partial^2 \mathcal{L}_{\mathcal{Q}_{d,i}^b}(s)}{\partial s^2} \bigg|_{s=0} - \left(\mathbb{E}[\mathcal{Q}_{d,i}^b] \right)^2$$

where $\mathcal{L}_{\mathcal{Q}_{d,i}^b}(s)$ is the Laplace transform of $\mathcal{Q}_{d,i}^b$ and $R_c = 1/2\sqrt{\lambda_1}$ is the cell radius. Means and variances of $\mathcal{Q}_{d,i}^m$, $\mathcal{Q}_{r,i}$ can be calculated similarly. □

From the above results for interference power statistics, the interference power is directly proportional to both the active users density, λ_1, and the transmission power levels represented by $\zeta_i, i \in [1:4]$ in Eqs. 26 - 29

5.2 Modelling Interference Power Distribution

A parameterized probability distribution, which includes a wide variety of curve shapes, is useful in the representation of data when the underlying model is unknown or difficult to obtain in closed form. A parameterized probability distribution is usually characterized by its flexibility, generality, and simplicity. Although distributions are not necessarily determined by their moments, the moments often provide useful information and are widely used in practice. It is shown that the Gamma distribution is a good approximation for the interference when the point under study is closer to the cell center, but fails to represent the actual interference distribution whenever the point under study is exactly at the cell edge. We use the same approach here and match a Gamma distribution to the first two moments of the interference power terms derived earlier in Theorem 2.

5.2.1 Gamma Distribution

The Gamma distribution is specified by a shape parameter k and a scale parameter θ. The pdf of a Gamma distributed RV $\gamma[k, \theta]$ is defined as

$$F_\gamma(q|k,\theta) = \frac{q^{k-1}e^{(-q/\theta)}}{\theta^k \Gamma(k)}$$

where the Gamma function $\Gamma(t)$ is defined as $\Gamma(t) = \int_0^\infty x^{t-1}e^{-x}dx$. The mean and variance of $\gamma[k, \theta]$ are $k\theta$ and $k\theta^2$ respectively.

Since we know mean and variance of interference powers, we can estimate the shape and scale parameters by using the formulae:

$$k_i = \frac{(\mathbb{E}[\mathcal{Q}_i])^2}{\text{var}[\mathcal{Q}_i]}, \theta_i = \frac{\text{var}[\mathcal{Q}_i]}{\mathbb{E}[\mathcal{Q}_i]} \tag{30}$$

6 Simulations and Results

7 Conclusions and Future Work

8 References