Sojourn Time of Moving Relays in Dual-Hop Cooperative Communication

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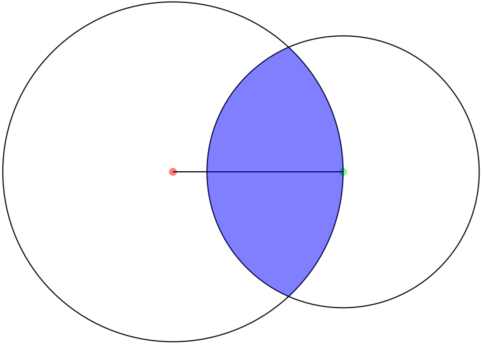
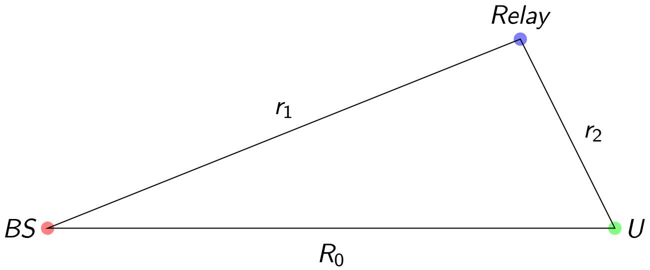
Problem



Downlink Cooperation policy



$$\{P_b r_1^{-\alpha} > P_b R_0^{-\alpha}, P_r r_2^{-\alpha} > P_b R_0^{-\alpha}\}$$

P_b and P_r are the transmission powers of base station and relay respectively.

$\{r_1 < R_0, r_2 < R_2\}$ where $R_2 = cR_0, c = \left(\frac{P_r}{P_b}\right)^{\frac{1}{\alpha}}$

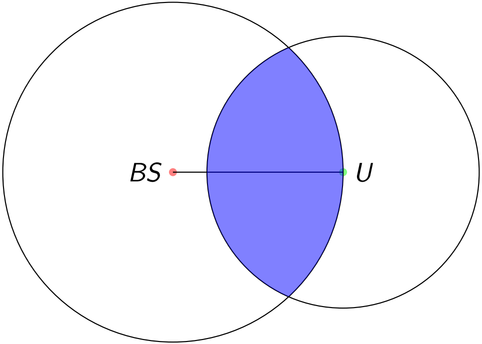


Figure: Feasible region

Motivation

- Relay selection

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- Effect on the whole network

Mobility Model

The *n*th transition of a node is defined by the quadruple

$$(\mathbf{X}_{n-1}, \mathbf{X}_n, V_n, S_n)$$

X_{n-1} and X_n are the starting and destination waypoints.

Velocity - V_n, Pause time or Thinking time at destination - S_n

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Mobility model

- Distribution of transition length($L_n = \|\mathbf{X}_{n-1} - \mathbf{X}_n\|$)
- Distribution of angle made by the vector $\mathbf{X}_n - \mathbf{X}_{n-1}$ w.r.t x-axis.
- Distributions of V_n and S_n

Rayleigh RWP

Angle is chosen uniformly from $[0, 2\pi]$

Transition length is Rayleigh distributed with parameter λ.

$$P(L > l) = exp(-\lambda \pi l^2), l \geq 0$$

$$V \equiv v \text{ and } S \equiv 0$$

Rayleigh RWP

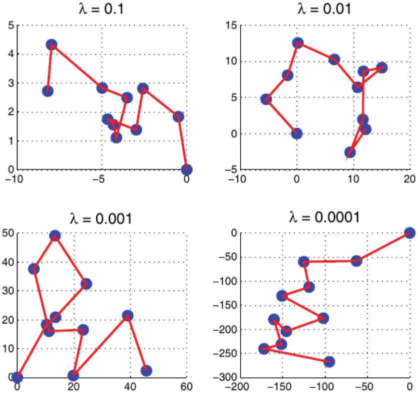


Figure: Sample traces of Rayleigh RWP[1]

[1] Image Credits: Xingqin Lin et al.

Rayleigh RWP

The mean transition length and time are as follows:

$$E[L] = \frac{1}{2\sqrt{\lambda}}$$

$$E[T] = \frac{1}{2v\sqrt{\lambda}}$$

Sojourn Time

Sojourn time is the amount of time a node resides in the region of interest. Calculating the mean sojourn time is challenging primarily because it involves finding node distribution during each transition. An expression for mean sojourn time of a cell user during one movement period starting from origin in a hexagonal cell was given in [**?**]. We have to note that a moving node usually makes more than one transition before it leaves the region. Also, starting from origin implies the node co-exists with BS at t=0 which is not representative of the distribution of relays/users. Even if we allow these two assumptions, the problem is still difficult to solve in this approach as the region has no definite shape like a polygon and finding integration limits is tedious.

Sojourn Time

If we know the expected number of transitions $E[N]$ a node makes before moving out of the region, then sojourn time can be given by

$$S_T = (E[N] - 1)E[T] + E[T_{last}]$$

where T_{last} is the time spent inside the region during the last transition. Since it is difficult to characterize T_{last}, we approximate S_T to $(E[N] - 1)E[T]$.

E[N]

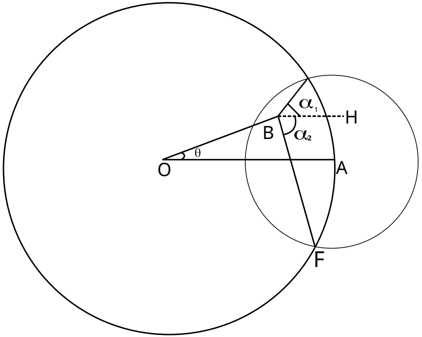
$$E(N) = \sum_{k=1}^{\infty} kPr(r, \theta, k)$$

where

$$Pr(r, \theta, k) = \int_{S-A} \int_{A} \dots \int_{A} f_{X_1/X_0}(x_1/x_0) \dots f_{X_{k+1}/X_k}(x_{k+1}/x_k) dA_1 \dots dA_{k+1}$$

is the probability that the node exits the region during $k+1$ th transition. $f_{X_n/X_{n-1}}(x_n/x_{n-1})$ is the probability density of the destination X_n given that the node's current position is X_{n-1}. $X_0 = (r, \theta)$ is where the node starts the movement at t=0, A is the feasibility region and S is the entire plane.

Leaving in One Transition



$$\rho(r,\theta) = Pr(-\alpha_2 < \alpha < \alpha_1, r_1 > r_{11}) + Pr(\alpha_1 < \alpha < 2\pi - \alpha_2, r_1 > r_{12})$$

$$= \int_{-\alpha_2}^{\alpha_1} \int_{r_{11}}^{\infty} f_{r_1,\alpha}(r_1,\alpha) dr_1 d\alpha + \int_{\alpha_1}^{2\pi-\alpha_2} \int_{r_{12}}^{\infty} f_{r_1,\alpha}(r_1,\alpha) dr_1 d\alpha$$

This is a general expresion that can be used for any mobility model.

In case of RWP, r_1 and α are chosen independently. Therefore, $f_{r_1,\alpha}(r_1,\alpha) = f_{r_1}(r_1) f_\alpha(\alpha)$

$$\begin{aligned}
\rho(r,\theta) &= \int_{-\alpha_2}^{\alpha_1} f_\alpha(\alpha) \int_{r_{11}}^{\infty} f_{r_1}(r_1) dr_1 \, d\alpha + \int_{\alpha_1}^{2\pi-\alpha_2} f_\alpha(\alpha) \int_{r_{12}}^{\infty} f_{r_1}(r_1) dr_1 \, d\alpha \\
&= \int_{-\alpha_2}^{\alpha_1} \frac{1}{2\pi} e^{-\lambda\pi r_{11}^2} d\alpha + \int_{\alpha_1}^{2\pi-\alpha_2} \frac{1}{2\pi} e^{-\lambda\pi r_{12}^2} d\alpha
\end{aligned}$$

Leaving in One Transition

The probability with which a node at (r, θ) moves out of the region of interest during the next transition is

$$\rho(r,\theta) = \int_{-\alpha_2}^{\alpha_1} \frac{1}{2\pi} e^{-\lambda\pi r_{11}^2} \, d\alpha + \int_{\alpha_1}^{2\pi-\alpha_2} \frac{1}{2\pi} e^{-\lambda\pi r_{12}^2} \, d\alpha$$

Where

$$r_{11} = -r\cos(\theta-\alpha) + \sqrt{R_0^2 - r^2\sin^2(\theta-\alpha)}$$

$$\begin{aligned} r_{12} &= R_0\cos\alpha - r\cos(\theta-\alpha) \\ &+ \sqrt{[R_0\cos\alpha - r\cos(\theta-\alpha)]^2 - r^2\sin^2\theta - [R_0 - r\cos\theta]^2 + R_2^2} \end{aligned}$$

Leaving in One Transition

$$\beta_2 = \cos^{-1}\left(1 - \frac{R_2^2}{2R_0^2}\right) \quad \beta_1 = \beta_2 - \theta$$

$$\alpha_1 = \theta + \tan^{-1}\left(\frac{R_0 \sin \beta_1}{R_0 \cos \beta_1 - r}\right)$$

$$\alpha_2 = \tan^{-1}\left(\frac{r \sin \theta + R_0 \sin \beta_2}{R_0 cos \beta_2 - r \cos \theta}\right)$$

Simulations

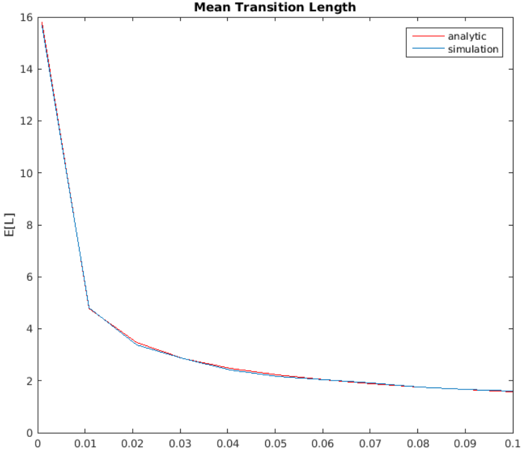
For all simulations, a 1000x1000 square is used to represent the whole 2-D plane. This size is good enough in the sense that using a larger square led to longer runtimes with little to no effect on the results. In the mobility model we used, transition length is Rayleigh distributed. To draw a length from Rayleigh distribution, the following method is used:

1. Draw a number N from Poisson distribution with density λA where A is the area of the square.
2. Distribute these N points uniformly on the square
3. Of these N points, choose the point that is closest to the point under consideration.

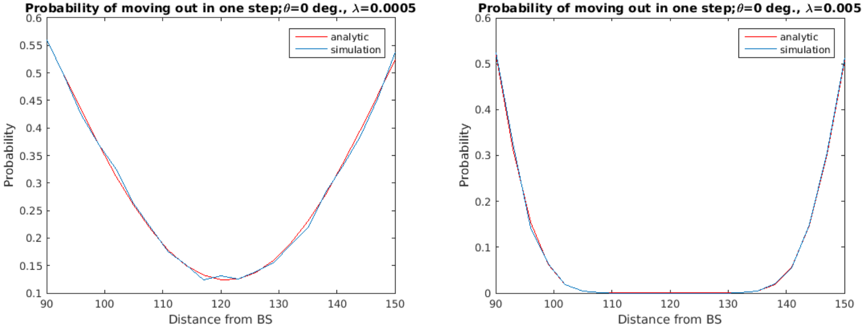
As discussed in [**?**], this leads to Rayleigh distribution of transition length and can be proved easily using null probability of a Poisson Point Process.

Simulations

To test if the above mentioned method introduces any artefacts, let us see how simulated $E[L]$ fares with the formula $E[L] = \frac{1}{2\sqrt{\lambda}}$. Simulated expected length is calculated by averaging transition lengths of 100 traces in each of which the node makes 1000 transitions.



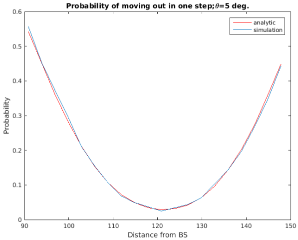
Simulations



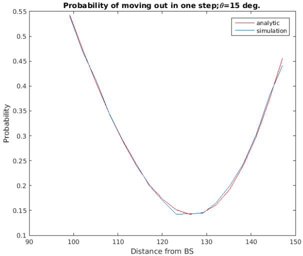
(a) $\lambda = 0.0005$ (b) $\lambda = 0.005$

Figure: $\rho(r, 0)$ vs. λ

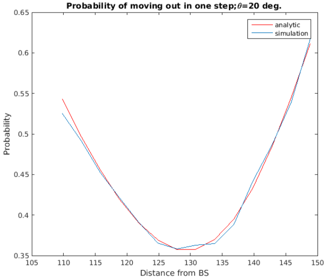
Simulations



(a) $\theta = 5^{\circ}$



(b) $\theta = 15^{\circ}$



E[N]

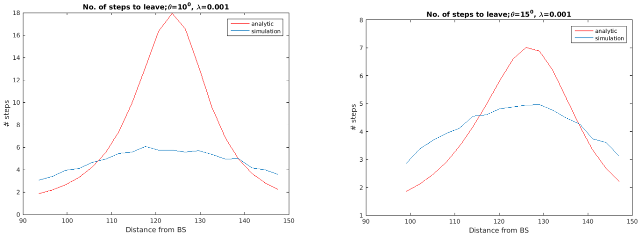
Let us make a gross approximation and use the same probability of leaving for all waypoints in the path. Then the average number of steps a node starting at (r, θ) takes to leave the region is given by

$$E[N] = \sum_{k=1}^{\infty} k(1 - \rho(r, \theta))^{k-1} \rho(r, \theta)$$

$$= \frac{1}{\rho(r, \theta)}$$

E[N]

The plots are for points along the radius at angles $\theta = 10^\circ$ and $\theta = 15^\circ$. We can see that the analytical formula agrees better for narrower regions. As discussed in Chapter 4, this can be improved by modelling the motion of a node as an absorbing Markov Chain and discretizing the state space.



(d) $\theta = 10^\circ$ (e) $\theta = 15^\circ$

Figure: Expected number of transitions

Ongoing Work

Markov Chain Process

The transitions in this mobility model have Markovian property in the sense that the next waypoint depends entirely on the current position.

$$f_{X_n/X_{n-1},X_{n-2},\ldots,X_0}(x_n/x_{n-1},x_{n-2},\ldots,x_0) = f_{X_n/X_{n-1}}(x_n/x_{n-1})$$

Where X_{n-1} is the current waypoint and X_n is the next waypoint.

Ongoing Work

Discretizing State Space

The idea is to discretize the state space of this Markov chain and model the motion as an Absorbing Markov Chain in which the node transitions among the non-absorbing states present inside the region before finally moving to the absorbing state. Let the whole space be represented by $n + 1$ states of which n states lie inside the region of interest and the $n + 1$th state represents the space outside the region. The transition probabilities among first n states depend on the distances between the nodes and the transition probabilities from these n states to the absorbing state is $\rho(r_i, \theta_i)$ where (r_i, θ_i) is the position of the ith state. We can then use the expressions given in [**?**] to find expected value and variance of the number of transitions a node makes before getting absorbed in the $(n + 1)$th state.

The transition probability matrix of this Markov Chain is

$$P = \begin{pmatrix} Q & R \\ \mathbf{0} & 1 \end{pmatrix}$$

Where $Q_{n\times n}$ is the transition probability matrix of n non-absorbing states and $R_{n\times 1}$ contains the probabilities of moving out in one step from each of those n states. $\mathbf{t}_{n\times 1}$, the vector which contains expected number of transitions, is given by

$$\mathbf{t} = F\mathbf{1}$$

and the variance vector V is given by

$$V = (2F - I)\mathbf{t} - \mathbf{t}_{sq}$$

where $F = \sum_{k=0}^{\infty} Q^k = (I - Q)^{-1}$ is the fundamental matrix.

References

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