# Sojourn Time of Moving Relays in Dual-Hop Cooperative Communication

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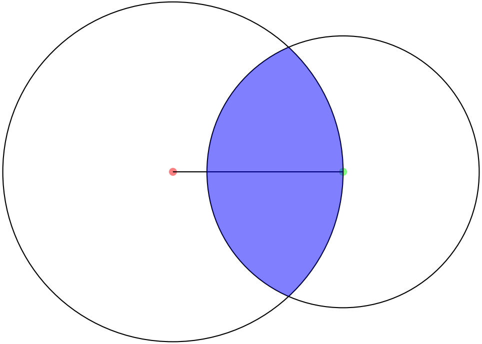
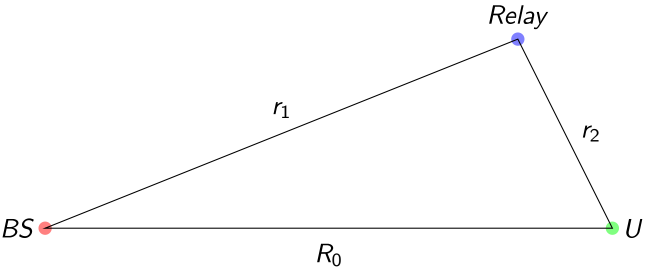
# Problem



# Downlink Cooperation policy




$$\{P_b r_1^{-\gamma} > P_b R_0^{-\gamma}, P_r r_2^{-\gamma} > P_b R_0^{-\gamma}\}$$

$P_b$ and $P_r$ are the transmission powers of base station and relay respectively.

$\{r_1 < R_0, r_2 < R_2\}$ where $R_2 = cR_0, c = \left(\frac{P_r}{P_b}\right)^{\frac{1}{\gamma}}$

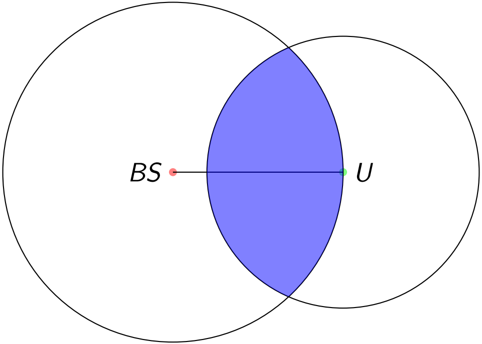



Figure: Feasible region

# Motivation

- Relay selection

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- Effect on the whole network

# Mobility Model

The $n$th transition of a node is defined by the quadruple

$$(\mathbf{X}_{n-1}, \mathbf{X}_n, V_n, S_n)$$

$X_{n-1}$ and $X_n$ are the starting and destination waypoints.

Velocity - $V_n$, Pause time or Thinking time at destination - $S_n$

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Mobility model

- Distribution of transition length($L_n = \|\mathbf{X}_{n-1} - \mathbf{X}_n\|$)
- Distribution of angle made by the vector $\mathbf{X}_n - \mathbf{X}_{n-1}$ w.r.t x-axis.
- Distributions of $V_n$ and $S_n$

# Rayleigh RWP

Angle is chosen uniformly from $[0, 2\pi]$

Transition length is Rayleigh distributed with parameter $\lambda$.

$$P(L > l) = exp(-\lambda\pi l^2), l \geq 0$$

$$V \equiv v \text{ and } S \equiv 0$$

# Rayleigh RWP

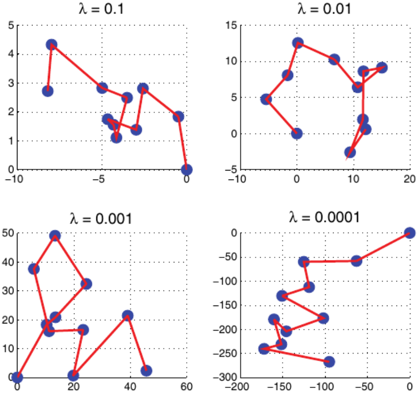


Figure: Sample traces of Rayleigh RWP[1]

[1]Image Credits: Xingqin Lin et al.

# Rayleigh RWP

The mean transition length and time are as follows:

$$E[L] = \frac{1}{2\sqrt{\lambda}}$$

$$E[T] = \frac{1}{2v\sqrt{\lambda}}$$

# Existing Literature

- Sojourn Time and Handover Rate in cellular networks, Xingqin Lin et al.

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- Analysis of Cell Sojourn Time in Heterogeneous Networks With Small Cells, S. Shin et al.
  Expected amount of time a mobile spends in macro-cell-only-area (MoA)

# Sojourn Time

If we know the expected number of transitions $E[N]$ in which a node moves out of the region, then sojourn time can be given by

$$S_T = (E[N] - 1)E[T] + E[T_{last}]$$

where $T_{last}$ is the time spent inside the region during the last transition. Since it is difficult to characterize $T_{last}$,

$$S_T \approx (E[N] - 1/2)E[T]$$

# E[N]

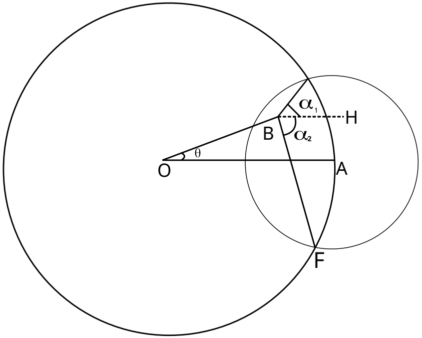
$$E(N) = \sum_{k=1}^{\infty} k Pr(r, \theta, k)$$

where

$$Pr(r, \theta, k) = \int_{S-A} \int_{A} \dots \int_{A} f_{X_1/X_0}(x_1/x_0) \dots f_{X_{k+1}/X_k}(x_{k+1}/x_k) dA_1 \dots dA_{k+1}$$

$f_{X_n/X_{n-1}}(x_n/x_{n-1})$ is the probability density of the destination $X_n$ given that the node's current position is $X_{n-1}$

# Leaving in One Transition




$$\rho(r,\theta) = Pr(-\alpha_2 < \alpha < \alpha_1, r_1 > r_{11}) + Pr(\alpha_1 < \alpha < 2\pi - \alpha_2, r_1 > r_{12})$$
$$= \int_{-\alpha_2}^{\alpha_1}\int_{r_{11}}^{\infty} f_{r_1,\alpha}(r_1,\alpha)dr_1 d\alpha + \int_{\alpha_1}^{2\pi-\alpha_2}\int_{r_{12}}^{\infty} f_{r_1,\alpha}(r_1,\alpha)dr_1 d\alpha$$

This is a general expression that can be used for any mobility model.

In case of RWP, $r_1$ and $\alpha$ are chosen independently. Therefore, $f_{r_1,\alpha}(r_1, \alpha) = f_{r_1}(r_1) f_\alpha(\alpha)$

$$\begin{aligned}\rho(r, \theta) &= \int_{-\alpha_2}^{\alpha_1} f_\alpha(\alpha) \int_{r_{11}}^{\infty} f_{r_1}(r_1) dr_1 d\alpha + \int_{\alpha_1}^{2\pi-\alpha_2} f_\alpha(\alpha) \int_{r_{12}}^{\infty} f_{r_1}(r_1) dr_1 d\alpha \\ &= \int_{-\alpha_2}^{\alpha_1} \frac{1}{2\pi} e^{-\lambda\pi r_{11}^2} d\alpha + \int_{\alpha_1}^{2\pi-\alpha_2} \frac{1}{2\pi} e^{-\lambda\pi r_{12}^2} d\alpha\end{aligned}$$

# Leaving in One Transition

The probability with which a node at $(r, \theta)$ moves out of the region of interest during the next transition is

$$\rho(r, \theta) = \int_{-\alpha_2}^{\alpha_1} \frac{1}{2\pi} e^{-\lambda \pi r_{11}^2} \, d\alpha + \int_{\alpha_1}^{2\pi - \alpha_2} \frac{1}{2\pi} e^{-\lambda \pi r_{12}^2} \, d\alpha$$

Where

$$r_{11} = -r \cos(\theta - \alpha) + \sqrt{R_0^2 - r^2 \sin^2(\theta - \alpha)}$$

$$\begin{aligned} r_{12} &= R_0 \cos \alpha - r \cos(\theta - \alpha) \\ &+ \sqrt{[R_0 \cos \alpha - r \cos(\theta - \alpha)]^2 - r^2 \sin^2 \theta - [R_0 - r \cos \theta]^2 + R_2^2} \end{aligned}$$

# Leaving in One Transition

$$\beta_2 = \cos^{-1}\left(1 - \frac{R_2^2}{2R_0^2}\right) \quad \beta_1 = \beta_2 - \theta$$

$$\alpha_1 = \theta + \tan^{-1}\left(\frac{R_0 \sin \beta_1}{R_0 \cos \beta_1 - r}\right)$$

$$\alpha_2 = \tan^{-1}\left(\frac{r \sin \theta + R_0 \sin \beta_2}{R_0 cos \beta_2 - r \cos \theta}\right)$$

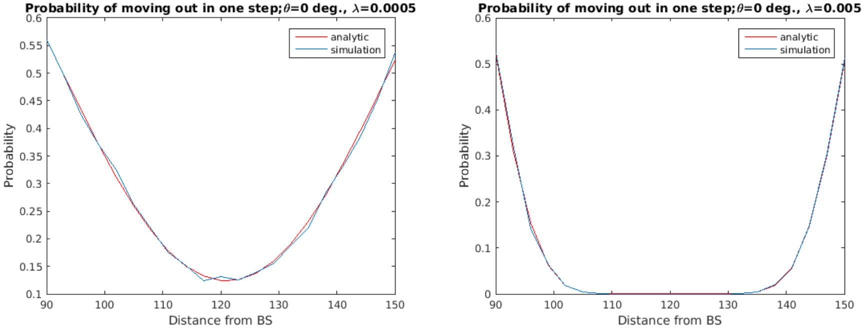
# Simulations

For all simulations, a 1000x1000 square is used to represent the whole 2-D plane. $R_0 = 150, R_2 = 0.4R_0$
To draw a length from Rayleigh distribution, the following method is used:

1. Draw a number N from Poisson distribution with density $\lambda A$ where $A$ is the area of the square.
2. Distribute these N points uniformly on the square
3. Of these N points, choose the point that is closest to the point under consideration.

This leads to Rayleigh distribution of transition length and can be proved using null probability of a Poisson Point Process.

# Simulations

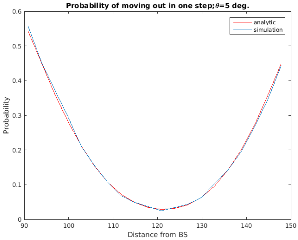



(a) $\lambda = 0.0005$

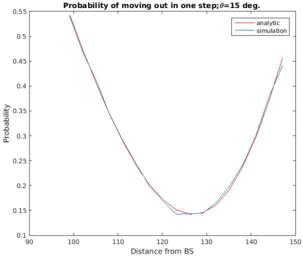
(b) $\lambda = 0.005$

Figure: $\rho(r, 0)$ vs. $\lambda$

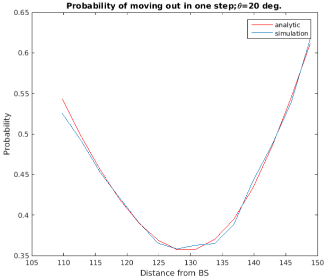
# Simulations




(a) $\theta = 5°$




(b) $\theta = 15°$



# E[N]

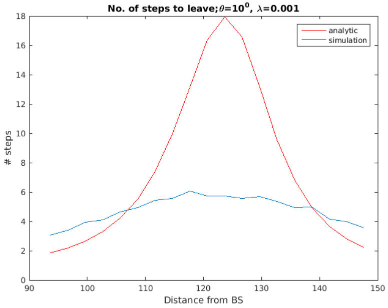
Let us make a gross approximation and use the same probability of leaving for all waypoints in the path. Then the average number of steps a node starting at $(r, \theta)$ takes to leave the region is given by

$$E[N] = \sum_{k=1}^{\infty} k(1 - \rho(r,\theta))^{k-1} \rho(r,\theta)$$

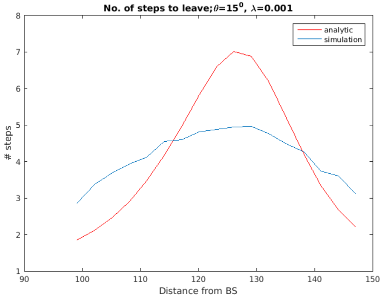
$$= \frac{1}{\rho(r,\theta)}$$

# E[N]

The plots are for points along the radius at angles $\theta = 10^\circ$ and $\theta = 15^\circ$. We can see that the analytical formula agrees better for narrower regions.




(d) $\theta = 10^\circ$




(e) $\theta = 15^\circ$

Figure: Expected number of transitions

# Ongoing Work

## Markov Chain Process

The transitions in this mobility model have Markovian property

$$f_{X_n/X_{n-1},X_{n-2},\ldots,X_0}(x_n/x_{n-1},x_{n-2},\ldots,x_0) = f_{X_n/X_{n-1}}(x_n/x_{n-1})$$

Where $X_{n-1}$ is the current waypoint and $X_n$ is the next waypoint.

# Ongoing Work

## Discretizing the State Space

- The state space of the Markov chain is continuous
- Discretise the space into n+1 states
  $n$ states lie inside the region of interest and the $n+1$th state represents space outside the region

# Ongoing Work

## Discretizing the State Space

- The state space of the Markov chain is continuous
- Discretise the space into n+1 states
  $n$ states lie inside the region of interest and the $n+1$th state represents space outside the region
- Model the motion as an Absorbing Markov Chain
  The transition probabilities among first $n$ states depend on the distances between the states and the transition probabilities from these $n$ states to the absorbing state is $\rho(r_i, \theta_i)$

The transition matrix is

$$P = \begin{pmatrix} Q & R \\ \mathbf{0} & 1 \end{pmatrix}$$

$Q_{n\times n}$ is the transition matrix of $n$ non-absorbing states
$R_{n\times 1}$ contains the probabilities of moving out in one step from each of those $n$ states.
Expected number of transitions $\mathbf{t}_{n\times 1}$

$$\mathbf{t} = F\mathbf{1}$$

and the variance vector $V$ is given by

$$V = (2F - I)\mathbf{t} - \mathbf{t}_{sq}$$

where $F = \sum_{k=0}^{\infty} Q^k = (I - Q)^{-1}$

# References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