Sojourn Time of Moving Relays in Dual-Hop Cooperative Communication

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Overview

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- Problem Formulation
- Simulations
- Further Work

Section 1

Introduction

Downlink Cooperation policy

A cooperation policy for downlink can be defined along the same lines as we did in uplink case. Let P_b be the transmission power used by the base station and P_r be relay power. A simple distance based policy would be

$$\{P_b r_1^{-\alpha} > P_b R_0^{-\alpha}, P_r r_2^{-\alpha} > P_b R_0^{-\alpha}\}$$

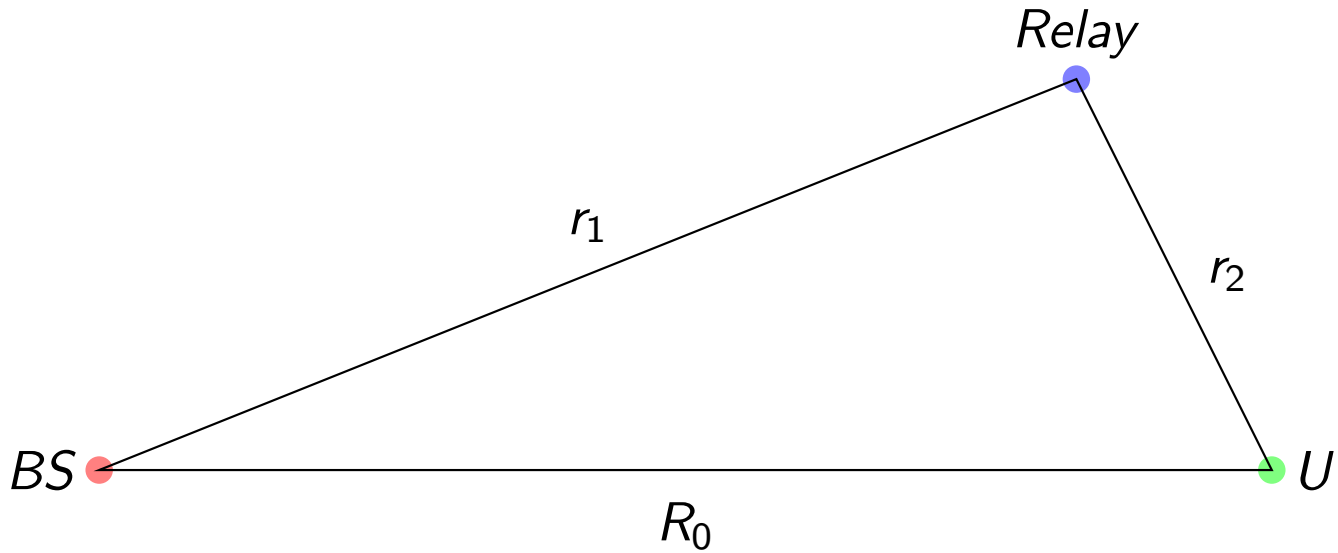


Figure: Distances for Downlink Cooperation

It can be rewritten as $\{r_1 < R_0, r_2 < R_2\}$ where $R_2 = cR_0, c = \left(\frac{P_r}{P_b}\right)^{\frac{1}{\alpha}}$ and r_1, r_2 are as defined in the figure 1. The two conditions in the policy ensure that both BS-relay and relay-user links are stronger than BS-user link. The policy dictates that the idle user should be located in the shaded part of figure 2 to be considered for relaying.

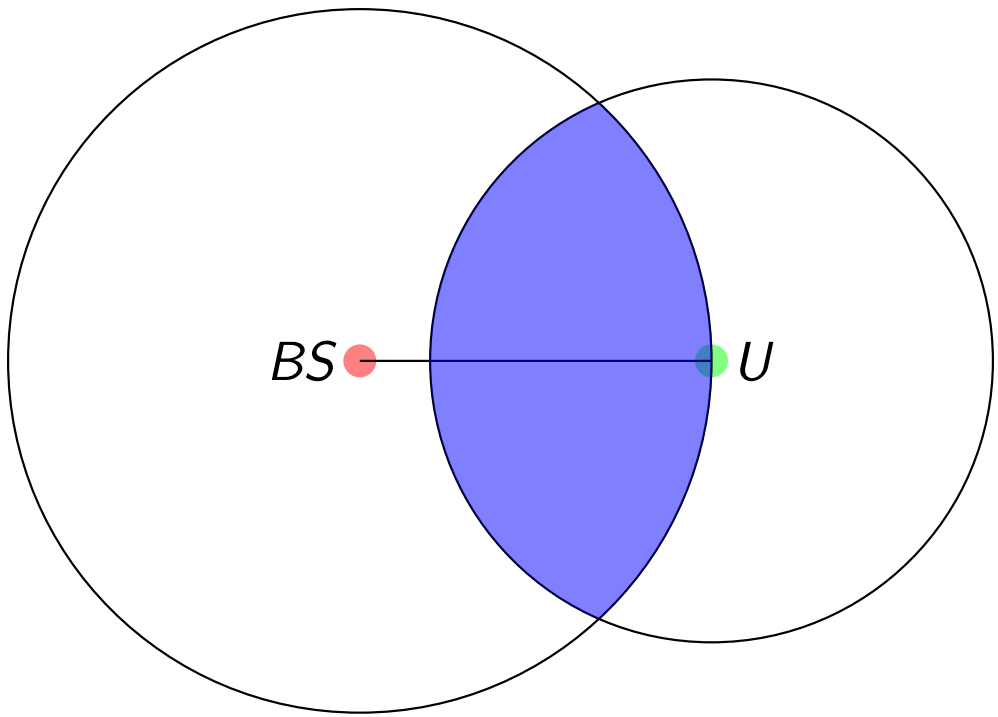


Figure: Feasible region

Mobile Relays

Definitions

We define mobility more formally and introduce the model proposed in [**?**]. The nth transition of a node can be denoted by the parameters set $(\mathbf{X}_{n-1}, \mathbf{X}_n, V_n, S_n)$. X_{n-1} denotes the starting waypoint and X_N denotes the destination. In addition to the transition time which can be obtained from velocity V_n, pause time or thinking time (S_n) at destination can also be included in the description of mobility. Different mobility models can be distinguished by the distribution of transition length$(L_n = \|\mathbf{X}_{n-1} - \mathbf{X}_n\|)$ and the distribution of angle made by the vector $\mathbf{X}_n - \mathbf{X}_{n-1}$ w.r.t x-axis.

Rayleigh RWP

In Rayleigh RWP, the angle is chosen uniformly from $[0, 2\pi]$ and the transition length is rayleigh distributed with parameter λ.

$$P(L > l) = exp(-\lambda\pi l^2), l \geq 0$$

We set $V_n \equiv v$ and S_n to 0 . What the above selection of distributions means is that when the node is at waypoint $\mathbf{X}_{n-1}$, a homogenous Poisson Point Process ϕ of intensity λ is generated and the nearest point in the set is chosen as the next waypoint $\mathbf{X}_n$. i.e., $\mathbf{X}_n = \arg\min_{x \in \phi} \|x - \mathbf{X}_{n-1}\|$. This can be proved from the null distribution of PPP. This gives better insight in to the role of parameter λ. Larger λ implies that the points are denser in generated PPP which in turn means the transition length is shorter. Figure **??** shows sample traces of rayleigh RWP for different λ. Please note that the figures are scaled differently.

Rayleigh RWP

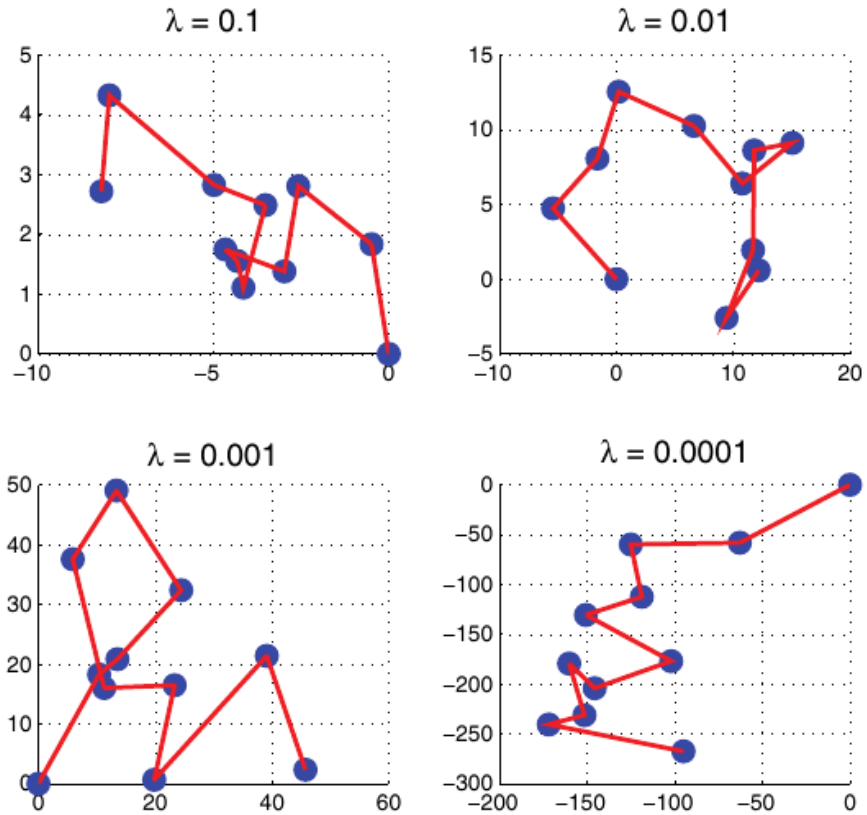


Figure: The transition lengths are statistically shorter with larger mobility parameter λ, and vice versa.[1]

[1] Image Credits: Xingqin Lin et al.

The mean transition length and time are as follows:

$$E[L] = \frac{1}{2\sqrt{\lambda}}$$

$$E[T] = \frac{1}{2v\sqrt{\lambda}}$$

Sojourn Time

Sojourn time is the amount of time a node resides in the region of interest. Calculating the mean sojourn time is challenging primarily because it involves finding node distribution during each transition. An expression for mean sojourn time of a cell user during one movement period starting from origin in a hexagonal cell was given in [**?**]. We have to note that a moving node usually makes more than one transition before it leaves the region. Also, starting from origin implies the node co-exists with BS at t=0 which is not representative of the distribution of relays/users. Even if we allow these two assumptions, the problem is still difficult to solve in this approach as the region has no definite shape like a polygon and finding integration limits is tedious.

Sojourn Time

If we know the expected number of transitions $E[N]$ a node makes before moving out of the region, then sojourn time can be given by

$$S_T = (E[N] - 1)E[T] + E[T_{last}]$$

where T_{last} is the time spent inside the region during the last transition. Since it is difficult to characterize T_{last}, we approximate S_T to $(E[N] - 1)E[T]$.

E[N]

$$E(N) = \sum_{k=1}^{\infty} kPr(r, \theta, k)$$

where

$$Pr(r, \theta, k) = \int_{S-A} \int_{A} \ldots \int_{A} f_{X_1/X_0}(x_1/x_0) \ldots f_{X_{k+1}/X_k}(x_{k+1}/x_k) dA_1 \ldots dA_{k+1}$$

is the probability that the node exits the region during $k+1$ th transition. $f_{X_n/X_{n-1}}(x_n/x_{n-1})$ is the probability density of the destination X_n given that the node's current position is X_{n-1}. $X_0 = (r, \theta)$ is where the node starts the movement at t=0, A is the feasibility region and S is the entire plane.

Leaving in One Transition

$$\rho(r,\theta) = Pr(-\alpha_2 < \alpha < \alpha_1, r_1 > r_{11}) + Pr(\alpha_1 < \alpha < 2\pi - \alpha_2, r_1 > r_{12})$$
$$= \int_{-\alpha_2}^{\alpha_1} \int_{r_{11}}^{\infty} f_{r_1,\alpha}(r_1,\alpha) dr_1 d\alpha + \int_{\alpha_1}^{2\pi-\alpha_2} \int_{r_{12}}^{\infty} f_{r_1,\alpha}(r_1,\alpha) dr_1 d\alpha$$

This is a general expresion that can be used for any mobility model. In case of RWP, r_1 and α are chosen independently. Therefore, $f_{r_1,\alpha}(r_1,\alpha) = f_{r_1}(r_1) f_\alpha(\alpha)$

$$\rho(r,\theta) = \int_{-\alpha_2}^{\alpha_1} f_\alpha(\alpha) \int_{r_{11}}^{\infty} f_{r_1}(r_1) dr_1 d\alpha + \int_{\alpha_1}^{2\pi-\alpha_2} f_\alpha(\alpha) \int_{r_{12}}^{\infty} f_{r_1}(r_1) dr_1 d\alpha$$
$$= \int_{-\alpha_2}^{\alpha_1} \frac{1}{2\pi} e^{-\lambda\pi r_{11}^2} d\alpha + \int_{\alpha_1}^{2\pi-\alpha_2} \frac{1}{2\pi} e^{-\lambda\pi r_{12}^2} d\alpha$$

Leaving in One Transition

The probability with which a node at (r, θ) moves out of the region of interest during the next transition is

$$\rho(r,\theta) = \int_{-\alpha_2}^{\alpha_1} \frac{1}{2\pi} e^{-\lambda\pi r_{11}^2}\, d\alpha + \int_{\alpha_1}^{2\pi-\alpha_2} \frac{1}{2\pi} e^{-\lambda\pi r_{12}^2}\, d\alpha$$

Where

$$r_{11} = -r\cos(\theta-\alpha) + \sqrt{R_0^2 - r^2\sin^2(\theta-\alpha)}$$

$$r_{12} = R_0\cos\alpha - r\cos(\theta-\alpha) + \sqrt{[R_0\cos\alpha - r\cos(\theta-\alpha)]^2 - r^2\sin^2\theta - [R_0 - r\cos\theta]^2 + R_2^2}$$

Leaving in One Transition

$$\beta_2 = \cos^{-1}\left(1 - \frac{R_2^2}{2R_0^2}\right) \quad \beta_1 = \beta_2 - \theta$$

$$\alpha_1 = \theta + \tan^{-1}\left(\frac{R_0 \sin\beta_1}{R_0 \cos\beta_1 - r}\right)$$

$$\alpha_2 = \tan^{-1}\left(\frac{r \sin\theta + R_0 \sin\beta_2}{R_0 cos\beta_2 - r\cos\theta}\right)$$

Simulations

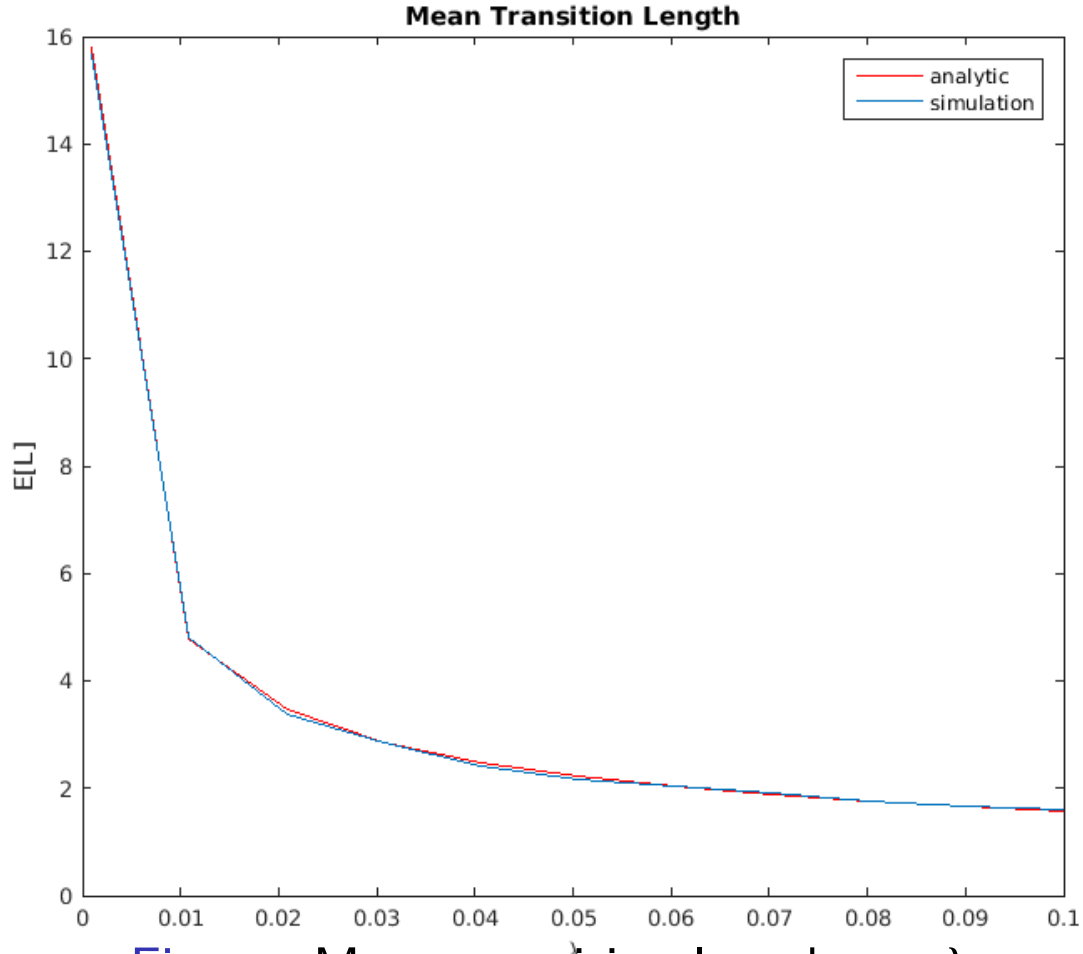
For all simulations, a 1000x1000 square is used to represent the whole 2-D plane. This size is good enough in the sense that using a larger square led to longer runtimes with little to no effect on the results. In the mobility model we used, transition length is Rayleigh distributed. To draw a length from Rayleigh distribution, the following method is used:

1. Draw a number N from Poisson distribution with density λA where A is the area of the square.
2. Distribute these N points uniformly on the square
3. Of these N points, choose the point that is closest to the point under consideration.

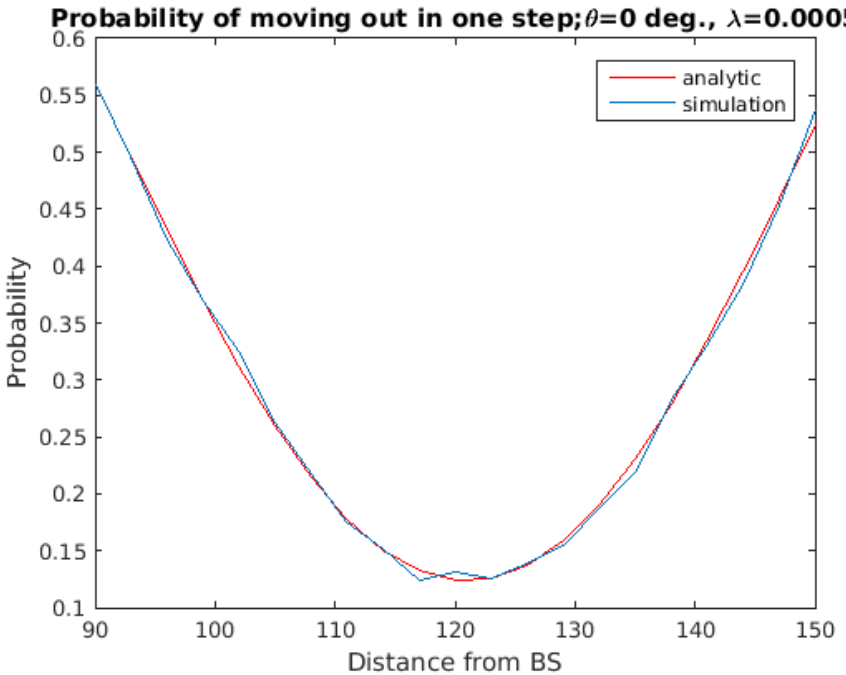
As discussed in [**?**], this leads to Rayleigh distribution of transition length and can be proved easily using null probability of a Poisson Point Process.

Simulations

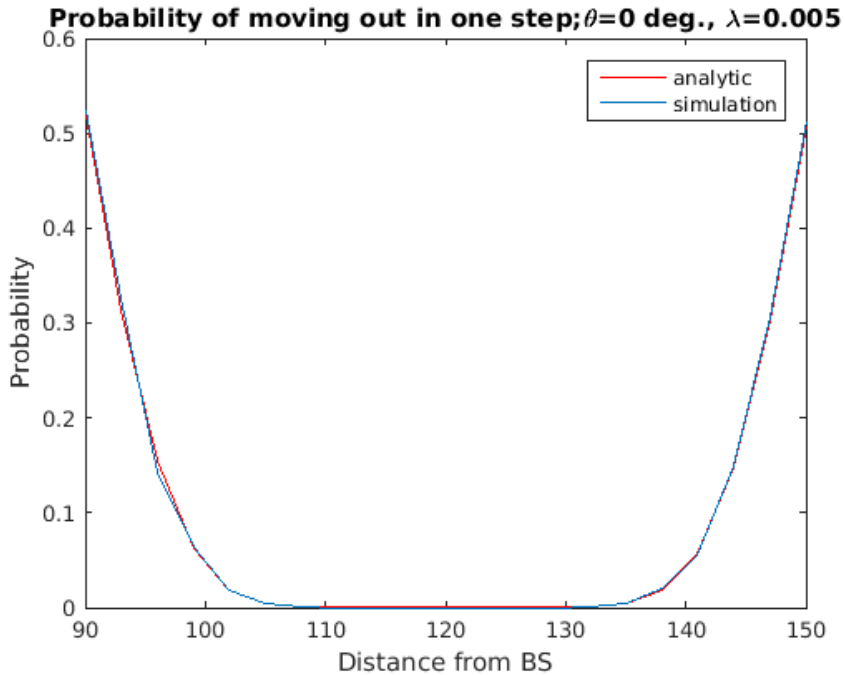
To test if the above mentioned method introduces any artefacts, let us see how simulated $E[L]$ fares with the formula $E[L] = \frac{1}{2\sqrt{\lambda}}$. Simulated expected length is calculated by averaging transition lengths of 100 traces in each of which the node makes 1000 transitions.



Simulations



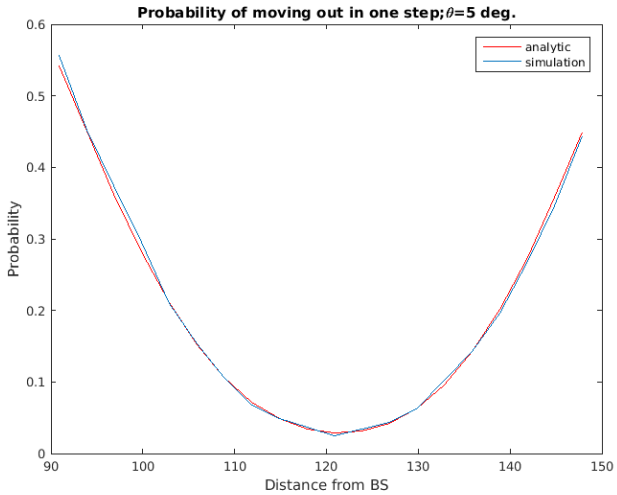
(a) $\lambda = 0.0005$



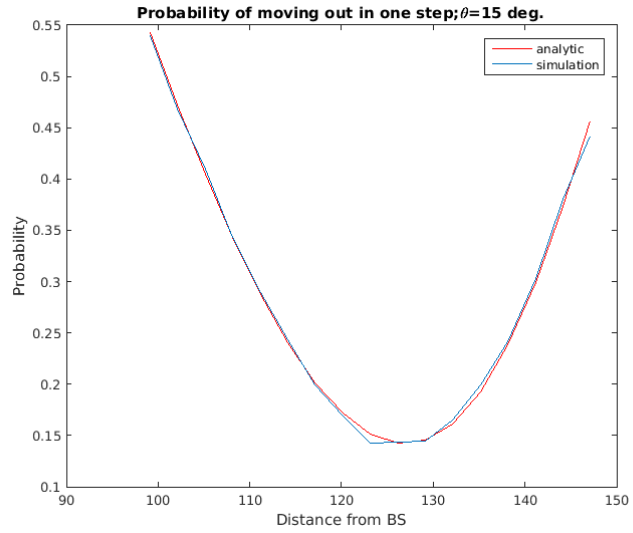
(b) $\lambda = 0.005$

Figure: $\rho(r, 0)$ vs. λ

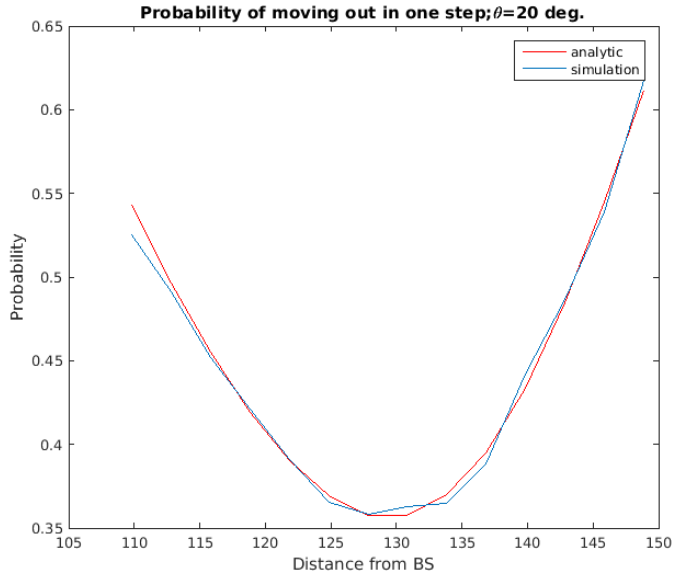
Simulations



(a) $\theta = 5^\circ$



(b) $\theta = 15^\circ$



E[N]

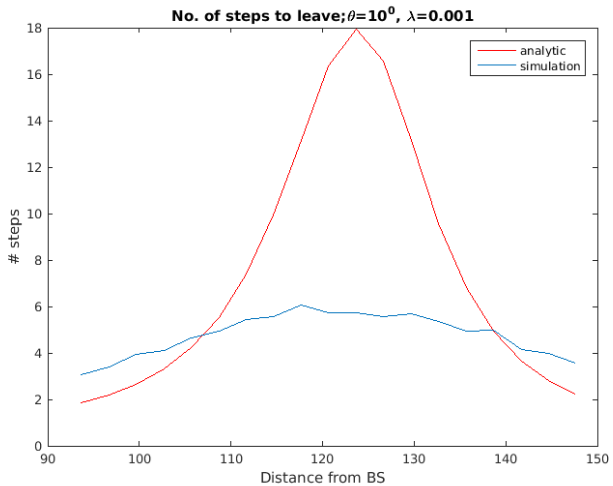
Let us make a gross approximation and use the same probability of leaving for all waypoints in the path. Then the average number of steps a node starting at (r, θ) takes to leave the region is given by

$$E[N] = \sum_{k=1}^{\infty} k(1 - \rho(r, \theta))^{k-1} \rho(r, \theta)$$

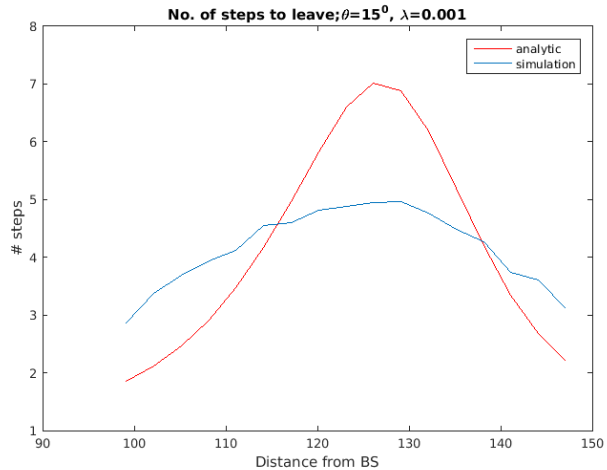
$$= \frac{1}{\rho(r, \theta)}$$

E[N]

The plots are for points along the radius at angles $\theta = 10^\circ$ and $\theta = 15^\circ$. We can see that the analytical formula agrees better for narrower regions. As discussed in Chapter 4, this can be improved by modelling the motion of a node as an absorbing Markov Chain and discretizing the state space.



(d) $\theta = 10^\circ$



(e) $\theta = 15^\circ$

Figure: Expected number of transitions

Ongoing Work

Markov Chain Process

The transitions in this mobility model have Markovian property in the sense that the next waypoint depends entirely on the current position.

$$f_{X_n/X_{n-1},X_{n-2},\ldots,X_0}(x_n/x_{n-1}, x_{n-2}, \ldots, x_0) = f_{X_n/X_{n-1}}(x_n/x_{n-1})$$

Where X_{n-1} is the current waypoint and X_n is the next waypoint.

Ongoing Work

Discretizing State Space

The idea is to discretize the state space of this Markov chain and model the motion as an Absorbing Markov Chain in which the node transitions among the non-absorbing states present inside the region before finally moving to the absorbing state. Let the whole space be represented by $n+1$ states of which n states lie inside the region of interest and the $n+1$th state represents the space outside the region. The transition probabilities among first n states depend on the distances between the nodes and the transition probabilities from these n states to the absorbing state is $\rho(r_i, \theta_i)$ where (r_i, θ_i) is the position of the ith state. We can then use the expressions given in [**?**] to find expected value and variance of the number of transitions a node makes before getting absorbed in the $(n+1)$th state.

The transition probability matrix of this Markov Chain is

$$P = \begin{pmatrix} Q & R \\ \mathbf{0} & 1 \end{pmatrix}$$

Where $Q_{n\times n}$ is the transition probability matrix of n non-absorbing states and $R_{n\times 1}$ contains the probabilities of moving out in one step from each of those n states. $\mathbf{t}_{n\times 1}$, the vector which contains expected number of transitions, is given by

$$\mathbf{t} = F\mathbf{1}$$

and the variance vector V is given by

$$V = (2F - I)\mathbf{t} - \mathbf{t}_{sq}$$

where $F = \sum_{k=0}^{\infty} Q^k = (I - Q)^{-1}$ is the fundamental matrix.

References

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