Power Allocation and Relay Selection in Amplify-and-Forward Relaying

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***Abstract*—Multihop communication is considered to be a standard in next generation cellular networks. There are several relaying schemes, Decode-and-forward(DF) and Amplify-and-Forward(AF) being the popular ones. In DF scheme, the relay decodes the message from the source, re-encodes and transmits it to the destination node whereas in AF the relay amplifies the received signal and transmits to the destination node. Relay selection and optimal power allocation are two important aspects in either scheme. In this work, we look at these two problems in 2-hop communication network in which relays employ AF scheme and show that power allocation and relay selection are interdependant and give conditions on channel conditions under which relays may be chosen irrespective of source power.**

I. Introduction

Cooperative communication is an efficient way to use network resources to increase rate and coverage in cellular networks. Currently, network provider places the relay in an optimum position so that the relay to base station link is always good. In next generation cellular networks, user-assisted relaying is being considered a standard. The idea is based on the fact that around a transmitting user there will be many idle user equipments which can be used as relays. That means the standard should allow users to send signals directly to nearby users without going through base station. There are many challenges that need to be solved to be able to do this correctly considering that virtually every user thinks this risks their privacy. New incentive mechanisms need to be developed to compensate the relaying users for the extra power spent. Apart from security and billing issues, there are also some engineering challenges like optimising power at both relay and source nodes, selecting the best relay etc.

Two popular relaying schemes are Decode-and-Forward and Amplify-and-Forward. In partial decode-and-forward scheme, the whole message is split in to a private part and a public part. In the first slot, the source node broadcasts the public part to relay node and destination node. The relay decodes the message and re-encodes it using an independent codebook and transmits to destination in the second slot. The source transmits the private part in second slot. The destination node receives public part from source in the first slot and in second slot it receives re-encoded public part from relay node and private part from source node. This scheme imposes severe restrictions on timing and synchronization at different stages of transmission. This is one of the reasons why amplify-and-forward scheme is considered more viable although rate is usually higher in DF scheme.

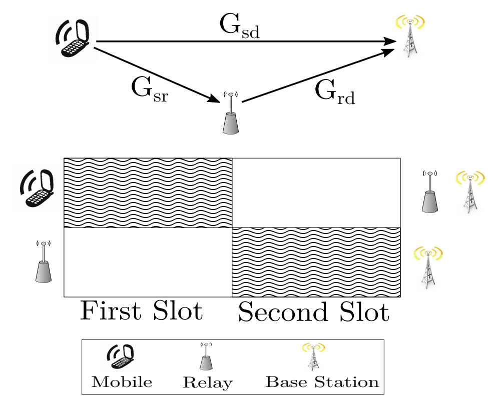

Fig. 1. Two slots in AF

In previous works, many relay selection schemes have been proposed but most of them are not based on SNR. For such schemes source power does not affect relay selection. In an SNR based relay selection policy, source power and relay power allocations affect relay selection while intrinsically depending on which relay is being used. However, this may not always be the case; under some channel conditions power allocation does not affect relay selection. In this work, we assume relay power is constant and provide an intuitive understanding of the conditions under which a relay switch over can take place and under what conditions the relay selection problem and power allocation problem are independent.

II. Amplify-and-Forward Relaying

Consider 2-hop relaying; we have a source node, the destination, and a relay node. In amplify-and-forward(AF) relaying scheme, the total transmission period is divided in to two slots. In the first slot, source node broadcasts the message to relay and destination nodes and in the second the relay amplifies the received signal and retransmits it to the destination node. This is illustrated in fig 1.

A. Received Signals

The signals received at relay and destination nodes in the two slots are as follows:

First Slot:

$$Y_{sd} = \sqrt{P_s G_{sd}} X_s + n_{sd} \tag{1}$$

$$Y_{sr} = \sqrt{P_s G_{sr}} X_s + n_{sr}$$

Second Slot:

$$Y_{rd} = \sqrt{P_r G_{rd}} X_{rd} + n_{rd} \quad (2)$$

$$Y_{rd} = \frac{\sqrt{P_r G_{rd} P_s G_{sr}}}{\sqrt{P_s G_{sr} + \sigma^2}} X_s + \frac{\sqrt{P_r G_{rd}}}{\sqrt{P_s G_{sr} + \sigma^2}} n_{sr} + n_{rd} \quad (3)$$

The final expression for Y_{rd} is obtained by substituting $X_{rd} = \frac{Y_{sr}}{|Y_{sr}|}$ in eq. 2. All symbols have usual meanings - s denotes source, P denotes power, $G_{sd}\big(= \dfrac{g_{sd}}{d_{sd}^2}\big)$ is the channel gain from source to destination, etc.

B. Rate

The rate/capacity of AF relaying scheme is given by

$$R = \frac{1}{2} w \log_2(1 + \Gamma_{sd} + \Gamma_{rd})$$

where Γ represents SNR

Substituting Γ_{sd} and Γ_{rd}, obtained from equations 1 and 3 respectively, in the above expression we get

$$R = \frac{1}{2} w \log_2 \left(1 + \frac{P_s G_{sd}}{\sigma^2} + \frac{P_s G_{sr} P_r G_{rd}}{\sigma^2(\sigma^2 + P_s G_{sr} + P_r G_{rd})} \right)$$

III. Source power and Relay selection

When there are multiple relays we have to select one of them for transmission. There are various relay selection techniques one of which is Max-Min selection- relay i which maximizes min$\{G_{sr_i}, G_{r_i d}\}$ is selected - and a continuous version of this scheme which uses harmonic mean of $G_{sr_i}, G_{r_i d}$ instead of *min*. However, these are not optimal schemes in the sense that the resulting relay may not give the highest possible rate. An optimal scheme will select a relay which gives maximum rate i.e., relay k is selected where

$$k = \arg\max_i \Gamma_{r_i d} \quad (4)$$

Since Γ_{sd} is same for all relays, this scheme selects the optimal relay.

Now that we've selected a relay for transmission the next step is to optimise the source power P_s. But changing P_s changes Γ_{rd} therefore it may so happen that different relays are optimal at different source powers. To see if this scenario is possible, consider two relays R_1 and R_2 and assume both use same constant power P_r. Γ_{rd} can be rewritten as

$$\Gamma(P_s) = \frac{P_s ab}{1 + P_s a + b} \quad (5)$$

where $a = \frac{G_{sr}}{\sigma^2}$ and $b = \frac{P_r G_{rd}}{\sigma^2}$ Now the question can be reframed as: if at some power P_1, R_1 is a better relay than R_2 i.e.,$\Gamma_1(P_1) > \Gamma_2(P_1)$, then can R_2 be a better relay than R_1 for some other power P_2. To answer this let us find the power at which both relays are equally good.

$$\Gamma_1(P_0) = \Gamma_2(P_0)$$

Solving the above equation, we get

$$P_0 = (1 + b_1)(1 + b_2) \frac{\frac{a_1 b_1}{1+b_1} - \frac{a_2 b_2}{1+b_2}}{a_1 a_2 (b_2 - b_1)} \quad (6)$$

For P_0 to be positive, both numerator and denominator should have same sign i.e., if $\frac{a_1 b_1}{1+b_1} > \frac{a_2 b_2}{1+b_2}$ then $b_2 > b_1$. To explain this intuitively, let us assume b_1, b_2 to be much larger than 1 which reduces the first inequality to $a_1 > a_2$. What this means is, source to relay channel is better for R_1 but relay to destination channel is stronger for R_2. Hence at low source powers R_1 gives better SNR but for source power greater than P_0, R_2 is a better relay than R_1. Same argument can be made for the case where inequalities are in the opposite direction.
For $P_0 < 0$, one of the relays is the desired one irrespective of source power.
To summarise, here are the conditions under which one relay is better than the other:

- $\frac{a_1 b_1}{1+b_1} > \frac{a_2 b_2}{1+b_2}$ and $b_2 > b_1$
  R_1 at source power less than P_0 and R_2 at power greater than P_0
- $\frac{a_1 b_1}{1+b_1} < \frac{a_2 b_2}{1+b_2}$ and $b_2 < b_1$
  R_2 at source power $< P_0$ and R_1 at power $> P_0$
- $\frac{a_1 b_1}{1+b_1} < \frac{a_2 b_2}{1+b_2}$ and $b_2 > b_1$
  R_2 is a better relay for all source powers
- $\frac{a_1 b_1}{1+b_1} > \frac{a_2 b_2}{1+b_2}$ and $b_2 < b_1$
  R_1 is a better relay for all source powers

Finally the algorithm to use when there are two relays is:

- Assume some initial source power and select the best relay at that power and optimise the source power
- Check if the current relay is still the best relay at this source power. If it is not, use the other relay and optimise the power

IV. Future Work

- Proving convergence of the above algorithm and finding out whether the initial assumed source power has any affect on the convergence.
- When the number of relays increases, this becomes more complex. A good way to overcome this is by finding a centralized way to optimise source power and selecting a relay i.e., given the channel gains of all relay links, the source node should be able to find the transmission power and the relay.
- Although source power optimisation outperforms relay power optimisation it is still important to consider the case when relay power is varying particularily when the relays are user equipments.

V. Conclusion

In this work, we have shown that relay selection and source power control problems are not independant and explained briefly under what conditions they are independent. We also proposed an algorithm that can be used to find the best relay while optimising source power when there are multiple relays.

REFERENCES

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