Amplify-and-Forward

The transmission has two slots. In th first, source node broadcasts the signal to relay and destinatio and in the senconf the realy amplifies the received signal and retransmits it to the destination node.

First Slot:

$$Y_{sd} = \sqrt{P_s G_{sd}} X_s + n_{sd} Y_{sr} = \sqrt{P_s G_{sr}} X_s + n_{sr} \tag{1}$$

Second Slot:

$$Y_{rd} = \sqrt{P_r G_{rd}} X_{rd} + n_{rd} X_{rd} = \frac{Y_{sr}}{|Y_{sr}|} \tag{2}$$

$$Y_{rd} = \frac{\sqrt{P_r G_{rd} P_s G_{sr}}}{\sqrt{P_s G_{rd} + \sigma^2}} X_s + \frac{\sqrt{P_r G_{rd}}}{\sqrt{P_s G_{rd} + \sigma^2}} n_{sr} + n_{rd} \tag{3}$$

Rate expression:

$$R = \frac{1}{2} w log_2(1 + \Gamma_{sd} + \Gamma_{srd}) \quad where \Gamma represents SNR \tag{4}$$

$$R = \frac{1}{2} w log_2(1 + \frac{P_s g_{sd}}{\sigma^2} + \frac{P_s g_{sr} P_r g_{rd}}{\sigma^2(\sigma^2 + P_s g_{sr} + P_r g_{rd})}) \tag{5}$$

Assume there are two relays and let $a_1 = \frac{g_{sr_1}}{\sigma^2}$ and $b_1 = \frac{P_r g_{r_1 d}}{\sigma^2}$ similarily a_2, b_2 for relay 2. At a particular source power P_s, relay 1 is chosen over relay 2 if $\frac{a_1 b_1}{1+P_s a_1 + b_1} > \frac{a_2 b_2}{1+P_s a_2 + b_2}$

Consider the function $f(p) = \frac{pab}{1+pa+b}$

$$f'(p) = \frac{(1+b)ab}{(1+pa+b)^2} \tag{6}$$

$f'(p)$ is positive and decreasing with p.

The power at which both the relays give same rate can be obtained by equating $f_1(p)$ and $f_2(p)$.

$$P_0 = (1+b_1)(1+b_2)\frac{\frac{a_1 b_1}{1+b_1} - \frac{a_2 b_2}{1+b_2}}{a_1 a_2 (b_2 - b_1)} \tag{7}$$

For P_0 to be positive, both numerator and denominator should have same sign i.e., if $\frac{a_1 b_1}{1+b_1} > \frac{a_2 b_2}{1+b_2}$ then $b_2 > b_1$. To explain this intuitively, let us assume b_0, b_2 to be much larger than 1 which reduces the first inequialitely ti $a_1 > a_2$. What this means is, source to relay cahennel is better for relay 1 but relay to destination channel is stronger for relay 2. Hence at low source powers relay 1 might givr better SNR but if we increase source power past P_0 relay 2 gives more rate than relay 1. Same arguments can be made for the case where inequalities are in the opposite direction. For $P_0 < 0$, one of the rel;ays is the desired one irrespec tve of source pwer.