Fourier Transform of sine - Gaussian

Define Functions

In[1]:= $\text{sineGaussian[t\_]} := A * e^{-\Gamma * t^2} * \text{Sin}[2 * \pi * f_c * t]$

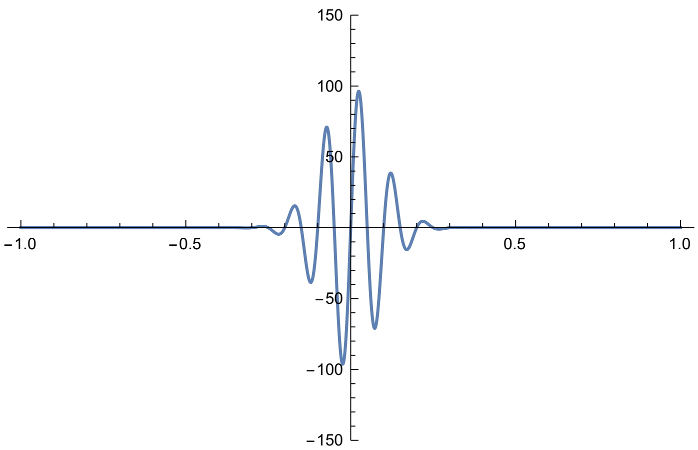
In[3]:= $\text{FT[f\_]} := \text{Abs}\left[\frac{1}{\sqrt{T}} \int_0^T \left(\text{sineGaussian[t]} * e^{2*\pi*f*i*t}\right) dt\right]$

Define constants

In[4]:= $T = 1;\ f_c = 10;\ Q = 1;\ \Gamma = \frac{2 * \pi * f_c}{Q};\ A = 100;$

In[5]:= `Plot[sineGaussian[t], {t, -1, 1}, PlotRange → {-150, 150}]`

Out[5]=



If I copy the output of typing FT[f] and turn it into a new function, and plot the new function, it is significantly faster than just plotting FT[g] - this takes much longer.

In[6]:= `FT[f]`

Out[6]= $$\frac{5}{2}\sqrt{5}\, e^{-\frac{1}{20}\pi\,\text{Re}\left[(10+f)^2\right]}\,\text{Abs}\left[e^{2 f \pi}\left(\text{Erfi}\left[\frac{1}{2}(-10+f)\sqrt{\frac{\pi}{5}}\right] - \text{Erfi}\left[\frac{1}{2}((-10+20\,i)+f)\sqrt{\frac{\pi}{5}}\right]\right) - \text{Erfi}\left[\frac{1}{2}(10+f)\sqrt{\frac{\pi}{5}}\right] + \text{Erfi}\left[\frac{1}{2}((10+20\,i)+f)\sqrt{\frac{\pi}{5}}\right]\right]$$

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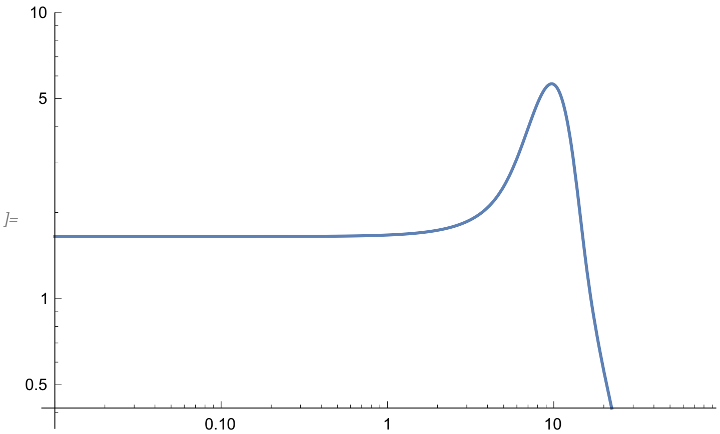
LogLogPlot[FT[g], {g, 0.01, 100}, PlotRange → {0, 10}] (*Much Slower!*)
```

Out[•]=



In[7]:= `new[f_] :=`

$$\frac{5}{2}\sqrt{5}\, e^{-\frac{1}{20}\pi\,\mathrm{Re}\left[(10+f)^2\right]}\,\mathrm{Abs}\left[e^{2\,f\,\pi}\left(\mathrm{Erfi}\left[\frac{1}{2}(-10+f)\sqrt{\frac{\pi}{5}}\right]-\mathrm{Erfi}\left[\frac{1}{2}((-10+20\,i)+f)\sqrt{\frac{\pi}{5}}\right]\right)-\mathrm{Erfi}\left[\frac{1}{2}(10+f)\sqrt{\frac{\pi}{5}}\right]+\mathrm{Erfi}\left[\frac{1}{2}((10+20\,i)+f)\sqrt{\frac{\pi}{5}}\right]\right]$$

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LogLogPlot[new[f], {f, 0.01, 100}, PlotRange → {0, 10}] (*Fast*)
```

Out[8]=