**Contents:**

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**Inequalities:**

> | a + b | <= |a| + |b|

> | a – b | >= |a| - |b|

> A.M. >= G.M. i.e. (a+b)/2 > sqrt(ab)

> a/b + b/a = 2 if both ‘a’ and ‘b’ are of same sign

> a2 + b2 + c2 >= a\*b + a\*c + b\*c

> (a + b)\*(b + c)\*(a + c) >= 8\*a\*b\*c if a>= 0, b >= 0, c >= 0

> For any 4 numbers:

If x12 + x22 =1 and y12 + y22 = 1. Then, |x1\*y1 + x2\*y2 | <= 1

> if a + b = 2, then a4 + b4 = 2

> |x| <= a, then -a <= x <= a for a > 0

> x > x4 only when x is between 0 and 1

> 2n > n2 for n >= 5

> We can subtract one inequality from another if they both faces the same direction. Hence, given x > y and a > b, we can’t conclude that (x-a) > (y-b)

> In “could be true” questions, any inequality which can satisfy a single equation will be included in the solution.

> try to combine two inequalities and get the solution algebraically, if possible.

> Check the following ranges while plugging in numbers:

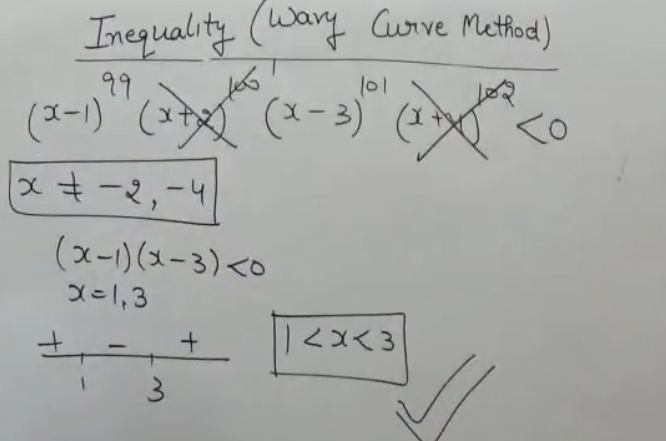
> 0 , < 0 , 0 , between 0 and 1, between 0 and -1

> **We can raise both parts of an inequality to an even power if we know that both parts of an inequality are non-negative (the same for taking an even root of both sides of an inequality**

> In case of linear equations with power being even, ignore them. For example, in (x+1)\*((x+3)^2)\*(x+5) (x+3) can be ignored.

**Wavy curve method to solve equations:**

Ignore the terms with exponents as even. While retaining the equations whose exponents are odd, draw them on number line. Draw a wave connecting the numbers on that number line. the alternate segment will be +ve and –ve on the line.



* Only multiply equalities when both the sides of inequality is +ve

Square root of x2 = |x|

When DS asks for a combination of variables, don't solve for each variable, just extract out the value of equation.

When a value of a variable is sought and you are getting 2 values, then the answer of the problem would be no solution even after combination of the 2 statements

\* In a question, when you are asked to divide or multiply by an integer in an equation, don’t do that blindly unless you know that you are going to divide or multiply only a positive number.

\* Take care of 0 while solving equations.

\* When equation provided in stem has modulus operator, you can interpret range of some values already i.e some are >0 because of that.

In DS questions for Inequality, it might happen that you have to check -ve and +ve sign of variables of given equation to arrive at a final answer.

* If you find difficult to simplify an expression then simply plug-in values of random numbers and you can check the result with all the answer choices you've got
* If the exponent is between 0 and 1 the number > than the original proper fraction

The retailer has less than twice as many radios as clocks in inventory.

r < 2\*c

Absolute equation both sides, consider only 2 cases ++ and +-

**Algebra:**

To solve quadratic equation without using formula : x2 + b\*x + c

Find the numbers, the sum of whom is -b and product is c

For example:

√(b2 – 4\*a\*c) is called the discriminant in the equation. If it is 0, equation has 1 solution, if > 0 then it gives 2 solution else equation has no solution.

A quadratic equation reaches its minimum when x = (-b)/2\*a

*For function problems*, pick number and try out equations

when you do not know the sign of numbers, you cannot take reciprocals

if x < y then 1/x > 1/y either if both are +ve or both are -ve

Do not flip sign when one is +ve and other one is -ve

Taking squares:

if both sides positive raise the square and sign will remain the same

if both side -ve flip the sign.

If not sure whether number could be +ve or -ve, you cannot do squares

Square of odd is odd and even is even

=========== Ratio and Proportion ============

> If a/b = c/d = e/f, then (a+c+e)/(b+d+f)

> To calculate % change in 2 ratios, find % change in numerator and % change in denominator(This needs to be reverse calculated) and then use 100 as reference applying these % changes to get the result.

> If a > b then (a+k)/(b+k) < a/b and (a-k)/(b-k) > a/b. Similarly, if a < b, aforementioned equalities will be reversed.

> 2 similar products are sold. One at x% loss and other at x% profit. The result is loss of [x/10]^2%

> When amount left and amount sold is given with a given profit, then profit is (goods\_sold/goods\_left)\*100

> To find maxima of a quadratic equation, differentiate it and equate the differentiation to 0

=========== Time, Speed and Distance ========

> To convert a value from km/h to m/s, multiply the number by 5/18 and to do vice-versa, multiply by 18/5

> Use proportionality as the key idea to solve a problem based on T,S&D when one of the three things is constant

> To solve questions based on clocks, consider speed of minute clock as 60km/h while speed of hour hand as 5km/h and use relative speed of 55km/h to find the solution.

================= Combinatorics ==========

In problems where it is required to insert things in odd positions use the below depiction

- | - | - | - | -

e.g - 4 boys, 3 girls arrange with no girls together - 4! \* 5\*4\*3\*2\*1

Formula to find rank of a word in dictionary

Write the word and put its relevant alphabetical rank above it(which is relative to the given alphabets)

For example for "RANDOM" it will be - R(6), A(1), N(4), D(2), O(5), M(3)

for each word write the numbers of integers - smaller than that integer on the right - below that number.

For RANDOM it will be : 5, 0, 2, 0, 1, 0

The rank would be 1 + n!\*first\_number + (n-1)!\*second\_number + ...

For "RANDOM" - 1 + 5\*5! + 0\*4! + 2\*3! + 0\*2! + 1\*1! + 0\*0! = 1+600+12+1 = 614

Number of lines that can be drawn between set of n points = nC2

Number of triangles that can be drawn between set of n points = nC3

Number of circular arrangements for n things = (n - 1)!

Desc - Fix the position of 1 person and find the max permutation of left out people

If there is a question asking about the number possible with 2 conditions in that case consider individual case of one condition combination

Total number of selections of 0 or more things out of ‘n’ different things:

nC0 + nC1 + nC2 + … + nCn = 2^n

Number of ways dividing (mn) things into m groups such that each group ‘n’ objects are:

(mn)!/(n^m)\*m!

> Number of ways of distributing ‘n’ identical things to ‘r’ persons in which each person can get any number of things: (n + r – 1)C(r – 1)

> Number of ways of distributing ‘n’ identical things to r distinct groups: (n-1)C(r-1)

> For nCr to be greatest, if ‘n’ is even, r = n/2 else r = (n+1)/2 or (n-1)/2.

> Number of selections of ‘k’ consecutive things out of ‘n’ items in a row : (n-k+1)

> Number of circular arrangements when clockwise and anti-clockwise are not considered different:

(n-1)!/2

> Number of terms in (a1 + a2 + a3 + an )^m is (m+n-1)C(n-1)

> Number of squares in a square of size N X N  N \* (N + 1) \* (2\*N + 1)/6 and similarly

number of rectangles(including number of squares)  N^2 \*( N + 1 )^2/4

**Principle of Symmetry:**

Let’s take 6 alphabets A to F. The number of combinations possible are 6! Among these combinations, the combination in which X is left to Y is same as number of combination in which Y is left to X.

Example:

**Question 7: 7 people (A, B, C, D, E, F and G) go to a movie and sit next to each other in 7 adjacent seats in the front row of the theatre. A will not sit to the left of F and B will not sit to the left of E. How many different arrangements are possible?**  
  
**Solution:** Number of ways of arranging 7 people in 7 seats is 7! (using Basic Counting Principle)  
  
Of these 7! arrangements, we want those arrangements in which A is sitting to the right of F and B is sitting to the right of E. A will sit to the right of F in half of the 7! arrangements. Of these 7!/2 arrangements, half will have B to the right of E and other half will have B to the left of E. So the number of arrangements in which A is to the right of F and B is to the right of E is (7!/2)/2 = 7!/4

**Regarding distribution of items in a group**

When the question says that you need to make n groups/bundles/teams that are not distinct, you need to divide by (n!). If the groups/bundles/teams are distinct then you do not divide by (n!).

For Example:

**In how many ways can one divide 12 different chocolate bars into four stacks of 3 bars each?**  
  
Here the 4 stacks are not distinct. You need to divide the answer you obtained above by 4! (similar to the simple example with just 4 chocolates we saw above).  
  
In this case, the required number of ways = 12!/(3!\*3!\*3!\*3!\*4!)

Sometimes in DS question based on combinatorics, we get 2 different values of 2 variables but still result is the same.

For Example, in the below question statement (A) is sufficient

**Question 1: There are x children and y chairs in a room where x and y are prime numbers. In how many ways can the x children be seated in the y chairs (assuming that each chair can seat exactly one child)?**  
  
Statement 1: x + y = 12  
Statement 2: There are more chairs than children.

This is because combinatorics does not distinguish between people and things. 7 children on 5 seats is the same as 5 children on 7 seats because in each case you have to select 5 out of 7 (either seats or children) and then arrange 5 children in 5! ways.

**FRACTIONS/DECIMALS/PERCENTAGES:**

Percentages:

\* If there is increment of 25% in value for a given commodity over a period of n years, the resultant value will be :

(original\_value)\* (1.25)^n

\* Similarly, in case where value if being decremented, say by 20% use the below reverse addition form

(original\_value) \* (0.8)^n

\* Use the above method to solve questions like: A commodity price first incremented by 25%, then inc. by 40% and

finally decremented by 20%

The solution will be like : (original\_value)\*(1.25)\*(1.4)\*(0.8)

\* F = P ( 1 + r/n)^nt, where

F = Final value, n = No. of compounding periods per year, t = No. of years

> We can have x% of negative number too

**Fractions:**

\* Incrementing the value of numerator and denominator by constant amount takes its value closer to 1 and vice versa.

\* Formula to find average speed - (2\*a\*b)/(a+b)

\* Rational number can be represented as a/b which can be written as terminating or repeating decimal. The numbers which cannot be represented in this way are called irrational numbers. such as pi, sq(2). There is no overlap between these sets. rational + irrational = real numbers

\* There are 25 primes between 1 and 100

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97

\* If n is a positive integer there is always a prime number p : n < p < 2n

\* If a number equals the sum of its proper divisors, it is said to be a perfect number. 6 = 1 + 2 + 3

\* if a is a factor of b and c, then a is a factor of (b+c)

\* A fraction will always yield a terminating decimal as long as the denominator has only 2 and 5 as its prime factors

\* In questions asking whether number can be limited to finite decimals check for '7' and '3' in denominator which should be eliminated to make this happen.

\* Fraction gets smaller when an integer is subtracted from numerator and denominator of the fraction.

**Decimals:**

*Rules for Rounding Off:*

**Rule I** — When the figure next beyond the last figure or place to be retained is less than 5, the figure in the last place retained shall be left unchanged.

**Rule II** — When the figure next beyond the last figure or place to be retained is more than 5 or is 5 followed by any figures other than zeros, the figure in the last place retained shall be increased by 1. --> 13.455 --> 13.5

**Rule III** — When the figure next beyond the last figure or place to be retained is 5 alone or 5 followed by zeros only, the figure in the last place retained shall be (a) increased by 1 if it is odd and (b) left unchanged if even (zero would be regarded as an even number for this purpose).

13.45 -> 13.4, 13.75 -> 13.8

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**To find solutions to problem of equal installment over a period of time:**

Louie must repay the loan in three equal monthly payments. To the nearest dollar, how much does Louie have to pay each month

Let X be the amount paid after one installment. Then after 1 month, amount needs to be repaid is 1.p\*Principal-X. Keep multiplying with 1.p and subtracting X till last installment when it will be X which is left.

**Solution to equation of type of percentage tagging:**

2 categories. Some items of cat1 marked for cat2 and some items of cat2 marked for cat1, use matrix method.

**Word Problems:**

Algebraic translations:

Using equations

Rates and Work

Make RT = D pattern for rate/distance problems.

For average speed calculation and all take distance of 100

Average speed of a person that walked to/ from for a distance with speeds x and y is 2\*x\*y/(x+y)

When dealing with tasks, take the task as 1 unit

\* Sometimes it is given that additional x% discount is given after $y, in that case consider only discount of x% not the discount % given if value bought/sold is below y

**Set:**

\* For all evenly spaced set, the average equals - (first+last)/2

\* The sum of that set is (average)\*(number\_of\_items)

\* If a given median is not integer then there are even number of elements for sure.

\* The product of k elements is always divisible by k!

\* x! is divisible by all integers up to and including x

\* For any consecutive set of odd number of integers, the sum is always divisible by number of elements.

\* Same is not applicable for set of even number of consecutive integers.

\* The probability of selecting number of elements in ascending order from a given set is 1/(n!) since n!

Represents the number of ways the elements can be arranged.

\*  The number of factors of perfect square is odd and all other positive integers have even number of factors.

\* Weighted average is like giving unequal precedence to the numbers in the set, example:

m\*(a/(a+b)) + n\*(b/(a+b)) = x

\* For evenly spaced numbers mean and median are same and that set is in arithmetic series.

\* 1 unit of standard deviation from mean: mean-s\_d , mean+s\_d

\* Alligation and Mixture : use cross formula

A1 A2

\ /

A

/ \

N1 N2

======================== Number Properties =======================

* If a number is not a prime number then it is a composite number.
* All prime numbers are of the form: 6\*n-1 and 6\*n+1
* If 'n' is a positive number then there is always a prime 'p' such that n < p < 2\*n
* Number of factors of an integer : 300 - 2^2 \* 5^2 \* 3 --> (2+1)\*(2+1)\*(1+1) = 18
* To find HCF - Factorize the number and take out the number with common powers
* To Find LCM - Factorize and pick the maximum power of the prime numbers (factors)
* integer - |integer| = remainder/divisor
* To solve divisibility/remainder kind of question write the number as:
* Quotient \* Divisor + Remainder
* The number of distinct factors and sum of factors of a perfect square are always odd
* Take **‘0’, negatives and fractional numbers** into consideration as well while solving DS questions.
* Sum of first positive odd numbers – n^2
* Sum of first +ve even numbers – n\*(n+1)
* A fraction will be non-terminating only if denominator is of the form 2^m\*5^n. In question while determining this, do reduce the fraction to prime factors
* ‘1’ is a perfect square and perfect cube and also a factor of all the numbers.
* To find a number which is perfect square, Divide that number into factors and check if count of those factors is divisible by 2.
* H.C.F of (a^m-1) and (a^n-1) is given by (a^(H.C.F of m and n) -1 )
* n!+1 cannot be divisible by any number less than or equal to n
* Remainder is always non-negative. So, -32%7 will be 7\*(-5) + 3 is correct and 7\*(-4) – 4 is not
* if ‘m’ and ‘n’ are divisible by ‘d’ independently, then (m+n) and (m-n) are also divisible by ‘d’
* If ‘a’ is divisible by ‘b’ and ‘c’ by ‘d’, then ‘ac’ is divisible by ‘bd’
* Highest power of a factor ‘p’ that is contained in a number N : [N/p] + [n/p^2] + [N/p^3] …
* Sum of factors of a number : Let N = \*, Then their sum is : (x^0+x^1+x^2+…x^n)\*(y^0+y^1+y^2…+y^n)\*(z^0+z^1+z^2+…+z^n)
* Number of factors for the above equation = (a+1)\*(b+1)\*(c+1)
* Number of even factors = (2^1+2^2+2^3…) we have removed 2^0 for including only even numbers. So, formula will be (a)\*(b+1)\*(c+1)
* Number of odd number of factors = (1)\*(b+1)\*(c+1) wherein we have removed all factors of 2
* If p is a prime number then (p^2-1) will be divisible by 24.
* Process to convert a rational number with pure recurring decimal: For x.<digit\_1, digit\_2…digit\_n><digit\_1, digit\_2…digit\_n>… = x + n/99…n. So, for x.yyyy = x+y/9 and for x.yzyzyzyz = x + (Y\*z)/99
* For value between 0 and 1, squaring the number will reduce the value of number.
* Formula for finding H.C.F and L.C.M of a given rational number
* H.C.F of fraction = H.C.F. of numerator / L.C.M. of denominator
* L.C.M. of fraction = L.C.M. of numerator / H.C.F. of denominator
* If H.C.F. of ‘x’ and ‘y’ is ‘g’ then H.C.F. of (x, (x+y)) , (x, (x-y)), (x+y, x-y) is also g
* A composite number is a number which has at least one divisor apart from ‘1’ and itself.
* Two consecutive integers are co-prime, which means that they don't share ANY common factor but 1
* Divisor means factor which divides the dividend.
* 0 is not a multiple of any integer.
* The number which is a factor and multiple of a number is the number itself
* 1 is not a prime number because a prime number has exactly 2 factors.
* If a number 'x' is divisible by 'y' and 'y' is divisible by 'z', then 'x' is also divisible by 'z'
* To solve questions like if 'n' is divisible by 'x' and 'y'. Is 'n' divisible by 'z'. To solve this question, create a box (factor box) and store all the max encountered prime factors of 'x' and 'y'. Now, divide 'z' into its prime factors and check if the count of prime factors in box satisfies the one we generated after factorization of 'z'. e.g. for numbers 10 and 12 we will store 2 only twice (as 12 contributes 2 2's and 10 1)
* '0' is an even integer.
* To solve questions involving numbers raised to power fractions. Take the LCM of denominator of both fractions raised to power.
* There are 4 primes each between 1-10(2, 3, 5, 7) and 11-20(11, 13, 17, 19) while 2 each in 21-30(23, 29) and 31-40(31, 37) and 3 in range 41-50 (41, 43, 47)
* Number Properties:
* To solve questions of type. If x is divided by y , 'remain' is the remainder and if x is divided by y, values is val. Then represent the equation as:

x = q\*y + r

* or x = val\*y => Divided val in q and r to get: (q)\*y + r\*y => equate r with given remainder 'remain'
* Don't factor an equation unless it is equated to 0.
* if a and b are multiples of k and are k units apart from each other, then k is greatest common divisor of aa and bb.
* When need to take consecutive numbers in AM/GM do use prior options too as in : consider a-d, a, a+d rather than a, a+d, a+2d
* Any positive perfect square can be expressed as the product of an even number of prime factors: 4=2\*2, 9=3\*3, 16=2\*2\*2\*2, 25=5\*5, ... each is written as the product of even number of prime factors.
* the positive roots (to any degree) of any integer greater than 1 are always greater than 1.
* Use fractions for multiplication and division and use percents or decimals for addition and subtraction.
* Adding the same number to both the numerator and the denominator brings the fraction closer to 1, regardless of the fraction's value.
* In GMAT, while rounding up, just look at the adjacent digit of the digit being dropped, rest all don't matter.
* Unit digit of any number depends on the Cyclicity i.e. the cycle of powers after which the Unit digits start repeating
* Numbers with Unit digits 2, 3, 7, 8 have Cyclicity 4 i.e. their unit digit repeates after every 4th power
* Numbers with Unit digits 0, 1, 5, 6 have Cyclicity 1 i.e. their unit digit repeates after every power
* Numbers with Unit digits 4, 9 have Cyclicity 2 i.e. their unit digit repeats after every 2nd power
* 0 is not a factor of any number

what is the remainder upon division -1 by 13? The answer to this question is 12: −1=13∗(−1)+12−1=13∗(−1)+12.

Relative prime numbers – Numbers which have unity as common factor. For Example – 25 and 49. If there are 2 relative prime numbers as denominators of 2 rational numbers, then addition/subtraction operation between those rational number cannot yield an integer.

2-consecutive numbers are always co-prime.

Two prime numbers with difference of 2 are called twin primes. e.g. 11 and 13, 17 and 19

A number is said to be perfect if sum of divisors (incl. number itself) is 2\*number

The difference between 2 numbers xy and yx is divisible by 9. e.g. 31-13 = 18%9 = 0

If ‘m’ and ‘n’ are 2 integers then (m+n)! is divisible by m! and n!

a^n/(a+1) will yield ‘a’ if ‘n’ is odd and ‘1’ if ‘n’ is even

For any natural number n^3-n is divisible by 6.

Remainder theorem:

(x\*y)%z = (x%z)\*(y%z)

In the above equation, we can use negative remainder too.

If we given an expression x\*y\*z… and asked to find last 2 digits then we can find it by dividing the number by 100 and taking its remainder.

e.g. 25^102 when divided by 17

=> (8^102)%7 => (512^34)%17 => (2^34)%17 => (-1)^8\*4/17 = 4

Also, We will have n^x as unit digit as n if ‘x’ is either divisible by 5 or 9.

To find the maximum power of a number that can divided a given number X. Find the power of largest prime in the divisor.

Like to find highest power of 63^n which can divide 13890! find the number of 7s in 13890 since 7 is highest prime in 63 and least repeated.

if sum of 2 primes is odd then one of the number is definitely will be 2

There are no guaranteed outcome in division operations so it can be odd,even or a decimal number

If |x| = x always, then x = 0

Specific to the case of 5, if there are 5 -ve integers in the set then there might be 0 in the set, so don't assume.

n! for arrangement of n things given all are unique and if there are some similar numbers also then the result will be n!/m! where m is the number of like things

\* GCF of number m and n X LCM of number m and n = m X n

\* All perfect squares have odd number of factors.

\* You can perform arithmetic on remainder itself:

\* If a number x leaves 5 as a remainder on division by 7 and y leaves remainder of 5 on divisible by 7. Then the total is : 10, correcting it will give: 10 - 7 = 3

If a number is given (variable) and another number is given + the LCM of both the numbers is given, to find the variable put both the number and the LCM in a table and factorize to find.

E <-- to find

45 - 5 X 9

225 - 5 X 5 X 9

Now to complete 225 E must contains two 5's and 0 or more 9's,

If it contains only 5's it can be 5 X 5 X 5 = 75

or 5 X 5 = 25

Sum of first n even numbers = n(n+1)

Sum of first n odd numbers = n^2

How to find tens digit of a number:

for 1, 3, 7, 9 - Convert number in such a way such that last digit of number is 1 then the tens digit will be the last digit of - first\_digit\_of\_base\*last\_digit\_of\_exponent

for 2, 4, 6, 8 - keep in mind 76\*xxx76 = zxy...76 and 24^any\_even\_power = xyz...76 and 24^odd\_power=24 .. here we will get exact last 2 digits

(1) if number's 10s digit is even, then last digit will always has to be 25.

(2) if number's 10s digit is odd, then last digit will always has to be in cyclic pattern of 25, 75,25,75 and so on...

\*\*\*\*\*\*\*\*\*\*\*\*\*\*

Last digits of a product of integers are last digits of the product of last digits of these integers.

For instance last 2 digits of 845\*9512\*408\*613 would be the last 2 digits of 45\*12\*8\*13=540\*104=40\*4=160=60

\*\*\*\*\*\*\*\*\*\*\*\*\*\*

Factors and Multiples

> The sum of the divisors of the square of a prime number x (excluding itself) is always 1 + sqrt(x)

> If p is a multiple of q and q is a multiple of p, p and q must be equal

Remainder:

in questions related to if a number divided by x leaves a remainder of a and y leaves remainder b. We can write the equation as xy + (first number common to both the equations)

In the questions which asks to find the remainder when dividing by 5, frame the equation as (x0 + p)^z now check what is the remainder when p^z will have when divided by 5. on expansion, this equation will have multiple of x0 except the last which will be p^z.

When the integer x is divided by the integer y, the remainder is 60. Which of the following is a possible

value of the quotient x/y?

x = ky + r ==> x/y = k + r/y ==> r/y = given options check if completely divisible : given were 15.15, 18.16 and 17.17 (incorrect)

\* Prime factors and normal factors are different.

Factorials:

Number of trialing 0s in a number:

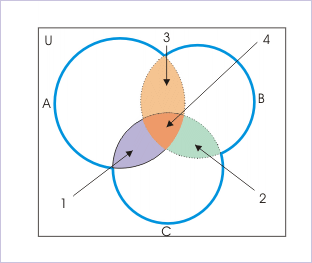
n/5 + n/5^2 + n/5^3 + … n/5^k such that 5^k < n

if N > 4, then unit digit of 1! + 2! + 3! + 4! ... + N! = 3

> In how many different ways can a group of 8 people be divided into 4 teams of 2 people each  
Then, Divide by 4! because we are not interested which team is chosen first or second.

It can be extended to find number of powers of a number p by replacing ‘5’ with ‘p’ in the above formula

Overlapping Problems:



In case 2-group is mentioned

AUBUC = A+B+C - ( A^B) – (A^C) – (B^C) + (A^B^C) + Neither

-----------------------<sum of 2 group overlaps>----(all three)

In case sum of EXACTLY 2 group (1+2+3 as in above diagram) is mentioned the formula will be come

AUBUC = A+B+C - ( A^B) – (A^C) – (B^C) - 2\*(A^B^C) + Neither

-----------------------<sum of 2 group overlaps>----(all three)

======= Modulus=======

For question involving finding out the number of solution to a equation which involves multiple Modulus based terms:

Solution : First, get value of x from each sub-equation so that value of that modulus becomes 0 if we plug-in the value of x, those values are called the critical values. One by one take that value of x and determine the sign of subequation by checking the sign of sub-equation inside the modulus. While solving change the sign of sub-equation and insert that value. Get the value of x and check if that falls in the range as specified by its sub-equation. If it doesn't fall in that put its value and get the outcome and that solution of x should satisfy the initial solution of the sub-equation was inserted.

This method is called the "critical values" method

like: |x+3| – |4-x| = |8+x| How many solutions will this equation have? It will have 0

Here, put values in the extreme and middle range of -3, 4, and -8 one-by-one and find the value of x, it won't satisfy the inserted value.

Range will be :

x<-8

-8<x<-3

-3<x<4

x>4

Taking range of -3 <= x <= 4

x+ will be positive and x-3 will be negative ..

eq becomes (x+2)=-(x-3)+1..

x+2=-x+3+1..

x=1.. valid value

b) two mods..

|x+2|=|x-3|+1..

critical values -2 and 3...

so regions are <-2, -2<=x<3, x>=3..

i) x<-2...

x+2 will be -ive and x-3 will be negative ..

eq becomes -(x+2)=-(x-3)+1.. both negative

-x-2=-x+3+1..... no values..

ii) −2<=x<3−2<=x<3..

x+2 will be positive and x-3 will be negative ..

eq becomes (x+2)=-(x-3)+1..

x+2=-x+3+1..

x=1.. valid value

iii)x>=3..

x+2 will be positive and x-3 will be positive ..

eq becomes (x+2)=(x-3)+1..

x+2=x-3+1..

no valid value..

so the solution is x=1

> must know properties: |xy|=|x|∗|y| and |xy|=|x||y|

========== Sequence and Series ========

\* Sum of n terms is : a\*n + n\*(n-1)/2

> To find sum of numbers in a given range, find the average between the range and multiply that with number count in that range.

\* A set of 3 consecutive number is divisible by 8 if middle number is odd number.

\* For any set of consecutive integers with an odd number of terms, the sum of the integers is always a multiple of the number of terms

> the standard deviation of any set is greater than or equal to zero: SD≥0  
> In order the range to be zero set T should have all identical elements,

> if all the integers are multiplied by any constant, the mean of all integers in the set will be multiplies that constant.  
> In order to have a set with the range of zero, all of the elements in the set must be the same.

> The product of k consecutive integers is divisible by k, k-1, k-2,...,2, and 1

============ Work/Rate Problems ============

In Work/rate problems, if equations are needed to be proved and numbers are given in term of as even or odd numbers then in that case change them into 2\*z for even and 2\*z + 1 for odd numbers.

In case combined rates are given, keep rates as 1/x and 1/y

> Use %ages rather than fractions to calculate the work done. For example, if a person can complete a work in 6 days, take its daily work rate as 16.6% rather tha 1/6.

> If (x men + y women) can complete a work in X days and (m men+n women) can complete a work in A days, then

xX + yX = mA + nA

> When a questions involves both men and women, convert man days to woman days and solve.

> If a machine can complete work in 'a' hours and another can in 'b' hours then stake of a is 1/a( 1/a + 1/b )

**Standard Deviation:**

\* Minimum number in a list of number with 2 standard deviations as -> s-2d, s-d, s, s+d, s+2d is "s-2d"

\* On multiplying list by a number X, standard deviation changes my multiple of X, if divided it dives else either it remains the same or pushed up

\* Decrease in values of a set by a given amount will reduce the standard deviation

=========== Geometry ========

**Triangle:**

\* A right angle triangle has the maximum area when it is an isosceles triangle

\* Similar triangles has areas in the form that one area is some multiple of another area.

\* The area of a rhombus is not determined by squaring a side, but rather by taking half the product of the diagonals,

\* The number of diagonals in a figure - n\*(n-1)/3

\* median = sqrt(2\*b\*b + 2\*c\*c – a\*a)/4

\* Sum of lengths of any 2 sides of a triangle has to be > the third side.

Important property: **median from right angle is half of the hypotenuse**

\* Difference between length of any 2 sides of triangle should be < the third side.

\* The point at which 3 altitude meet is called orthocenter and an angle made by any side at orthocenter and vertical angle make supplementary pair.

Area of triangle = (Perimeter\*inscribed\_circle\_radius)/2

For a given perimeter, the equilateral triangle has the largest area and for a given area the equilateral triangle has the smallest perimeter.

if a bisector and perpendicular coincide then we have an isosceles triangle.

Congruent triangle can be proven with:

SSS (All sides equal), SAS (Side, Angle, Side), ASA (Angle, Side, Angle), AAS (Angle, Angle, Side)

The radius of the circle inscribed in an equilateral triangle is r=a\*(sqrt(3)/6)

Any time a height is dropped from the right angle vertex of a right triangle to the opposite side of that right triangle, the three triangles that result have the same 3 angle measures. This means that they are similar triangles.

The side of triangle inscribed inside a circle of radius 'r' is sqrt(3)\*r

If the lengths of the sides of a triangle are a, b, and c, where the largest side is c  
> For a right triangle: a2 + b2=c2  
> For an acute (a triangle that has all angles less than 90°) triangle: a2 +b2 > c2  
> For an obtuse (a triangle that has an angle greater than 90°) triangle: a^2 + b^2 < c^2

**Equilateral triangle:**

The radius of circumscribed triangle is: sqrt(3)\*a/3  
The radius of inscribed triangle is : sqrt(3)\*a/6

Altitude – sqrt(3)\*a/2 where a is side of eq. triangle

circumradius = (2/3)\*h where h is altitude

in-radius = (1/3)\*h where h is altitude of eq. triangle

So, the radius of in-circle is half of the radius of circumscribed circle.

Among all of the triangles possible for given dimension, the equilateral triangle encompass much area.

Sum of lengths of all perpendiculars drawn on sides of any equilateral triangle is constant.

**Isosceles Triangle:**

For an isosceles triangle with the given length of equal sides, the right triangle has the largest area

For a given isosceles triangle : B^2/4 = L^2 – A^2

Where B is base, L is leg length (side which is equal to another one) and A is altitude

Radius of circle inscribed in right triangle = (a + b – c) /2

An altitude from right triangle to the other side creates 3 similar triangles. Also, that altitude = Hypotenuse/2

Sum of length of any 2 sides should be greater then 3rd side and difference between length of any 2 sides should < third side

*Angle equation:*

a/SinA = b/SinB = c/SinC

Given sides of triangle, we can find its area using cosine rule:

Cos C = (a^2 + b^2 – c^2)/(2\*a\*b)

While solving the problem, don’t go by the images. they might be misleading

Area of triangle: (in\_radius \* S) where S = (a + b + c)/2

Also, are of triangle is:

sqrt(s\* (s – a) \* (s – b)\* (s – c)) where s = (a + b + c)/2

Also, area = (1/2)\*(product\_of\_2\_sides)\*(sine-of\_included\_angle)

e.g. – (1/2)\*a\*c\*SinB = (a \* b \* c)/4 \* R where R is circumradius.

Congruent: Two triangles are said to be congruent when their sides and angles subtend.

Sample Conditions – SAS = ASA = SSS = AAS = SSA

Perpendicular bisector of triangle meets at a point called circumcenter wherein each vertex can be assumed to be on circle.

**Polygon**:

In a polygon, sum of all external angles is 360 degrees.

Area of a polygon = ( (n\*s^2)/4 \* cot(180/n) )

**Quadrilateral**:

Quadrilateral is a polygon with 4 sides.

Area of quadrilateral: (1/2) \* (length\_of\_a\_diagonal) \* (h1 + h2)

where h1 and h2 is the length of perpendiculars directed upon this diagonal from opposite sides.

Also, area = (1/2) \* (product\_of\_diagonals) \* (size\_of\_angle\_between\_them)

In a convex quadrilateral which in inscribed in a circle, the product of diagonals is sum of products of opposite sides:

(A \* C) + (B \* D) = (AC \* BD)

A kite is a quadrilateral with two distinct pairs of equal adjacent sides and whose diagonals intersect at 90 degrees.

**Parallelogram**:

Area of parallelogram = Base\*Height

A parallelogram inscribed in a circle is a rectangle.

A parallelogram circumscribed about a circle is rhombus.

Sum of the squares of the diagonals of parallelogram is equal to the sum of squares of its sides.

Rhombus : A rhombus will equal diagonals is a square, also diagonal of rhombus intersect at 90 degrees

Area of rhombus = (diagonal\_1\*diagonal\_2)/2

Trapezoid : two paralle lines

Area = ½\*(b1+b2)\*height

Where median(average length of 2 bases) = (b1+b2)/2

**Circle:**

To form a circle we should have 3 points which are not collinear

Length of chord = 2\*sqrt(r^r – d^d)

Where ‘r’ is the radius of circle and ‘d’ is the perpendicular distance from chord.

The angle that a chord makes with tangent is subtended by chord on circle in alternate segment.

radius of circle = (circumference \* 7)/44

Opposite angles of cyclic quadrilateral (a quadrilateral in circle) sum up 180 degree.

*Rule of tangents and secants*:

If a secant and a tangent of a circle are drawn from a point outside the circle, then the product of the lengths of the secant and its external segment equals the square of the length of the tangent segment.

c^2 = e \* (a + e) where c is length of tangent, e is distance of secant from circle and a is the secant extended from the point where it intersects the circle to centre of circle.

**Two Secants Intersecting**

If two secant segments are drawn from a point outisde a circle, the product of the lengths(C+D) of one secant segment and its exteranal segment(D) equals the product of the lengths (A+B) of the other secant segment and its external segment (B).

(A + B) \* B = (C + D) \* D

**Mensuration**

Surface area of cylinder : 2\*π\*r\*h + 2\*π\*r\*r

Surface area of cone/pyramid : π\*r\*l where l is the slant height = sqroot(h\*h + r\*r)

Total surface area of cone : π\*r\*l + π\*r\*r

Volume of cone : (1/3)\* π\*r\*r\*h

Surface area of sphere: 4\*π\*r\*r

Volume of sphere : (4/3)\* π\*r\*r\*r

Surface area of hemisphere : 3\*π\*r\*r

Volume of hemisphere : (2/3)\* π\*r\*r\*r

Frustum:

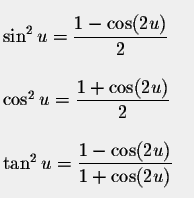
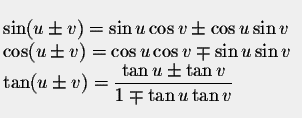
When cone is cut or chipped from top, the leftover is called frustum

Slant surface area of frustum : π\*(r1 + r2)\* l where l is slant height

Volume of frustum : (π/3)\*r1 + r2 + r1\*r2)

Do take care of hollow elements: for hollow, volume will be –(r1^n – r2^n) while in case of solid it will be –(r^n)

======================== Trigonometry ============================



======================= Coordinate Geometry ==============

We have the rectangle with dimensions 9\*6 (9 horizontal dots and 6 vertical). AB is parallel to y-axis and AC is parallel to x-axis.

Choose the (x,y) coordinates for vertex A: 9C1\*6C1;

Choose the x coordinate for vertex C (as y coordinate is fixed by A): 8C1, (9-1=8 as 1 horizontal dot is already occupied by A);

Choose the y coordinate for vertex B (as x coordinate is fixed by A): 5C1, (6-1=5 as 1 vertical dot is already occupied by A).

> Negative slop can be extremely negative or moderately negative.

9C1\*6C1\*8C1\*5C1=2160.

\* Perpendicular lines have -ve reciprocal slopes.If slope of line is m, then slope of line perpendicular to this line will be (-1/m)

\* The distance between two parallel lines = |b-c|(sqrt(m\*m + 1))

\* Distance between a point and a line = |ax0+by0+c|/sqrt(a\*a+b\*b) and when the point is origin the distance will be translated to |c|/sqrt(a\*a+b\*b)

Area of triangle in coordinate plane with given vertices

area = ( | Ax(By-Cy) + Bx(Cy-Ay) + Cx(Ay – By) | ) / 2

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Short Solutions:

For any prime number n > 3 , whats the remainder if n^2 is divisible by 12.

Consider the product of elements (n-1) and (n+1)

both are before and after the prime number and have a difference of 2. So, (n-1) is divisible by 2 and (n+1) by 4. Since n is divisible by 12 one of the n-1, n+1 will be divisible by 3 because n is a prime number, so the number will be divisible by 24 (2\*4\*3)

or (n-1)\*(n+1) or (n^2 - 1) will be divisible by 24. So dividing n^2 will us the remainder 1.

There are 15 prime numbers between 1-50 and 25 prime numbers between 1-100

Do use P(~A) = 1 - P(A) at appropriate places

Probability:

Probability of occurring a certain event P k times in n-time sequence

P = nCk \* p^k \* (1-p)^(n-k)

Cues:

When checking for options, reduce the terms so that there is not odd one out.

> Maximum pieces of cake in k cuts  
>> 0 line gives 1 whole piece;  
>> 1 line will give 2 pieces: 1+1=2;  
>> 2 lines will give 4 pieces: 2+2=4;  
>> 3 lines will give 7 pieces: 4+3=7;  
>> 4 lines will give 7+4=11 pieces.

Error log from GMC:

> Write the values clearly.  
> Do check if DISTINCT is mentioned before taking same number for different variables.  
> When finding solution to an equation, check if you can make 0 = 0 irrespective of values of given variables  
> When 2 solutions seem same, reject both  
> Combination of 1 and 1 not tried when it was given both numbers are odd.  
> For a given area equilateral triangle has the minimum possible perimeter.   
> For a given perimeter equilateral triangle has the maximum possible area.  
> When you have variables on both sides, don't struck them off. You might have 0 as a solution.  
  
  
> if x is a positive integer then √x is either a positive integer or an irrational number.  
> % increase - % decrease principle. If price of a commodity is increased by (100/x)% then we should decrease new price by (100/(x+1))% to get back the same price.  
> Check the following numbers: -2, -1, -1/2, 0, 1/2, 1, 2  
  
> if we have probability of finding k element in x as k/x then if we throw away some elements randomly then probability won't change.  
> Any positive integer root from a number more than 1 will be more than 1  
> In absolute mod question, check all variable values.  
> Greater the numerator/denominator, greater the fraction

> In case of square root symbol check the power. It might be 1/2 or 1/3  
> 0 is multiple of all numbers  
> Keep a track on overlapping subsets - I put value of all male and females in males.

> If √p is an integer, then p is also an integer.

> a handy way to compare standard deviations is to find the average difference of each term from the mean.

> 1/nth root of every integer will be > 1.

> The positive difference of the fourth powers of two consecutive positive integers must be divisible by one less than twice the larger integer

General:

> When not given, don't consider the number as integer.

> In DS question, evaluate both (A) and (B) independently

> If given that x, y and z are consecutive don't interpret as if x, y, z is a consecutive sequence. z, y, x or y, x, z can also be valid

> When there are 2 correct answers you can reject both the choices.

> When given a question about decimal repetition, do cross-check if the sequence is correct