**Divide and Conquer**

Multiplying Polynomials



Master theorem to find the complexity of the recurrence relation mentioned as:



Everything relies on values of log(b)a and d. We have to choose max. So if d > log(b)a, keep it n^d, else if log(b)^a > n^d keep it log(b)^a else if both are equal, keep both as: n^d\*logn

Proof of master theorem:







QuickSort

Quicksort has complexity of O(n^2), if all elements are equal.



Number of inversions:

An inversion of a sequence 𝑎0, 𝑎1, . . . , 𝑎𝑛−1 is a pair of indices 0 ≤ 𝑖 < 𝑗 < 𝑛 such

that 𝑎𝑖 > 𝑎*j.*

Use mergesort recursive calls and the following code:

/\* This function merges 2 arrays and count inversions of those \*/

int merge\_and\_count(vector<int>& vec, int left, int ave, int right) {

int tot\_sz = right-left+1;

int inversions = 0;

vector<int> result;

int i = left, j = ave+1;

while (i <= ave && j <= right) {

if (vec[i] <= vec[j]) { result.push\_back(vec[i]); ++i; }

else { inversions += (ave - i + 1); result.push\_back(vec[j]); ++j; }

}

if (i == ave+1) { for (int x = j; x <= right; ++x) result.push\_back(vec[x]); }}

else if (j == right+1) {for (int x = i; x < ave; ++x) result.push\_back(vec[x]);}}

// fill back

for (int i=left, j=0; i <= right && j<result.size(); ++i, ++j){vec[i] = result[j];}

return inversions;

}

**Covering the segments**

To capture the points which is covering the line segments, create a list of the form:

(a1l, l), (a2l, l), (p1, p), (a1r, r), (a3l, l), (a2r, r), (p2, p), (a3r, r)

sort it...

and then the Number of l's on the left side of point is the number of line segments it is covering.

**use this for solving problem on 1 axis**

**Dynamic Programming:**

DP is an approach for solving problems which seem recursive in nature but can be improvised in terms of time by building the solution from bottom to top rather than trying to solve the problem by solving the subproblems. One of the example to quote when DP wins over greedy approach is coin change problem. Suppose we have coins of denominations - 20, 8 and 1. How many coins are required for changing 24. In greedy, we will first select 20 and then 4 single cent coins, giving a total of 5 coins, but we can also take 3 coins of 8 cents too :)

Coin change problem



**Edit Distance Problem:**

In this problem, we are asked to find minimum number of insertions and deletion which need to be performed to make 1 string equal to another or in other words find the longest common subsequence.

Following is the algorithm to execute the necessary operations:



Following is the way we can backtrack and print the path:



**Knapsack without repetition (1-D array used):**

Characterize the structure of optimal solution by creating optimal solution to a subproblem using cut and paste technique. Take out an item and its value

and declare that remaining weight should have optimal value if take out this item. After that fix that w(i) again in the total weight.

￼￼

**Knapsack with repetition (2D-array used):**



For reconstructing the solution, or to actually calculate value compare the value of matrix[i-1][j] and matrix[i-1][j-weight[i-1]]

To Print all anagrams together

Sort those and store in hash if not exist else append in the form of hash chain.

To rotate an image by 90 degree get the transpose of the matrix and then reverse each row

**Graphs** :

Represented as adjacency matrix

struct graph{

int n\_v, n\_e

int degrees[MAX\_V]

int edges[MAX\_V][MAX\_EDGES];

}

//Make 0 the degrees of vertices degrees[0->MAX\_V]

void insertedge(int e1, int e2){

edges[e1][edges->degree[e1]] = e2;

edges[e2][edges->degree[e2]] = e1;

edges->degree[e1]++

edges->degree[e2]++

}

class Graph {

//A vector of vector...graph[i] denotes the vector of edges that are connected to graph[i]

vector< vector<int> > graphVector;

bool \*marked;

int \*edgeTo;

public :

int degree(int i);

void addVertex(int v);

void init(int v);

void dfs(int h);

void dfsWithStack(int v);

void bfs(int v);

void bfsWithQueue(int v);

void addEdge(int v, int w);

bool deleteEdge(int v, int w);

vector<int>::iterator searchEdge(int v, int w);

};

BFS can be used to find the shortest path, in the loop.

Strongly Connected Components can be detected by using Depth first search using which we can find if 2 vertices are connected by comparing their id's, id[] size is number of vertices since it might happen that all vertices are separated.

To go for connected components use an array id's that stores the same value for each vertex in the same connected components.

Uses of Directed Graphs ->

**Topological Sorting:**

It is used in precedence scheduling, where we put the vertices (tasks) in a sequential manner so that task preceding a given task is completed before that. For that, we go for DFS and put the vertices on stack so that while popping up we get correct sequence. Use recursive DFS for this purpose.

If there is a directed cycle, there cannot be a topological sorting because in that case we cannot be get done with the inner vertex so as to return to the originating vertex.

Application of digraph:

web crawling

Reachability and

Path finding

Difference between connected components and strongly connected components is that connected components are in relation with undirected graph while SCC deals with directed graph

Digraph processing is all about DFS

**Strongly Connected Components algorithm – Kosaraju's Algorithm**

> Find the topological order of reverse graph

> Run DFS on original graph using the topological sequence generated by the first one.

**Sorting**

Quicksort is better than HeapSort because there are no swaps made when the elements are in order.

nth order statistic or QuickSelect

function select(list, left, right, n)

if left = right // If the list contains only one element,

return list[left] // return that element

pivotIndex := ... // select a pivotIndex between left and right,

// e.g., left + floor(rand() \* (right - left + 1))

pivotIndex := partition(list, left, right, pivotIndex)

// The pivot is in its final sorted position

if n = pivotIndex

return list[n]

else if n < pivotIndex

return select(list, left, pivotIndex - 1, n)

else

return select(list, pivotIndex + 1, right, n)

**Trees**

Heavy Path problem

/\* Main function which truncates the binary tree. \*/

struct Node \*prune(struct Node \*root, int sum)

{

// Base Case

if (root == NULL) return NULL;

// Recur for left and right subtrees

root->left = prune(root->left, sum - root->data);

root->right = prune(root->right, sum - root->data);

// If we reach leaf whose data is smaller than sum,

// we delete the leaf. An important thing to note

// is a non-leaf node can become leaf when its

// chilren are deleted.

if (root->left==NULL && root->right==NULL)

{

if (root->data < sum)

{

free(root);

return NULL;

}

}

return root;

}

> Number of nodes = return (size(node->left) + 1 + (size(node->right)

Bottom view of tree: Store in hashmap left -- and right ++

Mirroring of tree is done using pre-order traversal

Use top-down approach to create a tree from array

Element Insertion in tree

if(element < ptr->element)

ptr->left = insertNode(ptr->left, element);

else

ptr->right = insertNode(ptr->right, element);

return ptr;

**if tree Equal:**

return ( ptr1->element == ptr2->element && checkIfEqual(ptr1->left, ptr2->left) && checkIfEqual(ptr1->right, ptr2->right));

**If foldable**

return ( ( (ptr1 && ptr2) || (ptr1 == NULL && ptr2 == NULL) ) && checkIfFoldable(ptr1->left, ptr2->right) && checkIfFoldable(ptr1->right, ptr2->left));

**Level order Traversal with Q :**

int nodeLevel(struct treeStruct \*ptr, int nodeToFind)

{

int width = 0, local = 0;

int level = 0;

if(ptr->element == nodeToFind){

return 1;

}

q.push(ptr);

while(!q.empty())

{

++level;

local = q.size();

if(local > width)

width = local;

for(int i=0; i <= local - 1; i++)

{

struct treeStruct \*dummy = q.front();

if(dummy->element == nodeToFind)

return level;

q.pop();

if(dummy->left != NULL)

q.push(dummy->left);

if(dummy->right != NULL)

q.push(dummy->right);

}

}

return 0;

}

**> The distance between two nodes can be obtained in terms of lowest common ancestor. Following is the formula.**

Dist(n1, n2) = Dist(root, n1) + Dist(root, n2) - 2\*Dist(root, lca)

**> To Print left and right view of binary tree ...take this as a reference**

left/Right\_ViewUtil(root->left, level+1, max\_level);

put check if(max\_level < level ) print the node and max\_level = level in this case when further elements are encountered at that particular level it stops

**Check if a tree is BST**

Do in-order traversal and check sorting order

Another way to do the same

isBSTUtil(node->left, min, node->data-1) && // Allow only distinct values

isBSTUtil(node->right, node->data+1, max); // Allow only distinct values

To find next successor in BST..use a flag that is set to true for the element we are searching and then check that flag on next traversal

> **To find if root to leaf path equals a certain sum:**

For root to leaf path sum do subsum = sum - node\_element if subsum is 0 in case left and right are NULL then that is the answer

**> To check if trees are similar :**

return ( areTreesSimilar(ptr1->left, ptr2->left) && areTreesSimilar(ptr1->right, ptr2->right) );

> **To check a sum tree:**

if(ptr->element != (isSumTree(ptr->left, var) + isSumTree(ptr->right, var)) )

=======================================================================

Distributed hashtables are those which are decentralized and for each key a value might exists on multiple nodes/clusters

DHTs only directly support exact-match search

1- 9 gives 100 puzzle - 123-45-67+89 = 100

To store just 100 numbers from incoming flow of numbers, use a circular buffer/heap can also be used

to sort a big array with many repetitions use AVL with repetitions

Anagrams

Sorting and counting the characters..we can use one array in which we increment one string and decrement for other. After operation, check if array is filled with 0s

To print 1 - 100

Either create 100 class objects or use recursive templates

To check if a string anagram can be palindrome

for this check if length is even then every character occurs even number of times

while if the length is odd, then one except one should appear even number of times

Fastest way to check 2 anagrams map each character with prime number ad find the multiplied result of both strings

if they are equal then it is or you can take a map / Character array[26] / Sorting

To find row wit hmax 1's first find the indes of 1 in first row and then update that index accordinly by going LHS and put max\_row\_index that row in case left entry

of original index is 1

to order one array based on another array

use a1 and put its values in hashmap with <ey, value> --> <number, occurences>

now traverse a2, search for that in hashmap if its found put that in p/p array and remove it from hashmap and rest of elements are sorted and appened to the o/p array

word break problem

if (dictionaryContains( str.substr(0, i) ) &&

wordBreak( str.substr(i, size-i) ))

return true;

#include<stdio.h>

/\* function to multiply two numbers x and y\*/

int multiply(int x, int y)

{

/\* 0 multiplied with anything gives 0 \*/

if(y == 0)

return 0;

/\* Add x one by one \*/

if(y > 0 )

return (x + multiply(x, y-1));

/\* the case where y is negative \*/

if(y < 0 )

return -multiply(x, -y);

}

1. Print the left boundary in top-down manner.

2. Print all leaf nodes from left to right, which can again be sub-divided into two sub-parts:

…..2.1 Print all leaf nodes of left sub-tree from left to right.

…..2.2 Print all leaf nodes of right subtree from left to right.

3. Print the right boundary in bottom-up manner.

We need to take care of one thing that nodes are not printed again. e.g. The left most node is also the leaf node of the tree.

Flyweight design pattern is basically for memory efficiency in which objects asked for, by multiple modules are returned from a common repository, while no new objects are created if such object doesn't exists then that one is created and stored in the repo. Example can be taken as a circles of specific color stored in hashmap

Memoization is basically type of flyweight pattern

LIS -

L(i) = 1 + max { L(j) } where a[i] > a[j] else this value is 1

Sum of subarray with a given sum divided by a number X... here in this suppose starting index is A and ending index is B then the remainder for the sum from 0->A and 0->b will be the same and between them will be the numbers with a given sum so for this

1, 9, 4, 5, 2, 8, 7, 11

sum array

1, 10 , 14, 19, 21, 29, 36, 47

now suppose X is 4 then mod array is

1 , 2, 2, 3, 1, 1, 0, 3

now create a hashmap with the <numbers\_given\_above, frequency>

total will be freq1 ! + freq2 ! + freq3 !... freqn !

here it will be - 0! + 3! + 1! + 1! = 6

quickselect is a selection algorithm to find the kth smallest element in an unordered list

**BitSets**

**To check the divisibility by x**

**for 2 : check if rightmost bit is 1 or not**

**for 3 : Total odd number of bits – even number of bits should be divisible by 3. Do it recursively**

**for 4 : Total number of set bit should be 1 and total 0's from left should be even**

**for 5 : ?**

**for 6 : Check if divisible by 2 and 3**

**for 7 : recursively check (n <<3) – n**

**for 8 : check if last 3 digits are 0**

To find out number of set bits by

doing n = n&(n-1) 'x' times till n becomes 0 where x is number of bits set.

To find next divisible number by 2, find if the number has only 1 set bit by using (n !(n&n-1)) , if it is then that is the number, else right shift one-by-one and get the count. Then 1<<count is the number.

Swap 2 numbers

x = x^y;

y = x^y;

x = x^y;

These method will fail if both variables are same so put a check before swapping

Get the rightmost set bit - set\_bit\_no = xor & ~(xor-1)

for(i = 0; i < n; i++)

{

if(arr[i] & set\_bit\_no)

\*x = \*x ^ arr[i]; /\*XOR of first set \*/

else

\*y = \*y ^ arr[i]; /\*XOR of second set\*/

}

Detect if two integers have opposite signs XOR the 2 numbers if they have opposite signs the result will have lftmost bit as 1 hence <0

**Max profit**

fill sum array in reverse direction with each entry filled with the max element on its right

now traverse from left->right and get sum[i] - a[i] and add them to get max profilt

> To find the position of rightmost bit do 2'complement then & it with original number that will be the answer or (n&(!(n-1)));

// Bitwise operator based function to check divisibility by 9

**bool** isDivBy9(**int** n)

{

// Base cases

**if** (n == 0 || n == 9)

**return** **true**;

**if** (n < 9)

**return** **false**;

// If n is greater than 9, then recur for [floor(n/9) - n%8]

**return** isDivBy9((**int**)(n>>3) - (**int**)(n&7));

}

Align the most-significant ones of N and D.

* Compute t = (N - D);.
* If (t >= 0), then set the least significant bit of Q to 1, and set N = t.
* Left-shift N by 1.
* Left-shift Q by 1.
* Go to step 2.

Inverse a bit string n^(~0)

for(int i = 0; i < s; i++)

{

b <<=1; // left shift b

b |= a & 0x1; //get unit bit

a >>= 1; // right shift a

}

Another trick to reverse a number

run loop from i = 0 to 16 (sizeof(data\_type) \* 4 )

check by right shifting number by & 1 | check by right shifting number by j & 1

set\_bit\_no = xor & ~(xor-1); ( rightmost set bit )

XOR of two different numbers x and y results in a number which contains set bits at the places where x and y differ. So if x and y are 10…0100 and 11…1001, then result would be 01…1101.

So the idea is to XOR all the elements in set. In the result xor, all repeating elements would nullify each other. The result would contain the set bits where two non-repeating elements differ.

Now, if we take any set bit of the result xor and again do XOR of the subset where that particular bit is set, we get the one non-repeating element. And for other non-repeating element we can take the subset where that particular bit is not set.

Find minimum using bitwise operators

y ^ ((x ^ y) & -(x < y))

Max

x ^ ((x ^ y) & -(x < y));

/\* This function will return n % d. d must be one of: 1, 2, 4, 8, 16, 32, … \*/

unsigned **int** getModulo(unsigned **int** n, unsigned **int** d)

{

**return** ( n & (d-1) );

}

2's complement of a number is 1's complement + 1

To add 1 to a number x (say 0011000111), we need to flip all the bits after the rightmost 0 bit (we get 001100**0**000). Finally, flip the rightmost 0 bit also (we get 0011001000) and we are done

To flip a number use XOR operator

/\* Flip all the set bits until we find a 0 \*/

while( x & m )

{

x = x^m;

m <<= 1;

}

check divisibility by 8

if (((x >> 3) << 3) == x) then done;

Boolean Array Puzzle

Given an array of 2 elements one is having value 0 and another one having value 1

make both 0.

Solution : a[ a[1] ] = a[ a[0] ]

=======================================================================

**Arrays**

**Maximum bitonic subArray**

The bitonic array is one in which there are elements which are first in increasing order and then in decreasing order and you have to find maximum length of that array

Solution:

Construct two arrays int[] and dec[] in inc[] array make each element

inc[i]=inc[i-1]+1 if a[i] > a[i-1]

and then do the same in dec[] array but in opposite direction

i.e dec[i-1]=dec[i]+1 if a[i-1]>a[i]

At last find max ( inc[i] + dec[i] -1) for each i .

MAX Sub SEQUENCE SUM =

max[0] = A[0];

max[1] = A[1];

max[i] = max{ A[i-1], A[i-2] + A[i] }

To merge k sorted arrays use min heap putting all elements at once then picking min element

Snake and ladder game can be implemented using a 2-D array... to find min nodes use shortest path.

**Strings**

**Longest Increasing Subsequence**

initialize all to 0 for i,0 and 0,j

if A[i] == B[j]

LIS[i, j] = 1 + LIS[i-1, j-1];

else

LIS[i, j] = max{ LIS(i-1, j) , LIS(i, j-1) }

**Longest Palindromic suubsequence**

if( A[i] == A[j])

LCS[i, j] = LCS[i-1, j-1] + 2

else if i == j

LCS[i, j] = 1

else

LCS[i, j] = max{ LCS[i-1, j] , LCS[i, j-1]}

**Karumanchi**

k =1 -> n for logk = nlogn

1^p + 2^p + 3^p .... + n^p = ( n^(p+1))/(p+1)

Master Theorem for run time analysis

======================= Binary Search ======================

Matrix Search

Method 1: Find appropriate position of element in left column and right column. If the element is found in left/right column itself, return true, else check if index of 2 arrays are common. If not, return false else check for the array at that index.

Method 2: Set l = 0 and r = m\*n-1

to find mid = (l+r)/2

**bool** searchMatrix(vector**<**vector**<int>** **>** **&**matrix, **int** target) {

**int** n **=** matrix.size();

**int** m **=** matrix[0].size();

**int** l **=** 0, r **=** m **\*** n **-** 1;

**while** (l **!=** r){

**int** mid **=** (l **+** r **-** 1) **>>** 1;

**if** (matrix[mid **/** m][mid **%** m] **<** target)

l **=** mid **+** 1;

**else**

r **=** mid;

}

**return** matrix[r **/** m][r **%** m] **==** target;

}

**Strings**

**Knuth-Morris-Pratt**

Worst case complexity – O(n)

A string searching algorithm that runs in O(n) times as compare to naive algorithm which can has O(n\*m) as the worst complexity.

To accomplish this, we use the method of degeneration by calculating the length of the longest proper prefix in the (sub)pattern that matches a proper suffix in the same (sub)pattern.

e.g. In 'ABCAB', the prefix and suffix which are equal and are of maximum length is 'AB'

To carry out this task, we use an auxiliary array (also called partial match table) in which we store the maximum length of that prefix/suffix at a given position in pattern. e.g. for pattern 'ABCABLAB', the array will be

A|B|C|A|B|L|A|B|

0|0|0|1|2|0|1|2|

Let the string is 'ABCAPABCGCABCABLAB'

this table will help us deciding about how much characters we should skip in case of mismatch. e.g if pattern is 'ABCABLAB'

then we will have match for 'ABCA' (4 chars) and we will have mismatch for 5th character (required 'P' , got 'B') and in that case we will check the value at:

array[partial\_match\_length - 1] which is array[3] = 1 and we have to skip the search by

partial\_match\_length - array[partial\_match\_length - 1]

=4-1 = 2(we have to skip 'AB')