Computed the minimal triangular graph over $\Omega := (0, 1)^2$, i.e find a function $q : \bar{\Omega} \to R$, which is piecewise linear and continuous on a given triangulation of Ω, and which satisfies the boundary condition $q|_{\partial\Omega} = \frac{1}{2} - |x_2 - \frac{1}{2}|$, and whose graph has minimal surface area among all piecewise linear and continuous functions defined on the same triangulation and satisfying the same boundary conditions.

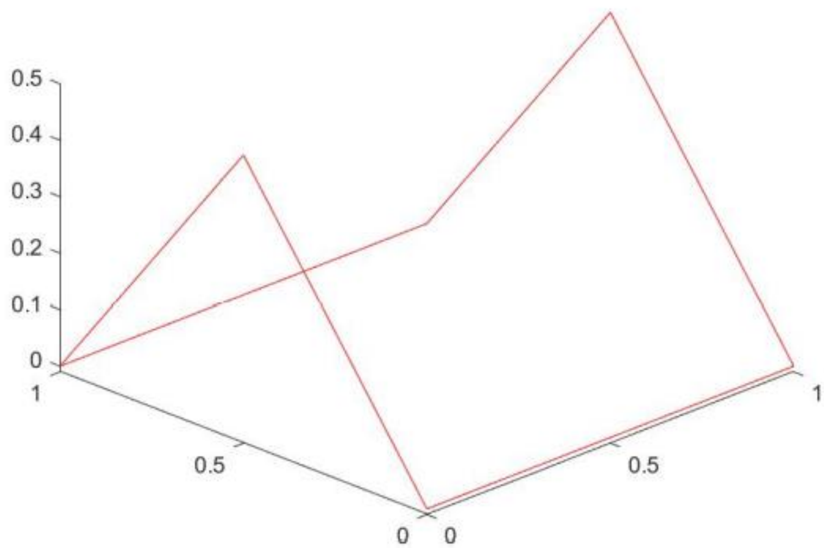

Figure 1: Boundary condition

Output: Surface area and a figure showing the triangulated graph.

Proceed as follows: Subdivide Ω into triangles (everything consists of triangles (Plato)) with corners $Q_1(x_1^1, x_2^1), \dots, Q_m(x_1^m, x_2^m)$. Consider functions q, which are continuous on $\bar{\Omega}$ and linear on each triangle. Construct a function $A : \mathbb{R}^n \to \mathbb{R}$ which defines the surface area of the graph of q (what is n?).

Solution algorithm: Steepest descent with Armijo step size rule:

a) Choose $x^0 \in \mathbb{R}^n$, and $\beta, \gamma \in (0,1)$,

b) for $k = 0, 1, 2, \dots$ do

  (i) $\nabla f(x^k) = 0 \to$ STOP with $x^* = x^k$.

  (ii) $s^k := -\nabla f(x^k)$,

  (iii) $\sigma_k := \max\{\sigma > 0; \sigma \in \{1, \beta, \beta^2, \dots\}\} : f(x^k) - f(x^k + \sigma s^k) \geq -\gamma\sigma\nabla f(x^k)^t s^k$

  (iv) $x^{k+1} = x^k + \sigma_k s^k$

Choose $\beta = \frac{1}{2}$, $\gamma \in [10^{-3}, 10^{-2}]$ and stop if in b) (ii) $\|s^k\| \leq 10^{-8}$ is satisfied.