Used Newton's method:

a) the root of

$$f(x) = \exp(-x) - 10^{-9}$$

with $x^0 = 0$, and compare the results $(x^k, f(x^k))$ for the stopping criteria

(i) $|f(x^k)| < \epsilon_a$,

(ii) $|f(x^k)| < \epsilon_r |f(x^0)| + \epsilon_a$,

(iii) $|x^{k+1} - x^k| < \epsilon_a$,

(iv) $|x^{k+1} - x^k| < \epsilon_r |f(x^0)| + \epsilon_a$,

with all possible combinations of $\epsilon_a, \epsilon_r = 10^{-3}$ and 10^{-10}.

b) the extremum of

$$F(x) = (x_1 - 1)^4 + 2(x_1 - 1)^2(x_2 - 1)^2 + (x_2 + 1)^4 - 2(x_2 - 1)^2 - (2x_2 + 1)^2 + 1$$

with $x^0 = (1.21, -1.15)^\top$ and the stopping criteria

(i) $\|x^{k+1} - x^k\|_\infty < 10^{-4}$,

(ii) $\|x^{k+1} - x^k\|_\infty < \epsilon_r \|\nabla F(x^0)\| + 10^{-4}$,

where $\|x\|_\infty := \max_{i=1,\dots,n} |x_i|$. In (ii) test different values for ϵ_r and compare the results.