# ASSIGNMENT 3

**PROBLEM STATEMENT: -**

Implement Union, Intersection, Complement and Difference operations on fuzzy sets. Also create fuzzy relations by Cartesian product of any two fuzzy sets and perform max-min composition on any two fuzzy relations.

**OBJECTIVE:**

3. To learn and understand the different operations on fuzzy sets.
4. To learn and understand the min-max composition on fuzzy set.

**THEORY:**

A fuzzy set is an extension of a classical set in which each element has a degree of membership between 0 and 1. Unlike classical sets where an element either belongs (membership degree 1) or does not belong (membership degree 0), fuzzy sets allow for gradual membership degrees, representing the degree to which an element belongs to the set. The membership degree is often described by a membership function.

**Operations on Fuzzy Sets**

Having two fuzzy sets A˜ and B˜, the universe of information U and an element $y$ of the universe, the following relations express the union, intersection and complement operation on fuzzy sets.

**Union of fuzzy sets:**

- The union of two fuzzy sets A and B with membership functions μA and μB, respectively, is a fuzzy set C, denoted C = A ∪ B, with the membership function μC.
- There are two definitions for the union operation: the max membership function and the product rule, as defined in following equations:

$$\mu C(x) = \max [\mu A(x), \mu B(x)] \quad (2.12)$$

$$\mu C(x) = \mu A(x) + \mu B(x) - \mu A(x)\mu B(x)$$

where x is an element in the universe of discourse X.

- Example:

Let A and B be two fuzzy sets in the universe of discourse X and (x1, x2, x3, x4) ∈ X defined as follows:

A = {0/x1 + 1/x2 + 0.7/x3 + 0.4/x4 + 0.2/x5 + 0/x6}

B = {0/x1 + 0.4/x2 + 0.7/x3 + 0.8/x4 + 1/x5 + 0/x6}

The union of fuzzy sets A and B using the max membership function is:

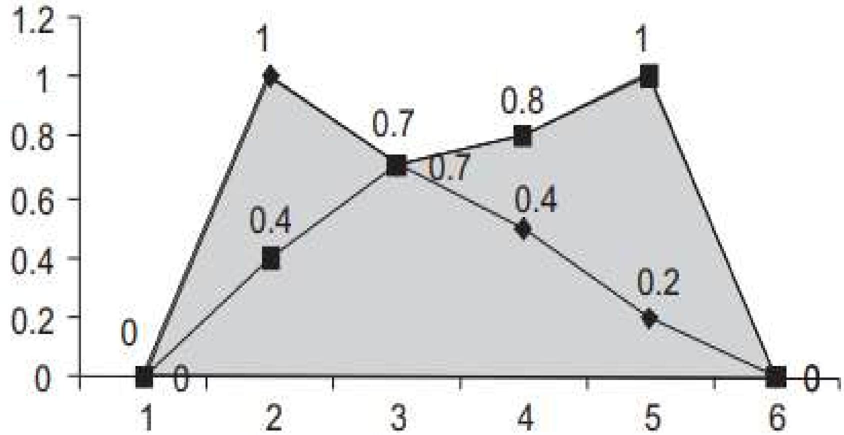
Cmax = A ∪ B = {0/x1 + 1/x2 + 0.7/x3 + 0.8/x4 + 1/x5 + 0/x6}

where μC(xi) is calculated from max [μA(xi), μB(xi)] for i = 1, 2, 3,..., 6.

Alternatively, using the product rule it is:

Cprod = A ∪ B = {0/x1 + 1/x2 + 0.91/x3 + 0.88/x4 + 1/x5 + 0/x6}

where μC(xi) is calculated using [μA(xi) + μB(xi) − μA(xi) ∗ μB(xi)] for i = 1, 2, 3,..., 6.




Union of fuzzy sets *A* and *B* using max operation

**Intersection of fuzzy sets:**

- The intersection of two fuzzy sets A and B with membership functions μA and μB, respectively, is a fuzzy set C, denoted C = A ∩ B, with membership function μC defined using the min membership function or the product rule as follows:

$$\mu C(x) = \min [\mu A(x), \mu B(x)]$$
$$\mu C(x) = \mu A(x) * \mu B(x)$$

- Example:

  Let A and B be two fuzzy sets in the universe of discourse X and (x1, x2, x3, x4) □ X defined as in the previous example. The intersection of fuzzy sets A and B using the min membership function is

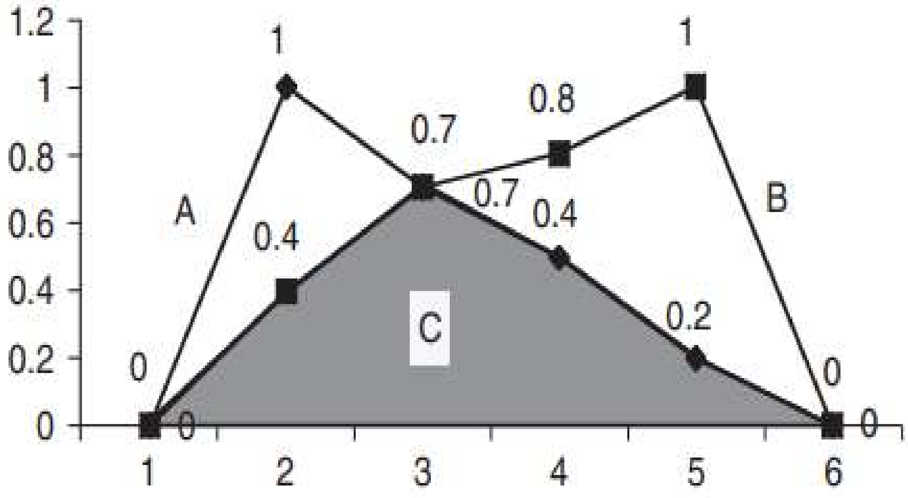
  Cmin = A ∩ B = {0/x1 + 0.4/x2 + 0.7/x3 + 0.4/x4 + 0.2/x5 + 0/x6}

  where μC(xi) is calculated from μC(x) = min [μA(x), μB(x)] for i = 1, 2, 3,..., 6.

  Alternatively using the product rule it is

  Cprod = A ∩ B = {0/x1 + 0.4/x2 + 0.49/x3 + 0.32/x4 + 0.2/x5 + 0/x6}

  where μC(xi) is calculated from μC(x) = μA(x) □ μB(x) for i = 1, 2, 3,..., 6.




Intersection of fuzzy sets *A* and *B* using the min operation

**Complement of fuzzy set:**

- The complement of a fuzzy set A with membership function μA is a fuzzy set, denoted ∼A, with membership function $\mu_{\sim A}$ defined as

$$\mu_{\sim A}(x) = 1 - \mu_A(x)$$

- Example:

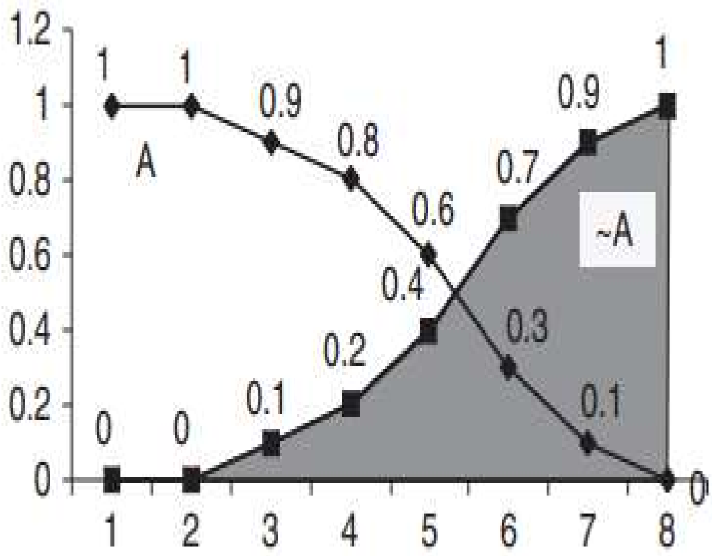
  Let A be a fuzzy set in the universe of discourse X and (x1, x2, x3, x4, x5, x6, x7, x8) ∈ X defined as follows

  A = {1/x1 + 1/x2 + 0.9/x3 + 0.8/x4 + 0.7/x5 + 0.3/x6 + 0.1/x7 + 0/x8}

  The complement of fuzzy set A is ~A :

  ~ A = {0/x1 + 0/x2 + 0.1/x3 + 0.2/x4 + 0.3/x5 + 0.7/x6 + 0.9/x7 + 1/x8}

  where μ~A(x) is calculated from [1 − μA(x)] for i = 1, 2, 3,..., 8.




Complement of fuzzy set *A*

**Fuzzy Relations**

- The concept of a relation has a natural extension to fuzzy sets and plays an important role in the theory of such sets and their applications.
- A fuzzy relation *R* from the fuzzy set *A* in *X* to the fuzzy set *B* in *Y* is a fuzzy set defined by the Cartesian product *A* × *B* in the Cartesian product space *X* × *Y* .
- R is characterized by the membership function expressing various degrees of strength of

relations:

$$R = A \times B = \sum \mu_R(x, y)/(x, y) = \sum \min(\mu_A(x), \mu_B(y))$$

$$R = A \times B = \sum \mu_R(x, y)/(x, y) = \sum \mu_A(x)^* \mu_B(y)$$

- In Equations the sum does not mean a mathematical summation operation, it means all possible combinations of all elements.
- R is also called the relational matrix. The Cartesian product is implemented in the same fashion, as is the cross product of two vectors. For example, fuzzy set A with 4 elements (a column vector of dimension 4×1) and fuzzy set B with 5 elements (a row vector of dimension 1×5) will provide the resulting fuzzy relation R which is represented by a matrix of dimension 4×5.
- Example:

  Let A and B be two fuzzy sets defined by

  A = {1/1 + 0.8/2 + 0.6/3 + 0.5/4}

  B = {0.5/1 + 1/2 + 0.3/3 + 0/4}

  The fuzzy relation (i.e., the Cartesian product of A and B using the min operation) will be

$$\mathbf{R} = \mathbf{A} \times \mathbf{B} = \begin{bmatrix} \{1, .5\} & \{1, 1\} & \{1, .3\} & \{1, 0\} \\ \{.8, .5\} & \{.8, 1\} & \{.8, .3\} & \{.8, 0\} \\ \{.6, .5\} & \{.6, 1\} & \{.6, .3\} & \{.6, 0\} \\ \{.5, .5\} & \{.5, 1\} & \{.5, .3\} & \{.5, 0\} \end{bmatrix} = \begin{bmatrix} 0.5 & 1 & 0.3 & 0 \\ 0.5 & 0.8 & 0.3 & 0 \\ 0.5 & 0.6 & 0.3 & 0 \\ 0.5 & 0.5 & 0.3 & 0 \end{bmatrix}$$

  The fuzzy relation using the product operation will be

$$\mathbf{R} = \mathbf{A} \times \mathbf{B} = \begin{bmatrix} \{1, .5\} & \{1, 1\} & \{1, .3\} & \{1, 0\} \\ \{.8, .5\} & \{.8, 1\} & \{.8, .3\} & \{.8, 0\} \\ \{.6, .5\} & \{.6, 1\} & \{.6, .3\} & \{.6, 0\} \\ \{.5, .5\} & \{.5, 1\} & \{.5, .3\} & \{.5, 0\} \end{bmatrix} = \begin{bmatrix} 0.5 & 1 & 0.3 & 0 \\ 0.4 & 0.8 & 0.24 & 0 \\ 0.3 & 0.6 & 0.18 & 0 \\ 0.25 & 0.5 & 0.15 & 0 \end{bmatrix}$$

**Max-min composition**

- If R is a fuzzy relation in X × Y and A is a fuzzy set in X then the fuzzy set B in Y is given by

  $$B = A \circ R$$

  B is inferred from A using the relation matrix R which defines the mapping between X and Y and the operation '∘' is defined as the max/min operation.

- Example:

  Let A be a fuzzy set defined by

  A = {0.9/1 + 0.4/2 + 0/3}

  With the fuzzy relation R given by the following relational matrix:

$$\mathbf{R} = \mathbf{A} \times \mathbf{B} = \begin{bmatrix} 1 & 0.8 & 0.1 \\ 0.8 & 0.6 & 0.3 \\ 0.6 & 0.3 & 0.1 \end{bmatrix}$$

Then the fuzzy output B in Y using the max/min operation will be

$$\mathbf{B} = \mathbf{A} \circ \mathbf{R} = \begin{bmatrix} \frac{0.9}{1} & \frac{0.4}{2} & \frac{0}{3} \end{bmatrix} \circ \begin{bmatrix} 1 & 0.8 & 0.1 \\ 0.8 & 0.6 & 0.3 \\ 0.6 & 0.3 & 0.1 \end{bmatrix}$$

$$\mathbf{B} = \begin{bmatrix} \{0.9, 1\} & \{0.9, 0.8\} & \{0.9, 0.1\} \\ \{0.4, 0.8\} & \{0.4, 0.6\} & \{0.4, 0.3\} \\ \{0, 0.6\} & \{0, 0.3\} & \{0, 0.1\} \end{bmatrix}$$

Taking the minimum values row-wise, we obtain

$$\mathbf{B} = \begin{bmatrix} 0.9 & 0.8 & 0.1 \\ 0.4 & 0.4 & 0.3 \\ 0 & 0 & 0 \end{bmatrix}$$

Taking the maximum values column-wise, we obtain the fuzzy set B from the compositional relation:

$$\mathbf{B} = \begin{bmatrix} 0.9 & 0.8 & 0.3 \end{bmatrix}$$

**CONCLUSION:**

In this way we have explored the operations on fuzzy sets, fuzzy relations by Cartesian product and max-min composition on fuzzy relations.

**ORAL QUESTION**

1. Explain the concept of a fuzzy set and its representation.
2. Define a fuzzy relation and its purpose in fuzzy logic.
3. How is the Cartesian product of two fuzzy sets computed?
4. Discuss the steps involved in performing max-min composition on fuzzy relations.