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Optimal theoretical building form to minimize direct solar irradiation

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Abstract

The aim of this paper is to explore the optimal geometric form of a building that minimizes direct solar irradiation incident on the envelope, and to find useful guidelines for building designers at the early decision-making stage. To achieve this goal, the mathematical theory of Calculus of Variation is used to find the theoretical optimal solutions and, based on this analytical approach, a simplified model to calculate annual solar heat gains is introduced. The benefit of using a totally analytical optimization is to provide simple and universal geometrical rules applicable in a general scenario, despite of using a numerical optimization that is more suitable to solve specific and complex cases by considering all of the aspects involved, e.g. the internal structure of the building or the optical/thermal properties of material used. The optimization is applied to various cases with different constraints.

Our results suggest some rules to follow, even in the general case whereby the plan is fixed by the building's designer, to achieve a reduction in the total amount of direct solar irradiation without necessarily reducing the annual solar heat gains. For this reason our results could be applied not only in hot regions but also in mild and cold ones.
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Keywords: Solar irradiation; Warm climates; Building form optimization; Calculus of Variation

1. Introduction

For a long time, since the 70 s oil crisis, great attention was paid worldwide to reducing energy consumption in buildings. The undoubted issue of energy consumption of highly glazed skyscrapers and the boom in construction in hot regions (Givoni, 1994; Bhattacharjee, 1982), e.g. in the city of Dubai, provides a new challenge for architects and other designers to develop new low-energy urbanism and architecture. The United Arab Emirates is currently the world's largest user of energy on a pro capita basis, with 70% of primary domestic energy usage being committed to buildings (Kazim, 2007).

Even in Europe, strategies to optimize the building envelope for solar energy utilization (e.g. local solar exposure conditions) are a crucial issue in the implementation of the EPB Directive 2010/31/EU (recast of Directive 2002/91/EC). For instance, in Italy, a country with a mild climate, peak rates of electricity, in summer 2006, were for the first time higher than those reached in winter of the same year. The same phenomenon occurred in 2008, 2010 and 2011 (Terna, 2012) (see Fig. 1).

Building orientation could provide reductions in cooling loads by minimizing solar penetration and absorption through windows, walls and roofs. Designers could follow some simple rules concerning solar exposure. Far from the polar and equatorial regions, for example, the highest intensity of solar radiation is on east- and west-facing walls in summer and south-facing walls in winter. This promotes a strong preference for the north–south orientation of main facades and glazing. Moreover shading systems, in particular for windows, are usually used to reduce solar heat gains but this leads to a reduction in natural daylight and hence an increase in energy consumption for artificial lighting, in

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Nomenclature

$\Phi_{sol,k}$	heat flow by solar gains through building element k (W)
$\Phi_{r,k}$	extra heat flow, due to thermal radiation to the sky from building element k (W)
Φ^{s}_{sol}	heat flow, by solar gains on a typical day, calculated with the simplified model (W)
$F_{sh,ob,k}$	shading reduction factor for external obstacles for the solar effective collecting area of surface k
$F_{r,k}$	form factor between the building element k and the sky
$A_{sol,k}$	effective collecting area of surface k (m^2)
$I_{sol,k}$	solar irradiance on surface k (W/m^2)
I_d	diffuse solar irradiance intensity (W/m^2)
I_b	direct solar irradiance intensity (W/m^2)
ρ	reflection coefficient of the ground
α_S	dimensionless absorption coefficient for solar radiation
$\bar{\alpha}_S$	average dimensionless absorption coefficient for solar radiation
β	solar altitude above the horizon angle
ϕ	solar azimuth angle
δ	declination angle
δ^s	equivalent declination angle of the simplified model
L	latitude angle
L^s	equivalent latitude angle of the simplified model
ω	hour angle
$\hat{n}$	normal vector
ψ	azimuth angle of a surface
Σ	tilt angle of a surface
θ	azimuth angle, describing building orientation
$\mathscr{L}$	Lagrangian operator
$S[\ldots]$	action operator

particular in case of high ceilings and shallow internal spaces where a well exploitation of daylight can considerably limit the use of artificial lighting (EN 15193, 2007; Kuhn et al., 2000; Kischkoweit-Lopin, 2002; Fontoynont, 2002; Scartezzini, 2003; Leccese et al., 2009).

These solutions are usually implemented with hindsight and studied when the building form is already defined, so it could be useful to investigate optimal building forms for solar energy utilization, to obtain also a priori design guidelines to support building designers. The problem of building's form optimization has been investigated in many publications especially with a numerical approach by using multicriteria optimization (Marks, 1997; Wright et al., 2002), genetic algorithms (Wright et al., 2002; Wang et al., 2006; Tuhus-Dubrow and Krarti, 2010; Kämpf and Robinson, 2010; Kämpf et al., 2010a,b), discrete polyoptimization (Jedrzejuk and Marks, 2002) and hierarchical geometry relations (Yi and Malkawi, 2009). In particular J.H. Kämpf and D. Robinson developed a method based on genetic algorithms to find the 3-dimensional form that maximizes solar energy utilization (Kämpf and Robinson, 2010; Kämpf et al., 2010a,b).

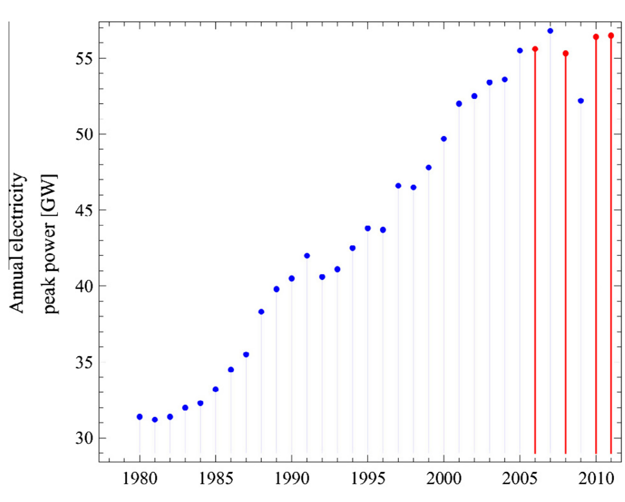

Fig. 1. Trend of the annual electricity peak power in Italy from 1980. In 2006, 2008, 2010 and 2011 (thick lines), the summer peak was higher than the one required in winter (Terna, 2012).

For computational reasons, those optimization algorithms need to select a particular parametrization in order to define the family of functions or forms to optimize, in particular with the definition of 'n' parameters. This corresponds to arbitrarily select the n-dimensional subspace where the optimization method is applied. Then 'optimized' solutions are obtained, but with respect to the particular parametrization chosen. Furthermore those algorithms do not ensure that global optimal solutions are achieved, but only local optimal solutions, neither in the restricted n-dimensional subspace.

However, the set of all possible forms, represented here as 3-dimensional surfaces, is an infinite-dimensional space. Therefore the probability that the global optimal solution is actually in the arbitrarily chosen n-dimensional subspace is zero. One could raise the objection that even if a subspace is selected, a well chosen parametrization could permit to approximate the optimal global form with arbitrary accuracy. But unfortunately a finite-dimensional subspace of functions cannot be dense in the infinite-dimensional space of functions.

In this work we apply an entirely analytical approach to finding 3-dimensional building forms minimizing direct solar irradiation by using the theory of Calculus of Variation that permits to explore the theoretical global optimal solution in the overall space of functions, in particular in the subspace of differentiable functions that is dense in the space of continuous functions. After the introduction of a simplified model to calculate solar heat gains, needed for a mathematical resolution of the problem, we find the optimal forms in various cases with various constraints. In the conclusions we summarize the results achieved and how this method could be further improved.

The authors have been studying the same problem by using genetic algorithms (Caruso, 2012).

2. Methodology

The method used consists in the development of a simplified model to calculate direct irradiation of the envelope. Simplifications and assumptions made are required in order to use the mathematical optimization method base on the Calculus of Variation, and, in particular, in order to attain a differential equation analytically solvable or easily interpretable. Finally a second and more accurate model to calculate irradiation of the envelope is introduced. This second model is base on the first one, but in addition it take into account diffuse irradiation and parameters are obtained by a best fit with experimental data.

The first simplified model is necessary to formalize the optimization method, whereas the second one is used to evaluate realistically the solar irradiation of the envelope of the optimal solutions found, and verify the validity of the optimization method.

2.1. Model of solar heat gains

In European technical standards, the heat flow by solar gains through a building element k, $\Phi_{sol,\,k}$ (W), is based on the effective collecting area and given by the following equation (EN ISO 13790, 2008):

$$\Phi_{sol,k} = F_{sh,ob,k} A_{sol,k} I_{sol,k} - F_{r,k} \Phi_{r,k} \tag{1}$$

where $F_{sh,ob,k}$ is the shading reduction factor for external obstacles, $A_{sol,k}$ (m^2) is the effective collecting area of surface k with a given orientation and tilt angle, in the considered zone or space, $I_{sol,k}$ (W/m^2) is the solar irradiance, i.e. the mean energy of the solar irradiation over the time step of the calculation, per square meter of collecting area of surface k, with a given orientation and tilt angle, $F_{r,k}$ is the form factor between the building element and the sky and $\Phi_{r,k}$ (W) is the extra heat flow, due to thermal radiation, to the sky from building element k.

The effective solar collecting area of an element of the building envelope, A_{sol} (m^2), is expressed by two different equations, respectively in the case of the glazed and opaque elements (EN ISO 13790, 2008), but in both cases A_{sol} is proportional to the projected area of the element A (m^2), in particular:

$$A_{sol,G} = \alpha_G A_G \tag{2}$$

$$A_{sol,O} = \alpha_O A_O \tag{3}$$

where the subscripts G and O refer to the glazed and opaque elements respectively, α_G is a factor that includes a shading reduction factor and solar energy transmittance of the glazed element considered and α_O is a factor that includes absorption coefficient for solar radiation, external surface heat resistance and thermal transmittance of the opaque element considered (EN ISO 13790, 2008).

Consistently with these standards, we introduce a simplified model to calculate solar heat gains, essential to apply the optimization with an analytical approach. We focus on direct solar irradiation. Diffuse irradiation and the last member of Eq. (1), that provides a correction for thermal radiation to the sky, are therefore not taken into account. The factor $F_{sh,ob,k}$ in Eq. (1), depending on external obstacles, is not considered, in order to not deal with the problem in a particular scenario. Following these hypotheses, the surface of the building envelope is considered homogeneous, representing on average all the elements. Moreover, we consider the case in which the glazed and opaque elements are uniformly distributed over the entire building envelope, and in particular we fix the percentage, r, of glazed surface in each portion of the envelope. Therefore the effective collecting area associated to a portion of the envelope, with area A, is calculated as follows:

$$A_{sol} = A_{sol,G} + A_{sol,O} = (\alpha_G r + \alpha_O(1-r))A \doteq \bar{\alpha}_S A \tag{4}$$

where we introduced the average dimensionless absorption coefficient for solar radiation, $\bar{\alpha}_S$, associated to each portion of the building envelope.

Hence, from Eq. (1), the simplified heat flow by solar gains ($\Phi^s_{sol,k}$) through a portion, k, of the envelope is calculated as follows:

$$\Phi^s_{sol,k} = \bar{\alpha}_S A_k I_{sol,k} \tag{5}$$

where A_k (m^2) is the area of the portion k.

The position of the Sun in the sky is defined by the altitude above the horizon β and the azimuth angle ϕ, using the standard trigonometric expressions, as follows (ASHRAE, 2009):

$$\begin{aligned} \beta &= \arcsin(\sin\delta\sin L + \cos\delta\cos L\cos\omega) \\ \phi &= \arcsin(\cos\delta\sin\omega/\cos\beta), \end{aligned} \tag{6}$$

where δ is the declination, L is the latitude, and ω is the hour angle. In our simplified model we consider the Sun's trajectory going exactly from East, at sunrise, to West, at sunset (see Fig. (2)). This corresponds to putting $\delta = 0$ and to considering an equivalent latitude L^s that changes every month. Hence, the position of the Sun in this simplified model is described by the following equations:

$$\begin{aligned} \beta &= \arcsin(\cos L^s\cos\omega) \\ \phi &= \arcsin(\sin\omega/\cos\beta), \end{aligned} \tag{7}$$

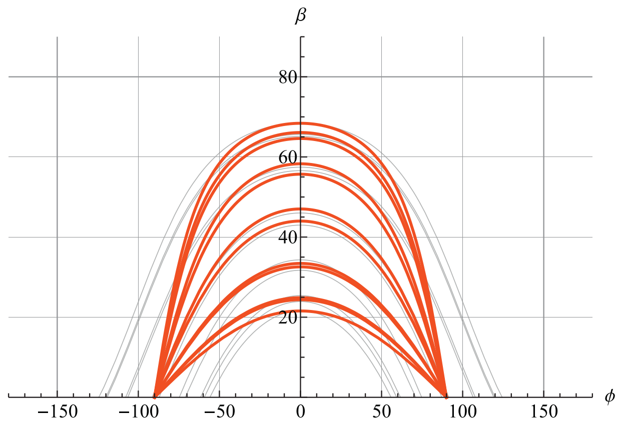

Fig. 2. The Sun's path during the year, expressed by the altitude above the horizon β and the azimuth angle ϕ. Comparison between the astronomical model using Cooper's formula for declination (Cooper, 1969) (thin lines) and the simplified model introduced hear (thick lines).

while the direction of the rays is represented by the vector:

$$\hat{r}(\omega) = \begin{pmatrix} \sin\omega \sin L^s \\ \cos\omega \\ -\sin\omega\cos L^s \end{pmatrix} \tag{8}$$

where the North is defined by $(1, 0, 0)$.

The building form is here considered as a 3-dimensional surface in which every point has coordinates $(x, y, z(x, y))$, where $z(x, y)$ is the continuous function (in particular, $C^2(\mathbb{R}^2)$) describing the building's form. Now, if we consider an infinitesimal element of this surface with area dA, we obtain from Eq. (5) the infinitesimal expression of the solar heat gain:

$$d\Phi^s_{sol,k} = \bar{\alpha}_S I(\omega)(-\hat{r}(\omega)\cdot\hat{n})^+ dA \tag{9}$$

where $I(\omega)$ is the solar irradiance intensity at time $\omega, \hat{n}$ is the normal vector of the infinitesimal surface considered, and the symbol $(\ldots)^+$ represents the positive part. Indeed we do not consider the case in which $\hat{r}(\omega)\cdot\hat{n} > 0$ because in this case the surface is definitely in the shade. The total Φ^s_{sol}, during one day on all the surfaces is then calculated with a double integral as follows:

$$\Phi^s_{sol} = \bar{\alpha}_S I \frac{T}{\pi}\int_S dA \int_0^\pi d\omega(-\hat{r}(\omega)\cdot\hat{n})^+, \tag{10}$$

where $T = 12$ h and I is the average irradiance intensity during the daytime.

2.2. Validity of the method

A simplified model is necessary for essentially two reasons: to well exploit the mathematical approach introduced below, and to obtain general rules on the optimal form, i.e. without defining thermal/optical characteristics of materials of the envelope, internal structure of the building or shading external obstacles and then avoiding to specialize the method only for specific scenarios. In particular the following assumptions have been made:

a. only direct irradiation is considered;
b. external obstacles are neglected;
c. the glazed and opaque elements are uniformly distributed over the entire building envelope;
d. Sun's paths go exactly from East to West;
e. the solar irradiance I is taken in average and constant throughout the day.

Those assumptions limit the validity of the optimization method in some cases and scenarios. In particular, by assuming (a.), the optimization method could perform worse in cloudy climates. A more detailed comment of this aspect is made in conclusion, since the irradiation of the envelope of the optimal solutions found is calculated below by using a second model that consider diffuse irradiation.

Assumption (b.) corresponds to considering low density urban scenario. Therefore this optimization method cannot be used in densely built urban settlement, and, in general, in the case in which shading from obstacles is relevant.

Assumption (c.) is valid in various scenarios, e.g. skyscrapers, and, in general, in case of buildings designed as a uniform repetition of the same architectural pattern all along the envelope. On the contrary that assumption exclude the possibility of heterogeneous distribution of transparent partition on the envelope, e.g concentrate glazed elements on a particular side of the building.

The last assumptions (d.) and (e.) are made for a relevant technical mathematical simplification for the development of the optimization method. The most relevant effect of those assumption is in summer, that is in fact the season when it is more useful to minimize direct irradiation. Indeed, the difference between the real and the simplified Sun's path is relevant in sunrise and sunset, and, in particular, the solar exposure time is shorter in those parts of the day if using the simplified model. On the other hand, the real value of I is lower in the beginning and in the end of the day in respect to the rest of the daytime. Those two effects are not separately negligible but opposed. Therefore the combination of those assumptions make a not relevant effect on the results, as it will be shown below, in Section 2.4.

2.3. Optimization with the Calculus of Variation

Eq. (10) represents the integral form of daily direct solar irradiance, depending on the building form and, in particular, on the function $z(x, y)$. I.e. Φ^s_{sol} is a functional operator expressed as $\Phi^s_{sol}[z]$, to indicate that it depends on the function $z(x, y)$. Our goal is to optimize the operator, i.e. to find the maximum/minimum value achievable and the corresponding functions. We then use the theory of Calculus of Variations (Goldstine, 1980; Dacorogna, 1989), and the Lagrangian formalism.

In general, let $S[z]$ be an integral operator, which is called *action*, expressed in terms of $z(x, y)$ as follows:

$$S[z] = \int dxdy\,\mathscr{L}\left[z, \frac{\partial z(x,y)}{\partial x}, \frac{\partial z(x,y)}{\partial y}, x, y\right] \tag{11}$$

where $\mathscr{L}$ is the Lagrangian operator which depends on x, y, $z(x,y)$ and its first derivatives. The function $z(x,y)$ that makes S maximum or minimum is solution of the Euler–Lagrange differential equation, which has the following expression (Dacorogna, 1989):

$$\frac{1}{\partial x}\left(\frac{\partial \mathscr{L}}{\partial\left(\frac{\partial z(x,y)}{\partial x}\right)}\right)+\frac{1}{\partial y}\left(\frac{\partial \mathscr{L}}{\partial\left(\frac{\partial z(x,y)}{\partial y}\right)}\right)=\frac{\partial \mathscr{L}}{\partial z}. \tag{12}$$

Given S and the resulting expression of $\mathscr{L}$, it is then possible to find the function $z(x,y)$ that is optimal for the given problem, at various boundary conditions, by solving the differential Eq. (12). Note that you cannot know in advance if $z(x,y)$ minimizes or maximizes the operator S.

The normal vector $\hat{n}$ of the surface, described by the function $z(x,y)$, has the following expression:

$$\hat{n}=\frac{1}{\sqrt{\left(\frac{\partial z}{\partial x}\right)^2+\left(\frac{\partial z}{\partial y}\right)^2+1}}\begin{pmatrix}-\frac{\partial z}{\partial x}\\ -\frac{\partial z}{\partial y}\\ 1\end{pmatrix}, \tag{13}$$

and $dA=dxdy\sqrt{\left(\frac{\partial z}{\partial x}\right)^2+\left(\frac{\partial z}{\partial y}\right)^2+1}$, hence by integrating Eq. (10), we obtain the action relative to our problem:

$$\begin{aligned}\Phi^s_{sol}[z]&=\bar{\alpha}_S I\frac{T}{\pi}\int_S dxdy\left(\sqrt{\bar{z}_x^2+\bar{z}_y^2}+\bar{z}_x\right)\\ &=\int dxdy\mathscr{L}_{sol}\left[z,\frac{\partial z}{\partial x},\frac{\partial z}{\partial y},x,y\right]\end{aligned} \tag{14}$$

where $\bar{z}_x=\frac{\partial z}{\partial x}\sin L^s+\cos L^s, \bar{z}_y=\frac{\partial z}{\partial y}$ and $\mathscr{L}_{sol}=\bar{\alpha}_S I\frac{T}{\pi}\left(\sqrt{\bar{z}_x^2+\bar{z}_y^2}+\bar{z}_x\right)$ is the Lagrangian operator describing the problem here considered. In order to find the function $z(x,y)$ that optimizes the operator $\Phi^s_{sol}[z]$, we consider $\bar{\alpha}_S, I$ and T as constant. Hence, the optimal solutions, obtained by using the Calculus of Variations, don't depend on $\bar{\alpha}_S, I$ and T, since they are factors of the integral operator.

2.4. Fitting the model with experimental data

After optimization, the solutions are verified with a more realistic model that calculates not only solar heat gains from direct solar irradiation but also from ground and sky-diffuse solar radiation. For instance, the solar heat gains on a flat surface with area A, with orientation in space described by the azimuth angle ψ and the tilt angle of the surface Σ, are calculated as follows:

$$\begin{aligned}\Phi^{check}_{sol}=&\bar{\alpha}_S AT[I_d\rho(1-\cos\Sigma)+I_d(1+\cos\Sigma)\\ &+\frac{1}{\pi}I_b(\cos\Sigma\cos L^s+\cos\psi\sin\Sigma\cos L^s\\ &+\sqrt{\sin^2\psi\sin^2\Sigma+(\cos\Sigma\cos L^s+\cos\psi\sin\Sigma\cos L^s)^2}\bigg)\bigg]\end{aligned} \tag{15}$$

where I_d (W/m^2) is the diffuse solar irradiance intensity, I_b (W/m^2) is the direct one and ρ is the reflection coefficient of the ground. The equivalent latitude is calculated putting $L^s=L+\delta^s$, where δ^s is a correction that depends on the specific month.

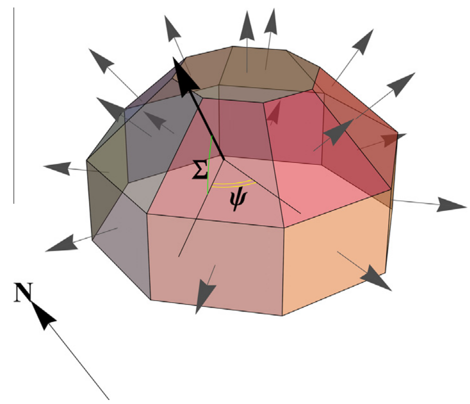

Fig. 3. Diagram of the 17 directions considered to extrapolate ENEA data for the fit.

To make this model as realistic as possible, we fit our model with real data of solar irradiation for each month. The empirical data are obtained from ENEA (xxxx) considering 17 different orientations of a flat surface, i.e. with $\Sigma = 0°, 45°, 90°$ and $\psi = -180°, -135°, -90°, -45°, 0°, 45°, 90°, 135°$ (see Fig. 3), for around 100 large Italian cities at different latitudes. The parameters used for the fit are I_d, I_b and δ^s, and depend on the month considered. The values obtained are listed in Table 1.

Finally, we test the model by comparing experimental data with the best fit, taking as examples the cities of Milan and Palermo, as shown in Fig. 4. Moreover, in Fig. 5 it is possible to compare the annual irradiation between the experimental data and the model used, for each of the 17 directions considered in Fig. 3 in Milan.

2.5. Constraints and boundary conditions: cases handled

The solutions of a variational problem depend on the boundary conditions and constraints, imposed accordingly with the problem considered. In architecture, the maximum volume value is one of the most significant parameters, usually regulated by laws and standards. As an alternative,

Table 1
Best fit parameters for each month.

Month	I_b (MJ/m^2 day)	I_d (MJ/m^2 day)	δ(s)
January	22.03	2.47	−24.27
February	29.64	3.68	−8.01
March	43.92	5.36	7.55
April	35.67	10.77	14.54
May	23.61	16.54	20.42
June	17.66	20.03	31.38
July	21.68	19.17	26.54
August	31.55	13.66	17.03
September	39.34	7.59	9.64
October	32.32	4.31	−0.32
November	22.07	2.84	−13.86
December	17.07	2.16	−19.91

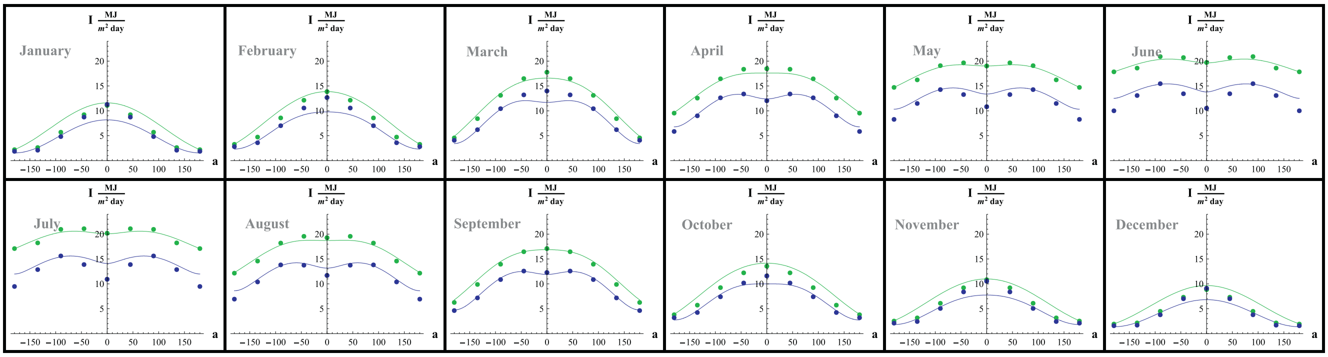

Fig. 4. Best fit graphs of Eq. (13) to ENEA data. The comparison is made for each month of the year, for 16 different orientations, respectively 8 with $\Sigma = 45°$ (clear/green lines and points) and 8 with $\Sigma = 90°$ (dark/blue lines and points), for Milan with $L = 45.48°$. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

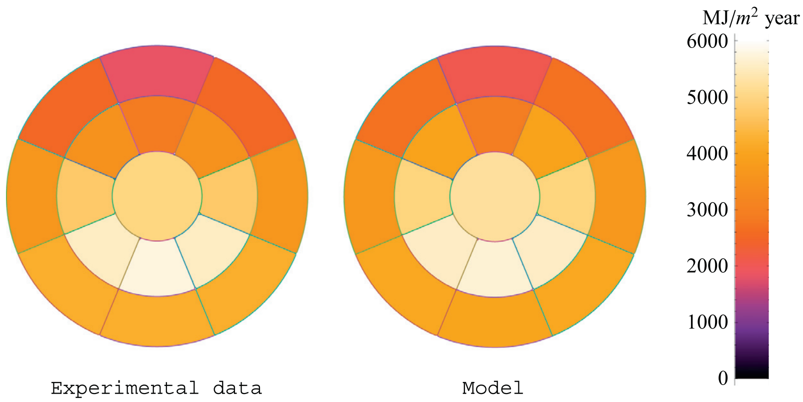

Fig. 5. Comparison of the annual irradiation between the experimental data (left) and the model used (right) for each of the 17 directions considered.

in same cases the local standards set the total floor area value, but this constraint can be considered equivalent to fixing the maximum volume because the height of the floors is usually regulated by local standards, e.g. it is generally equal to 3 m in residential construction. Hence, the maximum volume is here considered a constraint as follows:

$$V = \int dx\, dy\, z(x,y) \tag{16}$$

Note that it is possible to design a form with a surface area as big as is required, even if at fixed volume. This means that the least upper bound of the values of Φ_{sol} is equal to infinity. Therefore, the solutions that maximize Φ_{sol} are degenerate solutions (e.g. infinite plans, infinitely thin shapes) and they were not taken into account. The finite solutions that could be found are then minimal for the problem considered, as the results achieved confirm.

In the first case handled, we find a family of solutions for the general problem at fixed volume, without other constraints. Afterwards, we study the same problem with an additional boundary condition, i.e. fixing the board of the base of the building. This second case is closer to practice, in which building perimeter is shaped by the site geometry or by other technical needs.

3. Results

The differential equation describing the optimal form $z(x,y)$ is obtained by applying Eq. 12 to the Lagrangian operator $\mathscr{L}_{sol}$:

$$\frac{\partial^2 z(x,y)}{\partial y^2}\left(cot(L)+\frac{\partial z(x,y)}{\partial x}\right)^2 + \frac{\partial z(x,y)}{\partial y}\left(\frac{\partial z(x,y)}{\partial y}\frac{\partial^2 z(x,y)}{\partial x^2} - 2\left(cot(L)+\frac{\partial z(x,y)}{\partial x}\right)\frac{\partial^2 z(x,y)}{\partial x \partial y}\right) - \lambda = 0 \tag{17}$$

where λ is the Lagrange multiplier that refers to the constraint on the volume. It is possible to convert this

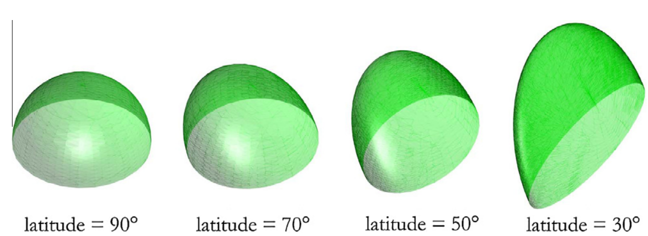

Fig. 6. A family of optimal solutions parametrized by the latitude of the site, i.e. 90° (north pole), 70°, 50° and 30°. The method used was developed in reference (Caruso, 2012).

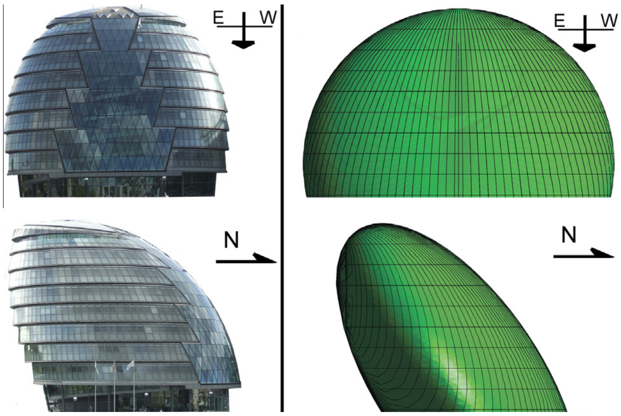

Fig. 7. Comparison between an optimal form, obtained theoretically for the latitude of London (51,5°), and the Greater London Authority building, designed and realized in 2002 by Sir Norman Foster (Caruso, 2012; Abel, 2004; Allison, 2006).

differential equation into an equation of the 2D curvature of the sections parallel to the ecliptic.

3.1. Optimal form

It is possible to demonstrate that, without constraints, the solution is trivial, but in the case in which the volume V is fixed we find more interesting solutions. In particular, we find a family of solutions with ovoidal form, parametrized with respect to the latitude, that solve the differential Eq. (12) (see Fig. 6). The optimal form of the solutions family found, at the latitude of London (51,5°N), is very close to an existing building, Greater London Authority-building (GLA), realized in 2002 by Sir Norman Foster with the scope to reduce direct solar irradiation (see Fig. 7).

The GLA building was designed as a sphere distorted to the south (Abel, 2004; Allison, 2006). The sphere is the optimal form that has the minimal surface area at fixed

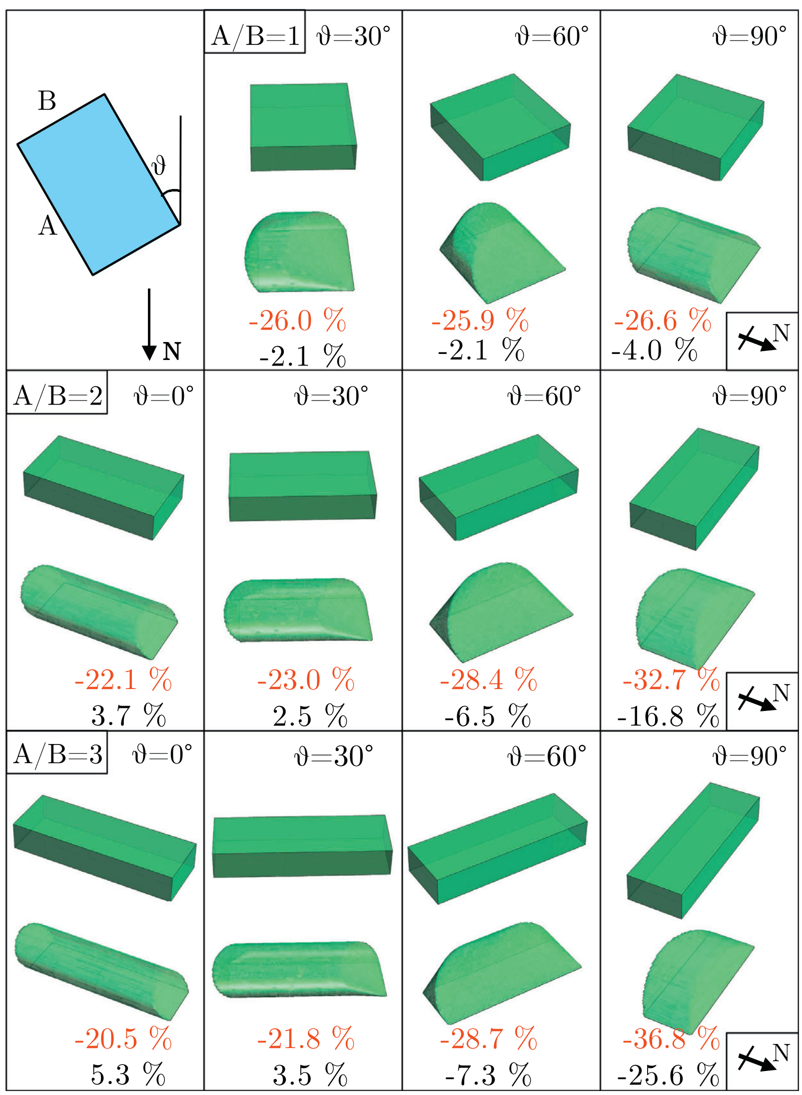

Fig. 8. Energy saving rate due to direct irradiation (clear/red) and to total irradiation (bold/black) between cuboid buildings and the corresponding optimal forms with the same rectangular plan, at various orientations (θ) and aspect ratios (A/B), with latitude $L = 45°$. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

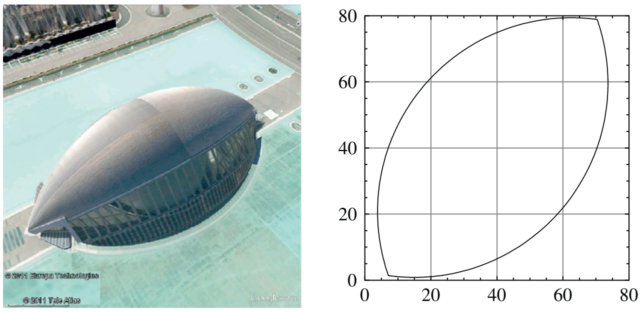

Fig. 9. The Hemisfèric building in Valencia (left) and its floor plan (right).

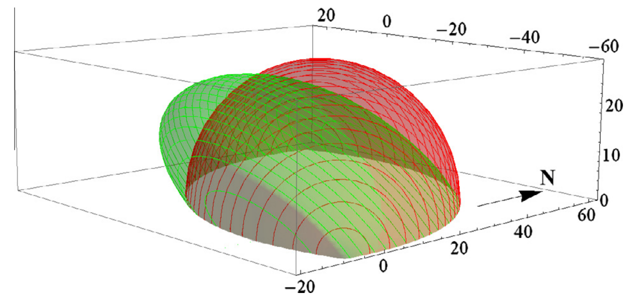

Fig. 10. Comparison by superposition of the Hemisfèric with the corresponding optimal form (dashed lines).

volume, and in fact compactness should reduce total heat gains on the envelope (Sun at Work in Europe, 1999). The distortion to the south intuitively improves this performance. Osaji et al. (2007) state that the Greater London Authority building's energy consumption level can be deduced to be less than half levels in the Department of the Environment, Transport and the Regions (DETR) good practice office guide. On the other hand, a building with optimal form has not necessarily a good energy performance.

3.2. Rectangular plan

In this application, we study the case in which the base perimeter of the building is rectangular, that is a common case, for a particular latitude $L = 45°$. The performance of the optimal forms are compared with the corresponding 'standard' building, i.e. a cuboid building with the same rectangular base and volume. First of all, solar heat gains of cuboid buildings are calculated as the sum of the roof and wall gains. Solar heat gains on the roof are calculated using Eq. (10) and putting the normal vector of the surface $\hat{n} = (0, 0, 1)$, as follows:

$$\Phi_{roof} = \bar{\alpha}_S\ ITAB/\pi \int_0^{\pi} d\omega(-\hat{r}(\omega)\cdot\hat{n})^+ = \sqrt{2}\bar{\alpha}_S\ ITAB/\pi,$$

where A is the length, B the depth, H the height and θ the orientation, i.e. the azimuth angle between the longest edge of the rectangular base direction and that of the north–south (see Fig. 8).

Solar heat gains on walls are similarly calculated as follows:

$$\Phi_{walls} = \bar{\alpha}_S ITH/\pi\left(A\sqrt{1+\sin^2\theta} + B\sqrt{1+\cos^2\theta}\right)$$

then the total solar heat gains on the cuboid is expressed by the following equation:

$$\Phi_{cub} = \bar{\alpha}_S IT/\pi\left(AB + V/B\sqrt{1+\sin^2\theta} + V/A\sqrt{1+\cos^2\theta}\right)$$

We take into account cuboids with the same volume at different base aspect ratios, respectively with $A/B = 1$, 2, 3, and four different orientations, $\theta = 0°$, 30°, 60°, 90°, then we calculate their performances and compare them with the corresponding optimal form ones. Fig. 8 shows the results and in particular that it is possible to reduce the solar heat gains due to direct irradiation by up to 36%, while the total solar heat gains (in black in Fig. 8) are not necessarily reduced. Hence, our optimization method could be applied even in mild and cold regions, in which direct solar irradiation must still be screened while the diffuse irradiation is useful for natural lighting and energy supplies.

3.3. Cases of study

Our optimization method has been extended to a more general case, i.e. with a general base form of the building. Our results show, in fact, that the optimal forms should have a particular characteristic, i.e. the sections of the building parallel to the ecliptic should be as compact as possible. Using this criterion, the idea was to consider an existing building and to find the corresponding optimal form with the same volume and the same ground floor plan.

The first case of study is a building of modern architecture and, in particular, the Hemisfèric in Valencia at $L = 39.47°$ (see Fig. 9), a building designed by the architect Santiago Calatrava (1996–1998). If we know the volume and the ground floor plan, the corresponding optimal form is obtained. A direct comparison is shown in Fig. 10, by superposition. Table 2 shows a reduction in direct solar irradiation of 9.63% and in total of 6.32%. This could be due to the compactness of the Hemisfèric building.

Table 2
Comparison of building solar heat gains with the optimal ones.

	Direct solar irradiation (GW h/year)	Diffuse solar irradiation (GW h/year)	Total solar irradiation (GW h/year)
Hemisfèric	2.75	4.12	6.87
Optimal	2.49	3.95	6.44
Comparison	−9.63%	−4.11%	−6.32%
Bel-Air tower	4.95	8.33	13.28
Optimal	3.06	5.93	9.00
Comparison	−38.03%	−28.77%	−32.22%

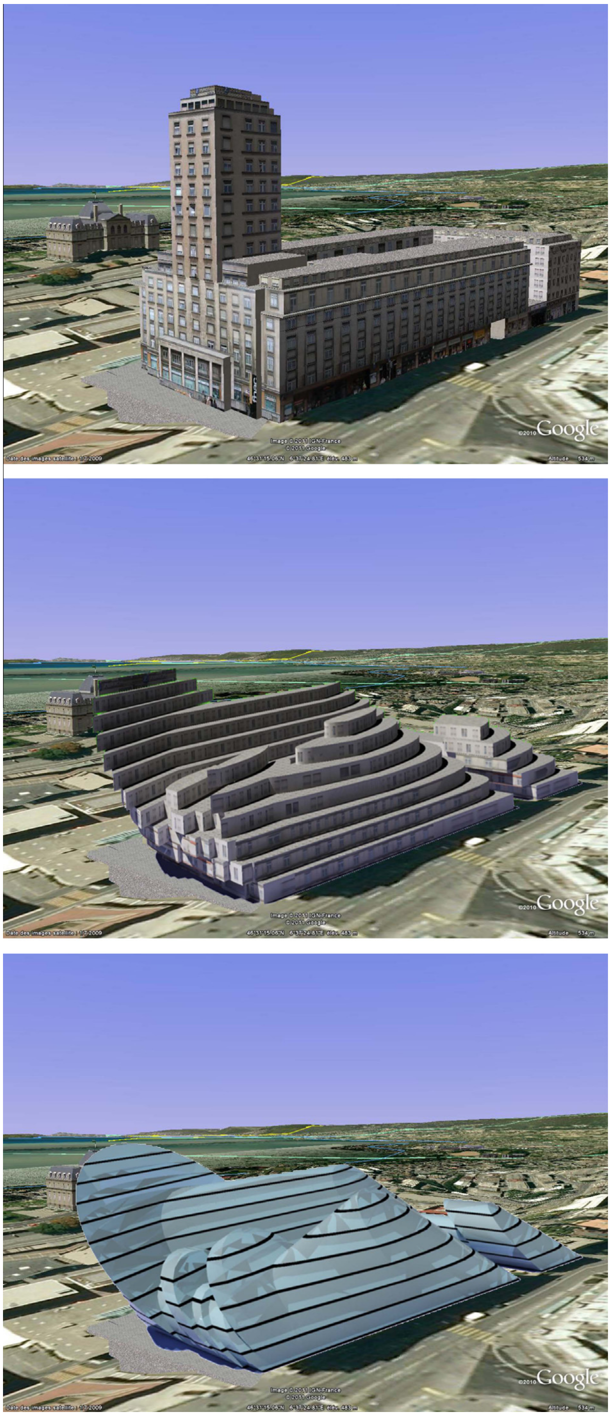

Fig. 11. Comparison of the Tour de Bel-Air building (top), in Lausanne, with the corresponding optimal form as “terraced building” (middle) and with curved envelope (bottom).

The second case of study is an office building realized in 1931: the Bel-Air Tower in Lausanne at $L = 46.52°$. As in the first case, the corresponding optimal form is obtained and compared with the existing one (see Fig. 11). In this case, as shown in Table 2, there is a remarkable reduction in solar heat gains, and in particular in the one due to direct solar irradiation, of 38%, showing that the existing building is not compact enough.

4. Conclusions

The reduction in solar gains on the building envelope, in particular due to direct solar irradiation, leads to a significant theoretical reduction in energy consumption not only in warm but also in mild regions, as shown in the cases of study considered. We found that the optimal form of the building, obtained with a mathematical approach, depends in general on the latitude and that there already exists some examples in modern architecture.

In particular, the theoretical optimal forms have a particular characteristic, i.e. the sections of the building parallel to the average ecliptic have a constant bi-dimensional curvature, and then they are circular. As a result, useful guidelines to building designers in the common professional practice could be given at the early decision-making stage: in order to minimize direct irradiation on the envelope, those particular sections should be as compact as possible.

In the case of a rectangular plan, the results show that an optimal form of the building can reduce direct solar gains by up to 20% without necessarily reducing the total solar gains. Therefore, the solutions found are also useful to reduce energy consumption of a building with a glassed envelope, even in mild and cold regions.

Other insights may be gained by integrating the analytical approach used here with numerical approaches, in order to address the problem of optimizing energy performance in more complex scenarios, in order to go beyond some assumptions of our theoretical approach.

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