Static Analysis for Various objects with different sizes

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a. Stiff objects vs soft objects?
b. Fragile/brittle properties?
c. Object's edge and overall part geometry?

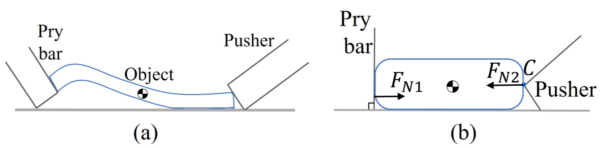

Fig. A4 (a) The diagram of prying the soft object. (b) The diagram of prying the thick object.

Thank you very much for your constructive questions. In this work, there are three assumptions about the target objects:

1. We treat the object as an ideal rigid body.
2. The chamfers in object corners are small enough.
3. The object is thin and h/l is estimated to be zero.

In this work, we did not consider modeling the soft objects or thick objects. We analyze the model of these objects in this responding letter as below:

(a) For the soft objects, the prying model differs from the rigid object model. For the rigid object, the tilt angle of the pry bar α and the tilt angle of objects φ follows:

$$w \cdot \sin\alpha = l \cdot \sin\varphi \tag{7}$$

And for the soft objects as shown in Fig. A4(a), φ will not follow Eq.(7) because the object may deform. In this case, φ depends on the object's material. Therefore, it is difficult to determine when the object can be pried up and inserted. We suppose to analyze the model of the soft object in the future.

(b) For fragile objects, it is easily got damaged due to deformation. And buckling a common situation for the deformation. The analysis of the buckling is shown below:

Buckling is the sudden change in the shape of objects under load. According to the Euler formula, the force F_{bkl} causing buckling is given below:

$$F_{bkl} = \frac{\pi^2 E_m I}{l^2} \tag{9}$$

where E_m is the modulus of elasticity, I is the smallest area moment of inertia.

And when the force is larger than F_{bkl}, the card deforms suddenly, and the object may get damaged.

where E_m is the modulus of elasticity, I is the smallest area moment of inertia.

(c) The contact point between the fingertips and the object will change for the different chamfer sizes and thicknesses of the objects. The analysis of the chamfer size and the thickness is shown below. The calculation results are very complex, and the space of the paper is limited. Therefore, We suppose we can show these results in future work.

The model analysis including the chamfer size and the the overall part geometry is shown below:

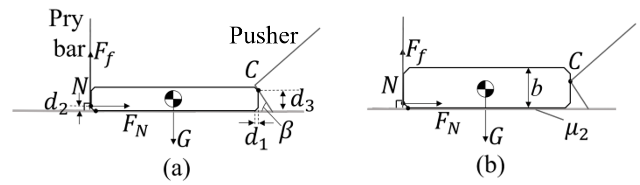

Fig. A5 (a) FBD model of object when the contact point C is lower than the edge of the pusher. (b) FBD model of object when the contact point C is higher than the edge of the pusher.

The Free body diagram (FBD) is shown in Fig. A5 of this letter. Fig. A5 shows the model that the object tends to be pried up. d_1 and d_2 represent the chamfer sizes of the object. b is the thickness of the object. C is the contact point between pusher and the object, and N is the contact point between pry bar and the object. β is the tilt angle of the pusher. μ_1 is the friction coefficient between fingertip and the object, and μ_2 is the friction coefficient between the object and surface. w is the width of the fingertip. From the static equilibrium of FBD, F_N can be written as:

$$F_N = \frac{BG}{\mu_1 - A} \tag{A2}$$

The forms of A and B depend on the thickness of the object. When $w\sin\beta > b$ (Fig. A5(a)):

$$A = \frac{(s_\beta + \mu_1 c_\beta)d_3 + (\mu_1 s_\beta - c_\beta)d_1 - Kd_2}{K(a - d_1) + (s_\beta + \mu_1 c_\beta)d_3 + (\mu_1 s_\beta - c_\beta)d_1} \tag{A3}$$

$$B = \frac{K(0.5a - d_1) + \mu_2(s_\beta + \mu_1 c_\beta)d_3 + \mu_2(\mu_1 s_\beta - c_\beta)d_1}{K(a - d_1) + (s_\beta + \mu_1 c_\beta)d_3 + (\mu_1 s_\beta - c_\beta)d_1} \tag{A4}$$

where

$$K = (\mu_1 - \mu_2)c_\beta + (1 + \mu_1\mu_2)s_\beta \tag{A5}$$

And when $wsin\beta < b$ (Fig. A5(b)):

$$A = \frac{d_3 + \mu_1 d_1 - (1 + \mu_1\mu_2)d_2}{(1 + \mu_1\mu_2)(a - d_1) + \mu_2 d_3 + \mu_1\mu_2 d_1} \tag{A6}$$

$$B = \frac{\mu_2 d_3 + \mu_1\mu_2 d_1 + (1 + \mu_1\mu_2)(0.5a - d_1)}{(1 + \mu_1\mu_2)(a - d_1) + \mu_2 d_3 + \mu_1\mu_2 d_1} \tag{A7}$$

And the insertion model is shown in Fig. A6.

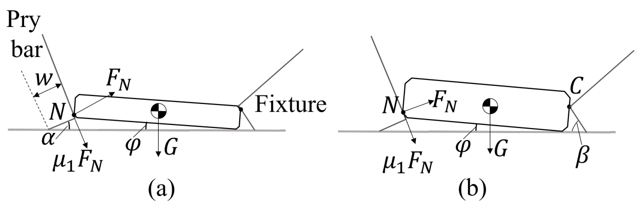

Fig. A6 (a)FBD of the thin object in the insert phase.(b)FBD of the thick object in the insert phase.

With the rotation of the pry bar, one end of the object is lifted gradually. FBD in inserting phase is shown in Fig. A6. From the static equilibrium of FBD, F_N can be written as:

$$F_N = mg\frac{0.5(c_\varphi a - s_\varphi b) + T \cdot d_3}{(s_{\alpha+\varphi} - \mu_1 c_{\alpha+\varphi}) \cdot (a - d_2 s_\varphi) + S \cdot d_3} \tag{A8}$$

When $wsin\beta > b$(Fig.A7(a)) :

$$S = -\frac{(1 - \mu_1\mu_2)c_\alpha + (\mu_1 + \mu_2)s_\alpha}{(1 + \mu_1\mu_2)s_\beta + (\mu_1 + \mu_2)c_\beta}(s_{\beta-\varphi} + \mu_1 c_{\beta-\varphi}) \tag{A9}$$

$$T = -\frac{\mu_2(s_{\beta-\varphi} + \mu_1 s_{\beta-\varphi})}{(1 - \mu_1\mu_2)s_\beta + (\mu_1 + \mu_2)c_\beta} \tag{A10}$$

where φ is the angle between the object and surface, $c_\varphi = cos(\varphi)$, $s_\varphi = sin(\varphi)$,$s_{\beta-\varphi}$=$sin(\beta - \varphi)$, $c_{\beta-\varphi}$=$cos(\beta - \varphi)$.

When $wsin\beta < b$ (Fig. A6(b)):

$$S = -\frac{((1 + \mu_1\mu_2)c_\alpha + (\mu_1 - \mu_2))}{c_\varphi((1 + \mu_1\mu_2)c_\beta + (\mu_2 - \mu_1)s_\beta)} \tag{A11}$$

$$T = -\frac{\mu_2}{((1 + \mu_1\mu_2)c_\beta + (\mu_2 - \mu_1)c_\beta)c_\varphi} \tag{A12}$$

The analysis in this part is based on the condition that the object has been pried up. Therefore, there is no relative sliding between the object and the pry bar before the insert phase. Based on this condition, we get the relationship between φ and α:

$$\varphi = arcsin(\frac{wsin\alpha + d_2 cos\alpha}{a}) \tag{A13}$$

From the Eq.(A2) to Eq.(A13), we can calculate F_N for the objects with different chamfer sizes and thickness. However, due to the limited space of the paper, we cannot put it in the paper.

References