Innovative Information System Approach for Robust Multi-criteria Decision Making with Unknown Weights

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Abstract

In an era where informed decision-making depends on reliable and robust information systems, this paper presents an innovative approach to the evaluation of multi-criteria decision-making problems with unknown criteria weights. Focused on selecting a city electric vehicle for personal use, the study extensively explores criteria weight scenarios within the Stable Preference Ordering Towards Ideal Solution (SPOTIS) method. Using a novel fuzzy ranking concept for ranking definition under multiple evaluation scenarios enhances decision reliability under varied input conditions. Emphasizing the significance of reliable information systems and customer support, this study aims to empower decision-makers with comprehensive insights into complex decision problems.

Keywords: uncertainty, decision support, unknown weights, fuzzy ranking, decision-making

1. Introduction

Operational research is crucial in today's complex decision-making landscape, providing a foundation for effective management [2]. Supported by Information Systems (IS) and Multi-Criteria Decision Analysis (MCDA) methods, decision-makers can tackle a variety of factors in complex problems more efficiently. MCDA methods allow building decision models tailored to specific criteria and their relevance and have proven effective across various applications such as business analytics, sustainable development, and information systems evaluations [1]. These methods are invaluable for structuring decision problems and generating robust solutions, making them essential tools in modern management practices.

Despite their popularity, MCDA methods present challenges, particularly in selecting appropriate evaluation methods and determining the relevance of criteria. Criteria weights significantly influence final results [4]. Moreover, they can be difficult to identify accurately. The selection of these weights often involves a consensus between objective and subjective methods, each with its limitations [5]. Objective methods may rely solely on data, potentially leading to misleading conclusions, while subjective methods can be biased by expert judgment. The challenge becomes even more visible when the relevance of criteria is unknown, necessitating a reliable approach to handle multiple scenarios for criteria weight values distribution. Addressing these issues is essential for ensuring reliable decision-making and generating robust, trustworthy recommendations.

To tackle these challenges, this paper proposes an approach to evaluate multi-criteria decision problems with unknown criteria weights, using the Stable Preference Ordering Towards Ideal Solution (SPOTIS) method. By exploring various weight scenarios, the study aims to enhance the reliability of the decision-making process, validated in selecting city electric vehicles. The study applies a novel fuzzy ranking concept to examine the robustness of evaluation outcomes, providing a detailed and reliable understanding of recommendations under varied conditions. The main contributions of the study are developing a comprehensive evaluation approach, applying a fuzzy ranking concept for robust recommendations, and verifying the method's performance in vehicle selection.

2. Preliminaries

2.1. Criteria weights determination

Exploring all possible vectors of criteria weights for a given decision problem enables a thorough examination of the potential relevance of every criterion within the problem space. The proposed approach for criteria weights generation uses a recursive calculation of the weight vectors according to the pseudo-code presented in Algorithm 1.

Algorithm 1 Algorithm of criteria weights vector generation.

```
Require: Step s, Number of criteria n
max_points = 1 / s
procedure R_WEIGHTS(n, max_points)
    if n == 2 then
        for for i in range(max_points + 1) do
            Write results(i, max_points - i)
        end for
        return results
    else
        for i in range(max_points + 1 do
            for rest in R_WEIGHTS(n - 1, i) do
                Write results(max_points - i, rest)
            end for
        end for
        return results
    end if
end procedure
```

2.2. Fuzzy ranking

Departing from conventional methods of aggregating MCDA results, the fuzzy ranking approach determines membership degrees of alternatives regarding the ranking positions, indicating the results' robustness. The fuzzy ranking produces a two-dimensional matrix that shows these membership degrees, giving decision-makers a nuanced understanding of the uncertainty and robustness associated with the rankings. This approach addresses the limitations of traditional aggregation methods, offering a more comprehensive and informed perspective in multi-criteria decision analysis. The formal representation of this matrix is defined as (1):

$$M = \begin{array}{c} \\ R_1 \\ R_2 \\ R_3 \\ \dots \\ R_m \end{array} \begin{array}{c} \begin{array}{ccccc} A_1 & A_2 & A_3 & \dots & A_m \end{array} \\ \begin{bmatrix} p_{11} & p_{21} & p_{31} & \cdots & p_{m1} \\ p_{12} & p_{22} & p_{32} & \cdots & p_{m2} \\ p_{13} & p_{23} & p_{33} & \cdots & p_{m3} \\ \dots & \dots & \dots & \dots & \dots \\ p_{1m} & p_{2m} & p_{3m} & \cdots & p_{mm} \end{bmatrix} \end{array} \tag{1}$$

where p_{ij} represents the frequency of placing $i-th$ alternative within $j-th$ position in ranking.

3. Study case and results

This study introduces an approach to handle unknown criteria weights in multi-criteria decision problems. By extensively exploring the criteria weights space, it examines MCDA outcomes under various assessment scenarios. The analysis resolution can be adjusted for accuracy or time efficiency, generating different combinations of criteria weights. These scenarios are applied within the SPOTIS method to ensure ranking stability and resistance to the Rank Reversal paradox. The fuzzy ranking concept then analyzes these rankings to present comprehensive and detailed recommendations to decision-makers. This approach enhances stable multi-criteria analysis and can be integrated into Information Systems for decision support. The study focuses on selecting city electric vehicles using the following criteria: C_1: Cost, C_2: Engine power, C_3: Maximum torque, C_4: Battery capacity, C_5: Range, C_6: Length, C_7: Maximum load capacity, C_8: Maximum speed, C_9: Charge time 10%-80% 50kW, C_{10}: Full charge time 22kW. The evaluation considered 15 alternatives representing city electric vehicles, defined from a publicly available dataset in [3].

The multi-criteria decision analysis of city electric vehicles was performed with defined input data. Conventional assessments assume that criteria weights are determined, but challenges arise when these weights are unknown. Objective weighting methods, commonly used to extract criteria relevance from decision matrix data, generate a single vector of criteria weights. This approach has limitations, as the generated weights may not accurately reflect the actual relevance of the criteria. A more appropriate solution is to analyze multiple scenarios to examine result robustness under varied conditions. This research addresses the gap by generating possible combinations of criteria weights with a defined resolution of 0.05. The obtained set of weights was used in the SPOTIS method to evaluate alternatives. Multiple rankings from different scenarios were then analyzed with a fuzzy ranking matrix to assess robustness. The procedure generated $92,378$ possible combinations for 10 criteria, as shown in Table 1. This extensive evaluation provides comprehensive insights into the decision problem under varied conditions.

Table 1. Sample criteria weights vectors generated with the proposed approach using step 0.05.

S_i	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}
S_1	0.05	0.40	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.20
S_2	0.05	0.35	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.20
S_3	0.15	0.30	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.20
...	...	...	...	...	...	...	...	...	...	...
S_{92378}	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.55

Table 2. Fuzzy ranking matrix defined based on the rankings obtained from SPOTIS method and criteria weights scenarios.

R_i	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9	A_{10}	A_{11}	A_{12}	A_{13}	A_{14}	A_{15}
R_1	0.13	0.10	0.01	0.00	0.00	0.00	1.00	0.12	0.36	0.00	0.00	0.00	0.17	0.00	0.08
R_2	0.16	0.18	0.01	0.00	0.00	0.05	0.32	0.13	1.00	0.00	0.04	0.02	0.12	0.03	0.15
R_3	0.34	0.63	0.05	0.00	0.00	0.17	0.08	1.00	0.07	0.00	0.05	0.11	0.40	0.06	0.60
R_4	0.70	0.38	0.06	0.00	0.00	1.00	0.03	0.28	0.08	0.00	0.10	0.23	0.40	0.18	0.25
R_5	0.44	0.78	0.22	0.00	0.00	0.26	0.02	0.30	0.03	0.00	0.16	0.70	0.85	0.10	0.60
R_6	0.76	0.42	0.33	0.00	0.00	0.38	0.02	0.17	0.04	0.00	0.26	0.81	0.67	0.17	0.40
R_7	0.42	0.30	0.41	0.00	0.00	0.24	0.01	0.12	0.02	0.01	0.38	1.00	1.00	0.14	0.53
R_8	0.43	0.29	0.56	0.00	0.00	0.19	0.02	0.09	0.03	0.01	0.44	1.00	0.84	0.20	0.47
R_9	0.43	0.36	0.69	0.01	0.01	0.18	0.02	0.07	0.02	0.01	0.65	0.72	0.78	0.26	0.29
R_{10}	0.56	0.40	1.00	0.01	0.01	0.14	0.01	0.05	0.02	0.02	1.00	0.37	0.32	0.19	0.38
R_{11}	1.00	0.45	0.61	0.04	0.03	0.10	0.01	0.04	0.01	0.04	0.68	0.17	0.15	0.21	1.00
R_{12}	0.18	1.00	0.50	0.09	0.08	0.06	0.00	0.02	0.02	0.14	0.62	0.01	0.00	1.00	0.07
R_{13}	0.01	0.07	0.05	0.95	0.33	0.00	0.00	0.00	0.00	0.96	0.01	0.00	0.00	0.12	0.12
R_{14}	0.00	0.04	0.01	1.00	1.00	0.00	0.00	0.00	0.01	0.23	0.00	0.00	0.00	0.15	0.01
R_{15}	0.00	0.01	0.01	0.57	0.71	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.13	0.00

The evaluation process used generated scenarios of criteria weights with the SPOTIS method. Each criteria weights vector from the possible combinations was used to evaluate the decision matrix, producing $92,378$ rankings for further analysis. A fuzzy ranking was then determined by calculating the frequency of each alternative's placement in specific positions and normalizing these values to indicate membership degrees, showing result robustness. Results presented in Table 2 indicated that the two most recommended city electric vehicles are robust selections under the given criteria weights. Alternative A_7 ranked 1st with the highest membership degree (1.00), significantly higher than A_9 (0.36). Other alternatives had lower membership degrees, not exceeding 0.17, indicating high uncertainty for the 1st position. Alternatives with a membership degree of 0.00 were not considered rational selections. For the 2nd position, A_9 was the most robust choice with a membership degree of 1.00, followed by A_7 . Although some criteria weights ranked these alternatives in reverse order, most scenarios favored A_7 as 1st. The fuzzy set representing the calculated values of membership degrees for the 1st position is shown in Equation (2).

$$\mu_{R_1} = \left\{ \frac{0.13}{A_1}, \frac{0.10}{A_2}, \frac{0.01}{A_3}, \frac{0.00}{A_4}, \frac{0.00}{A_5}, \frac{0.00}{A_6}, \frac{1.00}{A_7}, \frac{0.12}{A_8}, \frac{0.36}{A_9}, \frac{0.00}{A_{10}}, \frac{0.00}{A_{11}}, \frac{0.00}{A_{12}}, \frac{0.17}{A_{13}}, \frac{0.00}{A_{14}}, \frac{0.08}{A_{15}} \right\} \tag{2}$$

The fuzzy ranking for other positions showed multiple alternatives with high membership degrees, making it challenging to determine a clear order. In cases where criteria relevance is unknown, partial information from customers or decision-makers could narrow the analysis focus, such as by identifying partial criteria rankings to limit the explored space.

4. Conclusion

Making reliable recommendations is challenging, especially with unknown criteria weights. This study's evaluation approach extensively explores the decision problem space regarding criteria weights and offers a robust solution. By considering varied input conditions and using the fuzzy ranking concept, the proposed approach provides decision-makers with a more detailed view, enabling more informed decisions. The results for city electric vehicle selection confirm the effectiveness of this method for problems with unknown criteria weights.

Future studies could explore multi-criteria problems with partially known criteria relevance. By prioritizing certain criteria, this approach could offer more personalized and beneficial recommendations.

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