Matrix Factorization for Movie Recommendation Systems

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IIT Gandhinagar

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Today's Learning Journey

- **The Problem**: Why do we need recommendation systems?

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- **Step-by-Step**: Building intuition with examples
- **Algorithms**: ALS vs Gradient Descent
- **Practice**: Hands-on understanding

Problem Setup

The Movie Recommendation Challenge

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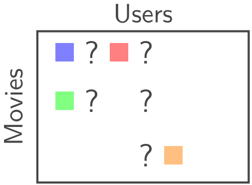

Sparse Rating Matrix

The Movie Recommendation Challenge

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- Netflix: 200M+ users, 15K+ titles
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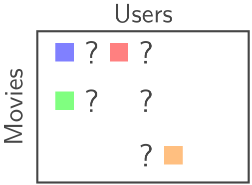
The Challenge:

- You've rated 100 movies out of 15,000
- How do we predict what you'll like?



Sparse Rating Matrix

Pop Quiz #1: Understanding the Scale

Answer this!

If Netflix has 200 million users and 15,000 movies, how many possible ratings exist?
Hint: Think about the total number of user-movie pairs

Pop Quiz #1: Understanding the Scale - Answer

Answer this!

Answer: $200 \times 10^6 \times 15 \times 10^3 = 3 \times 10^{12}$ possible ratings!
Follow-up: If typical users rate only 100 movies, what percentage of the matrix is filled?
Answer: $\frac{100}{15000} = 0.67\%$ - extremely sparse!

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Important: The Sparsity Challenge

99.33% of the rating matrix is empty! This is why we need smart algorithms.

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The Rating Matrix $\mathbf{A} \in \mathbb{R}^{N \times M}$ with proper labels:

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	Sholay	**Swades**	**Batman**	**Interstellar**	**...**
Arjun	a_{11}	?	a_{13}	?	$\cdots$
Priya	?	a_{22}	?	a_{24}	$\cdots$
Ravi	a_{31}	?	?	a_{34}	$\cdots$
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- **Challenge**: Predict missing entries ?
- **Notation**: $\Omega = \{(i, j) : a_{ij} \text{ is observed}\}$

Concrete Example: Our Movie Dataset

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- Arjun loves Bollywood films (Sholay, Swades)

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Observations:

- Arjun loves Bollywood films (Sholay, Swades)
- Ravi enjoys Sci-Fi (Interstellar)
- Can we predict Priya's rating for Sholay?
- Can we predict Ravi's rating for Swades?

Key Insight: Latent Features

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- Language (Hindi, English, Tamil)
- Era (90s classics, modern CGI)

Key Insight:

- Your taste = combination of preferences
- Movie appeal = combination of features
- But we don't know these explicitly!

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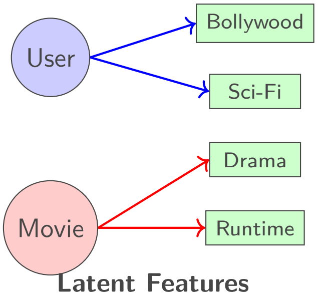

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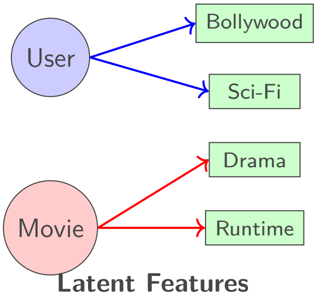

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Batman	0.05	0.80	0.30
Interstellar	0.05	0.95	0.70
Shawshank	0.05	0.15	0.95

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Movie Feature Matrix H $\in \mathbb{R}^{3\times 5}$:

$$\mathbf{H} = \begin{bmatrix} 0.95 & 1.00 & 0.05 & 0.05 & 0.05 \\ 0.10 & 0.20 & 0.80 & 0.95 & 0.15 \\ 0.85 & 0.90 & 0.30 & 0.70 & 0.95 \end{bmatrix}$$

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Next: Let's understand what each number means!

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The Magic: These values $\times$ movie features should recreate observed ratings!

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$w_{11} = 3.8$: Arjun's strong Bollywood preference!

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$w_{11} = 3.8$: Arjun's strong Bollywood preference!
This explains why he gave Sholay (high Bollywood content) a rating of 5.

Focus: Arjun's Sci-Fi Preference

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Understanding w_{12} - how much Arjun likes Sci-Fi:

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Focus: Arjun's Sci-Fi Preference

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Focus: Arjun's Sci-Fi Preference

Understanding w_{12} - how much Arjun likes Sci-Fi:

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$w_{12} = 0.8$: Arjun's weak Sci-Fi preference
This explains why he gave Interstellar (high Sci-Fi) only a rating of 3.

Focus: Arjun's Drama Preference

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Understanding w_{13} - how much Arjun likes Drama:

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Focus: Arjun's Drama Preference

Understanding w_{13} - how much Arjun likes Drama:

	Bollywood	Sci-Fi	Drama
Arjun	3.8	0.8	1.5
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$w_{13} = 1.5$: Arjun's low Drama preference

Focus: Arjun's Drama Preference

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$w_{13} = 1.5$: Arjun's low Drama preference

This explains why he gave Shawshank (high Drama) only a rating of 2.

Arjun's Complete Taste Profile

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Key Points:

Arjun's taste profile: [3.8 Bollywood, 0.8 Sci-Fi, 1.5 Drama]

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In Matrix Form:

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$$\boxed{\mathbf{A} \approx \mathbf{WH}}$$

$$\mathbf{A}_{3\times 5} = \begin{bmatrix} 5 & 4 & 2 & 3 & 2 \\ ? & 5 & 1 & 4 & ? \\ 4 & ? & 1 & 5 & ? \end{bmatrix} \approx$$

$$\begin{bmatrix} 3.8 & 0.8 & 1.5 \\ 1.8 & 4.1 & 3.2 \\ 2.4 & 4.8 & 2.1 \end{bmatrix} \begin{bmatrix} 0.95 & 1.00 & 0.05 & 0.05 & 0.05 \\ 0.10 & 0.20 & 0.80 & 0.95 & 0.15 \\ 0.85 & 0.90 & 0.30 & 0.70 & 0.95 \end{bmatrix} = \mathbf{W}_{3\times 3}\mathbf{H}_{3\times 5}$$

Step 4: Understanding the Calculation

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Sholay's DNA:

- Bollywood-ness: 0.95 (very high!)
- Sci-Fi-ness: 0.10 (low)
- Drama-ness: 0.85 (high)

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The Magic Formula:

$$\text{Arjun's rating} = \text{Arjun's preferences} \cdot \text{Sholay's features}$$

Step 4: Detailed Calculation Example

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Let's compute Arjun's predicted rating for Sholay:
Arjun's preferences: $\mathbf{w}_1 = [3.8, 0.8, 1.5]$
Sholay's features: $\mathbf{h}_1 = [0.95, 0.10, 0.85]^T$

Step 4: Detailed Calculation Example

Let's compute Arjun's predicted rating for Sholay:
Arjun's preferences: $\mathbf{w}_1 = [3.8, 0.8, 1.5]$
Sholay's features: $\mathbf{h}_1 = [0.95, 0.10, 0.85]^T$

$$\hat{a}_{11} = \mathbf{w}_1^T \mathbf{h}_1 \tag{1}$$
$$= 3.8 \cdot 0.95 + 0.8 \cdot 0.10 + 1.5 \cdot 0.85 \tag{2}$$
$$= 3.61 + 0.08 + 1.28 = \mathbf{4.97} \tag{3}$$

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$\hat{a}_{11} = 4.97 \approx 5$ ✓
This shows how matrix factorization works: find user preferences that recreate observed ratings!

Pop Quiz #2: Matrix Dimensions

Answer this!

If we have N users, M movies, and r latent features:

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4. How many parameters do we need to learn?

Pop Quiz #2: Matrix Dimensions - Answers

Answer this!

Answers:

1. $\mathbf{A} \in \mathbb{R}^{N \times M}$ (rating matrix)
2. $\mathbf{W} \in \mathbb{R}^{N \times r}$ (user preferences)
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4. Total parameters: $Nr + rM = r(N + M)$

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Key Points:

If $r \ll \min(N, M)$, we have huge parameter reduction!

Learning the Factorization

The Optimization Problem

Objective: Minimize prediction error on observed ratings only

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$$\boxed{\text{minimize}_{\mathbf{W},\mathbf{H}} \sum_{(i,j)\in\Omega} (a_{ij} - \mathbf{w}_i^T \mathbf{h}_j)^2}$$

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- Ω: set of observed (i, j) pairs

Why This is Challenging

Problem Characteristics:

- **Non-convex:** Multiple local minima exist

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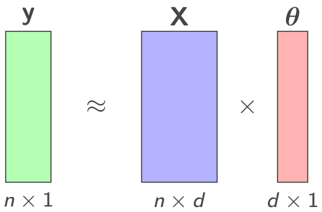
Key Insight: While non-convex jointly, it's convex in each matrix individually!

Alternating Least Squares (ALS): A Visual Derivation

Step 0: Recap of Standard Least Squares

Objective: Estimate $\boldsymbol{\theta} \in \mathbb{R}^{d \times 1}$ from data matrix $\mathbf{X} \in \mathbb{R}^{n \times d}$ and label vector $\mathbf{y} \in \mathbb{R}^{n \times 1}$.

$$\mathbf{y} \approx \mathbf{X}\boldsymbol{\theta} \quad \Rightarrow \quad \hat{\boldsymbol{\theta}} = \arg\min_{\boldsymbol{\theta}} \|\mathbf{y} - \mathbf{X}\boldsymbol{\theta}\|_2^2$$



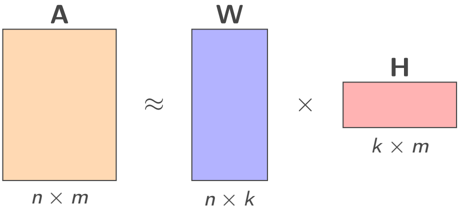
Solution: $\hat{\boldsymbol{\theta}} = (\mathbf{X}^\top \mathbf{X})^{-1} \mathbf{X}^\top \mathbf{y}$

ALS Problem Setup

Goal: Decompose $\mathbf{A} \in \mathbb{R}^{n \times m}$ as $\mathbf{WH}$ with:

$$\mathbf{W} \in \mathbb{R}^{n \times k}$$
$$\mathbf{H} \in \mathbb{R}^{k \times m}$$

Objective: $\mathbf{A} \approx \mathbf{WH}$ with $k \ll \min(n, m)$



Challenge: Both $\mathbf{W}$ and $\mathbf{H}$ are unknown.

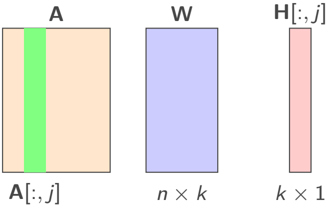
Step 1: Fix **W**, Learn **H** One Column at a Time

Idea: With **W** fixed, each column of **H** can be estimated via LS.

$$\mathbf{A}[:,j] \approx \mathbf{W} \cdot \mathbf{H}[:,j]$$

$$\mathbf{W} : n \times k \qquad \mathbf{H}[:,j] : k \times 1 \qquad \mathbf{A}[:,j] : n \times 1$$

This becomes: $\mathbf{y} \approx \mathbf{X}\theta$ (just like standard LS)



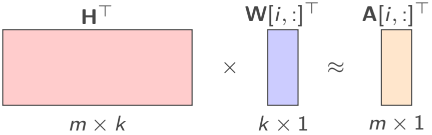
Repeat: for $j = 0$ to $m - 1$
$\mathbf{H}[:,j] \leftarrow \mathrm{LS}(\mathbf{W}, \mathbf{A}[:,j])$

Step 2: Fix $\mathbf{H}$, Learn $\mathbf{W}$ One Row at a Time

Goal: Learn $\mathbf{W}[i,:] \in \mathbb{R}^{1\times k}$ row by row.
Start from: $\mathbf{A}[i,:] \approx \mathbf{W}[i,:] \cdot \mathbf{H}$
Transpose both sides:

$$\mathbf{A}[i,:]^\top \approx \mathbf{H}^\top \cdot \mathbf{W}[i,:]^\top$$

Looks like LS again: $\mathbf{y} \approx \mathbf{X}\boldsymbol{\theta}$



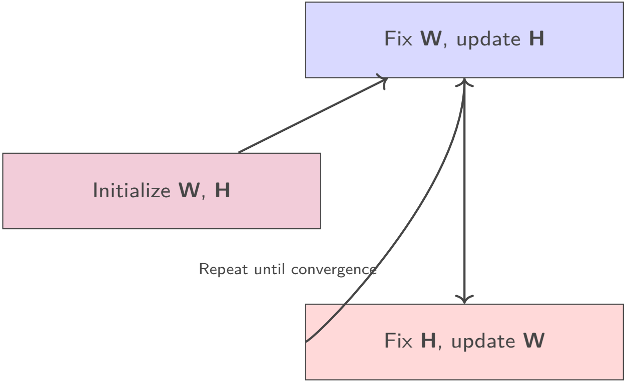
Solve: $\mathbf{W}[i,:]^\top \leftarrow \mathrm{LS}(\mathbf{H}^\top, \mathbf{A}^\top[:,i])$ for all i

Final ALS Loop Summary

Repeat for T iterations:

- Fix $\mathbf{W}$, then for each j, compute $\mathbf{H}[:,j] \leftarrow \text{LS}(\mathbf{W}, \mathbf{A}[:,j])$
- Fix $\mathbf{H}$, then for each i, compute $\mathbf{W}[i,:]^\top \leftarrow \text{LS}(\mathbf{H}^\top, \mathbf{A}^\top[:,i])$

All updates use standard least squares!



Algorithm 2: Gradient Descent

Gradient Descent: Matrix Factorization Setup

Goal: Find $\mathbf{W} \in \mathbb{R}^{n \times k}$ and $\mathbf{H} \in \mathbb{R}^{k \times m}$ such that:

$$\mathbf{A} \approx \mathbf{W}\mathbf{H}$$

Objective Function:

$$L(\mathbf{W}, \mathbf{H}) = \sum_{(i,j) \in \Omega} (a_{ij} - \mathbf{w}_i^\top \mathbf{h}_j)^2$$

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Initialize:

- $\mathbf{W}$: $n \times k$ matrix with small random values
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Gradient Descent Loop:

- For $t = 1$ to T:
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Note: Gradients can be computed either over all $(i, j) \in \Omega$ (Batch GD) or stochastically.

Algorithm Comparison and Practical Considerations

ALS vs SGD: Head-to-Head Comparison

Aspect	**ALS**	**SGD**
Updates	Alternating	Simultaneous
Convergence	Faster, more stable	Slower, can oscillate
Parallelization	Excellent	Limited
Memory	Higher	Lower
Implementation	Complex	Simple
Hyperparameters	Few (rank r)	Many (α, schedule)
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- **SGD:** Online learning, real-time updates, research

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Problem: Basic matrix factorization can overfit to training data

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$$\text{minimize}_{\mathbf{W},\mathbf{H}} \sum_{(i,j)\in\Omega} (a_{ij} - \mathbf{w}_i^T \mathbf{h}_j)^2 + \lambda(\|\mathbf{W}\|_F^2 + \|\mathbf{H}\|_F^2)$$

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What this does:

- Penalizes large feature values
- Prevents overfitting to observed ratings
- λ controls regularization strength
- Helps generalization to unseen ratings

Advanced Considerations: Bias Terms

Real-world insight: Not all ratings differences are due to preferences!

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Example: Example

Mean rating = 3.5, Arjun rates 0.5 higher, Sholay gets 0.8 higher

$\hat{a}_{\text{Arjun,Sholay}} = 3.5 + 0.5 + 0.8 + \mathbf{w}_{\text{Arjun}}^T \mathbf{h}_{\text{Sholay}}$

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Key Points:

Idea: More interactions = higher confidence in preference

Advanced Considerations: Cold Start Problem

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Important: Real-World Solution

Most production systems use hybrid approaches combining multiple signals

Hands-On Understanding

Let's Build Intuition: Small Example

Our 3×3 rating matrix:

$$\mathbf{A} = \begin{bmatrix} 5 & ? & 2 \\ 4 & 4 & ? \\ ? & 5 & 1 \end{bmatrix}$$

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Goal: Find $\mathbf{W} \in \mathbb{R}^{3\times 2}$ and $\mathbf{H} \in \mathbb{R}^{2\times 3}$ such that:

$$\mathbf{A} \approx \mathbf{WH} = \begin{bmatrix} w_{11} & w_{12} \\ w_{21} & w_{22} \\ w_{31} & w_{32} \end{bmatrix} \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \end{bmatrix}$$

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Constraint: Only minimize error on observed entries!

Step-by-Step ALS Solution

Iteration 1: Initialize randomly

$$\mathbf{W}^{(0)} = \begin{bmatrix} 0.5 & 0.3 \\ 0.4 & 0.6 \\ 0.2 & 0.8 \end{bmatrix}, \quad \mathbf{H}^{(0)} = \begin{bmatrix} 1.0 & 0.5 & 0.2 \\ 0.3 & 1.2 & 0.8 \end{bmatrix}$$

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Update User 1: Only use observed ratings (positions 1,3)

$$\mathbf{y}_1 = [5, 2]^T \tag{4}$$

$$\mathbf{X}_1 = \begin{bmatrix} 1.0 & 0.3 \\ 0.2 & 0.8 \end{bmatrix} \text{ (columns 1,3 of } \mathbf{H}^{(0)\,T}) \tag{5}$$

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Solve: $\mathbf{w}_1^{(1)} = \mathrm{LS}(\mathbf{X}_1, \mathbf{y}_1)$
Continue for all users and movies...

Pop Quiz #3: Netflix Engineering Challenge

Answer this!

You're Netflix's lead ML engineer. You have:

- 200M users, 15K movies
- 20B ratings (0.67% filled)
- Need real-time recommendations
- New users/movies arrive daily

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1. Which algorithm: ALS or SGD? Why?

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2. What rank r would you choose?

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Pop Quiz #3: Netflix Engineering Challenge

Answer this!

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- 20B ratings (0.67% filled)
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Suggested Solution:

Summary and Key Takeaways

Key Insights: Part 1

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1. **Sparsity** ⇒ **Factorization**: Sparse matrices can be approximated by low-rank factorizations
2. **Latent Features**: Users and items characterized by hidden factors
3. **Bilinear Problem**: Non-convex jointly, convex individually

Key Insights: Part 2

4. **Scale Matters**: Algorithm choice depends on data size

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The Mathematical Beauty:

$$\boxed{\text{Collaborative Filtering} = \text{Matrix Factorization} = \text{Dimensionality Reduct}}$$

Extensions: Interpretable Factorization

Non-negative Matrix Factorization (NMF):

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Example: Movie Example

If factor $1 =$ "Action-ness", then $h_{1j} \geq 0$ means movie j has some amount of action (never "negative action")

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- Deep Factorization Machines

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Enable richer context-aware recommendations

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- Examples: GraphRec, NGCF, LightGCN

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Recommend known good items vs explore new possibilities?

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Balance accuracy with discovery!

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Key Points:

Same principles apply across domains!

Pop Quiz #4: Matrix Factorization Fundamentals

Answer this!

True or False?

1. Matrix factorization can only work with explicit ratings
2. ALS always converges to the global optimum
3. A rank-1 factorization means all users have identical preferences

Pop Quiz #4: Answers - Fundamentals

Answer this!

Answers:

1. **False** - Works with implicit feedback too
2. **False** - Converges to local optimum only
3. **False** - Rank-1 means one pattern, not identical

Pop Quiz #5: Advanced Topics

Answer this!

True or False?

1. Adding regularization always improves recommendations
2. SGD is better than ALS for all applications

Bonus: What are the main trade-offs between ALS and SGD?

Pop Quiz #5: Answers - Advanced Topics

Answer this!

Answers:

1. **False** - Too much regularization causes underfitting
2. **False** - Choice depends on data scale and needs

Bonus: ALS: Fast, parallel. **SGD:** Online, low memory.