Gate 2015 ECE Q. 48

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Question

A plant transfer function is given as $G(s) = (K_P + \frac{K_I}{s})(\frac{1}{s(s+2)})$. When the plant operates in a unity feedback configuration, the condition for the stability of the closed loop system is

1
$$K_P > \frac{K_I}{2} > 0$$

②
$$2K_I > K_P > 0$$

3
$$2K_I < K_P$$

$$2K_I > K_P$$

Solution

The closed loop transfer function for unity feedback is

$$\frac{G(s)}{1+G(s)} = \frac{(K_p s + K_i)}{s^2(s+2) + (K_s s + K_i)}$$
$$= \frac{(K_p s + K_i)}{s^3 + 2s^2 + K_p s + K_i}$$

Using Routh's tabular form:

$$\begin{vmatrix}
S^3 & 1 & K_P \\
S^2 & 2 & K_I \\
S^1 & \frac{2K_P - K_i}{2} & 0 \\
S^0 & K_P
\end{vmatrix}$$

contd...

For system to be Stable: $K_P > 0$

and
$$\frac{2K_P-K_I}{2}>0$$

or

$$2K_P - K_I > 0$$

or

$$K_P > \frac{K_I}{2} > 0$$

So, 1st option is correct.