Control Systems

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Contents

***Abstract*—This manual is an introduction to control systems based on GATE problems.Links to sample Python codes are available in the text.**

Download python codes using

```
svn co https://github.com/gadepall/school/trunk/
    control/codes
```

1 Feedback Circuits

1.1. The CE BJT amplifier in Fig. 1.1 employs shunt–shunt feedback: Feedback resistor R_F senses the output Voltage V_o and provides a feedback current to the base node. $\left(R_f = 56k\Omega, R_C = 5.6k\Omega, R_S = 10k\Omega\right)$

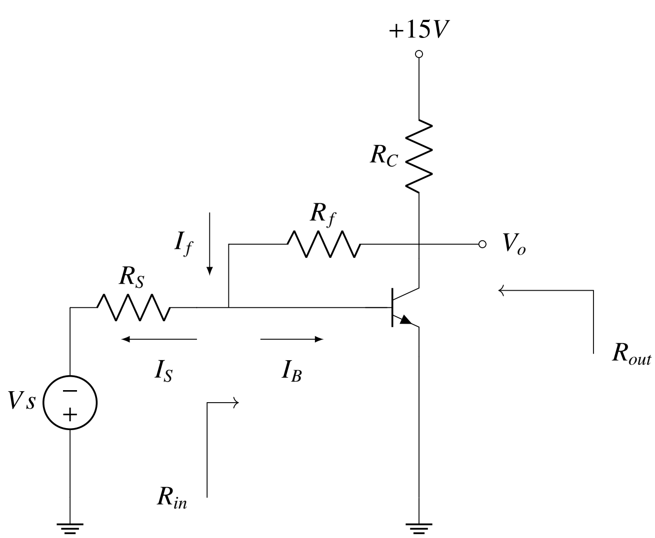

Fig. 1.1

1.2. Draw the equivalent control system for fig. 1.1
Solution:

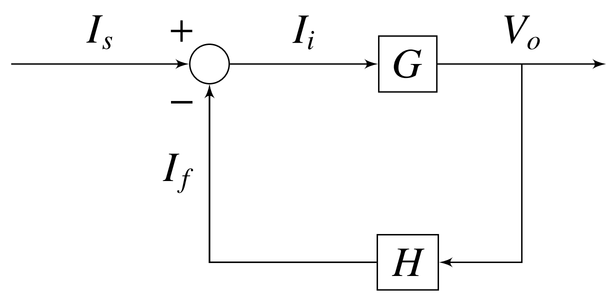

Fig. 1.2

1.3. If V_s has a zero dc component, find the dc collector current of the BJT. Assume the transistor H = 100.
Solution: Since, $V_E = 0$ and $V_S = V_{BE}$

$$I_S = \frac{V_{BE}}{R_S} = \frac{0.7}{10 * 10^3} \tag{1.3.1}$$

$$\implies I_S = 0.07mA \tag{1.3.2}$$

Applying KCL at feedback resistor output

$$-V_o + V_{BE} + I_f R_f = 0$$

$$\left(Since, I_f = I_B + I_S\right)$$

$$V_o = V_{BE} + (I_B + I_S) R_f$$

$$= 0.7 + \left(I_B + 0.07 * 10^{-3}\right)\left(56 * 10^3\right)$$

$$\implies V_o = \left(56 * 10^3\right) I_B + 4.62 \tag{1.3.3}$$

Applying KCL at collector node

$$\frac{V_o - 15}{5.6 * 10^3} + I_C + I_f = 0$$

$$(Since, I_C = HI_B)$$

$$\frac{V_o - 15}{5.6 * 10^3} + HI_B + (I_B + I_S) = 0$$

$$\frac{V_o - 15}{5.6 * 10^3} + (100 + 1) I_B + \left(0.07 * 10^{-3}\right) = 0$$

$$\implies V_o = 14.608 - \left(565.5 * 10^3\right) I_B \tag{1.3.4}$$

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Subtracting 1.3.3 from 1.3.4, we get,

$$I_B = 16.06\mu A \quad (1.3.5)$$
$$I_C = I_E = HI_B \quad (1.3.6)$$
$$\text{Dc collector Current}, I_C = 1.606mA \quad (1.3.7)$$

Parameter	Description
R_{in}	Total Input Resistance
R_{out}	Total Output Resistance
r_π	Output resistance of NPN
R_f	Feedback resistance
R_i	Input resistance of G circuit
R_o	Output resistance of G circuit
R_{if}	Input resistance of Feedback
R_{of}	Output resistance of Feedback
R_s	Resistance of Current Source
R_L	Output Load Resistance
g_m	Trans conductance
I_C	Collector current
I_E	Emitter Current
I_B	Base Current

TABLE 1.3

1.4. Find the small-signal equivalent circuit of the amplifier with the signal source represented by its Norton equivalent (as we usually do when the feedback connection at the input is shunt).
Solution: In fig 1.4

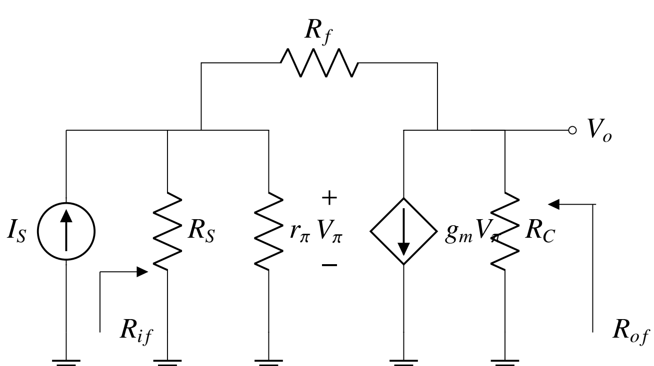

Fig. 1.4

1.5. Find the G circuit and determine the value of G, R_i , and R_o.
Solution: G circuit in fig. 1.5 and G Block in 1.5

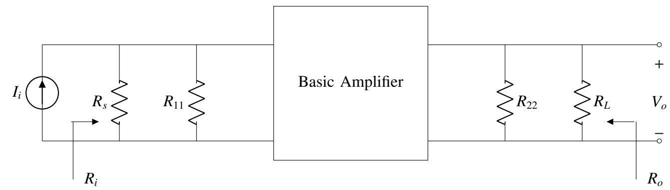

Fig. 1.5: Gain Block

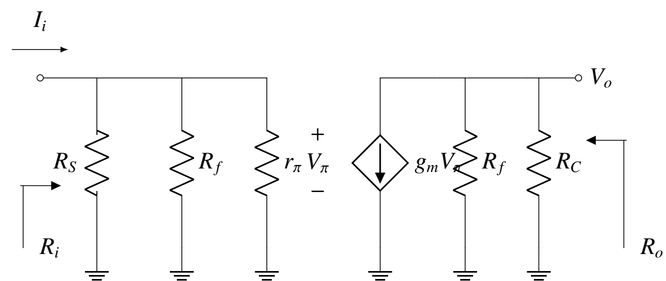

Fig. 1.5: G Circuit

$$g_m = \frac{I_C}{V_\pi} = \frac{1.606 * 10^{-3}}{25 * 10^{-3}} = 64mA/V \quad (1.5.1)$$
$$r_\pi = \frac{H}{g_m} = \frac{100}{64 * 10^{-3}} = 1.56k\Omega \quad (1.5.2)$$
$$V_o = -g_m V_\pi \left(R_f \| R_C\right) \quad (1.5.3)$$
$$V_\pi = I_i \left(R_S \| R_f \| r_\pi\right) \quad (1.5.4)$$

$$\text{Gain}, G = \frac{V_o}{I_l} \quad (1.5.5)$$
$$G = -g_m \left(R_f \| R_c\right)\left(R_s \| R_f \| r_s\right) \quad (1.5.6)$$
$$G = -429k\Omega \quad (1.5.7)$$

Input Resistance

$$R_i = \left(R_s \| R_f \| r_s\right) = 1.31k\Omega \quad (1.5.8)$$
$$\text{Output Resistance}, \quad (1.5.9)$$
$$R_o = R_C \| R_f \quad (1.5.10)$$
$$R_o = 5.09k\Omega \quad (1.5.11)$$

1.6. Find H and hence AH and 1+AH.
Solution:

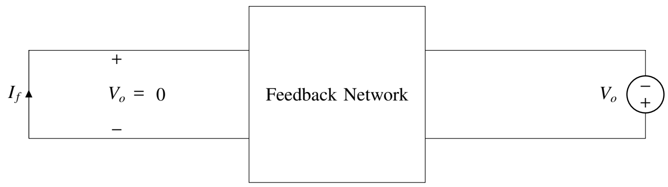

Fig. 1.6: H Block

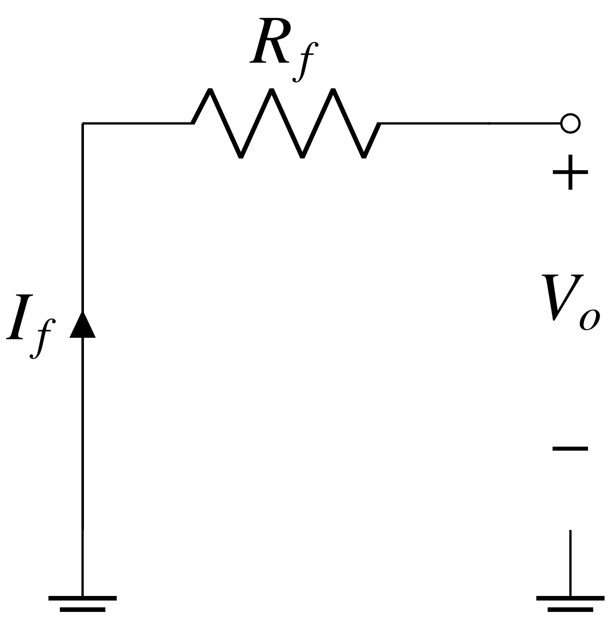

Fig. 1.6: H Circuit

$$H = \frac{I_f}{V_o} = -\frac{1}{R_f} \tag{1.6.1}$$
$$\implies H = -17.85 * 10^{-4} \tag{1.6.2}$$
$$GH = 7.662 \tag{1.6.3}$$
$$1 + GH = 8.66 \tag{1.6.4}$$

1.7. Find R_{11} and R_{22} from fig. 1.6
Solution:

$$R_{11} = R_f \tag{1.7.1}$$
$$R_{22} = R_f \tag{1.7.2}$$

1.8. Find T , R_{if} and R_{of} and hence R_{in} and R_{out} .
Solution:

$$T = \frac{G}{1+GH} \tag{1.8.1}$$
$$= -49.54k\Omega \tag{1.8.2}$$
$$R_{if} = \frac{R_i}{1+GH} \tag{1.8.3}$$
$$= \frac{1.31 * 10^3}{8.66} \tag{1.8.4}$$
$$= 151.27\Omega \tag{1.8.5}$$
$$R_{of} = \frac{R_o}{1+GH} \tag{1.8.6}$$
$$= \frac{5.09 * 10^3}{8.66} \tag{1.8.7}$$
$$= 587.7\Omega \tag{1.8.8}$$
$$R_{in} = \frac{1}{\frac{1}{R_{if}} - \frac{1}{R_s}} \tag{1.8.9}$$
$$= 153.2\Omega \tag{1.8.10}$$
$$R_{out} = \frac{1}{\frac{1}{R_{of}} - \frac{1}{R_L}} \tag{1.8.11}$$
$$= R_{of} \tag{1.8.12}$$

1.9. What voltage gain V_o/V_s is realized? How does this value compare to the ideal value obtained if the loop gain is very large and thus the signal voltage at the base becomes almost zero (like what happens in an inverting op-amp circuit).
Solution:

$$\frac{V_o}{V_s} = \frac{V_o}{I_s R_s} \tag{1.9.1}$$
$$= \frac{T}{R_s} \tag{1.9.2}$$
$$\text{Since, } T = \frac{V_o}{I_S} \tag{1.9.3}$$
$$\frac{V_o}{V_s} = \frac{-49.54 * 10^3}{10 * 10^3} \tag{1.9.4}$$
$$= -4.95V/V \tag{1.9.5}$$

If the loop gain is very large, then the gain with feedback T is:

$$T = \frac{1}{H} \tag{1.9.6}$$
$$= \frac{1}{(-17.85 * 10^{-6})} \tag{1.9.7}$$
$$= -56k\Omega \tag{1.9.8}$$

$\therefore$ the closed loop gain, T = - R_f

Parameter	Value
R_f	$56k\Omega$
R_S	$10k\Omega$
R_C	$5.6k\Omega$
I_S	$0.07mA$
I_B	$16.06\mu A$
I_C	$1.606mA$
I_E	$1.606mA$
g_m	$64mA/V$
r_π	$1.56k\Omega$
G	$-429k\Omega$
R_i	$1.31k\Omega$
R_o	$5.09k\Omega$
H	$-17.85 * 10^{-4}$
GH	7.662
$1 + GH$	8.66
T	$-49.54k\Omega$
R_{if}	$151.27k\Omega$
R_{of}	$587.7k\Omega$
R_{in}	153.2Ω

TABLE 1.9

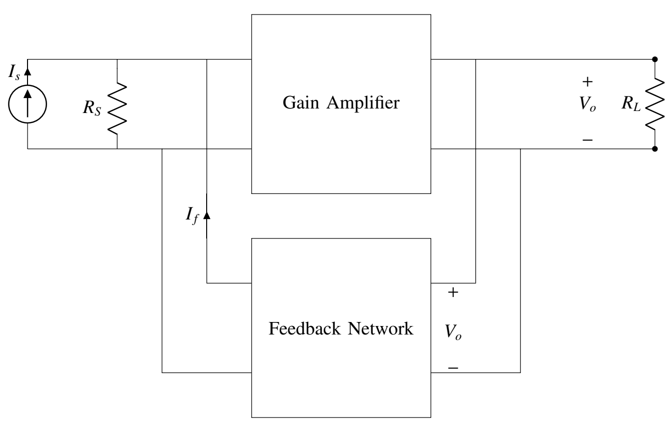

Fig. 1.9: Shunt-Shunt Amplifier Block Diagram

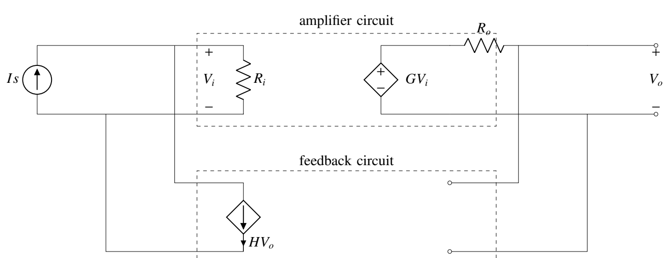

Fig. 1.9: Ideal structure for Shunt-Shunt

1.10. Verify your solution using spice
Solution: Doing operational Analysis on the Circuit 1.1

Parameter	Value
I_C	$2.1mA$
I_E	$2.1mA$
I_B	$2.1\mu A$
I_S	$0.07mA$

TABLE 1.10

Table 1.10 is close to the numerical Calculation done above.