Control Systems

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***Abstract*—This manual is an introduction to control systems based on GATE problems.Links to sample Python codes are available in the text.**

Download python codes using

```
svn co https://github.com/gadepall/school/trunk/
    control/codes
```

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8.1 Stability

8.1. The plant transfer function is given as

$$G(s) = \left(K_P + \frac{K_I}{s}\right)\left(\frac{1}{s(s+2)}\right) \tag{8.1.1}$$

When the plant operates in a unity feedback configuration, the condition for the stability of the closed loop system is

a) $K_P > \frac{K_I}{2} > 0$
b) $2K_I > K_P > 0$
c) $2K_I < K_P$
d) $2K_I > K_P$

Solution: The closed Loop Transfer function for unity feedback is

$$\frac{G(s)}{(1+G(s))} = \frac{(K_P s + K_I)}{s^2(s+2) + (K_P s + K_I)} \tag{8.1.2}$$

$$= \frac{(K_P s + K_I)}{s^3 + 2s^2 + K_P s + K_I} \tag{8.1.3}$$

Using the Routh Hurwitz tabular form:

$$\begin{vmatrix} s^3 \\ s^2 \\ s^1 \\ s^0 \end{vmatrix} \begin{vmatrix} 1 & K_P \\ 2 & K_I \\ \frac{2K_P-K_I}{2} & 0 \\ K_P & \end{vmatrix} \tag{8.1.4}$$

Sufficient condition for Routh-Hurwitz Stability Criterion:- The sufficient condition is that all the elements of the first column of the Routh array should have the same sign.
For a system to be stable:

$$K_P > 0 \tag{8.1.5}$$

$$2K_P - K_I > 0 \tag{8.1.6}$$

$$K_P > \frac{K_I}{2} > 0 \tag{8.1.7}$$

So, (8.1.7) is the required condition for system to be stable.

8.2. Verify your result by plotting the gain and phase plots of (8.1.3) for different values of K_P and K_I.
Solution: For a system to be stable Gain Margin should be greater than 1 and Phase Margin Should be Positive.
The following code plots for $K_P = 1$ and $K_I = 0.5$ Fig. 8.2

```
codes/ee17btech11031_1.py
```

- The Phase margin: 73.07475039260186
- The Gain Margin: ∞

The Phase plot is as shown,

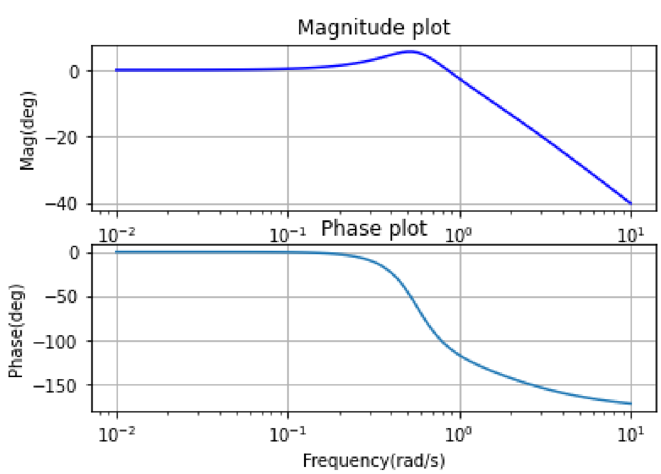

Fig. 8.2

For $K_P = -1$ and $K_I = 0.5$ Fig. 8.2

```
codes/ee17btech11031_2.py
```

- The Phase margin: -128.18015255977565
- The Gain Margin: ∞

The Phase plot is as shown,

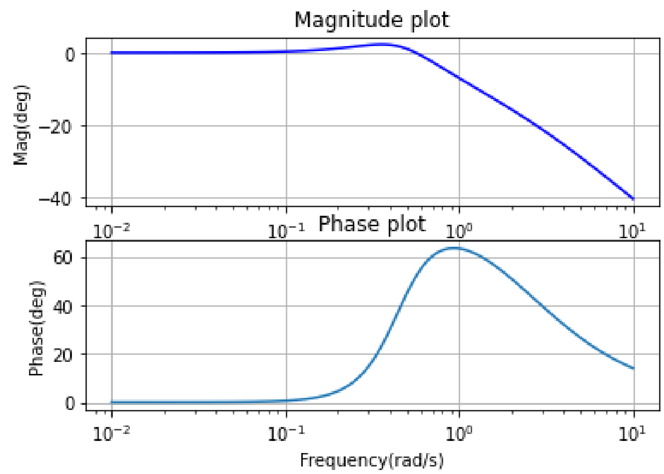

Fig. 8.2

The above problem can be found in PI controller as shown:

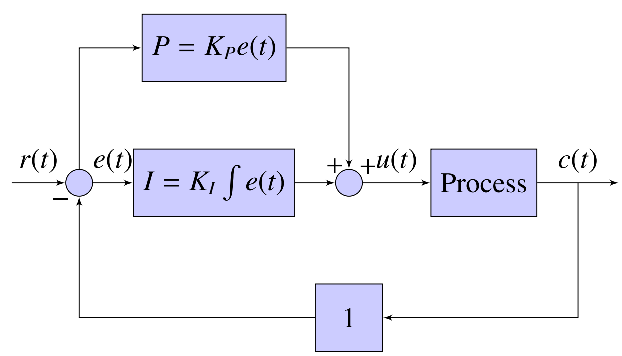

Fig. 8.2

$$u(t) = K_P e(t) + K_I \int e(t) \tag{8.2.1}$$

Applying Laplace Transform:

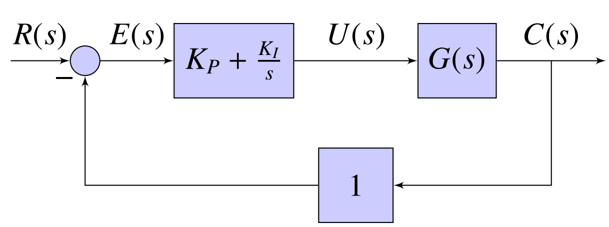

Fig. 8.2

$$U(s) = \left(K_P + \frac{K_I}{s}\right) E(s) \qquad (8.2.2)$$

In (8.1.1)

$$E(s) = \frac{1}{s(s+2)} \qquad (8.2.3)$$