Control Systems

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***Abstract*—The objective of this manual is to introduce control system design at an elementary level.**

Download python codes using

```
svn co https://github.com/gadepall/school/trunk/
    control/ketan/codes
```

1 PID Controller

1.1 Introduction

1.1. For a unity feedback system shown in Fig. 1.1

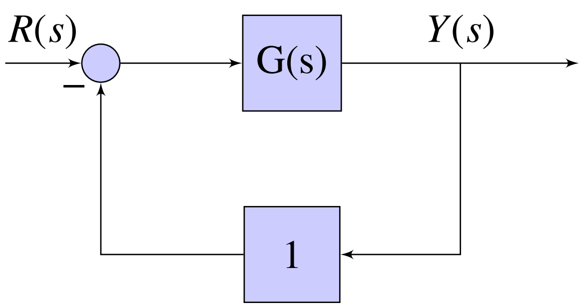

Fig. 1.1

$$G(s) = \frac{K}{s(s+1)} \tag{1.1.1}$$

Design a PD controller such that the phase margin is 45°and appropriate steady state error is less than or equal to $\frac{1}{15}$ units of the final output value. Further the gain crossover frequency of the system must be less than 7.5 rad/s.

Solution: Using TABLE **??** The gain after cascading the PD controller with G(s) is

$$G_c(s) = \frac{K_P(1+T_d s)K}{s(s+1)} \tag{1.1.2}$$

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Type	**Poles**	**Input**	**Steady State Error**
Type 0	0	Step	$e_{ss} = \frac{1}{1+\lim_{s\to 0} G(s)}$
Type 1	1	Ramp	$e_{ss} = \frac{1}{\lim_{s\to 0} sG(s)}$
Type 2	2	Parabolic	$e_{ss} = \frac{1}{1+\lim_{s\to 0} s^2G(s)}$

TABLE 1.1: System Types and Poles at Origin

Using TABLE 1.1, (1.1.2) is Type 1 system.

$$e_{ss} = \frac{1}{\lim_{s\to 0} sG_c(s)} \tag{1.1.3}$$

$$e_{ss} \leq \frac{1}{15} \lim_{s\to 0} sG_c(s) \tag{1.1.4}$$

$$\implies K_P K \geq \sqrt{15} \tag{1.1.5}$$

For Phase Margin 45°, at gain crossover frequency ω,

$$\tan^{-1}(T_d\omega) - \tan^{-1}(\omega) = -45° \tag{1.1.6}$$

$$\left|G_c(\jmath\omega)\right| = \frac{\sqrt{15}\sqrt{T_d^2\omega^2+1}}{\omega\sqrt{\omega^2+1}} = 1 \tag{1.1.7}$$

By Hit and Trial, one of the best combinations is

$$\omega = 2.893 \tag{1.1.8}$$

$$T_d = -0.71 \tag{1.1.9}$$

Parameters	**Required**	**Obtained**
ω	≤ 7.5	2.893
Phase Margin	45°	45°
T_d	Not Given	-0.71

TABLE 1.1

1.2. Verify using a Python Plot

Solution: The following code plots Fig. 1.2

```
codes/ee17btech11031_pd_ke.py
```

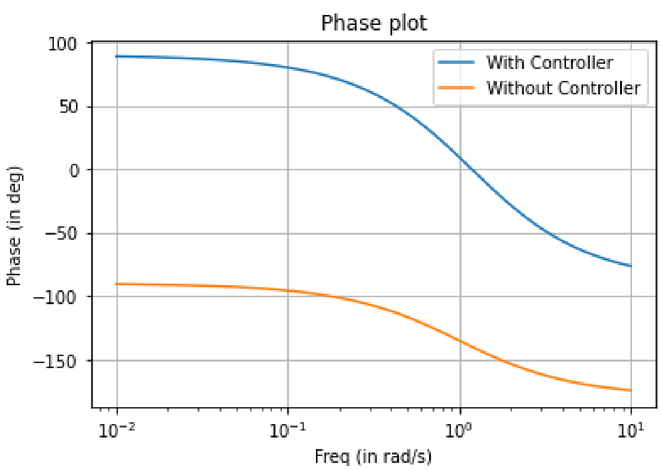
Phase plot
With Controller
Without Controller
Phase (in deg)
Freq (in rad/s)
100
50
0
−50
−100
−150
10^{-2}
10^{-1}
10^{0}
10^{1}

Fig. 1.2