Homework 4

习题6

4.

> ✎ Note
>
> 设总体$\boldsymbol{X} \sim N_p(\boldsymbol{\mu}, \boldsymbol{\Sigma})$,其中$\boldsymbol{\mu} = (\mu_1, \cdots, \mu_p)'$和$\boldsymbol{\Sigma} > 0$.假设$\boldsymbol{x}_1, \cdots, \boldsymbol{x}_n$为来自$p$元正态总体$\boldsymbol{X}$的一组独立同分布的简单随机样本,且$n > p$.记$\mathbf{C}$为$k \times p$的常数矩阵和$\boldsymbol{r}$为已知的$k$维向量,且要求$k < q$和$\mathrm{rank}(\mathbf{C}) = k$.试给出检验$H_0: \mathbf{C}\boldsymbol{\mu} = \boldsymbol{r}$的检验统计量及其分布.

$$Sol.\ \text{考虑似然比统计量}\lambda = \frac{\sup\limits_{\mathbf{C}\boldsymbol{\mu}=\boldsymbol{r}} f(\boldsymbol{x_1}, \cdots, \boldsymbol{x_n})}{\sup\limits_{\boldsymbol{\mu}} f(\boldsymbol{x_1}, \cdots, \boldsymbol{x_n})},$$

$$\text{其中} f(\boldsymbol{x_1}, \cdots, \boldsymbol{x_n}) = \frac{1}{(2\pi)^{np/2}|\boldsymbol{\Sigma}|^{n/2}} \mathrm{etr}\left\{-\frac{1}{2}\boldsymbol{\Sigma}^{-1}\left(\sum_{i=1}^n(\boldsymbol{x}_i - \boldsymbol{\mu})(\boldsymbol{x}_i - \boldsymbol{\mu})'\right)\right\}$$

$$= \frac{1}{(2\pi)^{np/2}|\boldsymbol{\Sigma}|^{n/2}} \mathrm{etr}\left\{-\frac{1}{2}\boldsymbol{\Sigma}^{-1}(\mathbf{V} + n(\overline{\boldsymbol{x}} - \boldsymbol{\mu})(\overline{\boldsymbol{x}} - \boldsymbol{\mu})')\right\}, \mathbf{V}\text{为样本离差阵},$$

$$\text{即}\mathbf{V} = \sum_{i=1}^n (\boldsymbol{x}_i - \overline{\boldsymbol{x}})(\boldsymbol{x}_i - \overline{\boldsymbol{x}})'.\ \text{已知}\widehat{\boldsymbol{\mu}}_{\mathrm{MLE}} = \overline{\boldsymbol{x}}, \widehat{\boldsymbol{\Sigma}}_{\mathrm{MLE}} = \frac{\mathbf{V}}{n}, \text{则} \sup_{\boldsymbol{\mu}} f(\boldsymbol{x_1}, \cdots, \boldsymbol{x_n}) = \frac{1}{\left(\frac{2e\pi}{n}\right)^{\frac{np}{2}}|\mathbf{V}|^{\frac{n}{2}}}.$$

$$\text{要使} \sup_{\mathbf{C}\boldsymbol{\mu}=\boldsymbol{r}} f(\boldsymbol{x_1}, \cdots, \boldsymbol{x_n})\text{最小, 考虑} \ln f' = -n\ln|\boldsymbol{\Sigma}| - \mathrm{tr}\left(\boldsymbol{\Sigma}^{-1}(\mathbf{V} + n(\overline{\boldsymbol{x}} - \boldsymbol{\mu})(\overline{\boldsymbol{x}} - \boldsymbol{\mu})')\right)$$

$$\frac{\partial \ln f'}{\partial \boldsymbol{\Sigma}} = -n\boldsymbol{\Sigma}^{-1} + \boldsymbol{\Sigma}^{-2}(\mathbf{V} + n(\overline{\boldsymbol{x}} - \boldsymbol{\mu})(\overline{\boldsymbol{x}} - \boldsymbol{\mu})') = 0, \text{则}\widehat{\boldsymbol{\Sigma}} = \frac{1}{n}\left(\mathbf{V} + n(\overline{\boldsymbol{x}} - \boldsymbol{\mu})(\overline{\boldsymbol{x}} - \boldsymbol{\mu})'\right),$$

$$\sup_{\mathbf{C}\boldsymbol{\mu}=\boldsymbol{r}} f(\boldsymbol{x_1}, \cdots, \boldsymbol{x_n}) = \sup_{\mathbf{C}\boldsymbol{\mu}=\boldsymbol{r}} \frac{1}{\left(\frac{2e\pi}{n}\right)^{\frac{np}{2}}|(\mathbf{V} + n(\overline{\boldsymbol{x}} - \boldsymbol{\mu})(\overline{\boldsymbol{x}} - \boldsymbol{\mu})')|^{\frac{n}{2}}}$$

$$\lambda = \sup_{\mathbf{C}\boldsymbol{\mu}=\boldsymbol{r}} \frac{|\mathbf{V}|^{\frac{n}{2}}}{|(\mathbf{V} + n(\overline{\boldsymbol{x}} - \boldsymbol{\mu})(\overline{\boldsymbol{x}} - \boldsymbol{\mu})')|^{\frac{n}{2}}} = \sup_{\mathbf{C}\boldsymbol{\mu}=\boldsymbol{r}} \frac{1}{|\mathbf{I}_p + n\mathbf{V}^{-1}(\overline{\boldsymbol{x}} - \boldsymbol{\mu})(\overline{\boldsymbol{x}} - \boldsymbol{\mu})')|^{\frac{n}{2}}}$$

$$= \sup_{\mathbf{C}\boldsymbol{\mu}=\boldsymbol{r}} \left(1 + n(\overline{\boldsymbol{x}} - \boldsymbol{\mu})'\mathbf{V}^{-1}(\overline{\boldsymbol{x}} - \boldsymbol{\mu})\right)^{-\frac{n}{2}}, \text{等价于求} \inf_{\mathbf{C}\boldsymbol{\mu}=\boldsymbol{r}} (\overline{\boldsymbol{x}} - \boldsymbol{\mu})'\mathbf{V}^{-1}(\overline{\boldsymbol{x}} - \boldsymbol{\mu})\text{即可}.$$

$$\text{令}L = (\overline{\boldsymbol{x}} - \boldsymbol{\mu})'\mathbf{V}^{-1}(\overline{\boldsymbol{x}} - \boldsymbol{\mu}) - \boldsymbol{t}'(\mathbf{C}\boldsymbol{\mu} - \boldsymbol{r}),$$

$$\begin{cases} \frac{\partial L}{\partial \boldsymbol{t}} = \mathbf{C}\boldsymbol{\mu} - \boldsymbol{r} = 0 \\ \frac{\partial L}{\partial \boldsymbol{\mu}} = -2\mathbf{V}^{-1}(\overline{\boldsymbol{x}} - \boldsymbol{\mu}) - \mathbf{C}'\boldsymbol{t} = 0 \end{cases}, \quad \text{解得} \begin{cases} \boldsymbol{t} = 2(\mathbf{CVC}')^{-1}(\boldsymbol{r} - \mathbf{C}\overline{\boldsymbol{x}}) \\ \widehat{\boldsymbol{\mu}} = \overline{\boldsymbol{x}} + \mathbf{VC}'(\mathbf{CVC}')^{-1}(\boldsymbol{r} - \mathbf{C}\overline{\boldsymbol{x}}) \end{cases}$$

$$\text{故}\lambda = \left(1 + n(\boldsymbol{r} - \mathbf{C}\overline{\boldsymbol{x}})'(\mathbf{CVC}')^{-1}\mathbf{CV} \cdot \mathbf{V}^{-1} \cdot \mathbf{VC}'(\mathbf{CVC}')^{-1}(\boldsymbol{r} - \mathbf{C}\overline{\boldsymbol{x}})\right)^{-\frac{n}{2}}$$

$$= \left(1 + n(\mathbf{C}\overline{\boldsymbol{x}} - \boldsymbol{r})'(\mathbf{CVC}')^{-1}(\mathbf{C}\overline{\boldsymbol{x}} - \boldsymbol{r})\right)^{-\frac{n}{2}}$$

$$\text{已知}\overline{\boldsymbol{x}} \sim N_p(\boldsymbol{\mu}, \boldsymbol{\Sigma}/n), \mathbf{V} \sim W_p(n-1, \boldsymbol{\Sigma}), \text{则}\sqrt{n}(\mathbf{C}\overline{\boldsymbol{x}} - \boldsymbol{r}) \sim N_p(\sqrt{n}(\mathbf{C}\boldsymbol{\mu} - \boldsymbol{r}), \mathbf{C\Sigma C}')$$

$$\mathbf{CVC}' \sim W_p(n-1, \mathbf{C\Sigma C}'),$$

$$\text{故}n(n-1)(\mathbf{C}\overline{\boldsymbol{x}} - \boldsymbol{r})'(\mathbf{CVC}')^{-1}(\mathbf{C}\overline{\boldsymbol{x}} - \boldsymbol{r}) \sim T^2\left(p, n-1, \sqrt{n}(\mathbf{C}\boldsymbol{\mu} - \boldsymbol{r})\right).$$

$$\text{令}T^2 = n(n-1)(\mathbf{C}\overline{\boldsymbol{x}} - \boldsymbol{r})'(\mathbf{CVC}')^{-1}(\mathbf{C}\overline{\boldsymbol{x}} - \boldsymbol{r})\text{为其检验统计量, 服从非中心}T^2\text{分布}.$$

$$\text{另}\frac{n-p-1}{(n-2)p}T^2 \sim F_{p,n-p-1}(\delta)\text{也可作为检验统计量, 服从非中心}F\text{分布},$$

$$\text{其中}\delta = n(n-1)(\mathbf{C}\boldsymbol{\mu} - \boldsymbol{r})'(\mathbf{C\Sigma C}')^{-1}(\mathbf{C}\boldsymbol{\mu} - \boldsymbol{r}).$$

5.

> ✎ Note

设$\boldsymbol{x}_1,\cdots,\boldsymbol{x}_n$为来自总体$\boldsymbol{X}\sim N_p(\boldsymbol{\mu},\boldsymbol{\Sigma})$的独立同分布的简单随机样本,其中$\boldsymbol{\mu}=(\mu_1,\cdots,\mu_p)$,$\boldsymbol{\Sigma}>0$和$n>p$.记样本均值为$\overline{\boldsymbol{x}}$,样本离差阵为$\mathbf{V}$.考虑下面的假设问题:

$$H_0:\mu_1=\mu_2=\cdots=\mu_p,\qquad H_1:\mu_1,\cdots,\mu_2,\cdots,\mu_p\text{至少有一对不相等}.$$

令$\mathbf{C}$为$(p-1)\times p$的矩阵,记为

$$\mathbf{C}=\begin{pmatrix}1&-1&0&\cdots&0\\1&0&-1&\cdots&0\\\vdots&\vdots&\vdots&&\vdots\\1&0&0&\cdots&-1\end{pmatrix}.$$

则上面的假设等价于

$$H_0:\mathbf{C}\boldsymbol{\mu}=\mathbf{0}_{p-1},\qquad H_1:\mathbf{C}\boldsymbol{\mu}\neq\mathbf{0}_{p-1},$$

其中$\mathbf{0}_{p-1}$为$p-1$的零向量.试求检验H_0的似然比统计量及其分布.

Sol. 由4.可知Hotelling T^2统计量$T^2=n(n-1)(\mathbf{C}\overline{\boldsymbol{x}})'(\mathbf{C}\mathbf{V}\mathbf{C}')^{-1}(\mathbf{C}\overline{\boldsymbol{x}})\sim T^2(p,n-1,\sqrt{n}\mathbf{C}\boldsymbol{\mu})$. 类似地,$\dfrac{n-p-1}{(n-2)p}T^2\sim F_{p,n-p-1}(\delta)$也可作为检验统计量, 其中$\delta=n(n-1)\boldsymbol{\mu}'\mathbf{C}'(\mathbf{C}\boldsymbol{\Sigma}\mathbf{C}')^{-1}\mathbf{C}\boldsymbol{\mu}$.

8.

Note

对两个p元正态总体$N_p(\boldsymbol{\mu}_1,\boldsymbol{\Sigma})$和$N_p(\boldsymbol{\mu}_2,\boldsymbol{\Sigma})$, $\boldsymbol{\Sigma}$均值向量的检验问题,试用似然比原理导出检验$H_0:\boldsymbol{\mu_1}=\boldsymbol{\mu}_2$的似然比统计量及其分布.

Sol. 设$\boldsymbol{x}_1,\cdots,\boldsymbol{x}_n$与$\boldsymbol{y}_1,\cdots \boldsymbol{y}_m$分别为来自总体$N_p(\boldsymbol{\mu}_1,\boldsymbol{\Sigma})$与$N_p(\boldsymbol{\mu}_2,\boldsymbol{\Sigma})$的独立同分布的简单随机样本.

$$f(\boldsymbol{x}_1,\cdots,\boldsymbol{x}_n,\boldsymbol{y}_1,\cdots \boldsymbol{y}_m)$$

$$=\frac{1}{(2\pi)^{\frac{(n+m)p}{2}}|\boldsymbol{\Sigma}|^{\frac{n+m}{2}}}\text{etr}\left\{-\frac{1}{2}\boldsymbol{\Sigma}^{-1}\left(\sum_{i=1}^{n}(\boldsymbol{x}_i-\boldsymbol{\mu}_1)(\boldsymbol{x}_i-\boldsymbol{\mu}_1)'+\sum_{i=1}^{m}(\boldsymbol{y}_i-\boldsymbol{\mu}_2)(\boldsymbol{y}_i-\boldsymbol{\mu}_2)'\right)\right\}$$

$$\lambda=\frac{\sup\limits_{\boldsymbol{\mu}_1=\boldsymbol{\mu}_2} f(\boldsymbol{x}_1,\cdots,\boldsymbol{x}_n,\boldsymbol{y}_1,\cdots \boldsymbol{y}_m)}{\sup\limits_{\boldsymbol{\mu}_1,\boldsymbol{\mu}_2} f(\boldsymbol{x}_1,\cdots,\boldsymbol{x}_n,\boldsymbol{y}_1,\cdots \boldsymbol{y}_m)}.\text{由4. }\sup_{\boldsymbol{\mu}_1,\boldsymbol{\mu}_2} f(\boldsymbol{x}_1,\cdots,\boldsymbol{x}_n,\boldsymbol{y}_1,\cdots \boldsymbol{y}_m)=\frac{1}{\left(\frac{2e\pi}{n+m}\right)^{\frac{(n+m)p}{2}}|\mathbf{V}_1+\mathbf{V}_2|^{\frac{n+m}{2}}}.$$

其中$\mathbf{V}_1=\sum_{i=1}^{n}(\boldsymbol{x}_i-\overline{\boldsymbol{x}})(\boldsymbol{x}_i-\overline{\boldsymbol{x}})',\mathbf{V}_2=\sum_{i=1}^{m}(\boldsymbol{y}_i-\overline{\boldsymbol{y}})(\boldsymbol{y}_i-\overline{\boldsymbol{y}})'$.

当$\boldsymbol{\mu}_1=\boldsymbol{\mu}_2=\boldsymbol{\mu}$时,

$$\ln f'=-(n+m)\ln|\boldsymbol{\Sigma}|-\text{tr}\left(\boldsymbol{\Sigma}^{-1}(\mathbf{V}_1+\mathbf{V}_2)\right)-n(\overline{\boldsymbol{x}}-\boldsymbol{\mu})'\boldsymbol{\Sigma}^{-1}(\overline{\boldsymbol{x}}-\boldsymbol{\mu})-m(\overline{\boldsymbol{y}}-\boldsymbol{\mu})'\boldsymbol{\Sigma}^{-1}(\overline{\boldsymbol{y}}-\boldsymbol{\mu})$$

令$\frac{\partial \ln f'}{\partial \boldsymbol{\mu}}=n(\overline{\boldsymbol{x}}-\boldsymbol{\mu})+m(\overline{\boldsymbol{y}}-\boldsymbol{\mu})=0$,得$\widehat{\boldsymbol{\mu}}=\frac{n\overline{\boldsymbol{x}}+m\overline{\boldsymbol{y}}}{n+m},\frac{\partial^2 \ln f'}{\partial \boldsymbol{\mu}'\partial \boldsymbol{\mu}}=-(n+m)\mathbf{I}_p<0$,

故取$\boldsymbol{\mu}=\widehat{\boldsymbol{\mu}}$时,$\ln f'=-(n+m)\ln|\boldsymbol{\Sigma}|-\text{tr}\left(\boldsymbol{\Sigma}^{-1}(\mathbf{V}_1+\mathbf{V}_2)\right)-\frac{nm}{n+m}(\overline{\boldsymbol{x}}-\overline{\boldsymbol{y}})'\boldsymbol{\Sigma}^{-1}(\overline{\boldsymbol{x}}-\overline{\boldsymbol{y}})$为极大值

令$\frac{\partial \ln f'}{\partial \boldsymbol{\Sigma}}=-(n+m)\boldsymbol{\Sigma}^{-1}+(\mathbf{V}_1+\mathbf{V}_2)\boldsymbol{\Sigma}^{-2}+\frac{nm}{n+m}(\overline{\boldsymbol{x}}-\overline{\boldsymbol{y}})(\overline{\boldsymbol{x}}-\overline{\boldsymbol{y}})'\boldsymbol{\Sigma}^{-2}=0$,

$$\widehat{\boldsymbol{\Sigma}}=\frac{\mathbf{V}_1+\mathbf{V}_2}{n+m}+\frac{nm}{(n+m)^2}(\overline{\boldsymbol{x}}-\overline{\boldsymbol{y}})(\overline{\boldsymbol{x}}-\overline{\boldsymbol{y}})',$$

从而$\sup\limits_{\boldsymbol{\mu}_1=\boldsymbol{\mu}_2} f(\boldsymbol{x}_1,\cdots,\boldsymbol{x}_n,\boldsymbol{y}_1,\cdots \boldsymbol{y}_m)=\frac{1}{\left(\frac{2e\pi}{n+m}\right)^{\frac{(n+m)p}{2}}|\mathbf{V}_1+\mathbf{V}_2+\frac{nm}{n+m}(\overline{\boldsymbol{x}}-\overline{\boldsymbol{y}})(\overline{\boldsymbol{x}}-\overline{\boldsymbol{y}})'|^{\frac{n+m}{2}}}$,

$$\lambda=\frac{1}{|\mathbf{I}_p+\frac{nm}{n+m}(\overline{\boldsymbol{x}}-\overline{\boldsymbol{y}})'(\overline{\boldsymbol{x}}-\overline{\boldsymbol{y}})(\mathbf{V}_1+\mathbf{V}_2)^{-1}|^{\frac{n+m}{2}}}=\left(1+\frac{nm}{n+m}(\overline{\boldsymbol{x}}-\overline{\boldsymbol{y}})(\mathbf{V}_1+\mathbf{V}_2)^{-1}(\overline{\boldsymbol{x}}-\overline{\boldsymbol{y}})'\right)^{-\frac{n+m}{2}}.$$

λ为似然比统计量,进一步,

已知$\overline{\boldsymbol{x}}\sim N_p(\boldsymbol{\mu}_1,\overline{\boldsymbol{y}}\sim N_p(\boldsymbol{\mu}_2,\boldsymbol{\Sigma}/n),\mathbf{V}_1\sim W_p(n-1,\boldsymbol{\Sigma}),\mathbf{V}_2\sim W_p(m-1,\boldsymbol{\Sigma})$,

则$\sqrt{\frac{nm}{n+m}}(\overline{\boldsymbol{x}}-\overline{\boldsymbol{y}})\sim N_p\left(\sqrt{\frac{nm}{n+m}}(\boldsymbol{\mu}_1-\boldsymbol{\mu}_2),\boldsymbol{\Sigma}\right),\mathbf{V}_1+\mathbf{V}_2\sim W_p(n+m-2,\boldsymbol{\Sigma})$.

令$T^2=\frac{(n+m-2)nm}{n+m}(\overline{\boldsymbol{x}}-\overline{\boldsymbol{y}})(\overline{\boldsymbol{x}}-\overline{\boldsymbol{y}})'\sim T^2\left(p,n+m-2,\sqrt{\frac{nm}{n+m}}(\boldsymbol{\mu}_1-\boldsymbol{\mu}_2)\right)$

另$\frac{n+m-p-2}{p(n+m-3)}T^2\sim F_{p,n+m-p-2}(\delta)$,其中$\delta=\frac{(n+m-2)nm}{n+m}(\boldsymbol{\mu}_1-\boldsymbol{\mu}_2)'\boldsymbol{\Sigma}^{-1}(\boldsymbol{\mu}_1-\boldsymbol{\mu}_2)$.

11.

Note

某项研究确定运功或膳食补充是否会减缓妇女的骨质流失,研究人员通过光子吸收法测量了实验前和实验1年后骨骼中的矿物质含量.表6.8是对参与该项目实验前25个个体和参与该项目实验1年后24个个体骨骼中的矿物质含量数据,记录了3个骨骼主力侧和非主力侧上矿物质含量,其中X_1表示主力侧的桡骨、X_2表示桡骨、X_3表示主力侧的肱骨、X_4表示肱骨、X_5表示主力侧的尺骨、X_6表示尺骨中矿物质的含量.假设

$\boldsymbol{X}=(X_1,\cdots,X_6)'\sim N_6(\boldsymbol{\mu},\boldsymbol{\Sigma})$.

(1)分别绘制实验前数据和实验1年后数据的矩阵散点图;

(2)给定显著性水平$\alpha=0.5$,检验经过实验后骨骼中的矿物质是否有流失?;

(3)构造均值差95%的同时置信区间和Bonferroni置信区间;

(4)给定显著性水平$\alpha=0.5$，分别对实验前和实验后的数据进行独立性检验.首先对随机向量$\boldsymbol{X}$和协方差矩阵$\boldsymbol{\Sigma}$进行如下剖分:

$$\boldsymbol{X}=\begin{pmatrix}\boldsymbol{X}^{(1)}\\ \boldsymbol{X}^{(2)}\\ \boldsymbol{X}^{(3)}\end{pmatrix},\quad \boldsymbol{\mu}=\begin{pmatrix}\boldsymbol{\mu}^{(1)}\\ \boldsymbol{\mu}^{(2)}\\ \boldsymbol{\mu}^{(3)}\end{pmatrix},\quad \boldsymbol{\Sigma}=\begin{pmatrix}\boldsymbol{\Sigma}_{11}&\boldsymbol{\Sigma}_{12}&\boldsymbol{\Sigma}_{13}\\ \boldsymbol{\Sigma}_{21}&\boldsymbol{\Sigma}_{22}&\boldsymbol{\Sigma}_{23}\\ \boldsymbol{\Sigma}_{31}&\boldsymbol{\Sigma}_{32}&\boldsymbol{\Sigma}_{33}\end{pmatrix},$$

其中$\boldsymbol{X}^{(1)}=(X_1,X_2)',\boldsymbol{X}^{(2)}=(X_3,X_4)',\boldsymbol{X}^{(3)}=(X_5,X_6)'$.考虑如下的检验问题:

$$H_0:\boldsymbol{\Sigma}_{12}=\mathbf{0},\boldsymbol{\Sigma}_{13}=\mathbf{0},\boldsymbol{\Sigma}_{23}=\mathbf{0},\qquad H_1:\boldsymbol{\Sigma}_{12},\boldsymbol{\Sigma}_{13},\boldsymbol{\Sigma}_{23}\text{不全为}\mathbf{0}\text{矩阵}.$$

表6.8骨骼中的矿物质含量数据

X_1	X_2	X_3	X_4	X_5	X_6	X_1	X_2	X_3	X_4	X_5	X_6
1.103	1.052	2.139	2.238	0.873	0.872	1.027	1.051	2.268	2.246	0.869	0.964
0.842	0.859	1.873	1.741	0.590	0.744	0.857	0.817	1.718	1.710	0.602	0.689
0.925	0.873	1.887	1.809	0.767	0.713	0.875	0.880	1.953	1.756	0.765	0.738
0.857	0.744	1.739	1.547	0.706	0.674	0.873	0.698	1.668	1.443	0.761	0.698
0.795	0.809	1.734	1.715	0.549	0.654	0.811	0.813	1.643	1.661	0.551	0.619
0.787	0.779	1.509	1.474	0.782	0.571	0.640	0.734	1.396	1.378	0.753	0.515
0.933	0.880	1.695	1.656	0.737	0.803	0.947	0.865	1.851	1.686	0.708	0.787
0.799	0.851	1.740	1.777	0.618	0.682	0.886	0.806	1.742	1.815	0.687	0.715
0.945	0.876	1.811	1.759	0.853	0.777	0.991	0.923	1.931	1.776	0.844	0.656
0.921	0.906	1.954	2.009	0.823	0.765	0.977	0.925	1.933	2.106	0.869	0.789
0.792	0.825	1.624	1.657	0.686	0.668	0.825	0.826	1.609	1.651	0.654	0.726
0.815	0.751	2.204	1.846	0.678	0.546	0.851	0.765	2.352	1.980	0.692	0.526
0.755	0.724	1.508	1.458	0.662	0.595	0.770	0.730	1.470	1.420	0.670	0.580
0.880	0.866	1.786	1.811	0.810	0.819	0.912	0.875	1.846	1.809	0.823	0.773
0.900	0.838	1.902	1.606	0.723	0.677	0.905	0.826	1.842	1.579	0.746	0.729
0.764	0.757	1.743	1.794	0.586	0.541	0.756	0.727	1.747	1.860	0.656	0.506
0.733	0.748	1.863	1.869	0.672	0.752	0.765	0.764	1.923	1.941	0.693	0.740
0.932	0.898	2.028	2.032	0.836	0.805	0.932	0.914	2.190	1.997	0.883	0.785
0.856	0.786	1.390	1.324	0.578	0.610	0.843	0.782	1.242	1.228	0.577	0.627
0.890	0.950	2.187	2.087	0.758	0.718	0.879	0.906	2.164	1.999	0.802	0.769
0.688	0.532	1.650	1.378	0.533	0.482	0.673	0.537	1.573	1.330	0.540	0.498
0.940	0.850	2.334	2.225	0.757	0.731	0.949	0.900	2.130	2.159	0.804	0.779
0.493	0.616	1.037	1.268	0.546	0.615	0.463	0.637	1.041	1.265	0.570	0.634
0.835	0.752	1.509	1.422	0.618	0.664	0.776	0.743	1.442	1.411	0.585	0.640
0.915	0.936	1.971	1.869	0.869	0.868						

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Note

自己设计一个模拟例子,编写程序,对6.5.2节介绍的球形检验问题进行模拟研究.