The basic rules of natural deduction:

	introduction	*elimination*
$\wedge$	$\dfrac{\phi \quad \psi}{\phi \wedge \psi}\ \wedge\mathrm{i}$	$\dfrac{\phi \wedge \psi}{\phi}\ \wedge\mathrm{e}_1 \qquad \dfrac{\phi \wedge \psi}{\psi}\ \wedge\mathrm{e}_2$
$\vee$	$\dfrac{\phi}{\phi \vee \psi}\ \vee\mathrm{i}_1 \qquad \dfrac{\psi}{\phi \vee \psi}\ \vee\mathrm{i}_2$	$\dfrac{\phi \vee \psi \quad \boxed{\begin{array}{c}\phi\\ \vdots\\ \chi\end{array}} \quad \boxed{\begin{array}{c}\psi\\ \vdots\\ \chi\end{array}}}{\chi}\ \vee\mathrm{e}$
$\rightarrow$	$\dfrac{\boxed{\begin{array}{c}\phi\\ \vdots\\ \psi\end{array}}}{\phi \rightarrow \psi}\ \rightarrow\mathrm{i}$	$\dfrac{\phi \quad \phi \rightarrow \psi}{\psi}\ \rightarrow\mathrm{e}$
$\neg$	$\dfrac{\boxed{\begin{array}{c}\phi\\ \vdots\\ \bot\end{array}}}{\neg\phi}\ \neg\mathrm{i}$	$\dfrac{\phi \quad \neg\phi}{\bot}\ \neg\mathrm{e}$
$\bot$	(no introduction rule for $\bot$)	$\dfrac{\bot}{\phi}\ \bot\mathrm{e}$
$\neg\neg$		$\dfrac{\neg\neg\phi}{\phi}\ \neg\neg\mathrm{e}$

Some useful derived rules:

$$\dfrac{\phi \rightarrow \psi \quad \neg\psi}{\neg\phi}\ \mathrm{MT} \qquad\qquad \dfrac{\phi}{\neg\neg\phi}\ \neg\neg\mathrm{i}$$

$$\dfrac{\boxed{\begin{array}{c}\neg\phi\\ \vdots\\ \bot\end{array}}}{\phi}\ \mathrm{PBC} \qquad\qquad \dfrac{}{\phi \vee \neg\phi}\ \mathrm{LEM}$$

Figure 1.2. Natural deduction rules for propositional logic.

Dessutom får copy-regeln användas.

	introduction	*elimination*
$=$	$\dfrac{}{t = t}\;{=}i$	$\dfrac{t_1 = t_2 \qquad \Phi[t_1/x]}{\Phi[t_2/x]}\;{=}e$
$\forall$	$\dfrac{\boxed{\begin{array}{c} x_0\colon \\ \vdots \\ \Phi[x_0/x] \end{array}}}{\forall x\, \Phi}\;\forall x\, i$	$\dfrac{\forall x\, \Phi}{\Phi[t/x]}\;\forall x\, e$
$\exists$	$\dfrac{\Phi[t/x]}{\exists x\, \Phi}\;\exists x\, i$	$\dfrac{\exists x\, \Phi \qquad \boxed{\begin{array}{c} x_0\colon\ \Phi[x_0/x] \\ \vdots \\ \chi \end{array}}}{\chi}\;\exists x\, e$

Figure: Natural deduction rules for predicate logic

Ett bevissystem för CTL

$$p \ \frac{-}{\mathcal{M}, s \vdash_{[]} p} \ p \in L(s) \qquad\qquad \neg p \ \frac{-}{\mathcal{M}, s \vdash_{[]} \neg p} \ p \notin L(s)$$

$$\wedge \ \frac{\mathcal{M}, s \vdash_{[]} \phi \qquad \mathcal{M}, s \vdash_{[]} \psi}{\mathcal{M}, s \vdash_{[]} \phi \wedge \psi}$$

$$\vee_1 \ \frac{\mathcal{M}, s \vdash_{[]} \phi}{\mathcal{M}, s \vdash_{[]} \phi \vee \psi} \qquad\qquad \vee_2 \ \frac{\mathcal{M}, s \vdash_{[]} \psi}{\mathcal{M}, s \vdash_{[]} \phi \vee \psi}$$

$$\mathsf{AX} \ \frac{\mathcal{M}, s_1 \vdash_{[]} \phi \quad \cdots \quad \mathcal{M}, s_n \vdash_{[]} \phi}{\mathcal{M}, s \vdash_{[]} \mathsf{AX}\ \phi}$$

$$\mathsf{AG}_1 \ \frac{-}{\mathcal{M}, s \vdash_U \mathsf{AG}\ \phi} \ s \in U \qquad\qquad \mathsf{AF}_1 \ \frac{\mathcal{M}, s \vdash_{[]} \phi}{\mathcal{M}, s \vdash_U \mathsf{AF}\ \phi} \ s \notin U$$

$$\mathsf{AG}_2 \ \frac{\mathcal{M}, s \vdash_{[]} \phi \qquad \mathcal{M}, s_1 \vdash_{U,s} \mathsf{AG}\ \phi \ \cdots \ \mathcal{M}, s_n \vdash_{U,s} \mathsf{AG}\ \phi}{\mathcal{M}, s \vdash_U \mathsf{AG}\ \phi} \ s \notin U$$

$$\mathsf{AF}_2 \ \frac{\mathcal{M}, s_1 \vdash_{U,s} \mathsf{AF}\ \phi \quad \cdots \quad \mathcal{M}, s_n \vdash_{U,s} \mathsf{AF}\ \phi}{\mathcal{M}, s \vdash_U \mathsf{AF}\ \phi} \ s \notin U$$

$$\mathsf{EX} \ \frac{\mathcal{M}, s' \vdash_{[]} \phi}{\mathcal{M}, s \vdash_{[]} \mathsf{EX}\ \phi} \qquad\qquad \mathsf{EG}_1 \ \frac{-}{\mathcal{M}, s \vdash_U \mathsf{EG}\ \phi} \ s \in U$$

$$\mathsf{EG}_2 \ \frac{\mathcal{M}, s \vdash_{[]} \phi \qquad \mathcal{M}, s' \vdash_{U,s} \mathsf{EG}\ \phi}{\mathcal{M}, s \vdash_U \mathsf{EG}\ \phi} \ s \notin U$$

$$\mathsf{EF}_1 \ \frac{\mathcal{M}, s \vdash_{[]} \phi}{\mathcal{M}, s \vdash_U \mathsf{EF}\ \phi} \ s \notin U \qquad\qquad \mathsf{EF}_2 \ \frac{\mathcal{M}, s' \vdash_{U,s} \mathsf{EF}\ \phi}{\mathcal{M}, s \vdash_U \mathsf{EF}\ \phi} \ s \notin U$$

Figur 1: Ett bevissystem för CTL.

I premisserna till A–reglerna betecknar $s_1, \ldots, s_n$ *alla* efterföljare till tillståndet s i modellen $\mathcal{M}$, medan i premisserna till E–reglerna betecknar s' *någon* efterföljare till s.

$$\frac{(\!|\phi|\!)\ C_1\ (\!|\eta|\!) \qquad (\!|\eta|\!)\ C_2\ (\!|\psi|\!)}{(\!|\phi|\!)\ C_1; C_2\ (\!|\psi|\!)}\ \text{Composition}$$

$$\frac{}{(\!|\psi[E/x]|\!)\ x = E\ (\!|\psi|\!)}\ \text{Assignment}$$

$$\frac{(\!|\phi \wedge B|\!)\ C_1\ (\!|\psi|\!) \qquad (\!|\phi \wedge \neg B|\!)\ C_2\ (\!|\psi|\!)}{(\!|\phi|\!)\ \texttt{if}\ B\ \{C_1\}\ \texttt{else}\ \{C_2\}\ (\!|\psi|\!)}\ \text{If-statement}$$

$$\frac{(\!|\psi \wedge B|\!)\ C\ (\!|\psi|\!)}{(\!|\psi|\!)\ \texttt{while}\ B\ \{C\}\ (\!|\psi \wedge \neg B|\!)}\ \text{Partial-while}$$

$$\frac{\vdash_{\mathrm{AR}} \phi' \to \phi \qquad (\!|\phi|\!)\ C\ (\!|\psi|\!) \qquad \vdash_{\mathrm{AR}} \psi \to \psi'}{(\!|\phi'|\!)\ C\ (\!|\psi'|\!)}\ \text{Implied}$$

Figure 4.1. Proof rules for partial correctness of Hoare triples.