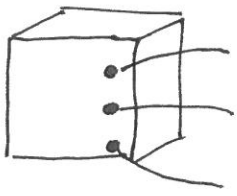
ÖVNING 1

Övn 1

Printer



grön lampa "Väntar på input" "Waiting for input"

gul lampa "Arbetar" "Working"

röd lampa "Papperstrassel" "Paper jam"

Variabler: (Propositional variables)

grön/green (grön lampa lyser) "green lamp is shining"

gul/yellow (gul — " —) "yellow — " — "

röd/red (röd — " —) "red — " — "

på/on (printern är påslagen) "printer is switched on"

Beskriv som en satslogisk formel:
Formalize by means of a propositional formula:

"Exakt en av de tre lamporna lyser när printern är påslagen."

"Exactly one of the three lamps is shining when the printer is switched on."

$$\text{på} \rightarrow (\text{grön} \wedge \neg\text{gul} \wedge \neg\text{röd}) \vee (\neg\text{grön} \wedge \text{gul} \wedge \neg\text{röd}) \vee (\neg\text{grön} \wedge \neg\text{gul} \wedge \text{röd})$$

EXERCISES 1.2, p. 78+

NATURAL DEDUCTION

1 (d) $p \to (p \to q), p \vdash q$ ← sequent

- Always try first to make sense of the sequent!
- Always make a proof plan in your head!

1	$p \to (p \to q)$	Premiss
2	p	Premiss
3	$p \to q$	$\to$e 2,1 ← "modus ponens"
4	q	$\to$e 2,3

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1 (i) $(p \to r) \wedge (q \to r) \vdash p \wedge q \to r$

Boxar

Boxes

1	$(p \to r) \wedge (q \to r)$	Premiss
2	$p \to r$	$\wedge$E, 1
3	$p \wedge q$	Antagande
4	p	$\wedge$e, 3
5	r	$\to$e 4,2
6	$p \wedge q \to r$	$\to$i 3-5

- Several possibilities here.

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1 (j) $(q \to r) \vdash (p \to q) \to (p \to r)$

Nästlade boxar

Nested boxes

1	$q \to r$	Premiss
2	$p \to q$	Antagande
3	p	Antagande
4	q	$\to$e 3,2
5	r	$\to$e 4,1
6	$p \to r$	$\to$i 3-5
7	$(p \to q) \to (p \to r)$	$\to$i 2-6

- Simplest proof strategy: syntax-driven introduction rules complemented suitably with elimination rules

Studenterna gör

1 (a) $p \to (q \to r) \vdash q \to (p \to r)$

1 (g) $p \vdash q \to (p \wedge q)$

1 (r) $p \to (q \wedge r) \vdash (p \to q) \wedge (p \to r)$

(end of 1st hour)

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1 (v) $p \vee (p \wedge q) \vdash p$

Disjunktion

1.	$p \vee (p \wedge q)$	Premiss	
2.	p	Antagande	
3.	p	Copy 2	
4.	$p \wedge q$	Antagande	
5	p	$\wedge e$, 4	
6	p	$\vee e$ 1, 2-3, 4-5	

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2 (a) $\neg p \to \neg q \vdash q \to p$

Falsum

Contradiction

1.	$\neg p \to \neg q$	Premiss	
2.	q	Antagande	
3.	$\neg p$	Antagande	
4.	$\neg q$	$\to e$ 3,1	
5.	$\bot$	$\neg e$ 2,4	
6.	$\neg\neg p$	$\neg i$ 3-5	Alt. 6. p PBC
7.	p	$\neg\neg e$ 6	
8.	$q \to p$	$\to i$ 2-7	

- Contradiction as a formula ($\bot$) represents syntactically the fact that the formulas above cannot be simultaneously true.

Studenterna gör:

3a $\neg p \to p \vdash p$

3b $\neg p \vdash p \to q$

2b $\neg p \vee \neg q \vdash \neg(p \wedge q)$

3f $p \to q \vdash \neg p \vee q$

$\neg p \to p \vdash p$

PBC

3a

1.	$\neg p \to p$	Premiss	
2.	$\neg p$	Antagande	
3.	p	$\to e$	2, 1
4.	$\bot$	$\neg e$	2, 3
(5.	$\neg\neg p$	$\neg i$	2-4)
6.	p	$\neg\neg e$	5 (PBC 2-4)

Indirect proof:

You assume the opposite of what you believe to be true, and try to derive a contradiction.

$\neg p \vdash p \to q$

!

3b

1.	$\neg p$	Premiss	
2.	p	Antagande	
3.	$\bot$	$\neg e$	2, 1
4.	q	$\bot e$	3
5.	$p \to q$	$\to i$	2-4

2 b

1.	$\neg p \vee \neg q$	Premiss
2.	$p \wedge q$	Antagande
3.	$\neg p$	Antagande
4.	p	$\wedge e_1$ 2
5.	$\bot$	$\neg e$ 4, 3
6.	$\neg q$	Antagande
7.	q	$\wedge e_2$ 2
8.	$\bot$	$\neg e$ 7, 6
9.	$\bot$	$\vee e$ 1, 3-5, 6-8
10.	$\neg(p \wedge q)$	$\neg i$ 2-9

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3 f $\quad p \to q \vdash \neg p \vee q$

1.	$p \to q$	Premiss
2.	p	Antagande
3.	q	$\to e$ 2, 1
4.	$\neg p \vee q$	$\vee i_2$ 3
5.	$\neg p$	Antagande
6.	$\neg p \vee q$	$\vee i$ 5
7.	$p \vee \neg p$	LEM
8.	$\neg p \vee q$	$\vee e$ 7, 2-4, 5-6

LEM

"Law of Excluded Middle"

Like a case analysis.